

# Stochastic Variation of a Sharp-Edged Re-entry Vehicle

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**A sharp-edged re-entry vehicle consisting of flat plates was analyzed. Starting from a nominal configuration the lift-to-drag ratio was increased using the stochastic design improvement method. This resulted in several enhanced designs. To ensure the quality and robustness of these designs a stochastic simulation was conducted around each of them.**

## Nomenclature

$AoA$	=	Angle of Attack
$CDF$	=	Cumulative Distribution Function
$CoG$	=	Center of Gravity
$CoV$	=	Coefficient of Variation
$DLR$	=	Deutsches Zentrum fuer Luft- und Raumfahrt (German Aerospace Center)
$hb$	=	ratio of vehicle bottom height to vehicle height
$hb2$	=	height parameter, s. Appendix A
$H$	=	height of vehicle
$KS$	=	Kolmogorov-Smirnov distance
$L$	=	length of vehicle
$L/D_{max}$	=	maximum lift-to-drag ratio
$\mu$	=	mean value
$m$	=	mass of vehicle
$n$	=	number of Monte Carlo samples
$NaN$	=	Not a Number, number of lost samples
$\rho$	=	density
$\sigma$	=	standard deviation
$SDI$	=	Stochastic Design Improvement
$t$	=	thickness
$TPS$	=	Thermal Protection System
$W$	=	half the vehicle width
$w3l$	=	ratio of nose width to vehicle width

## I. Introduction

AT DLR's Space Launcher Systems Analysis group a re-entry vehicle with a flat plate geometry and sharp edges was investigated from a system's point of view. At the time of the investigation the REX Free Flyer study was and still is in phase 0. Apart from the Institute of Space Systems, the Institute of Aerodynamics and Flow Technology in Braunschweig and the Institute of Structures and Design in Stuttgart have been involved in the REX study. The concept is described in detail in Ref. 1. REX is envisioned to serve as a platform for microgravity experiments as well as a technology demonstrator. The idea behind the flat plates is to reduce the costs for manufacturing and installing the thermal protection system (TPS). The sharp edges could improve the aerodynamic characteristics. They allow for the design of vehicles that have attached shocks during hypersonic flight while flying at low angles

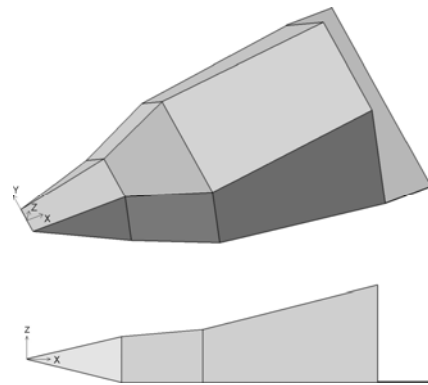


Figure 1. REX 111

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of attack (AoA). This in turn leads to high lift-to-drag ratios. For the purpose of this study the dimensions of the VEGA payload fairing were used but in principle any launcher of that payload class would suffice.

The aim of the herein presented work was to assess the general applicability of stochastic methods to a phase 0 study. In order to do so, the design REX111, shown in Fig. 1, was selected as a nominal configuration. Starting from there, the vehicle's maximum lift-to-drag ratio  $L/D_{\max}$  was improved using the stochastic design improvement (SDI) method<sup>2</sup>. This particular method was chosen because it is simple to apply, allows for a large number of design variables, and often leads to fast improvements. The method further generates several alternative designs. Following the improvement process, a stochastic simulation was conducted around the best design from every iteration. The aim was to check if the quality and robustness of the parameter  $L/D_{\max}$  were acceptable. The quality and robustness were analyzed using statistical methods. More precisely, the coefficient of variation (CoV) and the Kolmogorov-Smirnov distance (KS) were utilized.

Examples from the literature show that stochastic methods have been successfully applied to space systems. In Ref. 3 a stochastic analysis of a structural part of the Ariane 5 launcher is presented, whereas in Ref. 4 an uncertainty and reliability analysis of ESA's INTEGRAL satellite in conjunction with the Ariane 5 is undertaken. In Ref. 5 a telecommunication satellite is studied. Ref. 6 describes a probabilistic study of satellite microvibrations and finally Ref. 7 and 8 deal with space telescopes.

## II. Models

The calculation process consisted of three steps. First, the vehicle's shape and mechanical structure were modeled with CATIA V5<sup>9</sup>. The information about the vehicle's geometry and center of gravity (CoG) was then used to perform an aerodynamic analysis in the hypersonic regime. The DLR tool HOTSOSE v1.81, that applies a modified Newton plus shock-expansion method, was used for this purpose. At last, the trimmability of the vehicle was assessed. In the following, the three analysis steps are explained in more detail.

### A. Vehicle Shape

The CATIA model consisted of a fully parameterized outer vehicle shape. 16 geometrical parameters were necessary for a complete description of the shape. A detailed description of the parameters is given in Appendix A. First of all, the vehicle's length  $L$ , height  $H$  and half width  $W$  were defined as parameters. REX111 is 3420 mm long, 950 mm high and in total 2160 mm wide. The remaining 13 parameters were all defined with respect to the outer dimensions or other lengths in the model. The reason for this approach was to have parameters that vary between zero and one. The vehicle's CoG was approximated at being located in the center of the volume that is defined by the outer shape. An accommodation analysis for REX111 showed that this assumption is conservative. According to the accommodation analysis the CoG in x-direction is located at 63 % of the vehicle length whereas by using the before mentioned assumption the CoG is at 69 %. Please refer to Fig. 1 for a definition of the coordinate system. Stable trim can only be achieved if the CoG is in front of the aerodynamic pressure point. It is generally possible to place many of the subcomponents in the front of the vehicle. In addition, the heavier components can be placed close to the vehicle's nose.

### B. Mechanical Structure

The TPS concept<sup>10</sup>, that was assumed for this study, is given in Fig. 2. The necessary insulation thicknesses were estimated by first calculating a re-entry trajectory with the DLR tool TOSCA<sup>11</sup>. Several flight points along the trajectory were then selected for further analysis. For each of these flight points the heat fluxes on each of the vehicle's panels were computed using HOTSOSE. Based on these numbers the insulation thicknesses for each of the panels was calculated with an ANSYS model<sup>10</sup>. For the work presented in this paper the insulation thicknesses that will keep the vehicle's interior temperature at or below 75° C were selected.

The outer shell of the vehicle is made of 5 mm C/C-SiC plates. Beneath that is an insulation of variable thickness. The bottom plate in front of the vehicle was assumed to have an additional 2 mm layer of high temperature insulation. The substructure consists of 5 mm aluminum. The material densities are given in Table 1.

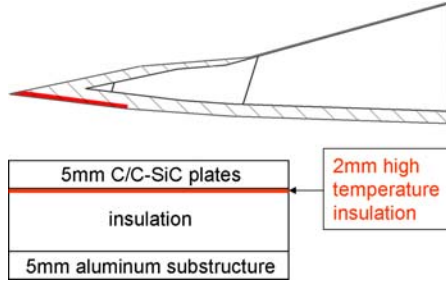


Figure 2. Thermal protection system

Table 1. Materials

material	$\rho$ , kg/m <sup>3</sup>	t, mm
CC-SiC	1900	5
high temperature insulation	240	2
insulation	120	variable
aluminum	2700	5

### C. Aerodynamics

After the shape was defined in CATIA, an aerodynamic analysis in the hypersonic regime was conducted using HOTSOSE. The necessary mesh was generated using the geometrical information from the CATIA model. The CoG value was inserted into the HOTSOSE input deck. All aerodynamic calculations were performed at a speed of Mach 10. Especially,  $L/D_{\max}$  was sought.

### D. Trimmability

Following a first aerodynamic analysis, the trimmability of the vehicle was assessed. The aim was to fly with  $L/D_{\max}$ . A flap was added to the back of the vehicle in order to achieve trim. The flap angle was then adjusted in steps of 1° until trim was achieved or the range of admissible flap angles was exceeded. A vehicle was deemed trimmable if the difference between the AoA at which the pitch moment is zero and the AoA necessary for  $L/D_{\max}$  was equal to or less than 2°. The flap angle was allowed to vary between -50° and +25°. A negative flap angle denotes a downward movement of the flap.

## III. Stochastic Methods

The software ST-ORM<sup>12</sup> was used for all stochastic processes. First, the SDI method was applied to increase the vehicle's  $L/D_{\max}$ . Thereby a number of designs was generated. Then, stochastic simulations were conducted for several designs to assess the quality and robustness of each design's  $L/D_{\max}$  and mass  $m$ .

### A. Stochastic Design Improvement

The idea behind SDI<sup>2</sup> is to move an entire cloud of designs towards a predefined target. After specifying the vehicle's target performance,  $n$  random designs are generated around the nominal design. In order to do so, the design variables, chosen by the user, are varied using uniform distributions. Then, the Euclidean distance between each design's performance and the target performance is calculated. The design that lies closest to the target becomes the starting point for the next iteration. The process is normally repeated three to five times or until the target is reached.

Table 2. Design variables of SDI

variable	min	max
L, mm	3000	5000
H, mm	950	1200
W, mm	475	1100
all other 13 geometrical parameters	0	1

In this case, the SDI process was run using five iterations with 24 Monte Carlo samples each. Normally, 8 to 16 samples per iteration are sufficient<sup>12</sup> but a higher number was chosen to account for samples that are not trimmable and thus not usable. The target was set to  $L/D_{\max} > 3$  and internal volume  $> 1$  m<sup>3</sup>. The internal volume was computed by subtracting the volume of the mechanical structure and the TPS from the volume of the outer shape. The volume requirement ensured that all subsystems could be accommodated. All geometrical parameters were selected as design variables. Their admissible ranges are given in Table 2. The maximum values of L and W are based on the size of the VEGA payload fairing<sup>13</sup>. The minimum values of H and W were chosen to allow a connection with the VEGA payload adapter, which measures 940 mm in diameter. The maximum value of H was limited to 1200 mm because designs with a low height tend to have a good  $L/D_{\max}$ . The remaining 13 geometrical parameters were all allowed to vary between 0 and 1.

## B. Stochastic Simulation

Stochastic simulations were performed in order to quantify the quality and robustness of the parameters  $L/D_{\max}$  and  $m$ . The method works by randomly creating  $n$  samples around a nominal design. The measures described in the following are taken from Ref. 14. The CoV was used as a measure of scatter and thus quality. It is defined as the ratio of standard deviation to mean value. The lower the CoV the lower the amount of scatter and thus the higher the quality. The value 1-KS was used in order to measure the robustness of a parameter. KS is the maximum distance between a given cumulative distribution function (CDF) and the CDF of the Gaussian distribution. Thus, the Gaussian distribution is deemed to be 100 % robust. According to this measure, a probability distribution that has, for example, more than one peak will have a substantially reduced robustness.

The stochastic simulations were performed using 400 samples each. ST-ORM utilizes Latin Hypercube Sampling. All parameters were assigned Gaussian distributions.  $L$ ,  $H$  and  $W$  were varied using a CoV of 5 %. For the remaining geometrical parameters a standard deviation of 0.05 was used. The material densities and thicknesses of the TPS all had a CoV of 5 %.

## IV. Results

The results will be presented in two steps. First, the results of the SDI process are given. Then, the results from the stochastic simulations for the designs with the highest  $L/D_{\max}$  in every iteration are shown.

### A. Stochastic Design Improvement

The highest value of  $L/D_{\max}$  in every iteration is given in Fig. 3. The first iteration of the SDI process improved  $L/D_{\max}$  from 1.91 to 2.45. Some more improvement was achieved during the next two iterations.  $L/D_{\max}$  increased to 2.71. Iterations four and five resulted only in small improvements but helped to generate more alternative designs. Overall,  $L/D_{\max}$  reached a value of 2.77 in the fifth iteration.

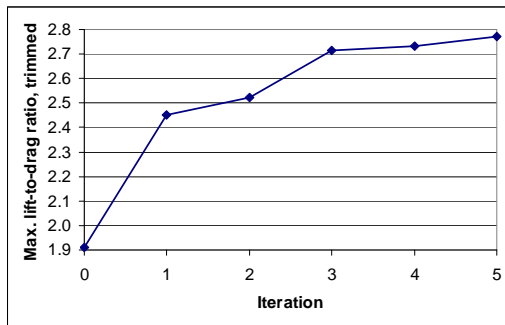
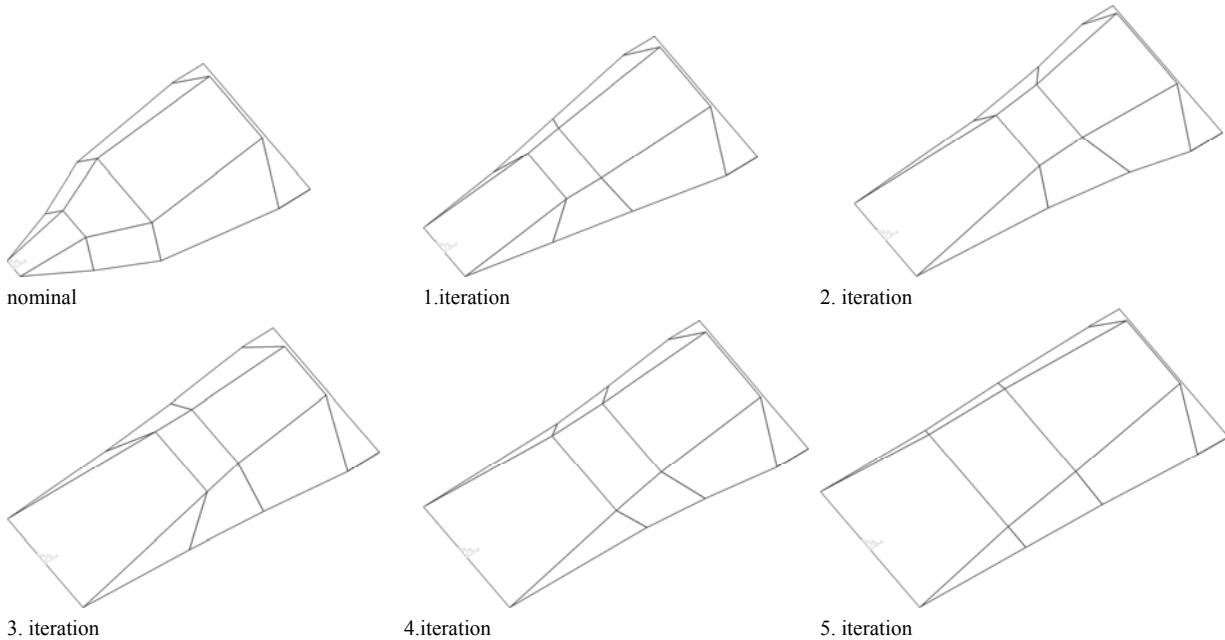


Figure 3. Highest value of  $L/D_{\max}$  in every iteration of the SDI process

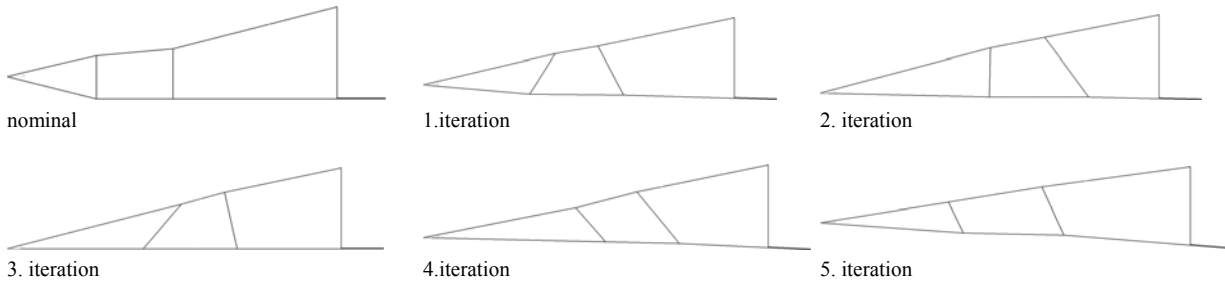
Table 3. Most important geometrical parameters of designs with highest  $L/D_{\max}$  in every SDI iteration

iteration	L, mm	H, mm	W, mm	w31	hb
nominal	3420	950	1080	0.12	0.24
1	3705	996	1041	0.40	0.17
2	4000	981	1100	0.57	0.07
3	3983	950	1071	0.71	0.00
4	4113	950	1100	0.79	0.12
5	4403	950	1100	0.91	0.29

Table 3 contains the dimensions of the designs with the highest  $L/D_{\max}$  in every iteration. The shapes of these vehicles are shown in Fig. 4 and 5. During the SDI process the vehicle length  $L$  increased. The nose width, which is described by parameter  $w31$ , increased as well. At first the bottom of the vehicle flattened. Especially, the vehicle from the third iteration is completely flat. Here the parameter  $hb$  is 0. The parameter  $hb$  is calculated by dividing the height of the vehicle's bottom by the vehicle's total height. During the fourth and fifth iteration the absolute value of  $hb$  increased again and the bottom of the vehicle developed a certain profile. The bottom plate closest to the back of the vehicle turned downwards to face into the air flow.

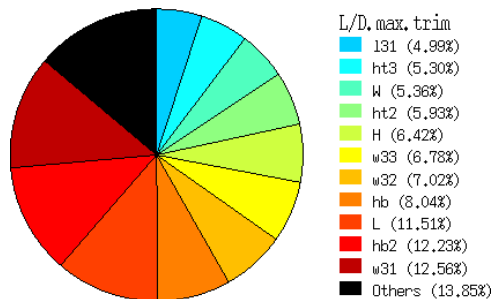


**Figure 4. Shape of vehicles with highest  $L/D_{max}$  in every SDI iteration**



**Figure 5. Shape of vehicles with highest  $L/D_{max}$  in every SDI iteration, side view**

The changes observed in the individual vehicle designs can also be seen over the entire population of vehicle designs. Fig. 6 shows the variables that drove the SDI process. The pie chart was generated by ST-ORM based on the nonlinear Spearman's rank correlation coefficients between  $L/D_{max}$  and all input variables. The influence of parameters L and w31 is about 12 % each. Parameter hb has an influence of 8 %. The relations of these parameters to  $L/D_{max}$  are displayed in the three scatter plots in Fig. 7 to 9. Again, L and w31 increase whereas hb decreases at first and then increases. The values for  $L/D_{max}$  for all samples are shown in Fig. 10.



**Figure 6. Variables influencing  $L/D_{max}$  during the SDI process; a definition of the parameters is given in Appendix A**

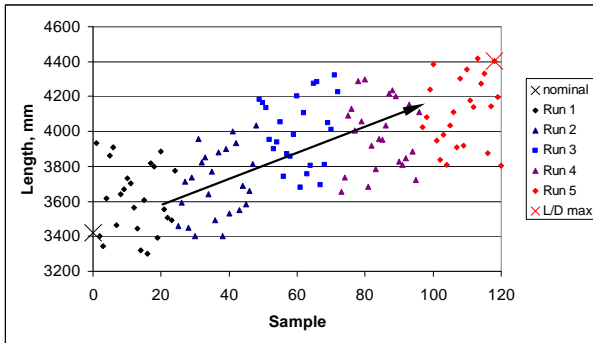


Figure 7. Vehicle length during SDI process

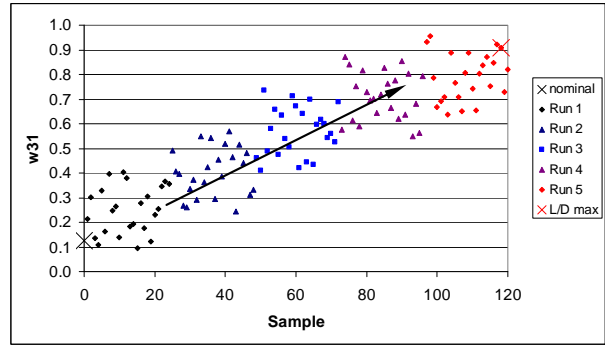


Figure 8. Parameter w31 during SDI process

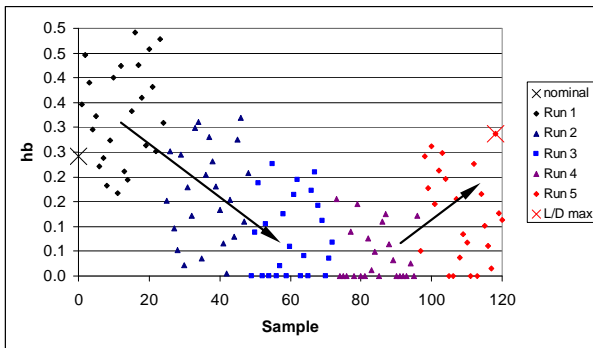


Figure 9. Parameter hb during SDI process

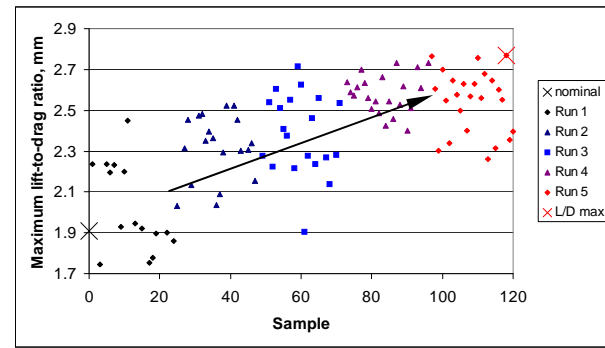


Figure 10. Maximum lift-to-drag ratio during SDI process

## B. Stochastic Simulation

It is not enough for a design to have a high  $L/D_{max}$ . The design also has to be robust with respect to changes in the input variables. Stochastic simulations were conducted for the nominal design as well as the designs with the highest  $L/D_{max}$  in every iteration. The goal was to assess the quality and robustness of these designs. The results regarding  $L/D_{max}$  are given in Table 4 whereas the results for  $m$  are shown in Table 5.

Table 4. Results of stochastic simulations for  $L/D_{max}$  of vehicles with highest  $L/D_{max}$  in every iteration

iteration	$\mu$	$\sigma$	min.	max.	CoV, %	1-KS, %	NaN
nominal	1.96	0.15	1.56	2.34	7.4	96.7	178
1	2.41	0.11	2.07	2.69	4.3	97.1	36
2	2.48	0.11	2.03	2.74	4.5	97.4	15
3	2.69	0.09	2.38	2.93	3.5	95.6	4
4	2.67	0.11	2.28	2.95	4.0	94.9	1
5	2.72	0.10	2.46	2.98	3.5	96.2	2

**Table 5. Results of stochastic simulations for vehicle mass in kg of vehicles with highest  $L/D_{max}$  in every iteration**

iteration	$\mu$	$\sigma$	min.	max.	CoV, %	1-KS, %	NaN
nominal	195	16.5	146	248	8.5	96.5	15
1	211	17.0	164	282	8.0	97.1	0
2	255	21.5	195	315	8.4	95.7	0
3	269	21.2	203	349	7.9	96.4	0
4	290	24.0	218	363	8.3	97.0	0
5	334	25.9	263	406	7.7	97.6	0

With regards to  $L/D_{max}$ , the stochastic simulations show that the nominal design has a higher amount of scatter than the other vehicle designs. The nominal design has a CoV of 7 % while the other designs have CoVs around 4 %. Thus, the quality of the nominal design is below the quality of all other designs. No big differences are found regarding the CoVs of  $m$ . It is around 8 % for all designs.

No significant differences in robustness are found for either  $L/D_{max}$  or  $m$ . The robustness is between 95 % and 98 %. All probability distributions have only one peak. The mass steadily increased during the SDI process.

## V. Discussion

### A. Stochastic Design Improvement

Five was a suitable number of iterations for the SDI process. In the beginning, the method led indeed to a fast increase of  $L/D_{max}$ . The additional iterations shaped the vehicle further. At the end of the process, the increase of  $L/D_{max}$  was very small. At that point it is no longer advisable to use the SDI method because SDI is a method that does not incorporate learning. If a further refinement of the vehicle shape is desired an optimization algorithm should be applied afterwards.

SDI has other advantages. First, the stochastic approach makes it unlikely to get stuck in a local minimum. Then, the method works no matter how linear or nonlinear the problem is. In addition, there is no limit to the number of design variables. Further, the improvement process itself contains valuable knowledge about the design driving variables. At last, SDI generates several alternative designs from which the user can choose. As can be seen in Fig. 10, there are several design with a high value of  $L/D_{max}$ . These advantages make SDI especially suitable for the pre-design of a vehicle. The tools applied in a phase 0 study contain algorithms that aim more at giving realistic than precise numbers. As the utilized computer models capture only the most important aspects of physics it would be of limited significance to optimize a vehicle beyond a certain threshold. Thus, the improvement achieved with the SDI method is good enough for a phase 0 study. In phase 0 capturing knowledge about the characteristics of a good design is more important than finding an optimal solution. Further refinement should take place during the later phases and with more advanced computer models.

The vehicle mass increased over the SDI process because the designs became longer and their nose widths increased. The weight is not a problem because configuration REX111 only uses about half of the lift capacity of VEGA. REX111 weights 1090 kg including subsystems and 15 % margin. REX is foreseen to be injected into a 300 km circular orbit at 30° inclination. According to the manual<sup>14</sup>, VEGA's lift capacity for this orbit is 2200 kg. Thus, there is plenty of margin.

### B. Stochastic Simulation

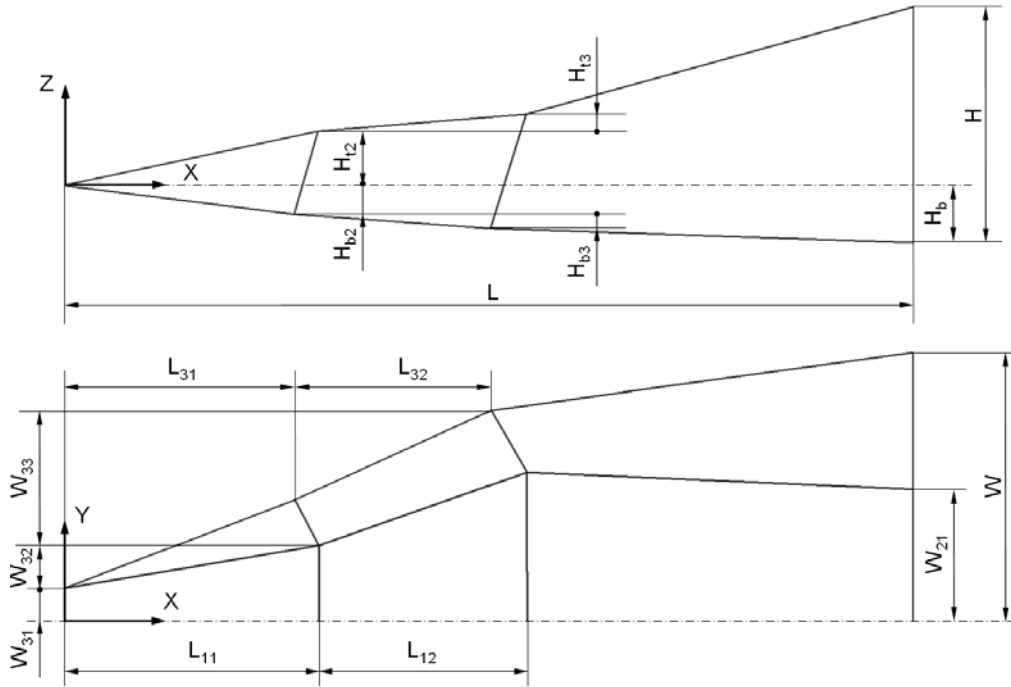
Except for the nominal design all designs proved to have an acceptable quality and robustness regarding the parameters  $L/D_{max}$  and  $m$ . The nominal design, REX111, was originally designed for a different CoG than assumed in the herein presented simulations, s. section II.A. When the CoG from the accommodation analysis is used REX111 fulfills the condition for stable trim over the entire range of flown AoAs. Under the new assumption the vehicle is still trimmable but unstable. All vehicles that were generated by the SDI process are unstable. This can be changed by including an accommodation analysis in the SDI process and in addition setting stability as a target.

## VI. Conclusion

This test of the stochastic design improvement method showed that it is a useful method for a phase 0 study. Before the resulting vehicle designs can be used, the multidisciplinary re-entry analysis needs to be completed. A critical point will most likely be the heat load at the vehicle's nose.

## Appendix

### A. Geometrical Parameters



$$l_{11} = \frac{L_{11}}{L}$$

$$l_{12} = \frac{L_{12}}{L - L_{11}}$$

$$l_{31} = \frac{L_{31}}{L}$$

$$l_{32} = \frac{L_{32}}{L - L_{31}}$$

$$w_{31} = \frac{W_{31}}{W}$$

$$w_{32} = \frac{W_{32}}{W - W_{31}}$$

$$w_{33} = \frac{W_{33}}{W - W_{31} - W_{32}}$$

$$w_{21} = \frac{W_{21}}{W}$$

$$h_b = \frac{H_b}{H}$$

$$h_{b2} = \frac{H_{b2}}{H_b}$$

$$h_{b3} = \frac{H_{b3}}{H_b - H_{b2}}$$

$$h_{t2} = \frac{H_{t2}}{H - H_b}$$

$$h_{t3} = \frac{H_{t3}}{H - H_b - H_{t2}}$$



## References

- <sup>1</sup>Essmann, O., Siemer, M., Longo, J.M.A., Weihs, H., "REX – Free Flyer: A Reusable Orbital Return Vehicle for Experiments Under Microgravity Conditions," *International Astronautical Congress*, 2008.
- <sup>2</sup>Marczyk, J., *Principles of Simulation-Based Computer-Aided Engineering*, FIM Publications, Madrid, 1999
- <sup>3</sup>Arroz Collado, S., "Stochastic Analysis of the ARIANE-V EPS-Structure Model," *Stochastic Analysis of Multivariate Systems in Computational Mechanics and Engineering*, edited by I. Doltsinis, International Center for Numerical Methods in Engineering (CIMNE), Barcelona, 1999, pp. 267-283.
- <sup>4</sup>Schueller, G., Calvi, A., Pradlwarter, H., Fransen, S., Pellissetti, M., Schenk, C.A., Klein, M., and Kreis, A., "Uncertainty and Reliability Analyses of Large Aerospace Structures," *Proceedings of the European Conference on Spacecraft Structures, Materials & Mechanical Testing*, SP-581, ESA Publication Division, Noordwijk, The Netherlands, 2005.
- <sup>5</sup>Marchante, E., "Stochastic Analysis in the Aerospace Industry," *Computational Stochastic Mechanics in a Meta-Computing Perspective*, edited by J. Marczyk, International Center for Numerical Methods in Engineering (CIMNE), Barcelona, 1997, pp. 39-58.
- <sup>6</sup>Marczyk, J., "Satellite Microvibrations: A Stochastic Problem," *Computational Stochastic Mechanics in a Meta-Computing Perspective*, edited by J. Marczyk, International Center for Numerical Methods in Engineering (CIMNE), Barcelona, 1997, pp. 131-148.
- <sup>7</sup>Koch, A., "Study and Employment of Stochastic Methods for the Design of Space Structures," Diplomarbeit, Faculty of Mechanical Engineering, Technical Univ. of Ilmenau, 2006.
- <sup>8</sup>Peacocke, T., Koch, A., "Pipeline Processing for Stochastic Studies of Telescope Performance Influenced by Thermo-Elastic Deformation," *Terahertz Technology and Applications*, edited by K. J. Linden and L. P. Sadwick, Proceedings of the SPIE, Vol. 6893, 2008.
- <sup>9</sup>CATIA, Ver. 5.18, Service Pack 6, Dassault Systèmes, Vélizy-Villacoublay, France, 2008.
- <sup>10</sup>Reimer, T., "REX Free Flyer: Thermalschutzsystem Vorauslegung", DLR-IB 435-2009/15, 2009.
- <sup>11</sup>Kopp, A., Koch, A., Sippel, M., "Voruntersuchung des Wiedereintritts des REX Free Flyers zur Vordimensionierung des Thermalschutzsystems", DLR SART TN-03/2009, 2009.
- <sup>12</sup>ST-ORM, Ver. 2.24, carhs GmbH, Alzenau, Germany
- <sup>13</sup>VEGA – User's Manual, Issue 3/ Revision 0, Arianespace, Evry, France, 2006.
- <sup>14</sup>Sippel, H., Marczyk, J., *Application Strategies of Robust Design & Complexity Management in Engineering: Current Status & Future Trends in Multi-Disciplinary Product Development*, CAEvolution GmbH, Garching, Germany, 2009