Competing Climate Policies

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Preface

Although the last five years have been spent working through numerous models from different branches of economics, building a model myself was a new and challenging experience. It was frustrating each time a dead end had to be abandoned, but in retrospect I am grateful for how the rigor of economics allowed me to study these matters in a systematic way, and gain insights I never would have gotten without this rigor.

I would like to thank my supervisor Bård Harstad for crucial encouragement and valuable feedback throughout the process from vague ideas to formalized models.

I am grateful to the *Oslo Centre for Research on Environmentally friendly Energy* (CREE) for awarding me their master scholarship.

I am indebted to Katinka Kristine Holtsmark for her support and patience - and impatience when necessary - through these months, and the years to come.

Contents

Abstract

In this thesis, I investigate the strategic provision of a public good in a dynamic setting. The public good in question is reduced emissions of greenhouse gases, which limits the public bad of climate change. Emissions can be reduced both through demand- and supply-side policies, and countries differ in which type of policy they prefer. Two stylized models are developed, with consumer countries that consume fossil fuels, but produce none, and producer countries that produce fossil fuels, but consume none.

In the first model, a producer country that does not care about the climate interacts with consumer countries that do. It is shown that even though the producer country does not care about the climate, it reduces production in order to limit emissions.

In the second model, both producer and consumer countries care about the climate. It is shown that the nature of future climate policies matters for emissions today. Between countries that prefer the same type of climate policy, the dynamic public good problem is aggravated. However, between countries that prefer different types of policies, the dynamic public good problem is alleviated.

1 Introduction and Summary

It was shown by Samuelson (1954) that the socially optimal level of a public good will generally not be provided through voluntary contributions. Furthermore, Fershtman and Nitzan (1991) states that in a dynamic setting even less is provided, because each agent contributes less when it is known that other agents will then contribute more in the future. However, these conclusions may change if we acknowledge that agents may have different preferences over the policies through which a public good is provided.

My thesis investigates the strategic provision of a public good in a dynamic setting, where future contributions depend upon past contributions, and agents have different preferences over the ways in which it is possible to contribute to the public good.

The public good in question is reduced emissions of greenhouse gases, which limits the public bad of climate change. It is generally acknowledged that the public good nature of a healthy climate is a major challenge to the prospects of preventing climate change. This is strikingly formulated by Hardin (1968) as a "Tragedy of the Commons".

However, emissions can be reduced through a range of policies, and it is unlikely that agents are indifferent to these. Policies that aim to reduce emissions can be grouped into demand and supply-side policies. A tax on consumption of fossil fuels is an example of the former, a tax on production of fossil fuels an example of the latter.

Both taxes would increase the price paid by consumers, and reduce the price received by producers. The tax would be equal to the difference between these prices. But, as an example, if these consumers and producers belonged to two different countries, a consumer country and a producer country, and each country returned tax revenue through lump-sum transfers, it is clear that the consumers would be better off with their country imposing a tax on consumption than with the producer country imposing a tax on production, and vice versa.

Also, the tax on consumption would reduce the market price of fossil fuels and thus benefit other consumers and hurt producers, while the tax on production would increase the market price and thus benefit other producers and hurt consumers.

It is clear that different types of future climate policies have different welfare implications for consumer and producer countries, and for net importers and exporters in a more general setting. If future climate policies depend upon the stock of greenhouse gases, it seems intuitive that countries take into account the future policy response to higher emissions today, and act strategically.

In this thesis, two models are developed to investigate such strategic interaction. They are both two-period models, and for each model I find the subgame perfect Nash equilibrium.

The first model, presented in Section 3, is used to investigate strategic emissions reductions by a price-setter producer country that produces fossil fuels, interacting with price-taker consumer countries that use demand-side climate policies.

The producer country could be thought of as a large country that controls the world's reserves of fossil fuels, or as a cartel of producer countries that cooperate in order to maximize their aggregate welfare, such as the *Organization of the Petroleum Exporting Countries* (OPEC). The consumer countries could be thought of as a large number of small countries that fail to cooperate.

The main result is that even though the producer country does not care about the climate, it takes into account how its production of fossil fuels affects the stock of greenhouse gases, and reduces production in order to limit emissions.

The producer country's motive is to avoid a future loss from a lower price due to demand-side climate policies. Higher first-period emissions would increase the second-period stock of greenhouse gases, and this would increase the consumer countries' marginal harm from emissions. Second-period demand for fossil fuels would be reduced. This would reduce the price the producer country could take for fossil fuels, and the producer country would incur a loss.

If the producer country puts greater weight on future welfare, it reduces first-period production more. The loss from a lower price in the second period then plays a bigger role for the producer country when first-period production is set.

If the future cost of production is higher, the producer country reduces first-period production less. The reason is that the a higher future cost makes the producer reduce second-period production, which implies that there are fewer second-period units of fossil fuels on which the producer country would lose from a lower price.

The effect of a higher survival rate, which implies that a greater part of first-period emissions remain in the atmosphere in the second period, is ambiguous. On one hand, the effect of higher first-period emissions on the second-period price is stronger, because first-period emissions then contribute more to the second period stock. This would make the producer country reduce first-period production more. But on the other hand, the increased amount of first-period emissions that remain in the atmosphere in the second period has a negative effect on second-period demand. The second-period quantity is reduced, which implies that there are fewer second-period units of fossil fuels on which the producer country would lose from a lower price. This would make the producer country reduce first-period production less.

Similarly, if the marginal damage of one consumer country increases in such a way that it also increases more rapidly, the effect is ambiguous. Because marginal damage increases more rapidly, the effect of higher first-period emissions on the second-period price is stronger, because marginal harm from an increase in emissions is bigger. Also, the price would have to be reduced more for this consumer country to be willing to buy more fossil fuels again. Both these effects would make the producer country reduce first-period production more. On the other hand, higher marginal damage from emissions implies lower demand and a lower second-period quantity. This would make the producer country reduce first-period production less.

The second model, presented in Section 4, is used to investigate the strategic response of both consumer and producer countries to future demand and supply-side policies.

There is a Cournot game with four groups of countries: 1. Price-setting consumer countries that are harmed by emissions and set quantities given the quantities set by other price-setting countries. 2. Price-setting producer countries that are harmed by emissions and set quantities given the quantities set by other price-setting countries. 3. Price-taking consumer countries that are not harmed by emissions. 4. Price-taking producer countries that are not harmed by emissions.

A price-setting consumer country could be thought of as a large country using demand-side climate policies, or as a demand-side coalition of countries such as the countries of the *European Union Emissions Trading System* (EU ETS).

A price-setting producer country could be thought of as a large country using supply-side policies, or as a supply-side coalition of countries. Such coalitions might not exist today, but supply-side policies are gaining interest, and the insights developed in the thesis could be valuable for our understanding of the impact of such coalitions.

The model can also be used to evaluate the strategies of price-setting consumer and producer countries that do not care about the climate, anticipating the future climate policies of price-setting consumer and producer countries that do care about the climate.

There are also price-taking consumer and producer countries that are not harmed from emissions. These could be thought of as small countries that perceive their own contribution to global emissions as negligible.

The main result is that consumer countries increase (decrease) consumption today if the future response of other countries to higher first-period emissions has a negative (positive) effect on the future price, and that producer countries reduce (increase) production today if the future response of other countries to higher first-period emissions has a negative (positive) effect on the future price.

The reason why a producer country reduces production today if the future response of other countries to higher first-period emissions has a negative effect on the future price is the same as it was in the first model. While in the first model all climate-policies were demand-side policies that reduced the price, in the second model there are both demand- and supply-side climate policies. If the demand-side policies have the strongest effect on the price, a producer country reduces first-period production just as in the first model.

However, if the supply-side policies of other producer countries have a stronger effect on the price than the demand-side policies of the consumer countries, a producer country increases first-period production. Higher first-period emissions make other producer countries reduce their second-period production, and this increases the price. A higher price increases profits.

If the future response of other countries to higher first-period emissions has a negative effect on the future price, a consumer country increases first-period consumption in order to gain from a lower price next period. This could be the case if the demand-side policies of other consumer countries have a stronger effect on the second-period price than the supply-side policies of the producer countries.

On the other hand, if the future response of other countries to higher first-period emissions has a positive effect on the future price, a consumer country reduces first-period consumption in order to avoid losing from a higher price next period. This could be the case if the demand-side policies of other consumer countries have a weaker effect on the second-period price than the supply-side policies of the producer countries.

Through this model, it is shown that the dynamic public good problem among countries which prefer the same type of climate policy, is aggravated when the effect of future climate policies on the price is taken into account. Not only does a country emit more in order to free-ride on greater future emissions reductions by other countries, but it also emits more in order to gain from the favorable effect higher emissions has on the future price, through the climate policy of other countries that prefer the same type of policy.

However, between countries that prefer different types of policies, the public good problem is alleviated when the effect of future climate policies on the price is taken into account. A country still has incentive to emit more in order to free-ride on greater future emissions reductions by other countries, but it also has incentive to emit less in order to avoid losses due to the unfavorable effect higher emissions has on the future price, through the climate policy of countries that prefer the other type of policy.

A brief review of the literature follows next, in Section 2, with emphasis on the relevance of the thesis to the different strands of literature. Then, in Section 3, the first model is presented. In Section 4, the second model is presented. In Section 5, further refinements and possible extensions are discussed. At last, some concluding remarks are given in Section 6.

There is an appendix at the end, containing proofs.

2 Related Literature

Much of the literature on policies that aim to limit climate change focuses on abatement in general, and does not distinguish between demand- and supply-side policies. Papers such as Barrett (1994) discuss participation in self-enforcing International Environmental Agreements (IEAs), Harstad (2012b,c) take participation as given and investigate the consequences of incomplete contracting within a coalition, and Battaglini and Harstad (2014) unites these two approaches, and shows how incomplete contracting can actually increase participation.

The literature on carbon leakage is of more direct relevance to this thesis, since it emphasizes the differences between policies, and their effects on the price of fossil fuels. Two papers, Hoel (1994) and Harstad (2012a), are reviewed in Subsection 2.1.

Wirl (1994) develops an insight similar to that of the first model of the thesis, that an oil cartel may pre-empt a climate coalition. This paper, and others building upon on it, are review in Subsection 2.2.

2.1 Carbon Leakage Is a Challenge That Can Be Overcome

Since fossil fuels that are traded internationally, it is generally acknowledged that an emissions reduction by a country or a coalition of countries does not necessarily lead to a one-to-one reduction in global emissions. A reduction in consumption is partly offset by increased consumption in other countries, and a reduction in production is partly offset by increased production in other countries. This is referred to as demand and supply-side carbon leakage, respectively.

Hoel (1994) develops an elegant solution to this challenge, exploiting that demand-side carbon leakage is caused by an initial price reduction, while supply-side carbon leakage is caused by an initial price increase: The optimal climate policy combines demand and supply-side policies in such a way that, except for terms-of-trade effects, the effects causing demand and supply-side carbon leakage offset each other.

The welfare consequences different types of policies have for the cooperating countries is a key point in this paper, which explains the terms-of-trade effects. But it is also noted that if then non-cooperating countries are importers of fossil fuels they would be better off if the cooperating countries used demand-side policies, and vice versa. This insight is a starting point for the second model of this thesis, developed in Section 4, where I let there be different countries, or different coalitions of cooperating countries, who all have climate policies and do not take the price as given, and investigate their strategic interaction in a dynamic setting.

At the end of this subsection, it is discussed how the game in Hoel (1994) is related to the the second period game of my first model, presented in Section 3. But first, other results from Hoel (1994) are presented, and bridge the way to a presentation of Harstad (2012a).

A note on the use of the term first best in the following paragraphs: In Hoel (1994), it is assumed that the non-cooperating countries take greenhouse gas emissions from other countries as given. It is also assumed that these countries have no climate policy, e.g. the demand and supply schedules of these countries are independent of total emissions. Thus, the non-cooperating countries behave as if they are not harmed by emissions. When I use the term first best in the following sections, this only applies to the case where the non-cooperating countries actually are not harmed by emissions. The term first best is not used in Hoel (1994). I do this because it is then easier to appreciate the results of Harstad (2012a), which is discussed subsequently. That paper assumes in the main part that non-cooperating countries are not harmed by emissions, and discusses the case were they are harmed by emissions in an extension.

Even though the problem of carbon leakage is overcome by the combination of demand and supply-side policies proposed by Hoel (1994), this solution is still not the first best. An obvious reason for this is that the price-setting cooperating countries exploit their market power. But even if they did not, the solution would not be first best: There would still be produced and consumed units of fossil fuels for which the the marginal social cost of production is greater than the marginal utility from consumption.

This is the case because: 1. Consumption in the non-cooperating countries is untaxed, and this consumption is satisfied by untaxed production in non-cooperating countries, and by production in cooperating countries which is taxed at less than the Pigouvian tax rate. 2. Production in the non-cooperating countries is untaxed, and this production is consumed by untaxed consumption in non-cooperating countries, and by consumption in cooperating countries which is taxed at less than the Pigouvian tax rate.

Hoel (1994) shows that the cooperating countries can do better if they can set the taxes in the non-cooperating countries directly, and compensate these countries through transfers such that they are equally well off. The taxes can then be set such that the sum of tax rates applying to a consumer and a producer trading fossil fuels is equal to the to the marginal harm from emissions. If there were no terms-of-trade effects, this solution would be first best.

However, this solution relies upon the possibility of setting tax rates in non-cooperating countries directly. It should also be noted again that it is not first best, since the cooperating countries do exploit their market power.

Harstad (2012a) shows that the first best can be achieved without dealing directly with non-cooperating countries, if trade in fossil fuel deposits is possible. The cooperating countries then buy or lease deposits in non-cooperating countries. This includes, but is not restricted to, the deposits that given the first best solution should be left unexploited, e.g. the deposits that are most expensive to extract.

Then, the cooperating countries tax their own production, including production from bought or leased deposits, at a tax rate equal to the marginal environmental cost of emissions, e.g. the Pigouvian tax rate. Marginal deposits are then left unexploited. The countries need not fear about carbon leakage, since fossil fuel supply is locally inelastic when all marginal deposits are controlled by the cooperating countries.

Also, in equilibrium no country is an importer or exporter, and because of this there are no distortions due to the market power of the cooperating countries. Thus, the first best solution is reached through supply-side policies.

It should be noted that these solutions are only first best if the non-participating countries do not take harm from emissions. This is discussed and shown in Harstad (2012a).

Next, the links between these works and the models in my thesis are further discussed.

If the non-cooperating countries of Hoel (1994) were allowed to have climate policies, e.g. set taxes based on their individual marginal harm from emissions, given a stock of greenhouse gases, the game would just be a slightly more complex version of the second period game of my first model, presented in Section 3. The cooperating countries would then take into account the effect of first-period emissions on the second-period price, just as in the model in Section 3. However, the effect would be conditional upon (1) whether the cooperating countries are net importers or exporters, and (2) the nature of future climate policies, which determine whether the price actually increases or decreases if first-period emissions increase.

In extensions of the subgame following trade in deposit fuels, Harstad (2012a) does actually let non-cooperating countries be harmed by greenhouse gases, and does also develop a dynamic model with two periods. However, this is done in separate extensions, and thus a strategic effect similar to the one in my model does not appear.

The model in Section 3 could be extended to provide such results. However, I have chosen to leave the development of results similar to these to the model in Section 4, because I want to show that this strategic effect is present even if all the countries with climate policies are price-setters.

Note that if the market for fuel deposits clears as described in Harstad (2012a), no country would be a net importer or a net exporter of fossil fuels, and such strategic effects would vanish. But there are considerable obstacles to such a market clearing, one of them political security considerations.

It is thus not difficult imagine a situation where supply-side policies, including buying of foreign deposits, are used to a much larger extent, but where countries are still net importers or exporters of fossil fuels. It is then important to understand how countries or coalitions with supply-side policies affect each other, and how they affect with countries which prefer demand-side policies. The model in Section 4 makes some steps towards such a better understanding of this.

2.2 An Oil Cartel May Pre-Empt a Climate Coalition

Wirl (1994) develops an insight similar to the main result of Section 3, and states that "energy suppliers may preempt energy taxation and thereby may raise the price at front". The interaction between a consumer's government and a cartel of energy suppliers is modeled as a differential game, e.g. with continuous and infinite time. This allows for the determination of paths for both the producer price and the tax on consumption. It is shown that that the producer price in the beginning of the game is high in order to pre-empt future taxes on consumption.

This non-cooperative outcome is compared to the situation where a benevolent social planner chooses the optimal path for extraction and consumption of fossil fuels, labelled as the cooperative outcome. It is found that in the non-cooperative outcome, extraction is delayed compared to the cooperative outcome. This is due both to the monopolist's standard exploitation of market power, and the pre-emption effect.

Although the two-period setup of the model in Section 3 does not allow for price, tax and extraction paths to be derived like in Wirl (1994), the relatively simple setup allows for a very clear cut exposition of the pre-emption effect. This setup does also facilitate comparative statics, which provide interesting results.

The model in Section 4 goes beyond Wirl (1994) in several aspects, by letting both consumer and producer countries be price-setters, and investigating both the interaction between countries which prefer the same policies and countries which prefer different policies.

Liski and Tahvonen (2004) is a paper in the tradition from Wirl (1994), which identifies a strategic effect that can not be derived from my two-period model. Their model does also exhibit a differential game. As in Wirl (1994), buyers of fossil fuels take the consumer price as given, while fossil fuels are sold by a cartel which set the producer price. However, there is a buyer's agency which sets a consumption tax and give the revenues to the consumers. This tax has a Pigouvian element, but also an element due to the fact that the agency takes into account the cartel's pre-emption strategy.

If damage from emissions is large, the cartel would like to delay production in order to pre-empt future demand-side policies. If the pre-emption effect is strong, which is the case if the consumers' damage from emissions is high, the producer price is decreasing over time, since gain from pre-emption is biggest if it is done at an early stage. But the buyers' agency knows this, and counteracts this with a consumption subsidy in the early phase of the game. This subsidy comes in addition to the Pigouvian element of the tax, and an import tariff. The tax on consumption, which is the sum of these elements, may thus be increasing over time. The subsidy element contributes to moving extraction to an earlier point in time. Then, there are less fossil fuels left to extract in the future, and thus less future taxation for the cartel to pre-empt.

The consumers, or more precisely their agency, act strategically, but not in order to pre-empt supply-side climate policies as in the second model of the thesis, presented in Section 4. The cartel does not care about the climate, and because of this such effects can not arise.

Another paper that investigates the strategic adaption of a seller of fossil fuels to changes in future demand is Gerlagh and Liski (2011). Here, the change in future demand is caused by the transition to a substitute that makes fossil fuels obsolete. This substitute takes time to develop, and the buyers have to rely on fossil fuels in the transition period. If there is little fossil fuels left, the price will be high and the transition period costly. This creates incentives for the buyers to initiate the transition earlier. The seller counteracts this by delaying extraction, such that supply increases, and the price decreases, as time passes. When fossil fuels become cheaper it is less attractive to initiate the transition, and this compensates the buyer for the increased transition cost.

3 Emissions Matter, Even If Climate Does Not

In this section, I develop a model of $N+1$ countries which interact over two time periods. N of these countries are referred to as consumer countries. These countries consume fossil fuels, but produce none. They also care about climate change. The last country is referred to as the producer country. This country produces fossil fuels, but consume none.

The policymaker in each country perfectly represents the preferences of the inhabitants. The policymakers set policy by setting quantities. In the following, I will just refer to the countries, not the policymakers or the inhabitants.

The consumer countries care about the climate, while the producer country does not. I will show that the producer country nevertheless chooses to reduce supply in order to limit emissions of greenhouse gases.

Consumer country i's utility from consuming x_{ti}^D fossil fuels at time t, $t = 1, 2$, is given by the function $U_{ti}(x_{ti}^D)$, which is increasing and concave. Fossil fuels are bought at a price p_t, which is taken as given. Country i's expenses on fossil fuels is given by $p_t x_{ti}^D$, and its welfare decreases linearly in these expenses.

Furthermore, the consumer countries are hurt by the stock of greenhouse gases in the atmosphere. In the first period, this stock is given by the aggregate consumption of fossil fuels, $x_1^D \equiv \sum_i x_{1i}^D$. In the second period, a fraction δ of these emissions are still in the atmosphere. Also, second-period emissions are added to the stock. Thus, the second period stock of greenhouse gases is given by $\delta x_1^D + x_2^D$, where $x_2^D \equiv \sum_i x_{2i}^D$ is aggregate consumption in the second period. Country i's harm from the stock of greenhouse gases at time t is given by a function H_{ti} of the stock, and H_{ti} is increasing and convex. In each period, consumer country i takes the consumption of other consumer countries as given.

Each consumer country does not take into account the negative external effect its emissions has on other consumer countries.

Thus, first period welfare of consumer country i is given by:

$$U_{1i}(x_{1i}^D) - p_1 x_{1i}^D - H_{1i}(x_1^D)$$

And second period welfare of consumer country i is given by:

$$U_{2i}(x_{2i}^D) - p_2 x_{2i}^D - H_{2i}\left(\delta x_1^D + x_2^D\right)$$

In the first period, consumer country i discounts second period welfare by a factor β_i. It is assumed that in the first period, each consumer country takes the second-period price and quantities as given, including its own second-period consumption.

The producer country's cost of producing x_t^S fossil fuels at time t, $t = 1, 2$, is given by the function $C_t(x_t^S)$, which is increasing and convex. Fossil fuels are sold at a price p_t. The producer country's income from the sale of fossil fuels is given by $p_t x_{2i}^S$, and its welfare increases linearly in this income. The producer country is not hurt by emissions.

First period welfare of the producer country is given by:

$$p_1 x_1^S - C_1(x_1^S)$$

Second period welfare of the producer country is given by:

$$p_2 x_2^S - C_2(x_2^S)$$

In the first period, the producer country discounts second period welfare by a factor β.

The producer country is a monopolist, and does not take the price as given. It could be thought of as a large country that controls the world's reserves of fossil fuels, or as a cartel of producer countries that cooperate in order to maximize their aggregate welfare.

One could think of the consumer countries as a large number of small countries that fail to cooperate. Due to the small size of each country, it is reasonable to assume price-taker behavior.

While clearly a simplification, the setup in this model is reminiscent of that in the real world. Every country in the world consumes fossil fuels, while only some countries have fossil reserves to extract. Some of these countries are even openly cooperating through a cartel, the *Organization of the Petroleum Exporting Countries* (OPEC).

Above, it was stated that each consumer country does not take into account the negative external effect its emissions has on other consumer countries. However, the setup allows for another interpretation of the harm function: It could be that a group of countries actually do take into account the external effect of emissions on each other. One should then interpret the harm function of one of these countries as the sum of the individual harm functions of these countries. Such a group of countries could be interpreted as a climate coalition, but in a restricted sense where it is only through the internalization of each others' harm from emissions that the countries cooperate.

To achieve simplicity, the model is built around a producer country that does not consume any fossil fuels. However, modeling this country as a net exporter of fossil fuels would not change the nature of the results.

There are two periods, and the equilibrium of the second period depends on the amount of emissions in the first period. Therefore, the model is solved by backward induction.

I assume that every maximization problem has an interior solution.

The next paragraphs describe an approximation that is made.

When maximizing welfare, a consumer country takes the consumption of the other countries as given, and thus acts as if an increase in its consumption of fossil fuels causes an equal increase in production.

Then, however, instead of expressing the second-period demand of each consumer country as a function of the price, given first-period production and the second-period production *of the other consumer countries*, an approximation is made: The second-period demand of each consumer country is assumed to be a function of the price, given first-period production and second-period production. This makes it possible to express aggregate second-period demand as a function of the price, given first-period production and second-period production.

This approximation is made because the producer country is assumed to set the quantity, from which follows the price that equates demand to the quantity set. This necessitates that aggregate second-period demand is a function of the price, first- and second-period production, not a sum of each country's demand given the price, first-period production and the consumption of other countries.

This approximation is unlikely to have qualitative consequences, but is nevertheless somewhat unsatisfactory. It could have been avoided by letting the producer country set the price, and then let the equilibrium quantity be given by the quantity the consumer countries choose to consume at that price. The second-period demand from each consumer country could then be expressed as a function of of the price, given first-period production and the second-period production of the other consumer countries.

An equivalent approximation is made with regard to first-period demand.

Note that none of these challenges arise in the second model, presented in Section 4, where no such approximations are made.

3.1 Second-Period Optimization

In the second period, each consumer country maximizes welfare given the price, p_2, and first-period emissions, x_1. For each consumer country, this defines demand as a function of the price, given first-period emissions. Aggregate demand is given by the sum of demand from all N consumer countries. Thus, aggregate demand is also a function of the price, given first-period emissions.

The price that clears the market is the one that makes the consumer countries demand exactly the amount that the producer country produces.

Thus, when the producer country maximizes its welfare by maximizing profits, it knows which price it can ask for in order to sell various amounts of fossil fuels.

The Consumer Countries

Consumer country i chooses its consumption of fossil fuels, x_{2i}^D, to maximize welfare. The price, p_2, and first-period emissions, x_1, are taken as given. The maximization problem:

$$\max_{x_{2i}^D} \; U_{2i}(x_{2i}^D) - p_2 x_{2i}^D - H_{2i}\left(\delta x_1 + x_2^D\right)$$

The first order condition:

$$U_{2i}'(x_{2i}^D) - p_2 - H_{2i}'\left(\delta x_1 + x_2^D\right) = 0 \tag{1}$$

Then, the approximation described in the introduction is done, and consumer country i's demand for fossil fuels is expressed as a function of the price, given first- and second-period emissions: $x_{2i}^D(p_2 \mid x_1, x_2^S)$.

At the optimum, marginal welfare from the consumption of fossil fuels is equal to the price. Marginal welfare is given by the difference between marginal utility from the consumption of fossil fuels and marginal harm from the stock of greenhouse gases. The consumption of other countries is taken as given, and thus the consumer sees emissions as increasing one-to-one with its consumption of fossil fuels.

Then, the aggregate demand function is $x_2^D(p_2 \mid x_1, x_2^S) \equiv \sum_i x_{2i}^D(p_2 \mid x_1, x_2^S)$.

However, when the partial effect of a marginal price increase on aggregate demand is needed, this effect is assumed to be given by the sum over the partial effects on the demand by each consumer country, obtained by differentiation of the first order condition, Equation (4), without the approximation. Thus:

$$\frac{\partial x_2^D}{\partial p_2} = \sum_i \frac{1}{U_{2i}'' - H_{2i}''} < 0$$

The following lemma establishes the effect of higher first-period emissions on second period demand:

Lemma 1. *Higher first-period emissions reduces second period demand for fossil fuels:*

$$\frac{\partial x_2^D(p_2 \mid x_1, x_2^S)}{\partial x_1} < 0$$

Proof. Differentiation of the aggregate demand function with respect to x_1 yields:

$$\frac{\partial x_2^D(p_2 \mid x_1, x_2^S)}{\partial x_1} = \sum_i \frac{\partial x_{2i}^D(p_2 \mid x_1, x_2^S)}{\partial x_1} = \sum_i \frac{\delta H_{2i}''}{U_{2i}'' - H_{2i}''} < 0$$

Where the expression for $\frac{\partial x_{2i}^D}{\partial x_1}$ was found by differentiating the consumer country first order condition, Equation (1), with respect to x_1. □

The intuition behind this result is straightforward: H_{2i} is a convex function of the stock of greenhouse gases. Higher first-period emissions implies that the stock of greenhouse gases is larger. Then, due to the convexity of H_{2i}, marginal harm from contributions to the stock of greenhouse gases is higher. It then follows that, at a given price, the consumer countries will demand less fossil fuels.

Market Clearing

The producer country is the only producer of fossil fuels, and thus the price that clears the market is the one that equates demand, $x_2^D(p_2 \mid x_1, x_2^S)$, to the quantity that the producer chooses to produce, x_2^S. This can be seen from the market clearing condition:

$$x_2^D(p_2 \mid x_1, x_2^S) = x_2^S \tag{2}$$

This defines $p_2(x_2^S \mid x_1)$, the price that clears the market as a function of the producer country's production of fossil fuels, given first-period emissions.

If the producer country wants to sell more, it must reduce the price for the consumer countries to be willing to buy more. This follows from differentiation of the market clearing condition with respect to x_2^S:

$$\frac{\partial p_2}{\partial x_2^S} = \frac{1 - \frac{\partial x_2^D}{\partial x_2^S}}{\frac{\partial x_2^D}{\partial p_2}} = \frac{1 - \sum_i^N H''_{2i} \left(U''_{2i} - H''_{2i}\right)^{-1}}{\sum_i \left(U''_{2i} - H''_{2i}\right)^{-1}} < 0$$

Furthermore, the price that the consumer countries are willing to pay for a given quantity of fossil fuels depends on first-period emissions, which contribute to the stock of greenhouse gases. The following lemma establishes the effect of higher first-period emissions on the price the producer country can take, if it wants to sell a given quantity of fossil fuels:

Lemma 2. *Higher first-period emissions reduces the price the consumers are willing to pay for a given quantity of fossil fuels:*

$$\frac{\partial p_2(x_2^S \mid x_1)}{\partial x_1} < 0$$

Proof. This follows from differentiation of the market clearing condition, Equation (2), with respect to x_1, keeping x_2^S constant:

$$\frac{\partial p_2(x_2^S \mid x_1)}{\partial x_1} = -\frac{\frac{\partial x_2^D}{\partial x_1}}{\frac{\partial x_2^D}{\partial p_2}} < 0$$

□

The consumer countries would never be willing to pay a more for a unit of fossil fuels than their marginal welfare from the consumption of it. Higher first-period emissions reduces marginal welfare from the consumption of fossil fuels. Thus, if the same quantity is to be sold, the producer country must reduce the price.

The Producer Country

As shown, the producer country knows which price it can ask for if it wants to sell a certain quantity of fossil fuels. To find the optimal quantity, the producer country must balance the following concerns:

If the producer country produces an additional unit, it earns a profit on that unit. On the other hand, in order to sell that additional unit it must reduce the price. Only then will the consumers be willing to buy it. Thus, producing another unit implies lower income from inframarginal units, since each of these is now sold at a lower price. This is the standard problem of a monopolist.

The maximization problem of the producer country:

$$\max_{x_2^S} \; p_2(x_2^S \mid x_1)x_2^S - C_2(x_2^S)$$

The first order condition:

$$p_2(x_2 \mid x_1) + x_2 \frac{\partial p_2(x_2 \mid x_1)}{\partial x_2^S} - C_2'(x_2) = 0 \tag{3}$$

This defines the second period monopoly quantity x_2 and price $p_2^m \equiv p_2(x_2 \mid x_1)$ as functions of the first-period quantity x_1.

At the optimum, the profit from selling another unit is equal to the loss from a lower price on inframarginal units.

3.2 First-Period Optimization

In the second period, each consumer country maximizes welfare given the price, p_2. For each consumer country, this defines demand as a function of the price. Aggregate demand is given by the sum of demand from all N consumer countries. Thus, aggregate demand is also a function of the price. The indirect aggregate demand function defines which price the consumers are willing to pay for various amounts of fossil fuels.

Then, the producer country maximizes its welfare by maximizing profits, knowing which price the consumer countries are willing to pay for various amounts of fossil fuels.

Each country takes into account how first period consumption or production of fossil fuels affect their own welfare in the second period. The consumer country takes the second-period price and quantity as given, while the producer country takes into account how first-period emissions affect second-period price and quantity.

The Consumer Countries

Consumer country i chooses its consumption of fossil fuels, x_{1i}^D, to maximize welfare. The price, p_1, is taken as given. The maximization problem:

$$\max_{x_{1i}^D} \; U_{1i}(x_{1i}^D) - p_1 x_{1i}^D - H_{1i}(x_1^D) + \beta_i \left[U_{2i}(x_{2i}) - p_2 x_{2i} - H_{2i}\left(\delta x_1^D + x_2\right)\right]$$

The first order condition, omitting arguments:

$$U'_{1i} - p_1 - H'_{1i} - \beta_i \delta H'_{2i} = 0 \tag{4}$$

Then, the approximation described in the introduction is done, and consumer country i's demand for fossil fuels is expressed as a function of the price, given first-period emissions: $x_{1i}^D(p_2 \mid x_1^S)$.

The consumer takes into account that a fraction δ of first-period emissions affects its second period welfare. Apart from that, the interpretation of the first order condition is similar to that in the second period.

The aggregate demand function $x_1^D(p_1 \mid x_1^S) \equiv \sum_i x_{1i}^D(p_1 \mid x_1^S)$, and its properties can be derived in the same way as for the second period.

As for the second period, when the partial effect of a marginal price increase on aggregate demand is needed, this effect is assumed to be given by the sum over the partial effects on the demand by each consumer country, obtained by differentiation of the first order condition, Equation (4), without the approximation. Thus:

$$\frac{\partial x_1^D}{\partial p_1} = \sum_i \frac{1}{U''_{1i} - H''_{1i}} < 0$$

Market Clearing

The price that clears the market is the one that equates demand, $x_1^D(p_1 \mid x_1^S)$, to the quantity that the producer chooses to produce, x_1^S. This can be seen from the market clearing condition:

$$x_1^D(p_1 \mid x_1^S) = x_1^S \tag{5}$$

This defines $p_1(x_1^S)$, the the price that clears the market as a function of the producer country's production of fossil fuels. If the producer country wants to sell more, it must reduce the price for the consumer countries to be willing to buy more. This follows from differentiation of the market clearing condition with respect to x_1^S:

$$\frac{\partial p_1}{\partial x_1^S} = \frac{1 - \frac{\partial x_1^D}{\partial x_1^S}}{\frac{\partial x_1^D}{\partial p_1}} = \frac{1 - \sum_i^N H''_{1i}\left(U''_{1i} - H''_{1i}\right)^{-1}}{\sum_i \left(U''_{1i} - H''_{1i}\right)^{-1}} < 0$$

The Producer Country

As in the second period, the producer must balance the profit from producing an additional unit against the loss from a lower price on inframarginal units. But in the first period, the producer country must also take into account how first-period emissions affect second period welfare. The reason is that the price consumer countries are willing to pay for fossil fuels in the second period depends on the stock of greenhouse gases. The maximization problem is then:

$$\max_{x_1^S}\ p_1(x_1^S)x_1^S - C_1(x_1^S) + \beta\left[p_2(x_2 \mid x_1)x_2 - C_2(x_2)\right]$$

The first order condition:

$$\begin{aligned}&p_1(x_1) + x_1 p_1'(x_1) - C_1'(x_1)\\ &+\beta\left[\left(p_2(x_2 \mid x_1) + x_2\frac{\partial p_2(x_2 \mid x_1)}{\partial x_2^D} - C_2'(x_2)\right)\frac{dx_2}{dx_1} + x_2\frac{\partial p_2(x_2 \mid x_1)}{\partial x_1}\right] = 0\end{aligned}$$

The first three terms of the first order condition concern the first period profits from the production of fossil fuels. The earnings from producing another unit must be balanced against the loss due to a lower price on inframarginal units.

The term beginning with β represents the effect of higher first-period emissions on second period welfare. However, the producer country knows that for any level of first-period emissions, it will choose the optimal quantity in the second period. Thus, a marginal change in the monopoly quantity does not affect welfare, since marginal welfare is zero at the optimum. This follows from the envelope theorem, and is given by the second-period first order condition for the producer country, Equation (3), which can be used to simplify the second-period first order condition:

$$p_1(x_1) + x_1 p_1'(x_1) - C_1'(x_1) + \beta x_2\frac{\partial p_2(x_2 \mid x_1)}{\partial x_1} = 0 \tag{6}$$

This defines the first period monopoly quantity x_1 and price $p_1^m \equiv p_1(x_1)$.

It is clear that the producer country cares about how its production in the first period contributes to a higher stock of greenhouse gases in the second period. The reason is that a higher stock of greenhouse gases reduces the price received for inframarginal units, because the consumer countries' marginal welfare from consumption of fossil fuels is lower.

The main result of this model concerns this strategic effect on the choice of first period production:

Proposition 1. *Even though the producer country does not care about the climate, it takes into account how its production of fossil fuels affects the stock of greenhouse gases, and reduces production in order to limit first-period emissions.*

Proof. This follows directly from the first-period first order condition of the producer country, Equation (6), and Lemma 2. □

Thus, the existence of consumer countries that use demand-side policies to reduce emissions, makes the producer country care about the stock of greenhouse gases, because it fears for its future profits.

Note that letting the producer country care about the climate would not affect the strategic effect stated in Proposition 1. However, the reduction in production due to this strategic effect would then come in addition to the reduction due to the producer country's own harm from the stock of greenhouse gases in both periods.

Through comparative statics, the following subsection investigates how the strategic effect stated in Proposition 1 depends upon the parameters of the model.

3.3 Comparative Statics

In order to facilitate the investigation of comparative statics, three simplifying assumptions are made.

1. It is assumed that $\beta_i = 0 \ \forall \ i \in N$. This removes the effect on first period demand through the effect on x_2 of the parameter in question. For the proposition concerning second period harm, it also removes the direct effect on first period demand of greater harm from emissions.
2. It is assumed that the second-period harm function is quadratic, and that $H''_{2i} = h_{2i}$, where $h_{2i} > 0$ is a parameter.
3. It is assumed that the second-period utility function is quadratic, and that $U''_{2i} = u_{2i}$, where $u_{2i} < 0$ is a parameter.

The first result of this subsection concerns the the producer country's discount factor β:

Proposition 2. *If the producer country puts greater weight on future welfare, e.g. its discount factor β is higher, it reduces production more:*

$$\frac{dx_1}{d\beta} < 0$$

Proof. This is proven in Appendix A.1. □

The producer country reduces production because it cares about its welfare next period. If it cares more about its welfare next period, it will reduce production more.

The second result concerns the effect of a higher second period marginal cost of production. To facilitate this analysis, let the second period cost function, C_2, be quadratic, e.g. that $C_2 = \frac{1}{2}c_2(x_2^S)^2$, where $c_2 > 0$ is a parameter.

Proposition 3. *If the second period marginal cost of production increases (decreases), the producer country reduces (increases) second-period production and increases (reduces) first-period production:*

$$\left.\frac{dx_2}{dc_2}\right|_{x_1} < 0 \quad \textit{and} \quad \frac{dx_1}{dc_2} > 0$$

Proof. This is proven in Appendix A.2. □

It is rather intuitive that the producer country reduces second-period production when the second period cost of production increases. But with fewer units produced and sold in the second period, the loss from a lower price due to higher first-period emissions is smaller. Thus, the producer country also chooses to produce and emit more in the first period.

The third result of this subsection concerns the fraction of first-period emissions that remains in the atmosphere in the second period, which is given by the survival rate δ. A higher δ affects the strategic effect presented in Proposition 1 in two ways. The second-period quantity is reduced, and the effect of higher first-period emissions on the second-period price is amplified.

The quantity effect makes the producer country reduce first period production less, since there are fewer second-period units on which it would lose from a price increase caused by higher first-period emissions.

On the other hand, the price effect makes the producer country reduce production more, because the higher first-period emissions causes a greater price increase in the second period.

The following proposition states under which conditions either of the effects dominates. Also, from the proof of the proposition it can be seen how the quantity and price effects are identified and signed. Here, $W_2'' = 2p_1' + x_1 p_1'' - C_1'' + \beta \frac{dx_2}{dx_1}\frac{\partial p_2}{\partial x_1} < 0$ is the second-period objective function of the producer country, twice differentiated with respect to x_1^S.

Proposition 4. *If a greater part of first-period emissions remains in the atmosphere in the second period, because the survival rate δ is higher, the producer country reduces first period production more, e.g. $\frac{dx_1}{d\delta} < 0$, if*

$$x_2 \;>\; x_1 (W_2'')^{-1} \frac{\partial p_2}{\partial x_1}$$

and less, e.g. $\frac{dx_1}{d\delta} > 0$, if

$$x_2 \;<\; x_1 (W_2'')^{-1} \frac{\partial p_2}{\partial x_1}$$

Proof. This is proven in Appendix A.3. □

These conditions have a clear intuition. A large x_2 means that there are many second period units on which the producer country would lose from a higher price.

This strengthens the price effect.

A large x_1 implies that an increase in δ causes a big increase in the second period stock of greenhouse gases, and this causes a large reduction in second-period consumption. This strengthens the quantity effect. Note that $\frac{\partial p_2}{\partial x_1}$ is independent of x_1 when utility and harm functions are quadratic. This is commented in the proof.

A big $\frac{\partial p_2}{\partial x_1}$ means that the loss from a higher second-period price is reduced more when the second-period quantity is reduced because of a higher δ, since the loss on each unit is greater. Thus, this strengthens the quantity effect.

Finally, the quantity reduction is small if marginal welfare increases rapidly when quantity is reduced, e.g. W_2'' is big. Thus, a big W_2'' weakens the quantity effect.

The last result concerns a consumer country's marginal harm from emissions, and how rapidly it increases. A higher h_{2i} also affect the strategic effect presented in Proposition 1 in two ways, just like a higher δ: The second-period quantity is reduced, and the effect of higher first-period emissions on the second-period price is amplified.

The quantity effect is the same as when the consequences of a higher δ was investigated: It makes the producer country reduce first period production less, since there are fewer second-period units on which it would lose from a price increase caused by higher first-period emissions.

But the price effect now actually consists of two effects, which both pull in the same direction. First, a higher h_{2i} implies that country i would reduce second-period consumption more if first-period emissions were to increase. Second, a higher h_{2i} does also imply that country i is less willing to increase consumption when the price is reduced, and thus the price must be reduced more for the consumers to be willing to buy the same quantity of fossil fuels when first-period emissions increase.

Proposition 5. *If harm from emissions increases more rapidly, the producer country will reduce first-period emissions more, e.g.* $\frac{dx_1}{dh_{2i}} < 0$*, if*

$$\left.\frac{dx_2}{dh_{2i}}\right|_{x_1} \frac{\partial p_2}{\partial x_1} < -x_2 \frac{d}{dh_{2i}} \left(\frac{\partial p_2}{\partial x_1}\right)$$

and less, , e.g. $\frac{dx_1}{dh_{2i}} > 0$*, if*

$$\left.\frac{dx_2}{dh_{2i}}\right|_{x_1} \frac{\partial p_2}{\partial x_1} > -x_2 \frac{d}{dh_{2i}} \left(\frac{\partial p_2}{\partial x_1}\right)$$

Proof. This is proven in Appendix A.4. □

The left hand side of these inequalities represents the quantity effect, while the right hand side represents the price effect.

As discussed in Section 5, if further work was to be done on this model, a priority would be to express the conditions of Propositions 4 and 5 in terms of exogenous parameters.

In the introduction to this section, a restricted type of climate coalition among consumer countries was mentioned. Countries would take into account the external effect of emissions on each other, but would still take the price as given such that there were no terms-of-trade effects. It should be noted here that the formation of such a coalition would imply that the marginal harm of each country would increase more rapidly.

4 The Nature of Future Climate Policy Matters for Emissions Today

In this section, I develop a model that is related to the model in Section 3, but which allows for analysis of the effect of the nature of future climate policy on consumption and production today.

In this model, there are $N_D^p + N_D^m$ consumer countries. N_D^p of them take the price as given, and do not care about the climate. The remaining N_D^m of them do not take the price as given, but do care about the climate.

There are $N_S^p + N_S^m$ producer countries. N_S^p of them take the price as given, and do not care about the climate. The remaining N_S^m of them do not take the price as given, but do care about the climate.

Thus, there are $N_D^p + N_S^p$ countries that take the price and given, and do not care about the climate, and $N_D^m + N_S^m$ countries that do not take the price as given, but do care about the climate.

The price-setter countries could be thought of as large countries, or coalitions of countries.

The price-taker countries could be thought of as small countries which due to their size take the price as given. Although it is not modeled, it could also be that some of these small countries are hurt by climate change, but that they due to their size take emissions as given and do not see how their own consumption or production contribute to the stock of greenhouse gases.

The price-taking countries are introduced first, then the price-setting countries.

Consumer country i is a price-taker, and its utility from consuming x_{ti}^{Dp} fossil fuels at time t, $t = 1, 2$, is given by the function $U_{ti}(x_{ti}^{Dp})$, which is increasing and concave. Fossil fuels are bought at a price p_t. Country i's expenses on fossil fuels is given by $p_t x_{ti}^{Dp}$, and its welfare decreases linearly in these expenses.

Producer country k is a price-taker, and its cost of producing x_{tk}^{Sp} fossil fuels at time t, $t = 1, 2$, is given by the function $C_{tk}(x_{tk}^{Sp})$, which is increasing and convex. Fossil fuels are sold at a price p_t. Country k's income from the sale of fossil fuels is given by $p_t x_{2k}^{Sp}$, and its welfare increases linearly in this income.

The price-setters are hurt by the stock of greenhouse gases in the atmosphere. In the first period, this stock is given by the first-period equilibrium quantity of fossil fuels, x_1^{eq}. In the second period, a fraction δ of these greenhouse gases are still in the atmosphere. Also, second-period emissions of greenhouse gases are added to the stock. Thus, the second period stock of greenhouse gases is given by $\delta x_1^{\text{eq}} + x_2^{\text{eq}}$, where x_2^{eq} is the second-period equilibrium quantity of fossil fuels.

Consumer country j is a price-setter, and its utility from consuming x_{tj}^{D} fossil fuels at time t, $t = 1, 2$, is given by the function $U_{tj}(x_{tj}^{D})$, which is increasing and concave. Fossil fuels are bought at a price p_t. Country j's expenses on fossil fuels is given by $p_t x_{tj}^{D}$, and its welfare decreases linearly in these expenses. Country j's harm from the stock of greenhouse gases at time t is given by a function H_{tj} of the stock, and H_{tj} is increasing and convex.

Producer country l is a price-setter, and its cost of producing x_{tl}^S fossil fuels at time t, $t = 1, 2$, is given by the function $C_{tl}(x_{tl}^S)$, which is increasing and convex. Fossil fuels are sold at a price p_t. Country l's income from the sale of fossil fuels is given by $p_t x_{2l}^S$, and its welfare increases linearly in this income. Country l's harm from the stock of greenhouse gases at time t is given by a function H_{tl} of the stock, and H_{tl} is increasing and convex.

Note that when I refer to some of the countries as price-setters, what is meant is that they take into account how their chosen quantities affect the price, given the quantities chosen by other countries. They do not actually set the equilibrium price. The equilibrium price is the price that clears the market, given the equilibrium quantities of each price-setting country.

The model is solved by backward induction.

4.1 Second-Period Optimization

In the second period, each price-taking consumer country maximizes welfare given the price, p_2. For each consumer country, this defines demand as a function of the price. Aggregate demand from the price-taking consumer countries is given by the sum of demand from all N_D^p of these countries. Thus, aggregate demand is also a function of the price.

Similarly, each price-taking producer country maximizes welfare given the price, p_2. For each producer country, this defines supply as a function of the price. Aggregate supply from the price-taking producer countries is given by the sum of supply from all N_S^p of these countries. Thus, aggregate supply is also a function of the price.

The price that clears the market is the one that makes net supply from the $N_D^p + N_S^p$ price-taking countries equal to net demand from the $N_D^m + N_S^m$ price-setting countries.

Thus, when the price-setting countries maximize their welfare, they know which price they will have to pay, or will receive, depending on their chosen quantity, given the quantities set by the other price-setting countries.

It is also shown how higher first-period emissions affects the price a price-setting country will have to pay, or will receive, for a certain quantity of fossil fuels, in the second period.

Price-Taking Consumer Countries

Consumer country i chooses its consumption of fossil fuels, x_{2i}^{Dp}, to maximize welfare. The price, p_2, is taken as given. The maximization problem:

$$\max_{x_{2i}^{Dp}} \; U_{2i}(x_{2i}^{Dp}) - p_2 x_{2i}^{Dp}$$

First order condition:

$$U'_{2i}(x_{2i}^{Dp}) - p_2 = 0 \tag{7}$$

This defines $x_{2i}^{Dp}(p_2)$, country i's demand for fossil fuels as a function of the price. At the optimum, marginal utility from the consumption of fossil fuels is equal to the price.

The aggregate demand function of the price-taking consumer countries is $x_2^{Dp}(p_2) \equiv \sum_i x_{2i}^{Dp}(p_2)$.

Price-Taking Producer Countries

Consumer country k chooses its production of fossil fuels, x_{2k}^{Sp}, to maximize welfare. The price, p_2, is taken as given. The maximization problem:

$$\max_{x_{2k}^{Sp}} \; p_{2k} x_{2k}^{Sp} - C_{2k}(x_{2k}^{Sp})$$

First order condition:

$$p_2 - C'_{2k}(x_{2k}^{Sp}) = 0 \tag{8}$$

This defines $x_{2k}^{Sp}(p_2)$, country k's supply of fossil fuels as a function of the price. At the optimum, the marginal cost of production of fossil fuels is equal to the price.

The aggregate supply function of the price-taking producer countries is $x_2^{Sp}(p_2) \equiv \sum_k x_{2k}^{Sp}(p_2)$.

Market Clearing

The price that clears the market is the one that makes net supply from the $N_D^p + N_S^p$ price-taking countries, $x_2^{Sp}(p_2) + x_2^{Dp}(p_2)$, equal to net demand from the $N_D^m + N_S^m$ price-setting countries. Let demand from the price-setting countries be given by $x_2^D \equiv \sum_j x_{2j}^D$, and supply by $x_2^S \equiv \sum_l x_{2l}^S$. The market clearing condition:

$$x_2^D + x_2^{Dp}(p_2) = x_2^S + x_2^{Sp}(p_2) \tag{9}$$

Define net demand from the price-setting countries as $x_2^{\text{Dnet}} \equiv x_2^D - x_2^S$. Then, the market clearing condition defines $p_2(x_2^{\text{Dnet}})$, the market clearing price as a function of net demand from the price-setting countries. This is better seen from a reformulation of the market clearing condition:

$$x_2^{Sp}(p_2) - x_2^{Dp}(p_2) = x_2^D - x_2^S \equiv x_2^{\text{Dnet}}$$

The market clearing price must increase if net demand from the price-setting countries increase, since a higher price is needed for the price-taking consumer countries to consume less, and the price-taking producer countries to produce more. This follows from differentiation of the market clearing condition, Equation (9), with respect to x_2^{Dnet}:

$$\frac{dp_2}{dx_2^{\mathrm{Dnet}}} = \frac{1}{\frac{dx_2^S}{dp_2} - \frac{dx_2^{Dp}}{dp_2}} > 0$$

Thus, when the price-setting consumer countries decide upon their consumption of fossil fuels, they know that higher consumption implies a higher price on their inframarginal units. Similarly, the price-setting producer countries know that higher production implies a lower price on their inframarginal units.

Next, it is derived how the second-period quantity of fossil fuels is affected by a marginal increase in consumption or production by the price-setting countries, when the quantities set by other price-setting countries are held constant.

It is clear that given a set of quantities (x_2^D, x_2^S) for the price-setting countries, the market clearing price will adjust to equate consumption with production. Thus, the total second-period consumption and production of fossil fuels can be written as a function $x_2^{\mathrm{tot}}(x_2^D, x_2^S) \equiv x_2^D + x_2^{Dp}\left(p_2(x_2^{\mathrm{Dnet}})\right) = x_2^S + x_2^{Sp}\left(p_2(x_2^{\mathrm{Dnet}})\right)$.

It then follows that the partial effect on total consumption and production, of higher consumption by the price-setting consumer countries, can by found by partial differentiation of $x_t^{\mathrm{tot}}(x_2^D, x_2^S)$:

$$\frac{\partial x_2^{\mathrm{tot}}(x_2^D, x_2^S)}{\partial x_2^D} = 1 + \underbrace{\frac{\partial x_2^{Dp}}{\partial p_2}\frac{dp_2}{dx_2^{\mathrm{Dnet}}}}_{<0} = \frac{\partial x_2^{Sp}}{\partial p_2}\frac{dp_2}{dx_2^{\mathrm{Dnet}}} > 0$$

Unsurprisingly, total consumption and production increases when consumption by the price-setting countries increase. But the increase is not one-to-one, since the price-taking consumers reduce their consumption.

And similarly, regarding the partial effect on total consumption and production, of higher production by the price-setting producer countries:

$$\frac{\partial x_2^{\mathrm{tot}}(x_2^D, x_2^S)}{\partial x_2^S} = 1 - \underbrace{\frac{\partial x_2^{Sp}}{\partial p_2}\frac{dp_2}{dx_2^{\mathrm{Dnet}}}}_{>0} = -\frac{\partial x_2^{Dp}}{\partial p_2}\frac{dp_2}{dx_2^{\mathrm{Dnet}}} > 0$$

Total consumption and production increases when production by the price-setting countries increase. But the increase is not one-to-one, since the price-taking producers reduce their production.

Price-Setting Consumer Countries

The price-setting consumer countries do not take the price as given, only the quantities set by other price-setting countries. As shown, when the price-setting consumer countries decide upon their consumption of fossil fuels, they know that higher consumption implies a higher price on their inframarginal units.

Thus, when choosing the optimal quantity, the price-setting consumer countries must balance the net utility from another unit of fossil fuels against the loss from having to pay a higher price on inframarginal units.

Furthermore, they have to take into account how their consumption of fossil fuels contribute to the stock of greenhouse gases, which they take harm from. The maximization problem for country j, a price-setting consumer country, is then:

$$\max_{x_{2j}^D} \; U_{2j}(x_{2j}^D) - p_2(x_2^{\mathrm{Dnet}})x_{2j}^D - H_{2j}\left(\delta x_1^{\mathrm{eq}} + x_2^{\mathrm{tot}}(x_2^D, x_2^S)\right)$$

First order condition:

$$U'_{2j} - p_2 - x_{2j}^D \frac{dp_2}{dx_2^{\mathrm{Dnet}}} - H'_{2j}\frac{\partial x_2^{\mathrm{tot}}}{\partial x_2^D} = 0 \tag{10}$$

This defines the optimal second-period consumption of fossil fuels by consumer country j as $x_{2j}^{Dm}\left(x_1^{\mathrm{eq}} \mid x_{2,-j}^D, x_2^S\right)$, a function of the first-period quantity x_1^{eq}, given the quantities set by the other price-setting consumer countries, $x_{2,-j}^D$, and the quantities set by the price-setting producer countries, x_2^S.

The two first terms represent the net utility from a marginal increase in consumption, while the third term represents the loss from a higher price on inframarginal units.

The last term represents the harm from the increase in the stock of greenhouse gases that this marginal increase in consumption leads to. As shown, emissions increase less than one-to-one with the increase in country j's consumption, since the price-taking consumer countries reduce their consumption when the price increases to clear the market.

Furthermore, the consumption of the price-setting consumer countries depends on first-period emissions, since these contribute to the second period stock of greenhouse gases. The following lemma establishes the effect of higher first-period emissions on the consumption of a price-taking consumer country, given the quantities set by other price-setting countries.

For notational simplicity, let

$$W_{2j}(x_{2j}^D) \equiv U_{2j}(x_{2j}^D) - p_2(x_2^{\mathrm{Dnet}})x_{2j}^D - H_{2j}\left(\delta x_1^{\mathrm{eq}} + x_2^{\mathrm{tot}}(x_2^D, x_2^S)\right)$$

Then, W'_{2j} and W''_{2j} refer to the first and second derivative, respectively, of consumer country j's objective function.

Note that it is assumed that the second order condition for a maximum holds, e.g. that $W''_{2j} < 0$ for country j.

Lemma 3. *Higher first-period emissions reduces the second-period consumption of a price-setting consumer country, given the quantities set by other price-setting coun-*

tries:

$$\frac{\partial x_{2j}^{Dm}}{\partial x_1^{eq}} < 0$$

This reduction is bigger if more of the first-period emissions remain in the atmosphere in the second period, e.g. if the fraction δ is higher.

Proof. Differentiation of the consumer country first order condition, Equation (10), with respect to x_1^{eq}, keeping constant the quantities set by other price-setting countries, yields:

$$\frac{\partial x_{2j}^{Dm}}{\partial x_1^{\text{eq}}} = \frac{\delta H_{2j}'' \frac{\partial x_2^{\text{tot}}}{\partial x_2^D}}{W_{2j}''} < 0$$

It is evident that consumption is reduced more if δ is larger. □

A marginal increase in first-period emissions increases the second period stock of greenhouse gases by a fraction δ, which increases the price-setting consumer countries' marginal harm from consumption of a given quantity of fossil fuels. Thus, marginal welfare from the consumption of a given quantity of fossil fuels is lower, and the price-setting consumer countries choose to consume less fossil fuels.

The effect of first-period emissions on second-period emissions is of course stronger if more of the first-period emissions remain in the atmosphere in the second period.

Price-Setting Producer Countries

The price-setting producer countries do not take the price as given, only the quantities set by other price-setting countries. As shown, when the price-setting producer countries decide upon their production of fossil fuels, they know that higher production implies a lower price on their inframarginal units.

Thus, when choosing the optimal quantity, the price-setting producer countries must balance the profit from producing and selling another unit of fossil fuels against the loss from being paid a lower price for inframarginal units.

Furthermore, they have to take into account how their production of fossil fuels contribute to the stock of greenhouse gases, which they take harm from. The maximization problem for country l, a price-setting producer country, is then:

$$\max_{x_{2l}^S} \; p_{2l} x_{2l}^S - C_{2l}(x_{2l}^S) - H_{2l}\left(x_1^{\text{eq}} + x_2^{\text{tot}}(x_2^D, x_2^S)\right)$$

First order condition:

$$p_2 - C_{2l}' - x_{2l}^S \frac{dp_2}{dx_2^{\text{Dnet}}} - H_{2l}' \frac{\partial x_2^{\text{tot}}}{\partial x_2^S} = 0 \tag{11}$$

This defines the second-period production of fossil fuels by producer country l as $x_{2j}^{Sm}\left(x_1^{\text{eq}} \mid x_2^D, x_{2,-l}^S\right)$, a function of the first-period quantity x_1^{eq}, given the quantities set by the other price-setting producer countries, $x_{2,-l}^S$, and the quantities set by the price-setting consumer countries, $x_2,^D$.

The two first terms represent the profit from a marginal increase in production, while the third term represents the loss from a lower price on inframarginal units.

The last term represents the harm from the increase in the stock of greenhouse gases that this marginal increase in production leads to. As shown, emissions increase less than one-to-one with the increase in country l's production, since the price-taking producer countries reduce their production when the price is reduced in order to clear the market.

Furthermore, the production of the price-setting producer countries depends on first-period emissions, since these contribute to the second period stock of greenhouse gases. The following lemma establishes the effect of higher first-period emissions on the production of a price-taking producer country, given the quantities set by other price-setting countries.

For notational simplicity, let

$$W_{2l}(x_{2l}^S) \equiv p_{2l}x_{2l}^S - C_{2l}(x_{2l}^S) - H_{2l}\left(x_1^{\text{eq}} + x_2^{\text{tot}}(x_2^D, x_2^S)\right)$$

Then, W_{2l}' and W_{2l}'' refer to the first and second derivative, respectively, of producer country l's objective function.

Note that it is assumed that the second order condition for a maximum holds, e.g. that $W_{2l}'' < 0$ for country l.

Lemma 4. *Higher first-period emissions reduces the second-period production of a price-setting producer country, given the quantities set by other price-setting countries:*

$$\frac{\partial x_{2j}^{Sm}}{\partial x_1^{eq}} < 0$$

This reduction is bigger if more of the first-period emissions remain in the atmosphere in the second period, e.g. if the fraction δ is higher.

Proof. Differentiation of the producer country first order condition, Equation (11), with respect to x_1^{eq}, keeping constant the quantities set by other price-setting countries, yields:

$$\frac{\partial x_{2j}^{Sm}}{\partial x_1^{\text{eq}}} = \frac{\delta H_{2j}'' \frac{\partial x_2^{\text{tot}}}{\partial x_2^S}}{W_{2j}''} < 0$$

It is evident that production is reduced more if δ is larger. □

A marginal increase in first-period emissions increases the second period stock of greenhouse gases by a fraction δ, which increases the price-setting producer countries' marginal harm from production of a given quantity of fossil fuels. Thus, marginal welfare from the production of a given quantity of fossil fuels is lower, and the price-setting producer countries choose to produce less fossil fuels.

The effect of first-period emissions on second-period emissions is of course stronger if more of the first-period emissions remain in the atmosphere in the second period.

The Second-Period Equilibrium Price

The market clearing condition defined the price that clears the market as a function of net demand from the price-setting countries. If net demand from the price-setting countries are given by their optimal quantities, the market clearing condition defines the second-period equilibrium price, p_2^{eq}. The market clearing condition, with optimal quantities set by the price-setters:

$$\sum_j x_{2j}^{Dm}\left(x_1^{\text{eq}} \mid x_{2,-j}^D, x_2^S\right) + x_2^{Dp}(p_2^{\text{eq}}) = \sum_l x_{2l}^{Sm}\left(x_1^{\text{eq}} \mid x_2^D, x_{2,-l}^S\right) + x_2^{Sp}(p_2^{\text{eq}}) \tag{12}$$

This defines $p_2^{\text{eq}}(x_1^{\text{eq}})$, the equilibrium price as a function of first-period emissions, x_1^{eq}.

The next lemma concerns the effect of higher first-period emissions on the second-period price. Since the price is a function of net demand from the price-setting countries, x_2^{Dnet}, the change in the price due higher first-period emissions is given by the change in x_2^{Dnet}. The change in x_2^{Dnet} depends on the partial reductions in consumption and production due to higher first-period emissions, dampened or propagated by second order effects.

The partial reduction in consumption due to a marginal increase in first-period emissions is given by the sum of partial reductions by price-setting consumer countries:

$$\sum_j \frac{\partial x_{2j}^{Dm}}{\partial x_1^{\text{eq}}}$$

A reduction in consumption by a price-setting consumer country would make other price-setting consumer countries increase their consumption, while price-setting producer countries would reduce their production. Thus, the change in x_2^{Dnet} due to the partial reduction in consumption is given by:

$$\sum_j \frac{\partial x_{2j}^{Dm}}{\partial x_1^{\text{eq}}}\Bigg[1 + \underbrace{\sum_{u\neq j} \frac{\partial x_{2u}^{Dm}}{\partial x_{2,-u}^D} + \sum_l \frac{\partial x_{2l}^{Sm}}{\partial x_2^D}}_{\equiv\, \psi_j^D}\Bigg] = \sum_j \frac{\partial x_{2j}^{Dm}}{\partial x_1^{\text{eq}}}[1 + \psi_j^D]$$

The partial reduction in production due to a marginal increase in first-period emissions is given by the sum of partial reductions by price-setting production countries:

$$\sum_l \frac{\partial x_{2l}^{Sm}}{\partial x_1^{\mathrm{eq}}}$$

A reduction in production by a price-setting producer country would make other price-setting producer countries increase their production, while price-setting consumer countries would reduce their consumption. Thus, the change in x_2^{Dnet} due to the partial reduction in production is given by:

$$-\sum_l \frac{\partial x_{2l}^{Sm}}{\partial x_1^{\mathrm{eq}}}\Bigg[1+\underbrace{\sum_j \frac{\partial x_{2j}^{Dm}}{\partial x_2^S}+\sum_{v\neq l}\frac{\partial x_{2v}^{Sm}}{\partial x_{2,-v}^S}}_{\equiv\, \psi_l^S}\Bigg] = -\sum_l \frac{\partial x_{2l}^{Sm}}{\partial x_1^{\mathrm{eq}}}[1+\psi_l^S]$$

Lemma 5. *A marginal increase in first-period emissions makes the second-period equilibrium price decrease (increase) if the reduction in x_2^{Dnet} due to the partial reduction in consumption, is bigger (smaller) than the increase in x_2^{Dnet} due to the partial reduction in production:*

$$\frac{dp_2^{eq}}{dx_1} < 0 \quad \textit{if} \quad \left|\sum_j \frac{\partial x_{2j}^{Dm}}{\partial x_1^{eq}}(1+\psi_j^D)\right| > \left|\sum_l \frac{\partial x_{2l}^{Sm}}{\partial x_1^{eq}}(1+\psi_l^S)\right|$$

$$\frac{dp_2^{eq}}{dx_1} < 0 \quad \textit{if} \quad \left|\sum_j \frac{\partial x_{2j}^{Dm}}{\partial x_1^{eq}}(1+\psi_j^D)\right| < \left|\sum_l \frac{\partial x_{2l}^{Sm}}{\partial x_1^{eq}}(1+\psi_l^S)\right|$$

Proof. This follows from differentiation of the second-period equilibrium condition, Equation (9), with respect to x_1. Differentiation yields:

$$\frac{dp_2^{\mathrm{eq}}}{dx_1} = \frac{dp_2^{\mathrm{eq}}}{dx_2^{\mathrm{Dnet}}}\left(\sum_j \frac{\partial x_{2j}^{Dm}}{\partial x_1^{\mathrm{eq}}}[1+\psi_j^D] - \sum_l \frac{\partial x_{2l}^{Sm}}{\partial x_1^{\mathrm{eq}}}[1+\psi_l^S]\right)$$

The term in parenthesizes, the change in x_2^{Dnet}, is negative if the reduction in net demand due to the partial reduction in consumption, is bigger than the increase in net demand due to the partial reduction in production, e.g.:

$$\left|\sum_j \frac{\partial x_{2j}^{Dm}}{\partial x_1^{\mathrm{eq}}}(1+\psi_j^D)\right| > \left|\sum_l \frac{\partial x_{2l}^{Sm}}{\partial x_1^{\mathrm{eq}}}(1+\psi_l^S)\right|$$

The term in parenthesizes, the change in x_2^{Dnet}, is positive if the reduction in net demand due to the partial reduction in consumption, is smaller than the increase in net demand due to the partial reduction in production, e.g.:

$$\left|\sum_j \frac{\partial x_{2j}^{Dm}}{\partial x_1^{\mathrm{eq}}}(1+\psi_j^D)\right| < \left|\sum_l \frac{\partial x_{2l}^{Sm}}{\partial x_1^{\mathrm{eq}}}(1+\psi_l^S)\right|$$

$\square$

Lastly, the second-period equilibrium quantity of fossil fuels produced and consumed is given by x_2^{eq}, which is a function of first-period emissions, x_1^{eq}. It follows from the analysis above that the second-period quantity of fossil fuels is reduced if first-period emissions increase:

$$\frac{dx_2^{\text{eq}}}{dx_1^{\text{eq}}} < 0$$

4.2 First-Period Optimization

In the first period, each price-taking consumer country maximizes welfare given the price, p_1. For each consumer country, this defines demand as a function of the price. Aggregate demand from the price-taking consumer countries is given by the sum of demand from all N_D^p of these countries. Thus, aggregate demand is also a function of the price.

Similarly, each price-taking producer country maximizes welfare given the price, p_1. For each producer country, this defines supply as a function of the price. Aggregate supply from the price-taking producer countries is given by the sum of supply from all N_S^p of these countries. Thus, aggregate supply is also a function of the price.

The price that clears the market is the one that makes net supply from the $N_D^p + N_S^p$ price-taking countries equal to net demand from the $N_D^m + N_S^m$ price-setting countries.

Thus, when the price-setting countries maximize their welfare, they know which price they will have to pay, or will receive, depending on their chosen quantity, given the quantities set by the other price-setting countries.

Each price-setting country take into account how first period consumption of fossil fuels affect their own welfare in the second period.

Price-Taking Consumer Countries

Consumer country i chooses its consumption of fossil fuels, x_{1i}^{Dp}, to maximize welfare. The price, p_1, is taken as given. The maximization problem:

$$\max_{x_{1i}^{Dp}} \; U_{1i}(x_{1i}^{Dp}) - p_1 x_{1i}^{Dp}$$

First order condition:

$$U'_{1i}(x_{1i}^{Dp}) - p_1 = 0 \tag{13}$$

This defines $x_{1i}^{Dp}(p_1)$, country i's demand for fossil fuels as a function of the price. At the optimum, marginal utility from the consumption of fossil fuels is equal to the price.

The aggregate demand function of the price-taking consumer countries is $x_1^{Dp}(p_1) \equiv \sum_i x_{1i}^{Dp}(p_1)$.

Price-Taking Producer Countries

Consumer country k chooses its production of fossil fuels, x_{1k}^{Sp}, to maximize welfare. The price, p_1, is taken as given. The maximization problem:

$$\max_{x_{1k}^{Sp}} \; p_{1k} x_{1k}^{Sp} - C_{1k}(x_{1k}^{Sp})$$

First order condition:

$$p_1 - C'_{1k}(x_{1k}^{Sp}) = 0 \tag{14}$$

This defines $x_{1k}^{Sp}(p_1)$, country k's supply of fossil fuels as a function of the price. At the optimum, the marginal cost of production of fossil fuels is equal to the price.

The aggregate supply function of the price-taking producer countries is $x_1^{Sp}(p_1) \equiv \sum_k x_{1k}^{Sp}(p_1)$.

Market Clearing

The price that clears the market is the one that makes net supply from the $N_D^p + N_S^p$ price-taking countries, $x_1^{Sp}(p_1) + x_1^{Dp}(p_1)$, equal to net demand from the $N_D^m + N_S^m$ price-setting countries. Let demand from the price-setting countries be given by $x_1^D \equiv \sum_j x_{1j}^D$, and supply by $x_1^S \equiv \sum_l x_{1l}^S$. The market clearing condition:

$$x_1^D + x_1^{Dp}(p_1) = x_1^S + x_1^{Sp}(p_1) \tag{15}$$

Define net demand from the price-setting countries as $x_1^{\text{Dnet}} \equiv x_1^D - x_1^S$. Then, the market clearing condition defines $p_1(x_1^{\text{Dnet}})$, the market clearing price as a function of net demand from the price-setting countries. This is better seen from a reformulation of the market clearing condition:

$$x_1^S(p_1) - x_1^{Dp}(p_1) = x_1^D - x_1^S \equiv x_1^{\text{Dnet}}$$

The market clearing price must increase if net demand from the price-setting countries increase, since a higher price is needed for the price-taking consumer countries to consume less, and the price-taking producer countries to produce more. This follows from differentiation of the market clearing condition, Equation (15), with respect to x_1^{Dnet}:

$$\frac{dp_1}{dx_1^{\text{Dnet}}} = \frac{1}{\frac{dx_1^S}{dp_1} - \frac{dx_1^{Dp}}{dp_1}} > 0$$

Thus, when the price-setting consumer countries decide upon their consumption of fossil fuels, they know that higher consumption implies a higher price on their inframarginal units. Similarly, the price-setting producer countries know that higher production implies a lower price on their inframarginal units.

Next, it is derived how the second-period quantity of fossil fuels is affected by a marginal increase in consumption or production by the price-setting countries, when the quantities set by other price-setting countries are held constant.

It is clear that given a set of quantities (x_1^D, x_1^S) for the price-setting countries, the market clearing price will adjust to equate consumption with production. Thus, the total second-period consumption and production of fossil fuels can be written as a function $x_1^{\text{tot}}(x_1^D, x_1^S) \equiv x_1^D + x_1^{Dp}\left(p_1(x_1^{\text{Dnet}})\right) = x_1^S + x_1^{Sp}\left(p_1(x_1^{\text{Dnet}})\right)$.

It then follows that the partial effect on total consumption and production, of higher consumption by the price-setting consumer countries, can by found by partial differentiation of $x_t^{\text{tot}}(x_1^D, x_1^S)$:

$$\frac{\partial x_1^{\text{tot}}(x_1^D, x_1^S)}{\partial x_1^D} = 1 + \frac{\partial x_1^{Dp}}{\partial p_1}\frac{dp_1}{dx_1^{\text{Dnet}}} = \frac{\partial x_1^{Sp}}{\partial p_1}\frac{dp_1}{dx_1^{\text{Dnet}}} > 0$$

Unsurprisingly, total consumption and production increases when consumption by the price-setting countries increase. But the increase is not one-to-one, since the price-taking consumers reduce their consumption.

And similarly, regarding the partial effect on total consumption and production, of higher production by the price-setting producer countries:

$$\frac{\partial x_1^{\text{tot}}(x_1^D, x_1^S)}{\partial x_1^S} = 1 - \frac{\partial x_1^{Sp}}{\partial p_1}\frac{dp_1}{dx_1^{\text{Dnet}}} = -\frac{\partial x_1^{Dp}}{\partial p_1}\frac{dp_1}{dx_1^{\text{Dnet}}} > 0$$

Total consumption and production increases when production by the price-setting countries increase. But the increase is not one-to-one, since the price-taking producers reduce their production.

Price-Setting Consumer Countries

The price-setting consumer countries do not take the price as given, only the quantities set by other price-setting countries. As shown, when the price-setting consumer countries decide upon their consumption of fossil fuels, they know that higher consumption implies a higher price on their inframarginal units.

Thus, when choosing the optimal quantity, the price-setting consumer countries must balance the net utility from another unit of fossil fuels against the loss from having to pay a higher price on inframarginal units.

Furthermore, they have to take into account how their consumption of fossil fuels contribute to the stock of greenhouse gases, which they take harm from.

In the first period, the price-setting consumer countries must also take into account how first period emissions affect second period welfare, which is discounted at a rate β.

The maximization problem for country j, a price-setting consumer country, with argument omitted for $x_1^{\text{tot}}(x_1^D, x_1^S)$ to simplify notation, is then:

$$\max_{x_{1j}^D} \; U_{1j}(x_{1j}^D) - p_1(x_1^{\text{Dnet}})x_{1j}^D - H_{1j}\left(x_1^{\text{tot}}\right) \\ + \beta\left[U_{2j}(x_{2j}^{Dm}) - p_2^{\text{eq}}(x_1^{\text{tot}})x_{2j}^{Dm} - H_{2j}\left(\delta x_1^{\text{tot}} + x_2^{\text{eq}}(x_1^{\text{tot}})\right)\right]$$

First order condition:

$$U'_{1j} - p_1 - x_{1j}^D \frac{dp_1}{dx_1^{\text{Dnet}}} - H'_{1j}\frac{\partial x_1^{\text{tot}}}{\partial x_1^D} \\ + \beta\left[(U'_{2j} - p_2)\frac{dx_{2j}^{Dm}}{dx_1^{\text{eq}}} - x_{2j}^{Dm}\frac{dp_2^{\text{eq}}}{dx_1^{\text{eq}}} - H'_{2j}\left(\delta + \frac{dx_2^{\text{eq}}}{dx_1^{\text{eq}}}\right)\right]\frac{\partial x_1^{\text{tot}}}{\partial x_1^D} = 0$$

The first four terms of the first order condition concern the first period welfare from the consumption of fossil fuels, and have the same interpretation as their second period counterparts.

The term beginning with β represents the effect of higher first-period emissions on second period welfare. However, the producer country knows that for any level of first-period emissions, it will choose the optimal quantity in the second period. Thus, a marginal change in its own consumption does not affect welfare, since marginal welfare is zero at the optimum. This is given by the second-period first order condition for a price-setting consumer country, Equation (10), which can be used to simplify the second-period first order condition:

$$U'_{1j} - p_1 - x_{1j}^D \frac{dp_1}{dx_1^{\text{Dnet}}} - H'_{1j}\frac{\partial x_1^{\text{tot}}}{\partial x_1^D} - \beta\left[x_{2j}^{Dm}\frac{dp_{2,-j}^{\text{eq}}}{dx_1^{\text{eq}}} + H'_{2j}\left(\delta + \frac{dx_{2,-j}^{\text{eq}}}{dx_1^{\text{eq}}}\right)\right]\frac{\partial x_1^{\text{tot}}}{\partial x_1^D} = 0 \tag{16}$$

Where $\frac{dx_{2,-j}^{\text{eq}}}{dx_1^{\text{eq}}} \equiv \frac{dx_2^{\text{eq}}}{dx_1^{\text{eq}}} - \frac{\partial x_2^{\text{tot}}}{\partial x_2^D}\frac{dx_{2j}^{Dm}}{dx_1^{\text{eq}}}$ and $\frac{dp_{2,-j}^{\text{eq}}}{dx_1^{\text{eq}}} \equiv \frac{dp_2^{\text{eq}}}{dx_1^{\text{eq}}} - \frac{\partial p_2}{\partial x_2^{Dnet}}\frac{dx_{2j}^{Dm}}{dx_1^{\text{eq}}}$.

This defines the optimal first period consumption of fossil fuels by consumer country j as $x_{1j}^{Dm}\left(x_{1,-j}^D, x_1^S\right)$, a function of the quantities set by the other price-setting consumer countries, $x_{1,-j}^D$, and the quantities set by the price-setting producer countries, x_1^S.

Evidently, a price-setting consumer country cares about how its consumption in the first period contributes to a higher stock of greenhouse gases because itself takes harm from climate change.

But in addition, such a country does also take into account how first-period emissions affects the price it has to pay for inframarginal units in the second period. Note that it is only the price change due to the policies of the *other* countries that matter, e.g. the demand-side policies of the other consumer countries, and the supply-side policies of the producer countries.

The following proposition establishes how the first-period consumption of a price-setting consumer country depends on the climate policies of other countries:

Proposition 6. *Price-setting consumer countries increase (reduce) first-period consumption if the second-period response of other countries to higher first-period emissions has a negative (positive) effect on the second-period price.*

Proof. This follows from the first order condition, Equation (16), and Lemma 5 and its proof. □

Note that it could very well be the case that higher first-period emissions led to a lower second-period price, but that there nevertheless were price-setting consumer countries which reduced their consumption due to the effect stated in Proposition 6. This is because each country only cares about the policies of other countries.

The demand-side policy of a large consumer country that cares strongly about the climate can cause the actual effect of higher emissions on the price to be negative, as the same time as this country reduces its first period consumption because the effect on the price due to the policies of other countries is negative.

Price-Setting Producer Countries

The price-setting producer countries do not take the price as given, only the quantities set by other price-setting countries. As shown, when the price-setting producer countries decide upon their production of fossil fuels, they know that higher production implies a lower price on their inframarginal units.

Thus, when choosing the optimal quantity, the price-setting producer countries must balance the net utility from another unit of fossil fuels against the loss from having to pay a lower price on inframarginal units.

Furthermore, they have to take into account how their production of fossil fuels contribute to the stock of greenhouse gases, which they take harm from.

In the first period, the price-setting producer countries must also take into account how first period emissions affect second period welfare, which is discounted at a rate β.

The maximization problem for country l, a price-setting producer country, with arguments omitted for $x_1^{\text{tot}}(x_1^D, x_1^S)$ to simplify notation, is then:

$$\begin{aligned}\max_{x_{1l}^S} \; & p_1(x_1^{\text{Dnet}})x_{1l}^S - C_{1l}(x_{1l}^S) - H_{1l}\left(x_1^{\text{tot}}\right) \\ & + \beta\left[p_2^{\text{eq}}(x_1^{\text{tot}})x_{2l}^{Sm} - C_{2l}(x_{2l}^{Sm}) - H_{2l}\left(\delta x_1^{\text{tot}} + x_2^{\text{eq}}(x_1^{\text{tot}})\right)\right]\end{aligned}$$

First order condition:

$$\begin{aligned} & p_1 - C_{1l}' - x_{1l}^S \frac{dp_1}{dx_1^{\text{Dnet}}} - H_{1l}' \frac{\partial x_1^{\text{tot}}}{\partial x_1^S} \\ & + \beta\left[(p_2 - C_{2l}')\frac{dx_{2l}^{Sm}}{dx_1^{\text{eq}}} + x_{2l}^{Sm}\frac{dp_2^{\text{eq}}}{dx_1^{\text{eq}}} - H_{2l}'\left(\delta + \frac{dx_2^{\text{eq}}}{dx_1^{\text{eq}}}\right)\right]\frac{\partial x_1^{\text{tot}}}{\partial x_1^D} = 0\end{aligned}$$

The first four terms of the first order condition concern the first period welfare from the production of fossil fuels, and have the same interpretation as their second period counterparts.

The term beginning with β represents the effect of higher first-period emissions on second period welfare. However, the producer country knows that for any level of first-period emissions, it will choose the optimal quantity in the second period. Thus, a marginal change in its own production does not affect welfare, since marginal welfare is zero at the optimum. This is given by the second-period first order condition for a price-setting producer country, Equation (11), which can be used to simplify the second-period first order condition:

$$p_1 - C'_{1l} - x^S_{1l}\frac{dp_1}{dx_1^{\text{Dnet}}} - H'_{1l}\frac{\partial x_1^{\text{tot}}}{\partial x_1^S} + \beta\left[x^{Sm}_{2l}\frac{dp^{\text{eq}}_{2,-l}}{dx_1^{\text{eq}}} - H'_{2l}\left(\delta + \frac{dp^{\text{eq}}_{2,-l}}{dx_1^{\text{eq}}}\right)\right]\frac{\partial x_1^{\text{tot}}}{\partial x_1^D} = 0 \tag{17}$$

Where $\frac{dx^{\text{eq}}_{2,-l}}{dx^{\text{eq}}_1} \equiv \frac{dx^{\text{eq}}_2}{dx^{\text{eq}}_1} - \frac{\partial x^{\text{tot}}_2}{\partial x^S_2}\frac{dx^{Sm}_{2l}}{dx^{\text{eq}}_1}$ and $\frac{dp^{\text{eq}}_{2,-l}}{dx^{\text{eq}}_1} \equiv \frac{dp^{\text{eq}}_2}{dx^{\text{eq}}_1} - \frac{\partial p_2}{\partial x_2^{Dnet}}\frac{dx^{Sm}_{2l}}{dx^{\text{eq}}_1}$.

This defines the first period production of fossil fuels by producer country l as $x^{Sm}_{1l}\left(x_1^D, x^S_{1,-l}\right)$, a function of the quantities set by the other price-setting producer countries, $x^S_{1,-l}$, and the quantities set by the price-setting consumer countries, $x_1,^D$.

Evidently, a price-setting producer country cares about how its production in the first period contributes to a higher stock of greenhouse gases because itself takes harm from climate change.

But in addition, such a country does also take into account how first-period emissions affects the price it will receive for inframarginal units in the second period. Note that it is only the price change due to the policies of the *other* countries that matters, e.g. the supply-side policies of the other producer countries, and the demand-side policies of the producer countries.

The following proposition establishes how the first-period production of a price-setting producer country depends on the climate policies of other countries:

Proposition 7. *Price-setting producer countries reduce (increase) first-period production if the second-period response of other countries to higher first-period emissions has a negative (positive) effect on the second-period price.*

Proof. This follows from the first order condition, Equation (17), and Lemma 5 and its proof. □

Note that it could very well be the case that higher first-period emissions led to a higher second-period price, but that there nevertheless were price-setting producer countries which reduced their production due to the effect stated in Proposition 7. This is because each country only cares about the policies of other countries.

The supply-side policy of a large producer country that cares strongly about the climate can cause the actual effect of higher emissions on the price to be negative, as the same time as this country reduces its first period consumption because the effect on the price due to the policies of other countries is negative.

5 Refinements and Possible Extensions

First, ideas for further work on the models of the thesis are presented. Then, a possible extension which allows for countries which combine demand- and supply-side policies is discussed. Finally, I draw up questions that could be answered through a signaling model inspired by the results of my thesis, with private information about the harm from the stock of greenhouse gases.

5.1 Refinement of the Current Models

With regard to the first model, presented in Section 3, a first task would be to rebuild the model around a producer country that sets the price, not the quantity. This would remove the need for the somewhat unsatisfactory approximation that had to be done to solve the model with the current setup. This new setup would likely yield similar results, but in a more consistent framework.

Next, with the assumptions made about quadratic functions in the comparative statics part, it should be possible to actually solve the model for x_1 and x_2, and express these variables in terms of exogenous parameters. Then, the conditions in Propositions 4 and 5 could expressed in terms of exogenous parameters. It would then also be possible to determine whether the effects of these propositions are likely to be of one sign or another, given reasonable parameter values.

Moreover, comparative statics should be carried out without the simplifying assumption that consumer countries put zero weight on future welfare, e.g. $\beta_i = 0 \; \forall \; i$ should be dropped.

The second model, presented in Section 4, is probably the one with the greatest potential. To my knowledge, this perspective is not investigated in the literature on climate policies. A first task with respect to this model would be to make a version with quadratic functions, and attempt to solve the model for all the endogenous variables, in terms of exogenous parameters. Then, the conditions in Lemma 5 could be expressed in terms of parameters. Also, the precise conditions for the effect of higher first-period emissions on the second-period price, as seen by a producer or consumer country that does not take into account its own second-period policy, could be precisely formulated. This would allow for a fruitful discussion of the interaction between the number of countries in each group and their harm functions, among other things.

The special case with identical countries in each group should also be explored. This should allow for a particularly stark presentation of the results.

5.2 Countries Combining Policies

An extension of the second model should investigate the role of countries who both consume and produce fossil fuels, and combine demand- and supply-side policies as shown by Hoel (1994). Two questions have to be answered: How does such a country act strategically to affect the future decisions of other countries which use either demand- or supply-side policies? And how does other countries act strategically to affect the future decisions of a country that combines demand- and supply-side policies?

The answer to the first question is likely that qualitatively, the first-period actions of such a country will be similar to those of a pure consumer or producer country, depending on whether the country that combines policies is a net importer or exporter. But this should be formally established. Sketching an answer to the second question is not straightforward, but this is my hypothesis: The terms-of-trade effects on the quantity set by such a country depends only on net imports or exports, and are not directly affected by an increase in first-period emissions. Apart from terms-of-trade effects, the composition of demand- and supply-side policies is chosen to minimize carbon leakage. My hypothesis is that if first-period emissions increase marginally, the reduce second-period consumption and production in such a way that the effect on the second-period price is neutral. To summarize: Countries that combine policies would themselves act strategically in similar ways as described in this thesis, but their climate policies would not motivate such strategic actions from other countries.

5.3 Asymmetric Information

If each country's harm from emissions is private information, the results of the thesis motivate an investigation of countries' incentives to signal lower/higher harm from emissions, given a preferred type of policy.

A country's incentives to signal lower/higher harm from emissions, given a preferred type of policy, could be investigated in a two-period model where first-period emissions affect second-period behavior through the stock of greenhouse gases, and where countries base their beliefs about the second-period policies of other countries upon the behavior they observe in the first period.

Label the country in question i, and let it be of type A, while the other type is named B. Two questions would have to be resolved: 1. How would it affect the other countries' first-period behavior if country i could successfully signal lower/higher harm? 2. How would country i be affected by these responses?

With respect to other countries of type A, the conclusion seems relatively straightforward: Country i would have incentive to signal lower harm, thereby reducing the amount of free-riding from other countries of type A, which otherwise would free-ride both on country i's future emissions reductions an the favorable effect of country i's future climate policies would have on the second-period price. Country i would benefit directly from lower first-period emissions from the other countries of type A, and indirectly through the favorable effect on the first-period price.

With respect to countries of type B, it seems to be more complicated. If country i signaled lower harm, lower expected future emissions reductions due to country

i's policies would make countries of type B reduce first-period emissions. At the same time, if the countries of type B believed that country i's harm from emissions was lower, they would not have to worry so much about higher first-period emissions having an unfavorable effect on the second-period price through the policy of country i.

Next, whether this ends with a reduction or increase in first-period emissions from countries of type B, it is not clear which of these country i would prefer. A reduction in emissions would be beneficial, while the unfavorable effect the climate policy of countries of type B would have on the first-period price would be costly.

Thus, the answer to the first question asked would seem to depend upon the countries of type B's valuation of second-period emissions reductions versus unfavorable effects on the second-period price, while the answer to the second question would seem to depend on country i's valuation of first-period emissions reductions versus unfavorable effects on the first-period price.

6 Concluding Remarks

In this thesis, I have studied the strategic provision of a public good, when this good can be provided in different ways, and contributors to the public good have different preferences over the ways through which it can be provided. The public good in question is reduced emissions of greenhouse gases, which can be achieved both through demand- and supply-side climate policies.

It has been shown that countries that would benefit from a lower future price of fossil fuels emit more if the future climate policy response has a negative effect on the price, and less if the effect is positive. Similarly, countries that would benefit from a higher future price of fossil fuels emit less if the future climate policy response has a negative effect on the price, and more if the effect is positive.

The dynamic good problem among countries which prefer the same type of climate policy is aggravated when each country not only has incentive to free-ride on the future emissions reductions of the others, but also benefits from the favorable effect their climate policies have on the future price of fossil fuels.

However, the dynamic public good problem among countries which prefer different types of policies is alleviated, since the the type of policy not preferred would have an unfavorable effect on the future price of fossil fuels.

A next step would be to investigate how countries, and other agents, could take into account, and exploit, such strategic effects. In Section 4, a signaling game was proposed, with countries that prefer different climate policies, and have private information about their harm from greenhouse gases.

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A Appendix: Proofs for Section 3

Here, proofs of the results from the comparative statics section are provided. First, note that because of the quadratic utility and harm functions, $\frac{\partial p_2}{\partial x_1}$ depends only on parameters, and is thus unaffected by a change in x_1:

$$\frac{\partial p_2}{\partial x_1} = -\frac{\partial x_2^D}{\partial x_1}\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1} = -\underbrace{\sum_i \delta \frac{h_{2i}}{u_{2i} - h_{2i}}}_{=\frac{\partial x_2^D}{\partial x_1}<0} \Bigg(\underbrace{\sum_i \frac{1}{u_{2i} - h_{2i}}}_{=\frac{\partial x_2^D}{\partial p_2}<0}\Bigg)^{-1} < 0$$

Second, let W_1 and W_2 be given by:

$$\begin{aligned} W_1 &\equiv p_2(x_2^S \mid x_1)x_2^S - C_2(x_2^S) \\ W_2 &\equiv p_1(x_1^S)x_1^S - C_1(x_1^S) + \beta\left[p_2(x_2 \mid x_1)x_2 - C_2(x_2)\right] \end{aligned}$$

Then, the first order condition for the producer country's maximization problem in period t can be written as $W_t' = 0$, and the second order condition as $W_t'' < 0$.

A.1 Proof of Proposition 2

Differentiation of the first-period first order condition for the producer country, Equation (3), with respect to β yields:

$$(2p_1' + x_1p_1'' - C_1'')\frac{dx_1}{d\beta} + x_2\frac{\partial p_2}{\partial x_1} + \beta\left[\frac{dx_2}{d\beta}\frac{\partial p_2}{\partial x_1} + x_2 \underbrace{\frac{d}{d\beta}\left(\frac{\partial p_2}{\partial x_1}\right)}_{=0}\right] = 0 \tag{18}$$

The second of the bracketed terms in Equation (18) is zero, because $\frac{\partial p_2}{\partial x_1}$ is a constant which does not depend upon β.

$\frac{dx_2}{d\beta}$ can be written as shown below, since x_2 depends on β only through x_1:

$$\frac{dx_2}{d\beta} = \frac{dx_2}{dx_1}\frac{dx_1}{d\beta}$$

Then, Equation (18) can be rewritten as:

$$\Bigg(\underbrace{2p_1' + x_1p_1'' - C_1'' + \beta\frac{dx_2}{dx_1}\frac{\partial p_2}{\partial x_1}}_{=W_1''<0}\Bigg)\frac{dx_1}{d\beta} + x_2\frac{\partial p_2}{\partial x_1} = 0$$

This equation can then be solved for $\frac{dx_1}{d\beta}$:

$$\frac{dx_1}{d\beta} = x_2\underbrace{\frac{\partial p_2}{\partial x_1}}_{<0}\underbrace{\left(-W_1''\right)^{-1}}_{>0} < 0$$

Here, $W_1'' < 0$ by assumption, and it has been shown earlier that $\frac{\partial p_2}{\partial x_1} < 0$.

A.2 Proof of Proposition 3

Note that for this proposition it was assumed that the second period cost function, C_2, is quadratic, e.g. that $C_2 = \frac{1}{2}c_2(x_2^S)^2$, where $c_2 > 0$ is a parameter. Differentiation of the first-period first order condition for the producer country, Equation (3), with respect to c_2 yields:

$$(2p_1' + x_1 p_1'' - C_1'')\frac{dx_1}{dc_2} + \beta\left[\frac{dx_2}{dc_2}\frac{\partial p_2}{\partial x_1} + x_2 \underbrace{\frac{d}{dc_2}\left(\frac{\partial p_2}{\partial x_1}\right)}_{=\,0}\right] = 0 \tag{19}$$

The second of the bracketed terms in Equation (19) is zero, because $\frac{\partial p_2}{\partial x_1}$ is a constant which does not depend upon c_2.

$\frac{dx_2}{dc_2}$ can be expanded as shown below, since x_2 depends on c_2 both through its direct effect on second-period production, and via its effect on x_1:

$$\frac{dx_2}{dc_2} = \left.\frac{dx_2}{dc_2}\right|_{x_1} + \frac{dx_2}{dx_1}\frac{dx_1}{dc_2}$$

Using the expanded version of $\frac{dx_2}{dc_2}$, Equation (19) can be rewritten as:

$$\underbrace{\left(2p_1' + x_1 p_1'' - C_1'' + \beta\frac{dx_2}{dx_1}\frac{\partial p_2}{\partial x_1}\right)}_{=\,W_1''\,<\,0}\frac{dx_1}{dc_2} + \beta\left.\frac{dx_2}{dc_2}\right|_{x_1}\frac{\partial p_2}{\partial x_1} = 0$$

This equation can then be solved for $\frac{dx_1}{dc_2}$:

$$\frac{dx_1}{dc_2} = \beta\underbrace{\left.\frac{dx_2}{dc_2}\right|_{x_1}\frac{\partial p_2}{\partial x_1}}_{>\,0}\underbrace{\left(-W_1''\right)^{-1}}_{>\,0} > 0$$

Here, $W_1'' < 0$ by assumption. That $\frac{\partial p_2}{\partial x_1} < 0$ has been shown earlier. That the sign of $\left.\frac{dx_2}{dc_2}\right|_{x_1}$ is negative is shown by differentiation of the second-period first order condition for the producer country, Equation (3), with respect to c_2, keeping x_1 constant:

$$\left.\frac{dx_2}{dc_2}\right|_{x_1} = x_2\Bigg/\underbrace{\left(2\frac{\partial p_2}{\partial x_2^S} + x_2\frac{\partial^2 p_2}{\partial (x_2^S)^2} - c_2\right)}_{=\,W_2''\,<\,0} < 0 \tag{20}$$

A.3 Proof of Proposition 4

Differentiation of the first-period first order condition for the producer country, Equation (3), with respect to δ yields:

$$(2p_1' + x_1 p_1'' - C_1'')\frac{dx_1}{d\delta} + \beta\left[\frac{dx_2}{d\delta}\frac{\partial p_2}{\partial x_1} + x_2\frac{d}{d\delta}\left(\frac{\partial p_2}{\partial x_1}\right)\right] = 0 \tag{21}$$

Here, $\frac{dx_2}{d\delta}$ can be expanded as shown below, since x_2 depends on δ both through its direct effect on second period demand, and via its effect on x_1:

$$\frac{dx_2}{d\delta} = \left.\frac{dx_2}{d\delta}\right|_{x_1} + \frac{dx_2}{dx_1}\frac{dx_1}{d\delta}$$

Using the expanded version of $\frac{dx_2}{d\delta}$, Equation (21) can be rewritten as:

$$\Bigg(\underbrace{2p_1' + x_1 p_1'' - C_1'' + \beta\frac{dx_2}{dx_1}\frac{\partial p_2}{\partial x_1}}_{=\,W_1''\,<\,0}\Bigg)\frac{dx_1}{d\delta} + \beta\left[\left.\frac{dx_2}{d\delta}\right|_{x_1}\frac{\partial p_2}{\partial x_1} + x_2\frac{d}{d\delta}\left(\frac{\partial p_2}{\partial x_1}\right)\right] = 0$$

This equation can then be solved for $\frac{dx_1}{d\delta}$:

$$\frac{dx_1}{d\delta} = \left(\beta\Bigg[\underbrace{\left.\frac{dx_2}{d\delta}\right|_{x_1}\frac{\partial p_2}{\partial x_1}}_{\equiv\,A\,>\,0} + \underbrace{x_2\frac{d}{d\delta}\left(\frac{\partial p_2}{\partial x_1}\right)}_{\equiv\,B\,<\,0}\Bigg]\right)\underbrace{(-W_1'')^{-1}}_{>\,0}$$

Here, $W_1'' < 0$ by assumption. A and B represent the quantity and price effects, respectively, mentioned in the introduction to the proposition. In the following, it is shown how the signs of A and B are determined.

An increase in δ would affect the second-period equilibrium quantity, and thereby the loss from a higher price on inframarginal units. This is represented by A. That $\frac{\partial p_2}{\partial x_1} < 0$ has been shown earlier. That the sign of $\left.\frac{dx_2}{d\delta}\right|_{x_1}$ is negative is shown by differentiation of the second-period first order condition for the producer country, Equation (3), with respect to δ, keeping x_1 constant:

$$\left.\frac{dx_2}{d\delta}\right|_{x_1} = -\left(\underbrace{\left.\frac{dp_2}{d\delta}\right|_{x_1,x_2}}_{<\,0} + \underbrace{x_2\frac{d}{d\delta}\left.\left(\frac{\partial p_2}{\partial x_2^S}\right)\right|_{x_1,x_2}}_{=\,0}\right)\Bigg/\Bigg(\underbrace{2\frac{\partial p_2}{\partial x_2^S} + x_2\frac{\partial^2 p_2}{\partial (x_2^S)^2} - C_2''}_{=\,W_2''\,<\,0}\Bigg) < 0 \tag{22}$$

Here, $W_2'' < 0$ by assumption. That the first term of the numerator in Equation (22) is negative follows from differentiation of the second-period market clearing condition, Equation (2), with respect to δ, keeping x_1 and x_2 constant. This yields:

$$\left.\frac{dp_2}{d\delta}\right|_{x_1,x_2} = -\frac{\partial x_2^D}{\partial \delta}\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1} = -\sum_i \frac{h_{2i}x_1}{u_{2i}-h_{2i}}\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1} < 0$$

Where $\frac{\partial x_2^D}{\partial \delta}$ was found by differentiation of the second-period first order condition for a consumer country, Equation (1), with respect to δ.

The second term of the numerator in Equation (22) is zero, because $\frac{\partial p_2}{\partial x_2^S}$ is a constant which does not depend upon δ. This can be seen from differentiation of the second-period market clearing condition, Equation (2), with respect to x_2^S, which yields:

$$\frac{\partial p_2}{\partial x_2^S} = \left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1} = \left(\sum_i \frac{1}{u_{2i}-h_{2i}}\right)^{-1}$$

Thus, it has been shown that the sign of A is positive.

Next, it is shown that the sign of B is negative.

$$\frac{d}{d\delta}\left(\frac{\partial p_2}{\partial x_1}\right) = \frac{d}{d\delta}\left(\underbrace{\sum_i \delta\frac{h_{2i}}{u_{2i}-h_{2i}}}_{=\frac{\partial x_2^D}{\partial x_1}<0}\underbrace{\left(\sum_i \frac{1}{u_{2i}-h_{2i}}\right)^{-1}}_{=\frac{\partial x_2^D}{\partial p_2}<0}\right)$$

$$= -\sum_i^N \frac{h_{2i}}{u_{2i}-h_{2i}}\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1} < 0$$

Thus, it has been shown that the sign of B is negative.

At last, using the expressions found so far, the conditions under which $\frac{dx_1}{d\delta}$ is positive or negative are found. If it is positive, this is true:

$$\underbrace{\left.\frac{dx_2}{d\delta}\right|_{x_1}\frac{\partial p_2}{\partial x_1}}_{\equiv A>0} > \underbrace{-x_2\frac{d}{d\delta}\left(\frac{\partial p_2}{\partial x_1}\right)}_{\equiv B<0}$$

$$-\left(-\sum_i \frac{h_{2i}x_1}{u_{2i}-h_{2i}}\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1}\right)(W_2'')^{-1}\frac{\partial p_2}{\partial x_1} > -x_2\left(-\sum_i^N \frac{h_{2i}}{u_{2i}-h_{2i}}\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1}\right)$$

$$\sum_i \frac{h_{2i}x_1}{u_{2i}-h_{2i}}\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1}(W_2'')^{-1}\frac{\partial p_2}{\partial x_1} > x_2\sum_i^N \frac{h_{2i}}{u_{2i}-h_{2i}}\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1}$$

$$x_1\sum_i \frac{h_{2i}}{u_{2i}-h_{2i}}(W_2'')^{-1}\frac{\partial p_2}{\partial x_1} < x_2\sum_i^N \frac{h_{2i}}{u_{2i}-h_{2i}}$$

$$x_1(W_2'')^{-1}\frac{\partial p_2}{\partial x_1} > x_2$$

Thus:

$$\frac{dx_1}{d\delta} > 0 \text{ if } x_2 \; < \; x_1 (W_2'')^{-1} \frac{\partial p_2}{\partial x_1}$$

And vice versa:

$$\frac{dx_1}{d\delta} < 0 \text{ if } x_2 \; > \; x_1 (W_2'')^{-1} \frac{\partial p_2}{\partial x_1}$$

A.4 Proof of Proposition 5

Differentiation of the first-period first order condition for the producer country, Equation (3), with respect to h_{2i} yields:

$$(2p_1' + x_1 p_1'' - C_1'') \frac{dx_1}{dh_{2i}} + \beta \left[\frac{dx_2}{dh_{2i}} \frac{\partial p_2}{\partial x_1} + x_2 \frac{d}{dh_{2i}} \left(\frac{\partial p_2}{\partial x_1} \right) \right] = 0 \tag{23}$$

Each of the terms $\frac{dx_2}{dh_{2i}}$ and $\frac{d}{dh_{2i}}\left(\frac{\partial p_2}{\partial x_1}\right)$ can be expanded. $\frac{dx_2}{dh_{2i}}$ can be expanded as shown below, since x_2 depends on h_{2i} both through its direct effect on second period demand, and via its effect on x_1:

$$\frac{dx_2}{dh_{2i}} = \left.\frac{dx_2}{dh_{2i}}\right|_{x_1} + \frac{dx_2}{dx_1} \frac{dx_1}{dh_{2i}}$$

Note that because of the quadratic utility and harm functions, $\frac{\partial p_2}{\partial x_1}$ depends only on parameters, and is unaffected by a change in x_1. This is the reason why $\frac{d}{dh_{2i}}\left(\frac{\partial p_2}{\partial x_1}\right)$ is not expanded in a similar way.

Using the expanded version of $\frac{dx_2}{dh_{2i}}$, Equation (23) can be rewritten as:

$$\Bigg(\underbrace{2p_1' + x_1 p_1'' - C_1'' + \beta \frac{dx_2}{dx_1} \frac{\partial p_2}{\partial x_1}}_{= W_1'' < 0} \Bigg) \frac{dx_1}{dh_{2i}} + \beta \left[\left.\frac{dx_2}{dh_{2i}}\right|_{x_1} \frac{\partial p_2}{\partial x_1} + x_2 \frac{d}{dh_{2i}} \left(\frac{\partial p_2}{\partial x_1} \right) \right] = 0$$

This equation can then be solved for $\frac{dx_1}{dh_{2i}}$:

$$\frac{dx_1}{dh_{2i}} = \left(\beta \Bigg[\underbrace{\left.\frac{dx_2}{dh_{2i}}\right|_{x_1} \frac{\partial p_2}{\partial x_1}}_{\equiv C > 0} + \underbrace{x_2 \frac{d}{dh_{2i}} \left(\frac{\partial p_2}{\partial x_1} \right)}_{\equiv D < 0} \Bigg] \right) \underbrace{\left(-W_1''\right)^{-1}}_{> 0}$$

Here, $W_1'' < 0$ by assumption. C and D represent the quantity and price effects, respectively, mentioned in the introduction to the proposition. In the following, it is shown how the signs of C and D are determined.

An increase in h_{2i} would affect the second-period equilibrium quantity, and thereby the loss from a higher price on inframarginal units. This is represented by C. That $\frac{\partial p_2}{\partial x_1} < 0$ has been shown earlier. That the sign of $\frac{dx_2}{dh_{2i}}\Big|_{x_1}$ is negative is shown by differentiation of the second-period first order condition for the producer country, Equation (3), with respect to h_{2i}, keeping x_1 constant:

$$\frac{dx_2}{dh_{2i}}\Big|_{x_1} = -\Bigg(\underbrace{\frac{dp_2}{dh_{2i}}\Big|_{x_1,x_2}}_{<\,0} + x_2 \underbrace{\frac{d}{dh_{2i}}\left(\frac{\partial p_2}{\partial x_2^S}\right)\Big|_{x_1,x_2}}_{<\,0} \Bigg) \Bigg/ \Bigg(\underbrace{2\frac{\partial p_2}{\partial x_2^S} + x_2 \frac{\partial^2 p_2}{\partial (x_2^S)^2} - C_2''}_{=\,W_2''\,<\,0} \Bigg) < 0 \tag{24}$$

Here, $W_2'' < 0$ by assumption. That the first term of the numerator in Equation (24) is negative follows from differentiation of the second-period market clearing condition, Equation (2), with respect to h_{2i}, keeping x_1 and x_2 constant. This yields:

$$\frac{dp_2}{dh_{2i}}\Big|_{x_1,x_2} = -\frac{\partial x_2^D}{\partial h_{2i}} \left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1} = -\frac{\delta x_1 + x_2}{u_{2i} - h_{2i}} \left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1} < 0$$

Where $\frac{\partial x_2^D}{\partial h_{2i}}$ was found by differentiation of the second-period first order condition for consumer country i, Equation (1).

That the second term of the numerator in Equation (24) is negative, is seen from differentiation of $\frac{\partial p_2}{\partial x_2^S}$ with respect to h_{2i}, where the expression for $\frac{\partial p_2}{\partial x_2^S}$ is found by differentiation of the second-period market clearing condition, Equation (2), with respect to x_2^S:

$$\frac{d}{dh_{2i}}\left(\frac{\partial p_2}{\partial x_2^S}\right)\Big|_{x_1,x_2} = \frac{d}{dh_{2i}}\left(\frac{1-\sum_i^N h_{2i}\,(u_{2i}-h_{2i})^{-1}}{\sum_i (u_{2i}-h_{2i})^{-1}}\right)$$

$$= -\left[(u_{2i}-h_{2i})^{-1} + h_{2i}(-1)\,(u_{2i}-h_{2i})^{-2}\,(-1)\right]\left[\sum_i (u_{2i}-h_{2i})^{-1}\right]^{-1}$$

$$+\left[1-\sum_i^N h_{2i}\,(u_{2i}-h_{2i})^{-1}\right](-1)\left[\sum_i (u_{2i}-h_{2i})^{-1}\right]^{-2}(-1)\,(u_{2i}-h_{2i})^{-2}\,(-1)$$

$$= -\left[(u_{2i}-h_{2i})^{-1} + h_{2i}\,(u_{2i}-h_{2i})^{-2}\right]\underbrace{\left[\sum_i (u_{2i}-h_{2i})^{-1}\right]^{-1}}_{<\,0}$$

$$-\underbrace{\left[1-\sum_i^N h_{2i}\,(u_{2i}-h_{2i})^{-1}\right]}_{>\,0}\underbrace{\left[\sum_i (u_{2i}-h_{2i})^{-1}\right]^{-2}(u_{2i}-h_{2i})^{-2}}_{>\,0} < 0$$

Since

$$\left[(u_{2i}-h_{2i})^{-1} + h_{2i}\,(u_{2i}-h_{2i})^{-2}\right] = \frac{u_{2i}-h_{2i}+h_{2i}}{(u_{2i}-h_{2i})^2} = \frac{u_{2i}}{(u_{2i}-h_{2i})^2} < 0$$

Thus, it has been shown that the sign of C is positive.

Next, the sign of D is determined.

$$\frac{d}{dh_{2i}}\left(\frac{\partial p_2}{\partial x_1}\right) = \frac{d}{dh_{2i}}\left(\underbrace{\sum_i \delta \frac{h_{2i}}{u_{2i}-h_{2i}}}_{=\frac{\partial x_2^D}{\partial x_1}<0}\underbrace{\left(\sum_i \frac{1}{u_{2i}-h_{2i}}\right)^{-1}}_{=\frac{\partial x_2^D}{\partial p_2}<0}\right)$$

$$\begin{aligned}
&= -\left[\delta(u_{2i}-h_{2i})^{-1} + \delta h_{2i}\left(-(u_{2i}-h_{2i})^{-2}(-1)\right)\right]\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1} \\
&\quad - \frac{\partial x_2^D}{\partial x_1}\left[-\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-2}\left(-(u_{2i}-h_{2i})^{-2}(-1)\right)\right] \\
&= -\left[\delta(u_{2i}-h_{2i})^{-1} + \delta h_{2i}(u_{2i}-h_{2i})^{-1}\right]\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1} \\
&\quad + \frac{\partial x_2^D}{\partial x_1}\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-2}(u_{2i}-h_{2i})^{-2} \\
&= -\delta\left[\frac{u_{2i}}{(u_{2i}-h_{2i})^2}\right]\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-1} + \frac{\partial x_2^D}{\partial x_1}\left(\frac{\partial x_2^D}{\partial p_2}\right)^{-2}(u_{2i}-h_{2i})^{-2} \; < 0
\end{aligned}$$

Thus, it has been shown that the sign of D is negative.
It is thus clear that

$$\frac{dx_1}{dh_{2i}} > 0 \text{ if } \underbrace{\left.\frac{dx_2}{dh_{2i}}\right|_{x_1}\frac{\partial p_2}{\partial x_1}}_{\equiv\, C\, >\, 0} > -\,x_2\underbrace{\frac{d}{dh_{2i}}\left(\frac{\partial p_2}{\partial x_1}\right)}_{\equiv\, D\, <\, 0}$$

And vice versa:

$$\frac{dx_1}{dh_{2i}} < 0 \text{ if } \underbrace{\left.\frac{dx_2}{dh_{2i}}\right|_{x_1}\frac{\partial p_2}{\partial x_1}}_{\equiv\, C\, >\, 0} < -\,x_2\underbrace{\frac{d}{dh_{2i}}\left(\frac{\partial p_2}{\partial x_1}\right)}_{\equiv\, D\, <\, 0}$$