

# A Theoretical and Numerical Analysis of Collateralized Mortgage Obligations

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## **Abstract**

This thesis approaches securitization of mortgage loans. In particular, the foremost objective of the thesis is to conduct a theoretical examination of *Collateralized Mortgage Obligations* (CMOs). The analysis of mortgages and mortgage-related securities gets complicated due to uncertainty concerning the amount and timing of the prepayment element of cash flows from mortgages. This thesis therefore examines how prepayments are dealt with in the valuation of such securities. The thesis also conducts a numerical illustration of the state-of-the-art valuation methodology – the Monte Carlo simulation model.

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## 1. Foreword

Over the last few decades, we have seen a rapid growth in new investment vehicles within the market for credit derivatives and mortgage-related securities. This growth has unveiled the need and desire to trade and exchange different types of risk. The emergence of such securities is beneficial for both investors who want to get rid of risk and investors who want more exposure to these risks in their portfolios.

Throughout 2007, we saw how troubles in the American housing market spread to the credit markets and the stock markets, and created what we today know as the subprime mortgage crisis. We also saw that the troubles were transmitted to countries and regions in other parts of the world. During the fall of 2007, I understood that one of the elements that contributed to this spill over effect was a mortgage-related investment vehicle known as a CMO. This was a bond backed by a portfolio of mortgages. Through the master program in financial economics I had taken multiple derivatives and risk management courses, but I had never heard about this product. A closer investigation of the CMO, unveiled that there were challenges in the valuation caused by the prepayment options embedded in the mortgages underlying the CMOs. This short investigation triggered the thought of writing a thesis on CMOs.

When looking at the final product, it is apparent that the project has been more comprehensive than I initially thought. Especially, I have felt the need to examine the current theory more thoroughly than imagined.

Though the project has been comprehensive and challenging, going through the process has been inspiring, instructive and profitable. I would like to thank my advisor, Zexi Wang, for all feedback and advice throughout the process.

Finally, I would like to use the opportunity to thank my wife and other family, for all the love and support throughout this process and the years at NHH.



## 2. An Introduction to Collateralized Mortgage Obligations

According to Bodie et al. (2005:1048), a *Collateralized Mortgage Obligation* (CMO) can be defined as:

*A mortgage pass-through security that partitions cash flows from underlying mortgages into classes called tranches that receive principal payments according to stipulated rules.*

Initially, when first introduced in 1983 (Fernald et al. 1994:2), CMOs were a response to the risk inherited in funding mortgages. Since then, they have grown rapidly in popularity and today CMOs are regarded as one of the most innovative investment vehicles available in the market.

The enchantment of the security springs from the underlying opportunities in the security's structure: "The CMO's major financial innovation is that it provides for redirecting underlying cash flows in order to create securities that much more closely satisfy the asset/liability needs of institutional investors. As Wall Street likes to quickly point out, these securities are truly *custom design*" (Fabozzi & Ramsey 1999:1).

Through CMOs, investors can design a wide variety of securities with different cash flow and maturity characteristics. In this way, investors can meet specific investment objectives more accurately than earlier. Though CMOs clearly provide advantages, they quickly become quite complex and difficult to analyze. It is therefore important for an investor who is thinking of including CMOs in his portfolio, to first understand the distinctive features of these securities.

Early research on CMOs includes a research paper for the Federal Reserve Bank of New York by Julia Fernald, Frank Keane and Martin Mair (1994), and an article by John J. McConnell and Manoj Singh (1994) published in *The Journal of Finance*. The paper by Fernald et al. provides a description of common CMO structures and considers pricing, valuation, regulatory and risk assessment issues raised by this innovation. The paper withholds judgement on whether the market's evolution can be viewed as an advance. The article by McConnell and Singh presents a more detailed procedure for evaluating CMO tranches. The

solution procedure is in the spirit of a dynamic programming problem and includes a Monte Carlo simulation method.

Valuation of *mortgage backed securities* (MBSs) and CMOs consists of many moving parts, each one drawing on expertise in a different field. In a more recent work, Alexander L. Belikoff, Kirill Levin, Harvey J. Stein and Xusheng Tian (2007) at Bloomberg LP, details the different components, describing the approach taken by Bloomberg in each area. The paper puts particular emphasises on the new interest rate modelling component introduced for computing OAS, and the methods used to calibrate it accurately.

A scientist that has done a lot of research on *fixed income securities* is Frank J. Fabozzi. Together with Chuck Ramsey, he wrote maybe the most comprehensive book on the current theory of CMOs. In *Collateralized Mortgage Obligations: Structures and Analysis* (1999) they thoroughly look at the various types of CMOs, how and why they are created, how they should be valued, and how to quantify their exposure to changes in interest rates.

This thesis follows the book by Fabozzi and Ramsey to some extent. As opposed to Fabozzi and Ramsey, who look at the width of the theory, I have focused mainly on agency CMOs and on modelling of prepayments. This is done to see more clearly how CMOs differ from other securities backed by some pool of debt. Furthermore, since the CMO and its market is best developed in the United States, and the fact that most of the theory is written for the U.S. market, the thesis has been written with the U.S. market in mind. I also conduct a numerical illustration following the valuation methodology outlined by Fabozzi and Ramsey.

In chapter 3, I will present an overview of the *history* and *size* of the mortgage market. Thereafter, in chapter 4, I start building the theory of CMOs by presenting the collateral of CMOs, namely the *mortgage loan* and the creation of *mortgage-backed securities*. The element separating the CMO from other securities is *prepayments*. Therefore, in chapter 5, I look closer at the current theory for modelling prepayments. Next, I turn towards the creation of CMOs and present the *most common structures* in chapter 6. In chapter 7 I present the *state-of-the-art valuation methodology*, and finally I use this to conduct the *numerical illustration* of chapter 8. *Summary* and *concluding comments* in chapter 9, completes the thesis.

### 3. The Mortgage Market and its History

As already mentioned, CMOs were initially introduced as a response to the risk inherited in funding mortgages. Furthermore, since their introduction, CMOs have grown rapidly in popularity. CMOs are today regarded as one of the most innovative investment vehicles available in the market and have been described by Wall Street as being ‘truly custom design’. Though they the recent year have experienced some bad publicity due to the subprime mortgage crisis, the CMO will probably continue to grow in popularity.

In the following chapter, I will take a review of the history of the mortgage market. Before focusing on the theory underlying CMOs, I will also take a closer look at the size of the market.

#### 3.1 History

##### *3.1.1 The Origination of the Mortgage Loan*

An attempt to exactly date the first loan secured by some sort of collateral is not easy, if possible at all. A search for the history of the word *mortgage* resulted in the following:

*“The great jurist Sir Edward Coke, who lived from 1552 to 1634, has explained why the term mortgage comes from the Old French words mort, ‘dead’, and gage, ‘pledge’. It seemed to him that it had to do with the doubtfulness of whether or not the mortgagor will pay the debt. If the mortgagor does not, then the land pledged to the mortgagee as security for the debt ‘is taken from him for ever, and so dead to him upon condition, &c. And if he doth pay the money, then the pledge is dead as to the [mortgagee].’ This etymology, as understood by 17th-century attorneys, of the Old French term morgage, which we adopted, may well be correct. The term has been in English much longer than the 17th century, being first recorded in Middle English with the form morgage and the figurative sense ‘pledge’ in a work written before 1393” (<http://dictionary.reference.com/browse/mortgage>).*

Apparently, the procedure of securing debt by means of land or other property has been used for at least 600 years. There is no reason to believe other than that the procedure originated even much earlier.

### *3.1.2 Problems with Funding Mortgage Loans*

The greatest concern in funding mortgages is the risk inherited in mortgage loans. In the United States mortgage loans are largely fixed-rate. This means that the homeowner can lock in a fixed interest rate and that they receive an option to prepay the loan, partly or all of it, at any time and for any reason, before the maturity date. This causes the mortgage to be radically shortened or lengthened relative to the expectations, depending on the payment behaviour of the individuals. The result is that the value of the mortgage loan is affected in unpredictable ways.

These problems caused lending institutions to look for ways to reallocate these risks: “Problems faced by (...) institutions in the 1970’s and 1980’s (...) led to the initial innovations in the mortgage market that broadened the mortgage investment base. Individual mortgages were pooled into securities that could be sold to investors other than the mortgage originator” (Fernald et al. 1994: 1).

### *3.1.3 Mortgage-Backed Securities and Collateralized Mortgage Obligations*

Securities based on pools of individual mortgage loans are known as *Mortgage-Backed Securities* (MBSs). The initial restructuring of the mortgages in such pools resulted in simple *pass-through securities* where all investors shared the cash flows from the underlying pool of mortgage loans. This restructuring had the objective to reduce the uncertainty in the cash flows caused by the prepayment risk of single mortgage loans. Pooling them together and letting investors acquire a share in the pool diversified the individual components of prepayments. In other words, pooling the mortgages together caused the prepayment risk to be divided equally among the investors owning a share in the mortgage pool and thereby reducing (but not eliminating) the prepayment uncertainty.

“Beginning in 1983, the emergence of CMOs further refined the allocation of prepayment risk across investors. CMOs direct cash flows from the underlying mortgages (or an underlying MBS) to different classes, called ‘tranches’, according to a predetermined schedule” (Fernald et al. 1994:2). The CMO innovation allowed for the construction of tranches that were relatively free from prepayment risk, causing other tranches to absorb this prepayment risk. In

so doing, institutions were able to create “high-risk” (absorbing prepayment risk) and “low-risk” (free from prepayment risk) securities.

We remember that the main problem with funding mortgages was the risks inherited in the mortgages, specially the prepayment risk. The CMO “(...) effectively unbundles the decision to fund mortgages from the decision to accept prepayment risk” (Fernald et al. 1994:2), and thereby became a solution to a great concern for many institutions and investors. Through the CMO they also became able to meet specific investment objectives in a more accurately way by custom designing the CMO’s tranches.

### 3.1.4 The Issuance of CMOs

During the first years of their existence, there was a tremendous development in the issuance of CMOs. As figure 3.1 shows, the number of new issues grew from \$9 billion in 1983 to over \$100 billion in 1990 and about \$320 billion in 1993. This corresponds to an average growth of over 38% per year. As we will see in section 3.2, where we will look closer at the size of the mortgage market, there were over \$1 trillion in new issues of CMOs in 2005.

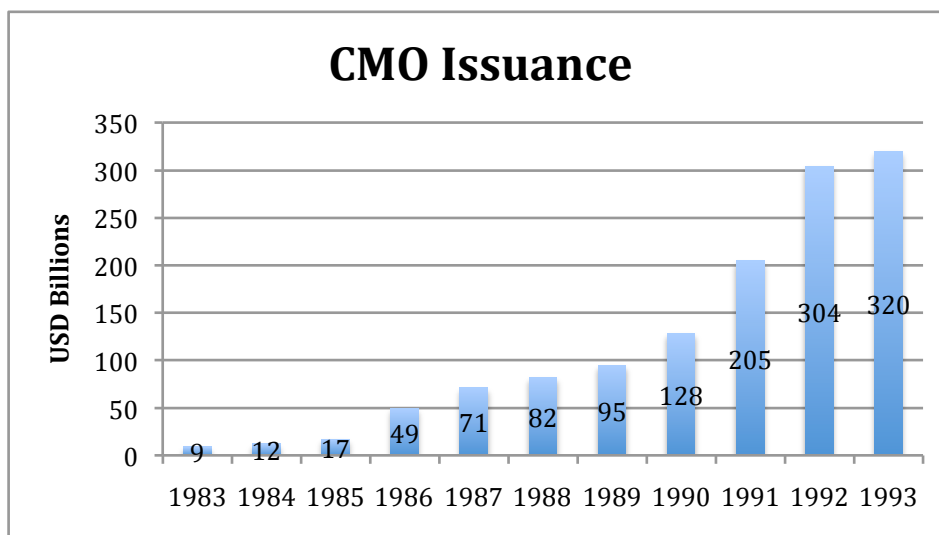


Figure 3.1: CMO Issuance, 1983-1993.<sup>1</sup>

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<sup>1</sup> Source: Fernald et al. (1994:3).

### 3.1.5 Taxation and REMICS

Because an entity that issues a mortgage-backed security simply acts as a conduit in passing interest and principal payments received from homeowners through to certificate holders, it is desirable to make sure that legal structures formed to allocate those payments are not taxed. If a pass-through is issued through a legal structure called a *grantor trust*, which is the arrangement used by issuers of pass-throughs, then there is tax laws providing that the issuers is not treated as a taxable entity. But, if there is more than one class of bonds (i.e., a multiclass pass-through such as a CMO), the trust does not qualify as a non-taxable entity.

It is possible to create structures that avoid adverse tax treatment. Due to the fact that the collateral of such instruments can be used to create securities with higher price, these securities are inefficient. Therefore issuers needed a new trust device so that mortgage-backed security structures with more than one class of bonds could be issued more efficiently.

The Tax Reform Act of 1986 created a new trust vehicle called the *Real Estate Mortgage Investment Conduit* (REMIC). This law allowed for the issuance of mortgage-backed securities with multiple bondholder classes without adverse tax consequences. It is quite common to hear market participants refer to a CMO as a REMIC, but not all CMOs are REMICs.

In this thesis, when I refer to a CMO I mean both REMIC and non-REMIC structures.

### 3.1.6 The Subprime Mortgage Crisis

During the spring and summer of 2007 we saw the emergence of what is today known as the *subprime mortgage crisis*. The crisis was amplified by the intense restructuring of mortgage loans, resulting in that a crisis that sprung from a fall in the U.S. housing market, had repercussion throughout the world.

A *subprime mortgage* is a kind of mortgage that is given to individuals that otherwise are not able to get an ordinary mortgage loan, either because they have had trouble servicing their debt or because they do not earn enough to qualify for ordinary mortgage loans. During the years up to 2007, there was a remarkable growth in such mortgage loans. While they in 2003 comprehended under 5% of outstanding U.S. mortgage debt, they had grown to over 13% in

March 2007 (Bjerkholt(3) 2007). As with other mortgage debt, these loans have been exposed to restructuring into new securities, such as CMOs, and then sold to other investors.

During 2006, the first indicators of a fall in the U.S. housing market surfaced. After almost 15 years of growth in the housing market (depending on which key figures you look at), there were indicators pointing at a housing-bubble about to burst. The rate of defaults and foreclosure on mortgages rose and of course the subprime mortgage section was the first to feel the heat. Come March 2007, the default rate on such loans had risen to over 13% compared to less than 5% in 2005 (Bjerkholt(3) 2007).

The initial consequences of the defaults on subprime mortgages were the bankruptcy of some American companies specialising in supplying such mortgages. But since many of these companies had restructured the mortgages and sold them to other investors such as hedge funds and insurance companies, domestic and foreign, the problems kept growing. Even though the original lenders of the fund were the ones who absorbed the main parts of the losses, the consequences spread throughout the credit market.

In addition, because the loans had been restructured and resold, there were great uncertainty about who was exposed to these bad loans. During the spring of 2007, it was mostly American investors that were struck, but by the end of the summer the crisis also reached European companies. Banks, such as IKB Deutsche Industriebank (Lund 2007) and UK's fifth largest mortgage lender, Northern Rock (Bjerkholt(4) 2007), went into financial distress and needed government help to avoid bankruptcy. During the summer of 2007, the crisis in the credit market also transmitted into the stock markets and many markets around the world went into the state of bear market.

The subprime mortgage crisis started with a housing-bubble bursting in late 2006, and through 2007 and the beginning of 2008 the problems rose to a global financial crisis. Still, we do not know if we have seen the end of it. The fact that the design of mortgage-related securities, such as the CMO, can strengthen and transmit problems from one market to another, and between countries and regions, shows the importance of understanding such securities and the creation of them.

### 3.2 Market Size

There is no doubt that the mortgage market is one of the largest debt markets in the United States. According to Belikoff et al. (2007:1) the total size of outstanding home mortgage debt (DOUTMORT), as of September 2005, was \$8.2 trillion, constituting almost 32% of total outstanding U.S. debt. In comparison, the U.S. federal debt (DOUTFED) weighed in at *only* \$4.6 trillion (near 18% of total outstanding debt).

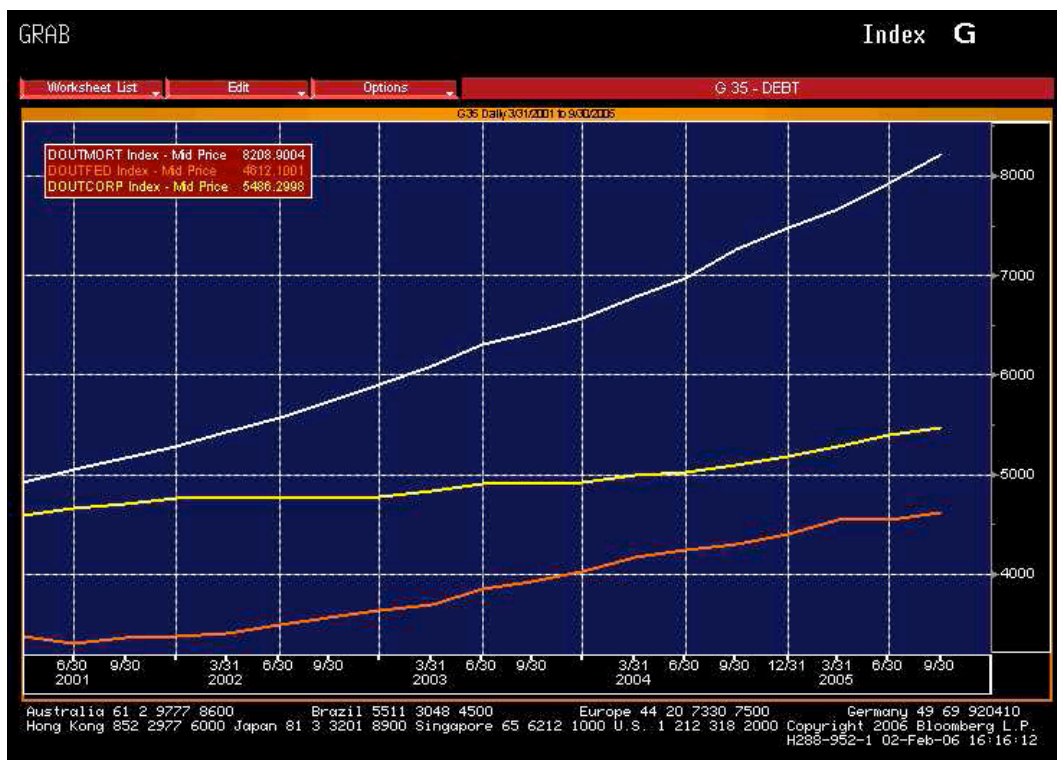


Figure 3.2: Debt Growth.<sup>2</sup>

As we see from the uppermost line in figure 3.2 (DOUTMORT Index), the mortgage market is also an expanding market, having grown over 11% a year in each of the last three years up to 2005, and over 13% the last year (Belikoff et al. 2007:1). The figure also shows that the growth in the mortgage market has outperformed the growth both in outstanding corporate debt (DOUTCORP Index) and in outstanding federal debt (DOUTFED Index).

<sup>2</sup> Source: Belikoff et al. (2007:2).



It is not only in outstanding notional that the mortgage market is large. In figure 3.3 and 3.4, we see the number of new issues of MBSs and CMOs during 2004 and 2005 sorted by issuer. We see that the largest issuers of MBSs, with approximately 90% of all new issues in 2004 and 2005, are the agencies Freddie Mac and Fannie Mae.<sup>3</sup> Totally, over \$1 trillion in agency pools were issued in 2004 and another \$987 billion in 2005.

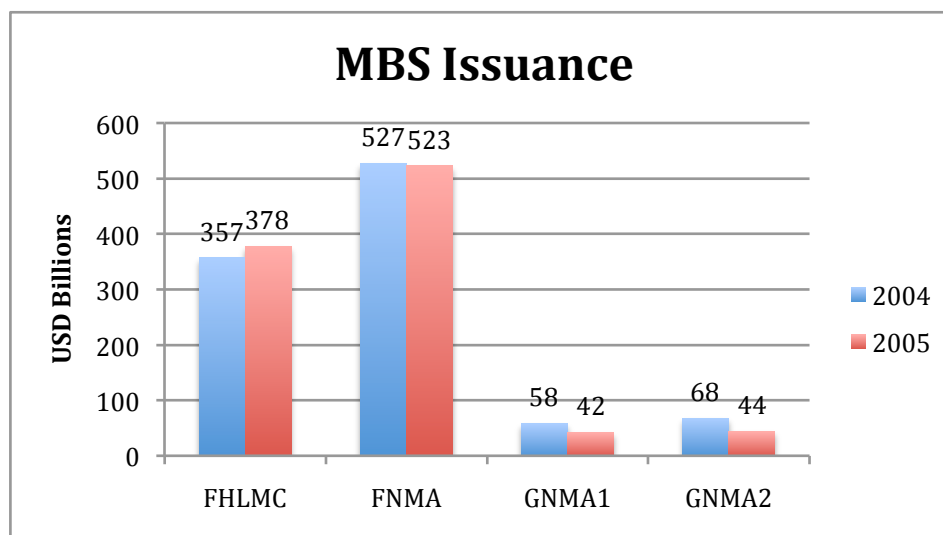


Figure 3.3: MBS Issuance, 2004 and 2005.<sup>4</sup>

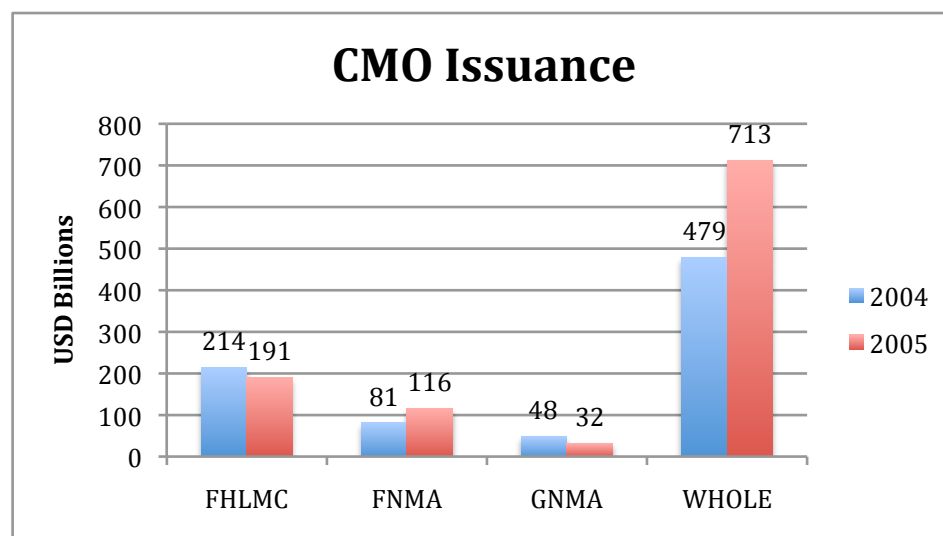


Figure 3.4: CMO Issuance, 2004 and 2005.<sup>5</sup>

<sup>3</sup> In section 4.2, I will describe these agency issuers in more detail. For now let's just note that the U.S. government guarantees for the agency issuers, though in different degrees.

<sup>4</sup> Source: Belikoff et al. (2007:3).

<sup>5</sup> Source: Belikoff et al. (2007:3).

Collateralization of mortgage debt kept up pace as well, with \$822 billion of CMOs issued in 2004 and over \$1 trillion in 2005. The difference between CMO issuances and MBS issuances is that the government agencies make up a much smaller part of the market. Non-agency CMOs, or *whole loans CMOs*, are issued by private entities and are not guaranteed by the government. These entities often use *whole loans* (i.e., unsecuritized loans), rather than pass-through securities. As we see from figure 3.4, non-agency CMOs constituted roughly 60-70% of all issuance of CMOs in 2004 and 2005.

Because of the subprime mortgage crisis the market encountered during the summer and fall of 2007, there is reason to believe that the growth of this market have slowed during the last year. Nevertheless, mortgage-related securities have obvious advantages for investors encountering specific investment objectives, but also for investors seeking diversification. It is therefore a strong possibility that the mortgage market and mortgage-related securities will continue to constitute a sizable and important part of the debt market.

## 4. Mortgage Loans and Mortgage-Backed Securities – the Collateral of CMOs

As mentioned earlier it is important and useful for investors to understand the typical characteristics of a CMO before including it in their portfolio. In trying to understand the structure of the CMOs, it is essential to appreciate the predominant features of the mortgage collateral underlying these instruments. In the following chapter I will therefore investigate how a mortgage-backed security is created from mortgage loans.

### 4.1 What is a Mortgage?

For many individuals, owning their own home is one of their biggest dreams. In most cases individuals do not have the funds needed and are therefore forced to borrow money to purchase one.

The market where these funds are borrowed is called the *mortgage market* and the funds are normally secured by the real estate purchased by the borrower. These kinds of loans are known as a *mortgage* and can be defined as: "... a loan secured by the collateral of specified real estate property, which obliges the borrower to make predetermined series of payments" (Fabozzi 2004:212). This collateral implies that the *mortgagee* (the lender) has the right to foreclosure on the loan if the *mortgagor* (the borrower) fails to make the contracted payments, which means that the mortgagee can seize the property to ensure that the debt is paid off.

Not all real estate properties can be *mortgaged* (that is, used as collateral for borrowing). The ones that can be mortgaged are divided in two wide categories: *residential* and *non-residential*. Residential properties consist of houses, condominiums, cooperatives and apartments. They can be subdivided into *single-family* (one-to-four-family) structures and *multifamily* structures. The non-residential properties consist of commercial and farm properties. In analysis of mortgage instruments, theorists usually focus on the first category.

The predetermined series of payments to the mortgagee are usually organized in *monthly* payments, but they can also be organized in *quarterly* payments, *semi-annual* payments or *annual* payments. In the case for monthly payments, the payments generate monthly cash

flows from the mortgage. As table 4.1 shows, these cash flows stems from three sources: a) *interest*, b) *scheduled principal payment* and c) *unscheduled principal payment*.

Instrument	Monthly Cash Flow
Mortgage Loan	Interest Scheduled principal payment Unscheduled principal payment

*Table 4.1: Cash Flow from a Mortgage Loan.*

The actual cash flows from mortgages can differ from the expected cash flows and this causes risk for the owner of the mortgage. The uncertainty can occur because of one of the following reasons: *credit-*, *prepayment-* and *interest rate risk*. In addition to uncertainty around the monthly cash flows, the owner of a mortgage loan is exposed to *liquidity risk*. In section 4.2, I will take a closer look at these risks.

#### *4.1.1 Participants in the Mortgage Market*

Naturally, there has to be ultimate lenders of the funds in the market, and as mentioned earlier, there are government agencies involved in the market. In addition to these two groups, there are three other groups of participants: *mortgage originators*, *mortgage servicers* and *mortgage insurers*.

##### *Mortgage Originators*

The *mortgage originator* is the original lender of the funds. Originators include thrifts, commercial banks, mortgage banks, life insurance companies and pension funds.

There are several ways an originator can generate income. First, originators charge an *origination fee* that is expressed in terms of points. Each point represents 1% of the borrowed funds. Secondly, the originators might generate profit from selling a mortgage in the *secondary market*. The profit earned in this way is called *secondary marketing profit*. Finally,

the originators might hold the mortgage in its investment portfolio and earn *interest* on the mortgage.

### *Mortgage Servicers*

A mortgage loan needs *servicing*. This implies collecting monthly payments and forwarding the proceeds to owners of the loan, sending payment notice to mortgagors, reminding mortgagors when payments are overdue, maintaining records of principal balances and other administrative tasks. It is also the servicers who initiate foreclosure proceedings if necessary. Mortgage servicers include bank-related entities, thrift-related entities and mortgage bankers.

The main source of revenues for mortgage servicers is the *servicing fee*, which is a fixed percentage of the outstanding mortgage balance. There are also other sources including *float* earned on the monthly payment (arises because of the delay permitted between the time the servicer receive the payment and the time that the payment must be sent to the investor) and several sources of *ancillary income*.

### *Mortgage Insurers*

Both the mortgagee and the mortgagor may take out *mortgage-related insurance*. Insurance originated by the lender to insure against default by the borrower, is called *mortgage insurance* or *private mortgage insurance*. The cost of the insurance is ironically born by the borrower usually through a higher contract rate. Mortgage insurance can be obtained from private mortgage insurance companies or, if the borrower qualifies, from government guaranteed mortgage insurance. The mortgage-related insurance originated by the borrower, is usually acquired with a life insurance company, and is typically called *credit life*. This type of insurance provides for a continuation of mortgage payments after the death of the insured individual. The mortgagee does not require this type of insurance.

Both mortgage-related insurances described above are beneficial to the creditworthiness of the mortgagor, but, as reflected in the requirements of the lender, the first type is more important from the perspective of the mortgagee.

## 4.2 Risks Associated with Investing in Mortgages

In section 4.1 we saw that there are three sources to uncertainty in the monthly cash flows received by an investor in mortgages: *credit*, *prepayment* and *interest rates*. We also acknowledged the fact that there is *liquidity risk* contained in owning a mortgage loan. In the following section I will discuss these risks in more detail.

### 4.2.1 Credit Risk

Credit risk emerges from the possibility that the homeowner/borrower will default on the mortgage and that the proceeds from the resale of the property fall short of the value of the mortgage.

When it comes to securities backed by a pool (or collection) of mortgage loans, which I will discuss in later sections, the concern with credit risk depends on the issuer of the security. If the security is issued by agencies (and thereby guaranteed), the investors are not concerned with credit risk. If, on the other hand, the security is issued by non-agencies, the investors are concerned with the credit risk associated with the borrowers whose loans are backing the security.

Agency issues include securities issued by the *Government National Mortgage Association* (GNMA – Ginnie Mae), the *Federal National Mortgage Association* (FNMA – Fannie Mae), and the *Federal Home Loan Mortgage Corporation* (FHLMC – Freddie Mac). Ginnie Mae is a federally related institution and the securities issued are backed by the full faith and credit of the U.S. government. Fannie Mae and Freddie Mac are government sponsored enterprises and the guarantee depends on their financial capacity to satisfy their obligations and the willingness of the U.S. government to bail them out should there be a default. This implies that the government guarantee is an implicit one and these agencies usually charge a small fee to agree to absorb all default losses on the underlying mortgages.

Non-agency securities are issued by private entities, usually mortgage conduits, commercial banks, savings and loan associations, mortgage lenders or investment banking related firms. These entities use mortgage loans that are not eligible for agency guarantees, e.g., mortgages

for commercial properties or multifamily homes and mortgages over a certain capped value (approximately \$200,000).

Such securities are rated by commercial rating agencies as Moody's, Standard & Poor's, Duff & Phelps, and Fitch IBCA. The rating of the mortgage-backed securities depends partially upon the rating of the issuer. Issuers of non-agency securities can augment the credit quality of securities in several ways according to standards set by these rating agencies.

Through the major parts of this thesis I will not focus on the credit risk of mortgage loans backing CMOs. The reason for this is that there is a different risk element of the cash flows from a mortgage that makes the analysis of CMOs complicated. By not focusing on the credit risk element, we are able to see the distinctive features that separate the CMOs from other securities backed by some pool of debt.

When I discuss different types of CMO structures later, I will take a quick look at the most popular method to augment the credit quality of non-agency issues.

#### *4.2.2 Prepayment Risk*

The illustration of the monthly cash flow from a mortgage loan in table 4.1 showed the different elements of the cash flow. The final element in the table, unscheduled principal payments covers the fact that homeowners sometimes pay off all or part of their mortgage balance prior to the maturity date. Effectively, someone who invests in a mortgage has granted the borrower an option to pay off the mortgage. These payments in excess of the scheduled payments are called *prepayments*.

Prepayment is the most important risk element in analysis of mortgages and mortgage-related instruments. "It is the amount and timing of this element of the cash flow from a mortgage that makes the analysis of mortgages and mortgage-backed securities complicated" (Fabozzi & Ramsey 1999:1-2). Therefore, this element is of utmost importance for the progressing analysis.

Though the single largest factor affecting prepayment behaviour is the interest rate level, there are other factors causing homeowners to prepay that are independent of the interest rates. We will discuss prepayments and the factors causing prepayments more thoroughly in chapter 5.

For now, we establish that the effect of prepayments is that the amount of the cash flow from a mortgage is not known with certainty.

#### 4.2.3 Interest Rate Risk

This risk category unavoidably overlaps the previous element due to the interest rate sensitivity of the prepayment option embedded in mortgages. Still, because a mortgage is a debt instrument, and long-term on average, its price will move in the opposite direction of the market interest rates. The phenomenon of mortgage prepayments will on the other hand, cause the prices of mortgage securities to behave differently than ordinary bonds. We can use the concepts of *duration* and *convexity* to contrast these bonds' price sensitivities to interest rate changes.

Duration is the negative of the first derivative of the price function with respect to a change in interest rates (the slope) divided by the price.

$$(4.1) \quad D = -\frac{1}{P} \sum_{t=1}^N \frac{t \cdot C_t}{(1 + \text{yield})^{t+1}}$$

Thus, duration provides a measure of the percentage price change of a security for a small change in interest rate.

Convexity is the second derivative of the price function divided by the price. The convexity thus captures the degree of curvature of the price-yield relationship, and hence is important to consider when large changes in yield occur. If the second derivative were zero, the price function would be a straight line and there would be no curvature.

#### 4.2.4 Liquidity Risk

The existence of an active secondary market for mortgage loans does not obstruct that bid-ask spreads are large compared with other debt instruments (i.e., mortgage loans tend to be rather



illiquid because they are large and indivisible). The bid-ask spread on a mortgage loan varies and for mortgage loans with abnormal collateral, spreads are wider. The more abnormal the collateral is, the higher is the bid-ask spread.

I will not focus on the liquidity risk of mortgages in this thesis.

### 4.3 Mortgage-Backed Securities

Starting from 1969 (Fabozzi & Ramsey 1999:1), the problems regarding prepayment and interest rate risk caused institutions to use mortgage loans as collateral for creation of new securities. Individual mortgages were pooled together into securities that could be sold to investors other than the mortgage originator. Popularly these instruments are referred to as *mortgage-backed securities* (MBSs). In the following section we will take a closer look at these securities and see how they are created.

#### *4.3.1 General Description of Mortgage-Backed Securities*

MBSs are created when a financial institution sell off parts of its residential mortgage portfolio to other investors. The financial institutions accomplish this by pooling the mortgages sold together and letting investors acquire a stake in the pool by buying units. It is these units that are known as mortgage-backed securities.

The cash flows from these securities are backed by the mortgage pool, which means that the investors receive monthly payments generated by homeowner paying down their home mortgage loans.

As for mortgage loans, the risk in these cash flows arise in the extent the actual flows differ from the expected flows.

#### *4.3.2 The Creation of a Mortgage-Backed Security*

In the following I will provide an illustration of how a MBS is created. As mentioned above, pooling together individual mortgages and letting investors acquire a stake in the pool, create

a MBS. In table 4.2 I have illustrated 1,000 mortgage loans and the cash flows from these loans. For the sake of simplicity, I will assume that the amount of each loan is NOK 1 million so that the aggregate value of all 1,000 loans is NOK 1 billion. We see that the cash flows for each loan correspond to the cash flows from a single mortgage as demonstrated in table 4.1. Note that I have changed the third element of the cash flow from *unscheduled principal payment* to *prepayments*. This is due to the importance of prepayments for the analysis of mortgage-related securities.

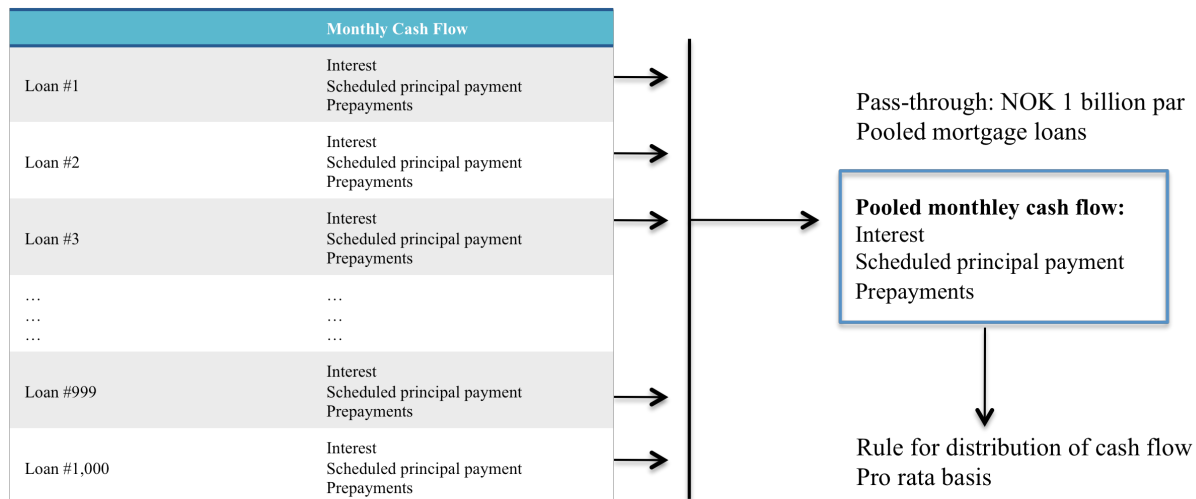
	Monthly Cash Flow
Loan #1	Interest Scheduled principal payment Prepayments
Loan #2	Interest Scheduled principal payment Prepayments
Loan #3	Interest Scheduled principal payment Prepayments
...	...
...	...
...	...
Loan #999	Interest Scheduled principal payment Prepayments
Loan #1,000	Interest Scheduled principal payment Prepayments

Table 4.2: Cash Flow from 1,000 Mortgage Loans.

An investor in one of the mortgage loans in table 4.2 is exposed to prepayment risk. For a single mortgage loan the uncertainty of prepayments are especially difficult to predict. If the individual investor bought all 1,000 loans, however, we expect the predictability to rise sharply due to historical prepayment experience. Buying all 1,000 loans will on the other hand imply an investment of NOK 1 billion (or even larger for bigger mortgage pools!).

Let us assume instead that some entity purchases all 1,000 loans and pools them together. Now the 1,000 individual loans can be used as collateral for the issuance of a security whose cash flow is based on the cash flow from the 1,000 individual loans. Figure 4.1 illustrates this process. Now the entity can issue 1 million certificates, each initially worth NOK 1,000,

resulting in that each certificate holder would be entitled to 0.000001% of the cash flows from the pool.



Each loan is for NOK 1 million  
Total loans: NOK 1 billion

*Figure 4.1: Creation of Pass-Through Security.*

The process just described is referred to as *securitization* and the security created is called a *mortgage pass-through security*, also referred to as a *pass-through*.

So what have we accomplished by this securitization? The total amount of prepayment risk has not changed. But, because the investor now can invest in a share of the pool, he is now exposed to the prepayment risk spread over 1,000 loans rather than one individual loan. This means that the investor gets the same risk exposure as owning all 1,000 loans, but at a much smaller cost than purchasing all 1,000 loans.

An investor in the pass-through described above is still exposed to the total prepayment risk associated with the underlying pool of mortgage loans, regardless of how many loans that is included in the pool. It is possible, however, to create securities where the investors do not share the prepayment risk equally. That is what we are going to look at when we turn our attention to the CMO innovation.

## 4.4 Types of Assets Backing a CMO

The purpose of the final section of chapter 4 is to show some of the different types of assets that are used to create the collateral for CMOs. By being ‘truly custom design’, there are numerous constructions backing CMOs. “The collateral may be comprised of as few as one or as many as 1,000 or more pools of passthrough securities, or a single pool of individual mortgage loans (called *whole loans*) that have not been securitized. (Of course, whole loans themselves comprise the collateral for passthrough securities.) There are also some CMOs that are backed by principal-only securities, interest-only securities, and other CMO tranches” (Fabozzi & Ramsey 1999:9).

### 4.4.1 Mortgage Loans

I have earlier explained the main features of a mortgage loan and looked at the risks associated with investing in one. Now I will shortly review some of the different types of mortgage loans. Many of them have been used as collateral for CMOs, either directly or through securitization with the resulting securities used as collateral. As mentioned earlier, most mortgages in the United States are fixed-rate. The assets backing CMOs are always fixed-rate, and therefore I restrict my review to fixed-rate mortgages.

#### *Level-Payment, Fixed-Rate Mortgage*

In a *level-payment, fixed-rate mortgage*, or simply *level-payment mortgage*, the borrower pays equal monthly instalments over an agreed-upon period of time, called the *maturity* or *term* of the mortgage. Each monthly mortgage payment for a level-payment mortgage is due on the first of each month and consists of: a) interest of roughly 1/12 of the fixed annual interest rate times the amount of outstanding mortgage balance at the beginning of the previous month and b) a repayment of a portion of the outstanding mortgage balance (principal). After the last scheduled monthly payment of the loan is made, the amount of the outstanding mortgage balance is zero.

### *Balloon Mortgage*

In the case of a *balloon mortgage loan*, or simply a *balloon loan*, the lender gives the borrower long-term financing, but at a specified future date the mortgage rate is renegotiated. At this specified future date, the initial loan is repaid and the origination of a new loan is established. When repaying the initial loan, the borrower makes what is called a *balloon payment*, which is the original amount borrowed less the amount amortized. In this way the lender provides long-term funds for what is essentially short-term borrowing. Effectively, a balloon mortgage is a short-term balloon loan in which the lender agrees to provide financing for the remainder of the term of the mortgage.

### *“Two-Step” Mortgage Loans*

The *two-step mortgage loan* is similar to a balloon mortgage in being a fixed-rate loan with a single rate reset at some point prior to maturity. Unlike a balloon mortgage, this rate reset occurs without specific action on the part of the borrower. In other words, the rate reset on the two-step does not consist of a repayment of the initial loan and the origination of a new one.

### *Tired Payment Mortgages*

The *tired payment mortgage* is designed with a fixed rate and a monthly payment that graduates over time. The initial monthly mortgage payments are below those of a level-payment mortgage and closer to the maturity of the mortgage the payments are higher.

## *4.4.2 Mortgage Pass-Through Securities*

As we have seen earlier there is three major agencies guaranteeing for pass-throughs. In addition there are non-agency pass-throughs comprising a small part of the pass-through market.

Agencies can provide one of two types of guarantees. They can guarantee the timely payment of both principal and interest, meaning that principal and interest are paid when they are due, even when the mortgagors fail to make their monthly instalments. Such pass-throughs are referred to as *fully modified pass-through*. The other type of guarantee also is a guarantee for

both principal and interest, but it guarantees for the timeliness of the interest payments only. These are referred to as *modified pass-throughs*.

#### *Government National Mortgage Association MBS*

Ginnie Mae pass-throughs are guaranteed by the full faith and credit of the United States government and therefore viewed as risk-free in terms of default risk, just like Treasury securities. The security guaranteed by Ginnie Mae is called mortgage-backed security and all Ginnie Mae MBSs are fully modified pass-throughs. A mortgage pool guaranteed by the Ginnie Mae includes only mortgage loans insured or guaranteed by either Federal Housing Administration, the Veterans Administration, or the Rural Housing Service.

#### *Federal Home Loan Mortgage Corporation PC*

The participation certificate (PC) issued by Freddie Mac is the second largest type of agency pass-throughs. Most market participants view Freddie Mac PCs almost as identical in creditworthiness to Ginnie Mae pass-throughs, although the government does not back the guarantee. Freddie Mac has two programs with which it creates PCs: *Cash Programs* and *Guarantor/Swap Program*. The first program creates regular constructions and the PCs are called *Cash PCs* or *Regular PCs*. In the second program the Freddie Mac allows originators to swap pooled mortgages for PCs in those same pools and the PCs created under this program is called *Swap PCs*. Another type of PCs is the *Gold PCs*, which has stronger guarantees, and is issued in both programs. It is expected that the Gold PCs will become the only type of PCs in the future. Freddie Mac offers both modified and fully modified pass-throughs.

#### *Federal National Mortgage Association MBS*

The third group of pass-throughs are the mortgage-backed securities issued by the Fannie Mae. As with the Freddie Mac PCs, Fannie Mae MBSs is not an obligation of the government. Fannie Mae also has a swap program similar to that of Freddie Mac. All Fannie Mae MBSs are fully modified pass-throughs.

#### *4.4.3 Stripped MBSs*

A pass-through distributes the cash flows from the underlying pool of mortgages on a pro rata basis. Redistributing the principal and interest from a pro rata distribution to an unequal distribution creates a stripped MBS. By allocating all the interest to one class and the entire principal to another, we are able to create what we call *IO classes* (interest-only) and *PO classes* (principal-only). The IO class receive no principal payments, and vice versa.

## 5. Modelling of Prepayment Behaviour

“The starting point in evaluation of any financial asset is estimation of its expected cash flow” (Fabozzi & Ramsey 1999:21). As mentioned earlier, the principal prepayments means that the cash flow of a pass-through cannot be known with certainty. Thus, the rate of principal prepayments is the dominant factor affecting the value of pass-throughs. To understand CMOs, it is therefore imperative that we understand the reasons for prepaying and how to project the cash flow of a pass-through.

In this chapter, I will discuss the prevailing industry conventions for projecting the cash flow of a pass-through. Later, I will refer to these conventions whenever I illustrate the cash flow of CMO structures. Therefore, my focus of this chapter is simply to illustrate the mechanics involved. Before turning the attention toward the CMO innovation in chapter 6, I will also take a closer look at the different factors affecting prepayments and briefly explain how to construct a prepayment model.

### 5.1 Prepayment Benchmark Conventions

“Estimating the cash flow from a passthrough requires making an assumption about future prepayments” (Fabozzi & Ramsey 1999:21). Over the years, there have been several conventions used as a benchmark for prepayment rates. Though some of them are no longer used, we discuss the following three conventions due to their historical significance: a) *Federal Housing Administration (FHA) experience*, b) the *conditional prepayment rate*, and c) the *Public Securities Association (PSA) prepayment benchmark*.

#### 5.1.1 Federal Housing Administration Experience

During the first years of the pass-through market’s development, calculations of cash flows assumed no prepayments for the first 12 years. At that time, they assumed that all the mortgages in the pool were prepaid. The “*FHA prepayment experience*” approach replaced this rather naive approach.



The FHA approach looks at the prepayment experience for 30-year mortgages derived from an FHA table on mortgage survival factors. The approach calls for the projection of the cash flow for a mortgage pool on the assumption that the prepayment rate will be the same as the FHA experience (referred to as “100% FHA”), or some multiple of FHA experience (faster or slower than FHA experience).

Though it was fairly popular, this method is no longer in use. As we already know, prepayments are tied to interest rate cycles. The FHA prepayments are for mortgages originated over all sorts of interest periods, indicating an average prepayment rate. Thereby the FHA prepayments are not necessarily indicative of the prepayment rate over various cycles for a particular pool. Another reason this method is no longer in use, is the fact that FHA tables are published periodically; causing confusion about which table the prepayments should be based on. Together with some other reasons not mentioned here, the prepayments based on FHA statistics therefore can be deceptive.

“Because estimated prepayments using FHA experience may be misleading, the resulting cash flow is not meaningful for valuing pass-throughs” (Fabozzi & Ramsey 1999:22).

### *5.1.2 Conditional Prepayment Rate*

The second benchmark for projecting prepayments and the cash flow of a pass-through is the *conditional prepayment rate*. This method requires assuming prepayment of some fraction of the remaining principal in the mortgage pool each month for the remaining term of the mortgage. The conditional prepayment rate (CPR) is the assumed rate for a pool, and is based on the characteristics of the pool (including its historical prepayment experience) and the current and expected future environment. The advantages of this approach are its simplicity and the fact that changes in economic conditions impacting the prepayment rate or changes in the historical prepayment pattern can be analyzed quickly.

#### *The Single-Monthly Mortality Rate*

Since the CPR is an annual prepayment rate, it is necessary to convert the CPR into a monthly prepayment rate to estimate monthly prepayments. This monthly prepayment rate is referred to as the *single-monthly mortality rate* and can be determined through the following formula:

$$(5.1) \quad SMM = 1 - (1 - CPR)^{1/12}$$

### *The SMM Rate and the Monthly Prepayment*

A given SMM rate indicates that approximately that portion of the remaining mortgage balance at the beginning of the month, less the scheduled principal payment, will be prepaid that month. That is,

$$(5.2) \quad P_t = SMM_t \cdot (MB_t - SPP_t)$$

where  $P_t$  = prepayment for month  $t$ ,  
 $MB_t$  = beginning mortgage balance for month  $t$ , and  
 $SPP_t$  = scheduled principal payment for month  $t$ .

### *5.1.3 PSA Prepayment Benchmark*

The last benchmark is the *Public Securities Association prepayment benchmark* and is expressed as a monthly series of annual prepayment rates. This model has a basic assumption that prepayment rates are low for newly originated mortgages and then speeds up as the mortgages becomes seasoned.

The PSA standard benchmark assumes the following prepayment rates for 30-year mortgages:

- 1) a CPR of 0.2% for the first month, increased by 0.2% per year per month for the next 29 months when it reaches 6% per year, and
- 2) a 6% CPR for the remaining years.

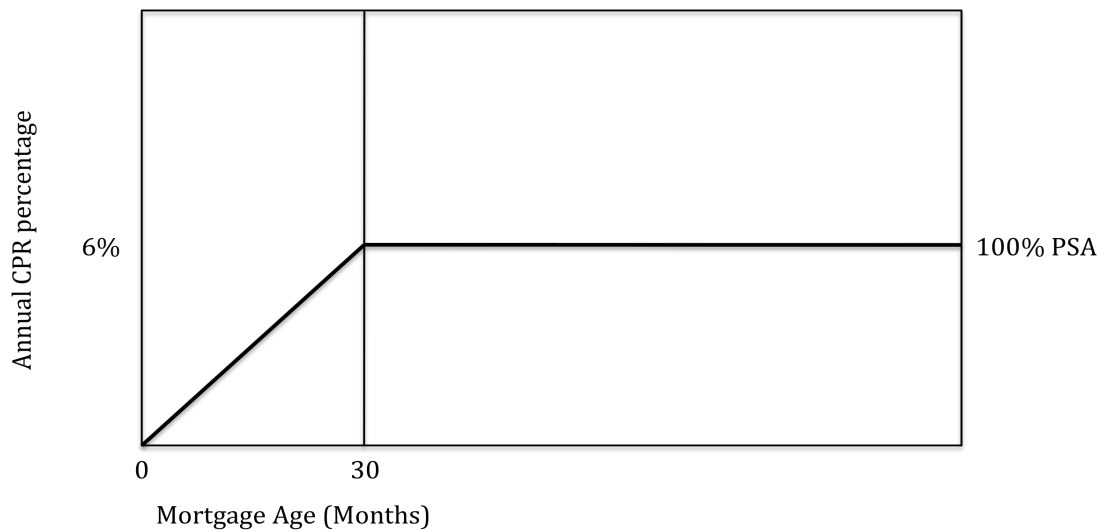
The benchmark above is referred to as “100% PSA” or simply “100 PSA”. The benchmark is illustrated in figure 5.1. Mathematically, 100 PSA can be expressed as follows:

$$(5.3) \quad CPR = 6\% \cdot \left(\frac{t}{30}\right)$$

when  $t$  is equal to, or less than, 30, and

$$(5.4) \quad CPR = 6\%$$

when  $t$  is larger than 30.



*Figure 5.1: 100% PSA*

Slower and faster speeds are referred to as some percentage of PSA. For example, 50 PSA means one-half the CPR of the PSA benchmark prepayment rate or 150 PSA means 1.5 times the CPR of the PSA benchmark prepayment rate. This is graphically depicted in figure 5.2 below.

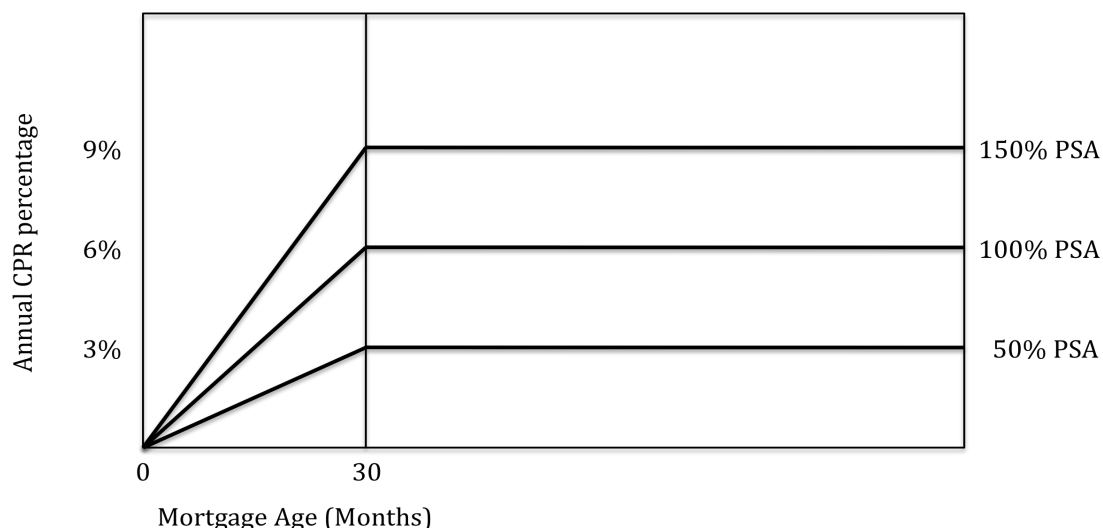


Figure 5.2: 50% PSA, 100% PSA and 150% PSA

#### 5.1.4 The PSA Prepayment Benchmark – a Market Convention Only

The PSA prepayment benchmark was originally introduced to provide a standard measure for pricing CMOs backed by 30-year fixed-rate, fully amortizing mortgages, and it is a product of a study by the PSA that evaluated the mortality rates of residential loans insured by the FHA.

In the data that the PSA committee examined, it looked like the mortgages became ‘seasoned’ (i.e., prepayment rates tended to level off) after 29 months at which time the CPR tended to hover at approximately 6%. Based on this, the PSA developed its prepayment benchmark through assumptions (like a linear increase in CPR from month 1 to month 30) and other simplifications.

Though many astute money managers recognizing the usefulness of CPR for quoting yield and/or price (showing the convenience of the convention), the CPR has many limitations in determining the value of a pass-through. “The message is that money managers must take care in using any measure that is based on the PSA prepayment benchmark. It is simply a market convention” (Fabozzi and Ramsey 1999:33).

### 5.1.5 Average Life and Macaulay Duration

When evaluating mortgage-backed securities, market participants desire some measure of the “life” of mortgage-backed securities. Typically, these measures are used to compare the MBS to a comparable Treasury security. The measures usually used are the *average life* and *Macaulay duration*.

#### *Average Life*

The average life of a MBS is the weighted average time to receipt of principal payments (scheduled payments and projected prepayments). The formula for the average life can be expressed as

$$(5.5) \quad \frac{(1 \cdot PP_1) + (2 \cdot PP_2) + \dots + (T \cdot PP_T)}{12 \cdot \sum_{t=1}^T PP_t}$$

where  $PP_t$  = principal payments at time  $t$ , and  
 $T$  = the number of months.

It is important to remember that the average life of a MBS differ under various prepayment assumptions. An investor might purchase a pass-through under the assumption that the prepayment speed would be 150 PSA, resulting in a given average life. Obviously, the average life of the pass-through can extend or contract considerably if the prepayment speed changes.

### *Macaulay Duration*

Macaulay duration is a weighted average term to maturity where the weights are the present value of cash flows. The yield used to discount the cash flow is the cash flow yield. Thus, the formula for Macaulay duration looks like this:

$$(5.6) \quad \frac{[1 \cdot PV(CF_1)] + [2 \cdot PV(CF_2)] + \dots + [T \cdot PV(CF_T)]}{G}$$

where  $PV(CF_t)$  = present value of cash flow at time  $t$ ,  
 $T$  = the number of months, and  
 $G$  = price plus accrued interest.

### *5.1.6 Vector Analysis*

A practice used by market participants to overcome the drawback of the PSA benchmark is to assume that the PSA speed can change over time. This technique is known as *variable prepay vector array analysis*, or for short just *vector analysis*. A vector analysis is simply a single prepay assumption that is held constant for one or more months. In the case of a CMO backed by 30-year mortgages, this type of analysis could have as many as 360 prepayment vectors.

Because differing levels of prepayment activity dramatically affect the cash flow of certain classes, vector analysis is crucial in evaluating many CMO structures.

## 5.2 Factors Affecting Prepayment Behaviour

Which factors cause mortgagors to prepay their home mortgage? There are several different factors, but in this thesis I will focus on four main categories: a) *the prevailing mortgage rate*, b) *characteristics of the underlying mortgage pool*, c) *seasonal factors*, and d) *general economic activity*.

### *5.2.1 Prevailing Mortgage Rate*

There are three ways the current mortgage rate can affect prepayments and in the following I will address each of them.

#### *Spread Between Contract Rate and Prevailing Mortgage Rate*

The first, the spread between the prevailing mortgage rate and the contract rate, affects the incentive to refinance and is the single most important factor affecting prepayments because of refinancing. The greater the difference between them, the greater is the incentive for the mortgagor to refinance the loan. To make sense, the refinancing must lead to interest savings greater than the total costs of the process.

“Historically, it has been observed that when mortgage rates fall to more than 200 basis points below the contract rate, prepayment rates increase” (Fabozzi & Ramsey 1999:35). On the other side, since mortgage originators have designed loans where the refinancing costs are folded into the amount borrowed and the fact that it has become more feasible to obtain 100% financing, the view that mortgage rates must plunge dramatically below the contract rate to make refinancing economically rational, has changed.

It is also important to note that the benefits from refinancing depend on the initial level of the contract rate. It is obvious that the present value of the benefit of a 200 basis point decline from an initial contract rate of 8%, is greater than an equal decline from an initial contract rate of 17%. In modelling prepayment behaviour, one would therefore expect that prepayments caused by refinancing might be highly correlated with a percentage change in the rates rather than a spread.

Due to lack of observations, it has not yet been possible to test empirically which of the two that better explains prepayment behaviour.

#### *Path of Mortgage Rates*

“The historical pattern of prepayments and economic theory suggests that it is not only the level of mortgage rates that affect prepayment behavior but also the path that mortgage rates take to get to the current level” (Fabozzi & Ramsey 1999:35).

We can illustrate this through a simple example. Three years after origination of a mortgage loan with a 9% contract rate, the mortgage rate has declined to 6%. The share of refinancing in this situation will depend on the path that the mortgage rate has taken over these three years. Let us consider two paths in getting to the current mortgage rate. In the first path, the rate drops to 6% the first year, rises to 10% the second year, and then falls to 6% again. In the second path, the mortgage rate rises to 10% the first year, falls to 9% again the second year, and then falls to 6%.

Those who benefit from refinancing will more than likely take advantage of the opportunity to refinance after the first year if the mortgage rate follows the first path. When the rate drops to 6% again at the end of the third year, the likelihood is that prepayments due to refinancing will not surge; those who can benefit from refinancing has already taken advantage due to the drop after the first year. The prepayment behaviour described above is referred to as the *refinancing burnout* (or simply, *burnout*) *phenomenon*.

If the mortgage rate follows the other path described above, the prepayments due to refinancing activity will most likely surge after the third year. Therefore, the burnout phenomenon is related to the path of the mortgage rates.

Because it is complicated to quantify this path dependency, it has been difficult to model prepayments. One way to deal with the path dependency is to use the *pool factor*, which is the ratio of the remaining mortgage balance outstanding for the pool to the original balance. The argument for this approach is that the lower the pool factor is, the greater have the prepayments historically been. It is therefore more likely that burnout will occur. An alternative adjustment for burnout is a nonlinear function, which is generated from the entire history of the ratio of the contract rate to the mortgage-refinancing rate since the mortgage was issued.

### *Level of Mortgage Rates*

The final way that the current mortgage rate can affect prepayments, is in contrast to the two others linked to housing turnover rather than refinancing. A lower mortgage rate can increase the affordability of homes. Due to such rate environments, mortgagors can be introduced to an opportune time to purchase a more expensive home (trade up), or to change location for other reasons. In this way, the level of mortgage rates can increase or decrease prepayments.



### *5.2.2 Characteristics of the Underlying Mortgage Pool*

The six characteristics of the underlying mortgage pool that affects prepayments are: a) the *contract rate*, b) whether the loans are *FHA/VA-guaranteed* or *conventional*, c) the amount of *seasoning*, d) the *type of loan*, e) the *pool factor*, and f) the *geographical location* of the underlying properties.

In the previous section, we discussed both the contract rate and the pool factor. Whether you have a 30-year level payment mortgage, a 5-year balloon mortgage, or some other type of mortgage, it is obvious that the type of mortgage loan will affect the prepayment behaviour of the mortgagor differently because of the different flexibility the mortgages offer. In the following, we will therefore focus on the three other characteristics mentioned above.

#### *FHA/VA Mortgages versus Conventional Mortgages*

The Federal Housing Administration (FHA) or the Veterans Administration (VA) guarantees for the mortgages backing GNMA pass-throughs, while most pass-throughs issued by FNMA and FHLMC are backed by conventional loans. We will look at four characteristics of mortgages guaranteed by FHA or VA that causes their prepayment characteristics to differ from those of conventional loans.

First, guaranteed mortgages are assumable. Consequently, prepayments should be lower than for otherwise comparable conventional loans when the contract rate is less than the current mortgage rate. The reason is that purchasers will assume the seller's mortgage loan in order to acquire the below-market interest rate, and, as a result, there will be no prepayment resulting from the sale of the property.

Next, the amount of FHA/VA mortgages is typically small, which in turn reduces the incentive to refinance as the mortgage rates decline. Thereby, FHA/VA mortgages produces a prepayment rate due to refinancing that is less than for conventional loans.

Third, the mortgagors that must obtain a mortgage loan guaranteed by FHA or VA, typically has a lower income level than that of mortgagors with conventional loans. They do not have

the same ability to take advantage of refinancing opportunities due to the fact that they do not have the funds necessary to carry out such a process.

In contrast to the above-mentioned characteristics, the last characteristic of FHA/VA-guaranteed mortgages suggests faster prepayments for such loans. Historically, default rates are greater for FHA/VA mortgages compared to conventional loans. Defaults cause faster prepayments. However, the factor of faster prepayments due to defaults is swamped by the three other characteristics.

FHA/VA-guaranteed mortgages therefore imply slower prepayment than conventional loans.

### *Seasoning*

Empirical evidence suggest that prepayment rates are low just after the mortgage is originated, and that there is an increase in prepayment rates after the mortgage is somewhat *seasoned*. Seasoning in this case refers to the aging of the mortgage loan. The prepayment rates tend to level off at some time, in which case the loans are referred to as *fully seasoned*. The theory of mortgage seasoning is the underlying theory for the PSA prepayment benchmark, which we discussed in section 5.1.

### *Geographical Location of the Underlying Properties*

The fact that differences in local economics can affect housing turnover, causes prepayment behaviour to be faster than the average national prepayment rate in some regions of the country, while other regions exhibit slower prepayment rates.

#### *5.2.3 Seasonal Factors*

Prepayments follow a well-documented seasonal pattern. In the primary housing market, it is easy to detect that home buying increases in the spring and gradually reaches a peak in the late summer. Then, there is a decline during the fall and winter. This activity is reflected in prepayment behaviour caused by turnover of housing as homebuyers sell their existing homes and purchase new ones. Prepayments are low in the winter months, increases during the

spring and early summer. Though the activity is reflected in prepayments, the peak may not be observed until the early fall due to delays in passing through prepayments.

#### *5.2.4 General Economic Activity*

It is no surprise that economic theory suggests that general economic activity has a distinct influence on prepayment behaviour through its effect on housing turnover. A growing economy causes an improvement in opportunities for worker migration and a rise in personal income. As a result, family mobility increases and causes in turn housing turnover to rise. For a weak economy, the exact opposite holds.

“Although some modelers of prepayment behaviour may incorporate macroeconomic measures of economic activity (...), the trend has been to ignore them or limit their use to specific applications” (Fabozzi & Ramsey 1999:38).

The reason that they have been ignored is two folded. First, empirical tests show that in the degree that the relationship between residuals of prepayment forecasting models (that does not include such macroeconomic measures) and various macroeconomic measures is statistically significant the explanatory power is low. Second, as showed later, prepayment models are based on projection of a path for future mortgage rates, and inclusion of macroeconomic measures would call for forecasting of the value of these variables over long time periods.

Still, some researchers have included macroeconomic measures when modelling prepayment.

### 5.3 Prepayment Models and Projections

After the assessment of different factors that are expected to affect prepayment behaviour, we are able to build a prepayment model.

The model building starts by modelling the statistical relationship between the factors found. Sometimes it might be tempting to think that the more factors we include, the more capable the model gets in explaining the variations in prepayment rates. This is often true, but at some point the gain from adding another factor is so small that it is not necessarily worth the extra

effort. In many cases it is sufficient to use only a few factors. In fact, one study quoted by Fabozzi & Ramsey (1999:39) suggests that the four factors that we focused on in section 5.2 above explain about 95% of the variation in prepayment rates.

After finding the factors that explain the variations in prepayment rates sufficiently, we combine them into one model.

As we will see in section 7.3, the practice in prepayment modelling has been to generate a path of monthly interest rates that is consistent with the prevailing term structure of interest rates. Based on these interest rate paths, and through an assumed relationship between short-term and long-term interest rates, we are able to generate monthly mortgage refinancing rates. Finally, prepayment rates caused by refinancing incentives and burnout are projected. Accordingly, the prepayment projection is contingent on the interest rate path projection.

Finally, we note that when conducting a prepayment forecast, the procedure is similar to that of prepayment modelling. As mentioned above, we generate a set of prepayment rates for each of the remaining months of the mortgage pool, and then convert the set of prepayment rates into a PSA speed.

## 6. Collateralized Mortgage Obligations

In chapter 4 we saw that investors are able to acquire a share in pass-throughs, and thereby achieve exposure to the prepayment risk spread over the mortgage loans of the pool rather than one individual loan. We also saw that the investor still is exposed to the total prepayment risk associated with the underlying pool of mortgage loans. Some investors do not want to be exposed to prepayment risk and therefore pass-through securities are not attractive products for these investors. Due to this fact, there have been developed securities where the investors do not share the prepayment risk equally.

In the following chapter, I will look closer at the security constituting the core of this thesis – the *Collateralized Mortgage Obligation*. This innovation distributes the prepayment risk of mortgages disproportionately between the investors.

I start by discussing how a CMO can be created from a pass-through security. Then I will present different CMO structures. The main focus of this thesis is on CMOs issued by agencies, the CMOs which redistribute or “tranche” prepayment risk. But, as mentioned in section 4.2, I will briefly discuss the most popular method to augment the credit quality of non-agency issues.

After this section, I will turn the attention towards the valuation of CMOs.

### 6.1 The CMO Innovation

In figure 4.3, we saw how a pass-through security is created. The cash flows from 1,000 mortgage loans were gathered into a mortgage pool and then distributed on a pro rata basis. We could instead distribute the principal (both scheduled and prepayments) on some prioritized basis.

Figure 6.1 illustrates how this can be done. Initially, we have the cash flow from the pass-through, which we remember from figure 4.3 is distributed on a pro rata basis. We then construct a CMO with three classes of bonds or tranches. The cash flow from the pass-through is then distributed to the three tranches according to a set of payment rules. We also



We have now created a mortgage-backed security called a *collateralized mortgage obligation*. The collateral for a CMO may be one or more pass-throughs or a pool of mortgages that have not been securitized. Anyhow, the ultimate source of the CMO's cash flow is a pool of mortgage loans.

Once again, the total prepayment risk has not changed. The total prepayment risk of the CMO is the same as the total prepayment risk of the original 1,000 mortgage loans. However, with this construction, the prepayment risk has been distributed disproportionately across the three tranches. Tranche A absorbs prepayments first, then Tranche B, and last Tranche C. This effectively makes Tranche A a shorter term security than the other two, and Tranche C the security with longest maturity.

The benefit of the CMO innovation is apparent: “(...), redirection of the cash flow from the underlying mortgage pool creates tranches that satisfy the asset/liability objectives of certain institutional investors better than pass-throughs” (Fabozzi & Ramsey 1999:5).

The CMO structure illustrated in figure 6.1 is a simple one. Today, there are much more complicated structures available. Still, “the basis objective is to provide certain CMO classes with less uncertainty about prepayment risk. Note, of course, that this can occur only if the reduction in prepayment risk for some tranches is absorbed by other tranches in the CMO structure” (Fabozzi & Ramsey 1999:5).

In the next section, I will discuss some of the different CMO structures available.

## 6.2 Different Types of CMO Structures

As we have seen, the creation of a CMO cannot eliminate prepayment risk; it can only redistribute the risk among different classes of bonds called *tranches*. This redistribution leads to instruments with different price performance characteristics that may be more suitable for the particular needs and expectations of investors.

In the following section, I will describe some of the most common structures. The first two – *sequential-pay* and *accrual* – are two simple types of tranches. *Floater*s and *inverse floater*s offer tranches with floating coupon rates. *Structured principal-only* and *structured interest-only* spilt the interest payments and the principal payments into different tranches. Finally, I

will look closer at *planned amortization class*. Planned amortization class have a principal repayment schedule that must be satisfied. It therefore offers greater predictability of the cash flow to the tranche.

Before we look closer at the different structures and tranches available, I will just make a comment regarding the coupon rate of a CMO structure. The coupon rate of all the tranches in a CMO structure can be the same. However, there is no reason for this to be true; in fact it is typically not the case. “The coupon rate is commonly set equal to a rate that will make the tranche’s price at issuance trade close to par. So, in an upward sloping yield curve environment, the longer the average life, the higher the coupon rate” (Fabozzi & Ramsey 1999:52). In the following I will assume that the coupon rate of different tranches is the same.

In addition to the structure discussed in this section, there exists a wide range of structures, for example, *TAC*, *VADM*, *support tranches*, and *re-REMICs*<sup>6</sup>, that I will not discuss in this thesis.

### 6.2.1 Sequential-Pay

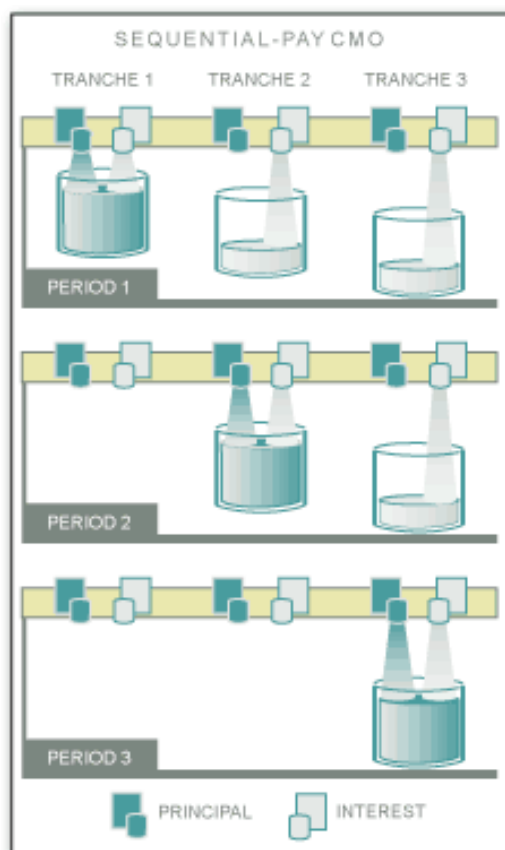
The *sequential-pay* was the first generation of CMOs and was structured so that each tranche (i.e., bond class) would be retired sequential. The CMO structure illustrated in figure 6.1 is an example of such a structure.

We saw in figure 6.1 that the disbursement of the monthly principal received from the underlying pass-through was made in a special way. In a sequential-pay structure, a tranche is not entitled to receive principal until the entire principal of the tranche before has been paid off. The procedure of the disbursement described above is illustrated in figure 6.2 below.

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<sup>6</sup> A description of these structures can be found in chapter 7 of Fabozzi & Ramsey (1999).





*Figure 6.2: Sequential-Pay CMO.*

While the priority rules of the payment of monthly principal is known, the precise amount is not. As thoroughly stated already, this will depend on the monthly payments of the collateral and thereby on the prepayment rate of the collateral. The cash flow can be projected as described in chapter 5, through an assumed PSA speed. With this given PSA speed, the cash flow from a given sequential-pay CMO can be exhibited graphically as in figure 6.3.

As we see from figure 6.3, all the tranches receive interest from day one. The principal payment, however, is first distributed to Tranche A, secondly Tranche B, and finally to Tranche C. This corresponds to figure 6.2. We see that the cash flow amount received each month peaks at about 30 months. This is in accordance with the PSA prepayment benchmark.

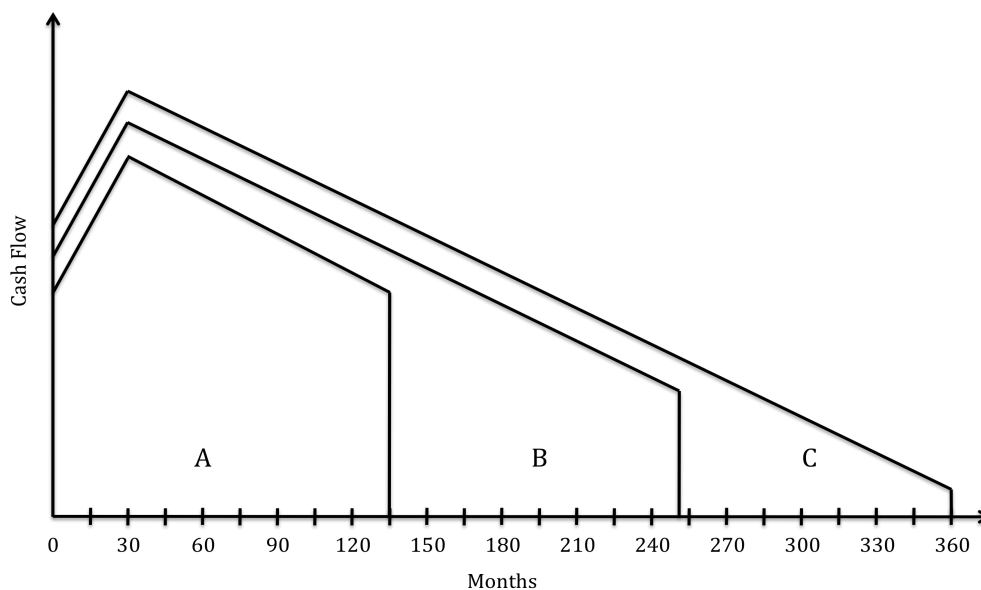


Figure 6.3: Projected Cash Flow for a Sequential-Pay CMO (assuming a given PSA speed).

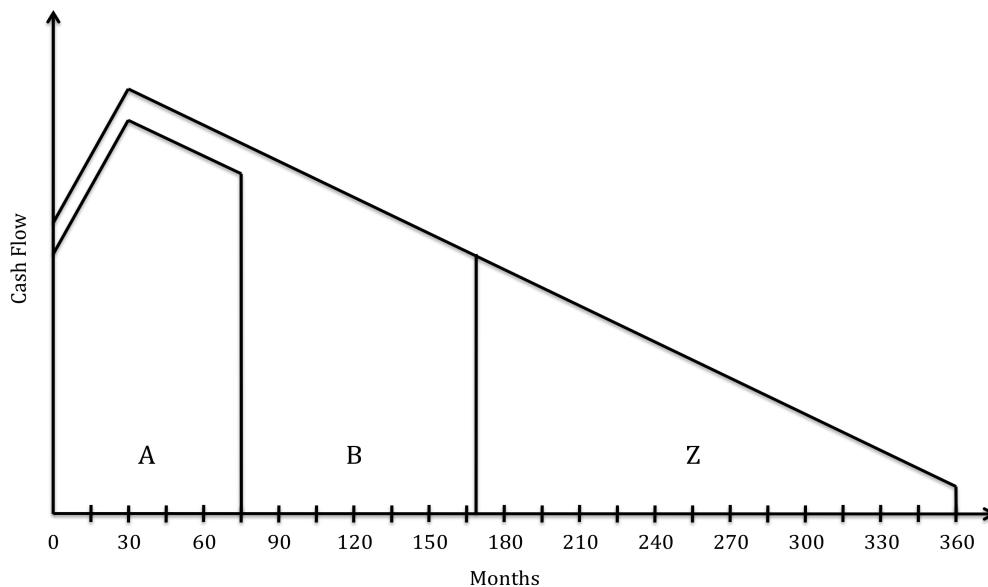
“The *principal paydown window* for a tranche is the time period between the beginning and the ending of the principal payments to that tranche” (Fabozzi & Ramsey 1999:46). For Tranche A in figure 6.3, the principal pay down window is 135 months.

By comparing the average life of the underlying mortgage pool and the average life of the different tranches, we are able to see the outcome of prioritizing the distribution of principal. The average life of shorter-term tranches will have an average life that is shorter than the underlying mortgage pool, and longer-term tranches will have longer average life. Establishing prioritized payment rules therefore effectively protects the shorter-term tranche in the structure against extension risk. This protection comes from the other tranches. Similarly, the longer-term tranche is protected against contraction risk.

### 6.2.2 Accrual Tranches

In the sequential-pay structure described above, the payment rules for interest provide for all tranches to be paid interest each month. This is not true for all sequential-pay CMO structures. In many cases there is at least one tranche that does not receive current interest. The interest that should have been received is instead added to the outstanding principal balance. Such tranches are commonly referred to as an *accrual tranche*, or a *Z bond* (because

they are similar to zero-coupon bonds). The effect of constructing accrued tranches is that the interest is instead used to speed up the pay down of the principal balance of the other tranches. Therefore the expected final maturity of the other tranches is shortened as a result of the inclusion of an accrued tranche. This is illustrated in figure 6.4.



*Figure 6.4: Projected Cash Flow of a Sequential-Pay CMO with an Accrued Tranche, Z.*

Due to this new structure, both the principal pay down window and the average life of the other tranches are reduced compared to the plain sequential-pay structure of figure 6.3. Correspondingly, the accrued tranche has a longer principal pay down window and a longer average life than Tranche C in the previous figure.

### *6.2.3 Floater and Inverse Floater Tranches*

The sequential-pay structure and the accrued tranches discussed above offered a fixed coupon rate for all tranches. Many financial institutions prefer floating-rate assets because they in many cases can offer a much better match for their liabilities. In this respect, the market for CMOs would be very limited if only fixed-rate coupon tranches could be created.

A *floater* is a tranche that receive a floating coupon rate based on some market interest rate, for example the LIBOR (London Interbank Offered Rate). Such a tranche will better suit for example depository institutions, because the liabilities float with market interest rates.

But can a floating-rate tranche be created from fixed-rate collateral? Given that the underlying collateral pays a fixed interest rate, this seems like an impossible task. However, it is in fact possible to construct such tranches.

To create a floating-rate tranche, we need to overcome the drawback that the cash flows are based on fixed-rate collateral. By using an inverse floating-rate tranche (i.e., *inverse floater*), we are able to overcome this drawback. “The coupon rate of an inverse floater changes in the opposite direction from the reference rate used to set the coupon rate for the corresponding floater” (Fabozzi & Ramsey 1999:56). Thereby, by including an inverse floater in the CMO structure, we are able to create a floater.

But there is a cap or maximum coupon rate that can be paid to the floater tranche. Unless there is a cap, the coupon rate of the inverse floater can become negative. This implies that the investor in an inverse floater tranche can be forced to pay interest on his investment. This is not reasonable. To deal with this, it is therefore necessary to set a floor (i.e., a minimum coupon rate) on the inverse floater tranche.

#### 6.2.4 *Principal-Only and Interest-Only Tranches*

In section 4.4 we looked at stripped MBSs. These were securities created by paying the entire principal to one bond class and all the interest to another bond class. These bonds are referred to as principal-only and interest-only bonds.

CMO structures that have tranches receiving only the principal or only the interest can also be created. The PO and IO tranches are commonly referred to as *structured POs* and *structured IOs* to distinguish them from IO mortgage strips. Structured POs and IOs can be constructed in several different ways.

### 6.2.5 Planned Amortization Class Tranches

*Planned amortization class bonds* or *PAC tranches* have the characteristic that as long as the prepayments stay within a specified range, a schedule of principal payments could be realized. This means that there is a principal repayment schedule that must be satisfied as long as the prepayments stay within the range. This leads to a greater predictability of the cash flow for these classes of bonds or tranches.

PAC bondholders have priority over all other classes in the CMO issue in receiving principal payments from the underlying collateral. The fact that this type of tranche offers greater predictability, must lead to greater uncertainty for some other non-PAC classes. These classes are referred to as *support tranches* or *companion tranches*.

The basic idea of the PAC tranche is that, should the actual principal repayment be greater than the scheduled amount, the support tranche receive the excess. If the actual principal repayment falls short of the scheduled amount, the PAC holders have priority on subsequent principal payments from the collateral. This protects the PAC tranche against both contraction and extension risk. It is therefore said that the PAC tranches provide *two-sided prepayment protection*.

## 6.3 Non-Agency CMOs

Non-agency securities are issued by private entities, usually mortgage conduits, commercial banks, savings and loan associations, mortgage lenders or investment banking related firms. These entities use mortgage loans that are not eligible for agency guarantees, e.g., mortgages for commercial properties or multifamily homes and mortgages over a certain capped value (approximately \$200,000).

Because non-agency securities are not backed by government guarantees, investors in these securities are exposed to credit risk. This further complicates the analysis of CMOs.

Non-agency securities are rated by commercial rating agencies like Moody's, Standard & Poor's, Duff & Phelps, and Fitch IBCA. Through standards set by these rating agencies, there are ways in which issuers of non-agency securities can augment the credit quality of their securities.

As mentioned earlier, the focus of this thesis is on agency CMOs. Still, in this section, I will take a closer look at non-agency CMOs. First, I will look at the collateral for such securities and discuss the difference between agency and non-agency CMOs. Before turning the attention towards the valuation methodology for CMOs, I will then take a quick look at the most popular method to augment the credit quality of non-agency issues.

### *6.3.1 The Collateral for Non-Agency CMOs*

The mortgage loans that are used as collateral for agency CMOs are conforming loans. This implies that they must meet the underwriting standards for the agency. The collateral for non-agency CMOs, on the other hand, consists of non-conforming loans. There are several reasons why a loan may be non-conforming:

- The mortgage balance exceeds the amount permitted by the agency.
- The borrower characteristics fail to meet the underwriting standards established by the agency.
- The loan characteristics fail to meet the underwriting standards established by the agency.
- The applicant fails to provide full documentation as required by the agency.

A mortgage loan that is non-conforming merely because the mortgage balance exceeds the maximum permitted by the agency, is called a *jumbo loan*.

When it comes to the characteristics of the borrower, a loan may fail to qualify because the borrower's credit history does not meet the underwriting standards or the *payment-to-income* (PTI) ratio exceeds the maximum permitted.

A characteristic that may result in a loan failing to meet the underwriting standards, is that the *loan-to-value* (LTV) ratio exceeds the maximum established by the agency, or that the loan is not a first-mortgage lien.

In assessing whether a loan qualifies for conforming classification, the agencies require documentation (verification) of the information provided in the loan application. These include documents to verify PTI and LTV. Failure to provide adequate documentation will result in a loan failing to conform.

For borrowers seeking non-conforming loans for any of the above-mentioned reasons, there exist alternative lending programs. For example, there are originators that have special lending programs for jumbo loans, and it exist lenders who specialize in loans that exceed the maximum LTV. There also exist originators who will provide a loan based on no documentation (*'no-doc loan'*) or limited documentation (*'low-doc loan'*) with respect to verification of income.

Although the borrowers of non-conforming loans have failed to meet the requirements of the agencies, they are not necessarily subprime borrowers. Only the worst borrowers qualify for this description.

### *6.3.2 Differences between Agency and Non-Agency CMOs*

In addition to the conforming and non-conforming loans already described, there are several differences between agency and non-agency CMOs. In the following section, I will shortly discuss some of these differences. The major differences have to do with *guarantees*, *dispersion of the characteristics of the underlying collateral*, *servicer advances*, *compensating interest* and *clean-up calls*.

Agency CMOs are created from pools of pass-through securities. In the non-agency market, a CMO can be created from either a pool of pass-throughs or unsecuritized mortgages loans. The most common in the non-agency market, is to use the latter, mortgage loans that have not been securitized as pass-throughs. Since a mortgage loan is commonly referred to as a whole loan, non-agency CMOs are commonly referred to as *whole-loan CMOs*.

As already mentioned several times, a non-agency CMO has no explicit or implicit government guarantee of payment of interest and principal, as there is for agency securities. This absence of guarantee means that the investor in a non-agency security is exposed to credit risk.

While both agency and non-agency CMOs are backed by one-to-four-single family residential mortgages, the underlying loans for non-agency securities will typically be more heterogeneous with respect to coupon rate and maturity of the individual loans. Non-agency CMOs may have both 15-year and 30-year mortgages included in the underlying mortgage pool. This result in much more uncertainty regarding the prediction of prepayments due to

refinancing based on the pool's *weighted average coupon* (WAC). In addition, there are issues in which the underlying collateral is mixed with various types of mortgage-related loans, for example collateral that is a combination of first-lien mortgages, home equity loans, manufactured housing loans, and home improvement loans.

Earlier we saw that the servicer is responsible for the collection of interest and principal, which is passed along to the trustee. The servicer also handles delinquencies and foreclosure. There are typically a master servicer and sub-servicers dealing with a CMO. These entities play a critical role in assessing the credit risk of a non-agency CMO, and therefore the quality of the servicers is examined closely by rating agencies.

Compensating interest is a feature that is unique for non-agency CMOs. Due to the fact that non-agency CMOs are not guaranteed by the government, and that homeowners may prepay their mortgage on any day throughout the month, investors may end up with less than a full month of interest. It is this phenomenon that is known as *payment interest shortfall* or *compensating interest*. Different issuers handle compensating interest differently.

All non-agency CMOs are issued with '*clean-up*' *call provisions*. This clean-up call provides the servicers, or the residual holders (typically the issuer), a call option on all the outstanding tranches of the CMO structure when the CMO balance is paid down to a certain percentage of the original principal balance.

There are also different types of structures and tranches for non-agency CMOs. Since my focus in this thesis is on the agency CMOs, I will not discuss them here.<sup>7</sup>

### 6.3.3 How to Augmenting the Credit Rating of Non-Agency CMOs

The nationally recognized statistical rating organizations rate non-agency securities. The rating of the securities is affected by different factors, one of them being the rating of the issuer of the security. The issuers of non-agency securities can augment the credit quality of the bonds in several ways according to standards set by the rating agencies.

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<sup>7</sup> A description of these structures can be found in chapter 8 of Fabozzi & Ramsey (1999).



In this final section of chapter 6, I will take a look at the most popular method used to increase the credit quality of mortgage-backed securities, namely through the recognizable structure of another type of pool-backed security, the *Collateralized Debt Obligation* (CDO).

A CDO is “a way of packing credit risk” (Hull 2006:744). As with the creation of agency CMOs, several classes of tranches is created from a portfolio of bonds and there are rules for determining how the costs of defaults are allocated to classes. It is usual to separate the tranches into three main categories:

- Senior tranche
- Mezzanine tranche
- Subordinate tranche

Furthermore, each tranche can be divided into one or more junior tranches.

In accordance with the rules for allocation of defaults, the tranches have different priority regarding the cash flow from the underlying portfolio of mortgages (and thereby different credit risk characteristics). The senior tranche has the highest priority and the subordinate tranche has the lowest. This implies that the senior tranche has the lowest credit risk and the lowest expected return.

The rules for allocation of defaults typically imply that the different tranches are given *strike levels*. These strike levels determine the credit risk, and thereby the expected return, inherit in the tranche. Typically, the subordinate tranche has a strike level of 0%, the mezzanine has a strike level of 3%, and the senior tranche has a strike level of 10% (and usually a *strike-out level* of 30%). This means that subordinate tranche must absorb all defaults within the interval 0-3%. If the total loss, due to defaults, in the underlying portfolio exceeds 3%, then the tranche will be worthless.

If the total loss in the underlying portfolio exceeds the strike-out level of the senior tranche (usually 30%), the issuer of the CDO usually is the one who must absorb the exceeding losses.

In addition to the above-described method, issuers can also obtain insurance, corporate guarantees or letters of credit from insurance companies or banks. The rating is then partially dependent upon the rating of the insurer.

## 7. Valuation Methodology

So far, I have focused on explaining what a CMO is and how you can construct a CMO from a set of mortgage loans. Hopefully, the reader has acquired a basic knowledge of CMOs, which in turn enables him to keep up when we now turn our attention towards the theoretical valuation of CMOs.

When it comes to fixed income valuation modelling, there are commonly two methodologies that are used to value securities with embedded options:

- The Binomial Model
- The Monte Carlo Model

For some securities, the decision to exercise an option is not dependent on how interest rates evolve over time. “That is, the decision of an issuer to call a bond will depend on the level of the rate at which the issuer can be refunded relative to the issue’s coupon rate, and not the path interest rates took to get to that rate” (Fabozzi & Ramsey 1999:166). For others, the periodic cash flows are interest rate path-dependent, meaning that the cash flows also is determined by the path that interest rates took to get to the current level.

The Monte Carlo model is the most flexible valuation methodology for valuing interest sensitive instruments where the history of interest rates is important. In the following, I will therefore concentrate on the Monte Carlo model.

“Conceptually, the valuation of passthroughs using Monte Carlo model is simple. In practice, however, it is very complex” (Fabozzi & Ramsey 1999:166). The procedure implies generating a set of cash flows to the holder of the pass-through. These cash flows are based on simulated future mortgage refinancing rates and prepayment rates.

The valuation procedure gets even more difficult when we implement it for CMOs. Through the securitization of the pass-through, both the prepayment risk and the interest risk have been separated into different tranches. “The sensitivity of the passthrough comprising the collateral to these two risks is not transmitted equally to every tranche” (Fabozzi & Ramsey 1999:166). The result is that some tranches wind up more sensitive of prepayment risk and interest risk than the collateral, while others are much less sensitive.

For investors in CMOs, the objective is to find out which tranches they want to purchase. To do this, it is crucial to find out how the value of the collateral gets transmitted to the CMO tranches, more specifically where the value and risk go. Only in this way, the investors are able to identify the tranches with low risk and high value.

As we have seen earlier, non-agency CMOs can incorporate credit risk, and thus the pool might also be sliced into tranches absorbing defaults as well. A valuation model for non-agency CMOs therefore requires another input, namely a default model. This complicates the valuation modelling even further. The principles of valuation of default absorbing tranches are therefore best covered in a study of CDOs.<sup>8</sup> I will consequently focus on the valuation methodology of agency CMOs.

## 7.1 Motivating Monte Carlo Simulation

The prepayment of a pass-through security for a given month depends on whether there have been prior opportunities to refinance since the underlying mortgage was originated. In this way, the prepayments of pass-through securities are interest rate path-dependent. Earlier, we have seen that pools of pass-throughs are used as collateral for CMOs and therefore they are fundamental for the creation of CMOs. Consequently, there is path-dependency in a CMO tranche's cash flow as well.

The path-dependency in the cash flows has two sources. First, there is the path-dependency in the collateral prepayments as discussed above. Secondly, the cash flows of a tranche are dependent on the outstanding balance of the other tranches. Therefore we also need the history of prepayments to estimate balances.

In the finance literature there is great consensus that the Monte Carlo model is a good tool for valuing securities where the value is path-dependent. In the introduction to a chapter on Monte Carlo valuation, for example, McDonald says: "There is no simple valuation model for such options<sup>9</sup>, and the binomial pricing approach is difficult because the final payoff depends on the specific path the stock price takes through the tree – i.e., the payoff is path-dependent.

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<sup>8</sup> For Norwegian readers interested in the valuation principles of CDOs, I recommend the thesis by Myklebust & Li (2007).

<sup>9</sup> McDonald is here referring to arithmetic Asian options, which are path-dependent.

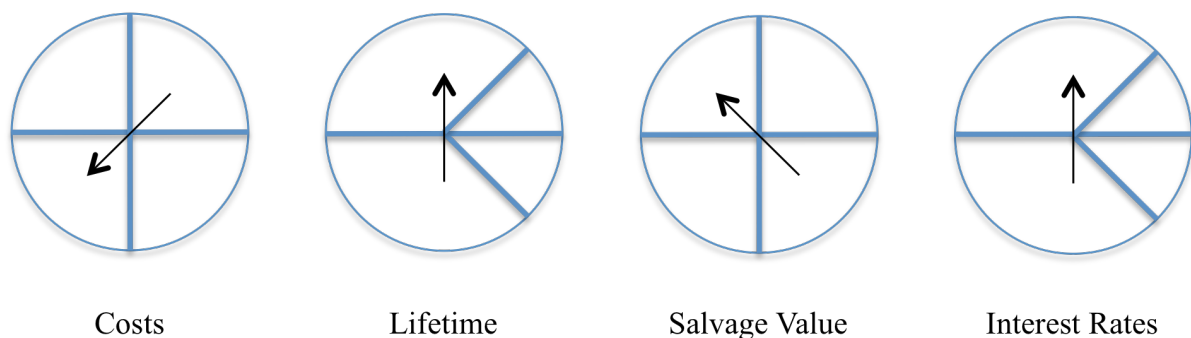
A pricing method that can be used in such cases is Monte Carlo valuation” (McDonald 2006:617).

Also in the case of CMOs, the Monte Carlo simulation model is an appropriate valuation method. In fact, Fabozzi & Ramsey (1999:163) describes the Monte Carlo simulation model as “... the state-of-the-art methodology for valuing CMOs ...”

## 7.2 Monte Carlo Simulation

Monte Carlo simulation is a simulation technique that has been used in various areas of risk analysis for decades, for instance for capital investment analysis and for valuation of derivatives. The technique draws its name from the use of randomly drawn variables, but with probability of each draw controlled to approximate the actual probability of occurrence.

To illustrate the main idea of Monte Carlo methodology, let’s think of an investment decision where we have four variables that the management are uncertain about: a) *costs*, b) *lifetime*, c) *salvage value* and d) *interest rates*. Furthermore, let’s say that each variable can have four outcomes. For each of the variables we create a roulette wheel and divide them in parts depending on the probability of each outcome as shown in the figure below.



*Figure 7.1: Roulette Wheels.*

For the variables *costs* and *salvage value*, we see that the probability of each of the four outcomes is 25%. For the other two variables, we have two outcomes with probability 37.5%

and two outcomes with probability 12.5%. When the wheel is spun, the probability of the wheel stopping on a particular outcome is the same as the probability of that outcome.

The main idea of the methodology is as follows: Each of the wheels is spun once to provide values of each variable. Based on these values the net present value of the project is computed. The wheels are spun again, and a new net present value is computed. This procedure is repeated several hundred, or even thousand, times. Each of these repetitions is referred to as *iteration*. After a large number of iterations, the portion of iterations that result in a particular net present value (or range of net present values) approximately equals the probability of that net present value (or range) occurring. For derivatives, it would be the value of the security instead of the net present value we would like to find. In this case, the variables could for instance be interest rates, volatility and stock prices.

It would be tedious to go through thousands of iterations like the ones explained above, and naturally; it would be even more tedious the more variables there are. Luckily, we have the computer!

In the case of Monte Carlo simulation for valuing CMOs, a sufficiently large number of potential interest rate paths are simulated. This is done in order to assess the value of the security along different paths. I will now explain this procedure in more detail.

### 7.3 Simulating Interest Rate Paths and Cash Flows

The starting point of the Monte Carlo valuation methodology is to generate random interest rate paths. To do this, we need a model of the evolution of interest rates. The typical approach used by Wall Street and commercial vendors is to use a model that takes as input today's term structure of interest rates, and to make an assumption about the volatility of interest rates. The term structure of interest rates is the theoretical spot rate (or zero coupon) curve implied by today's Treasury securities (or similar security).

Each interest rate model has its own underlying belief concerning the evolution of future interest rates and their own volatility assumptions. Typically, there is small or no significant divergence between the different interest rate models used by dealer firms and vendors, although their volatility assumptions can be significantly different.

“The volatility assumption determines the dispersion of future interest rates in the simulation” (Fabozzi & Ramsey 1999:167). It is no longer typical to use one volatility number for the yield of all maturities on the yield curve. Instead, many vendors today typically use either a short/long yield volatility or a term structure of yield volatility. The first type means that volatility is specified for maturities up to a certain number of years (short yield volatility) and a different yield volatility for greater maturities (long yield volatility). The short yield volatility is assumed to be greater than the long yield volatility. The second type, a term structure of yield volatilities, means that the yield volatility is assumed for each maturity.

The model used to generate random paths of interest rates should replicate today’s term structure of interest rates, and for all future dates there is no possible arbitrage within the model. This means that the random paths are generated from an arbitrage-free model of the future structure of interest rates.

As stated earlier, the simulation starts by generating many scenarios of future interest rate paths. This means that in each month of the scenario (i.e., path), a monthly interest rate and a mortgage-refinancing rate are generated. The monthly interest rates are used to determine the refinancing rates and to discount the projected cash flows in the scenario. The mortgage-refinancing rate is needed to determine the cash flow because it represents the opportunity cost the mortgagor is facing at that time.

The link between the mortgage refinancing rates and the mortgagor’s actual incentive to refinance is strongly related to the mortgagor’s original coupon rate (i.e., the rate on the mortgagor’s loan). If the refinancing rates are high relative to the original coupon rate, the mortgagor will have less incentive to refinance, or even a positive disincentive (i.e., the homeowner will avoid moving in order to avoid refinancing). If, on the other side the refinancing rates are low relative to the original coupon rate, the mortgagor has an incentive to refinance.

Prepayments are projected according to the procedures described in section 5.3 (i.e., by feeding the refinancing rate and the loan characteristics into a prepayment model). Given these projected prepayments, the cash flows along an interest rate scenario can be determined.

To make the procedure more concrete we will look at a simple illustration. Consider a pass-through security newly issued, with a maturity of 360 months. Table 7.1 below, illustrates  $N$  simulated interest rate scenarios. As we see, each scenario consists of a path of 360 1-month

future interest rates. This indicates that the first assumption we would make in our analysis is about the volatility of interest rates.

Month	Interest Rate Path Number						
	1	2	3	...	$n$	...	$N$
1	$f_1(1)$	$f_1(2)$	$f_1(3)$	...	$f_1(n)$	...	$f_1(N)$
2	$f_2(1)$	$f_2(2)$	$f_2(3)$	...	$f_2(n)$	...	$f_2(N)$
3	$f_3(1)$	$f_3(2)$	$f_3(3)$	...	$f_3(n)$	...	$f_3(N)$
...	...	...	...	...	...	...	...
$t$	$f_t(1)$	$f_t(2)$	$f_t(3)$	...	$f_t(n)$	...	$f_t(N)$
...	...	...	...	...	...	...	...
358	$f_{358}(1)$	$f_{358}(2)$	$f_{358}(3)$	...	$f_{358}(n)$	...	$f_{358}(N)$
359	$f_{359}(1)$	$f_{359}(2)$	$f_{359}(3)$	...	$f_{359}(n)$	...	$f_{359}(N)$
360	$f_{360}(1)$	$f_{360}(2)$	$f_{360}(3)$	...	$f_{360}(n)$	...	$f_{360}(N)$

Table 7.1: Simulated Paths of 1-Month Future Interest Rates.

Month	Interest Rate Path Number						
	1	2	3	...	$n$	...	$N$
1	$r_1(1)$	$r_1(2)$	$r_1(3)$	...	$r_1(n)$	...	$r_1(N)$
2	$r_2(1)$	$r_2(2)$	$r_2(3)$	...	$r_2(n)$	...	$r_2(N)$
3	$r_3(1)$	$r_3(2)$	$r_3(3)$	...	$r_3(n)$	...	$r_3(N)$
...	...	...	...	...	...	...	...
$t$	$r_t(1)$	$r_t(2)$	$r_t(3)$	...	$r_t(n)$	...	$r_t(N)$
...	...	...	...	...	...	...	...
358	$r_{358}(1)$	$r_{358}(2)$	$r_{358}(3)$	...	$r_{358}(n)$	...	$r_{358}(N)$
359	$r_{359}(1)$	$r_{359}(2)$	$r_{359}(3)$	...	$r_{359}(n)$	...	$r_{359}(N)$
360	$r_{360}(1)$	$r_{360}(2)$	$r_{360}(3)$	...	$r_{360}(n)$	...	$r_{360}(N)$

Table 7.2: Simulated Paths of Mortgage-Refinancing Rates.

For each of the interest rate path in table 7.1, we need to generate corresponding paths of simulated mortgage-refinancing rates. This is exhibited in table 7.2 above. To do this we need to make an assumption about the relationship between the Treasury rates and refinancing rates. The assumption taken is that there is a spread between the mortgage-refinancing rate and the Treasury yield. These mortgage-refinancing rates give us the fundament to generate the cash flows from the pass-through on each interest rate path. To do this we need a prepayment model.

After construction of a prepayment model, we need to make another assumption, namely that the prepayment model is correct. We then generate cash flows as depicted in table 7.3 below.

Month	Interest Rate Path Number						
	1	2	3	...	$n$	...	$N$
1	$C_1(1)$	$C_1(2)$	$C_1(3)$	...	$C_1(n)$	...	$C_1(N)$
2	$C_2(1)$	$C_2(2)$	$C_2(3)$	...	$C_2(n)$	...	$C_2(N)$
3	$C_3(1)$	$C_3(2)$	$C_3(3)$	...	$C_3(n)$	...	$C_3(N)$
...	...	...	...	...	...	...	...
$t$	$C_t(1)$	$C_t(2)$	$C_t(3)$	...	$C_t(n)$	...	$C_t(N)$
...	...	...	...	...	...	...	...
358	$C_{358}(1)$	$C_{358}(2)$	$C_{358}(3)$	...	$C_{358}(n)$	...	$C_{358}(N)$
359	$C_{359}(1)$	$C_{359}(2)$	$C_{359}(3)$	...	$C_{359}(n)$	...	$C_{359}(N)$
360	$C_{360}(1)$	$C_{360}(2)$	$C_{360}(3)$	...	$C_{360}(n)$	...	$C_{360}(N)$

Table 7.3: Simulated Cash Flows on Each of the Interest Rate Paths.

#### 7.4 Calculating the Present Value for a Scenario Interest Rate Path

After simulation of the interest rate paths and the corresponding cash flows, we are able to calculate the present value of each of the paths. When calculating the present value of the paths, we need a discount rate. The discount rate used is the simulated spot rate for each month on the interest rate path plus an appropriate spread. The simulated spot rate can be determined from the simulated future monthly rates. We have the following relationship between the simulated spot rate for month  $T$  on path  $n$  and the simulated future 1-month rate:

$$(7.1) \quad z_T(n) = \left\{ [1 + f_1(n)][1 + f_2(n)] \dots [1 + f_T(n)] \right\}^{1/T} - 1$$

where  $z_T(n)$  = simulated spot rate for month  $T$  on path  $n$ , and

$f_j(n)$  = simulated future 1-month rate for month  $j$  on path  $n$ .



By using equation (7.1) above, we can convert the interest rate path for the simulated future 1-month rates as shown in table 7.4.

Month	Interest Rate Path Number						
	1	2	3	...	$n$	...	$N$
1	$z_1(1)$	$z_1(2)$	$z_1(3)$	...	$z_1(n)$	...	$z_1(N)$
2	$z_2(1)$	$z_2(2)$	$z_2(3)$	...	$z_2(n)$	...	$z_2(N)$
3	$z_3(1)$	$z_3(2)$	$z_3(3)$	...	$z_3(n)$	...	$z_3(N)$
...	...	...	...	...	...	...	...
$t$	$z_t(1)$	$z_t(2)$	$z_t(3)$	...	$z_t(n)$	...	$z_t(N)$
...	...	...	...	...	...	...	...
358	$z_{358}(1)$	$z_{358}(2)$	$z_{358}(3)$	...	$z_{358}(n)$	...	$z_{358}(N)$
359	$z_{359}(1)$	$z_{359}(2)$	$z_{359}(3)$	...	$z_{359}(n)$	...	$z_{359}(N)$
359	$z_{360}(1)$	$z_{360}(2)$	$z_{360}(3)$	...	$z_{360}(n)$	...	$z_{360}(N)$

Table 7.4: Simulated Paths of Monthly Spot Rates

The present value of the cash flows for month  $T$  on interest rate path  $n$ , can now be calculated by discounting the cash flows at the simulated spot rate for month  $T$ , plus some spread:

$$(7.2) \quad PV[C_T(n)] = \frac{C_T(n)}{[1 + z_T(n) + K]^T}$$

where  $PV[C_T(n)]$  = present value of cash flow for month  $T$  on path  $n$ ,

$C_T(n)$  = cash flow for month  $T$  on path  $n$ ,

$z_T(n)$  = spot rate for month  $T$  on path  $n$ , and

$K$  = spread.

The sum of the present value of the cash flows for each month on path  $n$ , gives us the present value for the entire path  $n$ . That is,

$$(7.3) \quad PV[Path(n)] = PV[C_1(n)] + PV[C_2(n)] + \dots + PV[C_{360}(n)]$$

where  $PV[Path(n)]$  is the present value of the interest rate path  $n$ .

## 7.5 Determining the Theoretical Value

The present value of interest rate path  $n$  that we calculated in the previous section can be thought of as the theoretical value of a pass-through security if, and only if, path  $n$  was actually realized. We can use this to determine the theoretical value of the pass-through by calculating the average of the theoretical values of all interest rate paths:

$$(7.4) \quad V_0 = \frac{PV[Path(1)] + PV[Path(2)] + \dots + PV[Path(N)]}{N}$$

where  $V_0$  is the theoretical value of the pass-through and  $N$  is the number of interest rate paths.

For a CMO tranche, the same procedure is used for valuation. The principal repayment and interest distribution rules of the deal provide the cash flows for each month on each interest rate path. To do this, a structuring model is needed.

## 7.6 Distribution of Path Present Values

Both in business decisions and in the valuation of derivatives, the outcome depends on several random variables. As we have seen earlier, the Monte Carlo model is a commonly used, and functional, management science tool for such situations.

When conducting the simulation of the Monte Carlo model, the product is the average value and the probability distribution of the possible outcomes. Unfortunately, when used to value

fixed income securities like a CMO tranche, it has been limited to simply the reporting of the average value. This is referred to as the theoretical value of the security. In doing so, all the information about the distribution of the path present values have been ignored.

This information is quite important. “Therefore, before using the theoretical value for a mortgage-backed security generated from the Monte Carlo model, information about the distribution of the path present value should be obtained” (Fabozzi & Ramsey 1999:171).

## 7.7 Spread Measures

“The yield on any financial instrument is the interest rate that makes the present value of the expected cash flow equal to its market price plus accrued interest. For mortgage-backed securities, the yield is called a *cash flow yield*” (Fabozzi & Ramsey 1999:165). Due to the prepayment element of the cash flow from a mortgage-backed security, it is quite complicated to calculate the cash flow yield. To do this, it is necessary to make an assumption about the prepayment rate.<sup>10</sup>

After computing the cash flow yield and the average life, the next step is to compare the yield to the yield for a comparable Treasury security. That is, a Treasury security with the same maturity as the average life of the MBS. The difference between the cash flow yield and the yield on a comparable Treasury security is called the *nominal spread*.

In this section, I will look closer at the nominal spread. I will also discuss two other spread measures, namely the *zero-volatility spread* and a by-product of the Monte Carlo simulation model, the *option-adjusted spread*.

### 7.7.1 Nominal Spread

As already mentioned, the *nominal spread* is the difference between the cash flow yield and the yield on a comparable Treasury security.

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<sup>10</sup> A discussion of how to compute the cash flow yield of a mortgage-backed security can be found on page 164 in Fabozzi & Ramsey (1999).

Unfortunately, the nominal spread is the measure that some managers want to use as a measure of relative value. The problem with this is that the nominal spread masks the fact that a portion of this spread is compensation for accepting prepayment risk.

When deciding if they want to invest in a security, investors need a spread measure that indicates whether there is adequate compensation for the prepayment risk faced in that particular security.

### *7.7.2 Zero-Volatility Spread*

Another spread measure commonly quoted for MBSs is called the *zero-volatility spread*. This is a measure of the spread that the investor would realize over the entire Treasury spot rate curve if the mortgage-backed security were held to maturity. It is not a spread off one point on the Treasury yield curve, as is the nominal spread.

The zero-volatility spread (also called the *Z-spread* and the *static spread*) is the spread that will make the present value of the cash flow from the MBS, when discounted at the Treasury spot rate plus the spread, equal to the (market) price of the MBS. A trial-and-error procedure (or search algorithm) is required to determine the zero-volatility spread.

The difference between the nominal spread and the zero-volatility spread depends on the average life of the MBS (shorter average life indicates less difference between the two) and the shape of the yield curve (steeper yield curve indicates greater difference).

### *7.7.3 Option-Adjusted Spread*

The *option-adjusted spread* (OAS), a by-product of the Monte Carlo simulation model, indicates whether there is adequate compensation for the prepayment risk faced in a particular security. The OAS is the spread  $K$  that when added to all the spot rates (see equation (7.2)) on all interest rate paths will make the average present value of all paths equal to the observed market price (plus accrued interest). That is,

$$(7.5) \quad M_0 = \frac{PV[Path(1)] + PV[Path(2)] + \dots + PV[Path(N)]}{N}$$

where  $M_0$  is the observed market value of the pass-through and  $N$  is the number of interest rate paths. We see that equation (7.5) is almost exactly identical to equation (7.4). As with the zero-volatility spread, the procedure for determining the OAS is straightforward, but time consuming.

How can we interpret the OAS? Basically, as we have seen, the OAS is used to reconcile value with the observed market price. In equation (7.5), the left-hand side is the observed market value, and on the right-hand side of the equation, we have the output of the valuation model.

A basic principle for investors is to buy securities that have greater value than its price. By using the Monte Carlo model described in this chapter, investors are able to determine if the theoretical value is greater than the observed market price, and thereby determine if the security is an attractive investment object. In addition, through the OAS, the model converts the divergence between value and market price into a spread measure. Many investors see this as more convenient than looking at price differences.

The OAS is superior to the nominal spread measure because it gives recognition to the prepayment risk inherited in the bonds. “The OAS is ‘option adjusted’ because the cash flow on the interest rate paths are adjusted for the option of the borrowers to prepay” (Fabozzi & Ramsey 1999:172). It is possible to find the implied cost of this embedded option by calculating the difference between the OAS at the assumed volatility of interest rates and the zero-volatility spread. That is,

$$(7.6) \quad C_0 = Z_0 - OAS_0$$

where  $C_0$  is the option cost,  $Z_0$  is the zero-volatility spread and  $OAS_0$  is the option-adjusted spread. The option cost is a measure of the prepayment risk or option risk embedded in the

MBS. Note that the cost of the option is a by-product of the OAS analysis, not valued explicitly with some option-pricing model.

## 7.8 Selecting the Number of Interest Rate Paths

Before turning the attention towards an illustration of this valuation methodology, we address the question of the number of scenario paths,  $N$ , needed to value a CMO tranche. According to Fabozzi & Ramsey (1999:172), a typical analysis might be for 256 to 1,024 interest rate paths.

They also state that “the number of interest rate paths determines how ‘good’ the estimate is, not relative to the truth but relative to the model used” (Fabozzi & Ramsey 1999:173). By using more interest rate paths, the average spread tends to settle down. This means that we are faced with a statistical sampling problem.

One way to cut down on the sample paths necessary to get a good statistical sample, is to employ some form of *variance reduction*. Such techniques enables you to derive price estimates within a tick, meaning that if the model is used to generate more scenarios, price estimate from the model will not change by more than a tick. That is, if you for example use 1,024 paths to obtain the estimated price for a tranche, there will be little additional information from the model by generating more than that number of paths.

In valuing a CMO tranche using Monte Carlo simulation, there is no such thing as a correct number of paths. There is no doubt that the more paths you generate, the ‘better’ your estimate (relative to the model used) gets. However, by increasing the number of paths, you also increase the workload in the analysis. There may also be restrictions due to limitations in tools used to conduct the analysis. Therefore, different types of reduction techniques, as the variance reduction mentioned above, may be useful.

The important thing when determining the number of paths, is to generate enough paths, or use adequate reduction techniques, so that you are able to get a good statistical sample.

## 8. A Numerical Illustration of the Valuation Methodology

In the final chapter of this thesis, I will conduct a numerical valuation using the methodology introduced in chapter 7. To do this I will need to look at a specified CMO structure.

As we recall from section 6.2, the CMO structure of figure 6.1 is an illustration of a sequential-pay structure. Let us call this hypothetical structure for BAU-1 and use this to illustrate the Monte Carlo simulation model for a CMO.

As we have seen, the collateral for this structure is a hypothetical pass-through with a total par value of NOK 1 billion. Let us assume the following characteristics for the underlying collateral: a) the pass-through coupon rate is 7% and b) the *weighted average coupon* (WAC) of the underlying mortgages is 8%. We then use the NOK 1 billion collateral to create three tranches. The structure and payment rules for BAU-1 are shown in figure 8.1 below.

**BAU-1: Underlying collateral is pass-through with total par value of NOK 1 billion, pass-through coupon rate is 7%, and WAC of underlying mortgages is 8%.**

Tranche	Interest	Principal
A (NOK 400 million)	Pay each month based on par amount outstanding	Receive all monthly principal until completely paid off
B (NOK 300 million)	Pay each month based on par amount outstanding	After Tranch A paid off, receive all monthly principal
C (NOK 300 million)	Pay each month based on par amount outstanding	After Tranch B paid off, receive all monthly principal

*Figure 8.1: The Hypothetical Three-Tranche Sequential-Pay Structure BAU-1.*

As mentioned in the last section of chapter 4, there are numerous constructions backing CMOs. To project the cash flow from these underlying mortgages, we also need to make an assumption about which type of asset that is backing BAU-1. In an attempt to not make the illustration more complex than necessary, I will assume that the underlying mortgages are

level-payment mortgages. This means that they pay equal monthly instalments over the term of the mortgage. These instalments consist of roughly 1/12 of the fixed annual interest times the amount of outstanding balance at the beginning of the previous month, and a repayment of a portion of the outstanding balance (the principal). Furthermore, as mentioned several times, I assume that the underlying MBS is guaranteed by the full faith of the U.S. government (indicating a GNMA MBS), thereby excluding credit risk.

I now have a specified CMO structure to analyse. Before conducting the valuation through the Monte Carlo simulation model described in chapter 7, I need to determine how many interest rate scenarios to simulate. As we recall from section 7.8, the estimated value of a tranche becomes ‘better’ the more scenarios that are simulated. Without any variance reducing techniques, this implies using as many scenarios as possible. In section 7.8, we also saw that according to Fabozzi and Ramsey, a typical analysis might be for 256 to 1,024 interest rate paths.

The objective for this numerical analysis is to show the principles of the Monte Carlo simulation model and not to get ‘the best possible estimate of the value’. This would of course be of outmost importance for an investor thinking of investing in a given CMO. For simplicity, I therefore restrict the analysis to 256 interest rate scenarios.

I am now ready to conduct the valuation of the hypothetical CMO structure. I start by discussing different interest models and the simulation of interest rate paths.

## 8.1 Interest Rate Models and Interest Rate Simulation

There are several different models of the short-term interest rate. The simplest models are those in which the interest rate follows arithmetic or geometric Brownian motion. That is, for example

$$(8.1) \quad dr = a dt + \sigma dZ$$

In this specification, the short-rate is normally distributed with mean  $r_0 + at$  and variance  $\sigma^2 t$ .



When modelling the interest rate through such simple models, several undesirable characteristics occur. Some of the objectives against the model are:

- In this framework the short-rate can be negative. If this occurs in the real world, investors would prefer to hold money under a mattress instead of holding bonds. Therefore it is not reasonable to think that the *nominal* short-rate can be negative.
- The drift term in the short-rate is constant. This indicates that if  $a > 0$ , the short-rate will drift up over time forever. In practice, if the short-rate rises, we expect it to fall; i.e., it is *mean reverting*.
- The volatility of the short rate is the same whether the rate is high or low. In practice, however, we expect the short-rate to be more volatile if rates are high.

Because of these objections, there have been developed several different interest rate models. These models differ in their specifications of drift and volatility, and the differences can result in very different pricing implications.

In the following, I will discuss three different models of the short-term interest rate, namely the short-rate motions assumed in the bond pricing models by *Rendleman-Bartter*, *Vasicek* and *Cox-Ingersoll-Ross*.<sup>11</sup>

### 8.1.1 The Rendleman-Bartter Model

The Rendleman-Bartter model (Rendleman and Bartter 1980) assumes that the short-rate follows the geometric Brownian motion

$$(8.2) \quad dr = ar \, dt + \sigma r \, dz$$

In this model, interest rates can never be negative. However, the interest rate based on equation (8.2) can be arbitrary high. As already mentioned, we would expect rates to demonstrate mean reversion; if rates are high, we expect them on average to decrease. In the

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<sup>11</sup> The discussion of these models follows chapter 24.2 in McDonald, Robert L. (2006): *Derivatives Markets*, and not the original papers.

Rendleman-Bartter model, on the other hand, the probability of rates going up or down is the same whether rates are 100% or 1%.

### 8.1.2 The Vasicek Model

The Vasicek model (Vasicek 1977), on the other hand, takes a step towards dealing with mean reversion. The model assumes that the short-rate follows the geometric Brownian motion

$$(8.3) \quad dr = a(b - r) dt + \sigma dz$$

The equation above is an Ornstein-Uhlenbeck process.

It is the  $a(b-r)dt$  term that induce mean reversion. The parameter  $b$  is the level to which short-term interest rates revert. If  $r > b$ , the short-rate is expected to decrease, and vice versa. The parameter  $a$ , reflects the speed with which the interest adjusts to  $b$ . If  $a = 0$ , the short-rate is a random walk. If  $a = 1$ , the gap between the short-rate and  $b$  is expected to close in a year.

From the  $dz$ -term of equation (8.3) we see that interest rates can become negative. This is due to the fact that the term multiplying  $dz$  is simply the volatility, independent of the level of the interest rate. This also says that the variability of interest rates is independent of the level of rates.

Because both mean and variance in the Rendleman-Bartter model are proportional to the level of the interest rate, the interest rate could never be negative. Therefore, as the rate approaches zero, both the mean and variance also approaches zero. Thereby it is never possible for the rate to fall below zero. In the Vasicek model rates can become negative because the variance does not vanish as  $r$  approaches zero.

This causes the Vasicek model to have some unreasonable pricing implications; in particular negative yields for long-term bonds.

### 8.1.3 The Cox-Ingersoll-Ross Model

The Cox-Ingersoll-Ross model (Cox et al. 1985b) assumes a short-term interest rate model of the form

$$(8.4) \quad dr = a(b - r) dt + \sigma\sqrt{r} dz$$

The only difference from the Vasicek model is that in the Cox-Ingersoll-Ross (CIR) model the variance is proportional to the square root of the interest rate, instead of being constant. Due to this subtle difference, the CIR model satisfies all the objections to the earlier models:

- Interest rates cannot be negative. If  $r = 0$ , the drift in the rate is positive and the variance is zero, so the rate will become positive.
- As the short-rate rises, the volatility of the short-rate also rises.
- The short-rate exhibits mean reversion.

Therefore, the CIR model is the most convenient model to use for the simulation of interest rate paths.

### 8.1.4 Simulation of Interest Rate Paths

Before simulating the interest rate paths, we need to clarify some notational matters. In equations (8.1) – (8.4), we see that the interest rate is denoted  $r$ . In table 7.2, this notation is used for the refinancing rate. To avoid confusions, we therefore denote the interest rate as  $f$  from now on.<sup>12</sup>

In addition, because the differential equation (8.4) is just notation, we need to rewrite it before we can implement the equation to simulate interest rate paths.

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<sup>12</sup> This is equivalent to the notation for the 1-month future interest rate in table 7.1.

$$(8.5) \quad f_t = f_0 + \int_0^t a(b-r) du + \int_0^t \sigma \sqrt{r} dz_u$$

In equation (8.5), we have rewritten the differential equation (8.4) into a more familiar expression. Furthermore, we can rewrite the equation (8.5) to

$$(8.6) \quad f_t = f_0 \cdot e^{a(b-f_0)t + \sigma \sqrt{f_0} \sqrt{t} \varepsilon}$$

where  $\varepsilon$  equals a  $N(0,1)$  distributed random variable. Details regarding the simulation of this random variable can be found in appendix A.

With equation (8.6), we have arrived at an equation that we can use to simulate the 1-month future interest rate paths.

In my simulation, I will conduct two analogue valuations based on different assumptions about today's term structure of interest rates. First, I assume a rising term structure and call this CMO for BAU-1A. Secondly, I assume a falling term structure and call this CMO for BAU-1B. Naturally, one would expect the prepayment rates to be smaller for the rising term structure and larger for the falling term structure.

The input data for the two simulations can be found in table 8.1 below.

*8.1A: Rising Term Structure*

$t$	1/12
$a$	20%
$b$	10%
$\sigma$	2%
$f_0$	6.5%

*8.1B: Falling Term Structure*

$t$	1/12
$a$	-20%
$b$	10%
$\sigma$	2%
$f_0$	6.5%

*Table 8.1: Input Data for the Simulation of 1-Month Future Interest Rates.*

In both cases, the current spot rate is set to 6.5%. As we recall, the WAC of the underlying mortgages was 8%. It is reasonable to assume a spot rate that is lower than the WAC. As we will see in section 8.2, the spread of 150 basis points are not randomly picked.

Since I am simulating monthly interest rates, the time unit used in the equation is one month. That is, since the other parameters are given in yearly terms, the time unit  $t$  equals 1/12 of a year.

Furthermore, the 'drift' of BAU-1A is assumed at 20% per year, and the 'drift' of BAU-1B is assumed at -20% per year, implying a rising term structure for BAU-1A and a falling term structure for BAU-1B. In both cases, the level of mean reversion  $b$  in the interest rate is assumed to be 10% per year and the volatility is assumed to be 2% per year.

The 'drift' and level of mean reversion are randomly chosen and not based on any statistical research. The volatility, on the other hand, is based on historical volatility for interest rates. Statistical research shows that the volatility of interest rates is much lower than stock market volatility; in the region of 1-2% over the last 50 years. Therefore, the volatility is assumed to be in the region of 2%.

A description of how the simulation through the CIR-model was implemented in an Excel spreadsheet can be found in appendix B.

## 8.2 Refinancing Rate Simulation

We recall from section 7.3, that for each interest rate path simulated, a corresponding mortgage refinancing rate path is generated. This is done through making an assumption about the relationship between the Treasury rate and the refinancing rate. Typically, the assumption taken is that the mortgage-refinancing rate has a spread above the Treasury.

Usually, when an individual is seeking a loan, he needs to accept a contract rate that is above the Treasury yield. The reason for this is that the original lenders of the funds want to make a margin on the loan. Therefore, the assumption about a spread above the Treasury seems reasonable.

Throughout my illustration of the Monte Carlo simulation model, I will therefore assume a spread above the interest rate of 150 basis points. Given the assumption of a current spot rate

of 6.5%, this causes the initial mortgage-refinancing rate to be equal to the WAC of the underlying mortgages. It seems reasonable that at the time of origin, the mortgages can be refinanced at the same rate as the contract rate.

The assumed relationship between the interest rate and the mortgage-refinancing rate is therefore:

$$(8.7) \quad r_t = f_t + 1.5\%$$

where  $r_t$  is the mortgage refinancing rate at time  $t$ , and  $f_t$  is the interest rate at time  $t$ .

Therefore, after simulating the interest rate scenarios, all we have to do to generate corresponding mortgage refinancing rates, is to add 150 basis points to each of the interest rates simulated.

### 8.3 Prepayment Model and Prepayment Rate Simulation

As we saw in section 5.3, building a prepayment model normally starts by modelling the statistical relationship between the factors that are expected to affect prepayment behaviour. The valuation I am conducting in this thesis is just a hypothetical illustration and not an empirical research. This means that I am not able to model the statistical relationship between the factors, and therefore, I have to make assumptions about these relations.

Furthermore, we also saw that the four factors discussed in section 5.2 explained about 95% of the variation in prepayment rates. Normally, I should therefore include at least those four factors in my prepayment model. But, given my hypothetical world, including macroeconomic-, geographical- and seasonal factors and so on, would complicate the analysis further.

To deal with this challenge, I have built a prepayment model that has a 50% PSA speed as a basis. The idea is that this PSA speed is reflecting prepayments that are caused by the above-mentioned factors.

This leaves us with the prevailing mortgage rate as the main factor included in the model. As we can recall, there are three ways that the mortgage rate can affect prepayment behaviour: spread between contract rate (or WAC) and the mortgage rate, path of mortgages rates, and through housing turnover caused by the level of the mortgage rate.

The biggest concern is associated with the path of the mortgage rates. But, since I have assumed linear, rising and falling, term structures for BAU-1A and BAU-1B respectively, the refinancing burnout phenomenon described in section 5.2 will not occur and will therefore be overlooked in this valuation. The other two ways that the prevailing mortgage rate can affect the prepayment behaviour of mortgagors, is easier to model.

The prepayment model used for my valuation is:

$$(8.8) \quad CPR_t = 50PSA + 1_{\{0 < (WAC - r_t) < 200bp\}} [(WAC - r_t) \cdot F_1] + 1_{\{(WAC - r_t) > 200bp\}} [(WAC - r_t) \cdot F_2]$$

where  $CPR_t$  = conditional prepayment rate at time  $t$ ,

$WAC$  = weighted average contract rate,

$r_t$  = refinancing rate at time  $t$ ,

$F_1$  = first level factor, and

$F_2$  = second level factor.

In addition we have an indicator function that becomes 1 if the condition (A) is true and 0 otherwise. That is,

$$(8.9) \quad 1_{\{X > A\}} = \begin{cases} 1 & \text{if } X > A \\ 0 & \text{if } X \leq A \end{cases}$$

From equation (8.8), the CPR has an assumed 50% PSA speed. In addition to this basis, there are two elements that are conditioned on the spread between the WAC and the mortgage-

refinancing rate. As long as the spread is smaller than 200 basis points, the spread is multiplied by the factor  $F_1$ . If the spread becomes larger than 200 basis points, the spread is multiplied by the factor  $F_2$ .

The critical element of 200 basis points has been chosen due to the fact that prepayment rates have tended to increase when mortgage rates fall by more than 200 basis points.<sup>13</sup> This implies that  $F_1 < F_2$ .

Obviously, prepayments caused by the spread between the WAC and the prevailing mortgage rate are reflected in these two elements. But, also the prepayments due to housing turnover caused by the level of the mortgage rates can be reflected in these elements. For example, let us assume that the statistical relationship between the level of mortgage rates and prepayments due to housing turnover is as follows: for every percent that the mortgage rate drops, prepayments increase by one percent. If  $F_1$  have been assumed to be 1 and  $F_2$  have been assumed to be 3, all we have to do to include the housing turnover effect is to increase  $F_1$  and  $F_2$  by 1, bringing them up to 2 and 4 respectively.

Before implementing the prepayment model in equation (8.8), I have to give a strong emphasis to the simplification of the prepayment model used in this simulation. As underlined several times throughout this thesis, the most important element in deciding the value of MBSs and CMOs is the prepayment risk of the cash flow. Therefore, the modelling of prepayments is of outmost importance in real life valuation. Simplifications, of the kind done to build the prepayment model (8.8), are not good. However, for the purpose of this thesis, which is to illustrate the Monte Carlo simulation model and how prepayments are dealt with, it may be excused.

As mentioned above, I need to make some assumptions about the relationship between the factors and the refinancing rates. These assumptions and the other input data for the simulation of prepayment rates based on the prepayment model (8.8) can be found in table 8.2 below.

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<sup>13</sup> Fabozzi & Ramsey (1999:35).



<b><i>“Critical Month”</i></b>	30
<b><i>Monthly Increment</i></b>	0.1%
<b><i>Max CPR</i></b>	3%
<b><i>WAC</i></b>	8%
<b><i>Basis Points</i></b>	200
<b><i>F<sub>1</sub></i></b>	1
<b><i>F<sub>2</sub></i></b>	2.5

*Table 8.2: Input Data for the Simulation of Prepayment Rates.*

The first three input data given is used to generate a 50% PSA speed. Recall from section 5.1 that the 100% PSA starts at 0.2%, then rises by a monthly increment of 0.2% up to month 30 (“critical month”), and from then stays constant at a CPR of 6% (max CPR). The last four are used to model the prepayment caused by the prevailing mortgage rate.

A more detailed description of how the simulation of prepayment rates was implemented in an Excel spreadsheet can be found in appendix C.

## 8.4 Calculation and Structuring of the Cash Flows

After the prepayment rates have been simulated according to the prepayment model, we turn our attention towards the calculation of the cash flow from the mortgage pool, and the structuring model used to redistribute the cash flows to the three tranches.

### *8.4.1 Calculation of the Cash Flows from the Underlying Mortgage Pool*

I will now show how the monthly cash flow for the hypothetical pass-through underlying the CMO structures examined is constructed. Remember that the underlying mortgages for the pass-through are assumed to be fixed-rate level-payment mortgages with a WAC rate of 8%.

Furthermore, I assumed that the pass-through rate is 7% and the total size of the mortgage pool is NOK 1 billion.

In table 8.3, I have illustrated the different elements needed to construct the monthly cash flow on a given interest rate path. The cash flow is broken down into three main components: a) *interest* (based on the pass-through rate), b) the *regularly scheduled principal repayment*, and c) *prepayments* based on the prepayment model. Remember that this is the same three components that we discussed in chapter 4.

Path $n$							
Outstanding Balance	SMM	Mortgage Payment	Net Interest	Scheduled Principal	Prepayment	Total Principal	Cash Flow

Table 8.3: Calculation of Cash Flow from the Underlying Mortgage Pool.

Let us walk through table 8.3 column by column.

**Column 1:** This column gives the outstanding mortgage balance at the beginning of the month. It is equal to the outstanding balance at the beginning of the previous month reduced by the total principal payment in the previous month.

**Column 2:** This column shows the single-monthly mortality rate (SMM), and is determined according to equation (5.1) based on the prepayment rates simulated through the prepayment model (equation (8.8)).

**Column 3:** The total monthly mortgage payment is shown in this column. There is a formula to determine what the monthly mortgage balance will be for each month given prepayments.<sup>14</sup> This is,

$$(8.10) \quad MP_t = MB_t \left[ \frac{WAC(1+WAC)^{n-t+1}}{(1+WAC)^{n-t+1} - 1} \right]$$

<sup>14</sup> The formula is presented in Chapter 20 of Fabozzi (1993:363).

where  $MP_t$  = mortgage payment for month  $t$ ,  
 $MB_t$  = outstanding mortgage balance at the beginning of month  $t$ ,  
 $WAC$  = weighted average (monthly) contract rate,  
 $n$  = number of months for mortgage.

**Column 4:** The monthly interest paid from the pass-through is determined by multiplying the outstanding mortgage balance at the beginning of the month by the pass-through rate. To obtain the monthly interest, the (yearly) pass-through rate of 7% needs to be converted into a monthly interest rate (remember that for a fixed-rate level-payment mortgage, the monthly interest constitutes approximately 1/12 of the yearly interest).

**Column 5:** This column gives the regularly scheduled principal repayment. That is the difference between the total monthly mortgage payment (the amount shown in column (3)) and the gross coupon interest for the month. The gross coupon rate is the WAC multiplied by the outstanding mortgage balance (again the (yearly) WAC rate of 8% is converted into a monthly rate). That is,

$$(8.11) \quad SPP_t = MP_t - (MB_t \cdot WAC)$$

**Column 6:** In this column, the prepayment of the month is reported. The prepayment is determined according to equation (5.2)

**Column 7:** The total principal is the sum of column (5) and (6).

**Column 8:** The monthly cash flow for the pass-through on a given interest rate path, is shown in this last column. The monthly cash flow is the sum of the interest paid from the pass-through (column (4)) and the total principal payments for the month (column (7)).

I have now calculated the cash flow from the underlying pass-through. In a valuation procedure of a pass-through, I now could have started to calculate the present value of the cash flow on each path and the theoretical value of the pass-through. In this thesis I am conducting a numerical illustration of the valuation of CMO tranches, and therefore there is still some work to be done before I can compute the theoretical values.

#### *8.4.2 Structuring Model for the Cash Flow from the Underlying Mortgage Pool*

Before I could start calculating the present value of the cash flows, and thereby the theoretical value of the tranches, I need to distribute the cash flows among the three tranches according to the prioritized payment rules.

In table 8.4, I have illustrated the different elements needed for each tranche.

Path $n$			
Outstanding Balance	Total Principal	Net Interest	Cash Flow

*Table 8.4: Cash Flow to the Tranches.*

Let us walk through table 8.4 column by column.

**Column 1:** This column gives the outstanding mortgage balance of the tranche at the beginning of the month. It is equal to the outstanding balance at the beginning of the previous month reduced by the total principal payment in the previous month.

**Column 2:** This column shows the total principal received for a given month. We remember that the payment rules stated that all principal from the underlying pass-through should be distributed to Tranche A until it has been totally paid off. Then, Tranche B starts receiving principal. When Tranche B also has been paid off, Tranche C starts receiving principal.

A more detailed description of how the distribution of principal was implemented in an Excel spreadsheet can be found in appendix D.

**Column 3:** The payment rules stated that all tranches receive interest on the outstanding balance. In addition, I have assumed that the interest rate for the tranches is equal and the same as the pass-through rate of 7%. This makes it fairly easy to distribute the interest rate from the underlying pass-through. The procedure is straightforward the same as for the mortgage pool; multiply the outstanding balance at the beginning of the month by the pass-through rate for each of the tranches (again the pass-through rate has to be converted into a monthly rate).

**Column 4:** The monthly cash flow for a tranche on a given interest rate path, is shown in this last column. The monthly cash flow is the sum of the total principal payment to the tranche (column (2)) and the interest received each month (column (3)).

Now, I have the cash flows for each tranche on each interest rate path and I am now able to start calculating present values.

## 8.5 Calculating Present Value and Theoretical Value

After I have simulated the interest rate paths and the corresponding cash flows, I am able to calculate the present value of the cash flow on each of the paths. This enables me to calculate the theoretical value of the tranches.

### *8.5.1 Calculating the Present Value of Each Interest Rate Scenario*

When calculating the present value of the paths, we need a discount rate. From section 7.4, we remember that the rate used is the simulated spot rate for each month on the interest rate path plus an appropriate spread. Furthermore, we remember that this spread, the option-adjusted spread, is used to level the theoretical value with the market price. Since the CMO structure I

am valuing in this illustration is a hypothetical structure, there does not exist any market price and I set this spread equal to zero.

This means that the discount rate I use in my valuation is the simulated spot rate for each month on the interest rate path. Recall from chapter 7 that the simulated spot rate can be determined from the simulated future monthly rates according to equation (7.1):

$$(7.1) \quad z_T(n) = \left\{ [1 + f_1(n)][1 + f_2(n)] \dots [1 + f_T(n)] \right\}^{1/T} - 1$$

After converting the simulated future monthly rates, I am able to calculate the present value of the cash flow on each of the paths according to equation (7.2):

$$(7.2) \quad PV[C_T(n)] = \frac{C_T(n)}{[1 + z_T(n) + K]^T}$$

Remember that I assumed the spread,  $K$ , to be equal to zero.

### 8.5.2 Calculating the Theoretical Value of the Tranches

The present value of all the simulated interest rate scenarios enables me to calculate the theoretical value of the different tranches according to equation (7.4) from chapter 7:

$$(7.4) \quad V_0 = \frac{PV[Path(1)] + PV[Path(2)] + \dots + PV[Path(N)]}{N}$$

## 8.6 The Results of the Numerical Valuation

I have now finally reached the point where I can report the results of the numerical valuation conducted in this chapter. In this section, I will first report the theoretical value of the BAU-1 structure, and then I will report the distribution of the path present values.

Note that I will not report the option-adjusted spread. We saw in chapter 7, that this spread measure is normally reported in connection with the valuation of a CMO. The OAS is a by-product of the Monte Carlo simulation model connecting the theoretical value to the market price. The BAU-1 structure analyzed in this chapter is a hypothetical CMO and consequently it does not exist a market price for this structure. Therefore, I have no OAS to report.

### 8.6.1 Theoretical Value of BAU-1

Remember that I assumed two different term structures of interest rates. We will first look at the results of the numerical valuation when I assumed a rising term structure.

#### ***BAU-1A: Rising Term Structure***

<b><i>t</i></b>	1/12
<b><i>a</i></b>	20%
<b><i>b</i></b>	10%
<b><math>\sigma</math></b>	2%
<b><i>f</i><sub>0</sub></b>	6.5%

*Table 8.5: Assumptions about the Term Structure of Interest Rates for BAU-1A.*

As we see from table 8.5, I assumed for BAU-1A that we had a rising term structure of interest rates. We would expect the prepayment rates to be low for this scenario due to the fact that future rates will be higher than the prevailing rate.<sup>15</sup>

The theoretical values for the three tranches in BAU-1A are reported in table 8.6 below.

Tranche	Theoretical Value
Tranche A	NOK 400,317,303
Tranche B	NOK 291,902,987
Tranche C	NOK 281,128,915

*Table 8.6: Theoretical Values of the Tranches in BAU-1A.*

We see that if there were issued certificates with par value of NOK 1,000 for each of the three tranches (that is 400,000 of Tranche A, 300,000 of Tranche B and 300,000 of Tranche C), the value of the certificates would be NOK 1000.79, NOK 973.01 and NOK 937.10 for Tranche A, Tranche B and Tranche C respectively. This seems reasonable. Tranche A is the one that benefit the most from the prepayments at the cost of the other two tranches. Furthermore, since the majority of the cash flows to Tranche B and C are received later than Tranche A, they are discounted ‘harder’ (especially with a rising term structure) and therefore have a lower value compared to the par value of the certificate.

Next, we turn to the results of the numerical valuation when I assumed a falling term structure.

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<sup>15</sup> In fact, with an assumption about a rising term structure of interest rates, we would almost never observe the simulated interest rate to fall below the prevailing rate. Correspondingly, the mortgage-refinancing rate will not fall below the prevailing mortgage rate and there will be little or no incentives for borrowers to refinance their mortgage. Still there are incentives related to general economic activity, geographical location of the underlying properties and so on. Therefore, when assuming rising term structure, we would expect some prepayments to occur.

Remember that in the prepayment model I built in section 8.3 (equation (8.8)), I have tried to reflect prepayments due to factors such as general economic activity and geographical locations, by including a 50% PSA speed as a basis for the model.



***BAU-1B: Falling Term Structure***

$t$	1/12
$a$	-20%
$b$	10%
$\sigma$	2%
$f_0$	6.5%

*Table 8.7: Assumptions about the Term Structure of Interest Rates for BAU-1B.*

As we see from table 8.7, I assumed for BAU-1B that we had a falling term structure of interest rates. We would expect the prepayment rates to be high for this scenario due to the fact that future rates will be lower than the prevailing rate. Higher prepayment naturally means that each tranche would receive payments earlier than expected. This also indicates that for BAU-1B, we would expect the value of the different tranches to be higher than for BAU-1A.

The theoretical values for the three tranches in BAU-1B are reported in table 8.8 below.

Tranche	Theoretical Value
Tranche A	NOK 413,100,271
Tranche B	NOK 331,137,043
Tranche C	NOK 351,319,309

*Table 8.8: Theoretical Values of the Tranches in BAU-1B.*

In this case, if there were issued certificates with par value of NOK 1,000 for each of the three tranches, the value of the certificates would be NOK 1032.75, NOK 1103.79 and NOK 1171.06 for Tranche A, Tranche B and Tranche C respectively. Intuitively, these results might not be as reasonable as the ones for BAU-1A.

The first thing that we observe, as expected, is that the value of all three tranches is higher than for BAU-1A. In light of higher prepayments expected for a falling term structure, this seems reasonable. Secondly, we observe that the certificates for Tranche C are the most valuable of the three tranches, Tranche B second most valuable and Tranche A the least valuable. This does not correspond to the picture we saw for BAU-1A. In light of the prepayments, should not Tranche A be the most valuable since it benefits most from prepayments? Normally, we would expect this to be correct. But with falling term structure, cash flows received early are discounted 'harder' than cash flows received later. Tranche C, which receive the majority of the cash flow in the distant future, therefore becomes more valuable today, and Tranche A, which receive the majority of the cash flows in the near future, becomes less valuable today (i.e., relative to each other!).

### *8.6.2 Distribution of Path Present Values for BAU-1*

As we saw in section 7.6, in the valuation of derivatives, the outcome depends on several random variables. The Monte Carlo model is a commonly used, and functional, management science tool for such situations.

Unfortunately, when used to value fixed income securities like a CMO tranche, it is normal simply to report the first product of the Monte Carlo simulation model, the theoretical or average value. In so doing, all information about the second product of the model, the distribution of the path present values, has been ignored. This information is important and should be obtained.

In this sub-section, I will therefore report the path present value distribution of both BAU-1A and BAU-1B. The values I will report are the average value of the path (that is, the theoretical value), the standard deviation of the average value, the minimum path present value and the maximum path present value.

Let us first look at the scenario where I have assumed a rising term structure of interest rates. The descriptive data can be seen in table 8.9 below.

Tranche	Average Value	Standard Deviation	Minimum	Maximum
Tranche A	NOK 400,317,303	NOK 942,560	NOK 397,969,671	NOK 402,785,088
Tranche B	NOK 291,902,987	NOK 2,087,062	NOK 286,742,697	NOK 298,136,509
Tranche C	NOK 281,128,915	NOK 2,840,570	NOK 273,647,851	NOK 289,707,243

*Table 8.9: Distribution of Path Present Values for BAU-IA*

We see that the path present values for Tranche A has the smallest distribution interval (i.e., [minimum, maximum]), and thereby the smallest standard deviation. Furthermore, we see that the path present values for Tranche C has the largest distribution interval, and correspondingly it also has the largest standard deviation. Again, the reason is related to the timing of the cash flows to the tranches; the nearer the timing of the cash flows, the more accurate the estimate of the average value gets.

Next, let us turn to the scenario where I have assumed a falling term structure of interest rates, and see whether the distribution has changed. The data describing the distribution of path present values can be seen in table 8.10 below.

Tranche	Average Value	Standard Deviation	Minimum	Maximum
Tranche A	NOK 413,100,271	NOK 687,053	NOK 411,091,998	NOK 414,930,908
Tranche B	NOK 331,137,043	NOK 1,505,640	NOK 327,184,761	NOK 335,319,536
Tranche C	NOK 351,319,309	NOK 2,420,179	NOK 344,787,989	NOK 357,840,198

*Table 8.10: Distribution of Path Present Values for BAU-IB*

We see that for all three tranches, the intervals have shrunk and the standard deviations have decreased. This seems reasonable. As mentioned earlier, we would expect higher prepayments when we assume a falling term structure. This means that a larger weight of the total cash

flows to the tranches is received earlier, and thereby the estimates become more accurate. Beside this difference between BAU-1A and BAU-1B, we see that the internal differences between the tranches of BAU-1B show the same picture as for BAU-1A; smallest distribution interval and standard deviation for Tranche A and largest for Tranche C.

## 8.7 Final Comments to the Numerical Illustration

Before summing up and drawing the conclusions from this thesis, I will make some final comments on the numerical illustration conducted in this chapter. These comments are related to the fact that the numerical valuation in this chapter is an illustration of the methodology, and that the CMO structures valued are hypothetical. This means that there are simplifications done in the analysis, which I do not necessarily recommend for a real life valuation of a CMO.

The following simplifying elements are the most important to keep in the back of our head:

- **Term Structure of Interest Rates** – the assumptions made about the term structure is relatively rigid. When I have assumed a rising or falling term structure, I have assumed that it will rise or fall for 360 months. In real life, the term structure will shift over a time period as long as 30 years. Furthermore, the parameters used to simulate interest rate paths through the Cox-Ingersoll-Ross model (i.e, current spot rate, drift, level of reversion, variance, etc.) are arbitrarily picked. In real life, these parameters need to be estimated through statistical procedures.
- **Prepayment Model** – when building the prepayment model, I have made it simple to illustrate the valuation methodology and how prepayments are dealt with in such valuations. By being the most important element when valuating CMOs, it is of outmost importance to find the factors affecting prepayments and the statistical relationships between them and the prepayment rates. By doing this properly, we are able to model the real life as best we can.
- **Number of Paths Simulated** – in an attempt of not making the excel-document used for the valuation to large, I have restricted the number of paths to 256. To make the estimate of the value as accurately as possible, it might be necessary to do more path simulations and/or use variance-reducing techniques.

## 9. Summary and Conclusion

Since 1969, mortgage loans have been used as collateral for creation of mortgage-backed securities. That is, securities that are backed by numerous mortgages, often in form of a pool of mortgages. Since then, the market for such mortgage-related securities has experienced a tremendous growth.

Common for mortgages and mortgage-related securities is that there is uncertainty concerning the cash flows from the mortgages as a result of four risk elements: *credit-*, *prepayment-*, *interest rate-* and *liquidity risk*.

In being what Wall Street describes as ‘truly custom designed’, collateralized mortgage obligations maybe the most exciting of the mortgage-related securities. “The CMO’s major financial innovation is that it provides for redirecting underlying cash flows in order to create securities that much more closely satisfy the asset/liability needs of institutional investors” (Fabozzi & Ramsey 1999:1).

Through the subprime mortgage crisis, the CMO has showed us that securities can strengthen and transmit problems from one market to another, and between countries and regions. It is therefore important for investors and other market participants to understand the creation and structure of such securities.

There are mainly two categories of issuers of CMOs: *agency* and *non-agency*. Agency issues include CMOs guaranteed by institutions such as Ginnie Mae, Fannie Mae and Freddie Mac, and investors in these securities do not have to worry about credit risk. Non-agency issues are CMOs not guaranteed by any institution. They therefore induce credit risk on investors. However, in both cases, it is the amount and timing of the prepayments that separates CMOs from other securities backed by some portfolio of securities, such as the CDO.

The main objective of this thesis has therefore been to study how prepayments are dealt with in the valuation of the security. It has also been important to show how this is implemented in practice.

In the literature, there have been several conventions used as a benchmark for prepayment rates. The *Federal Housing Administration (FHA) approach* looks at the prepayment experience for 30-year mortgages derived from a FHA table on mortgage survival factors.

The approach calls for the projection of the cash flow for a mortgage pool on the assumption that the prepayments will be the same as the FHA experience (referred to as “100% FHA”), or some multiple of FHA experience (faster or slower than FHA experience). Though it was fairly popular, this method is no longer used because it does not give emphasis to the fact that prepayments are tied to interest rate cycles.

The *conditional prepayment rate* (CPR) requires assuming prepayment of some fraction of the remaining principal in the mortgage pool each month for the remaining term of the mortgage. The CPR is based on the characteristics of the pool (including its historical prepayment experience) and the current and expected future. The advantages of this approach are its simplicity and the fact that changes in economic conditions impacting the prepayment rate or changes in the historical prepayment pattern can be analyzed quickly.

The *Public Securities Association* (PSA) *prepayment benchmark* is expressed as a monthly series of annual prepayment rates. This model has a basic assumption that prepayment rates are low for newly originated mortgages and then speeds up as the mortgages becomes seasoned. The PSA standard benchmark (referred to as “100% PSA” or simply “100 PSA”) assumes the following prepayment rates for 30-year mortgages: 1) a CPR of 0.2% for the first month, increased by 0.2% per year per month for the next 29 months when it reaches 6% per year, and 2) a 6% CPR for the remaining years. The PSA benchmark is today the most popular method to project prepayments.

In this thesis, we have seen that there are four main categories of factors that affect the prepayment behaviour of mortgagors. The *prevailing mortgage rate* can affect the prepayment behaviour through the spread between the current rate and the contract rate on the mortgage, through the path that mortgage rates take to get to the current level, and through how the level of the current rate affects housing turnover. In addition to the characteristics related to the prevailing mortgage rate, other *characteristics of the underlying mortgage pool* that affects prepayment behaviour include whether the loans are FHA/VA-guaranteed or conventional, the amount of seasoning, the type of loan and the geographical location of the underlying properties. Furthermore, prepayment behaviour follows a well-documented *seasonal pattern* caused by housing turnover; fairly low in the winter months, increasing in the spring and early summer, and gradually reaching a peak in the early fall. Finally, it is no surprise that economic theory suggest that *general economic activity* has a distinct influence on prepayment behaviour through its effect on housing turnover.

The assessment of factors that are expected to affect prepayment behaviour enables one to build a prepayment model for the purpose of valuation. The model building starts by modelling the statistical relationship between the factors found. It is not always true that adding additional factors makes the model more capable of explaining the variations in prepayment rates. Often, it is sufficient to include only a few factors. In this thesis, we have seen that the four factors discussed actually explain about 95% of the variation in prepayment rates.

The literature suggests that, when it comes to fixed income valuation, there are commonly two methodologies that are used to value securities with embedded options: the *binomial model* and the *Monte Carlo Model*. We have seen that the prepayments of CMOs are interest rate path-dependent, meaning that cash flows also is determined by the path that interest rates took to get to the current level. The Monte Carlo model is the most flexible valuation methodology for valuing interest sensitive instruments where the history of interest rates is important. Furthermore, we have seen that the Monte Carlo simulation model has been described as the state-of-the-art methodology for valuing CMOs.

Conceptually, the valuation of pass-through securities and CMOs using the Monte Carlo model is simple. Through the numerical illustration, however, we have seen that in practice, the procedure is very complex. The procedure implies generating a set of cash flows to the holder of the security. These cash flows are based on simulated future mortgage refinancing rates and prepayment rates. In simulating these rates, we have seen the importance of the assumptions made about the term structure of interest rates and estimation of the statistical relationships between prepayments and the factors affecting such behaviour. Furthermore, the number of paths simulated also needs to be taken into consideration. In practice, such a procedure would be extremely time consuming and almost impossible without the computer power available today.

Although there is reason to believe that the subprime mortgage crisis has slowed down the growth of the market for CMOs, the obvious advantages for investors encountering specific investment objectives, but also for investors seeking diversification, there is a strong possibility that the market will continue to constitute a sizable and important part of the debt market in the future.

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## Appendix A

### *Box-Müller Transformation*<sup>16</sup>

To generate the  $N(0,1)$  distributed numbers needed in equation (8.6) to simulate interest rates, the following, so-called Box-Müller transformation, is very useful: if  $U_1$  and  $U_2$  are two independent uniformly distributed numbers over the interval  $(0,1)$ , then  $X_1$  and  $X_2$  defined as

$$X_1 = \sqrt{-2\ln(U_1)} \cos(2\pi U_2)$$

$$X_2 = \sqrt{-2\ln(U_1)} \sin(2\pi U_2)$$

are two independent  $N(0,1)$  distributed random variables.

Hence, for each simulated interest rate I have written the formula

$$=+\text{SQRT}(-2*\text{LN}(\text{RAND()}))*\text{SIN}(2*\text{PI}()*\text{RAND()}),$$

into my spreadsheet, to generate the independent  $N(0,1)$  distributed random variables needed. Whenever new information is added to the spreadsheet or the recalculation button (F9) is pressed, all the  $N(0,1)$  random variables are sampled again.

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<sup>16</sup> The Box-Müller transformation described, follows a project-text from the course ECO423 Risk Management at the Norwegian School of Economics and Business Administration. See references under Persson (2007).

## Appendix B

### *Simulation of Interest Rate Paths*

The input data used in the simulation of the interest rate paths, are placed in the following cells:

Column/ Row	C
2	Time interval, $t$
3	Drift, $a$
4	Level of reversion, $b$
5	Volatility, $\sigma$
6	Current spot rate, $f_0$

Using the equation (8.6), we can generate interest rate paths. That is,

$$(8.6) \quad f_t = f_0 \cdot e^{a(b-f_0)t + \sigma \sqrt{f_0} \sqrt{t} \epsilon}$$

Hence, to simulate the first 1-month future interest rate on the first path, I have written the following formula into cell F11 of the spreadsheet:

`=+$C$6*EXP($C$3*($C$4-$C$5)*$C$2+$C$5*SQRT($C$6)*SQRT($C$2)*N(0,1)!F11),`

where `N(0,1)!F11` refers to a cell in the spreadsheet where I have generated  $N(0,1)$  distributed random variables according to the Box-Müller transformation described in appendix A.

To complete the first path, I then write the following formula into cell F12:

`=+F11*EXP($C$3*($C$4-$C$5)*$C$2+$C$5*SQRT(F11)*SQRT($C$2)*N(0,1)!F12)`

The next step is to copy the formula down 360 cells to generate a complete interest rate path. The interest rate path created is then copied 256 times to generate the set of interest rate paths (in accordance with table 7.1).

## Appendix C

### *Prepayment Rate Simulation*

The input data used in the simulation of the prepayment rates, are placed in the following cells:

Column/ Row	B
2	“Critical Month”
3	Monthly Increment
4	Max CPR
5	WAC
6	Basis Points
7	Factor 1
8	Factor 2

Using the equation (8.8), we can generate prepayment rates

$$(8.8) \quad CPR_t = 50PSA + 1_{\{0 < (WAC - r_t) < 200bp\}} [(WAC - r_t) \cdot F_1] + 1_{\{(WAC - r_t) > 200bp\}} [(WAC - r_t) \cdot F_2]$$

Hence, to simulate the first prepayment rate, I have written the following formula into cell F11 of the spreadsheet:

$$= +IF(E11 < \$B\$2; E11 * \$B\$3; \$B\$4) \\ +IF((\$B\$5 - RefinancingRates!F11) > 0; IF((\$B\$5 - RefinancingRates!F11) < \$B\$6; \$B\$7 * (\$B\$5 - RefinancingRates!F11); 0); 0) \\ +IF((\$B\$5 - RefinancingRates!F11) >= \$B\$6; \$B\$8 * (\$B\$5 - RefinancingRates!F11); 0)$$

where “E11” refers to the given month on the path and “RefinancingRates!F11” refers to a cell in the spreadsheet where I have generated the mortgage-refinancing rates based on equation (8.7).

The first IF-function of the formula generates the basis PSA speed of the prepayment model, the second and third IF-function generates the additional prepayment due to refinancing if the mortgage rate falls up to 200 basis points, and the last IF-function generates the additional prepayment due to refinancing if the mortgage rate falls with more than 200 basis points.

The next step is to copy the formula down 360 cells to generate a complete prepayment rate path. The prepayment rate path created is then copied 256 times to generate the set of prepayment rate paths corresponding to the interest rate paths described in appendix B.

## Appendix D

### *Structuring of the Principal Payments from the Underlying Pass-Through*

The prioritizing of the principal payments from the underlying pass-through is structured so that Tranche A receive all principal until it is completely paid off. When Tranche A has been paid off, Tranche B start receiving principal, and finally, when Tranche B has been paid off, Tranche C starts receiving principal.

1. Hence, to structure principal to Tranche A, I have written the following formula into cell G11 of the spreadsheet:

= +IF(F11>0;IF(F11>CashFlowPool!L11;CashFlowPool!L11;F11);0)

where “F11” refers to the outstanding balance of the tranche and “CashFlowPool!F11” refers to a cell in the spreadsheet where I have calculated the cash flow from the underlying pass-through.

2. To structure principal to Tranche B, I have written the following formula into cell G11 of the spreadsheet:

= +IF(TrancheA!G11<CashFlowPool!L11;IF(F11>0;  
IF(F11>CashFlowPool!L11;CashFlowPool!L11-TrancheA!G11;F11);0);0)

where “TrancheA!G11” refers to the cell described above (principal to Tranche A). “F11” refers to the outstanding balance of the tranche and “CashFlowPool!F11” refers to the same as described above.

3. To structure principal to Tranche C, I have written the following formula into cell G11 of the spreadsheet:

= +IF(TrancheA!G11=0;IF(TrancheB!G11<CashFlowPool!L11;  
IF(F11>0;IF(F11>CashFlowPool!L11;  
CashFlowPool!L11-TrancheB!G11;F11);0);0);0)

where “TrancheB!G11” refers to the cell described above (principal to Tranche B).  
“F11” refers to the outstanding balance of the tranche, “TrancheA!G11” and  
“CashFlowPool!F11” refers to the same as described above.