Research Article

Cognitive Multihop Wireless Sensor Networks over Nakagami-$m$ Fading Channels

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Considering the effect of imperfect channel state information (CSI), we study the performance of a cluster-based cognitive multihop wireless sensor network with decode-and-forward (DF) partial relay selection over Nakagami-$m$ fading channels. The closed-form expressions for the exact outage probability and bit error rate (BER) of the secondary system are derived and validated by simulations. Asymptotic outage analysis in high SNR regime reveals that the diversity order is determined by the minimum fading severity parameter of all the secondary transmission links, irrespective of the CSI imperfection. It is shown that the fading severity of the secondary transmission links has more influence on the outage performance than that of the interference links. We also conclude that, for secondary nodes whose transmit power is restricted by the interference constraint of the primary user, increasing the number of relaying hops is an effective way to improve their transmission performance. Besides, increasing the number of available relays in each relay cluster can mitigate the performance degradation caused by CSI imperfection.

1. Introduction

Cognitive radio (CR) is a promising technology to improve the spectrum utilization for wireless systems [1]. The underlay paradigm in CR has drawn much attention due to its flexibility [2]. By allowing simultaneous transmissions of secondary users (SU) and primary users (PU), as long as the interference caused by SU to PU is below a tolerable threshold, the underlay CR can take full advantage of the spectrum resources. However, since the transmit power of SU is strictly limited by the interference power constraint, the secondary transmission often suffers from a short-range drawback. To deal with this problem, cooperative relaying techniques [3] are introduced to secondary systems to extend their coverage area. Furthermore, two-hop or multihop sensor (relaying) can help the secondary systems, for example, mobile ad hoc wireless networks and wireless sensor networks, in achieving broader coverage and better transmission performance. As the combination of cognitive radio and cooperative communications, cognitive dual-hop or multihop networks benefit from both the high spectrum utilization and the cooperative diversity, which have been intensively studied recently [4–8].

In [5], the authors investigated the performance of multihop cognitive decode-and-forward (DF) networks over Rayleigh fading channels. In [6], performance for multihop cognitive amplify-and-forward (AF) networks over Nakagami-$m$ fading was presented. Cognitive multihop networks in the presence of multiple primary receivers with AF and DF protocols were studied in [7, 8], respectively, over Nakagami-$m$ fading channels. All the aforementioned references assumed that only one single relay node is available in each hop of the secondary transmission. When there are multiple relays available for one-hop transmission, relay selection can bring significant performance improvement. Reference [9] proposed a cluster-based multihop network where relay selection can be applied to multihop transmission. In [10], performance of a cluster-based multihop
sensor network with partial relay selection without spectrum sharing was analyzed. Power allocation and relay selection problems for cluster-based cognitive multihop networks were studied in [11].

In cognitive radio networks, channel state information (CSI) plays an important role. On one hand, to restrict the interference from SU to PU, SU must adjust their transmit power according to the instantaneous CSI of the interference links between SU and PU. On the other hand, the relay selection for each hop depends on the instantaneous CSI of the corresponding secondary transmission links. Most existing works assumed that perfect CSI could be acquired. In practice, however, due to channel estimation errors, mobility, feedback delay, limited feedback, and feedback quantization, the acquired CSI may sometimes be imperfect. In [12, 13], the impact of imperfect CSI of the interference links between SU and PU was studied on the outage performance of cognitive dual-hop networks with AF and DF protocols, respectively. In [14, 15], the outage performance of cognitive relay networks (CRN) for the Nth best relay selection and orthogonal space-time block coding (OSTBC) in Rayleigh channels are considered. For multihop relaying, the authors in [16] analyzed the performance of multihop cognitive DF networks, considering the effect of imperfect CSI of the interference links. However, [12, 13, 16] all focused on Rayleigh fading channels. To the best of our knowledge, there have been no prior works on the impact of imperfect CSI on cognitive multihop networks in a more general fading environment, such as Nakagami-\(m\) fading environment. Motivated by these considerations, we extend our previous research on a cognitive dual-hop network over Rayleigh fading channels in [13] to a cognitive multihop wireless sensor network over Nakagami-\(m\) fading channels.

In this paper, we investigate the performance of a cluster-based cognitive multihop wireless sensor network with DF partial relay selection over Nakagami-\(m\) fading channels. The effect of imperfect CSI of the secondary transmission links is taken into account. We derive the closed-form expressions for the exact outage probability and bit error rate (BER) and the asymptotic outage probability of the secondary system and validate them by simulations. The impact of various factors (e.g., the CSI imperfection, the fading severity of all links, the number of hops, and the number of sensors/relays in clusters) on the performance of the secondary system is analyzed.

The remainder of this paper is organized as follows: in Section 2, we present the system model for the analysis of cognitive multihop wireless sensor networks; then, based on this model, in Section 3, the outage probability and bit-error-rate (BER) are derived over Nakagami-\(m\) fading channels; meanwhile, the asymptotic outage probability for large system SNR is derived to study the diversity order in such a system; in Section 4, simulation results are given and compared; finally, we conclude this paper in Section 5.

2. System Model

The system model we consider in this paper is illustrated in Figure 1. The secondary system shares the same spectrum with a primary user in an underlay approach. Like most studies, we only consider the existence of the primary receiver (PR) and assume the primary transmitter (PT) is far away from SU, so the interference from PT can be ignored. The secondary system is a cognitive cluster-based multihop sensor network whose transmission consists of \(K\) hops. Between the secondary source (SS) and the secondary destination (SD), there are \(K - 1\) sensor clusters where cluster \(k\) (\(k = 1, 2, \ldots, K - 1\)) contains \(N_k\) available relay nodes. To simplify the notation, we denote SS and SD by cluster 0 and cluster \(K\), respectively. So we have \(N_0 = 1\).

Due to the half-duplex mode of the relays, the transmission of the \(K\) hops happens in \(K\) separate time slots. We assume that all channels experience Nakagami-\(m\) quasistatic fading; that is, the channel fading coefficients remain constant during one slot but change independently for every slot. For the reason that the CSI of the next hops cannot be obtained in the current hop, the relay selection for the current hop merely depends on the CSI of the current hop. To this end, partial relay selection (PRS) can be adopted in the secondary transmission as a feasible strategy, since the relay selection process is based on the channel quality of the current hop only, instead of the end-to-end signal-to-noise ratio (SNR). Besides, DF protocol is exploited in the secondary relays where the receiver for each hop decodes the signal and then reencodes it before forwarding it to the next hop.

To guarantee PU’s communication, the maximum tolerable interference power constraint \(Q\) at PR must be satisfied. In other words, the transmitter in each hop should restrict its transmit power so that the interference it causes to PR does not exceed \(Q\).

In the \(k\)th hop (\(k = 1, 2, \ldots, K\)), the transmitter in cluster \(k - 1\) is denoted by \(t_k\) while the \(N_k\) potential receivers in cluster \(k\) (i.e., the available relays included in cluster \(k\)) are represented as \([r_1^{(k)}, r_2^{(k)}, \ldots, r_{N_k}^{(k)}]\). We denote the channel fading coefficient of the interference link between transmitter \(t_k\) and the PR by \(h_{k,P}\) and the channel fading coefficients of the secondary transmission links between transmitter \(t_k\) and receiver \(r_i^{(k)}\) (\(i = 1, 2, \ldots, N_k\)) by \(h_{kj}\). According to the PRS protocol, the best relay with the maximum received SNR is chosen from the potential receivers \([r_1^{(k)}, r_2^{(k)}, \ldots, r_{N_k}^{(k)}]\) as the transmitter \(t_{k+1}\) in the \((k + 1)\)th hop.
For Nakagami-$m$ fading, the channel power gains $g_{XY} = |h_{XY}|^2$ ($X = k; Y = 1, 2, \ldots, N_k, p$) are independent and non-identically distributed (INID) random variables, following Gamma distribution with fading severity parameters $m_{X,Y}$ ($m_{X,Y}$ is positive integer) and mean $\Omega_{X,Y}$. The probability density function (PDF) and cumulative distributed function (CDF) of $g_{XY}$ are given by

$$f_{g_{XY}}(x) = \frac{\rho_{XY}^{m_{XY}-1} \Gamma(m_{XY})}{\Gamma(m_{XY})^{m_{XY}}} \exp(-\beta_{XY}x), \quad (1)$$

$$F_{g_{XY}}(x) = 1 - \exp(-\beta_{XY}x) \frac{m_{XY}^{-1} \Gamma(m_{XY})}{n!}, \quad (2)$$

respectively, where $\beta_{XY} = m_{XY}/\Omega_{XY}$ and $\Gamma(\cdot)$ and $\gamma(\cdot, \cdot)$ represent the Gamma function [17, (8.339.1)] and the lower incomplete Gamma function [17, (8.350.1)]. We assume that all relays in the same cluster are relatively centralized, so the channel parameters pertaining to relays in the same cluster are identical; that is, we have $m_{k,i} = m_k, \Omega_{k,i} = \Omega_k$, and $\beta_{k,i} = \beta_k (i = 1, 2, \ldots, N_k)$. In addition, the thermal noise at each receiver is modeled as independent complex additive white Gaussian noise (AWGN) with variance $\sigma^2$.

In this paper, we consider the imperfect CSI of the secondary transmission links. Specifically, in the $k$th hop, the perfect channel gain of the transmission link between transmitter $t_k$ and the potential receiver $r_i$ ($i = 1, 2, \ldots, N_k$) and its imperfect counterpart are denoted by $g_{k,i}$ and $\hat{g}_{k,i}$, respectively. According to [18, equation (9.398)], the joint PDF of the perfect channel gain $g_{k,i}$ and its imperfect counterpart $\hat{g}_{k,i}$ which follows the same distribution as $g_{k,i}$ is given by

$$f_{g_{k,i},\hat{g}_{k,i}}(x, y) = \frac{\rho_{k}^{m_{k,i}+1}}{\Gamma(m_k)} \left(\frac{xy}{\rho_k}\right)^{(m_k-1)/2} \times \exp\left(-\frac{\beta_k (x+y)}{1-\rho_k}\right) I_{m_k-1}\left(\frac{2\beta_k \sqrt{\rho_k xy}}{1-\rho_k}\right), \quad (3)$$

where $I_n(\cdot)$ denotes the $n$th-order modified Bessel function of the first kind and $\rho_k = \text{Cov}(g_{k,i}, \hat{g}_{k,i})/\sqrt{\text{Var}(g_{k,i}) \text{Var}(\hat{g}_{k,i})} \in [0, 1]$ represents the correlation coefficient between $g_{k,i}$ and $\hat{g}_{k,i}; \rho_k$ reflects the degree of CSI imperfection. For instance, $\rho_k = 1$ indicates that the CSI is absolutely perfect while $\rho_k = 0$ represents that the CSI estimation is totally random.

### 3. Exact Performance Analysis

In this section, considering the imperfect CSI of the secondary transmission links, we derive the exact outage probability and BER of the cognitive multihop sensor network.


Since the secondary users suffer from low transmit power, each secondary node should try its best to transmit the signal. In the $k$th hop, to satisfy the interference power constraint of PU, the transmit power of $t_k$ should be set to $P_k = Q/g_{k,p}$. Due to the imperfect CSI of the secondary transmission links, the imperfect received SNR at receiver $r_i^{(k)}$ ($i = 1, 2, \ldots, N_k$) is calculated as $\hat{\gamma}_{k,i} = Q\hat{g}_{k,i}/N_0 g_{k,p}$. According to the PRS strategy, the “best” receiver with the maximum imperfect received SNR is selected as transmitter $t_{k+1}$ for the next hop so that the imperfect SNR for the $k$th hop satisfies

$$\hat{\gamma}_k = \max_{i=1,2,\ldots,N_k} \{\hat{\gamma}_{k,i}\} = \max_{i=1,2,\ldots,N_k} \left\{\frac{Q\hat{g}_{k,i}}{N_0 g_{k,p}}\right\}. \quad (4)$$

The imperfect channel gain between transmitter $t_k$ and the selected receiver $t_{k+1}$ is given by

$$\hat{g}_k = \max_{i=1,2,\ldots,N_k} \{\hat{g}_{k,i}\}. \quad (5)$$

From (5), we obtain the CDF of $\hat{g}_k$ as

$$F_{\hat{g}_k}(x) = \left[F_{\hat{g}_{k,i}}(x)\right]^{N_k}. \quad (6)$$

By taking the derivative of (6), we can get the PDF of $\hat{g}_k$ as

$$f_{\hat{g}_k}(x) = N_k f_{\hat{g}_{k,i}}(x)^{N_k-1} f_{\hat{g}_{k,i}}(x). \quad (7)$$

The actual SNR for the $k$th hop is given by

$$\gamma_k = \frac{Q\hat{g}_k}{N_0 g_{k,p}}, \quad (8)$$

where $g_k$ is the actual channel gain between transmitter $t_k$ and the selected receiver $t_{k+1}$. The end-to-end SNR of the secondary system can be expressed as [10]

$$\gamma_{ee} = \min_{k=1,2,\ldots,K} \{\gamma_k\}. \quad (9)$$

Next, we will first derive the PDF of the actual channel gain for the $k$th hop $g_k$ by the similar approach to [19]. From [19, equations (29)-(30)], we have

$$f_{g_k}(x) = \int_{0}^{\gamma_k} f_{g_k|g_{k-1}}(x | y) f_{g_{k-1}}(y) dy \quad (10)$$

where $f_{g_k|g_{k-1}}(x | y)$ is the conditional PDF of $g_k$ given $g_{k-1}$. By substituting (7) into (10), we obtain

$$f_{g_k}(x) = N_k \int_{0}^{\gamma_k} f_{g_{k-1}|\hat{g}_{k-1}}(x | y) \left[F_{\hat{g}_{k-1}}(y)\right]^{N_k-1} dy. \quad (11)$$
By substituting (3) and (2) into (11) and using the binomial expansion, we can get
\[
f_{g_k}(x) = \frac{N_k \beta_k^{m_k+1}}{(1 - \rho_k) \Gamma(m_k)} \left( \frac{x}{\rho_k} \right)^{(m_k-1)/2} \exp\left( -\frac{\beta_k x}{1 - \rho_k} \right) \times \sum_{j=0}^{N_k-1} \left( \frac{N_k - 1}{j} \right) (-1)^j \sum_{n=0}^{m_k-1} \left( \frac{\beta_k y}{n!} \right)^n \int_0^\infty \frac{2 \beta_k \sqrt{\rho_k x y}}{1 - \rho_k} \left( \sum_{n=0}^{m_k-1} \left( \frac{\beta_k y}{n!} \right)^n \right)^j dy.
\]

According to the multinomial theorem, the term \(\sum_{n=0}^{m_k-1} \left( \frac{\beta_k y}{n!} \right)^n\) in (12) can be expanded as
\[
\left[ \sum_{n=0}^{m_k-1} \left( \frac{\beta_k y}{n!} \right)^n \right]^j = j! \sum_{k_1+k_2+\cdots+k_{m_k}=j} \prod_{i=0}^{m_k} \left( \frac{1}{i!} \right)^{k_{i+1}} \left( \frac{\beta_k y}{i!} \right)^{k_i},
\]
where \(h = \sum_{i=0}^{m_k-1} t_{i+1}\). Then (12) can be rewritten as
\[
f_{g_k}(x) = \frac{N_k \beta_k^{m_k+1}}{(1 - \rho_k) \Gamma(m_k)} \left( \frac{x}{\rho_k} \right)^{(m_k-1)/2} \exp\left( -\frac{\beta_k x}{1 - \rho_k} \right) \times \sum_{j=0}^{N_k-1} \left( \frac{N_k - 1}{j} \right) (-1)^j \sum_{n=0}^{m_k-1} \left( \frac{\beta_k y}{n!} \right)^n \int_0^\infty \frac{2 \beta_k \sqrt{\rho_k x y}}{1 - \rho_k} \left( \sum_{n=0}^{m_k-1} \left( \frac{\beta_k y}{n!} \right)^n \right)^j dy.
\]

By using [17, (8.406.3)], we can rewrite the term \(\Phi\) in the above equation as
\[
\Phi = \int_0^\infty y^{(m_k-1)/2+h} \exp\left( -\frac{\beta_k y}{1 - \rho_k} \right) \times \sum_{m=0}^{h} \left( \frac{h + m_k - 1}{h - m} \right) \sum_{m=0}^{h} \left( \frac{\beta_k \rho_k x}{1 - \rho_k} \right) \left( \frac{\beta_k \rho_k x}{1 - \rho_k} \right)^m.
\]

where \(i\) is the imaginary unit and \(J_n(\cdot)\) denotes the nth-order Bessel function of the first kind. From [17, equation (6.643.4)], (15) can be simplified as
\[
\Phi = h! \beta_k^{h-1} \left( \frac{1}{1 - \rho_k} + j \right)^{-h-m_k} \left( \frac{\sqrt{\rho_k x y}}{1 - \rho_k} \right)^{m_k-1} \times \exp\left( \frac{\beta_k \rho_k x}{1 - \rho_k} \right) \times \sum_{m=0}^{h} \left( \frac{\beta_k \rho_k x}{1 - \rho_k} \right)^m.
\]

By substituting (17) into (14), we get the closed-form expression for the PDF of \(g_k\) as follows:
\[
f_{g_k}(x) = \frac{N_k}{\Gamma(m_k)} \sum_{j=0}^{N_k-1} \left( \frac{N_k - 1}{j} \right) (-1)^j j! \]
\[
\left( \frac{\beta_k \rho_k x}{1 - \rho_k} \right)^m \sum_{m=0}^{h} \left( \frac{\beta_k \rho_k x}{1 - \rho_k} \right)^m.
\]
\[
\frac{\gamma}{N} = \frac{Q}{N_0g_{k,p}} < \gamma_{th} \\
F_{\gamma_k}(\gamma_{th}) = \Pr \left\{ \frac{Qg_k}{N_0g_{k,p}} < \gamma_{th} \right\} \\
= \int_0^{\gamma_{th}} f_{g_k}(x) f_{g_{k,p}}(x) \, dx,
\]

where the system SNR is defined as \( \frac{Q}{N_0} \). By substituting (19) and (1) into (20), we have

\[
F_{\gamma_k}(\gamma_{th}) = \frac{N_k}{\Gamma(m_k)} \frac{1}{\Gamma(m_{k,p})} \sum_{j=0}^{N_k-1} \left( \frac{N_k - 1}{j} \right) \, (-1)^j \, j! \\
\times \sum_{k_1 + k_2 + \cdots + k_n = j} \frac{1}{m_k!} \frac{\rho_k^m (1 - \rho_k)^{h-m}}{[1 + j (1 - \rho_k)]^{m_k+m} x_{k_1+1}^{k_1+1} h!} \\
\times \gamma(\gamma + m, \frac{(j+1) \beta_k x}{1 + j (1 - \rho_k)}) .
\]

For the last hop, namely, the \( K \)th hop, by setting \( N_k = 1 \), we can simplify \( F_{\gamma_k}(\gamma_{th}) \) as

\[
F_{\gamma_k}(\gamma_{th}) = \frac{N_k}{\Gamma(m_k)} \frac{1}{\Gamma(m_{k,p})} \sum_{j=0}^{N_k-1} \left( \frac{N_k - 1}{j} \right) \, (-1)^j \, j! \\
\times \sum_{k_1 + k_2 + \cdots + k_n = j} \frac{1}{m_k!} \frac{\rho_k^m (1 - \rho_k)^{h-m}}{[1 + j (1 - \rho_k)]^{m_k+m} x_{k_1+1}^{k_1+1} h!} \\
\times \gamma(\gamma + m, \frac{(j+1) \beta_k x}{1 + j (1 - \rho_k)}) .
\]

3.2. End-to-End Outage Probability. The outage probability of the multihop secondary system which characterizes the probability that the end-to-end SNR falls below a predetermined threshold \( \gamma_{th} \) is given by

\[
P_{out} = \Pr \{ \gamma_{ele} < \gamma_{th} \} = 1 - \prod_{k=1}^{K} \left( 1 - F_{\gamma_k}(\gamma_{th}) \right) .
\]
3.3. End-to-End Average BER. The end-to-end average BER of the multihop secondary system is given by [8]

\[
\text{BER}_{e2e} \quad = \quad \sum_{k=1}^{K} \text{BER}_k \prod_{l=k+1}^{K} (1 - 2\text{BER}_l) \tag{25}
\]

\[
= \frac{1}{2} \left[ 1 - \sum_{k=1}^{K} (1 - 2\text{BER}_k) \right],
\]

where BER\(_k\) denotes the average BER of the \(k\)th hop. We derive the average BER for different kinds of modulations.

3.3.1. Binary Transmissions. The single-hop average BER of coherent, differentially coherent, and noncoherent detection of binary signals is given by [8]

\[
\text{BER}_k = \frac{a^b}{2\Gamma(b)} \int_0^\infty y^{b-1} e^{-ay} F_{\gamma_k}(y) \, dy, \tag{26}
\]

where the parameters \(a\) and \(b\) depend on the particular form of modulation and detection [20].

We define the integral term in (26) as a function \(X(a, b)\) which is expressed as

\[
X(a, b) = \int_0^\infty y^{b-1} e^{-ay} F_{\gamma_k}(y) \, dy. \tag{27}
\]

Then (26) can be rewritten as

\[
\text{BER}_k = \frac{a^b}{2\Gamma(b)} X(a, b). \tag{28}
\]

Now we will derive the closed-form expression for \(X(a, b)\). By exploiting [17, equation (9.131.1)], we can rewrite the CDF of \(\gamma_k\) (22) as follows:

\[
F_{\gamma_k}(\gamma) = \frac{N_k}{\Gamma(m_k) \Gamma(m_k,p)} \sum_{j=0}^{N_k-1} \binom{N_k - 1}{j} (-1)^j j!
\times \sum_{k_1+k_2+\cdots+k_m=j} \prod_{t=0}^{m-1} \frac{1}{k_{t+1}!} (\frac{1}{t!})^{k_{t+1}} h!
\times \sum_{m=0}^{h} \frac{(h + m - 1) \Gamma(m_k + m_k + m)}{h! (m_k + m)}
\times a^b \Gamma(m_k) \Gamma(m_k,p) \sum_{j=0}^{N_k-1} \binom{N_k - 1}{j} (-1)^j j!
\times \sum_{k_1+k_2+\cdots+k_m=j} \prod_{t=0}^{m-1} \frac{1}{k_{t+1}!} (\frac{1}{t!})^{k_{t+1}} h!
\times \sum_{m=0}^{h} \frac{(h + m - 1) \Gamma(m_k + m_k + m)}{h! (m_k + m)}
\times \left\{ \frac{\beta_k}{(1 + j (1 - \rho_k))^{m_k+m}} \right\}.
\]

Then, by substituting (22) into (27) and using the variable transformation \(x = (j+1)\beta_k y/[1 + j (1 - \rho_k)]\beta_k \nu\), we have

\[
X(a, b) = \frac{N_k}{\Gamma(m_k) \Gamma(m_k,p)} \sum_{j=0}^{N_k-1} \binom{N_k - 1}{j} (-1)^j j!
\times \sum_{k_1+k_2+\cdots+k_m=j} \prod_{t=0}^{m-1} \frac{1}{k_{t+1}!} (\frac{1}{t!})^{k_{t+1}} h!
\times \sum_{m=0}^{h} \frac{(h + m - 1) \Gamma(m_k + m_k + m)}{h! (m_k + m)}
\times a^b \Gamma(m_k) \Gamma(m_k,p) \sum_{j=0}^{N_k-1} \binom{N_k - 1}{j} (-1)^j j!
\times \sum_{k_1+k_2+\cdots+k_m=j} \prod_{t=0}^{m-1} \frac{1}{k_{t+1}!} (\frac{1}{t!})^{k_{t+1}} h!
\times \sum_{m=0}^{h} \frac{(h + m - 1) \Gamma(m_k + m_k + m)}{h! (m_k + m)}
\times \left\{ \frac{\beta_k}{(1 + j (1 - \rho_k))^{m_k+m}} \right\}.
\]

By using [17, equation (7.522.1)], we can obtain the closed-form expression for \(X(a, b)\) as follows:

\[
X(a, b) = \frac{1}{a^b \Gamma(m_k) \Gamma(m_k,p)} \sum_{j=0}^{N_k-1} \binom{N_k - 1}{j} (-1)^j j!
\times \sum_{k_1+k_2+\cdots+k_m=j} \prod_{t=0}^{m-1} \frac{1}{k_{t+1}!} (\frac{1}{t!})^{k_{t+1}} h!
\times \sum_{m=0}^{h} \frac{(h + m - 1) \Gamma(m_k + m_k + m)}{h! (m_k + m)}
\times \left\{ \frac{\beta_k}{(1 + j (1 - \rho_k))^{m_k+m}} \right\}.
\]
With the help of (32), the single-hop average BER for binary transmissions (28) can be calculated. Then, by substituting it into (25), we can obtain the end-to-end average BER for binary transmissions.

### 3.3.2. M-QAM Transmissions

The single-hop average BER of M-QAM transmissions is given by [8]

\[
\text{BER}_k = \frac{1}{\sqrt{M} \log_2 \sqrt{M}} \sum_{q=1}^{\log_2 \sqrt{M}} \sum_{a=0}^{\sqrt{M}-1} \phi_{qa} \sqrt{\frac{\omega_q}{\pi}} \quad (33)
\]

where \( \phi_{qa} = (-1)^{q+1} \sqrt{M} \left( 2^{q-1} - \left( \frac{q}{2^{q-1}} \right) + (1/2) \right) \).

From (27), the integral term in (33) can be rewritten as \( X(\omega_q, 1/2) \). Thus, (33) can be expressed as

\[
\text{BER}_k = \frac{1}{\sqrt{M} \log_2 \sqrt{M}} \sum_{q=1}^{\log_2 \sqrt{M}} \sum_{a=0}^{\sqrt{M}-1} \phi_{qa} \sqrt{\frac{\omega_q}{\pi}} X(\omega_q, 1/2). \quad (34)
\]

With the help of (32) and by substituting (34) into (25), we can get the end-to-end average BER for M-QAM transmissions.

### 3.3.3. M-PSK Transmissions

The BER of 2/4/\ldots /M-PSK transmissions can be obtained by recursive algorithms based on the generalized 2/4-PSK [21]. Therefore, we only need to derive the BER of 2/4-PSK transmissions.

According to [8], the BER for the most significant bit (MSB) and the least significant bit (LSB) with 2/4-PSK constellation through a set of angles \( (\theta = [\theta_1, \theta_2]) \) are given by

\[
\text{BER}(y) = \frac{1}{2} \text{erfc}(\sin \theta_2 \sqrt{y}), \quad (35)
\]

\[
\text{BER}(y) = \frac{1}{2} \text{erfc}(\cos \theta_2 \sqrt{y}), \quad (36)
\]

respectively.

\[
\times E \left( m_k + m_{k,p} + m_k + m_{k,p} + m_k + b : \frac{a [1 + j (1 - \rho_k)] \beta_{k,p} \gamma}{(j + 1) \beta_k} \right),
\]

\[m_k + m + 1 : \frac{a [1 + j (1 - \rho_k)] \beta_{k,p} \gamma}{(j + 1) \beta_k}, \quad (31)\]

where \( E(\cdot) \) denotes MacRobert’s \( E \)-Function. Since MacRobert’s \( E \)-Function can always be expressed in terms of Meijer’s \( G \)-function (which can be computed in MATLAB), we can rewrite (31) as

\[
X(a, b) = \frac{1}{a^b} \frac{N_k}{\Gamma(m_k) \Gamma(m_{k,p})} \sum_{j=0}^{N_k-1} \left( \begin{array}{c} N_k - 1 \\ j \end{array} \right) (-1)^j j! \sum_{k_1+k_2+\ldots+k_m=n}^{m_k-1} \frac{1}{k_1+1} \frac{1}{k_2+1} \ldots \frac{1}{k_m+1} \frac{1}{k_{n+1}+1} \frac{1}{k_{n+2}+1} \ldots \frac{1}{k_{n+m}+1} \frac{1}{(j + 1) \beta_k}.
\]

\[m_k + m + 1 : \frac{a [1 + j (1 - \rho_k)] \beta_{k,p} \gamma}{(j + 1) \beta_k}, \quad (32)\]

By applying integration by parts, we can obtain the single-hop average BER for the MSB with 2/4-PSK as

\[
\text{BER}_k = \frac{\sin \theta_2}{2 \sqrt{\pi}} \int_0^\infty y^{-1/2} e^{-y \theta_2} F_{\gamma_k}(y) \, dy = \frac{\sin \theta_2}{2 \sqrt{\pi}} X(\sin^2 \theta_2, 1/2).
\]

The single-hop average BER for the LSB with 2/4-PSK can be obtained in a similar way.

Given the single-hop average BER for the LSB and the MSB cases, the end-to-end average BER for \( M \)-PSK transmissions can be written in closed form with the help of (32) and (25).

### 4. Asymptotic Outage Performance Analysis

In this section, to study the diversity performance for the cognitive multihop sensor network, we derive the asymptotic outage probability of the secondary system in high SNR regime.

According to the asymptotic behavior of the lower incomplete Gamma function,

\[
y(n, x) \xrightarrow{x \to 0} x^n \frac{x^n}{n}, \quad (38)
\]

(21) can be approximately calculated as

\[
F_{\gamma_k}(\gamma_\text{th}) \approx \frac{N_k \beta_{k,p} m_k}{\Gamma(\gamma)} \left( \sum_{j=0}^{N_k-1} \left( \begin{array}{c} N_k - 1 \\ j \end{array} \right) (-1)^j j! \sum_{k_1+k_2+\ldots+k_m=n}^{m_k-1} \frac{1}{k_1+1} \frac{1}{k_2+1} \ldots \frac{1}{k_m+1} \frac{1}{k_{n+1}+1} \frac{1}{k_{n+2}+1} \ldots \frac{1}{k_{n+m}+1} \frac{1}{(j + 1) \beta_k} \right).
\]
\begin{equation}
\times \sum_{m=0}^{h} \left( \frac{h + m_k - 1}{h - m} \right) \frac{1}{m!} \frac{\rho_k^m (1 - \rho_k)^{h-m}}{[1 + j(1 - \rho_k)]^{h + m_k} + m_k + m} \times \frac{1}{m_k + m} \left( \frac{(j + 1) \beta_k y_{th}}{[1 + j(1 - \rho_k)]^\gamma} \right)^{m_k + m} \times \int_0^\infty x^{m_k + m + \gamma - 1} \exp(-\beta_k x) \, dx.
\end{equation}

With the help of the Gamma function and by omitting the higher order terms of \(1/\gamma\), we obtain the asymptotic expression for the CDF of \(y_{th}\) as
\begin{equation}
F_{y_{th}}(y_{th}) \approx \left( \frac{\beta_k y_{th}}{\beta_k \gamma} \right)^{m_k} \frac{N_k \Gamma(m_k + m_k p)}{\Gamma(m_k + 1) \Gamma(m_k p)} \sum_{j=0}^{N_k-1} \left( \frac{N_k - 1}{j} \right) \times (-1)^j j! \sum_{k_1+k_2+\cdots+k_m=j} \frac{1}{k_1+1} \left( \frac{1}{\gamma} \right)^{k_1+1} \times h! \left( \frac{h + m_k - 1}{h} \right) \left( 1 - \rho_k \right)^h \left[ 1 + j(1 - \rho_k) \right]^{m_k + h}.\end{equation}

By substituting (40) into (24), we can get the asymptotic expression for the outage probability of the secondary system. Diversity order is an important performance metric for cooperative relay system. It is defined as
\begin{equation}
d = - \lim_{\gamma \to \infty} \log \frac{P_{out}(y_{th})}{P_{out}(\gamma)}.
\end{equation}

From (40), we can see that, in high SNR regime,
\begin{equation}
F_{y_{th}}(y_{th}) \propto \left( \frac{1}{\gamma} \right)^{m_k}.
\end{equation}

According to (24), we can conclude that the lowest order of \(1/\gamma\) in the outage probability expression is determined by the minimum among the lowest order of \(1/\gamma\) in \(F_{y_{th}}(y_{th})\) \(k = 1, 2, \ldots, K\), that is, \(\min_{k=1,2,\ldots,K}\{m_k\}\). In other words, the diversity order of the secondary system is \(\min_{k=1,2,\ldots,K} m_k\). This indicates that the diversity order of the secondary system is merely determined by the fading severity parameters of the secondary transmission links but not affected by other factors such as the fading severity parameters of the interference links and the CSI imperfection.

5. Numerical Results

In this section, analytical results are presented and validated by simulations. We study the effect of various factors on the outage and error performance of the cognitive multihop sensor network, such as the CSI imperfection, the fading severity of the interference links and the secondary transmission links, the number of hops, and the number of relays in clusters. The predetermined SNR threshold for successful decoding is set as \(y_{th} = 3\) dB.

We denote the distance between cluster \(k - 1\) and cluster \(k\) by \(d_k\) and the distance between cluster \(k - 1\) and PR by \(d_{k,p}\) \(k = 1, 2, \ldots, K\). To consider the impact of path loss, we set \(\Omega_k = d_k^{\eta}\) and \(\Omega_{k,p} = d_{k,p}^{\eta}\), where \(\eta\) is the path loss exponent and is set to 3 in this paper.

To study the effect of the number of hops, we normalize the distance between SS and SD. Specifically, SS and SD are located at coordinates \((0,0)\) and \((1,0)\), respectively. We assume all relay clusters are uniformly distributed on the straight line connecting SS and SD; that is, cluster \(k\) is placed at \((k/K, 0)\), so we have \(d_k = 1/K\). PR is located at \((0.5,1)\).

For the implementation of simulations, we use Monte Carlo method on MATLAB platform. During the simulations, we have to generate pairs of correlated random variables following Nakagami-\(m\) distribution by a modified inverse transform method. The detailed procedure of generating a pair of correlated random variables following Nakagami-\(m\) distribution by a modified inverse transform method is referred to in [22].

Figure 2 plots the exact and asymptotic outage probability curves for a cognitive three-hop sensor network with \(N_k = 3\) relay nodes in each cluster. It shows that the exact analytical results match well the simulation results. Besides, the asymptotic curves are very close to the exact ones in high SNR regime. The slope of the curves which indicates the diversity order rises with the increase in \(m_k\) but remains unchanged with the change of \(m_{k,p}\) or \(\rho_k\). This reveals that the diversity order is affected by the fading severity of the secondary
Table I: Simulation configuration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of hops</td>
<td>$K$</td>
</tr>
<tr>
<td>Number of relays in cluster $k$ ($k = 1, 2, \ldots, K$)</td>
<td>$N_k$ ($N_k = 1$)</td>
</tr>
<tr>
<td>Correlation coefficient between $g_{ij}$ and $\hat{g}_{ij}$ ($i = 1, 2, \ldots, N_k$)</td>
<td>$\rho_k$</td>
</tr>
<tr>
<td>System SNR</td>
<td>$Q$/$N_0$</td>
</tr>
<tr>
<td>SNR threshold for successful decoding</td>
<td>$\gamma_{th} = 3$ dB</td>
</tr>
<tr>
<td>Path loss exponent</td>
<td>$\eta = 3$</td>
</tr>
<tr>
<td>Distance between cluster $k-1$ and cluster $k$ ($k = 1, 2, \ldots, K$)</td>
<td>$d_k = \frac{1}{K}$</td>
</tr>
<tr>
<td>Fading severity parameter of the secondary transmission link between cluster $k-1$ and cluster $k$ ($k = 1, 2, \ldots, K$)</td>
<td>$m_k$</td>
</tr>
<tr>
<td>Average power of the secondary transmission link between cluster $k-1$ and cluster $k$ ($k = 1, 2, \ldots, K$)</td>
<td>$\Omega_k = d_k^{-\eta}$</td>
</tr>
<tr>
<td>Average power of the interference link between cluster $k-1$ and PR ($k = 1, 2, \ldots, K$)</td>
<td>$\Omega_{k,p} = d_k^{-\eta}$</td>
</tr>
</tbody>
</table>

Figure 3 illustrates the outage probability of the secondary system versus $\rho_k$ with $m_k = 3$, $Q/N_0 = 5$ dB, $K = 3$, and $N_k = 3$.

transmission links, irrespective of the fading severity of the interference links and the CSI imperfection, which is consistent with our analysis in Section 4. From Figure 2, we can also observe that the severer the fading gets (i.e., the smaller $m_k$ or $m_{k,p}$ is), the worse the outage performance gets. Furthermore, the fading severity of the interference links has much less impact on the outage performance than that of the transmission links. In particular, when $m_k$ is small, $m_{k,p}$ almost has no effect on the outage performance of the secondary system.

Figure 3 illustrates the outage probability versus $\rho_k$ for a cognitive three-hop sensor network with $N_k = 3$ relay nodes in each cluster. We can see that the outage performance improves with the increase in $\rho_k$. Specifically, the descent of the curves is steep at first but then turns gentle when $\rho_k$ gets close to 1. This indicates that the outage performance is quite good even though the CSI is slightly imperfect; for example, $\rho_k = 0.8$. From Figure 3, we can also observe that as $m_{k,p}$ gets larger, the improvement of the outage performance gets smaller.

In Figure 4, the impact of the number of hops, the number of relays in each cluster, and the CSI imperfection on the outage performance is presented. We can observe that the outage performance improves with the increase in the number of hops. When there is only one hop (i.e., $K = 1$), $\rho_k$ and $N_k$ have no effect on the outage performance since the direct transmission does not involve the relay selection process. For the cases of $K \geq 2$, it is indicated that when the CSI is perfect (i.e., $\rho_k = 1$), assigning only two relays to each cluster can achieve the best outage performance and more relays would not help to achieve better performance. It is also shown that when the CSI estimation is totally random (i.e., $\rho_k = 0$), no matter how many relays each cluster has, the outage performance is the same with the case of $N_k = 1$ since the relay selection does not work anymore. However, when the CSI is imperfect but not totally random (i.e., $0 < \rho_k < 1$), the outage performance improves with the increase in $N_k$ and gradually approaches the case of perfect CSI. In this case, adding more relays to each cluster can mitigate the performance degradation caused by CSI imperfection.

Figure 5 plots the average end-to-end BER curves of a cognitive three-hop sensor network with $N_k = 3$ relay nodes in each cluster. We show four different kinds of modulations, that is, orthogonal coherent BFSK ($a = 1/2$, $b = 1/2$), orthogonal noncoherent BFSK ($a = 1/2$, $b = 1$), antipodal coherent BPSK ($a = 1$, $b = 1/2$), and antipodal differentially coherent BPSK ($a = 1$, $b = 1$). It can be observed that...
6. Conclusion

In this paper, we study the performance of a cluster-based cognitive multihop wireless sensor network with DF partial relay selection over Nakagami-\(m\) fading channels. The imperfect channel knowledge of the secondary transmission links is taken into account. We derive the closed-form expressions for the exact outage probability and BER of the secondary system as well as the asymptotic outage probability in high SNR regime. We also analyze the influence of various factors, for example, the fading severity parameters, the number of relaying hops, the number of available relays in clusters, and the CSI imperfection, on the secondary transmission performance.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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