

Modified Gravity on the Brane and Dark Energy

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Abstract

We analyze the dynamics of an AdS_5 braneworld with matter fields when gravity is allowed to deviate from the Einstein form on the brane. We consider exact 5-dimensional warped solutions which are associated with conformal bulk fields of weight -4 and describe on the brane the following three dynamics: those of inhomogeneous dust, of generalized dark radiation, and of homogeneous polytropic dark energy. We show that, with modified gravity on the brane, the existence of such dynamical geometries requires the presence of non-conformal matter fields confined to the brane.

1 Introduction

According to the Randall-Sundrum (RS) braneworld scenario [1, 2] the visible Universe is a 3-brane boundary of a Z_2 symmetric 5-dimensional (5D) anti-de Sitter (AdS) space. In the RS1 model [1] the AdS_5 orbifold has a compact fifth dimension and two branes. Gravity is localized on the hidden positive tension brane and decays towards

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the visible negative tension brane. In this configuration the RS scenario allows the hierarchy problem to be reformulated as an exponential hierarchy between the weak and Planck scales [1]. In the RS2 model [2] the fifth dimension is infinite and there is just one visible positive tension brane to which gravity is exponentially bound.

On the visible brane, the low energy theory of gravity is 4D Einstein general relativity and the cosmology may be Friedmann-Robertson-Walker [1]-[11]. With two branes this requires the stabilization of the radion mode by introducing, for example, a bulk scalar field [3, 6, 9, 11]. Using the Gauss-Codazzi formulation [12, 13] several other braneworld solutions have been discovered although a number of them have not yet been associated with exact 5D spacetimes [14]-[17].

In this paper we continue the analysis of the dynamics of a spherically symmetric RS 3-brane when conformal matter fields propagate in the bulk [18]-[21] (see also [22]). Some time ago, a new class of exact 5D dynamical warped solutions was discovered which is associated with conformal fields characterized by an energy-momentum tensor of weight -4. These solutions were shown to describe on the brane the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter [18, 19]. In addition, the radion mode was stabilized by a sector of the conformal bulk fluid with a constant negative 5D pressure [20, 21]. In this work we explore the possibility of modifying the theory of gravity on the brane [23, 24] and we analyze if, after that change, the model is still able to generate dynamics of these kinds, in particular the relevant one of a perfect dark energy fluid describing the accelerated expansion of our Universe. As we shall see, the answer will be in the affirmative, under some conditions that will be made explicit in what follows.

2 5D equations with modified gravity and conformal fields

In a 5D RS orbifold mapped by a set of comoving coordinates, (t, r, θ, ϕ, z) , the most general dynamical metric consistent with the Z_2 symmetry in z and with 4D spherical symmetry on the brane is given by

$$d\tilde{s}_5^2 = \Omega^2 \left(dz^2 - e^{2A} dt^2 + e^{2B} dr^2 + R^2 d\Omega_2^2 \right), \quad (1)$$

where $A = A(t, r, z)$, $B = B(t, r, z)$, $R = R(t, r, z)$ and $\Omega = \Omega(t, r, z)$ are Z_2 symmetric functions. R represents the physical radius of the 2-spheres, Ω is the warp factor which defines a global conformal transformation on the 5D metric, and $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$.

Let us consider a dynamical RS action with matter fields and allow gravity on the brane to differ from the standard Einstein-Hilbert form. In the RS2 setting, we write

$$\tilde{\mathcal{S}} = \int d^4x dz \sqrt{-\tilde{g}} \left\{ \frac{\tilde{R}}{2\kappa_5^2} - \Lambda_B + \left[-\frac{\lambda}{\sqrt{\tilde{g}_{55}}} + \frac{\tilde{\mathcal{K}}}{\kappa_5^2} + f(\tilde{R}_4) \right] \delta(z - z_0) \right\}$$

$$+ \int d^4x dz \sqrt{-\tilde{g}} \tilde{\mathcal{L}}_{\text{M}}, \quad (2)$$

where Λ_{B} is the negative bulk cosmological constant, $\kappa_5^2 = 8\pi/M_5^3$ with M_5 the fundamental 5D Planck mass, and f is some arbitrary function of the 4D Ricci scalar, \tilde{R}_4 . A special example is the modified gravity theory with

$$f(\tilde{R}_4) = -\frac{a}{\tilde{R}_4} + b\tilde{R}_4^2, \quad (3)$$

which as been considered as a gravitational alternative for the cosmic dark energy fluid [23]. The brane is located at $z = z_0$ and has tension λ . Note that λ is in fact the zero mode of f . The contribution of the matter fields is defined by the lagrangian $\tilde{\mathcal{L}}_{\text{M}}$. These may include sectors which are confined to the brane,

$$\tilde{\mathcal{L}}_{\text{M}} = \tilde{\mathcal{L}}_{\text{B}} + \frac{\tilde{\mathcal{L}}_{\text{b}}}{\sqrt{\tilde{g}_{55}}} \delta(z - z_0). \quad (4)$$

In the Hawking-Gibbons term the boundary scalar curvature is $\tilde{\mathcal{K}} = \tilde{g}^{\mu\nu} \tilde{\mathcal{K}}_{\mu\nu}$ where the extrinsic curvature is defined by $\tilde{\mathcal{K}}_{\mu\nu} = \tilde{\nabla}_{\mu} \tilde{n}_{\nu}$. The vector normal to the brane boundary is $\tilde{n}_{\nu} = \delta_{\nu}^5 \sqrt{\tilde{g}_{55}}$. In the RS1 model, the brane with tension λ and gravity f is the hidden Planck brane. To represent the visible brane we introduce another Dirac delta source at $z = z'_0$ with a tension λ' and a theory of gravity defined by a new function $g(\tilde{R}_4)$. The analysis that follows is valid for both RS models. For simplicity, we omit explicit reference to the second brane in the RS1 setting.

A Noether variation on the action (2) gives the 5D classical field equations

$$\begin{aligned} \tilde{G}_{\mu}^{\nu} = & -\kappa_5^2 \left[\Lambda_{\text{B}} \delta_{\mu}^{\nu} + \delta(z - z_0) \left(\lambda - \frac{\tilde{\mathcal{K}}}{\kappa_5^2} - f(\tilde{R}_4) \right) \tilde{\gamma}_{\mu}^{\nu} \right] \\ & - \kappa_5^2 \left[\delta(z - z_0) \frac{2}{\sqrt{\tilde{g}_{55}}} \left(\frac{\tilde{\mathcal{K}}_{\mu}^{\nu}}{\kappa_5^2} + f'(\tilde{R}_4) \tilde{R}_{4\mu}^{\nu} \right) - \tilde{\mathcal{T}}_{\text{M}\mu}^{\nu} \right], \end{aligned} \quad (5)$$

where the prime denotes differentiation with respect to the argument x of the function $f(x)$. The induced metric on the brane is

$$\tilde{\gamma}_{\mu}^{\nu} = \frac{1}{\sqrt{\tilde{g}_{55}}} \left(\delta_{\mu}^{\nu} - \delta_{\mu}^5 \delta_5^{\nu} \right) \quad (6)$$

and the stress-energy tensor $\tilde{\mathcal{T}}_{\text{M}\mu}^{\nu}$ associated with the matter fields is defined by

$$\tilde{\mathcal{T}}_{\text{M}\mu}^{\nu} = \tilde{\mathcal{L}}_{\text{M}} \delta_{\mu}^{\nu} - 2 \frac{\delta \tilde{\mathcal{L}}_{\text{M}}}{\delta \tilde{g}^{\mu\alpha}} \tilde{g}^{\alpha\nu}, \quad \tilde{\mathcal{T}}_{\text{M}\mu}^{\nu} = \tilde{\mathcal{T}}_{\text{B}\mu}^{\nu} + \frac{\tilde{\mathcal{T}}_{\text{b}\mu}^{\nu}}{\sqrt{\tilde{g}_{55}}} \delta(z - z_0). \quad (7)$$

The bulk stress-energy tensor is conserved,

$$\tilde{\nabla}_{\mu} \tilde{\mathcal{T}}_{\text{B}\nu}^{\mu} = 0. \quad (8)$$

Note that, in general, $\tilde{\mathcal{T}}_{\text{b}\mu}^\nu$ is not conserved.

For a general 5D metric, $\tilde{g}_{\mu\nu}$, (5) and (8) is difficult to solve system of partial differential equations. Let us introduce some simplifying assumptions on the field variables involved in the problem. First, consider that the bulk matter is described by conformal fields with weight s . Under the conformal transformation $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$, the stress-energy tensor satisfies $\tilde{\mathcal{T}}_{\text{B}\mu}^\nu = \Omega^{s+2} \mathcal{T}_{\text{B}\mu}^\nu$. Consequently, (5) and (8) may be re-written as [18]

$$\begin{aligned}
G_\mu^\nu &= -6\Omega^{-2} (\nabla_\mu \Omega) g^{\nu\rho} \nabla_\rho \Omega + 3\Omega^{-1} g^{\nu\rho} \nabla_\rho \nabla_\mu \Omega - 3\Omega^{-1} \delta_\mu^\nu g^{\rho\sigma} \nabla_\rho \nabla_\sigma \Omega \\
&\quad - \kappa_5^2 \Omega^2 \Lambda_{\text{B}} \delta_\mu^\nu + \kappa_5^2 \Omega^{s+4} \mathcal{T}_{\text{B}\mu}^\nu - \kappa_5^2 \Omega^2 \delta(z - z_0) \left(\lambda - \frac{\tilde{\mathcal{K}}}{\kappa_5^2} - f(\tilde{R}_4) \right) \tilde{\gamma}_\mu^\nu \\
&\quad - \kappa_5^2 \Omega^2 \delta(z - z_0) \frac{2}{\sqrt{\tilde{g}_{55}}} \left(\frac{\tilde{\mathcal{K}}_\mu^\nu}{\kappa_5^2} + f'(\tilde{R}_4) \tilde{R}_{4\mu}^\nu - \frac{\tilde{\mathcal{T}}_{\text{b}\mu}^\nu}{2} \right), \tag{9}
\end{aligned}$$

$$\nabla_\mu \mathcal{T}_{\text{B}\nu}^\mu + \Omega^{-1} \left[(s+7) \mathcal{T}_{\text{B}\nu}^\mu \partial_\mu \Omega - \mathcal{T}_{\text{B}\mu}^\mu \partial_\nu \Omega \right] = 0. \tag{10}$$

If we split the bulk tensor $\tilde{\mathcal{T}}_{\text{B}\mu}^\nu$ into two sectors, $\tilde{T}_{\text{B}\mu}^\nu$ and $\tilde{U}_{\text{B}\mu}^\nu$, with the same weight s , $\tilde{\mathcal{T}}_{\text{B}\mu}^\nu = \tilde{T}_{\text{B}\mu}^\nu + \tilde{U}_{\text{B}\mu}^\nu$, where $\tilde{T}_{\text{B}\mu}^\nu = \Omega^{s+2} T_{\text{B}\mu}^\nu$ and $\tilde{U}_{\text{B}\mu}^\nu = \Omega^{s+2} U_{\text{B}\mu}^\nu$, and take $s = -4$, then it is possible to separate (9) in the following way [21]

$$G_\mu^\nu = \kappa_5^2 T_{\text{B}\mu}^\nu, \tag{11}$$

$$\begin{aligned}
6\Omega^{-2} \nabla_\mu \Omega \nabla_\rho \Omega g^{\rho\nu} - 3\Omega^{-1} \nabla_\mu \nabla_\rho \Omega g^{\rho\nu} + 3\Omega^{-1} \nabla_\rho \nabla_\sigma \Omega g^{\rho\sigma} \delta_\mu^\nu = \\
- \kappa_5^2 \left[\Omega^2 \Lambda_{\text{B}} \delta_\mu^\nu - U_{\text{B}\mu}^\nu + \Omega^2 \delta(z - z_0) \left(\lambda - \frac{\tilde{\mathcal{K}}}{\kappa_5^2} - f(\tilde{R}_4) \right) \tilde{\gamma}_\mu^\nu \right] \\
- \kappa_5^2 \Omega^2 \delta(z - z_0) \frac{2}{\sqrt{\tilde{g}_{55}}} \left(\frac{\tilde{\mathcal{K}}_\mu^\nu}{\kappa_5^2} + f'(\tilde{R}_4) \tilde{R}_{4\mu}^\nu - \frac{\tilde{\mathcal{T}}_{\text{b}\mu}^\nu}{2} \right). \tag{12}
\end{aligned}$$

Within this splitting, the Bianchi identity implies

$$\nabla_\mu T_{\text{B}\nu}^\mu = 0. \tag{13}$$

Then, (10) is in fact

$$\nabla_\mu U_{\text{B}\nu}^\mu + \Omega^{-1} \left(3\mathcal{T}_{\text{B}\nu}^\mu \partial_\mu \Omega - \mathcal{T}_{\text{B}\mu}^\mu \partial_\nu \Omega \right) = 0. \tag{14}$$

In (11) and (13) we have 5D equations with conformal bulk fields but without a brane or bulk cosmological constant. These equations do not depend on the warp factor which is dynamically defined by (12) and (14). Consequently, in this simplified setting the warp is the only effect reflecting the existence of the brane or of the bulk cosmological constant. Note that this separation is only possible with the special set of bulk fields which have a stress-energy tensor with conformal weight $s = -4$.

Although the system of dynamical equations is now partly decoupled, it still remains difficult to solve, for Ω depends non-linearly on A , B and R . Furthermore, it is affected by $\mathcal{T}_{B\mu}^\nu$ and $\mathcal{T}_{b\mu}^\nu$. So, let us assume that $A = A(t, r)$, $B = B(t, r)$, $R = R(t, r)$ and $\Omega = \Omega(z)$. Then (11) and (12) lead to [21]

$$G_a^b = \kappa_5^2 T_{Ba}^b, \quad (15)$$

$$G_5^5 = \kappa_5^2 T_{B5}^5, \quad (16)$$

$$6\Omega^{-2}(\partial_z \Omega)^2 + \kappa_5^2 \Omega^2 \Lambda_B = \kappa_5^2 U_{B5}^5, \quad (17)$$

$$\begin{aligned} (3\Omega^{-1}\partial_z^2 \Omega + \kappa_5^2 \Omega^2 \Lambda_B) \delta_a^b = & -\kappa_5^2 \Omega^2 \delta(z - z_0) \left(\lambda - \frac{\tilde{\mathcal{K}}}{\kappa_5^2} - f(\tilde{R}_4) \right) \delta_a^b \\ & - \frac{2\kappa_5^2}{\sqrt{g_{55}}} \Omega^2 \delta(z - z_0) \left(\frac{\tilde{\mathcal{K}}_a^b}{\kappa_5^2} + f'(\tilde{R}_4) \tilde{R}_{4a}^b - \frac{\tilde{\mathcal{T}}_{ba}^b}{2} \right) + \kappa_5^2 U_{Ba}^b, \end{aligned} \quad (18)$$

where the latin indices represent the 4D coordinates t , r , θ and ϕ . Since, according to (15) and (16), $T_{B\mu}^\nu$ depends only on t and r , (13) becomes

$$\nabla_a T_{Bb}^a = 0. \quad (19)$$

On the other hand (17) and (18) imply that $U_{B\mu}^\nu$ must be diagonal, $U_{B\mu}^\nu = \text{diag}(-\bar{\rho}, \bar{p}_r, \bar{p}_T, \bar{p}_T, \bar{p}_5)$, with the density $\bar{\rho}$ and pressures \bar{p}_r , \bar{p}_T satisfying $\bar{\rho} = -\bar{p}_r = -\bar{p}_T$. In addition, $U_{B\mu}^\nu$ must only depend on z . Consequently, $\nabla_a U_{Bb}^a = 0$ is an identity. Then, using (14) and noting that $T_{B\mu}^\nu = T_{B\mu}^\nu(t, r)$, we find

$$\partial_z U_{B5}^5 + \Omega^{-1} \partial_z \Omega (2U_{B5}^5 - U_{Ba}^a) = 0, \quad 2T_{B5}^5 = T_{Ba}^a. \quad (20)$$

If $U_{B\mu}^\nu(z)$ is a conserved tensor field like $T_{B\mu}^\nu$ then U_{B5}^5 must be constant. So (20) leads to the following equations of state

$$2T_{B5}^5 = T_{Ba}^a, \quad 2U_{B5}^5 = U_{Ba}^a. \quad (21)$$

We obtain that $\bar{p}_5 = -2\bar{\rho}$ and $U_{B\mu}^\nu$ is constant. On the other hand, if $T_{B\mu}^\nu = \text{diag}(-\rho, p_r, p_T, p_T, p_5)$ where ρ , p_r , p_T and p_5 are, respectively, the density and pressures, then its equation of state is re-written as

$$\rho - p_r - 2p_T + 2p_5 = 0. \quad (22)$$

Note that ρ , p_r , p_T and p_5 must be independent of z but may be functions of t and r . The bulk matter is, however, inhomogeneously distributed along the fifth dimension because the physical energy density, $\tilde{\rho}(t, r, z)$, and pressures, $\tilde{p}(t, r, z)$, are related to $\rho(t, r)$ and $p(t, r)$ by the scale factor $\Omega^{-2}(z)$. The bulk sector $T_{B\mu}^\nu$ determines the dynamics on the branes, and $U_{B\mu}^\nu$ influences how the gravitational field is warped around the branes. In the RS1 model $U_{B\mu}^\nu$ also acts as a stabilizing sector [21].

3 Exact 5D warped solutions and modified gravity

The AdS₅ braneworld dynamics are defined by the solutions of (15) to (19) and (22). Let us first solve (17) and (18). As we have seen $U_{B\mu}^\nu$ is constant with $\bar{\rho} = -\bar{p}_r = -\bar{p}_T = -\bar{p}_5/2$. After integrating (17), by using the cartesian coordinate y related to z by $z = le^{y/l}$, and taking into account the Z_2 symmetry, we obtain [21]

$$\Omega(y) = e^{-|y|/l} \left(1 + p_B^5 e^{2|y|/l} \right), \quad (23)$$

where l is the AdS radius given by $l = 1/\sqrt{-\Lambda_B \kappa_5^2/6}$ and $p_B^5 = \bar{p}_5/(4\Lambda_B)$. This set of solutions must also satisfy (18) which contains the Israel jump conditions. These boundary conditions imply that the brane energy-momentum \tilde{T}_{ba}^b is given by

$$\tilde{T}_{ba}^b = \left[\lambda_{RS} (R - 2) + R \left(\lambda - \frac{\tilde{\mathcal{K}}}{\kappa_5^2} - f(\tilde{R}_4) \right) \right] \delta_a^b + 2 \left(\frac{\tilde{\mathcal{K}}_a^b}{\kappa_5^2} + f'(\tilde{R}_4) \tilde{R}_{4a}^b \right), \quad (24)$$

where $\lambda_{RS} = 6/(l\kappa_5^2)$ and $R = \Omega(0) = p_B^5 + 1$. \tilde{T}_{ba}^b , $\tilde{\mathcal{K}}_a^b$ and \tilde{R}_{4a}^b are calculated at the brane boundary $y = 0$.

To determine the dynamics on the brane we need to solve (15) and (16) when $T_{B\mu}^\nu$ satisfies (19) and (22). First note that if $T_{B\mu}^\nu = 0$ then the 5D metric is given by

$$d\tilde{s}_5^2 = dy^2 + \Omega^2 d\Omega_4^2, \quad (25)$$

where $d\Omega_4^2 = d\chi^2 + \sin^2\chi d\Omega_3^2$ with $d\Omega_3^2 = d\xi^2 + \sin^2\xi d\Omega_2^2$. The boundary conditions (24) then imply that

$$\tilde{T}_{ba}^b = \left[\frac{\lambda_{RS}}{3R} (R - 2)(R + 1) + R \left(\lambda - f \left(\frac{12}{R^2} \right) \right) + \frac{6}{R^2} f' \left(\frac{12}{R^2} \right) \right] \delta_a^b. \quad (26)$$

Since the boundary condition depends on the arbitrary function f , the matter confined to the brane is ambiguous. To lift such an ambiguity some fundamental physical principle should be added to the RS scenario, in order to fix the corresponding theory of gravity on the brane [25].

With a nonzero $T_{B\mu}^\nu$ it is possible to consider more general 5D geometries. Let us first note that as long as p_5 balances ρ, p_r and p_T according to (16) and (22), the 4D equation of state of the bulk fluid is not constrained. In previous work, three examples corresponding to inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter were analyzed [18, 19, 21]. The latter describes the dynamics on the brane of dark energy in the form of a polytropic fluid. The diagonal conformal matter can be defined by

$$\rho = \rho_P + \Lambda, \quad p_r + \eta\rho_P^\alpha + \Lambda = 0, \quad p_T = p_r, \quad p_5 = -\frac{1}{2}(\rho_P + 3\eta\rho_P^\alpha) - 2\Lambda, \quad (27)$$

where ρ_P is the polytropic energy density, Λ is a bulk quantity which mimics a brane cosmological constant and the parameters (α, η) characterize different polytropic phases.

Solving the conservation equations, we find [19, 26]

$$\rho_P = \left(\eta + \frac{a}{S^{3-3\alpha}} \right)^{\frac{1}{1-\alpha}}, \quad (28)$$

where $\alpha \neq 1$, a is an integration constant and $S = S(t)$ is the Robertson-Walker scale factor of the braneworld which is related to the physical radius by $R = rS$. For $-1 \leq \alpha < 0$ the fluid is in its generalized Chaplygin phase (see also [26]). With this density the Einstein equations lead to the following 5D dark energy polytropic solutions [19]

$$d\tilde{s}_5^2 = \Omega^2 \left[-dt^2 + S^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right) \right] + dy^2, \quad (29)$$

where the brane scale factor S satisfies

$$\frac{\ddot{S}}{S} = -\frac{\kappa_5^2}{6} (\rho_P - 3\eta\rho_P^\alpha - 2\Lambda) \quad (30)$$

and

$$\dot{S}^2 = \frac{\kappa_5^2}{3} (\rho_P + \Lambda) S^2 - k. \quad (31)$$

The global evolution of the observable universe is then given by [17, 19]

$$S\dot{S}^2 = V(S) = \frac{\kappa_5^2}{3} \left[\left(\eta S^{3-3\alpha} + a \right)^{\frac{1}{1-\alpha}} + \Lambda S^3 \right] - kS. \quad (32)$$

Let us now consider the boundary conditions (24) which define the brane tensor \tilde{T}_{ba}^b . Using the metric (29) we obtain

$$\tilde{T}_{bt}^t = \frac{\lambda_{RS}}{3R} (R-2)(R+1) + R \left[\lambda - f(\tilde{R}_4) \right] + \frac{6}{R^2} \frac{\ddot{S}}{S} f'(\tilde{R}_4), \quad (33)$$

$$\tilde{T}_{br}^r = \frac{\lambda_{RS}}{3R} (R-2)(R+1) + R \left[\lambda - f(\tilde{R}_4) \right] + \frac{2}{R^2} f'(\tilde{R}_4) \left(\frac{\ddot{S}}{S} + 2 \frac{\dot{S}^2 + k}{S^2} \right), \quad (34)$$

where $\tilde{R}_4 = 6(S\ddot{S} + \dot{S}^2 + k)/(S^2R)$. We also have $\tilde{T}_{b\theta}^\theta = \tilde{T}_{b\phi}^\phi = \tilde{T}_{br}^r$. All the other components of $\tilde{T}_{b\mu}^\nu$ are equal to zero. Subtracting (33) and (34) we find

$$\tilde{T}_{bt}^t - \tilde{T}_{br}^r = \frac{4}{R^2} f'(\tilde{R}_4) \left(\frac{\ddot{S}}{S} - \frac{\dot{S}^2 + k}{S^2} \right). \quad (35)$$

If $f = 0$ the theory of gravity on the brane is that of Einstein. Then we obtain $\tilde{T}_{bt}^t = \tilde{T}_{br}^r$ which implies that

$$\tilde{T}_{ba}^b = \left[\frac{\lambda_{RS}}{3R} (R-2)(R+1) + R\lambda \right] \delta_a^b. \quad (36)$$

For $\tilde{\mathcal{T}}_{b\mu}^\nu = 0$ we obtain the brane tension $\lambda = \lambda_{\text{RS}}(2 - R)(R + 1)/(3R^2)$. Note that if f is a constant then the same is true with the appropriate shift in the brane tension, $\lambda \rightarrow \lambda - f$. If f is not constant and there is no matter confined to the brane or if $\tilde{\mathcal{T}}_{bt}^t = \tilde{\mathcal{T}}_{br}^r$ then we are lead to

$$\frac{\ddot{S}}{S} = \frac{\dot{S}^2 + k}{S^2}. \quad (37)$$

Using (30) and (31) we find that equation (37) restricts ρ_{P} to be zero or a constant, $\rho_{\text{P}} = \eta^{1/(1-\alpha)}$. Under this circumstances, the potential $V(S)$ is given by

$$V(S) = \frac{\kappa_5^2}{3} (\rho_{\text{P}} + \Lambda) S^3 - kS, \quad (38)$$

and describes a global evolution determined by an effective brane cosmological constant Λ or $\Lambda + \eta^{1/(1-\alpha)}$.

In general, when f is not constant and $\tilde{\mathcal{T}}_{bt}^t$ is different from $\tilde{\mathcal{T}}_{br}^r$, the boundary equations (33) and (34) define state constraints relating the theory of gravity represented by f and the matter confined to the brane characterized by $\tilde{\mathcal{T}}_{b\mu}^\nu$. For example if f is given by (3) we find

$$\tilde{\mathcal{T}}_{bt}^t = \frac{\lambda_{\text{RS}}}{3R} (R - 2)(R + 1) + R \left(\lambda + \frac{a}{\tilde{R}_4} - b\tilde{R}_4^2 \right) + \frac{6}{R^2} \frac{\ddot{S}}{S} \left(\frac{a}{\tilde{R}_4^2} + 2b\tilde{R}_4 \right), \quad (39)$$

$$\tilde{\mathcal{T}}_{bt}^t - \tilde{\mathcal{T}}_{br}^r = \frac{4}{R^2} \left(\frac{a}{\tilde{R}_4^2} + 2b\tilde{R}_4 \right) \left(\frac{\ddot{S}}{S} - \frac{\dot{S}^2 + k}{S^2} \right). \quad (40)$$

In this perspective, the gravitational brane dynamics are defined by the conformal bulk fields of weight -4 and are supported on the brane by a simple brane tension, if f is a constant, and by more general matter, if f is some arbitrary function of \tilde{R}_4 . Such dynamics may be of the dark energy polytropic type or even more general and are independent of f . What actually depends on f is the kind of matter that is confined to the brane. As is clear, in (24) each f is balanced by a certain $\tilde{\mathcal{T}}_{b\mu}^\nu$ which in general must be associated with non-conformal fields.

4 Conclusions

We have analyzed in this paper exact 5D solutions which describe the dynamics of AdS₅ braneworlds when conformal fields of weight -4 propagate in the bulk. We have considered the possibility of a general deviation of the theory of gravity on the brane from the Einstein form and analyzed the effect this can have on the capability of the AdS₅ braneworld to describe on the brane the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic dark energy, respectively. These solutions are determined by the conformal bulk fields of weight -4. While a sector of the fields generates the dynamics on the brane, another sector affects the

way how gravity is warped around the brane. In the RS1 model, this latter sector also provides a way to stabilize the radion mode. In this work we have shown that, within our model of modified gravity, the existence of such exact 5D geometries requires the propagation of non-conformal matter fields confined to the brane. The corresponding energy-momentum tensor is defined in terms of the gravitational elements composing the relevant theory of gravity on the brane. Thus, these non-conformal fields play the role of supporting the dynamics on the brane when modified gravity is present. Their state explicitly depends on how gravity deviates from the Einstein form and on the AdS₅ braneworld dynamics generated by the conformal bulk fields. In this context, the ambiguity associated with the brane theory of gravity is reflected on the kind of matter that is confined to the brane. Note also that, related with this, one can consider a generalized non-minimal coupling of gravity with matter on the brane, of the form $f(\tilde{R}_4)\tilde{\mathcal{L}}_d$, where $\tilde{\mathcal{L}}_d$ is some matter lagrangian including also a kinetic term (see [27]). In the case of usual 4D gravity, this provides an explanation for dark energy dominance. Such effect on the brane will be studied elsewhere.

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