Self-Triggered Set-Valued Observers

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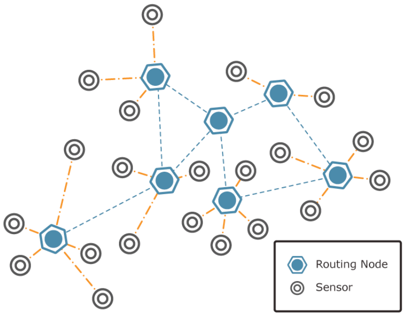
Outline

Motivation

- Networked Control Systems - Systems sharing a network with controller/observer in a different physical location.
- Fault Detection - The uncertainty regarding node communication increases the complexity.

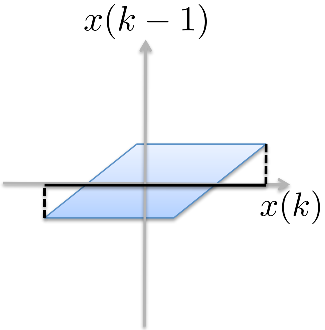


Set-valued estimates

- The objective is to determine and maintain set-valued estimates of the state of the system.
- Available measurements at the sensor nodes are shared with the estimator.
- Low-battery usage by the nodes and low-computational power are key requirements.
- Two main issues: the Fourier-Motzkin projection method is computationally heavy and the size of the matrices grows rapidly.

Motivating Example

- The set description relies on $x(k)$ and previous time instants.
- Exact projection relies on the expensive Fourier-Motzkin elimination method.
- When using polytopes the number of hyper-edges increases.
- Then, we use an overapproximation that is of low complexity.



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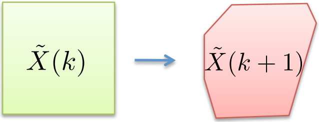

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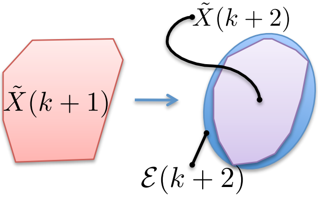


Problem Outline

- For an Uncertain Linear Parameter-Varying System (LPV), a set-valued estimate $X(k)$ is typically non-convex.
- Consider a Set-Valued Observer (SVO) that generates polytopic approximations, $\tilde{X}(k)$, of the optimal $X(k)$.

Self-Triggered SVOs Problem

How can we provide low-complexity overapproximations for the set $\tilde{X}(k)$.

Problem Model

- Take a Uncertain LPV of the form

$$S : \begin{cases} x(k+1) = A(\Delta(k))x(k) + B(\Delta(k))u(k) \\ y(k) = C(\Delta(k))x(k) \end{cases}$$

- n_Δ number of uncertainties
- $\Delta(k)$ is assumed to be polytopic i.e.,

$$A(\Delta(k)) = \sum_{\ell=1}^{n_\Delta} \Delta_\ell(k) A_\ell, \ |\Delta_\ell(k)| \leq 1, \forall k \geq 0.$$

- A_ℓ are constant matrices

Proposed Solution

Ellipsoidal Overbound

- Use SVOs to compute $\tilde{X}(k)$;
- Ensure a *symmetry* condition on $\tilde{X}(k)$;
- Then, Ellipsoidal overbound $\mathcal{E}(k)$ is simply the square of matrix M, where $\tilde{X}(k) := \{x : Mx \leq 1\}$.

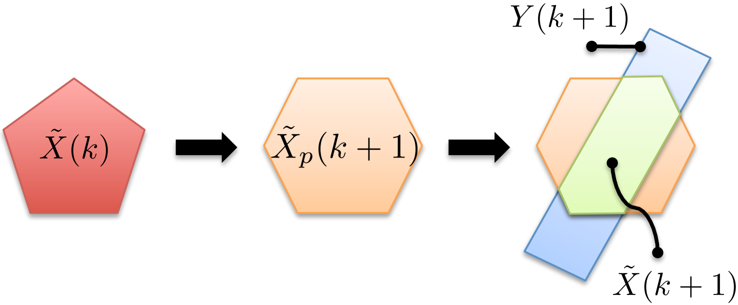
Self-Triggered SVOs

- Use standard SVOs to get $\tilde{X}(k)$ and the above ellipsoidal overbound $\mathcal{E}(k)$;
- Update $\mathcal{E}(k)$ using methods in the literature for ellipsoids;
- When $\mathcal{E}(k)$ is *too conservative* reduce the conservatism by running the standard SVOs.

SVOs

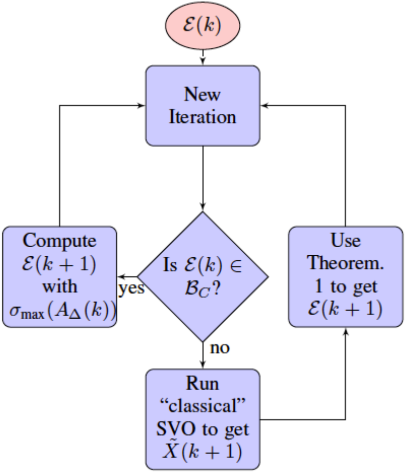
Given the previous set $\tilde{X}(k)$:

- Using SVOs, the algorithm predicts $\tilde{X}_p(k+1)$ using the dynamics;
- Then, the set is intersected with the measurement set $Y(k+1)$.



Self-triggered Strategy

- Each iteration the algorithm checks if the current ellipsoid is within the error bound $\mathcal{B}_C$.
- If yes, propagate it using the worst-case dynamics.
- If not, run classical SVO to get a better approximate.
- Compute the new ellipsoidal approximation.



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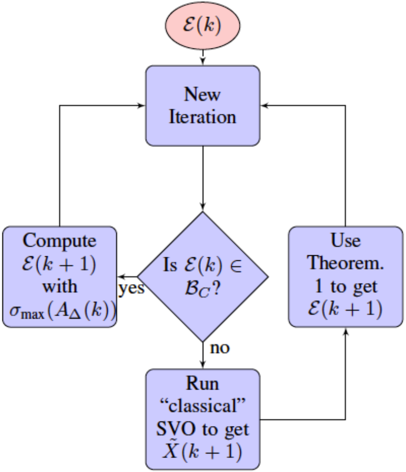

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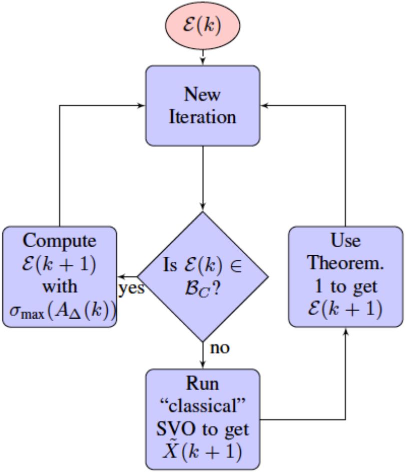
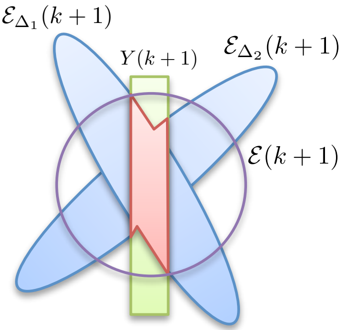


Special Case of distributed systems

- In gossip algorithms all the dynamics matrices are equal up to a permutation of row and columns.
- An overbound for the worst-case norm is found by selecting the ellipsoid that aligns with the singular vectors of $Y(k+1)$.
- For the case of ellipsoid overbound, a similar strategy to make the set symmetric can be used.



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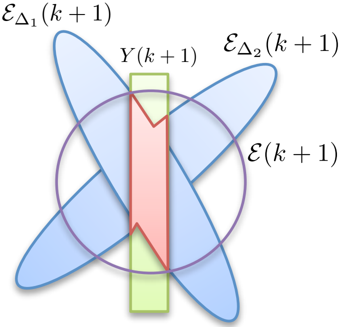


Properties

- Matrix $M \in \mathbb{R}^{\ell \times n}$, where $\ell \gg n$ (high number of hyper-plane restrictions) whereas the proposed overbound matrix belongs to $\mathbb{R}^{n \times n}$;
- Running the SVO computations at some time instants allows to use the idle moments to pre-compute the necessary combinations of matrices products;
- In distributed systems, it is possible to discard dynamics matrices based on their singular vectors structure and compute the worst estimate with minimal processing effort.

Main Result

- The time to the next trigger, τ, is given by

$$\tau = \left\lceil \log_\gamma \frac{\sigma_{\text{min}}(M(T))C}{\sqrt{n}} \right\rceil$$

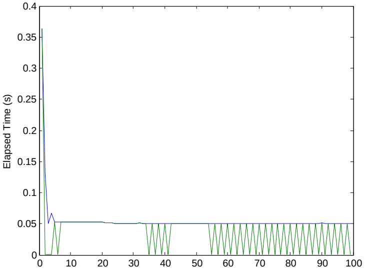
- T is the last triggering time;
- C is the maximum norm at the previous trigger

$$\gamma = \max_i \sigma_{\text{max}}(A_0 + A_{\Delta_i})$$

Simulation Results (1/2)

Setup: We considered a simplified wind turbine model with uncertainty in the initial state and bounded disturbances and measurement noise.

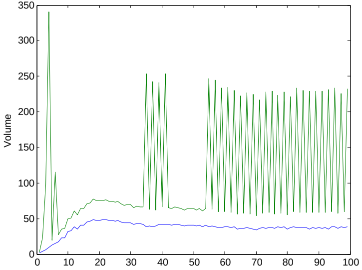
- In a typical run, the standard SVO (blue line) after the initial allocation of data structures, has each iteration taking around 50 ms.
- The Self-Triggered SVO (green line) makes it suitable for this application with sampling period of 10 ms.



Simulation Results (2/2)

The algorithm for having a polytope with symmetric edges has a severe impact on the performance after 50 sampling times.

- The volume metric degraded as the origin of the polytope moved away from the origin, which introduced conservatism.
- Additional research is required for ensuring the symmetry condition, which was outside the scope of this paper.



Final Remarks

Contributions:

- given a symmetric structure for polytope matrix, we show how to compute an overbounding ellipsoid or ball;
- a self-triggered mechanism that uses ellipsoidal approximations, triggering the computation of the SVO only when necessary to ensure convergence;
- results are provided for the worst-case triggering frequency for a Linear Parameter-Varying (LPV) system;
- For gossip algorithms, it is shown that the overbounds are efficient to compute and propagate, since its complexity is constant.

The end

- Thank you for your time.

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