Finite-time Average Consensus in a Byzantine Environment Using Set-Valued Observers

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***Abstract*— This paper addresses the problem of consensus in the presence of Byzantine faults, modeled by an attacker injecting a perturbation in the state of the nodes of a network. It is firstly shown that Set-Valued Observers (SVOs) attain finite-time consensus, even in the case where the state estimates are not shared between nodes, at the expenses of requiring large horizons, thus rendering the computation problem intractable in the general case. A novel algorithm is therefore proposed that achieves finite-time consensus, even if the aforementioned requirement is dropped, by intersecting the set-valued state estimates of neighboring nodes, making it suitable for practical applications and enabling nodes to determine a stopping time. This is in contrast with the standard iterative solutions found in the literature, for which the algorithms typically converge asymptotically and without any guarantees regarding the maximum error of the final consensus value, under faulty environments. The algorithm suggested is evaluated in simulation, illustrating, in particular, the finite-time consensus property.**

I. Introduction

The problem of consensus in a Byzantine environment relates to a set of agents agreeing on a common value, using a distributed algorithm, and considering that an unknown number of those nodes can be malfunctioning or attacked by an intruder. We are interested in randomized gossip average consensus in which nodes are allowed to send messages to a random neighbor in order to compute the average of their initial state. By nature, such algorithms are designed to cope with "crash type" faults by using redundancy and randomization. However, Byzantine faults, such as an intruder in the system, can prevent convergence or drive the steady state of the system to any value [1].

The consensus problem has been widely studied when considering rather non-antagonistic failure models which include packet drops and nodes crashing but, to enable a more comprehensive model, Byzantine faults must be considered. The work [1] considers the problem of detecting and correcting the state of the system in the presence of a Byzantine fault. The case of malicious agents and faulty agents is studied and the authors provide, in both cases, bounds on the number of corrupted nodes to ensure detectability of the fault. In [1], the system dynamics are described by a linear time-invariant model that constrains the communication network to be fixed in all time slots. Here, however, a randomized gossip algorithm is considered, thus dropping the assumption that the same sets of nodes are involved in message exchanges at every time instant.

The problem of finite-time consensus in the presence of malicious agents have been addressed in [2], where the authors show that the topology of the network categorizes its ability to deal with attacks. Both the number of corrupted nodes and vertex-disjoint paths in the network influence its resilience. In [2], it is assumed a broadcast model where, at each transmission time, the nodes send to all their neighbors the same value and the agents objective is to compute some function of the initial state. The main difference is the communication model which we assume to be gossip where random pairs of nodes exchange information.

The choice for representing the set of possible states depends on a mathematical formulation that enables fast and non-conservative intersections and unions of sets, as those are major and normally time-consuming operations when implemented in a computer. One approach is to use the concept of zonotopes, described in [3] and further developed in [4] and [5]. In this article, an alternative approach is adopted, based on the concept of Set-Valued Observers (SVOs) first introduced in [6] and [7] and further information can be found in [8] and [9] and the references therein.

In [10] an SVO-based overlay technique is used to detect Byzantine faults in gossip randomized average consensus algorithms. The main interest is on considering the stochastic information to detect faults and on providing bounds on the attacker signal magnitude. In [10], each node runs an independent SVO to construct a set-valued state estimation. The focus herein is given on, in finite-time, either detecting a fault or returning the average consensus value using estimations obtained from local information and exchanged during communication time. We do not consider the case of compensating for faults.

The main contributions of the present work are as follows:

- Finite-time consensus is shown to be a property of SVOs for a sufficiently large horizon when not communicating estimates, in non-faulty scenarios;
- An algorithm is introduced that intersects neighbor estimates to produce less conservative bounds and reduce the time required to detect faults;
- Finally, it is also shown that this algorithm has the property of achieving finite-time consensus without the need to consider large horizons.

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