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# Abstract

Doctor of Philosophy

## Precision bounds in noisy quantum metrology

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Quantum metrology is a vividly developing topic of current research in both theoretical and experimental physics. Its main goal is to explore the capabilities of quantum systems that, when employed as probes sensing physical parameters, allow to attain resolutions that are beyond the reach of classical protocols. Spectacularly, one may show that in an idealistic scenario, by utilising the phenomena of quantum entanglement and super-classical correlations, a parameter of interest may be in principle determined with mean squared error that scales as  $1/N^2$  with the number of particles the system consists of—surpassing the  $1/N$ -scaling characteristic to classical statistics. However, a natural question arises, whether such an impressive quantum enhancement persists when one takes into account the decoherence effects, i.e. the noise that distorts the system and is unavoidably present in any real-life implementation.

In this thesis, we resolve a major part of this issue by describing general techniques that allow to quantify the attainable precision in metrological schemes, while accounting for the impact of uncorrelated noise-types—ones that independently disturb each constituent particle (atom, photon) in the system. In particular, we show that the abstract geometrical structure of a *quantum channel* describing the noisy evolution of a single particle dictates critical bounds on the achievable quantum enhancement. Importantly, our results prove that an infinitesimal amount of noise is enough to restrict the precision to scale classically in the asymptotic  $N$  limit, what then constrains the maximal improvement to a constant factor. Although for relatively low numbers of particles the decoherence may be ignored, for large  $N$  the presence of noise heavily alters the form of both states and measurements that should be employed to achieve the ultimate resolution. Crucially, however, the established bounds are then typically attainable with use of states and detection techniques that are natural to current experiments.

As we thoroughly introduce the necessary concepts and mathematical tools lying behind the quantum metrological tasks, including the estimation theory techniques in both *frequentist* and *Bayesian* frameworks, we hope that this work may be found attractive by researchers coming from the quantum information theory background and willing to become more familiar with the current approaches to quantum metrology problems. Throughout the work, we provide examples of applications of the methods presented to typical *qubit noise models*, yet we also discuss in detail the phase estimation task in *Mach-Zehnder interferometry* both in the classical and quantum setting, with particular emphasis given to the photonic-losses model while analysing the impact of decoherence.