ON-LINE EVALUATION OF EQUALIZER PERFORMANCE BASED ON HIGH-ORDER STATISTICS

J. B. Destro Filho\textsuperscript{1}, J.P.Breda Destro\textsuperscript{2} and J. M. Travassos Romano\textsuperscript{1}

\textsuperscript{1} DECOM – FEEC - UNICAMP
State University of Campinas
Caixa Postal 6101
13081-970 Campinas - SP - BRAZIL
Tel.: (+55) 19 788-3703; Fax: (+55) 19 289-1395

\textsuperscript{2} CTA / IEAv / EIN-A
Rodovia dos Tamoios km 5,5
12228-904
São José dos Campos – SP - BRAZIL
Tel.: (+55) 12 347-5319

\textbf{ABSTRACT}: In this paper, a simple recursive method is developed in order to estimate the bit-error-rate (BER) at the output of an adaptive equalizer. This is accomplished in two steps. Firstly, a theoretical procedure for detecting decision errors is derived. This test is then used with an adaptive blind channel identification scheme, based on high-order statistics theory, in order to derive the final adaptive BER estimator. Simulation results point out that our method provides on-line and reliable BER estimation with a low computational burden for linear channels models.

1. INTRODUCTION

Adaptive equalization is currently used in the receiver of several communication systems in order to provide a reliable information exchange, for example, in computer networks, HDTV transmissions and mobile communications [Pro95]. As a consequence, the correct operation of the equalizer is essential to assure the global system performance.

On the other hand, according to the current trends of the literature, the classical supervised training algorithms are being replaced with blind schemes, such as Bussgang algorithms [DJK94]. In this case, equalizer adaption is quite difficult due to local minima problems, which lead often to poor performance. Besides, these problems increase as the signal-to-noise ratio (SNR) gets lower, which is a common situation in mobile communications.

Therefore, the monitoring of adaptive equalizers performance is a very important issue for current communication systems. Nevertheless, to our knowledge, there are few works of literature devoted to this crucial problem. In [DoK97], the authors propose a binary hypothesis test in order to detect errors due to an incorrect decision of the equalizer. Although such technique is quite effective, it presents high computational complexity and it does not work "on-line". In [CSF98], the authors estimate the bit-error-rate (BER) at the equalizer output by means of a neural network. Although this method may be applied to non-linear channels, the authors did not discuss the transient performance of the BER estimates, which may be affected by local minima problems connected with the neural network learning.

In this paper, our main goal is to develop a simple adaptive method in order to estimate the BER at the output of an adaptive equalizer. Section 2 presents the mathematical model and some basic concepts, which are used in section 3 to derive a theoretical procedure in order to check whether decision errors have taken place. This test is then used in section 4 with an adaptive channel identification scheme, leading to the final BER estimator. Section 5 presents simulation results and the final conclusions are summarized in section 6.

2. THEORETICAL FRAMEWORK AND PREVIOUS RESULTS

Consider in fig.1 the classical mathematical model used for the analysis of adaptive equalizers.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Communication system model.}
\end{figure}

Where \( \{h(n)\} , \{c(n)\} \) and \( \{v(n)\} \) denotes respectively the impulsive response of the channel, the linear equalizer and of the global system (channel plus linear equalizer). Besides,
\[
\{v(n)\} = \{h(n)\} \ast \{c(n)\}
\]

where the operator $$\ast$$ is the discrete convolution.

We suppose that the following hypotheses hold:

(H1) The communication system model is baseband.

(H2) The signal-to-noise ratio is high, such that the additive noise may be neglected.

(H3) The information signal \( x(n) \) is zero-mean, iid and M-PAM (where M is the number of modulation levels).

(H4) The communication system is linear and stable.
(H5) The global system \( \{v(n)\} \) is time-invariant and stable.

Notice that, although hypotheses (H1)-(H5) are restrictive, they have been extensively used in the past [BeG84,ShW90] in order to analyse adaptive equalization. Besides, (H1),(H2),(H4) and (H5) are currently used [DJK94] in order to derive important results in the field of blind equalization.

It may be demonstrated that the output of the linear equalizer is given by:
\[
y(n) = x(n-d) + \text{dist}(n)
\]  
(2)

Where \( d \) is the equalization delay, \( \text{dist}(n) \) is the distortion or intersymbol interference, \( N \) is the channel length; \( L \) the equalizer length and \( v(j) \) the \( j \)-th coefficient of \( \{v(n)\} \).

The main goal of the linear equalizer is to recover the information signal, such that at the output of the decision device the "open-eye" condition is verified [DJK94]:
\[
\hat{x}(n) = x(n-d) \Leftrightarrow |\text{dist}(n)| < Q/2
\]  
(4)

\[
\Delta = \begin{cases} 
0; & \text{if } x(n-d) = \hat{x}(n) \Leftrightarrow |\text{dist}(n)| < Q/2 \\
1; & \text{if } x(n-d) \neq \hat{x}(n) \Leftrightarrow |\text{dist}(n)| \geq Q/2 
\end{cases}
\]  
(5)

Where \( Q \) is the distance between two adjacent levels of the \( M \)-ary PAM signal and \( e(n) \) is the equalization error or decision error. Notice that when (4) holds, the intersymbol interference \( \text{dist}(n) \) may be different from zero, but the output of the decision device is equal to the transmitted signal. As a consequence, no decision error has occurred \( (e(n) = 0) \). Conversely, if (4) does not hold, then a decision error has taken place \( (e(n) = 1) \).

3. DETECTION OF EQUALIZATION ERRORS

Since the distortion (4) is a function of the global system coefficients \( v(j) \), and since each coefficient \( v(j) \) is bounded due to the stable character of \( \{v(n)\} \), we may define an upper bound for the distortion \( \text{dist}(n) \), which will be represented by the symbol \( \text{Sup}\{\text{dist}(n)\} \). (The operator \( \text{Sup}\{ (.) \} \) represents the maximum value of (.)).

In fact, there is no unique choice for this upper bound, which will be established following the classical approach suggested in [Luc65,DJK94]. These authors study the worst case of maximum intersymbol interference, which enables to cope with the general situation where the distortion may take any value lower than \( \text{Sup}\{\text{dist}(n)\} \).

A particular bound of the distortion is then established by imposing the following hypotheses:

(H6) The bound \( \text{Sup}\{\text{dist}(n)\} \) is derived by assuming the worst case, where the intersymbol interference is maximum.

(H7) The bound is defined by taking into account both effects of the transmitted signal \( x(n) \) and of the channel on the distortion, and such that \( \text{Sup}\{\text{dist}(n)\} \) is a function of only \( \{v(n)\} \).

In appendix I, we demonstrate the following theorem.

**THEOREM 1 (MAIN RESULT):** "Suppose that (H1)-(H7) are verified. Then:
\[
|v(d)| > (M-1)(N+L-2)\text{Sup}\{v(j)\} \Leftrightarrow \text{for } j,j \neq d
\]  
(6)

\[
|v(d)| \leq (M-1)(N+L-2)\text{Sup}\{v(j)\} \Leftrightarrow \text{for } j,j \neq d
\]  
(7)

Where \( \text{Sup}\{v(j)\} \) is the maximum absolute value of the set of coefficients \( \{v(0), v(1), ..., v(d-1), v(d+1), ..., v(N-L-2)\} \).

Equations (6)-(7) may be used as a simple procedure in order to check whether equalization errors have occurred, provided that the global system coefficients \( \{v(n)\} \), \( d \), \( M \) and \( L \) are known.

4. A NEW METHOD FOR THE BER ESTIMATION

Based on theorem 1, we propose the following recursive procedure in order to estimate the BER at the output of an adaptive equalizer (\( n \) denotes the iteration number).

Given \( d, L, M \) and \( N \) for \( n = 1 \) to the total number of iterations

**Step 1:** Equalize the input signal and calculate the recovered signal \( \hat{x}(n) \).

**Step 2:** Estimate the channel model \( \{\hat{h}(n)\} \).

**Step 3:** Estimate the global system
\[
\{\hat{v}(n)\} = \{\hat{h}(n)\}^* \{c(n)\}.
\]

**Step 4:** From \( \{\hat{v}(n)\} \), select \( |\hat{v}(d)| \) and \( \text{Sup}\{v(j)\} \).

**Step 5:** Compare \( |\hat{v}(d)| \) with \( \text{Sup}\{v(j)\} \) according to (6)-(7) and calculate \( e(n) \).

**Step 6:** Update the following estimator of the BER (expressed in percentage):
\[
\text{BER}(n) = \frac{n-1}{n} \text{BER}(n-1) + \left( \frac{100}{n} \right) e^2(n)
\]  
(8)

We propose to perform step 2 by means of a blind identification algorithm based on high-order statistics.
5. SIMULATION RESULTS

Figure 2 presents a comparison of our BER estimator to the classical BER estimator, for which the decision error $e(n)$ is obtained by comparing the pilot signal $x(n)$ to the recovered signal $\hat{x}(n)$. In fig. 2, we suppose $x(n)$ 2-PAM, the channel model $H(z) = 0.35z^{-1} + 0.35z^{-2}$, SNR = 20 dB and the equalizer is trained by the CMA algorithm initialized according to the "center-spike" strategy [DJJK94]. The plots have been obtained by averaging the results attained by the equalization of 50 different sequences $x(n)$, according to the Monte Carlo method.

Figure 2: Comparison of the new BER estimator (plot "NEW") with the classical one (plot "OLD"). Vertical axis: BER(n), in percentage. Horizontal axis: iterations.

Based on fig. 2, one may conclude that our estimator is effective, since it follows the classical estimator closely. Similar results have been obtained for SNR values ranging from 15 dB up to 40 dB. We have also simulated different linear channel models, for different M-PAM modulations and for different SNR values. These results, which are not presented here due to space limitations, are similar to fig. 2. However, for SNR values under 15 dB, convergence is quite slow and the steady-state difference between the two estimators reveals limitations of our method.

6 – CONCLUSION

In this paper, we have proposed a new method for the estimation of the bit-error-rate at the output of an equalizer. Such method is based on the blind identification of the channel, followed by a simple checking procedure, which recognizes equalization errors. Simulations point out that the new method is effective for high and moderate levels of the signal-to-noise ratio, and its implementation is simple. Theoretical analysis is under course in order to tackle with the error-propagation effects associated to inaccurate cumulants estimation, as well as the case of low signal-to-noise ratio.

ACKNOWLEDGEMENTS

The authors would like to thank CNPq, Fapesp and CAPES for the financial support of this work.

APPENDIX I – DEMONSTRATION OF THEOREM 1

We will just demonstrate (6), since demonstration of (7) may be carried out by a similar procedure. Please see [Des98] for further details.

We assume that hypotheses (H6), (H7) and the following condition hold:

$$\left|v(d)\right|>(M-1)(N+L-2), \sup_{j,j\neq d}\left\{|v(j)|\right\} \tag{9}$$

Then we will show that (9) leads to the open-eye, i.e.,

$$x(n-d) \equiv \hat{x}(n) \Rightarrow e(n)=0 \tag{10}$$

Step 1: The open-eye condition (4)-(5) in the context of hypotheses (H6)-(H7) and some useful inequalities.

According to (H6), we are just considering the worst case of maximum intersymbol interference. Thus (4)-(5) may be rewritten as below.

$$\Delta x(n-d) \Leftrightarrow \sup_{\text{dist}(n)}<Q/2 \tag{11}$$

$$e(n)=\{0; \text{ if } x(n-d) \equiv \hat{x}(n) \Leftrightarrow \sup_{\text{dist}(n)}<Q/2 \tag{12}$$

The following equations define three vectors.

$$V(n)=[v(0) v(1) \ldots v(N+L-2)]^T \tag{13a}$$

$$V(1)=[v(0) v(1) \ldots v(d-1) v(d+1) \ldots v(N+L-2)]^T \tag{13b}$$

$$X(n)=[x(n) \ldots x(n+d-1) x(n-d) \ldots x(n-N+L-2)]^T \tag{13c}$$

Now consider (H3). According to [Pro95], a M-PAM signal may take the discrete values established by the following set $A$:

$$A = \left\{ \frac{-1}{2} \ldots \frac{-1}{2} , \frac{-1}{2} , \ldots \frac{-1}{2} , \frac{M-1}{2} , \ldots \frac{M-1}{2} \right\} \tag{14}$$

As a consequence, we may define a bound for $x(n)$:
Step 2: An upper bound for the intersymbol interference.

Application of the triangle inequality to (3) leads to:

\[
\text{Sup} \left\{ \| x(n) \| : \forall n \right\} \leq \frac{\| \mathbf{M} - 1 \| \cdot \mathbf{Q} }{2} ; \forall n, j
\]  

(14)

According to (H7), we may calculate a bound for (15) with respect to the vector \( \mathbf{X}(n) \):

\[
\text{dist}(n) = \frac{\sum_{j=0}^{N+L-2} v(j) x(n-j) }{v(d)} \leq \frac{\sum_{j=0}^{N+L-2} v(j) \| x(n-j) \| }{v(d)} ; \forall n
\]

(15)

Notice that all quantities in (16) are positive. In this way, the bound should consider that all terms \( |x(n-j)| ; \forall n, j \) in (16) attain their maximum value according to (14).

\[
\text{Sup} \left\{ \text{dist}(n) \right\} \leq \frac{\| \mathbf{M} - 1 \| \cdot \mathbf{Q} }{2} ; \forall n
\]

(16)

Consider now a bound which also includes the effects of the channel, by considering (17) with respect to the vector \( \mathbf{V}(n) \):

\[
\text{Sup} \left\{ \text{dist}(n) \right\} \leq \frac{\| \mathbf{M} - 1 \| \cdot \mathbf{Q} }{2} ; \forall n
\]

(17)

Since all the terms in the sum of (18) are positive, this equation may be rewritten as below.

\[
\text{Sup} \left\{ \text{dist}(n) \right\} \leq \frac{\| \mathbf{M} - 1 \| \cdot \mathbf{Q} }{2} ; \forall n
\]

(18)

Step 3: Final demonstration

Since (9) holds, one may multiply both sides of this equation by the factor \( \mathbf{Q}/2 \) and arrange terms such that:

\[
\frac{\| \mathbf{M} - 1 \| \cdot \mathbf{Q} }{2} \cdot \frac{\| x(n) \| }{v(d)} < \frac{\mathbf{Q} }{2} ; \forall n, j = 0, 1, ..., N+L-2 ; j \neq d
\]

(21)

Applying (20) to inequality (21), we get:

\[
\text{Sup} \left\{ \text{dist}(n) \right\} < \frac{\mathbf{Q} }{2} ; \forall n
\]

(22)

Since \( \text{Sup} \left\{ \text{dist}(n) \right\} \) is a particular form of \( \text{Sup} \left\{ \text{dist}(n) \right\} \), comparison of (22) and (11)-(12) leads to the conclusion that the eye is open, which completes the demonstration.

References