In this paper we study the tracking performance of the momentum LMS (MLMS) algorithm in adaptive prediction for a time-varying chirped sinusoidal signal. The momentum term of the algorithm not only helps to speed up the convergence rate, but also improves the tracking capability for a nonstationary signal. We compare the simulation results of the MLMS with the conventional LMS to highlight the tracking performance. The simulation results show that the MLMS algorithm with an additional momentum term has a better tracking capability in an 8-tap adaptive predictor under noise-free conditions, especially when the filter tracks a fast time-variant signal. However, the MLMS does not have significant improvement when tracking a noise corrupted chirp signal. The normalised MLMS (NMLMS) algorithm has similar simulation results as the ordinary MLMS algorithm for the tracking performance.

2 NONSTATIONARY CHIRPED SINUSOIDS

Chirp-type signals are used in many applications, such as, radar, sonar and in FM communication systems. The linear chirped sinusoidal signal is described as

\[ s(n) = \sqrt{P_s} \exp(j(2\pi fn + \psi n^2/2 + \phi)) \]  

where \( \psi \) is the chirp rate of the signal, \( f \) is the centre frequency, \( n \) is the time period and \( \phi \) is the constant phase of the signal. The signal amplitude \( P_s \) is constant but the signal is nonstationary due to the chirping process (frequency sweeps linearly with time). The higher the chirp rate, the faster the signal changes in frequency and the harder it is for the adaptive filter to search for its minimum error surface.

The input signal \( x(n) \) is then given by

\[ x(n) = s(n) + \eta(n) \]  

where \( \eta(n) \) is a zero mean white Gaussian noise process with noise power \( P_\eta \).
3 MOMENTUM LMS ALGORITHM

The MLMS algorithm is schematically represented in Fig.1.

\[ w(n+1) = w(n) + 2\mu_e(n)x(n) + \alpha[w(n) - w(n-1)] \quad |\alpha| < 1 \quad (4) \]

where \( w(n) = [w_0(n), \cdots, w_N(n)]^T \) and \( N \) is the filter length. The output error is derived from the desired response \( d(n) \) according to

\[ e(n) = d(n) - \beta^T(n)X(n) \quad (5) \]

However, for a nonstationary input signal, the condition in (6) and (7) does not apply to the MLMS. This is due to the misadjustment of the MLMS filter under the tracking situation is dominated by the lag misadjustment rather than the noise misadjustment in the steady-state condition. The lag misadjustment is the result of the time variation of the optimal filter weight \( w_{opt}(n) \). Hence, high chirp rates will cause larger residual error at the tracking filter output.

As the optimum weight for the tracking filter can never be reached, the filter always stays in the tracking mode. In this case, the momentum term in the MLMS filter helps to accelerate the search towards the optimum weight solution. Thus, a smaller weight-error vector in the MLMS can be expected compared to the ordinary LMS algorithm.

4 SIMULATION RESULTS

In this section, the improvement of the tracking performance for the MLMS algorithm for a nonstationary input chirped sinusoidal signal is confirmed by simulation results. The adaptive predictor in Fig.2 is used to model and track the chirped sinusoidal signal. This configuration is well known for noise reduction filtering of noisy signals and investigation of tracking behaviour for chirped sinusoidal signals [3], [5] and [6].

Figure 1: Block diagram of MLMS

The momentum constant (\( \alpha \)) in the algorithm controls the scale of the gradient of previous filter weight. If the previous gradient is relative large, then this additional gradient term will accelerate the search towards the global minimum (MSE).

3.1 MLMS tracking behaviour

An algorithm with good convergence properties does not necessarily possess a good tracking capability. For a stationary input signal, the MLMS offers an improvement in the convergence rate relative to the LMS. However, this increase is offset by a corresponding increase in the misadjustment. The additional misadjustment is directly proportional to the fraction of the previous weight-vector increment in the momentum term [8].

\[ M_{MLMS} = \frac{k_\alpha}{1 - k_\alpha} \quad (6) \]

where \( k_\alpha = \sum_{i=1}^{N} \frac{\mu \lambda_i (1 + \alpha)}{1 - \alpha^2 - 2 \mu \lambda_i} < 1 \)

And as compared to the LMS

\[ M_{MLMS} = (1 + \alpha)M_{LMS} \quad (7) \]

The mean-squared error equation for the predictor is

\[ J(w) = E[s(n) - y(n)]^2 \quad (8) \]

where \( y(n) = W^T(n)X(n) \)

We compare MLMS tracking performance with the conventional LMS algorithm for noise-free (\( P_n = 0 \)) and noise-corrupted input signal with different SNR levels (10 dB and 30 dB). Each experiment is repeated 150 times, then the results are averaged and illustrated.

(i) Noise-free input chirp signal: For the first simulation, the input signal \( x(n) \) is a clean chirp signal with no noise added. The simulations are performed for different chirp rates \( \psi \) to differentiate tracking performance for fast time-varying signals and slow time-varying signals.
Figure 3: Simulation results for 8-tap adaptive predictor for different chirp rates under noise-free condition.

Fig. 3 shows the MSE learning curves of the LMS algorithm and the MLMS algorithm for two different values of chirp rate \( \psi \). The simulation results demonstrate that for a slower chirp rate (\( \psi = 10^{-7} \)), there is a 5 dB improvement in the tracking performance of the MLMS algorithm over the LMS algorithm. For a higher chirp rate (\( \psi = 5 \times 10^{-4} \)), the filter outperforms the LMS solution by about 8 dB. In both cases, the MLMS not only exhibits better tracking capability, but also has a superior convergence speed over the LMS.

(II) Noisy conditions: In this section, different levels of Gaussian white noise are added to the input chirp signal (SNR = 10 dB, 30 dB) to determine the tracking ability under different chirp rates \( \psi \). Fig. 4 shows the predictor response to the noise-corrupted nonstationary input signal.

Figure 4: The predictor performance under different SNR and chirp rates (\( \psi \)). The momentum term (\( \alpha \)) = 0.5 and step size (\( \mu \)) = 0.005

Part (a) and Part (b) in Fig. 4 show that the MLMS algorithm has no advantage in tracking a slow time-varying chirp signal in noisy conditions. Nevertheless, a better convergence rate can be obtained as compared to the standard LMS algorithm. Whereas, for a fast time-varying chirp signal, the MLMS algorithm has a slight improvement for a low SNR (10dB) and a significant improvement for a high SNR (30dB). The relative simulation results are shown in Part (c) and Part (d). The misadjustment for the MLMS is almost the same as the LMS for chirp rate \( \psi = 10^{-7} \) and has a lower value for chirp rate \( \psi = 10^{-4} \). This also shows that the MLMS has a better performance for a nonstationary signal than a stationary signal.

(III) Normalised MLMS: The NMLMS algorithm is shown as follow

\[
 w(n+1) = w(n) + \beta e(n)x(n) + \alpha[w(n) - w(n-1)] \quad |\alpha| < 1 \quad (9)
\]

where \( \beta = \frac{\mu}{\delta + \sum_{i=1}^{N} x^2(n-i)} \) and \( \delta = 0.01 \)

This yields an algorithm which has a stable convergence gain term which is not dependent on input power. The simulation is performed under noise-free and noisy conditions for a fixed amplitudne chirp signal.
The simulation results of the NMLMS are shown in Part(a) and Part(b) are very similar to the original MLMS. It is clearly seen that, the NMLMS does not gain any advantage in tracking a low SNR input chirp signal. However, a better convergence rate can be obtained.

5 CONCLUSION

In this paper we have been concerned with the steady-state performance characteristics of the MLMS algorithm. The improvement in tracking capability of the MLMS and NMLMS algorithms is presented by simulation. The filters do not gain significant improvement in tracking for a low SNR input chirp signal, however there is an obvious gain in speed of the convergence rates.

The lag misadjustment of the MLMS for a nonstationary chirp signal is significantly less than the original LMS. This is because the momentum term helps to accelerate the search process towards the time-variant optimal filter weight of the predictor.

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References


