Essays in Empirical Asset Pricing: Liquidity, Idiosyncratic risk, and the Conditional Risk-Return Relation

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to my parents

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Introduction

What kinds of risk do systematically drive stock returns? This question has prompted vast amounts of research and is still one of the main challenges in finance. It has not only been of interest in the finance literature, but it also concerns investors across the globe. In general, investors aim at avoiding risky stocks but are keen on earning high returns. But which stocks are considered to be risky? Does a premium exist for risky stocks? High returns and low risk - do these two goals conflict with each other? The following dissertation addresses these questions empirically. Studying the German and the US stock market, we investigate the risk-return relationship and evaluate which kind of stocks yield a significant risk premium.

The first model that gave an answer to these questions was the Capital Asset Pricing Model (CAPM). The model was developed by Sharpe (1964), Lintner (1965), and Mossin (1966) in the 1960s and set the foundation for modern asset pricing theory. Its central implication is that every asset's expected return is a linear increasing function of its market risk or market beta. According to the CAPM, the market excess return is the only systematic risk factor and the market beta, the slope of an asset return on the market excess return, embodies the systematic risk of the asset. Early empirical evidence such as that of Black et al. (1972) and Fama and MacBeth (1973) finds support for the model. However, during the 1980s and 1990s it turned out that market risk is not the only systematic risk. The so-called anomaly literature provides a large amount of evidence that the CAPM does not hold empirically and that other variables also influence stock prices. Banz (1981) documents that small firms have on average higher market risk adjusted returns than large firms in the US. This anomaly is entitled as the size effect. Further, Rosenberg et al. (1985) and Fama and French (1992) show that stocks with a high book-to-market equity ratio outperform stocks with a low one, which is the so-called book-to-market effect. The CAPM fails to explain the size and book-to-market effect. Fama and French (1993) show that portfolios constructed to mimic risk factors

related to size and book-to-market equity add substantially to the variation in stock returns explained by the market factor. For this reason, they argue in favor of a three-factor model. Besides the inclusion of the market excess return as in the CAPM, the Fama-French three-factor model considers the size and book-to-market factor. The size factor is the return of a portfolio of small firms minus the return of a portfolio of big firms. The book-to-market factor is the difference between the return of a portfolio of high book-to-market equity stocks minus the return of a portfolio of low book-to-market equity stocks. Fama and French (1993, 1995) suggest that the size and book-to-market factor mimic combinations of two underlying risk factors or state variables of special hedging concern for investors. Furthermore, Fama and French (1995) argue that the book-to-market beta is a proxy for relative distress. Firms with persistently low earnings have low book-to-market equity and negative slopes on the book-to-market factor. Fama and French (1996) find that the three-factor model absorbs most of the anomalies that have plagued the CAPM. Since its inception, the Fama-French three-factor model has been the standard empirical asset pricing model in the finance literature. For this reason, it is used as a standard of comparison throughout this dissertation.

This dissertation pursues two main goals. The first goal is to examine the relation between risk and return and to develop an appropriate test procedure to evaluate whether significant risk premia prevail. Early tests of the risk-return relation by Lintner[1] and Black et al. (1972) use a cross-sectional approach regressing mean returns for each asset on beta estimates. Fama and MacBeth (1973) introduce an alternative for estimating the risk-return relation. Instead of taking sample average returns, they regress asset returns on beta estimates for each month of the sample period. The sample mean of the slope coefficient represents the risk premium. Since its inception, the Fama-MacBeth test has been one of the standard econometric methodologies in the empirical asset pricing literature. In the first chapter of my dissertation, we question the Fama-MacBeth test and evaluate the risk-return relation by applying a conditional approach to the Fama-French model. Subsequently, we develop a procedure to test if the risk is also priced according to the conditional approach. This procedure is compared to the Fama-MacBeth test.

Second, we investigate whether other risk factors, which cannot be captured by the Fama-French factors, also influence stock returns. Although the Fama-French factors are well-established in the literature, there is some evidence that the Fama-French factors cannot explain all asset pricing effects. Jegadeesh and Titman (1993) discover that past

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[1]Douglas (1969) summarizes some of Lintner's unpublished results.

winners earn higher returns than past losers, the so-called momentum effect. Fama and French (1996) and Grundy and Martin (2001) find that the momentum effect cannot be captured by the Fama-French factors. As a result of this anomaly, many papers also consider the momentum factor and use a four-factor model. The momentum factor has been first proposed by Carhart (1997) and is the return of a portfolio of past winners minus the return of a portfolio of past losers. There are plenty of other asset pricing variables and risk factors that have been endorsed to be helpful in explaining stock returns in the literature. For example, Chen et al. (1986) propound macroeconomic risk factors as, e.g., interest rates, inflation, and industrial production. Cochrane (1996) suggests a production factor, Jagannathan and Wang (1996) a factor for human capital, Harvey and Siddique (2000) a coskewness factor, Gervais et al. (2001) a trading volume factor, Lamont et al. (2001) a financial constraint factor, Ang et al. (2001) a downside correlation factor, Easley et al. (2002) a measure for information risk, Vassalou and Xing (2004) a measure for default risk, and Ang et al. (2006a) downside beta. This dissertation evaluates the impact of illiquidity (chapter II) and idiosyncratic risk (chapter III) on stock returns. Liquidity measures the ability to trade large quantities quickly at low costs with little price impact. Idiosyncratic or unsystematic risk is the company or industry specific risk that is uncorrelated to the systematic risk. The final goal of chapters II and III is to test whether illiquidity and idiosyncratic risk yield significant risk premia.

Chapter I.[2] The first chapter challenges the widely used Fama-MacBeth test. According to asset pricing theory, in expectation there is a positive reward for taking risks. Investors are assumed to be risk averse and demand a compensation for holding risky assets. For this reason, riskier assets should yield higher expected returns. For instance, the expected market excess return, the difference between the market return and the risk-free rate, should be positive. To be in line with theory, empirical tests should find a positive relation between risk and expected returns. However, empirical tests are based on realized returns instead of expectations and realized returns are frequently negative. During periods of negative returns, the risk-return relation should be reversed, which is neglected by the standard Fama-MacBeth procedure. In order to take this into account, we make use of a conditional approach differentiating between periods with positive risk factor realizations and negative ones to test the risk-return relation. The conditional approach follows Pettengill et al. (1995). In contrast to the existent literature, we apply

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[2]This chapter is based on joint work with Christian Westheide (Koch and Westheide (2009)).

the conditional approach to the Fama-French three-factor model. We condition not only on the sign of the market return, but on that of each of the three factors, and test if the book-to-market and size betas retain their explanatory power once the conditional nature of the relation between betas and return is taken into account. As predicted by theory, our results yield strong support for a positive risk-return relation when risk factor realizations are positive and a negative one when risk factor realizations are negative. However, at this stage results are not comparable to the Fama-MacBeth test as the Fama-MacBeth approach tests if beta risk is priced. Thus, as a further contribution to the literature, we derive a test based on the conditional approach to evaluate if beta risks are priced making the two tests comparable. This test extends the approach by Freeman and Guermat (2006) to multi-factor models. Our results provide evidence that the FG test produces very similar results as the standard Fama-MacBeth test. This finding does not only hold for empirical data from the US stock market, but it is confirmed through simulations based on different return distributions. Therefore, the results of the first chapter justify the application of the Fama-MacBeth test in the next chapters of this dissertation.

In addition, our results stress the importance of the selection of test portfolios in empirical asset pricing. We detect that estimates for risk premia strongly rely on the choice of test portfolios. Results in chapters II and III confirm this finding, emphasizing the lack of robustness of asset pricing models to alternative portfolio formation.

The following two chapters study the German stock market. Although empirical asset pricing is an extensive research field, there are only a few studies dealing with the German stock market. This is mainly due to the fact that a comprehensive set of accounting data and numbers of shares outstanding is not electronically available back to the 1970s, which makes the construction of a long time-series for the book-to-market and size factors impossible. The empirical analyses of chapters II and III are based on a unique data set covering about 1000 German firms. We make use of hand collected data on the number of shares outstanding as well as accounting data from the Hoppenstedt Aktienführer allowing us to construct the size and book-to-market factor for Germany. Daily prices and trading volume are obtained from Deutsche Kapitalmarktdatenbank in Karlsruhe. The sample period runs from January 1974 to December 2006.

Chapter II.[3] Numerous episodes of financial market distress have underscored the importance of the smooth functioning of markets for the stability of the financial system. These episodes have been characterized by sudden and drastic reductions in market liquidity, which have led, amongst others, to disorderly adjustments in asset prices and a sharp increase in the costs of executing transactions. For instance, in October 1987, stock markets around the world crashed. Especially, on October 19, denoted as the Black Monday, the S&P 500 plummeted by over 20% creating the greatest loss Wall Street had ever suffered on a single day. Insufficient liquidity had a significant effect on the size of the price drop. Even recent events underline the importance of liquidity in stock markets. The subprime crisis was mainly triggered by the sharp fall in housing prices in the United States. From 2007 to 2009 the crisis rapidly developed and spread into a global economic shock, causing uncertainty across financial institutions. Liquidity dried up, resulting in a number of bank failures, large reductions in the market value of equities and declines in various stock market indices.

These extreme events illustrate that a lack of liquidity in financial markets can cause a decline in asset prices. However, liquidity is not only a concept that is related to the whole market, so-called aggregate market-wide liquidity risk. It can also aim at the risk resulting from a single investment, individual stock liquidity risk. When investors face tight liquidity positions, they may be forced to convert assets into cash. This is relatively more costly and more difficult when liquidity is lower. In order to reduce costs and to avoid the risk that arises from the difficulty of buying or selling an asset, investors should prefer liquid assets. In turn, this implies that investors buying illiquid assets should be compensated by higher expected returns. In the second chapter of this dissertation, we address the question whether illiquidity is a priced risk.

Unfortunately, estimating illiquidity is not straightforward as there is hardly a single measure that captures all of its aspects. Illiquidity is a multi-dimensional concept consisting of four dimensions: trading quantity, trading speed, trading costs, and price impact. In this study, we cover all of them. Our measure for trading quantity is turnover following Datar et al. (1998). Trading speed is measured by the number of days with zero trading volume as suggested by Liu (2006). Trading costs are approximated by the limited dependent variable model as proposed by Lesmond et al. (1999) and price impact by the Amihud (2002) measure.

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[3]This chapter is based on the working paper "Illiquidity and Stock Returns: Evidence from the German Stock market" (Koch (2010)).

Although there is evidence for the US market, e.g., Amihud and Mendelson (1986), Pastor and Stambaugh (2003), Acharya and Pedersen (2005), and Liu (2006) that illiquidity is a priced risk, other papers like Mazouz et al. (2009) show that the existence of a liquidity premium outside the US seems to be unclear and requires further analysis. Instead of concentrating on one liquidity measure and one econometric approach as often done so in the literature, this chapter covers all dimensions of liquidity and applies a multitude of different methodologies. Our results reveal that an illiquidity effect prevails. There exists a positive relation between stock returns and illiquidity. Further, we discover a significant risk premium on illiquidity independent of the measure chosen. Yet, the illiquidity premium is not consistent as it strongly relies on the selection of test portfolios. Furthermore, we analyze the link between the size of the firm and the illiquidity of the corresponding stock. Although the two concepts are correlated, we draw the conclusion that the two measures are no substitutes for each other.

Chapter III.[4] The third chapter deals with a widely accepted measure of risk, volatility, the standard deviation of returns per time unit. Volatility is often used to identify how risky an investment is or as a measure of the security's stability. In classical finance theory it is assumed that investors are risk averse and, hence, dislike high volatility. Therefore, they require a compensation for holding volatile stocks. Not only most of the empirical and theoretical asset pricing literature predicts a positive relationship between volatility and expected returns, but also many practitioners believe in the trade-off between volatility and expected returns. They share the view that high volatility must be connived in order to earn higher expected returns.

Volatility consists of two components: systematic and idiosyncratic risk. The largest component is idiosyncratic risk, which represents over 80% of the total volatility on average for single stocks. The last chapter of this dissertation investigates whether idiosyncratic volatility is a priced risk. Our results provide evidence that low idiosyncratic volatility stocks outperform high idiosyncratic volatility stocks. Further, our empirical findings do not support the positive relation between total volatility and expected returns, but show that the trade-off is negative.

Although this finding is in line with papers like Ang et al. (2006b, 2009), it stands in sharp contrast to most of the empirical and theoretical finance literature. Theoretical

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[4]This chapter is based on the working paper "Low Risk and High Returns: Evidence from the German Stock Market" (Koch (2009)).

studies like Merton (1987), Jones and Rhodes-Kropf (2003), and Malkiel and Xu (2006) predict that investors demand a premium for holding stocks with high idiosyncratic risk. A large number of empirical papers confirm this prediction on the US market. Malkiel and Xu (2006), Spiegel and Wang (2005), and Fu (2009) provide unambiguous evidence that portfolios with higher idiosyncratic volatility earn higher average returns. In contrast to estimating idiosyncratic volatility based on daily data over the last month as done by Ang et al. (2006b, 2009), they obtain estimates for idiosyncratic risk based on monthly data. Studying the US market, Huang et al. (2010) find that the negative relation between idiosyncratic risk and returns is driven by monthly stock return reversals and, thus, disappears after controlling for past returns. Bali and Cakici (2008) detect that the negative relation vanishes for equally-weighted portfolios on the US market.

In contrast to the existent literature, we construct an idiosyncratic risk factor and explicitly estimate the risk premium on the German stock market controlling for the market, size, book-to-market, and momentum factors. The results reflect the existence of a negative premium for idiosyncratic risk. The estimated factor risk premium is 10% per year after controlling for the other factors. Idiosyncratic risk is negatively significant in almost all specifications not only for the Fama-MacBeth test, but also for the GMM procedure, different test portfolios, different subperiods, and individual returns. Motivated by the US evidence, we use equally-weighted portfolios and also control for short-term reversal. However, low idiosyncratic risk stocks still outperform high idiosyncratic risk stocks. Given these counterintuitive results, we undertake a multiplicity of new robustness checks. First of all, we evaluate the existence of a monotonic relation between expected returns and idiosyncratic risk applying the Monotonic Relation test proposed by Patton and Timmermann (2010). Further, we differentiate between upside and downside idiosyncratic volatility, apply an (E)GARCH approach, use Dimson Betas as well as different market models to estimate idiosyncratic volatility. We also change the data frequency and use monthly data. However, the puzzle still prevails.

Chapter 1

The Conditional Relation between Fama-French Betas and Return

1.1 Introduction

How does beta risk cross-sectionally affect asset returns? This question has inspired vast amounts of empirical research. However, this issue has not been sufficiently answered. Several recent articles put the standard Fama and MacBeth (1973) test procedure into question and argue that a conditional approach as developed in Pettengill et al. (1995) is more appropriate. While many papers applying the conditional approach find a systematic conditional relationship between risk and return, most of this literature neglects to investigate if beta risk is a priced factor. This study considers the conditional cross-sectional risk-return relationship in a three-factor model and tests subsequently if beta risks based on the three factors are priced. Finally, we compare the power of this test to the widely used Fama-MacBeth test.

The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965), and Mossin (1966) is the first model which theoretically illustrates that market risk systematically affects returns. This model sets the foundation for modern asset pricing theory. Its central implication is that every asset's return is a linear function of its systematic risk or market beta. Early research such as that of Black et al. (1972) and Fama and MacBeth (1973) empirically confirms the CAPM. In the following, several studies yield contradicting results. For example, Reinganum (1981), Fama and French (1992), and Lettau and Ludvigson (2001) find that a systematic relationship between market beta and average returns across assets does not exist.

On top of this, the so-called anomaly literature provides a vast amount of evidence in the 80s and 90s that the CAPM does not hold empirically. Banz (1981) documents that small firms have on average higher risk-adjusted returns than large firms in the US. This anomaly is entitled as the size effect. Moreover, Fama and French (1992) show that the estimated market beta and the average returns are not systematically related once the size and book-to-market factor are included. Finally, Fama and French (1993, 1996) argue that many of the CAPM anomalies are captured by the Fama-French three-factor model. Besides the inclusion of the market excess return as in the CAPM, the three-factor model considers the size and book-to-market factor. Since its inception the Fama-French three-factor model has been the dominant model in empirical asset pricing.

However, Pettengill et al. (1995) propose a potential explanation of the observed weak relationship between market beta and stock returns. They point out that using realized returns implies that there exists a negative risk-return relationship in down-markets. Therefore, Pettengill et al. (1995) modify the Fama and MacBeth (1973) test procedure and develop a conditional approach incorporating the presumption that the risk-return relationship should be negative in down-markets. This is done by differentiating between periods with a positive realized risk premium (up-market) and a negative one (down-market). The conditional approach only tests the risk-return relation and is not related to conditional asset pricing models producing time-varying risk premia as proposed by Jagannathan and Wang (1996). As predicted by the conditional approach, the authors find a positive risk-return relationship in up-markets but an inverse relationship in down-markets for US data. Many other authors have followed the conditional test procedure. For instance, Fletcher (2000) also reports a positive significant relationship between market beta and returns in up-markets as well as a negative significant relationship in down-markets for international stocks. The conditional approach has been applied for several other countries and regions.[1]

However, the standard Fama-MacBeth procedure and the conditional approach test different hypotheses. Although both verify if there exists a systematic relationship between

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[1]Faff (2001) applies the conditional approach for Australia, Crombez and Vennet (2000) for Belgium, Lilti and Montagner (1998) for France, Elsas et al. (2003) for Germany, Lam (2001), Ho et al. (2006), and Tang and Shum (2006) for Hong Kong, Hodoshima et al. (2000) for Japan, Sandoval and Saens (2004) for Latin America, Wihlborg and Zhang (2004) for Poland, Tang and Shum (2004) for Singapore, Isakov (1999) for Switzerland, Sheu et al. (1998) for Taiwan, Karacabey and Karatepe (2004) for Turkey, Hung et al. (2004) for the UK as well as Huang and Hueng (2007) for daily instead of monthly US data. Basher and Sadorsky (1991) use the conditional approach to examine the impact of oil prices on emerging market stock returns.

risk and return, the Fama-MacBeth procedure additionally tests if investors receive a positive reward for holding risk, i.e., it tests if the risk premium is positive. According to Pettengill et al. (1995) this is the case if the following two conditions are satisfied: 1) the average market excess return is positive, 2) there is a symmetric relationship between the market risk premium in down- and in up-markets. However, Freeman and Guermat (2006) derive the inaccurateness of the second condition and clarify that, instead, market risk is not priced if a specific asymmetric relationship holds. Again, we want to emphasize that the detection of a conditional relationship between beta and return does not mean that the risk factor beta refers to is priced.

We make three contributions to the literature. Firstly, we apply the conditional approach to the predominant model in empirical asset pricing, the Fama-French three-factor model. We exceed the existent literature by not only conditioning on the sign of the market return, but on that of each of the three factors, and test if the book-to-market beta and size beta retain their explanatory power once the conditional nature of the relation between betas and return is taken into account. Our empirical results yield strong support for the conditional approach. All three factors exhibit a strong positive risk-return relationship in up-markets as well as an inverse relationship in down-markets. While other studies do not find a relationship between market beta and return in the presence of the size and book-to-market factor, e.g. Fama and French (1992), this study detects a strong one. Results are consistent for subperiods and a multiplicity of different test portfolios.

Secondly, we do not only test if there is a systematic relationship between beta risk and return, but we extend a test proposed by Freeman and Guermat (2006) (FG test in the following) to multi-factor models and test if beta risk is priced within the conditional approach. The FG test simultaneously tests both hypotheses. Thus, it enables us to compare the standard Fama-MacBeth test with the conditional test procedure and to shed some light on previous studies dealing with the conditional approach. Within the framework of the CAPM Freeman and Guermat (2006) show that the FG test has a power similar to that of the standard Fama-MacBeth test under the assumption of normally distributed returns. However, they conjecture that the FG test is more powerful when applied to empirical data because of the unconditional leptokurtosis in observed stock returns. In order to evaluate their conjecture, we use empirical stock market data and run simulations creating returns with fat tails. Using empirical data, our results show that the FG test and the Fama-MacBeth test produce qualitatively identical results. Our

simulations confirm these results. We consider three different distributions: the Normal distribution as well as the Pearson type IV distribution with and without skewness. Independent of the underlying distribution, we find that both tests exhibit a similar power and size. Thus, we cannot confirm the conjecture that the FG test has higher power even when modeling the unconditional leptokurtosis in stock returns. Our study conflicts with other studies like, e.g., Pettengill et al. (1995), who base their test on the above mentioned hypothesis that there is a symmetric relationship between the expected market excess return in down- and in up-markets. For most of our test portfolios we find an insignificant market risk premium within the conditional approach.

Thirdly, our results accentuate how crucial the choice of test portfolios in empirical asset pricing is. In contrast to most of the literature we make use of a variety of test portfolios. Applying both the Fama-MacBeth and the FG test, we find that the significance of market, size and book-to-market risk strongly depends on the selection of test portfolios. For the same risk factor we find positive, insignificant, and even negative risk premia.

The remainder of this chapter is organized as follows. In the next section we introduce the conditional approach in the setting of the Fama-French three-factor model and the econometric methodology. Section 1.3 discusses the data and the construction of the size and book-to-market factor. Section 1.4 reports the empirical results of the standard Fama-MacBeth and the conditional test. Subsequently, we present the derivation of the FG test in a multi-factor setting as well as its empirical results. In section 1.6 we compare the size and the power of the Fama-MacBeth to the power and the size of the FG test. Section 1.7 concludes.

1.2   Methodology

We consider the Fama-French three-factor model and, in contrast to most of the existing literature, allow for time-varying betas. The decision to allow the sensitivities to the risk factors to change over time is made in view of the several decades long data set used and the apparent change in asset and portfolio betas over time that is found in the data. The relevance of time-varying betas is emphasized in several papers, e.g., Harvey (1989), Ferson and Harvey (1991, 1993), and Jagannathan and Wang (1996). The three risk factors of the Fama-French model are denoted by m for market risk, smb for the size risk factor ('small minus big') relating to the market value of equity, and hml for the

book-to-market factor ('high minus low'). Thus, the sensitivities of a portfolio i to the risk factors at time t are denoted $\beta_{i,t}^m, \beta_{i,t}^{smb}, \beta_{i,t}^{hml}$. Our estimation results are based on the Fama-MacBeth (1973) approach. Besides the advantage of an easy implementation it automatically corrects standard deviations for heteroscedasticity, which is a widespread problem among asset returns. We estimate the Fama-French betas for every portfolio from the following time-series regression,

$$r_{i,\tau}^e = \alpha_{i,t} + \beta_{i,t}^m r_{m,\tau}^e + \beta_{i,t}^{smb} r_{smb,\tau} + \beta_{i,t}^{hml} r_{hml,\tau} + \epsilon_{i,\tau} \quad \tau = t - 60....t - 1, \qquad (1.1)$$

where $r_{i,\tau}^e$ denotes the excess return of portfolio i during time period τ, $r_{m,\tau}^e$ the market excess return, $r_{smb,\tau}$ and $r_{hml,\tau}$ the returns on the SMB and HML factors, respectively. This procedure is repeated by rolling the window of 60 months of observations one month ahead. Rolling windows of five years make an appropriate compromise between adjusting to the latest changes and avoiding of noise in the monthly estimations. The rolling five year windows have also been suggested in earlier literature such as Groenewold and Fraser (1997), Brennan et al. (1998), and Fraser et al. (2004). The next step consists of estimating the risk premia $\lambda_{0,t}, \lambda_{m,t}, \lambda_{smb,t}$ and $\lambda_{hml,t}$ using the estimated betas $\hat{\beta}_{i,t}^m, \hat{\beta}_{i,t}^{smb}$ and $\hat{\beta}_{i,t}^{hml}$ from equation 1.1 , i.e. computing cross-sectional regressions for every month,

$$r_{i,t}^e = \lambda_{0,t} + \lambda_{m,t}\hat{\beta}_{i,t}^m + \lambda_{smb,t}\hat{\beta}_{i,t}^{smb} + \lambda_{hml,t}\hat{\beta}_{i,t}^{hml} + \eta_{i,t}. \qquad (1.2)$$

The factor risk premium, λ_j with $j = 0, m, smb, hml$, is estimated as the average of the cross-sectional regression estimate, $\hat{\lambda}_j = \frac{1}{T}\sum_{t=1}^{T} \hat{\lambda}_{j,t}$. λ_j is the factor risk premium which compensates the investors for the risk taken. λ_m is interpreted as the market price of risk, λ_{smb} and λ_{hml} as the price of size and book-to-market risk.[2] Since the betas are estimated from a first-step regression, standard errors for the second regression can be misleading. In order to circumvent the presence of this errors-in-variables problem we apply a correction to the standard errors as proposed by Shanken (1992). Yet, the Shanken correction has to be treated critically as shown by Shanken and Weinstein (2006) because in practical applications it often yields a modified cross-product matrix of the estimated beta vectors that is not positive definite as it should be.

Estimating equation 1.2 by the Fama-MacBeth procedure leads to conclusions on whether the risk factors are priced. For instance, if $\lambda_{m,t}$ is nonzero, market risk is a priced factor.

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[2]The interpretation of the size and book-to-market risk is discussed in the literature. For instance, according to Amihud and Mendelson (1986, 1991) size may proxy for liquidity risk and Vassalou and Xing (2004) argue that the book-to-market ratio captures default risk.

If, on the other hand, $\lambda_{m,t}$ is not distinguishable from zero, then market risk is not priced. This can be the case either if there does not exist any relationship between beta and return or if it does exist but the market risk premium is not distinguishable from zero. Therefore, it is possible that beta is not priced despite the existence of a risk-return relationship. On this account we apply a procedure that has been suggested by Pettengill et al. (1995) in the context of the CAPM, which exclusively tests the relationship between market beta and realized returns conditional on whether the market excess return, i.e. the realized market risk premium, is positive or negative. This test takes into account that empirical tests are based on realized returns although the CAPM is stated in expectational terms. According to the CAPM the expected market excess return is positive[3] and, thus, there should exist a positive risk-return relation. However, the realized market excess return can also be negative implying a negative relation between beta and return. In order to test the systematic relationship between risk and return, the following equation is estimated:

$$r_{i,t}^e = \lambda_{0,t} + \lambda_{m,t}^+ \delta_{m,t} \hat{\beta}_{i,t}^m + \lambda_{m,t}^- (1 - \delta_{m,t}) \hat{\beta}_{i,t}^m + \eta_{i,t}. \tag{1.3}$$

While Pettengill et al. (1995) conduct this procedure for the CAPM and for beta constant over time, we apply the Fama-French three-factor model and allow for time-varying betas. That is, we estimate the following equation:

$$\begin{aligned}
r_{i,t}^e = \lambda_{0,t} &+ \lambda_{m,t}^+ \delta_{m,t} \hat{\beta}_{i,t}^m && + \lambda_{m,t}^- (1 - \delta_{m,t}) \hat{\beta}_{i,t}^m \\
&+ \lambda_{smb,t}^+ \delta_{smb,t} \hat{\beta}_{i,t}^{smb} && + \lambda_{smb,t}^- (1 - \delta_{smb,t}) \hat{\beta}_{i,t}^{smb} \\
&+ \lambda_{hml,t}^+ \delta_{hml,t} \hat{\beta}_{i,t}^{hml} && + \lambda_{hml,t}^- (1 - \delta_{hml,t}) \hat{\beta}_{i,t}^{hml} + \eta_{i,t}.
\end{aligned} \tag{1.4}$$

The δs are dummy variables with the value 1 if the market, the SMB and the HML factors, respectively, yield a positive excess return and 0 otherwise. We conduct cross-sectional regressions for each month as in the unconditional case. Our conditional estimates are $\hat{\lambda}_j^+ = \frac{1}{\sum_{t=1}^T \delta_{j,t}} \sum_{t=1}^T \hat{\lambda}_{j,t} \delta_{j,t}$ and $\hat{\lambda}_j^- = \frac{1}{\sum_{t=1}^T (1-\delta_{j,t})} \sum_{t=1}^T \hat{\lambda}_{j,t}(1-\delta_{j,t})$, respectively. That means, the parameters are averaged conditional upon the sign of the risk factors. We would like to stress that the conditional approach sharply differs from the way of estimating conditional asset pricing models since we do not estimate conditional betas in the first-step regression. Furthermore, the conditional approach differs from studies differentiating between upside and downside betas such as Ang et al. (2006a). Instead, we

---

[3]This follows from the assumption that agents are risk averse and that there is a positive net supply of market risk.

split our sample into different subsamples depending on positive or negative risk factors when conducting cross-sectional regressions in the second step.

While the Fama-MacBeth procedure tests whether betas are priced risk factors, the conditional approach as applied here only enables us to test whether there is a systematic relation between a risk factor and the realized returns. In other words, finding a significant relation between beta risk and return does not automatically imply that beta risk is priced and the model holds.

1.3 Data

This study uses monthly data from July 1926 through June 2008. The entire dataset is taken from Kenneth French's homepage. We deploy the 25 portfolios formed according to the same criteria as those used in Fama and French (1992, 1993), i.e., the portfolios are value-weighted for the intersections of five size and five book-to-market equity portfolios. The portfolios are constructed at the end of June, and size is measured by market capitalization of equity at the end of June. The book-to-market ratio is book equity at the last fiscal year end of the prior calendar year divided by the market capitalization at the end of December of the prior year. Additionally, we include 25 portfolios sorted by size and momentum, 10 portfolios sorted by momentum, 10 portfolios sorted by short-term reversal, 10 portfolios sorted by the earnings-price ratio, 10 by the cash flow-price ratio, and 10 by the dividend yield. 25 size and momentum portfolios are the intersections of five portfolios sorted by size and five portfolios formed on the previous eleven months return lagged by one month (past 2-12 return). In the same way, 10 momentum portfolios are constructed. 10 short-term reversal portfolios are constructed monthly formed on the return of the previous month. 10 portfolios sorted by the earnings-price and cash flow to price ratio are formed in June of year t based on the fiscal year $t-1$. Earnings are measured as earnings before extraordinary items. Cash flow are earnings before extraordinary items plus equity's share of depreciation plus deferred taxes. Finally, 10 portfolios are formed on dividend-price ratio at the end of each June using NYSE breakpoints. The dividend yield used to form portfolios in June of year t is the sum of dividends paid from July of $t-1$ to June of t per dollar of equity in June of t.

Furthermore, this study employs the three Fama-French factors. Although the composition of the market portfolio is not observable, we approximate the market excess return by the return on the value-weighted CRSP index compromising all NYSE, AMEX and

NASDAQ stocks minus the one-month Treasury bill-rate (from Ibbotson Associates). The size and book-to-market factor base on six portfolios, which are the intersections of two portfolios formed on size and three portfolios formed on the book-to-market ratios. Portfolios consisting of small (big) firms are denominated as small (big) portfolios, whereas portfolios consisting of firms with a low (high) book-to-market value are denoted as growth (value) portfolios. The size factor (SMB) is constructed as the difference between the average return on three small firm portfolios and the average return on three big firm portfolios. The book-to-market factor (HML) is the average return on the two value portfolios minus the average return on the two growth portfolios. The returns are based on all NYSE, AMEX and NASDAQ stocks, for which book and market equity data are available.

1.4 Empirical Results

1.4.1 Fama-MacBeth Test

Before presenting the results of the unconditional test resulting from conducting the Fama-MacBeth procedure, we want to stress the importance of using time-variant betas. Figures 1.1, 1.2, and 1.3 illustrate the variation in time of market, size and book-to-market betas from 1931:07 to 2008:06. As dependent variables we use the 25 size and book-to-market portfolios. Betas are calculated using equation 1.1. For the sake of clearness, we only illustrate portfolios 1, 25, and 10. Portfolio 1 contains the smallest growth stocks and is used as an example for large changes in betas over time. Portfolio 25 consists of the biggest value stocks and is an example for medium changes in betas over time. Portfolio 10 comprises stocks with the second smallest market capitalization and the highest book-to-market ratios. Its betas display small changes over time.

The dashed lines represent the 95% confidence interval. In particular, portfolio 1 indicates a strong variation in the betas across time. Although the betas of portfolio 25, figure 1.2, and particularly portfolio 10, figure 1.3, appear to be much less variable, even in the latter case market beta varies between 0.62 and 1.25, size beta between 0.69 and 1.23, and book-to-market beta between 0.50 and 1.24.

Table 1.1 shows the results of the Fama-MacBeth estimation for the whole period using equation 1.2. The monthly estimates of the coefficients are averaged and a t-test is applied to determine the statistical significance of the mean of the estimated coefficients. The

Figure 1.1: Fama-French Betas for Portfolio 1
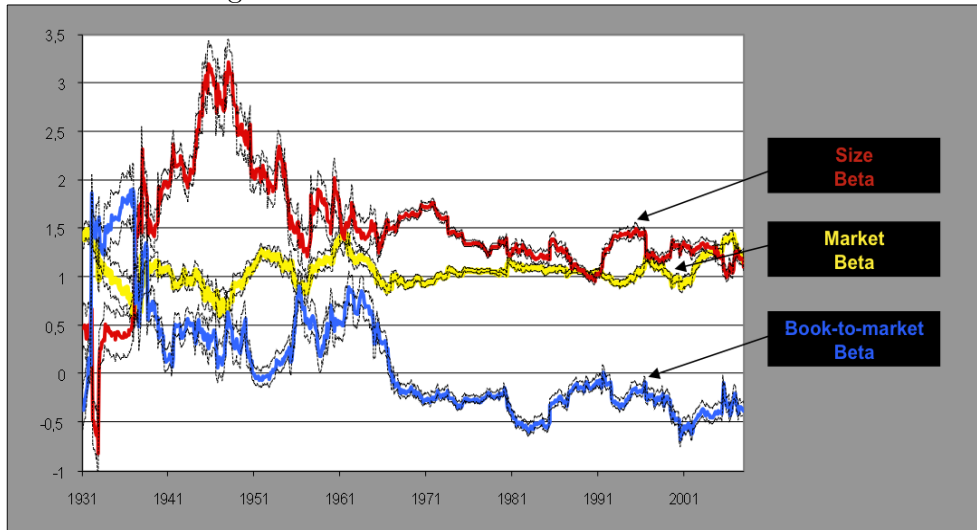
Figure 1.2: Fama-French Betas for Portfolio 25
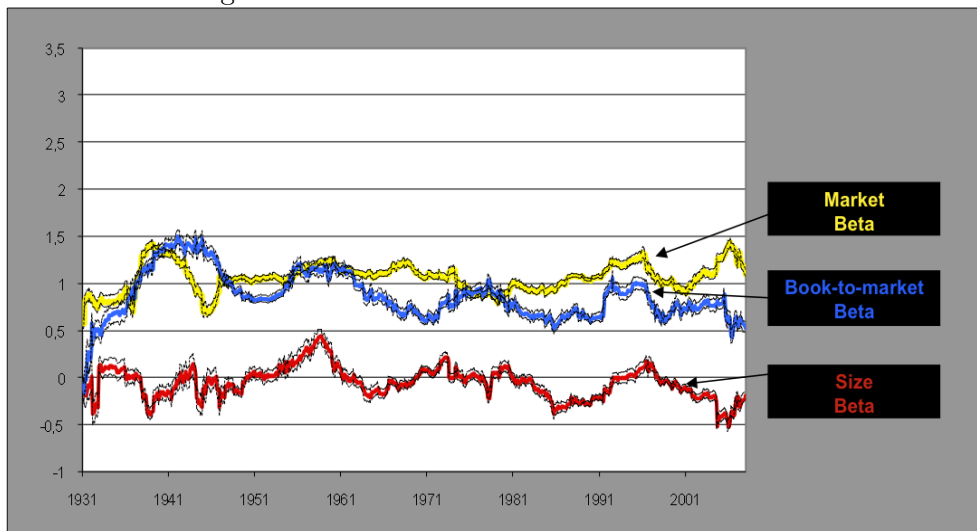
Figure 1.3: Fama-French Betas for Portfolio 10
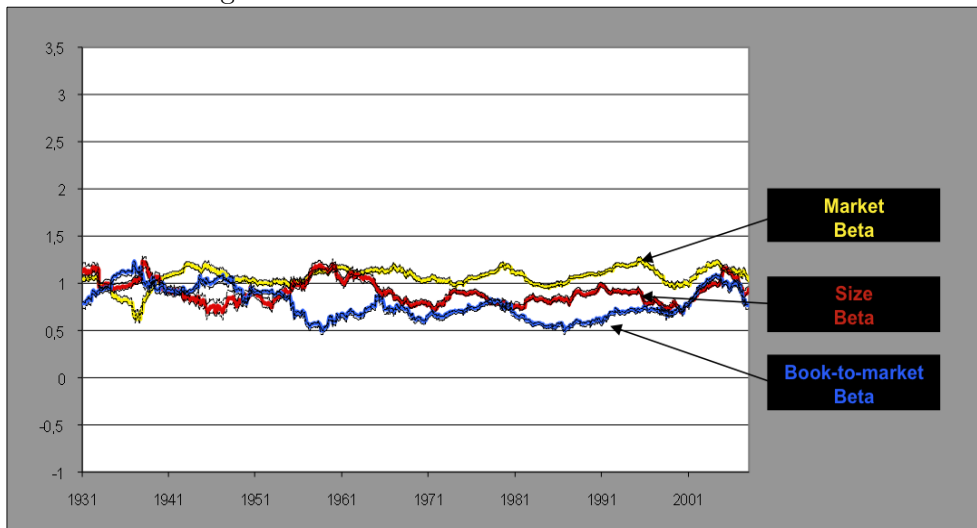
market risk premium is negative but insignificant and, thus, market risk is not found to be priced. Size risk is not found to be significant, either, while the coefficient of the book-to-market risk premium is highly significant. The constant is implausibly high. However, this is a feature occurring in most empirical studies that report the constant, for example Jagannathan and Wang (1996) and Lettau and Ludvigson (2001). In the

Table 1.1: Fama-MacBeth Test

Variable	λ	t-stat	$\bar{R}^2$
Cons	1.04	4.63***	0.47
Market	-0.31	-1.35	
SMB	0.18	1.64	
HML	0.41	3.49***	

*** significant (1-percent level)

This table depicts the results for the Fama-French three-factor model given by equation 1.2. Cons denotes the constant term, Market the risk premium of the market risk, SMB that of the size, and HML that of the book-to-market risk. Our dependent variables are 25 portfolios sorted by size and book-to-market equity. The coefficients are given as percentage points per month. $\bar{R}^2$ is the average cross-sectional R^2. The sample period runs from 1931:07 to 2008:06.

following we conduct the same analysis for four subperiods, the results being detailed in table 1.2. The subperiods are chosen such that they are of equal length. We observe that the size risk premium is not significant in any of the subperiods. Market risk is negatively priced in the fourth subperiod but insignificant in all other subperiods. The significance of the book-to-market premium varies, though, it is priced at the 1% level in the third period, while its coefficient, as that of the size premium, has the expected sign in all subperiods. Generally, though rarely done so in the literature, applying the Shanken (1992) correction to the standard errors would be advisable in order to overcome the errors-in-variables problem. We follow the heuristic in Shanken (1992) for the case of time-varying betas. The Shanken correction factors are negligible, increasing the standard errors by only 0.5% for the whole sample and by 1.7% on average for the subsamples, such that the significance of the coefficients is not changed. In the following, we disregard the correction factor.

Table 1.2: Fama-MacBeth Test - Subperiods

Variable	1931:07-1950:09			1950:10:1969:12		
	λ	t-stat	$\bar{R}^2$	λ	t-stat	$\bar{R}^2$
Cons	0.80	1.17	0.42	1.14	3.98**	0.40
Market	0.30	0.43		-0.32	-1.09	
SMB	0.37	1.29		0.14	0.93	
HML	0.49	1.41		0.21	1.63	
Variable	1970:01-1989:03			1989:04-2008:06		
	λ	t-stat	$\bar{R}^2$	λ	t-stat	$\bar{R}^2$
Cons	0.89	2.46**	0.55	1.31	3.74***	0.51
Market	-0.49	-1.03		-0.75	-1.91*	
SMB	0.16	0.86		0.05	0.22	
HML	0.64	3.49***		0.28	1.33	

* significant (10-percent level)

** significant (5-percent level)

*** significant (1-percent level)

This table depicts the results for the Fama-French three-factor model for four subperiods. Cons denotes the constant term, Market the risk premium of the market risk, SMB that of the size, and HML that of the book-to-market risk. The coefficients are given as percentage points per month. Our dependent variables are 25 portfolios sorted by size and book-to-market equity. $\bar{R}^2$ is the average cross-sectional R^2.

Table 1.3: Conditional Relation between Fama-French Betas and Returns

Variable	λ	t-stat
Cons	1.04	4.63***
Market-up	1.62	5.11***
Market-down	-3.20	-8.54***
SMB-up	2.17	14.82***
SMB-down	-1.95	-9.13***
HML-up	2.26	16.19***
HML-down	-1.96	-8.17***

*** significant (1-percent level)

This table depicts the results of the conditional relation between Fama-French betas and return given by equation 1.4 for the entire sample. Cons denotes the constant term, Market-up (-down) the risk premium of the market given that the excess market return is positive (negative), SMB-up (-down) that of the size given that the SMB factor is positive (negative), and HML-up (-down) that of the book-to-market risk given that the HML factor is positive (negative). Our dependent variables are 25 portfolios sorted by size and book-to-market equity. The coefficients are given as percentage points per month. The sample period runs from 1931:07 to 2008:06.

1.4.2   Conditional Relationship

First of all we check how frequently the realized excess return is negative. If it were hardly ever negative, the conditional relationship would have an negligible impact on tests of the relationship between beta and return. The risk-free rate exceeds the market return in 40.2% of the observations for the entire period. Moreover, in 48.4% of the observations the size factor and in 44.0% the book-to-market factor is negative, which accentuates the relevance of the distinction between up-markets and down-markets. Table 1.3 depicts the results of the conditional test for the entire sample. All coefficients are highly significant. The fact that we observe a strong relationship between market risk and returns is, among others, consistent with Pettengill et al. (1995) and Fletcher (2000). Moreover, our results clarify that there also exists a strong conditional relationship between returns and size as well as book-to-market beta.

Market beta is associated with increasing absolute returns, i.e. positively increasing returns in up- and negatively increasing returns in down-markets. The same applies to the size and book-to-market risk factors while the constant, as expected, does not change compared to the results of the Fama-MacBeth method. In contrast to Pettengill et al. (1995)

Table 1.4: Conditional Relation between Fama-French Betas and Returns - Subperiods

Variable	1931:07-1950:09		1950:10:1969:12		1970:01-1989:03		1989:04-2008:06	
	λ	t-stat	λ	t-stat	λ	t-stat	λ	t-stat
Cons	0.80	2.38**	1.14	3.98***	0.89	2.46**	1.31	3.74***
Market-up	2.38	2.37**	0.85	2.40**	2.06	4.13***	1.29	2.75***
Market-down	-2.89	-2.98**	-2.37	-4.24***	-3.39	-5.01***	-4.07	-5.21***
SMB-up	2.37	5.91***	1.64	9.37***	2.20	11.02***	2.46	8.46***
SMB-down	-2.21	-4.37***	-1.36	-4.39***	-2.00	-4.61***	-2.25	-4.71***
HML-up	3.24	6.52***	1.32	10.01***	2.24	14.97***	2.35	10.24***
HML-down	-2.44	-3.95***	-1.29	-4.38***	-2.02	-3.97***	-2.04	-4.24***

** significant (5-percent level)

*** significant (1-percent level)

This table depicts the results of the conditional relation between Fama-French betas and return given by equation 1.4 for the four subperiods. Cons denotes the constant term, Market-up (-down) the risk premium of the market given that the excess market return is positive (negative), SMB-up (-down) that of the size given that the SMB factor is positive (negative), and HML-up (-down) that of the book-to-market risk given that the HML factor is positive (negative). The coefficients are given as percentage points per month. Our dependent variables are 25 portfolios sorted by size and book-to-market equity.

the coefficients show asymmetry concerning the market risk. Returns increase less with beta when the market excess return is positive than they decrease when it is negative. This might intuitively explain why, while the market on average increases and beta relates asset returns to market returns, there is no significant risk premium for the market risk. In contrast, the coefficients for the size and book-to-market risk are not significantly asymmetric.[4] The results for the subperiods show similar results. In contrast to the findings of the standard Fama-MacBeth procedure the conditional approach leads to results that are consistent over time. All variables retain their significance in each of the four subperiods as illustrated in table 1.4.

We do not report the $\bar{R}^2$ since they comply with the values of the Fama-MacBeth procedure.[5]

---

[4] Testing for asymmetric coefficients results in the following test values: -3.2*** (Market), 0.87 (SMB) and 1.09 (HML). The null hypothesis is $\lambda_j^+ + \lambda_j^- = 0$.

[5] The conditional approach is based on the same regressions as the Fama-MacBeth test but it splits up the variables in positive and negative factor realizations. Therefore, the constant and the cross-sectional $\bar{R}^2$ are identical.

1.5   Testing for Priced Betas

1.5.1   Derivation of the FG Test

Our findings in the last section exclusively provide strong evidence for a systematic relationship between Fama-French betas and return. In this section we go one step further and test not only if there exists a systematic relationship between beta and return but also if beta risk is priced within the conditional approach. Besides the existence of a systematic relationship a priced beta would require a reward to compensate investors for the risk taken. In the following we generalize the FG test to a multi-factor framework and test if Fama-French betas are priced. Since the FG test and the standard Fama-MacBeth procedure are now based on the same hypothesis, it is possible to compare both procedures and to judge the relevance of the conditional approach. Freeman and Guermat (2006) base their test on the CAPM. We extend the test to multi-factor models. Moreover, we allow for time-variant betas. Consider the following return generating process:

$$r_{i,t}^e = E(r_{i,t}^e) + \beta_{i,t}^m [r_{m,t}^e - E(r_{m,t}^e)] + \beta_{i,t}^{smb}[r_{smb,t} - E(r_{smb,t})] + \beta_{i,t}^{hml}[r_{hml,t} - E(r_{hml,t})] + \epsilon_{i,t}.$$
(1.5)

The error term $\epsilon_{i,t}$, $E[\epsilon_{i,t}] = 0$, is assumed to be uncorrelated with both the betas and the excess returns. Yet, the error terms can be cross-sectionally correlated. Additionally, consider the expected return process:

$$E(r_{i,t}^e) = \alpha_{i,t} + \beta_{i,t}^m \pi^m + \beta_{i,t}^{smb}\pi^{smb} + \beta_{i,t}^{hml}\pi^{hml}$$
(1.6)

$\alpha_{i,t}$ represents a compensation for other risk factors that are orthogonal to the three included factors. Hence, it is assumed that $\alpha_{i,t}$ and $\beta_{i,t}$ are uncorrelated. Choosing $\alpha_{i,t} = 0$, $\pi^m = E[r_{m,t}^e]$, $\pi^{smb} = E[r_{smb,t}]$ and $\pi^{hml} = E[r_{hml,t}]$ would imply that the return process equals the Fama-French three-factor model. To put it differently, if $\pi^j = 0$, the risk factor j is not priced. This approach enables us to verify if beta risk is priced. For instance, testing the sole hypothesis that market risk is not priced under the assumption of a three-factor model, corresponds to the null hypothesis $\pi^m = 0$. We begin with the linear regression equation of our model:

$$r_{i,t}^e = \lambda_{0,t} + \lambda_{m,t}\beta_{i,t}^m + \lambda_{smb,t}\beta_{i,t}^{smb} + \lambda_{hml,t}\beta_{i,t}^{hml} + \eta_{i,t}.$$
(1.7)

According to the Fama-MacBeth procedure ordinary least squares regressions are conducted for all t.

Denote $\boldsymbol{\beta'_{i,t}} = [\beta^m_{i,t}\beta^{smb}_{i,t}\beta^{hml}_{i,t}]$, $\boldsymbol{f'_t} = [r^e_{m,t}r_{smb,t}r_{hml,t}]$, $\boldsymbol{\pi'} = [\pi^m\pi^{smb}\pi^{hml}]$ and $\boldsymbol{\lambda'_t} = [\lambda_{m,t}\lambda_{smb,t}\lambda_{hml,t}]$. According to the properties of ordinary least squares we obtain

$$
\begin{aligned}
\boldsymbol{\lambda_t} &= \mathrm{var}(\boldsymbol{\beta_{i,t}})^{-1}\mathrm{cov}(\boldsymbol{\beta_{i,t}}, r^e_{i,t}) \\
&= \mathrm{var}(\boldsymbol{\beta_{i,t}})^{-1}\mathrm{cov}(\boldsymbol{\beta_{i,t}}, E[r^e_t] + \boldsymbol{\beta'_{i,t}}(\boldsymbol{f_t} - E[\boldsymbol{f_t}])) \\
&= \mathrm{var}(\boldsymbol{\beta_{i,t}})^{-1}\mathrm{cov}(\boldsymbol{\beta_{i,t}}, \alpha_{i,t} + \boldsymbol{\beta'_{i,t}\pi}) + \boldsymbol{f_t} - E[\boldsymbol{f_t}] \\
&= \boldsymbol{\pi} + \boldsymbol{f_t} - E[\boldsymbol{f_t}],
\end{aligned}
$$

where $\mathrm{var}(\boldsymbol{\beta_{i,t}})$ is the 3×3 matrix of the variances and covariances of beta. Testing, e.g., the null hypothesis that market risk is not priced we obtain the following equations:

$$
\begin{aligned}
\lambda^+_m &= E[r^e_{m,t}|r^e_{m,t} > 0] - E[r^e_{m,t}] \\
\lambda^-_m &= E[r^e_{m,t}|r^e_{m,t} < 0] - E[r^e_{m,t}] \\
\lambda^+_m + \lambda^-_m &= E[r^e_{m,t}|r^e_{m,t} > 0] + E[r^e_{m,t}|r^e_{m,t} < 0] - 2E[r^e_{m,t}].
\end{aligned}
$$

This formula shows that our generalization of the Freeman and Guermat (2006) test procedure to multi-factor models leads to the same test equation. As the formula illustrates, the relation between λ^+_m and λ^-_m is generally asymmetric under the null hypothesis. By contrast, Pettengill et al. (1995) assume that priced beta risk corresponds to a symmetric relationship between λ^+_m and λ^-_m. Our test equation shows that this does not hold true as there is no reasonable argument why the expected value of the risk premium conditional on it being positive or negative should have the same absolute expected size. The Fama-MacBeth test is a special case of the FG test, disregarding the differentiation between λ^+_m and λ^-_m. In this case, we only consider unconditional expected values and, hence, under the null hypothesis, $\pi^m = 0$, we obtain $\lambda_m = 0$. This is the usual equation testing for the significance of market risk within the Fama-MacBeth framework.

In order to avoid messy notation, the right hand side of the last equation is denoted as $\theta_m = E[r^e_{m,t}|r^e_{m,t} > 0] + E[r^e_{m,t}|r^e_{m,t} < 0] - 2E[r^e_{m,t}]$.

1.5.2 The Bootstrap

Since $\lambda^+_j + \lambda^-_j - \theta_j = 0$ holds under the null hypothesis that risk factor j, $j \in \{m, smb, hml\}$, is not priced, this condition can be tested by a simple t-test:

$$
t = \frac{\hat{\lambda}^+_j + \hat{\lambda}^-_j - \hat{\theta}_j}{\hat{std}_j}. \tag{1.8}
$$

θ_j can be consistently estimated by taking sample averages. Provided that the standard deviation of the numerator std_j can also be consistently estimated, the Central Limit Theorem can be applied and hence, the asymptotic normality of the statistic follows from White (1999). However, since the components of θ_j are based on different sample sizes, the covariances cannot be estimated directly. One way to overcome this obstacle is to apply a bootstrap. It helps us learn about the sample characteristics by taking resamples and using this information to infer about the population. As shown by Babu and Singh (1984) the bootstrap can be used to consistently estimate a wide range of statistics, including not only the sample mean, but also the sample variance and smooth transforms of these statistics. In our setting the bootstrap is applied as follows. T observations are independently drawn with replacement. This gives us a new sample $(r_{j,t}^{e*}, \lambda_j^*)$. By calculating $\hat{\lambda}_j^{+*}$, $\hat{\lambda}_j^{-*}$ and $\hat{\theta}_j^*$ from the new sample, we obtain an estimate for the numerator. This result is saved and the whole procedure is repeated S times. Finally, the bootstrap variance is the sample variance of the S estimates of the numerator. In order to choose S sufficiently large, we take S equal to 10,000.

However, this procedure relies on the assumption that returns are identically and independently distributed. In order to account for possible autocorrelation and clusterings we additionally conduct a block bootstrap. The Moving Block Bootstrap developed by Künsch (1989) draws blocks of length l instead of drawing T observations independently. Lahiri (1999) shows that the Moving Block Bootstrap performs better than other block bootstraps in terms of the mean squared error. With respect to this criterion, Künsch (1989) shows that $l = T^{\frac{1}{3}}$ is the optimal block length.

1.5.3   Empirical Results

This subsection presents the test results of the FG test developed in subsection 1.5.1 based on the simple bootstrap and the Moving Block Bootstrap. Although θ_j is unknown and has to be estimated as well, we also consider the case of a known θ_j as a benchmark. By assuming a known θ_j the bootstrap becomes dispensable since the standard deviation can be solely calculated from the variances of $\hat{\lambda}_j^+$ and $\hat{\lambda}_j^-$. Under this simplifying assumption Freeman and Guermat (2006) reinterpreted the results in Pettengill et al. (1995), Fletcher (2000) and Hung et al. (2004) by testing if the market beta is a priced risk factor within the conditional approach. In the case of Pettengill et al. (1995), which is the only study dealing with monthly US data, they draw the conclusion that market risk is a priced risk

factor. Therefore, comparing the benchmark with the case of an unknown θ_j enables us to shed some light on the results in Freeman and Guermat (2006).

Table 1.5 illustrates the results of the FG test for the entire period. Neither the market nor the size risk can be shown to be priced, independent of the method used for the computation of standard errors. The t-values generally decrease in absolute values when choosing the Moving Block Bootstrap rather than the simple bootstrap. Under the assumption of known θ_j the t-values rather decrease in absolute values since the positive covariance between $\lambda_j^+ + \lambda_j^-$ and θ_j is neglected. The finding that the pricing of market beta cannot be confirmed stands in contrast to that of Freeman and Guermat (2006). Apart from the different sample periods, the most plausible reason for this finding is that the inclusion of size and book-to-market distinctly decreases the explanatory power of the market factor and causes insignificance of the coefficient. Thus, with respect to the market risk the results of the FG test are in line with previous tests, e.g., Fama and French (1992).

Although the results from the block bootstrap are qualitatively identical in comparison to the simple bootstrap, the t-values change, i.e. the book-to-market and size coefficient exhibit slightly lower t-values. In the following, results are exclusively based on the block bootstrap. Table 1.6 illustrates the results from the FG test for the four subperiods. The coefficient for the market risk turns from positive to negative over time. Though, each of the coefficients is insignificant. In contrast to the standard Fama-MacBeth test the FG test provides lower t-values for market risk except for the third period, in which they almost coincide. Concerning the size risk all coefficients are positive but insignificant in each subperiod. The book-to-market risk factor is significant at the 1% level in the third period and insignificant in the others, which confirms the results from the Fama-MacBeth test. Moreover, both tests indicate large standard deviations and hence, smaller t-values for the subperiods, which leads to less significant and partly to inconsistent results.[6]

All in all, our results show that the book-to-market beta is a priced risk factor, size beta cannot be shown to be significant and market beta is not priced. Furthermore, we can subsume that the results from the FG test and the Fama-MacBeth test are qualitatively similar. Therefore, our findings place emphasis on the results of

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[6]Additionally, we consider the same period as in Fama and French (1992) running from 1963 to 1990. Both, the Fama-MacBeth and the FG test, find insignificant premia for market and size risks but a priced book-to-market risk. These results differ from those in Fama and French (1992) who use firm characteristics instead of factor mimicking portfolios.

Table 1.5: FG Test

Variable	$\lambda_j^+ + \lambda_j^- - \theta_j$	t-stat (known θ_j)	t-stat (simple B.)	t-stat (Block B.)
Market	-0.07	-0.14	-0.14	-0.14
SMB	0.37	1.43	1.64	1.62
HML	0.86	3.09***	3.58***	3.27***

*** significant (1-percent level)

This table depicts the results of the FG test for the entire period assuming a constant θ_j as well as applying the simple bootstrap and the block bootstrap, respectively. Market is the risk premium of the market, SMB that of the size, and HML that of the book-to-market risk. $\lambda_j^+ + \lambda_j^- - \theta_j$ are defined as presented in subsection 1.5.1. Our dependent variables are 25 portfolios sorted by size and book-to-market equity. The sample period runs from 1931:07 to 2008:06.

Freeman and Guermat (2006), but stand in sharp contrast to the results of Pettengill et al. (1995). Basing their test on the inaccurate hypothesis that beta risk is priced if there is a symmetric relationship between the expected market excess return in down- and in up-markets and if a positive market excess return exists, Pettengill et al. (1995) draw the conclusion that the market risk premium is positively priced.

1.5.4 Robustness

In addition to the analysis based on portfolios sorted by size and book-to-market, we conduct the same procedure for other portfolios not or only partly sorted by the risk factors contained in the Fama-French three-factor model in order to verify the results we obtained previously. First, we choose 10 portfolios sorted by momentum since most asset pricing models come off badly in explaining momentum portfolios. For example, Fama and French (1996) and Grundy and Martin (2001) find that controlling for the market, the size effect and the book-to-market effect even increases the profitability of momentum strategies. Thus, this sorting appears to be an intuitive contrast to that with respect to size and book-to-market ratio, and it is a useful robustness check of our existing test results. As it is desirable to have a larger number of data points in the cross-sectional regressions in order to reduce the standard errors of the estimates, we also choose to try and explain the returns of 25 portfolios sorted by momentum and size. Additionally, we consider portfolios based on other characteristics and include 10

Table 1.6: FG Test - Subperiods

Variable	$\lambda_j^+ + \lambda_j^- - \theta_j$ 1931:07-1950:09	t-stat	$\lambda_j^+ + \lambda_j^- - \theta_j$ 1950:10:1969:12	t-stat
Market	1.79	1.29	0.05	0.09
SMB	0.82	1.64	0.26	0.69
HML	1.01	1.48	0.46	1.43
	1970:01-1989:03		1989:04-2008:06	
Market	-0.85	-1.04	-1.19	-1.62
SMB	0.33	0.74	0.10	0.22
HML	1.28	3.07***	0.58	1.03

*** significant (1-percent level)

This table depicts the results of the FG test for the four subperiods based on the block bootstrap. Market is the risk premium of the market, SMB that of the size, and HML that of the book-to-market risk. $\lambda_j^+ + \lambda_j^- - \theta_j$ are defined as presented in subsection 1.5.1. Our dependent variables are 25 portfolios sorted by size and book-to-market equity.

cash-flow price portfolios, 10 earnings-price portfolios, 10 dividend-price portfolios and 10 short-term reversal portfolios.

Table 1.7 depicts the results of the Fama-MacBeth test and the FG test. In the case of the 25 momentum-size portfolios size risk is positively priced whereas market risk is negatively priced and book-to-market risk is insignificant. The results for the FG test are similar except that market risk is not significant at the 10% level. In contrast to the 25 size and book-to-market portfolios used in the the previous sections, the book-to-market factor is not priced when considering the 25 size and momentum portfolios. This is confirmed for the 10 portfolios exclusively sorted with respect to momentum. Size risk as well as market risk are negatively priced, whereas book-to-market risk is unpriced. Again, the FG test finds a priced size factor, though, an insignificant coefficient for market and book-to-market risk. The relevance of size risk suggests that the risk of buying stocks of small firms has a negative influence on the momentum returns. An intuitive explanation is the following. This observation may be caused by the fact that winner stocks, in particular portfolios seven to nine, are negatively correlated with the size factor. After a period of exceptional performance small firms possibly have significant opportunities to continue their fast growth while bigger ones may be limited in their capacity to create further growth. Therefore, bigger companies may be considered riskier and, thus, require

Table 1.7: Fama-MacBeth Test and FG Test for Diverse Test Portfolios

	λ	t-stat	$\lambda_j^+ + \lambda_j^- - \theta_j$	t-stat
Variable	25 momentum-size (1932:01-2008:06)			
Market	-0.63	-2.18**	-0.90	-1.57
SMB	0.35	2.82***	0.72	2.84***
HML	-0.13	-0.67	0.09	0.24
	10 momentum (1932:01-2008:06)			
Market	-0.71	-1.68*	-0.88	-1.07
SMB	-0.37	-1.90*	-0.69	-1.76*
HML	-0.37	-1.51	-0.36	-0.75
	10 cash flow-price (1956:07-2008:06)			
Market	0.79	1.89*	2.20	2.60***
SMB	0.07	0.36	0.20	0.46
HML	0.36	2.66***	0.75	2.80***
	10 earnings-price (1956:07-2008:06)			
Market	0.66	1.91*	1.79	2.61***
SMB	0.42	1.98**	0.88	2.09**
HML	0.41	2.93***	0.90	3.19***
	10 dividend-price (1932:06-2008:06)			
Market	-0.26	-0.90	-0.04	-0.06
SMB	0.02	0.10	0.11	0.23
HML	0.09	0.61	0.38	1.19
	10 short-term reversal (1931:02-2008:06)			
Market	1.25	2.55**	3.18	3.43***
SMB	-0.36	-1.21	-0.64	-1.13
HML	0.28	0.99	0.91	1.61

* significant (10-percent level)

** significant (5-percent level)

*** significant (1-percent level)

This table depicts the results for the Fama-French three-factor model given by equation 1.2 using the returns of 25 portfolios sorted by momentum and size, 10 portfolios sorted by momentum, 10 portfolios sorted by the cash flow-price ratio, 10 portfolios sorted by the earnings-price ratio, 10 portfolios sorted by the dividend-price ratio, and 10 portfolios sorted by short-term reversal as dependent variables. λ represents the estimate based on the Fama-MacBeth approach and $\lambda_j^+ + \lambda_j^- - \theta_j$ the estimate based on the FG test. Market denotes the risk premium of the market risk, SMB that of the size and HML that of the book-to-market risk. The coefficients are given as percentage points per month.

a higher return due to their size. The negative pricing of market risk may be explained by a negative correlation between momentum and market betas. For the other four test portfolios we find very similiar results in terms of the significance of beta risk. As in the last subsections the results of the FG test affirm the results of the Fama-MacBeth test. Still, the similarity of the two tests can be accidental. In order to gain deeper insights we conduct a simulation to evaluate the power and the size of the two tests in the next section.

An interesting by-product is the finding that the significance of the risk factor highly depends on the way test portfolios are sorted. For instance, book-to-market risk is highly significant for the 10 cash flow-price portfolios and 10 earnings-price portfolios, though, it is not for the 10 dividend-price portfolios and 10 short-term reversal portfolios.

For the sake of completeness we also present the results of the conditional approach using different test portfolios as dependent variables. As depicted in table 1.8, most coefficients are significant. Exceptions are the HML-up λ for the pure momentum portfolios, the Market-down λ for the cash flow portfolios and the SMB-up λ for the 10 short-term reversal portfolios. All other coefficients are in line with our presumption finding positive coefficients for positive factor realizations and negative coefficients for negative factor realizations such that we can draw the conclusion that there exists a strong relationship between Fama-French betas and return independent of the construction of test portfolios.[7]

1.6   Simulation

So far, our results suggest that the Fama-MacBeth and the FG test lead to qualitatively similar results. However, this might occur merely by coincidence. The number of ways test portfolios are sorted is limited and does not allow us to draw any firm conclusions. In order to compare the performance of the two tests in a more general way, a simulation approach seems appropriate. We calibrate a Monte Carlo simulation in order to determine the power and the size of the Fama-MacBeth and the FG test. Our simulation works as follows. Initially, we estimate betas from 25 portfolios sorted by size and book-to-market. We

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[7]The results for the subperiods are consistent.

Table 1.8: Conditional Relation between Fama-French Betas and Returns for Diverse Test Portfolios

Variable	25 momentum-size (1932:01-2008:06)		10 momentum (1932:01-2008:06)		10 cash flow-price (1956:07-2008:06)	
	λ	t-stat	λ	t-stat	λ	t-stat
Cons	1.47	4.92***	1.51	3.61***	-0.31	-0.74
Market-up	1.64	4.18***	1.25	2.26**	2.14	4.17***
Market-down	-4.04	-8.65***	-3.64	-5.40***	-1.15	-1.60
SMB-up	2.28	12.95***	1.14	4.31***	1.18	4.37***
SMB-down	-1.71	-7.70***	-1.98	-6.63***	-1.08	-3.29***
HML-up	0.69	2.68**	0.32	0.94	1.99	13.62***
HML-down	-1.18	-4.16***	-1.26	-3.53***	-1.87	-6.31***
	10 earnings-price (1956:07-2008:06)		10 dividend-price (1932:06-2008:06)		10 short-term reversal (1931:02-2008:06)	
Cons	-0.12	-0.34	1.04	3.56***	-0.63	-1.32
Market-up	2.34	5.31***	1.78	4.85***	2.97	4.35***
Market-down	-1.75	3.01***	-3.34	-6.63***	-1.29	-1.85*
SMB-up	1.69	5.53***	1.26	4.19***	0.65	1.45
SMB-down	-0.91	-2.95***	-1.29	-3.41***	-1.43	-3.63***
HML-up	1.90	12.28***	1.41	7.31***	1.06	2.70***
HML-down	-1.63	-5.59***	-1.62	-6.35***	-0.71	-1.68*

* significant (10-percent level)

** significant (5-percent level)

*** significant (1-percent level)

This table depicts the results for the Fama-French three-factor model given by equation 1.4 when using the returns of 25 portfolios sorted by momentum and size, 10 portfolios sorted by momentum, 10 portfolios sorted by the cash flow-price ratio, 10 portfolios sorted by the earnings-price ratio, 10 portfolios sorted by the dividend-price ratio, and 10 portfolios sorted by short-term reversal as dependent variables. Cons denotes the constant term, Market-up (-down) the risk premium of the market given that the excess market return is positive (negative), SMB-up (-down) that of the size given that the SMB factor is positive (negative), and HML-up (-down) that of the book-to-market risk given that the HML factor is positive (negative). The coefficients are given as percentage points per month.

assume that our time-varying betas are predetermined for the entire simulation.[8] As cross-sectional correlation among portfolio returns is to be suspected, we use the residuals of the 25 portfolios to estimate a cross-sectional correlation matrix. By multiplying the Cholesky decomposition of the correlation matrix with the generated residuals we incorporate cross-sectional correlation into our framework.

In the second step, we generate error terms. We consider three different ways to specify residuals. As a benchmark case we assume that residuals are normally distributed with mean zero. We obtain the variance by calculating the empirical variance of the residuals for each portfolio. Since normally distributed stock returns cannot model the observed unconditional leptokurtosis in stock returns, we examine two further approaches. In order to model residuals in a more realistic fashion, we generate residuals by the Pearson distribution. The Pearson system, developed by Pearson (1895), is a family of continuous probability distributions that is fully specified by its first four standardized moments. It enables us to construct probability distributions, which exhibit considerable skewness and kurtosis. The Pearson system can be subdivided into seven types. Our focus is on Pearson type IV, which is not related to any standard distribution.[9] In order to model the observed fat tails, we compute the empirical kurtosis in addition to the standard deviation and generate error terms.[10] Finally, we go one step further and model the skewness. The distributions so far are based on the assumption of symmetry, which is not fulfilled, e.g., for the size and book-to-market factors. The same holds true for some of the 25 portfolio residuals. Applying the test by Ekström and Jammalamadaka (2007) we find that the size and book-to-market risk factors and some portfolio residuals exhibit an asymmetric distribution. Therefore, we calculate the empirical skewness. In each iteration, we draw random variables from the Pearson distribution based on the estimated standardized moments of our residuals.

In the third step, we generate the market, size and book-to-market factors drawing random numbers in the same way as for the residuals. Again, we consider three distributions: Normal distribution, Pearson type IV distribution with and without skewness.

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[8]Keeping the betas constant and modeling all factors alike enables us to evaluate if the power of the test depends on the way portfolios are sorted. For instance, using the 25 portfolios sorted by size and book-to-market we expect that both tests have lower power to detect a priced market factor than a book-to-market factor just because the variation in market betas is lower.

[9]The density function is proportional to $(1 + ((x - a)/b)^2)^{-c} * \exp(-d * \arctan((x - a)/b))$.

[10]Numbers are generated by MATLAB using the "pearsrnd" command. Given the first four moments, the parameters a,b,c and d can be identified.

Figure 1.4: Distribution of the HML Factor
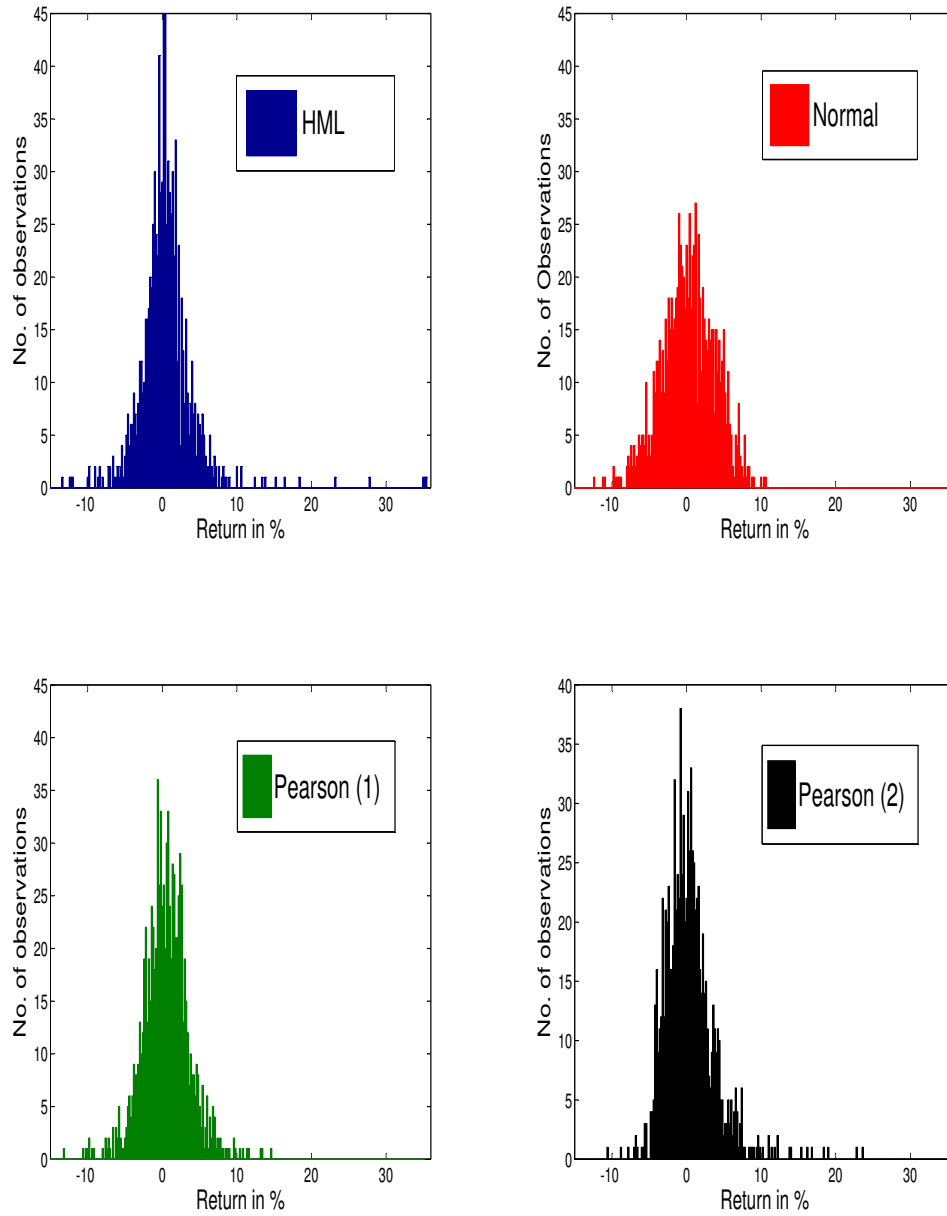
The first figure depicts the histogram of the HML factor (HML). The other figures illustrate the histograms of the generated HML factors based on random numbers drawn from three different distributions. Version 1 is based on the normal distribution (Normal), version 2 on the Pearson type IV distribution without skewness (Pearson (1)), and version 3 on the Pearson type IV distribution with skewness (Pearson (2)). The generated factors are taken arbitrarily.

The only difference is that we have to add the empirical mean of the factors to the generated values. Figure 1.4 depicts the histogram of the HML factor and compares it to the histogram of the simulated factor. The simulated factor realizations are chosen randomly. Subsequently, we have all ingredients to specify the portfolio excess returns. In the case that market beta risk is priced, stock returns are generated from:

$$r_{i,t}^e = \beta_{i,t}^m r_{m,t}^e + \beta_{i,t}^{smb} r_{smb,t} + \beta_{i,t}^{hml} r_{hml,t} + \epsilon_{i,t}. \tag{1.9}$$

Alternatively, when market beta risk is not priced, we obtain the following equation:

$$r_{i,t}^e = \mu_m + \beta_{i,t}^m (r_{m,t}^e - \mu_m) + \beta_{i,t}^{smb} r_{smb,t} + \beta_{i,t}^{hml} r_{hml,t} + \epsilon_{i,t}, \tag{1.10}$$

where $\mu_m = E[r_{m,t}^e]$. Analogously, returns for priced and not priced size and book-to-market risk are generated. The number of generated returns coincides with the number of observations (924) in section 1.4 and 1.5. The simulation exercise is based on 1000 replications. In each replication, factors and residuals are produced and equation 1.9 and 1.10 are used to generate portfolio excess returns.

The results of the simulation are depicted in tables 1.9 and 1.10 and convey some very interesting insights. Results vary across factors. Testing for a priced market factor the Fama-MacBeth test offers a slightly higher power than the FG test at the 5% level but a smaller power at the 10% level independent of the choice of the distribution. In the case of the size and book-to-market factors, the FG test surpasses the power of the Fama-MacBeth test. However the differences are small. The size of the two tests is almost identical. All in all, our findings indicate that the differences between the two tests are marginal, supporting the results in the last section. Moreover, we cannot support the conjecture of Freeman and Guermat (2006). They reckon that the power of the FG test exceeds the power of the Fama-MacBeth test in the presence of stock returns with fat tails.

Another insightful feature is that the power of the tests behaves very differently when we pass on to fat-tailed distributions. Both tests exhibit considerably lower power for fat-tailed distributions when using the market factor whereas the power tends to rise for the size and book-to-market factor. Including skewness slightly increases the power of the two tests no matter which factor we consider.

There is another noteworthy feature. Test results suggest that both tests have more difficulties to detect a priced market factor than a priced book-to-market factor. This finding could be due to the fact that the first four moments are different. Though,

Table 1.9: Power and Size of the Fama-MacBeth and the FG Test (5% two-sided)

		Market		SMB		HML	
Distribution		FM	FG test	FM	FG test	FM	FG test
Normal	Size	0.042	0.041	0.057	0.056	0.051	0.052
	Power	0.672	0.670	0.678	0.678	0.941	0.945
Pearson (1)	Size	0.066	0.067	0.051	0.053	0.063	0.058
	Power	0.607	0.591	0.690	0.694	0.941	0.941
Pearson (2)	Size	0.031	0.033	0.053	0.053	0.050	0.047
	Power	0.612	0.610	0.708	0.710	0.956	0.957

Table 1.10: Power and Size of the Fama-MacBeth and the FG Test (10% two-sided)

		Market		SMB		HML	
Distribution		FM	FG test	FM	FG test	FM	FG test
Normal	Size	0.097	0.090	0.110	0.107	0.095	0.101
	Power	0.779	0.780	0.794	0.797	0.974	0.974
Pearson (1)	Size	0.117	0.121	0.098	0.099	0.102	0.102
	Power	0.716	0.720	0.797	0.797	0.965	0.967
Pearson (2)	Size	0.077	0.075	0.010	0.094	0.099	0.097
	Power	0.726	0.726	0.811	0.814	0.979	0.980

These tables depict the power and the size of the Fama-MacBeth test (FM) and the FG test for each risk factor. Market denotes the market excess return, SMB the size factor and HML the book-to-market factor. Factors and residuals are generated drawing random numbers from three different distributions: Normal distribution (normal), Pearson type IV distribution without skewness (Pearson (1)), and Pearson type IV distribution with skewness (Pearson (2)). The value of the t-statistic in each case is then tested for significance at the 5 % (table 1.9) and at the 10% (table 1.10) two-sided level.

even if all factors are identically constructed with the same first four moments, this phenomenon prevails because of the differences in betas. Variation in book-to-market beta across portfolios is much higher than the variation in market beta across portfolios, which suggests that a wide spread in betas substantially raises the power of the test independent of the distribution. This finding underlines how crucial the sorting criteria are.

1.7 Conclusion

Our results provide evidence that there exists a systematic relationship between the three Fama-French betas and returns. Despite the inclusion of the size and book-to-market factors, we detect a systematic conditional relationship between market beta and return. Furthermore, the two additional factors of the three-factor model amplify their explanatory power once the conditional nature of the relation between beta and return is considered. This finding is consistent for different subperiods and test portfolios. Thus, the use of the conditional three-factor model betas estimated from historical price data by portfolio managers seems to be appropriate.

The main drawback of this procedure is that it does not test if risk factors entail a priced risk. On this account, we go one step further in this chapter and generalize the FG test to multi-factor models in order to test for priced betas within the conditional approach. We compare the results of the FG test to the results of the classical Fama-MacBeth test. Based on different test portfolios we find qualitatively similar results for both tests. The same holds true when we run simulations specifiying distributions with excess kurtosis and skewness. Hence, this study shows that the results of the FG test based on the conditional approach coincide with those from the standard Fama-MacBeth test procedure. Our findings suggest that the power of a test is not improved by the application of the conditional approach. To put it differently, our results confirm the standard Fama-MacBeth procedure. Because of the additional complexity of using the FG test the standard Fama-MacBeth test is favored.

Our findings stress the importance of the use of different test portfolios. Applying diverse test portfolios, we find starkly differing results. For some test portfolios risk factors seem positively priced, for some negatively priced, and for others they appear not to be priced at all. Thus, focusing on one selection of test portfolios, as often done so in the literature, can cause misleading results.

Many previous studies have applied the conditional approach as proposed by Pettengill et al. (1995). The conditional approach takes into account that the use of realized returns leads to a negative risk-return relationship in down-markets. Thus, the conditional approach appears to be more appropriate. However, either previous studies test if beta risk is priced within the framework of the conditional approach but based on a flawed hypothesis (symmetry of the λ coefficients) or they only test if there exists a conditional relationship between beta and return. In either case, the results of the test cannot be related to the results from the standard Fama-MacBeth procedure. In this study, we make these tests comparable by using the conditional approach to derive the FG test for priced beta and discover that the FG test leads to qualitatively similar findings as the classical Fama-MacBeth test. We do not want to claim that the conditional approach is irrelevant, but we want to point out that the choice of the test procedure depends on the research question. Testing for priced beta risk does not make the conditional approach necessary. Nevertheless, if we only focus on testing for a systematic relationship between beta risk and return, then the conditional approach is suitable.

Chapter 2

Illiquidity and Stock Returns

2.1 Introduction

One of the most active areas of research in empirical asset pricing over the recent years has been the examination of the influence of liquidity on asset prices. The concept of liquidity does not only provide an attraction for academic research, it is also of great interest among practitioners. Investment consultants and managers tailor portfolios to fit their clients' investment horizons and liquidity objectives. In the following chapter, we investigate the impact of liquidity on returns on the German stock market. The German stock market is one of the largest stock markets in the world by both market value and trading volume and has been insufficiently considered in the literature.[1] Our data set comprises 33 years making it comparable to corresponding US studies and is more comprehensive than most studies dealing with stock markets outside the US. This study does not concentrate on one liquidity measure and one econometric approach, but in contrast to the existent literature it covers all dimensions of liquidity and applies a multitude of different methodologies to evaluate the robustness of its results.

Liquidity is an elusive concept and has many facets. The major problem in estimating the effect of liquidity on returns is how to measure liquidity since there is hardly a single measure that captures all of its aspects. Illiquidity consists of four dimensions: trading quantity, trading speed, trading costs, and price impact. This study incorporates all four dimensions and separately tests if they entail a priced risk in the cross-section of the

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[1]One exception is Hagemeister and Kempf (2010). Although the main goal of their study is to model expected returns using earnings' expectations, they make use of a CAPM augmented with the bid-ask spread.

German stock market. Our analysis makes use of different empirical procedures and test portfolios to cover a large set of empirical approaches used in the literature. First, we sort stocks into portfolios based on illiquidity as, for instance, done so in Liu (2006), and test if the most illiquid stocks outperform the most liquid stocks. Further, we investigate the existence of a monotonic relation between illiquidity and returns conducting a test recently proposed by Patton and Timmermann (2010). Subsequently, we conduct regressions to test if illiquidity drives stock returns while controlling for other asset pricing effects like market capitalization, book-to-market equity ratio, and past returns. We apply a refinement of the Fama and MacBeth (1973) approach proposed by Litzenberger and Ramaswamy (1979) using individual returns. In the context of liquidity and asset pricing, this procedure has been applied by Datar et al. (1998). Finally, we estimate the illiquidity premium, which is in line with studies such as Pastor and Stambaugh (2003) and Acharya and Pedersen (2005). In contrast to these studies, we construct factor mimicking portfolios that capture the risk of illiquidity. We also take into account different test portfolios. Following Acharya and Pedersen (2005), we make use of size and book-to-market portfolios as well as portfolios sorted, amongst others, by illiquidity.

Our regression results provide evidence that a positive relation between stock returns and illiquidity exists after controlling for a multitude of different asset pricing effects. Further, we discover a significant risk premium on illiquidity independent of the measure chosen. However, this effect relies on the choice of test portfolios. The risk premium disappears when using the 16 size and book-to-market portfolios as test portfolios. This finding reflects the lack of robustness of asset pricing model tests to alternative portfolio formation.

There is a large number of empirical studies dealing with the relation between expected stock returns and illiquidity for the US market. One of the first studies was by Amihud and Mendelson (1986) focusing on trading costs measured by the bid-ask spread. They find a positive and concave relationship between expected returns and illiquidity. There are plenty of other measures capturing trading costs in the literature. For example, Roll (1984) develops an estimator of the effective spread based on the serial covariance of the change in price. Lesmond et al. (1999) create an estimator of the effective spread based on the idea of informed trading on non-zero return days and the absence of informed trading on zero return days. Holden (2009) develops a proxy of the effective spread based on observable price clusterings. It assumes that trade prices are clustered in order to minimize negotiation costs between potential traders. Apart from

the proxies regarding trading costs, researchers have employed several proxies measuring other dimensions. Amihud (2002) constructs a price impact measure representing the daily price response associated with one dollar of trading volume. He provides evidence that the price impact measure has a positive effect on stock returns. Another price impact measure is the Amivest measure, which is the ratio of the average trading volume and the average absolute return. For instance, this measure has been applied by Amihud et al. (1997) and Berkman and Eleswarapu (1998). Pastor and Stambaugh (2003) develop the Gamma measure which captures the return reversal in response to volume shocks. They find that market-wide illiquidity is a priced risk and is compensated by a risk premium. An additional price impact measure is used by Brennan and Subrahmanyam (1996). They make use of intraday trade and quote data and estimate Kyle's (1985) λ by regressing the trade-by-trade price change on the signed transaction size applying the model by Glosten and Harris (1988). The slope coefficient from this regression are taken to calculate the marginal cost of trading. Brennan and Subrahmanyam (1996) detect that the marginal cost of trading has a positive effect on returns adjusted by the Fama-French factors. Further, Chordia et al. (2009) propose a theoretically motivated illiquidity measure using the structure of Kyle's (1985) λ and find that it is priced in the cross-section.

Another dimension of illiquidity is trading quantity, which is widely used as it is relatively easy to construct. Brennan et al. (1998) use the stock's dollar trading volume as a measure of liquidity and show that trading volume has a negative effect on risk-adjusted stock returns. Alternatively, Datar et al. (1998) propose the use of stock turnover measured by the stock trading volume divided by the market capitalization and find that the cross-section of stock returns is negatively related to turnover. There is little research devoted to capturing the trading speed dimension of illiquidity. One exception is Liu (2006). Liu (2006) measures trading speed by the number of days of zero trading volume and argues that it captures the continuity of trading and the potential difficulty in executing an order. Constructing a factor mimicking portfolio reflecting the illiquidity premium, Liu (2006) reinforces earlier results. The number of days with zero returns proposed by Lesmond et al. (1999) is a similar measure. However, in the case of informationless trades results can diverge. Informationless trades should not create price changes in liquid markets.

Despite the large number of papers studying the US market, there is only a small number of papers studying the effect of liquidity on returns on other stock markets. Hwang and

Lu (2009) study the UK market from 1984 to 2004 and provide evidence that liquid stocks outperform illiquid stocks, a reversed illiquidity effect. Mazouz et al. (2009) show that systematic liquidity risk is not priced on the LSE studying the period from 1992 to 2007. Martinez et al. (2005) analyze the Spanish stock market from 1991 to 2000. Although a positive illiquidity premium surfaces in some of their regressions, results all in all indicate that a relation between illiquidity and returns does not exist on the Spanish stock market. The existence of a liquidity premium outside the US seems to be unclear and requires further analysis. In this study, we investigate whether an illiquidity premium prevails on the German stock market. The German stock market is one of the largest stock markets in the world. Our dataset covers 33 years and, thus, the number of time-series observations is similar to comparable US studies. The focus on the German stock market is of considerable interest. Results for the German stock market suggest that big firms tend to earn higher returns than small firms. Schrimpf et al. (2007) construct a size factor and find out that it is on average negative for the period from 1969 to 2002. Breig and Elsas (2009) confirm this finding for the period from 1990 to 2006. Studies like Amihud and Mendelson (1986) argue that size serves as a reasonable proxy for illiquidity. Liu (2006) finds a strong negative correlation between the market capitalization and illiquidity. Small firms tend to be firms with illiquid stocks and vice versa. Given the relation between illiquidity and size, we suspect a reversed or absent illiquidity effect in Germany.

Despite the absence of a size effect on the German stock market, our results provide evidence for a positive relation between stock returns and illiquidity, but a negative one to the size of the firm. Although the size of the firm and the illiquidity of the corresponding stock are correlated, our findings indicate that the two concepts grasp different risks.

The remainder of this chapter is organized as follows. In the next section we introduce four illiquidity measures. Section 3 describes the data and the econometric methodology in detail. Subsequently, we provide the empirical results. Section 5 concludes.

2.2 Illiquidity Measures

In general, liquidity mirrors the ability to trade large quantities quickly at low cost with little price impact. Implicitly, this description entails four dimensions of liquidity: trading quantity, trading speed, trading cost, and price impact. In order to consider the full spectrum of liquidity we include one measure for each of the four dimensions.

2.2.1 Trading Quantity

Trading volume is an obvious measure to cover the first dimension of liquidity: trading quantity. However, most of the literature favors a different measure since trading volume is closely related to the size of the firm. One way to separate between these two variables is by taking the ratio of trading volume to the market capitalization. This measure of illiquidity is well known as turnover and has been proposed by Datar et al. (1998) in the context of empirical asset pricing. It can be interpreted as the reciprocal of the average holding period. Several theoretical studies find that less frequently traded stocks are less liquid. For instance, Amihud and Mendelson (1986) show that less liquid stocks are allocated to investors with longer holding periods. Constantinides (1986) provides evidence that investors reduce their trading frequency of illiquid stocks. We define our trading quantity measure (TQ) of stock i in month t as

$$TQ_{i,t} = \frac{\frac{21}{\sum_{k=1}^{12} d_{t-k}} \sum_{k=1}^{12} \sum_{j=1}^{d_{t-k}} VOL_{i,j,t-k}}{\frac{1}{12} \sum_{k=1}^{12} SIZE_{i,t-k}}. \tag{2.1}$$

$VOL_{i,j,t-k}$ is the trading volume in EUR of stock i in month $t-k$ at day j, $SIZE_{i,t-k}$ is the market capitalization at the end of month $t-k$ and d_{t-k} is the number of trading days in month $t-k$. As for all other illiquidity measures we calculate $TQ_{i,t}$ over the previous twelve months. The numerator represents the average daily trading volume over the last twelve months. By multiplying this term by 21 we calculate the average monthly trading volume for a standardized month of 21 trading days. The denominator is the monthly average of the market capitalization.

2.2.2 Trading Speed

The second dimension is trading speed. We measure trading speed by the number of zero trading volume days over the last twelve months. This measure grasps the continuity and the potential delay in executing an order. The absence of a trade indicates the degree of illiquidity: the more frequently trade volume is zero, the more illiquid is the stock. In extreme cases, zero trading volume reflects a lock-in risk, which embodies the danger that stocks cannot be sold. On this account, in equilibrium investors should receive a premium for holding these stocks. The idea of this measure follows Liu (2006). Trading speed (TS) of stock i in month t is measured via:

$$TS_{i,t} = \frac{\sum_{k=1}^{12} \sum_{j=1}^{d_{t-k}} \mathbb{I}\{VOL_{i,j,t-k} = 0\}}{\sum_{k=1}^{12} d_{t-k}}. \tag{2.2}$$

$\mathbb{I}\{VOL_{i,j,t-k} = 0\}$ is an indicator variable, which is one if trading volume is equal to zero and zero otherwise. $TS_{i,t}$ is the ratio of zero trading days divided by the number of trading days. However, when it comes to the creation of portfolios sorted by the illiquidity of stocks, it is important to uniquely determine the order of stocks based on the degree of their illiquidity. Since many stocks exhibit the same number of zero trading days, in particular the most liquid stocks have zero non-trading days, we incorporate a second measure to explicitly characterize the degree of illiquidity for these stocks. Following Liu (2006) we use turnover. Our sorting criterion is constructed in a way that the number of zero trading volume days is the prime criterion. In case they coincide, we use the stock's turnover as a tie-breaking rule.

A similar measure as the number of zero trading days is the number of zero return days as applied by Lesmond et al. (1999) and Bekaert et al. (2007). If volume data are not available, as it is the case in many emerging markets, it can be an useful proxy. However, it produces different findings, e.g., in the case of informationless trades. Informationless trades should not create price changes in liquid markets.

2.2.3 Trading Costs

Although the most natural and intuitive way to model trading costs is the use of bid-ask spreads, a data series back to the 70's is unavailable for the German stock market. For such cases, Lesmond et al. (1999) propose to model transaction costs using a Tobit model. They develop an estimator of the effective spread based on the idea that in the presence of transaction costs, the marginal informed investor will trade only if the value of information exceeds transaction costs. They assume that a standard market model (CAPM) holds on non-zero return days, but a flat horizontal segment applies on zero return days. In the following description we fix a twelve months window. The true but unobserved return $r_{i,d,t}^*$ for stock i at day d in month t is given as:

$$r_{i,d,t}^* = \beta_i r_{m,d,t} + \epsilon_{i,d,t}, \qquad (2.3)$$

where β_i is the sensitivity of stock i to the market return $r_{m,d,t}$ and $\epsilon_{i,d,t}$ is a public information shock at day d in month t. $\epsilon_{i,d,t}$ is normally distributed with mean zero and variance σ_i^2. The observed return is given by:

$$
\begin{aligned}
r_{i,d,t} &= r_{i,d,t}^* - \alpha_{1,i} &&\text{when} & r_{i,d,t}^* &< \alpha_{1,i}, \\
r_{i,d,t} &= 0 &&\text{when} & \alpha_{1,i} &< r_{i,d,t}^* < \alpha_{2,i}, \\
r_{i,d,t} &= r_{i,d,t}^* - \alpha_{2,i} &&\text{when} & r_{i,d,t}^* &> \alpha_{2,i}.
\end{aligned}
\qquad (2.4)
$$

$\alpha_{1,i}$ is the percentage transaction cost of selling stock i and $\alpha_{2,i}$ of buying stock i. Thus, $\alpha_{1,i}$ is the threshold for trades on negative information and $\alpha_{2,i}$ on positive information. If $\alpha_{1,i} < \beta_i r_{m,d,t} + \epsilon_{i,d,t} < \alpha_{2,i}$, the measured return will be zero. Given this, transaction costs (TC) of stock i can be calculated by the difference between $\alpha_{2,i}$ and $\alpha_{1,i}$:

$$TC_i = \alpha_{2,i} - \alpha_{1,i}. \tag{2.5}$$

We estimate TC_i based on the previous twelve months. Subsequently, we shift the twelve months window one month ahead and estimate TC_i for the next window. Although, $\alpha_{1,i}$, $\alpha_{2,i}$, and β_i are fixed within a certain window, they vary from one window to the other. Usually, the intercept in the market model captures any misspecifications of the market model. Thus, differences in the alphas are not necessarily due to differences in transaction costs. However, since we are interested in the difference $\alpha_{2,i} - \alpha_{1,i}$ to determine the round trip transaction costs, the effect of any misspecification should cancel out. As a robustness check, we assume that the Fama-French three-factor model is the market model and calculate the correlation between the two measures. The average cross-sectional correlation between the trading costs using the CAPM as the market model and trading costs using the Fama-French three-factor model as the market model is 0.997. As we explain in more detail in section 2.3.2, we construct risk factors based on our different illiquidity measures. Using trading costs based on the two market models, we find a time-series correlation between the illiquidity factors of 0.998. In the following, we assume that the parsimonious CAPM is the appropriate market model.

The parameters are obtained by maximizing the following likelihood function:

$$
\begin{aligned}
L(\alpha_{1,i}, \alpha_{2,i}, \beta_i, \sigma_i | r_{i,d,t}, r_{m,d,t}) \;=\; & \prod_{d \in R_1} \frac{1}{\sigma_i} \phi\left(\frac{r_{i,d,t} + \alpha_{1,i} - \beta_i r_{m,d,t}}{\sigma_i} \right) \\
* \; & \prod_{d \in R_2} \frac{1}{\sigma_i} \phi\left(\frac{r_{i,d,t} + \alpha_{2,i} - \beta_i r_{m,d,t}}{\sigma_i} \right) \\
* \; & \prod_{d \in R_0} \left[\Phi\left(\frac{\alpha_{2,i} - \beta_i r_{m,d,t}}{\sigma_i} \right) - \Phi\left(\frac{\alpha_{1,i} - \beta_i r_{m,d,t}}{\sigma_i} \right) \right],
\end{aligned}
\tag{2.6}
$$

where ϕ represents the standard normal density and Φ the cumulative distribution function (cdf) of the standard normal distribution. Moreover, R_1 and R_2 denote the regions where the measured return $r_{i,d,t}$ is negative and positive, respectively. R_0 is the zero return region. This construction slightly diverges from Lesmond et al. (1999)'s approach

in so far as they define region 1 as $r_{i,d,t} \neq 0$ and $r_{m,d,t} > 0$ and region 2 as $r_{i,d,t} \neq 0$ and $r_{m,d,t} < 0$. The approach used in this paper follows Goyenko et al. (2009).[2]

2.2.4 Price Impact

The last dimension we deal with is price impact. As defined by Amihud (2002), we calculate the price impact measure (PI) of stock i in month t as follows:

$$PI_{i,t} = \frac{1}{\sum_{k=1}^{12} \tilde{d}_{t-k}} \sum_{k=1}^{12} \sum_{j=1}^{\tilde{d}_{t-k}} \frac{|r_{i,j,t-k}|}{VOL_{i,j,t-k}} * 10^5. \tag{2.7}$$

Since $PI_{i,t}$ is not defined for zero trading volume days, it only considers days with positive trading volume, $\tilde{d}_{t-k}$. This ratio gives the absolute percentage price change per EUR of daily trading volume, or the daily price impact of order flow. It follows Kyle's (1985) concept of illiquidity - the response of price to order flow. Because market makers cannot distinguish between order flow that is generated by informed traders and noise traders, they set prices as an increasing function of the order flow imbalance, which may indicate informed trading. This creates a positive relationship between the transaction volume and price change.

There are quite a few other proxies for price impact in the literature. The Amivest liquidity ratio calculates the ratio of the average volume to the average absolute value of daily returns. It has been used by Amihud et al. (1997) and Berkman and Eleswarapu (1998). As the absolute return is in the denominator, it can only be calculated for all non-zero return days. Another widely used measure is the Gamma measure developed by Pastor and Stambaugh (2003). It is based on the idea that in a regression of a stock's daily return on its signed volume, the coefficient which captures the bounce in the stock price following a given trading volume is more negative for less liquid stocks. Intuitively, it measures the reversal of the previous day's order flow shock. Goyenko et al. (2009) show that the Amihud measure outperforms the Amivest and the Gamma measure when compared to high frequency price impact benchmarks.

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[2]Goyenko et al. (2009) run a horse race of a multitude of liquidity measures based on daily data gauging their abilities to match the salient features of high frequency based benchmarks. They identify the best proxies and find that the measure applied in this study dominates the measure constructed by Lesmond et al. (1999).

2.3 Data & Methodology

2.3.1 Data

Our sample extends from January 1974 to December 2006 and incorporates firms that are listed at some point during the sample period at the Frankfurt Stock Exchange including the segments "Amtlicher Handel" and "Neuer Markt". Daily price and volume data are obtained from Deutsche Kapitalmarktdatenbank in Karlsruhe. Trading volume is the aggregated EUR (DM) volume at all German exchanges including XETRA. With respect to the construction of trading quantity, trading speed, and price impact, the question arises whether it is more reasonable to use the trading volume from the Frankfurter Stock Exchange or the total German trading volume. In order to compare the differences in our trading quantity, trading speed, and price impact measure when using the trading volume from the Frankfurter Stock Exchange and the total German trading volume, we calculate both. Table 2.1 provides the correlation. The cross-sectional correlation is 96.5% for trading quantity, 97.5% for trading speed and 91.7% for price impact from 1975:01 to 2006:12. The time-series correlation between illiquidity factors based on trading volume from the Frankfurter Stock Exchange and total German trading volume is even higher. It is above 99.4% for all three measures.[3] However, we suspect correlation to be lower since the inception of XETRA. For this reason, the second part of table 2.1 concentrates on the period from 1997:12 to 2006:12. Regarding trading speed and trading quantity the correlation only decreases marginally. The cross-sectional correlation between the two price impact measures shrinks to 84.1%. Though, using the two price impact factors we still find a time-series correlation of 99.3%. All in all, these findings indicate that illiquidity measures based on different trading volumes are very similar. In the following analysis, trading quantity, trading speed, and price impact are calculated based on the total German trading volume.

Prices are adjusted for dividends and equity offerings. Yearly book equity and the number of shares outstanding come from Hoppenstedt Aktienführer. We only examine common stocks. All illiquidity measures are calculated over a twelve months horizon. Daily stock returns and trading volume must be available for at least 200 days over the previous twelve months. We only include stocks if we can calculate each of the four

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[3]The construction of these return factor mimicking portfolios will be explained in detail in section 2.3.2.

Table 2.1: Overall German Trading Volume vs. Trading Volume from the Frankfurter Stock Exchange

1975:01-2006:12	Cross-Sec. Correlation				Time-series Correlation		
Illiquidity	TQ	TS	PI	F_{Illiq}	F_{TQ}	F_{TS}	F_{PI}
TQ_{ff}	0.965			$F_{TQ_{ff}}$	0.994		
TS_{ff}		0.975		$F_{TS_{ff}}$		0.994	
PI_{ff}			0.917	$F_{PI_{ff}}$			0.995

1997:12-2006:12	Cross-Sec. Correlation				Time-series Correlation		
Illiquidity	TQ	TS	PI	F_{Illiq}	F_{TQ}	F_{TS}	F_{TO}
TQ_{ff}	0.955			$F_{TQ_{ff}}$	0.993		
TS_{ff}		0.958		$F_{TS_{ff}}$		0.993	
PI_{ff}			0.841	$F_{PI_{ff}}$			0.993

This table shows the correlation between illiquidity measures (factors) based on the trading volume from all German Stock Exchanges and the trading volume from the Frankfurter Stock Exchange. First, we calculate trading quantity (TQ), trading speed (TS), and price impact (PI) based on the trading volume from all German Stock Exchanges including XETRA. Next, we determine trading quantity (TQ_{ff}), trading speed (TS_{ff}), and price impact (PI_{ff}) based on trading volume from the Frankfurter Stock Exchange. The first part of the table shows the average cross-sectional correlation. We also construct illiquidity factors as explained in subsection 2.3.2. F_{TQ} is the trading quantity, F_{TS} the trading speed, and F_{PI} the price impact factor. All three factors are based on the overall German trading volume. By contrast, $F_{TO_{ff}}$, $F_{TS_{ff}}$, and $F_{PI_{ff}}$ are factors based on liquidity measures using the trading volume from the Frankfurter Stock Exchange. We calculate the time-series correlation. The lower part of the table considers the liquidity measures since the inception of the XETRA.

illiquidity measures.[4] Our data set comprises 235 firms in the 1970s and remains almost constant in the early 1980s. From the mid 1990s onwards the number of observations soars due to the hot IPO wave. The average number of firms is 572 from 2000 to 2006. Table 2.2 shows the evolution of cross-sectional observations over time.

As the proxy for the market portfolio the CDAX from the Deutsche Börse AG is used.[5] The risk-free rate is measured by the one-month money market rate reported by Frankfurt

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[4]Unfortunately, trading volume is not available for the following three months: 1983:02, 1983:03, and 1983:10. We assume that the illiquidity measures behave in the same way as for the other months in these twelve months windows. Stocks with less than 140 daily returns and trading volumes during this time window are excluded.

[5]The CDAX is a value-weighted performance index and includes all German stocks listed in the General Standard and Prime Standard at the Frankfurter Stock Exchange. The correlation between the CDAX and a value-weighted index of all stocks in this sample is over 98%.

banks.[6] Since the influential works of Fama and French (1992, 1993) it has been common practice in the empirical asset pricing literature to control for the size and book-to-market effect. Size is measured by the market capitalization of a firm and book-to-market is the ratio of the book value and market capitalization. The size and book-to-market portfolios are constructed in the same way as in Fama and French (1993). The two factors are based on six portfolios, which are the intersections of two portfolios formed on size and three portfolios formed on book-to-market. Portfolios consisting of small (big) firms are denominated as small (big) portfolios, whereas portfolios consisting of firms with a low (high) book-to-market value are denoted as growth (value) portfolios. The size factor is constructed as the difference between the average return on three small portfolios and the average return on three big portfolios. The book-to-market factor is the average return on the two value portfolios minus the average return on the two growth portfolios. Supplementary, the momentum factor (WML) proposed by Carhart (1997) is included into our analysis. The momentum effect has been detected by Jegadeesh and Titman (1993) and illustrates that past winners earn higher returns than past losers. We follow Carhart (1997)'s way of constructing the momentum factor and sort stocks into three equally-weighted portfolios based on eleven month returns lagged by one month. Portfolio 1 contains past winners and portfolio 3 past losers. The breakpoints are the 30th and 70th percentiles. Portfolio 1 minus portfolio 3 is the WML factor.

Table 2.2: Number of Observations

Year (y)	Average No. of Firms
$1974 \leq y < 1980$	235
$1980 \leq y < 1985$	237
$1985 \leq y < 1990$	262
$1990 \leq y < 1995$	326
$1995 \leq y < 2000$	362
$2000 \leq y \leq 2006$	572

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[6]Money market rate is downloaded from the homepage of the German Bundesbank.

2.3.2   Methodology

Monotonic Relation Test

The goal of this section is to reveal the potential existence of a cross-sectional relation between illiquidity and expected returns. Sorting stocks into portfolios is the simplest way to verify this relation and is extensively used in the empirical asset pricing literature. Standard practice is to sort stocks into multiple portfolios and consider the mean return differential between the top and bottom portfolios and to evaluate by a simple t-test if the mean return differential is equal to zero. However, this proceeding does not provide a sufficient way to test for a monotonic relation between expected returns and illiquidity. On this account, we additionally conduct the Monotonic Relation (MR) test proposed by Patton and Timmermann (2010) testing for a monotonic relationship between expected returns and our sorting variables. Let μ_i be the population value of the expected return on the ith portfolio ($i = 1, ..., N$) obtained from a ranking of stocks. Suppose we intend to test the null hypothesis that there is no difference in the expected returns of the portfolios against the alternative hypothesis that the expected return is decreasing when we move from the top portfolio to the bottom portfolio. The null hypothesis (H_0) is written as

$$H_0 : \mu_1 = \mu_2 = ... = \mu_N \tag{2.8}$$

and the alternative hypothesis H_1 as

$$H_1 : \mu_1 < \mu_2 < ... < \mu_N. \tag{2.9}$$

Although the null hypothesis is written as an equality, finding $\mu_j > \mu_{j+1}$ makes rejections of the null against the alternative hypothesis less likely. To test the null hypothesis, we construct the average return differential of adjacent portfolios $\hat{\Delta}_i = \hat{\mu}_i - \hat{\mu}_{i-1}$. The test statistic for the MR test is given as follows:

$$J_T = \min_{i=2,...,N} \hat{\Delta}_i \tag{2.10}$$

In order to obtain critical values for the MR test, Patton and Timmermann (2010) suggest a bootstrap approach, which circumvents the problem of estimating the covariance matrix of the sample average returns for the N portfolio returns. Let $\left\{ r^e_{i,\tau(t)}, 1, ..., T; i = 1, ..., N \right\}$ be the original set of portfolio returns over T time periods. Now, we randomly draw with replacement a new sample of returns $\left\{ r^{eb}_{i,\tau(t)}, \tau(1), ..., \tau(T); i = 1, ..., N \right\}$, where $\tau(t)$ is the new time index which is a random draw from the original set $1, ..., T$ and $r^{eb}_{i,\tau(t)}$ is the

return of test portfolio i at time $\tau(t)$. b runs from $b = 1$ to B and represents the number of Bootstrap iterations. In our application, we set $B = 10,000$. The randomized time index, $\tau(t)$, is common across portfolios in order to preserve any cross-sectional dependences in returns. In order to account for time-series dependences as well, we apply a block bootstrap drawing blocks of returns instead of drawing returns independently. Politis and Romano (1994) model the block length based on a Geometric distribution with a parameter that controls the average block length. Following Patton and Timmermann (2010), we set the block length equal to ten, which seems suitable for return data displaying limited time-series dependences at the monthly horizon.

Subsequently, the randomly drawn return series is demeaned by subtracting the original sample mean. The new time series ensures that the bootstrapped data satisfy the null hypothesis by construction. Finally, we calculate J_T^b and count the number of times where $J_T < J_T^b$. The p-value is this number divided by the number of bootstraps, B.

Individual Stock Returns (GLS)

To control for different characteristics at the same time, we run multivariate regressions. Two approaches to test asset pricing models are widely applied in the literature. The first one is based on individual asset returns and makes use of firm characteristics. The second one builds upon the construction of portfolios and the creation of risk factors. In order to derive robust results, both approaches are considered.

Firstly, we test if individual returns are driven by illiquidity while controlling for firm characteristics like the well known determinants of stock returns: Firm size, book-to-market, and lagged returns. Additionally, we include the market beta. The use of firm characteristics has been supported by Daniel and Titman (1997). They provide evidence that firm characteristics rather than factor loadings determine expected returns. Regressing individual stock returns on characteristics has been applied in numerous studies like Datar et al. (1998), Ang et al. (2009), Fu (2009), and Huang et al. (2010).

The econometric methodology in this study is based on a refinement of the Fama and MacBeth (1973) approach proposed by Litzenberger and Ramaswamy (1979). While the classical Fama-MacBeth procedure places equal weight on all slope coefficients, the Litzenberger and Ramaswamy (1979) methodology places more (less) weight on slope coefficients that are estimated more (less) precisely. Amongst others, Shanken (1992) and Kandel and Stambaugh (1995) argue in favor of GLS over OLS.

While both approaches lead to the same results under classic Gauss-Markov assumptions, the GLS test is more powerful to the extent these assumptions are violated in practice. The following methodology is applied by, e.g., Datar et al. (1998) when testing if turnover drives stock returns. In a first step, we estimate market betas based on daily returns over the previous twelve months. In the second step, we estimate the following cross-sectional equation for every month:

$$r_{i,t}^e = r_{i,t} - r_{f,t} = \lambda_{0,t} + \sum_{k=1}^{K} \lambda_{k,t} x_{k,t} + v_{i,t} \quad i = 1, ..., N_t, t = 1, ..., T, \tag{2.11}$$

where $r_{i,t}$ is the return of stock i, $r_{f,t}$ is the risk-free rate, $r_{i,t}^e$ is the excess return of stock i, $x_{k,t}$ is characteristic or factor loading k, and $v_{i,t}$ is the error term at time t. $\lambda_{k,t}$ denotes the monthly coefficients, which are estimated from the cross-sectional regressions in every time period t. N_t is the number of cross-sectional observations at time t. Final estimates of λ are given by:

$$\hat{\lambda}_k = \sum_{t=1}^{T} Z_{k,t} \hat{\lambda}_{k,t} \quad \text{,where} \quad Z_{k,t} = \frac{[Var(\hat{\lambda}_{k,t})]^{-1}}{[\sum_{k=1}^{K} Var(\hat{\lambda}_{k,t})]^{-1}}. \tag{2.12}$$

The variance of $\hat{\lambda}_k$, is estimated by:

$$Var(\hat{\lambda}_k) = \sum_{t=1}^{T} Z_{k,t}^2 Var(\hat{\lambda}_{k,t}). \tag{2.13}$$

Portfolio Approach (GMM)

In addition, we investigate the role of illiquidity based on a portfolio approach while controlling for risk factors like size (smb), book-to-market (hml), and momentum (wml). The methodology described in the following enables us to estimate the illiquidity risk premium and to draw conclusions whether illiquidity risk is priced. We create factor mimicking portfolios based on each of the four illiquidity measures. The construction follows Carhart (1997). We rank stocks by their illiquidity and form three portfolios: 30 percent with the lowest illiquidity, the middle 40 percent, and 30 percent with the highest illiquidity. Portfolios are updated every month. The high minus low portfolio is our illiquidity factor.

As our dependent variables we construct 16 equally-weighted portfolios. We independently sort stocks into four portfolios by momentum, i.e. past 2-12 months returns, and by price impact. The intersections of the four momentum and the four price impact portfolios are 16 portfolios. Sorting by price impact and momentum creates a wide spread in returns and betas across portfolios raising the power of the test. The conventional sorting

in the US with respect to size and market beta or size and book-to market only creates a small variation in returns and betas for Germany as the spread across size and beta portfolios is very low.[7] Nonetheless, as a robustness check, we also present results based on 16 independently and value-weighted test portfolios sorted by size and book-to-market. The average portfolio returns are presented in table 2.3. It reveals that sorting by momentum and illiquidity creates a larger spread in average returns across portfolios and is thus a more reasonable choice.

Table 2.3: Test Portfolios

		Momentum						Book-to-market			
		Losers	2	3	Winners			Low	2	3	High
	Low	0.19	0.42	0.76	1.43		Small	0.73	0.49	0.82	1.05
		[8.24]	[6.03]	[5.12]	[5.37]			[5.20]	[4.56]	[4.99]	[4.89]
	2	-0.27	0.48	0.51	1.46		2	0.13	0.74	0.56	0.97
Illiquidity		[7.34]	[5.18]	[4.25]	[4.87]	Size		[5.16]	[4.37]	[4.30]	[4.83]
	3	0.31	0.54	0.97	1.58		3	0.39	0.57	0.80	1.33
		[6.39]	[4.16]	[3.46]	[4.75]			[4.83]	[4.16]	[4.28]	[5.06]
	High	0.99	0.94	0.97	1.65		Big	0.66	0.82	1.04	1.32
		[5.98]	[3.77]	[3.24]	[5.68]			[5.71]	[5.64]	[5.25]	[5.79]

This table illustrates the average returns of the test portfolios. The left panel presents the average returns of 16 equally-weighted momentum and price impact portfolios. The right panel depicts 16 value-weighted size and book-to-market portfolios. Values are measured in monthly percentage terms. Standard deviations are given in square brackets. The sample period runs from 1975:01-2006:12.

In order to test if the illiquidity risk factors are priced we apply GMM as proposed by Hansen (1982) based on a stochastic discount factor framework. GMM has the advantage that it is a one-step procedure and, thus, avoids the error-in-variables problem. As a robustness check, we also consider the cross-sectional approaches applied by Fama and MacBeth (1973) and Black et al. (1972).

Any asset pricing model can be written in a stochastic discount factor form:

$$E_t[m_{t+1}r^e_{i,t+1}] = 0. \tag{2.14}$$

The focus in this study is on linear factor models, which express the pricing kernel as a linear function of factors: $m_{t+1} = a - F'_t b$, where F_t denotes a vector of risk factors. The moment restriction displayed in equation 2.14 does not separately identify the parameters a and b. We follow the normalization by Kan and Robotti (2008) and choose demeaned

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[7]Lewellen et al. (2010) criticize the common practice to evaluate models exclusively based on the size and book-to-market portfolios.

factors. The stochastic discount factor is rewritten as $m_{t+1} = \xi[1 - (F_t - \mu)'\dot{b}]$, where μ is the unconditional mean of F_t, ξ is a scalar $\xi = a - \mu'b$ and $\dot{b} = \frac{b}{a - \mu'b}$. Finally, we divide m_{t+1} by ξ. As shown by Kan and Robotti (2008) this specification is advantageous because the outcome cannot be affected by affine transformations of the factors. Further, competing models can be compared as they exhibit the same mean. This normalization is also favored by Burnside (2008) finding that demeaned risk factors improve the performance of GMM.

Finally, we are interested if risk factors are priced, though, our GMM coefficients $\dot{b}$ only tell us if factors are marginally useful in asset pricing tests, given the presence of the other included factors. As proven by Cochrane (2005), p. 106, there exists an equivalent representation between the stochastic discount factor approach and beta pricing models. Risk premias λ are related to $\dot{b}$ by the following formula[8]:

$$\hat{\lambda} = \hat{\sum}_f \hat{\dot{b}}, \tag{2.15}$$

where $\hat{\sum}_f = \frac{1}{T}\sum_{t=1}^{T}(F_t - \mu)(F_t - \mu)'$. In order to obtain estimates for $\dot{b}$, we consider the following moment condition $g_T(\dot{b})$:

$$g_T(\dot{b}) = \frac{1}{T}\sum_{t=1}^{T}[(1 - (F_t - \mu)'\dot{b})r_{i,t+1}^e]. \tag{2.16}$$

$g_T(\dot{b})$ is a $N \times 1$ vector including one moment condition for each portfolio. The GMM optimization problem is as follows:

$$\hat{\dot{b}} = \arg\min_{\dot{b}} g_T(\dot{b})'W_T^{-1}g_T(\dot{b}), \tag{2.17}$$

where W_T is a weighting matrix. Hansen and Jagannathan (1997) argue in favor of the second moment matrix $W_T = \frac{1}{T}\sum_{t=1}^{T} r_t^e r_t^{e\prime}$. r_t^e represents a $N \times 1$ vector of test portfolio excess returns. Relative to the efficient two-stage procedure proposed by Hansen (1982) using the variance-covariance matrix of moment conditions as the weighting matrix, the second moment matrix approach entails a number of advantages. Since we intend to compare different asset pricing models, it is important that the weighting matrix remains constant. The better model should be better because it improves on the pricing errors rather than just blowing up the weighting matrix. Further, the GMM objective function evaluated at the estimated parameters has an intuitively appealing interpretation as the squared distance between a candidate discount factor and the space of true discount factors.

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[8]Burnside (2008) provides a detailed derivation of $\hat{\lambda}$ and its asymptotic distribution.

However, Kan and Robotti (2008) point out that the traditional HJ distance is inappropriate while imposing a constraint on the mean of the stochastic discount factor. On this account, they suggest a modified version of the HJ distance measure using the inverse of the covariance matrix rather than the inverse of the second moment matrix of the excess returns. The modified HJ distance measure is given by:

$$\tilde{HJ} = \sqrt{g_T(\hat{b})' \left(\frac{1}{T} \sum_{t=1}^{T} \tilde{r}_t^e \tilde{r}_t^{e'} \right)^{-1} g_T(\hat{b})}, \tag{2.18}$$

where $\tilde{r}_t^e$ are the demeaned excess returns of the test portfolios. This measure can be interpreted as the squared distance between a candidate discount factor and an admissible stochastic discount factor that has unit mean. If the asset pricing model is correctly specified, both measures have the same asymptotic distribution. The statistic $T * \tilde{HJ}^2$ is asymptotically distributed as a weighted sum of $\chi_{(1)}^2$-distributed random variables. We run the simulation suggested by Jagannathan and Wang (1996) 100,000 times in order to determine the p-value for testing the null hypothesis $\tilde{HJ} = 0$. We report the modified version of the HJ distance measure and use the inverse of the covariance matrix as our weighting matrix. For the sake of completeness, we also calculate the classical J-test, which uses the estimated variance-covariance matrix of moment conditions as the weighting matrix.

2.4 Empirical Results

2.4.1 Portfolio Returns

As a first step, we analyze if the four illiquidity measures described in section 2.2 are qualitatively similar. For this reason, we calculate the cross-sectional correlation, which is provided in table 2.4. Trading quantity exhibits a relatively strong correlation to the other illiquidity measures. The correlation is negative because a higher trading quantity reflects a higher degree of liquidity, whereas a high value for the other measures indicates a low degree of liquidity. The correlation is highest to trading speed (-0.77). Between the other three measures there also exists high average cross-sectional correlation. Correlation between trading speed and trading costs is 0.69, between trading speed and price impact 0.52, and between trading costs and price impact 0.62.

Further, we have a look at the cross-sectional correlations between the four illiquidity measures and the size of the firm. We find a negative correlation between size and trading

Table 2.4: Cross-Sectional Correlation

Variable	Correlations				
	TQ	TS	TC	PI	Size
TQ	1.00	-0.77	-0.56	-0.40	0.13
TS		1.00	0.69	0.52	-0.43
TC			1.00	0.62	-0.40
PI				1.00	-0.39
Size					1.00

This table shows the average cross-sectional correlations between the four illiquidity measures and size. $Size$ is the logarithm of the market capitalization. TQ represents the logarithm of the trading quantity, TS trading speed, TC trading costs, and PI price impact. The sample period runs from 1975:01-2006:12.

speed (-0.43), between size and trading costs (-0.40), and between size and price impact (-0.39). By contrast, trading quantity is hardly correlated with firm size (0.13), which is not very surprising given that trading quantity has been constructed to be relatively uncorrelated to firm size. The results so far suggest that our illiquidity measures partly capture a size effect.

In the following, we evaluate if more illiquid or less illiquid stocks earn higher returns. We categorize stocks into five portfolios based on the degree of illiquidity and calculate the average return of each portfolio. Following Liu (2006), we use equally-weighted portfolios.[9] We rebalance portfolios every month. Table 2.5 documents the results for the four illiquidity measures described in section 2.2. Portfolio "Low" includes the least illiquid stocks or the most liquid stocks whereas portfolio "High" contains the most illiquid stocks. All measures detect that more illiquid stocks tend to earn higher returns than less illiquid stocks, even though, the difference between "High" and "Low" is insignificant. The difference is highest for the trading speed and price impact dimension, 0.35% per month, followed by trading quantity and costs, 0.27% per month. We also report the results of the MR test. The hypothesis of a flat pattern in expected returns when moving from portfolio 1 to portfolio 5 is not rejected at a 10% level for all measures except in the case of trading speed. Table 2.5 also provides information about the standard deviation of the portfolio returns and the average market capitalization of the stocks included in the portfolios. The liquid portfolios have a higher standard deviation than the illiquid portfolios. This effect is very pronounced for trading quantity and trading speed. Moreover, the results suggest that illiquid stocks tend to be stocks from small firms across all

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[9]Results for value-weighted portfolios are qualitatively similar.

measures and vice versa. To control for size, we also conduct a double sort. Each month, we sort stocks based on size into five portfolios. Within each of the five size portfolios, stocks are again sorted into five portfolios based on one of the four illiquidity measures. The returns of the five illiquidity portfolios are then averaged over the five size portfolios such that we receive five illiquidity portfolios controlling for size. However, we still find that the most illiquid portfolio includes the smallest firms. Thus, the use of a double sort cannot completely seperate between the size and illiquidity effect. Results are not reported.

Additionally, we differentiate between two subperiods. The first one runs from 1975:01 to 1991:12 and the second one from 1992:01 to 2006:12. Subperiods are characterized by two main differences. First of all, the number of observations in the cross-section is about twice as high in the second subperiod. This is primarily due to the dotcom bubble and the resultant rise of newly listed firms. The increase in the number of cross-sectional observations creates a greater diversity in the level of illiquidity in the second period and enables us to differentiate between liquid and illiquid stocks in a more precise way. Furthermore, relative to the first subperiod, the second subperiod is characterized by a distinctly higher volatility of stock returns.

Results are summarized in table 2.6. In the first subperiod liquid and illiquid stocks earn similar returns independent of the illiquidity measure. The only exception is price impact, for which we detect a monotonically increasing relation. In the second subperiod, portfolio 5 has higher average returns than portfolio 1 independent of the measure chosen, though, the difference does not significantly deviate from zero. We only find a monotonically increasing relation between expected returns and illiquidity in the case of trading speed. In comparison to the other measures, the difference between the average returns of portfolio 5 and 1 is highest, 0.81%. Using trading costs and price impact, we find that the most illiquid stocks perform best but followed by portfolio 1.

2.4.2 Regressions - Individual Stock Returns

The construction of portfolios possesses the drawback that it only creates dispersion in one or two dimensions or, using more than two dimensions, it results in some portfolios with only a few stocks and, thus, a lot of noise. On this account, we run regressions taking

Table 2.5: Portfolios Sorted by Illiquidity

Portfolio	TQ			TS		
	Mean	Std Dev	Size	Mean	Std Dev	Size
Low	0.68	7.89	21.44	0.65	8.02	21.95
2	0.71	6.77	21.17	0.70	6.45	20.79
3	0.91	5.96	20.31	0.95	5.76	19.64
4	1.01	4.69	20.06	0.96	4.84	19.30
High	0.95	3.26	19.89	1.00	3.66	18.81
High-Low	0.27			0.35		
t-value	0.87			1.07		
MR	0.28			0.10*		

Portfolio	TC			PI		
	Mean	Std Dev	Size	Mean	Std Dev	Size
Low	0.85	6.16	22.12	0.87	6.26	22.17
2	0.65	6.56	20.17	0.55	6.82	19.88
3	0.76	6.18	19.46	0.70	5.36	19.39
4	0.87	4.95	19.10	0.91	4.87	19.04
High	1.12	4.50	18.51	1.23	5.40	18.23
High-Low	0.27			0.35		
t-value	1.20			1.53		
MR	0.80			0.95		

** significant (5-percent level)

* significant (10-percent level)

We form quintile portfolios every month by sorting stocks based on one of the four illiquidity measures. TQ represents trading quantity, TS trading speed, TC trading costs, and PI price impact. All measures are calculated over the previous twelve months. Portfolio 1 is the portfolio with the most liquid stocks, whereas portfolio 5 contains the most illiquid stocks. The statistics in the columns Mean and standard deviation (Std Dev) are measured in monthly percentage terms. Size reports the average log market capitalization. Critical values are based on robust Newey and West (1987) t-statistics. In the last row, we display the p-value of the Monotonic Relation test proposed by Patton and Timmermann (2010).

Table 2.6: Portfolios Sorted by Illiquidity - Subperiods

1975:01-1991:12	TQ		TS		TC		PI	
Portfolio	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Low	1.06	5.49	1.08	5.47	0.99	5.20	1.01	6.26
2	1.00	4.95	0.96	4.96	1.00	5.03	1.00	6.82
3	1.12	4.55	1.16	4.72	1.09	4.83	0.99	5.36
4	1.11	3.90	1.05	3.81	1.13	3.88	1.08	4.87
High	1.00	3.37	1.02	3.51	1.06	3.64	1.21	5.40
High-Low	-0.06		-0.06		0.07		0.20	
t-value	-0.22		-0.23		0.26		0.74	
MR	0.52		0.50		0.24		0.05*	
1992:01-2006:12	TQ		TS		TC		PI	
Low	0.25	9.82	0.17	7.64	0.69	7.10	0.72	7.16
2	0.39	8.22	0.39	8.08	0.26	7.76	0.04	8.18
3	0.67	6.98	0.71	7.84	0.40	7.13	0.37	6.21
4	0.90	5.21	0.86	5.59	0.58	5.67	0.73	5.40
High	0.90	2.97	0.97	3.61	1.19	5.01	1.25	6.20
High-Low	0.65		0.81		0.50		0.53	
t-value	1.07		1.32		1.33		1.37	
MR	0.14		0.05**		0.94		0.99	

** significant (5-percent level)

* significant (10-percent level)

We form quintile portfolios every month by sorting stocks based on one of the four illiquidity measures. All measures are calculated over the previous twelve months. Portfolio 1 is the portfolio with the most liquid stocks, whereas portfolio 5 contains the most illiquid stocks. The statistics in the columns Mean and standard deviation (Std Dev) are measured in monthly percentage terms. Critical values are based on robust Newey and West (1987) t-statistics. In the last row, we display the p-value of the Monotonic Relation test proposed by Patton and Timmermann (2010).

multiple characteristics into account at the same time.[10] In this section, we examine the influence of illiquidity measured by trading quantity, trading speed, trading costs, and price impact on individual stock returns. We control for characteristics such as size, book-to-market, past returns as well as the market beta. For each month we run a cross-sectional regression following the approach discussed in subsection 2.3.2. Table 2.7 displays the results. We find significant coefficients for trading quantity and price impact suggesting that more illiquid stocks offer higher returns. Trading speed and trading costs are not quite significant. Our findings underpin the results in Datar et al. (1998) showing that trading quantity negatively drives stock returns in the US market. However, we cannot conclude that each dimension of illiquidity significantly drives stock returns. The negative sign on market beta is in line with the findings for the US, see Datar et al. (1998).[11] Potentially, the negative sign of the market beta could be caused by the fact that beta is measured with error which depends on the efficiency of the market proxy used, see Kandel and Stambaugh (1995), as well as the length of the measurement interval and procedure, see Handa et al. (1989) for a detailed discussion. However, results also remain stable when we use portfolio betas or change the measurement interval. Furthermore, firm characteristics such as size, book-to-market, and past returns positively influence returns. For the latter two this confirms the findings for the US. However, the fact that bigger firms earn higher returns than smaller firms is a specific characteristic of the German stock market and stresses the absence of the size effect on the German stock market.[12] The average cross-sectional R^2 is about 8%.

Table 2.8 presents the results for the two subperiods. Results substantially deviate

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[10]A detailed discussion of the advantages and disadvantages of portfolio sortings and regressions can be found in Fama and French (2008).

[11]Additionally, we also estimate Dimson betas. When we compute betas based on daily data, we have to be wary of biases induced by infrequent trading. Infrequently traded securities have a beta estimate which is biased downwards while beta estimates for frequently traded securities are upward biased. In order to avoid biases in betas, we incorporate lags and leads of the market return following the approach by Dimson (1979). A consistent estimate of beta is obtained by aggregating the slope coefficients. We also follow the approach by Fama and French (1992) and only include lags of the market return. In both cases, the negative coefficient for market beta still shows up. The other coefficients remain qualitatively unchanged.

[12]As a further robustness check, we follow Ang et al. (2009) and include the size and book-to-market betas in the regressions. The coefficients for illiquidity do not qualitatively change. Size and book-to-market betas are insignificant, only the size beta is significant in the trading speed equation. Regressing returns on a constant and illiquidity, i.e., we ignore all other characteristics, we find slightly higher t-values for the illiquidity coefficients.

Table 2.7: GLS Regressions on Firm Characteristics

		Cons	β^M	Size	BM	Ret	Illiq	$\bar{R}^2$
TQ	λ	-0.43	-0.34	0.02	0.30	0.66	-0.05	0.082
	t-stat	-1.43	-6.64**	1.42	7.36**	11.05**	-3.15**	
TS	λ	-0.34	-0.42	0.03	0.30	0.63	0.12	0.081
	t-stat	-1.01	-8.27**	1.79*	7.19**	10.63**	1.06	
TC	λ	-0.50	-0.41	0.04	0.30	0.63	0.66	0.081
	t-stat	-1.58	-8.31**	2.28**	7.30**	10.66**	1.43	
PI	λ	-0.47	-0.43	0.04	0.28	0.65	0.05	0.080
	t-stat	-1.51	-9.00**	2.45**	6.90**	10.97**	6.32**	

* significant (10-percent level)

** significant (5-percent level)

This table depicts the results of GLS regressions of returns on firm characteristics. Cons is the average cross-sectional constant. β^M is the market beta, which is estimated over the previous twelve months using daily data. *Size* is the logarithm of the market capitalization of the previous month, *BM* is the log ratio of the book-value divided by the market capitalization measured at the end of the year, *Ret* is the past return over the previous two to twelve months, and *Illiq* represents one of the four illiquidity measures estimated over the previous twelve months. *TQ* denotes trading quantity, *TS* is our trading speed measure, *TC* our trading costs measure, and *PI* our price impact measure. All coefficients (λ) are multiplied by 100. The column $\bar{R}^2$ reports the average of the cross-sectional adjusted R^2. The sample period runs from 1975:01-2006:12.

from one subperiod to the other. None of the four illiquidity measures is significant in the first subperiod, whereas we find a significant and positive relation between illiquidity and returns across all measures in the second subperiod. The size effect is absent in both subperiods. In the second subperiod, the size of the firm is even positively related to stock returns. Excluding size slightly reduces the t-value of the illiquidity measures except for trading quantity. However, all coefficients remain significant.

2.4.3 Regressions - Portfolio Approach

It is very common in empirical asset pricing to use risk factors in order to evaluate if certain variables are priced in the market. Table 2.9 summarizes the descriptive statistics of the four illiquidity factors (F_{TQ}, F_{TS}, F_{TC}, F_{PI}) and the size factor (SMB). The trading quantity factor (F_{TQ}) has a mean of 0.29%, the trading speed factor (F_{TS}) of 0.37%, the

Table 2.8: GLS Regressions on Firm Characteristics - Subperiods

		Cons	β^M	Size	BM	Ret	Illiq	$\bar{R}^2$
					1975:01-1991:12			
TQ	λ	0.26	-0.18	-0.01	0.33	0.62	-0.01	0.090
	t-stat	0.65	-3.17**	-0.78	5.76**	5.89**	-0.56	
TS	λ	0.79	-0.22	-0.03	0.32	0.61	-0.17	0.090
	t-stat	1.68*	-3.71**	-1.21	5.51**	5.82**	-1.24	
TC	λ	0.47	-0.21	-0.02	0.33	0.61	0.01	0.091
	t-stat	1.15	-3.72**	-0.78	5.82**	5.78**	0.02	
PI	λ	0.24	-0.21	-0.005	0.32	0.60	0.001	0.088
	t-stat	0.58	-3.69**	-0.21	5.66**	5.65**	-0.03	
					1992:01-2006:12			
TQ	λ	-1.47	-0.83	0.08	0.28	0.68	-0.13	0.073
	t-stat	-3.14**	-7.89**	3.18**	4.67**	9.45**	-4.82**	
TS	λ	-1.61	-1.00	0.10	0.27	0.65	0.74	0.071
	t-stat	-3.27**	-9.94**	3.99**	4.62**	8.97**	3.72**	
TC	λ	-2.04	-0.98	0.12	0.26	0.65	3.83	0.071
	t-stat	-4.03**	-10.07**	4.56**	4.47**	9.04**	3.42**	
PI	λ	-1.45	-1.06	0.10	0.24	0.69	0.058	0.072
	t-stat	-3.06**	-11.37**	4.03**	4.06**	9.51**	6.78**	

* significant (10-percent level)

** significant (5-percent level)

This table depicts the results of the GLS regressions of returns on firm characteristics. Cons is the average cross-sectional constant. β^M is the market beta, which is estimated over the previous twelve months using daily data. $Size$ is the logarithm of the market capitalization of the previous month, BM is the log ratio of the book-value divided by the market capitalization measured at the end of the year, Ret is the past return over the previous two to twelve months, and $Illiq$ represents one of the four liquidity measures estimated over the previous twelve months. TQ denotes the logarithm of trading quantity, TS trading speed, TC trading costs, and PI price impact. All coefficients (λ) are multiplied by 100. The column $\bar{R}^2$ reports the average of the cross-sectional adjusted R^2. The upper part of the table presents the results for the first subperiod running from 1975:01-1991:12, the lower part the second subperiod running from 1992:01-2006:12.

trading cost factor (F_{TC}) of 0.26%, and the price impact factor (F_{PI}) of 0.34%. Moreover, the trading quantity and speed factors share the properties of a higher standard deviation, a lower minimum, and a higher maximum than the other two illiquidity factors. By contrast, the size factor is on average negative, has a smaller standard deviation, a higher minimum, and a lower maximum than the four illiquidity factors. Next, we consider the time-series correlations between our illiquidity factors. All factors are highly correlated. The correlation ranges from 81% between the trading quantity and the price impact factor and 97% between the trading quantity and trading speed factor. The correlation to the size factor is highest for the price impact factor (58%) and the trading costs (55%). The correlation to the trading speed (35%) and trading quantity factor (27%) is distinctly lower.

Table 2.9: Descriptive Statistics of the Factors

Variable	Mean	Std Dev	Min	Max	Correlations					Correlations		
					F_{TQ}	F_{TS}	F_{TC}	F_{PI}	SMB	MKT	HML	WML
F_{TQ}	0.29	5.18	-30.19	22.32	1.00	0.97	0.85	0.81	0.27	-0.72	0.19	0.50
F_{TS}	0.37	5.28	-29.87	21.06		1.00	0.91	0.88	0.35	-0.76	0.18	0.47
F_{TC}	0.26	3.98	-14.58	15.56			1.00	0.95	0.55	-0.79	0.12	0.28
F_{PI}	0.34	4.07	-16.64	16.75				1.00	0.58	-0.75	0.19	0.26
SMB	-0.35	3.19	-12.96	10.61					1.00	-0.54	-0.02	-0.14

This table shows the summary statistics of the factors. MKT is the market excess return, HML and SMB are the size and the book-to-market factor constructed in the same way as in Fama and French (1993). WML is the momentum factor constructed analogously to Carhart (1997). Our illiquidity factors are constructed as explained in subsection 2.3.2. F_{TQ} is the trading factor, F_{TS} the trading speed factor, F_{TC} the trading costs factor, and F_{PI} the price impact factor. The first four columns show the mean, the standard deviation, the minimum as well as the maximum expressed as monthly percentages. The next five columns report the correlation among the illiquidity factors and the SMB factor. The last three columns give the correlation to the market, book-to-market, and momentum factor. The sample period runs from 1975:01-2006:12.

Additionally, we provide the correlation to other asset pricing factors as the market (MKT), book-to-market (HML) and momentum factor (WML). Studies analyzing the US market like, e.g., Liu (2006) and Pastor and Stambaugh (2003) find a strong negative correlation between market-wide illiquidity and the market return. For example, Liu (2006) finds a correlation of -79.9% for the sample period from 1984 to 2003. This is in line with our findings for Germany. The correlation of the illiquidity and the market factor varies between -72% and -79%. This finding suggests that during bear markets a "flight

to quality" takes place creating a higher premium for less liquid stocks, whereas during bull markets the liquidity premium shrinks. During the stock market crash in October 1987, the market index (CDAX) plunged by about 26% from October to November. At the same time, we recognize an average illiquidity premium of almost 19%.[13] Similarly, we find an illiquidity premium of over 12% from August to September 1998. At this time liquidity dried up because of the collapse of Long-Term Capital Management and the Russian debt crisis. On the other hand, just before the peak of the dotcom bubble, the market surged by 46% from September 1999 to February 2000. The illiquidity premium was about -23%. All factors collectively detect a negative correlation with the market factor and capture a risk premium when markets dry up during extreme events. Furthermore, all illiquidity factors are hardly correlated to the book-to-market factor. Correlation to the momentum factor is higher, in particular for the trading speed and trading quantity factor.

Entire Sample

Table 2.10 presents the results of the GMM regressions for the four illiquidity measures. We differentiate between three models for each measure: the CAPM augmented with one of the illiquidity factors, the Fama-French three-factor model augmented with one of the illiquidity factors and the Carhart four-factor model augmented with one of the illiquidity factors. Independent of the model and the choice of the illiquidity measure, we find that the illquidity premium is significantly priced.[14] The only exception is the risk premium of the trading quantity factor in the five-factor model. Furthermore, we detect a positively priced market and momentum risk premium. The book-to-market factor is insignificant for all, the size factor for most specifications. Finally, we consider the $\tilde{HJ}$ and the J-Test to test if the models are rejected. Under the null hypothesis that the model is correct, $\tilde{HJ}$ and the value for the J-Test should be equal to zero. As shown in the last two columns of table 2.10 all models are rejected.

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[13]In this context, the illiquidity premium is calculated as follows. For each time period, we calculate the average return of the four illiquidity factors. We obtain a new time-series representing the average of the four illiquidity factors. Based on the returns of this new factor, we create an index, which is 100 in January 1975. The illiquidity premium is the percentaged change of this index.

[14]We also discover a significant illiquidity premium when we consider the first principal component of the four factors.

Table 2.10: GMM Regressions of 16 Price Impact & Momentum Portfolios

1975:01-2006:12		MKT	SMB	HML	WML	F_{ILLIQ}	$\tilde{HJ}$	J-Test
TQ	b	8.89				8.57	0.45	55.38
	t-stat	3.55**				2.56**	[0.000]	[0.000]
CAPM+ILLIQ	λ	0.60				0.64		
	t-stat	2.15**				2.22**		
	b	15.30	5.44	-13.01		14.10	0.43	45.00
	t-stat	3.52**	1.75*	-1.74*		3.20**	[0.000]	[0.000]
3F+LILLIQ	λ	0.74	-0.10	-0.84		0.79		
	t-stat	2.35**	-0.44	-1.32		2.52**		
	b	10.79	9.28	-5.20	6.82	5.38	0.38	42.32
	t-stat	2.68**	3.32**	-0.72	3.40**	1.14	[0.000]	[0.000]
4F+ILLIQ	λ	0.50	0.11	-0.23	1.16	0.51		
	t-stat	1.65	0.49	-0.36	4.48**	1.57		
TS	b	11.66				11.09	0.341	44.24
	t-stat	4.39**				4.20**	[0.000]	[0.000]
CAPM+ILLIQ	λ	0.68				0.78		
	t-stat	2.42**				2.71**		
	b	19.54	2.58	-20.21		18.83	0.37	24.54
	t-stat	4.09**	0.79	-2.31**		3.86**	[0.009]	[0.017]
FF+LILLIQ	λ	0.94	-0.27	-1.40		0.95		
	t-stat	2.83**	-1.07	-1.93*		2.93**		
	b	16.37	6.35	-14.70	4.50	12.81	0.35	28.09
	t-stat	3.65**	1.85*	-1.75*	1.88*	2.38**	[0.004]	[0.003]
4F+ILLIQ	λ	0.76	-0.08	-0.96	1.06	0.80		
	t-stat	2.25**	-0.32	-1.26	3.89**	2.44**		

1975:01-2006:12		MKT	SMB	HML	WML	F_{ILLIQ}	$\tilde{H}J$	J-Test
TC	b	12.60				15.58	0.41	42.00
	t-stat	6.25**				6.45**	[0.000]	[0.000]
CAPM+ILLIQ	λ	0.68				0.49		
	t-stat	2.49**				2.33**		
	b	17.68	-9.51	-13.50		26.71	0.37	25.18
	t-stat	4.96**	-2.44**	-1.81*		4.92**	[0.004]	[0.014]
FF+LILLIQ	λ	1.06	-0.61	-0.90		0.61		
	t-stat	3.24**	-2.33**	-1.38		2.74**		
	b	17.81	-1.65	-14.20	6.23	21.21	0.29	23.92
	t-stat	5.34**	-0.38	-1.92*	3.01**	3.96**	[0.008]	[0.006]
4F+ILLIQ	λ	0.92	-0.35	-0.93	1.09	0.55		
	t-stat	2.89**	-1.28	-1.41	4.02**	2.43**		
PI	b	9.28				11.28	0.43	50.41
	t-stat	5.61**				5.88**	[0.000]	[0.000]
CAPM+ILLIQ	λ	0.58				0.45		
	t-stat	2.16**				2.10**		
	b	13.98	-11.34	-15.77		23.03	0.39	28.44
	t-stat	5.09**	-2.85**	-2.14**		5.12**	[0.000]	[0.005]
FF+LILLIQ	λ	0.99	-0.59	-0.95		0.46		
	t-stat	3.20**	-2.31**	-1.55		2.10**		
	b	15.27	-2.71	-16.67	6.80	18.46	0.31	19.70
	t-stat	2.90**	-0.62	-2.13**	3.47**	3.86**	[0.013]	[0.050]
4F+ILLIQ	λ	0.88	-0.32	-1.02	1.12	0.45		
	t-stat	2.81**	-1.19	-1.54	4.24**	2.02**		

* significant (10-percent level)

** significant (5-percent level)

This table reports GMM estimates based on the stochastic discount factor form using the inverse of the covariance matrix of the test portfolios as the weighting matrix. Portfolios are sorted by price impact and the momentum strategy. Average returns of the test portfolios are presented in table 2.3. MKT is the market excess return, SMB and HML are the size and book-to-market factors, WML is the momentum factor and F_{ILLIQ} is the illiquidity factor based on trading quantity (TQ), trading speed (TS), trading costs (TC), and price impact (PI), respectively. CAPM denotes the Capital Asset Pricing Model, 3F represents the Fama-French three-factor model and 4F the Carhart four-factor model. b represents the coefficients from the stochastic discount factor model and λ the corresponding risk premia. Risk premia are expressed as monthly percentages. The J-Test is Hansen's (1982) χ^2 test statistics on the overidentifying restrictions of the model. $\tilde{H}J$ denotes a modification of the Hansen and Jagannathan (1997) distance measure as proposed by Kan and Robotti (2008), which is defined in equation 2.18. p-values of the J-Test and the $\tilde{H}J$ distance measure are provided in square brackets.

Orthogonalized Factors

Our illiquidity factors indicate a strong negative correlation to the market factor and partly a high correlation to the size and momentum factors. Thus, it is possible that the significance of the illiquidity factors is due to information that is already captured by other factors. Liu (2006) also finds a high correlation between the illiquidity and the market factor. In contrast to Liu (2006) we take this problem into account and orthogonalize the illiquidity factors with respect to the Carhart four-factor model.[15] Table 2.11 depicts the estimation results of the four-factor model augmented with one of the orthogonalized illiquidity measures. Apart from the trading quantity factor, the illiquidity premium is significant and the t-value is even higher than before. This provides evidence that illiquidity entails a risk premium that cannot be captured by other prominent risk factors.

Subperiods

Next, we consider two subperiods. Table 2.12 shows the results of the first subperiod for the Carhart four-factor model augmented with one of the illiquidity factors. The illiquidity factors are not orthogonalized. None of the illiquidity premia is significant. The only priced factor is momentum. Neither model is rejected with respect to the $\tilde{HJ}$ measure nor with respect to the J-Test at the 5% level. Although the t-values for the risk premia are slightly higher in the second subperiod presented in table 2.13, the illiquidity premia are insignificant except for the price impact factor. In the second period, all models are rejected. Results for orthogonalized factors are not reported for the subperiods. Illiquidity risk premia are all insignificant for the first subperiod. In the second subperiod, illiquidity premia measured by price impact (t-value=3.25) and by trading costs (t-value=2.85) are priced. Trading speed (t-value=0.92) and trading quantity (t-value=0.56) premia are insignificant, though.

Different Test Portfolios

It is widely known that cross-sectional asset pricing heavily relies on the choice of test portfolios. As another robustness check, we use 16 independently sorted size and book-to-market portfolios as our test portfolios. Table 2.14 reports the results. The illiquidity

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[15]Results are similar when we orthogonalize the illiquidity factors with respect to the CAPM or the Fama-French three-factor model.

Table 2.11:   GMM Regressions of 16 Price Impact & Momentum Portfolios - Orthogonalized Illiquidity Factors

1975:01-2006:12		MKT	SMB	HML	WML	F^O_{ILLIQ}	$\tilde{H}J$	*J*-Test
TQ	*b*	7.07	8.97	-4.08	8.67	5.41	0.38	42.32
	t-stat	4.05**	3.22**	-0.63	4.88**	1.15	[0.000]	[0.000]
	λ	0.50	0.11	-0.24	1.16	0.51		
	t-stat	1.65	0.49	-0.36	4.47**	1.20		
TS	*b*	7.49	7.32	-12.09	8.93	12.82	0.35	28.06
	t-stat	3.74**	2.19**	-1.59	4.56**	2.38**	[0.003]	[0.003]
	λ	0.76	-0.08	-0.97	1.06	1.16		
	t-stat	2.25**	-0.32	-1.26	3.89**	2.41**		
TC	*b*	7.03	4.26	-11.67	8.56	21.23	0.29	23.89
	t-stat	3.59**	1.21	-1.66*	4.21**	3.97**	[0.024]	[0.006]
	λ	0.93	-0.35	-0.93	1.09	1.11		
	t-stat	2.89**	-1.28	-1.41	4.02**	3.87**		
PI	*b*	6.98	4.53	-12.71	8.84	18.48	0.31	19.68
	t-stat	3.57**	1.34	-1.77*	4.47**	3.86**	[0.013]	[0.050]
	λ	0.88	-0.32	-1.02	1.12	1.06		
	t-stat	2.82**	-1.19	-1.54	4.24**	3.62**		

* significant (10-percent level)

** significant (5-percent level)

This table reports GMM estimates based on the stochastic discount factor form using the inverse of the covariance matrix of the test portfolios as the weighting matrix. Portfolios are sorted by price impact and the momentum strategy. Average returns of the test portfolios are presented in table 2.3. MKT is the market excess return, SMB and HML are the size and book-to-market factors, WML is the momentum factor, and F^O_{ILLIQ} is the illiquidity factor based on either trading quantity (TQ), trading speed (TS), trading costs (TC), or price impact (PI), respectively. All illiquidity factors are orthogonal to the MKT, SMB, HML, and WML factors. *b* represents the coefficients from the stochastic discount factor model and λ the corresponding risk premia. Risk premia are expressed as monthly percentages. The *J*-Test is Hansen's (1982) χ^2 test statistics on the overidentifying restrictions of the model. $\tilde{H}J$ denotes a modification of the Hansen and Jagannathan (1997) distance measure as proposed by Kan and Robotti (2008), which is defined in equation 2.18. p-values of the *J*-Test and the $\tilde{H}J$ distance measure are provided in square brackets.

Table 2.12: GMM Regressions of 16 Price Impact & Momentum Portfolios - Subperiod 1

1975:01-1991:12		MKT	SMB	HML	WML	F_{ILLIQ}	$\tilde{H}J$	J-Test
TQ	b	11.78	9.07	12.89	9.00	12.41	0.23	10.55
	t-stat	2.71**	2.03**	1.66*	2.77**	1.73*	[0.273]	[0.481]
	λ	0.34	0.31	0.71	0.77	0.16		
	t-stat	1.08	1.24	1.65	3.87**	0.55		
TS	b	10.73	6.61	12.05	9.33	10.32	0.24	11.02
	t-stat	2.68**	1.26	1.58	2.85**	1.73*	[0.235]	[0.441]
	λ	0.33	0.30	0.70	0.78	0.17		
	t-stat	1.05	1.22	1.67*	3.93**	0.62		
TC	b	10.29	4.45	10.45	10.22	11.40	0.22	11.23
	t-stat	3.18**	0.88	1.43	3.03**	4.24**	[0.296]	[0.424]
	λ	0.33	0.29	0.63	0.77	0.25		
	t-stat	1.05	1.20	1.53	3.88**	0.99		
PI	b	8.34	6.74	7.83	11.00	7.29	0.25	10.48
	t-stat	2.46**	1.21	0.88	3.45**	1.47	[0.234]	[0.488]
	λ	0.29	0.34	0.59	0.81	0.28		
	t-stat	0.92	1.36	1.27	4.19**	1.14		

* significant (10-percent level)

** significant (5-percent level)

This table reports the GMM estimates for the first subperiod based on the stochastic discount factor form using the covariance matrix of the test portfolios as the weighting matrix. Portfolios are sorted by price impact and momentum. Average returns of the test portfolios are presented in table 2.3. MKT is the market excess return, SMB and HML are the size and book-to-market factors as constructed by Fama-French (1993), WML is the momentum factor, and F_{ILLIQ} is the illiquidity factor based on trading quantity (TQ), trading speed (TS), trading costs (TC), and price impact (PI), respectively. b represents the coefficients from the stochastic discount factor model and λ the corresponding risk premia. Risk premia are expressed as monthly percentages. The J-Test is Hansen's (1982) χ^2 test statistics on the overidentifying restrictions of the model. $\tilde{H}J$ denotes a modification of the Hansen and Jagannathan (1997) distance measure as proposed by Kan and Robotti (2008), which is defined in equation 2.18. p-values of the J-Test and the $\tilde{H}J$ distance measure are provided in square brackets.

Table 2.13: GMM Regressions of 16 Price Impact & Momentum Portfolios - Subperiod 2

1992:01-2006:12		MKT	SMB	HML	WML	F_{ILLIQ}	$\tilde{HJ}$	J-Test
TQ	b	2.80	5.14	8.21	8.47	-3.82	0.61	38.64
	t-stat	0.72	1.49	1.48	4.09**	-1.04	[0.000]	[0.000]
	λ	0.27	-0.14	1.02	1.88	0.45		
	t-stat	0.59	-0.41	1.60	4.08**	0.83		
TS	b	6.87	5.39	2.01	6.52	1.58	0.62	42.45
	t-stat	1.77*	1.62	0.36	2.84**	0.38	[0.000]	[0.000]
	λ	0.42	-0.16	0.45	1.79	0.80		
	t-stat	0.91	-0.46	0.67	3.91**	1.49		
TC	b	12.90	-0.95	-5.42	4.96	14.72	0.58	33.29
	t-stat	3.77**	-0.22	-0.97	2.15**	2.53**	[0.000]	[0.000]
	λ	0.84	-0.54	-0.20	1.74	0.58		
	t-stat	1.75*	-1.37	-0.29	3.67**	1.60		
PI	b	11.94	-3.23	-7.89	4.77	14.71	0.57	31.19
	t-stat	4.23**	-0.71	-1.39	2.12**	3.02**	[0.000]	[0.001]
	λ	0.89	-0.60	-0.41	1.69	0.68		
	t-stat	1.84*	-1.48	-0.61	3.58**	1.80*		

* significant (10-percent level)

** significant (5-percent level)

This table reports the GMM estimates for the second subperiod based on the stochastic discount factor form using the covariance matrix of the test portfolios as the weighting matrix. Portfolios are sorted by price impact and momentum. Average returns of the test portfolios are presented in table 2.3. MKT is the market excess return, SMB and HML are the size and book-to-market factors as constructed by Fama-French (1993), WML is the momentum factor, and F_{ILLIQ} is the illiquidity factor based on trading quantity (TQ), trading speed (TS), trading costs (TC), and price impact (PI), respectively. b represents the coefficients from the stochastic discount factor model and λ the corresponding risk premia. Risk premia are expressed as monthly percentages. The J-Test is Hansen's (1982) χ^2 test statistics on the overidentifying restrictions of the model. $\tilde{HJ}$ denotes a modification of the Hansen and Jagannathan (1997) distance measure as proposed by Kan and Robotti (2008), which is defined in equation 2.18. p-values of the J-Test and the $\tilde{HJ}$ distance measure are provided in square brackets.

premium is found to be priced for none of the four measures, which stresses the limitations of the illiquidity risk factor in explaining stock returns. All models are rejected with respect to the J-Test but none with respect to the $\tilde{HJ}$ measure at the 5% level. In contrast to the 16 price impact and momentum portfolios, we find significant premia for size and book-to-market risk. The size factor is negatively priced reflecting the absence of the size effect in the German stock market. Momentum risk becomes insignificant. Results also hold for subperiods and orthogonalized illiquidity factors.

Different Methodology

In order to evaluate the robustness of our methodology, we additionally run cross-sectional regressions as proposed by Fama and MacBeth (1973) and estimate the risk premia. Results are presented for both the 16 price impact and momentum portfolios and the 16 size and book-to-market portfolios. Table 2.15 presents the results. Our findings support the results found so far. Using 16 price impact and momentum portfolios as test portfolios we detect significant risk premia for all illiquidity factors except for the trading quantity factor. Replacing these test portfolios by the 16 size and book-to-market portfolios dramatically changes the results. None of the illiquidity factors appears to be priced. Furthermore, the results hold in the same way when we apply the Fama-MacBeth approach with time-varying betas or the cross-sectional approach applied by Black et al. (1972), respectively.

2.5 Conclusion

In this chapter, we investigate the role of illiquidity risk in the cross-section of the German stock market. To tackle this research target we apply a great variety of different approaches and consider four well-established proxies for illiquidity. Each of these proxies covers a different dimension of illiquidity. We make use of various test procedures, which are widely used in the literature. As a starting point, we sort stocks into portfolios. In contrast to the existing literature, we do not only test if the most illiquid stocks outperform the most liquid ones but we also test for a monotonic relation. Furthermore, we regress individual stock returns on firm characteristics applying the procedure proposed by Litzenberger and Ramaswamy (1979). Supplementary, we conduct the GMM approach to verify if our four illiquidity factors entail significant risk premia while controlling for

Table 2.14: GMM Regressions of 16 Size & Book-to-Market Portfolios

1975:01-2006:12		MKT	SMB	HML	WML	F_{ILLIQ}	$\tilde{H}J$	J-Test
TQ	b	0.69	-0.81	8.50	4.50	-3.48	0.26	21.80
	t-stat	0.16	-0.24	3.94**	1.02	-0.71	[0.058]	[0.026]
	λ	0.61	-0.42	0.77	0.70	-0.32		
	t-stat	1.97*	-2.32**	3.94**	0.72	-0.58		
TS	b	-1.53	-2.80	8.81	-0.39	-2.86	0.26	21.75
	t-stat	-0.36	-0.84	2.18**	-0.09	-0.58	[0.059]	[0.026]
	λ	0.42	-0.33	0.73	-0.08	-0.46		
	t-stat	1.57	-1.99**	4.60**	-0.12	-0.92		
TC	b	2.29	-0.26	7.68	2.91	-1.34	0.26	23.59
	t-stat	0.53	-0.07	3.44**	0.74	-0.22	[0.061]	[0.015]
	λ	0.61	-0.40	0.74	0.59	-0.34		
	t-stat	1.99*	-2.26**	3.88**	0.63	-0.84		
PI	b	4.20	-1.46	6.95	1.75	2.85	0.26	23.48
	t-stat	1.26	-0.40	2.99**	0.45	0.61	[0.064]	[0.015]
	λ	0.61	-0.35	0.73	0.44	-0.04		
	t-stat	2.06**	-1.94*	3.85**	0.46	-0.11		

* significant (10-percent level)

** significant (5-percent level)

This table reports GMM estimates based on the stochastic discount factor form using the inverse of the covariance matrix of the test portfolios as the weighting matrix. Portfolios are sorted by size and book-to-market. Average returns of the test portfolios are presented in table 2.3. MKT is the market excess return, SMB and HML are the size and book-to-market factors, WML is the momentum factor, and F_{ILLIQ} is the illiquidity factor based on trading quantity (TQ), trading speed (TS), trading costs (TC), and price impact (PI), respectively. b represents the coefficients from the stochastic discount factor model and λ the corresponding risk premia. Risk premia are expressed as monthly percentages. The J-Test is Hansen's (1982) χ^2 test statistics on the overidentifying restrictions of the model. $\tilde{H}J$ denotes a modification of the Hansen and Jagannathan (1997) distance measure as proposed by Kan and Robotti (2008), which is defined in equation 2.18. p-values of the J-Test and the $\tilde{H}J$ distance measure are provided in square brackets.

Table 2.15: Fama-MacBeth Regressions

16 price impact and momentum portfolios								
1975:01-2006:12		Cons	MKT	SMB	HML	WML	F_{ILLIQ}	R^2
TQ	λ	-0.36	0.81	0.55	-0.43	1.13	0.51	0.35
	t-stat	-0.81	1.73*	1.80*	-0.57	4.20**	1.56	
TS	λ	-0.36	1.09	0.28	-1.18	1.03	0.69	0.35
	t-stat	-0.80	2.30**	0.88	-1.58	3.85**	2.17**	
TC	λ	-0.36	1.09	-0.24	-1.35	0.98	0.53	0.34
	t-stat	-0.19	2.18**	-0.76	-2.11**	3.77**	2.38**	
PI	λ	0.15	0.80	-0.25	-1.72	1.03	0.40	0.35
	t-stat	0.34	1.69	-0.79	-2.60**	3.95**	1.85*	
16 size and book-to-market portfolios								
TQ	λ	0.01	0.46	-0.26	0.77	0.45	-0.26	0.21
	t-stat	0.01	0.89	-1.53	4.50**	0.62	-0.39	
TS	λ	-0.02	0.53	-0.26	0.74	0.46	-0.02	0.21
	t-stat	-0.04	1.01	-1.52	4.31**	0.64	-0.03	
TC	λ	-0.04	0.58	-0.27	0.73	0.60	0.03	0.22
	t-stat	-0.09	1.11	-1.59	4.18**	0.82	0.07	
PI	λ	-0.06	0.61	-0.28	0.71	0.52	0.18	0.23
	t-stat	-0.14	1.15	-1.65	4.07**	0.73	0.44	

* significant (10-percent level)

** significant (5-percent level)

This table reports the results of the cross-sectional regressions following Fama and MacBeth (1973). Portfolios are sorted by size and book-to-market. Average returns are presented in table 2.3. MKT is the market excess return, SMB and HML are the size and book-to-market factors, WML is the momentum factor, and F_{ILLIQ} is the illiquidity factor based on trading quantity (TQ), trading speed (TS), trading costs (TC), and price impact (PI), respectively. λ denotes the risk premia. Risk premia are expressed as monthly percentages. R^2 is the cross-sectional adjusted coefficient of determination.

other potential asset pricing factors. Thereby, we take into account several robustness checks such as different test portfolios, subperiods, orthogonalized factors, and different methodologies.

Our results show that all illiquidity measures and factors are highly correlated. Sorting stocks into portfolios, our empirical results do not reflect a significant premium for illiquidity. However, after controlling for other characteristics a significantly positive relation between illiquidity and individual stock returns surfaces. This finding is robust for the second but disappears for the first subperiod. Additionally, we construct test portfolios and use risk factors to test if illiquidity entails a positive risk premium. Our results mirror a significant illiquidity premium across all measures using 16 price impact and momentum portfolios as test portfolios. Yet, the premium disappears when considering 16 size and book-to-market portfolios. This result indicates that the choice of test portfolios matters and is a crucial aspect in asset pricing.

Our findings document that the size of the firm and the illiquidity of the corresponding stock are correlated. Despite this correlation and the fact that the size effect is absent or even tends to be reversed, we find that illiquidity positively drives stock returns. Including market capitalization and illiquidity into one regression, we discover positive coefficients for both measures. In contrast to the illiquidity factor, the size factor is insignificant or even negatively priced. This also holds when the illiquidity factor is excluded. Although the two concepts are similar, our results clearly demonstrate that the two measures are no substitutes for each other.

Chapter 3

The Idiosyncratic Risk Puzzle

3.1 Introduction

According to classical finance theory there is a positive trade-off between risk and expected returns in equilibrium. Volatility of returns has been widely used as a proxy for risk.[1] These two statements imply that there should be a positive relation between volatility and expected returns. This view is also shared by most investment managers. Volatility and returns are conjoined by the hip - you simply do not get one without the other. Volatility consists of two components: systematic and idiosyncratic risk. The largest component is idiosyncratic risk, which represents over 80% of total volatility on average for a single stock. In this study, we analyze the relation between idiosyncratic risk and expected returns and find that idiosyncratic risk yields a negative risk premium on the German stock market. This effect even persists after controlling for several new robustness checks. Further, our results indicate that the total volatility-expected return relationship is reversed. Low volatility stocks outperform high volatility stocks.

Ang et al. (2006b) detect that high (idiosyncratic) volatility stocks earn low returns and vice versa. Whereas Ang et al. (2006b) is restricted to the US market, Ang et al. (2009) deliver further evidence that it is a global phenomenon. Despite the global evidence there are several papers, both theoretically and empirically, indicating that the relationship between idiosyncratic risk and returns should be the opposite way. From the theoretical perspective Malkiel and Xu (2006) as well as Jones and Rhodes-Kropf (2003) show that if investors are not able to diversify risk, then they will demand a premium for holding stocks

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[1]Volatility is a latent variable. In this context, volatility refers to the standard deviation of returns over a specified time period with the last observation on a date in the past.

with high idiosyncratic risk. Merton (1987) suggests that in an information-segmented market, firms with larger firm-specific variances require higher returns to compensate investors for holding imperfectly diversified portfolios. Some behavioral models, like Barberis and Huang (2001), show that higher idiosyncratic volatility stocks should earn higher expected returns. One exception is a recent paper by Barberis and Xiong (2010). They find the opposite constructing a model that is based on the concept of realization utility. Suppose, for instance, an investor buys a stock and then a few month later sells the stock. At the moment of selling the stock the investor receives a jolt of utility, the so-called realization utility.

One of the first empirical papers testing the relevance of idiosyncratic risk is by Fama and MacBeth (1973), which rejects its role in the CAPM. Lehmann (1990) studies the significance of residual risk in the context of statistical testing methodology. A statistically significant positive coefficient on idiosyncratic risk surfaces over the full sample period. More recent papers also find a positive relationship. Goyal and Santa-Clara (2003) document a significant positive relationship between average idiosyncratic risk and the return on the market. However, Bali et al. (2005) show that this phenomenon disappears for an extended sample period. Malkiel and Xu (2006), Spiegel and Wang (2005), and Fu (2009) provide unambiguous evidence that portfolios with higher idiosyncratic volatility earn higher average returns. In contrast to estimating idiosyncratic volatility based on daily data over the last month as done by Ang et al. (2006b), they obtain estimates for idiosyncratic risk based on monthly data. Furthermore, a recent paper by Huang et al. (2010) finds that the negative relation between idiosyncratic risk and returns is driven by monthly stock return reversals and, thus, disappears after controlling for past returns. Bali et al. (2009) show that the negative relation disappears when controlling for the maximum daily return over the previous month. Sorting stocks into portfolios based on idiosyncratic volatility, Bali and Cakici (2008) detect that the negative relation vanishes for equally-weighted portfolios. Jiang et al. (2009) show that idiosyncratic volatility is inversely related to future earnings. Bover et al. (2010) find that expected idiosyncratic skewness helps to explain the fact that high idiosyncratic risk stocks earn low returns and vice versa. Kapadia (2006) argues in favor of cross-sectional skewness as an explanation for this puzzle.

This paper includes two main novelties relative to the current literature. The first con-

tribution is the construction of an idiosyncratic risk factor, which enables us to explicitly estimate the risk premium in the cross-section and to compare it to established risk factors like size, book-to-market, and momentum.

Secondly, we conduct a multitude of new robustness checks. To test if low idiosyncratic risk stocks possess higher expected returns, it is a very common approach in the empirical asset pricing literature to sort stocks into multiple portfolios and consider the mean return differential between the top and bottom portfolios. In contrast to the existing literature, we also evaluate the existence of a monotonic relation between expected returns and idiosyncratic risk. We apply the Monotonic Relation test, which has been recently proposed by Patton and Timmermann (2010). Moreover, we differentiate between upside and downside idiosyncratic risk. Downside risk measures are well established in the recent asset pricing literature. Ang et al. (2006a) argue that the downside beta reflects the loss aversion of investors and show that it is a priced risk. Ang et al. (2001) provide evidence that downside correlation between individual stocks and the market portfolio is priced. Both studies find that stocks with more downside risk are compensated by higher returns. Investors care about downside rather than upside risk.

Our idiosyncratic risk estimates are based on static OLS regressions using the CAPM as the market model. In order to control if results are driven by the methodology, we apply dynamic approaches like GARCH and EGARCH, use Dimson betas and consider the Fama-French three-factor and the Carhart four-factor model to estimate idiosyncratic risk. Finally, we investigate if the finding that high (idiosyncratic) stocks earn low returns disappears if we use monthly data and calculate idiosyncratic risk over a 3 to 5 years horizon. For instance, Malkiel and Xu (2006) and Spiegel and Wang (2005) detect a positive relationship between returns and idiosyncratic risk in the US using monthly data. In this study, we analyze if such an effect also exists in Germany and, thus, we contrast the results from the daily analysis with the results from the monthly analysis within one study.

This study exclusively analyzes the German stock market. Considerable recent attention has been paid to data snooping problems in empirical studies. Since tests of asset pricing models have mostly been based on the same data source, i.e., the CRSP tape for US stock data, such a problem is inevitable at least conceptually. In order to obtain a different perspective, we study the German stock market. Limited attention has been paid to the German stock market in the asset pricing literature, which is partly because a comprehensive set of accounting data is not electronically available back to the 70s. Exceptions are

for instance by Wallmeier (2000) finding an impact of book-to-market equity and cash flow to price on stock returns and Ziegler et al. (2007) testing the performance of the Fama-French model in comparison to the CAPM. Furthermore, looking at the German stock market is of interest because it exhibits features that stand in contradiction to the results of other markets, in particular the US market. For instance, Breig and Elsas (2009) show that size and default risk are negatively priced.

Our empirical results provide strong evidence that low idiosyncratic risk stocks yield significantly higher returns than high idiosyncratic risk stocks creating an excess return of over 9% per annum. Idiosyncratic risk is a priced risk factor in the cross-section of the German stock market. The risk premium totals over 10% after controlling for market, size, book-to-market, and momentum risk. Potential asset pricing effects like coskewness, short-run momentum, and illiquidity cannot capture the idiosyncratic risk effect, either. Furthermore, other robustness checks fail as well. We find that a differentiation between upside and downside idiosyncratic risk is not reasonable since daily return distributions do not exhibit considerable skewness. The average cross-sectional correlation between idiosyncratic volatility and the upside and downside measure totals 94.8% and 90.5%, respectively. Thus, both measures capture essentially the same risk. Estimating idiosyncratic risk by a GARCH or an EGARCH model cannot resolve the problem, either. Both approaches produce the same qualitative results as the OLS approach. Measuring idiosyncratic risk relative to the CAPM with Dimson Betas, the Fama-French or the Carhart model still generates a premium for low idiosyncratic risk stocks.

Additionally, we find conflicting results to those from the US. Our results yield evidence that the phenomenon that low idiosyncratic risk stocks outperform high idiosyncratic stocks holds for equally-weighted portfolios and after controlling for the short-term reversal effect. When using monthly instead of daily data, we do not find that idiosyncratic risk is positively priced. Sorting portfolios with respect to the idiosyncratic risk over the last three to five years, we find negative but mostly insignificant differences between the lowest and highest idiosyncratic risk portfolios.

The remainder of this chapter is organized as follows. In the next section we describe the data and the econometric methodology in detail. Section 3 provides the empirical results. Section 4 contains further analysis. This section provides robustness checks regarding the differentiation between downside and upside idiosyncratic risk, an (E)GARCH estimation, the use of different market models, and the use of monthly data. Section 5 concludes.

3.2 Data & Methodology

3.2.1 Data

The sample period of German listed firms extends from January 1974 to December 2006 and incorporates firms listed at the Frankfurter Stock Exchange including the segments "Amtlicher Handel" and "Neuer Markt". Daily prices and trading volume are obtained from Deutsche Kapitalmarktdatenbank in Karlsruhe. Prices are adjusted for dividends and equity offerings. Yearly book equity and the number of shares outstanding are hand collected and come from Hoppenstedt Aktienführer. We only examine common stocks except the firm has exclusively listed preference stocks. Companies are only included in the monthly portfolio construction if at least 14 daily returns exist and at least seven of them are unequal to zero. Including firms with zero returns for almost all days of the month would implicitly incorporate an illiquidity effect into our measure. The reason is as follows. Let us consider an extreme case, in which a stock has only zero daily returns over the entire month. A stock that has exclusively zero returns exhibits zero idiosyncratic risk and zero total volatility. Further, Bekaert et al. (2007) argue that the number of zero returns reflect the illiquidity of a stock. In order to avoid that results are driven by an illiquidity effect, we disregard stocks with a multitude of zero returns.[2]

Table 3.1: Number of Observations

Year (y)	Average no. of Firms
$1974 \leq y < 1980$	171
$1980 \leq y < 1985$	178
$1985 \leq y < 1990$	235
$1990 \leq y < 1995$	293
$1995 \leq y < 2000$	354
$2000 \leq y \leq 2006$	576

The number of firms satisfying these requirements increases from 171 in the late 70s to 576 at the beginning of the new century. The surge of observations in the late 90s is particularly due to the dotcom bubble and the resultant rise of listed companies. Table 3.1 documents how the number of companies evolves over time.

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[2]Results are qualitatively the same without this restriction.

As the proxy for the market portfolio the CDAX from Deutsche Börse AG is used.[3] The risk-free rate is measured by the one-month money market rate reported by Frankfurt banks.[4] Since the influential work of Fama and French (1993) it has been common practice in the empirical asset pricing literature to incorporate the size (SMB) and book-to-market (HML) factor. The portfolios are constructed in the same way as in Fama and French (1993). Financial firms are excluded. The size and book-to-market factors are based on six value-weighted portfolios, which are the intersections of two portfolios formed on size and three portfolios formed on book-to-market equity. Portfolios consisting of small (big) firms are called small (big) portfolios, whereas portfolios consisting of firms with a low (high) book-to-market equity value are denoted as growth (value) portfolios. The size factor is constructed as the difference between the average return on three small portfolios and the average return on three big portfolios. The book-to-market factor is the average return on the two value portfolios minus the average return on the two growth portfolios. We also add the momentum factor (WML) as proposed by Carhart (1997). WML is based on the finding by Jegadeesh and Titman (1993) that past winners earn higher returns than past losers. We follow Carhart's (1997) way of constructing the momentum factor and sort stocks into three equally-weighted portfolios based on eleven months returns lagged one month (previous 2-12 months). Portfolio 1 contains past winners and portfolio 3 past losers. The breakpoints are the 30th and 70th percentiles. Portfolio 1 minus portfolio 3 is the WML factor. Similarly, we construct a short-term reversal (STR) factor based on the previous month return. The STR factor is included due to the evidence by Huang et al. (2010) that the negative relation between idiosyncratic risk and returns is explained by the short-term reversal effect. The significance of the short-term reversal effect has been detected by Jegadeesh (1990).

3.2.2   Methodology

Although models, such as Merton (1987), have a precise definition of idiosyncratic risk, they do not offer a way to estimate it. From the theoretical point of view it equals the return innovation's standard deviation beyond what investors expected given that period's factor realizations. These models do not even provide an empirical solution of how the

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[3]The CDAX is a value-weighted performance index and includes all German stocks listed in the General Standard and Prime Standard at the Frankfurter Stock Exchange. The correlation between the CDAX and a value-weighted index of all stocks in this sample is over 98%.

[4]Money market rate is downloaded from the homepage of the German Bundesbank.

market generates its expectations. In order to overcome this problem, this study follows Malkiel and Xu (2006) and assumes that the CAPM is the model used by the market. Other empirical studies for the US like Spiegel and Wang (2005), Ang et al. (2006b), and Ang et al. (2009) compute the idiosyncratic risk relative to the Fama-French three-factor model since it seems to be likely that market makers employ vehicles to hedge out established risk factors like the size and book-to-market risk. Because of the limited empirical evidence of relevant risk factors in Germany and the limits in data availability, this argument does not fully hold for Germany.[5] Therefore, selecting the CAPM as the market model seems to be a reasonable choice.[6] Given this, idiosyncratic risk equals the standard deviation of the residuals of the following regression:

$$r_{i,t,d} - r_{f,t,d} = \alpha_{i,t} + \beta_{i,t}^m(r_{m,t,d} - r_{f,t,d}) + \epsilon_{i,t,d}. \tag{3.1}$$

Following standard notation, $r_{i,t,d}$ is the return on stock i, $r_{f,t,d}$ is the risk-free rate, $r_{m,t,d}$ is the market return, $\beta_{i,t}^m$ is the market factor loading of stock i, and $\epsilon_{i,t,d}$ is the error term of stock i in month t at day d. For each month we estimate this equation using daily returns. Our approach follows Ang et al. (2006b). Idiosyncratic risk is defined by $\sigma_{i,t} = \sqrt{var(\epsilon_{i,t,d})}$. To include idiosyncratic risk as a risk factor we create a hedge portfolio that captures the risk of idiosyncratic volatility. Based on monthly updated estimates we rank stocks by their idiosyncratic risk and form three equally-weighted portfolios: 30 percent with the lowest idiosyncratic volatility, which is denoted as S^-, the middle 40 percent, and 30 percent with the highest idiosyncratic volatility, which is called S^+. The difference between S^+ and S^- is a proxy for idiosyncratic risk, which is denoted IR. This factor is conceptually similar to the momentum factor used in Carhart (1997).

Monotonic Relation Test

In the following three subsections, we introduce procedures to evaluate the relation between idiosyncratic risk and expected returns and whether idiosyncratic volatility is a

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[5]The Fama-French factors are neither freely downloadable as in the US nor exists a comprehensive database, which contains German accounting data back to the 60s or 70s.

[6]As a robustness check, we also apply the Fama-French and the Carhart model. Results are presented in section 3.4.4. Additionally, we consider potential estimation errors as beta estimates can be biased. Dimson (1979) discusses the problem of biased beta estimates due to infrequent trading. Results based on Dimson betas are shown in section 3.4.3.

priced risk. A simple and widely used approach in the empirical asset pricing literature is to sort stocks into idiosyncratic risk portfolios and consider the mean return differential between the top and bottom portfolios. Subsequently, one conducts a t-test to evaluate if the mean return differential is equal to zero. If a significant mean return differential emerges, it will indicate the existence of a risk premium. However, this procedure does not provide a sufficient way to test for a monotonic relation between expected returns and idiosyncratic risk as the top and bottom portfolios are exclusively taken into account. For this reason, we additionally conduct the Monotonic Relation (MR) test proposed by Patton and Timmermann (2010). This test works as follows. Let μ_i be the population value of the expected return on the ith portfolio ($i = 1, ..., N$) obtained from a ranking of stocks. Assume we would like to test the null hypothesis that there is no difference in the expected returns of the portfolios against the alternative hypothesis that the expected return is decreasing when we move from the top portfolio to the bottom portfolio. The null hypothesis (H_0) is written as

$$H_0 : \mu_1 = \mu_2 = ... = \mu_N \tag{3.2}$$

and the alternative hypothesis H_1 as

$$H_1 : \mu_1 > \mu_2 > ... > \mu_N. \tag{3.3}$$

Although the null hypothesis is written as an equality, finding $\mu_j < \mu_{j+1}$ makes rejections of the null against the alternative hypothesis less likely. To test the null hypothesis, we construct the average return differential of adjacent portfolios $\hat{\Delta}_i = \hat{\mu}_i - \hat{\mu}_{i-1}$. The test statistic for the MR test is given by:

$$J_T = \max_{i=2,...,N} \hat{\Delta}_i \tag{3.4}$$

In order to obtain critical values for the MR test, we apply the block bootstrap as proposed by Patton and Timmermann (2010).

Fama-MacBeth

The Fama-MacBeth procedure offers an approach to estimate the idiosyncratic risk premium while controlling for several other variables. It is a two step approach. In the first step, $\beta_{i,t}$, a $M \times 1$ vector of factor loadings, is estimated from the following time-series regression:

$$r_{i,\tau}^e = r_{i,\tau} - r_{f,\tau} = \delta_{i,t} + F_\tau' \beta_{i,t} + \eta_{i,\tau} \quad \tau = t - 60 .... t - 1, \tag{3.5}$$

where $r_{i,\tau}^e$ represents a vector of monthly excess returns of test portfolio i during time period τ. $\delta_{i,t}$ is a scalar, F_τ is a $M \times 1$ vector of factors, and $\eta_{i,\tau}$ is the error term of test portfolio i during time period τ. Our test portfolios are constructed as follows. We split stocks into 15 portfolios sorted by the momentum strategy and idiosyncratic risk in order to receive a wide spread in returns and betas in the portfolios.[7] A wide spread in betas and returns raises the power of the test essentially. Sorting with respect to size and market beta or size and book-to market, as usually done so in the US literature, only creates a small variation in returns and betas on the German stock market. This holds in particular for the size and market beta portfolios as both variables only cause little variation in the average returns of test portfolios. Instead, we first sort stocks into three momentum portfolios based on the past 2-12 months return. Within each momentum portfolio, we sort stocks into five portfolios with respect to their idiosyncratic risk. We sort by momentum and idiosyncratic risk because these variables create the largest spread in average return and betas across portfolios.[8] As a robustness check we also use 16 independently sorted value-weighted size and book-to-market portfolios. Our 15 momentum and idiosyncratic risk portfolios are not independently sorted as the number of stocks with low idiosyncratic risk and low past returns is relatively small. In some months the number of stocks is even zero and, hence, we cannot determine a complete time-series for the low idiosyncratic risk & past losers portfolio.

In contrast to most of the existing literature, we allow for time-varying betas. The decision to allow the sensitivities to the risk factors to change over time is made in view of the long data set used and the apparent change in portfolio betas over time that is found in the data. The relevance of time-varying betas is emphasized in several papers, e.g. Harvey (1989), Ferson and Harvey (1991, 1993) as well as Jagannathan and Wang (1996). These time-series regressions are repeated by rolling the window of 60 months of observations one month ahead. Rolling windows of five years make an appropriate compromise between adjusting to the latest changes and avoiding of noise in the monthly estimations. In the second step, we run a cross-sectional regression over N portfolios at each time t:

$$r_{i,t}^e = \lambda_{0,t} + \lambda_t' \hat{\beta}_{i,t} + v_{i,t}, \tag{3.6}$$

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[7]Lewellen et al. (2010) criticize the common practice to evaluate models exclusively based on the size and book-to-market portfolios.

[8]Cochrane (2005), p. 444, mentions the following argument. "If your portfolios have no spread in average returns - if you choose 25 random portfolios, then there will be nothing for the asset pricing model to test."

where $\lambda_{0,t}$ is a scalar, which should be zero if the model is correctly specified. λ_t is a $M \times 1$ vector of factor risk premia.[9] The factor premia λ are estimated as the averages of the cross-sectional regression estimates:

$$\bar{\hat{\lambda}} = \frac{1}{T} \sum_{t=1}^{T} \hat{\lambda}_t. \tag{3.7}$$

The covariance matrix of λ, $\sum_\lambda$, is estimated by:

$$\hat{\sum}_\lambda = \frac{1}{T^2} \sum_{t=1}^{T} (\hat{\lambda}_t - \bar{\hat{\lambda}})(\hat{\lambda}_t - \bar{\hat{\lambda}})'. \tag{3.8}$$

Since the factor loadings are estimated from a first-step regression, standard errors for the second regression can be misleading. In order to remedy the presence of this errors-in-variables problem, we could multiply the covariance matrix by an adjustment factor, the so-called Shanken correction, as proposed by Shanken (1992). Yet, the Shanken correction has to be treated critically as mentioned by Shanken and Weinstein (2006) because in practical applications it often yields a modified cross-product matrix of the estimated beta vectors that is not positive definite as it should be. For the sake of completeness we also present the adjusted t-values using the Shanken correction.[10]

GMM

In addition to the classical Fama-MacBeth approach, we conduct asset pricing tests in the GMM framework as proposed by Hansen (1982) in order to demonstrate that results do not heavily rely on the econometric methodology. The GMM framework also allows us to estimate the idiosyncratic risk premium in a multi-factor setting. In contrast to Fama-MacBeth, GMM is a one-step procedure and, thus, the error-in-variables problem does not occur. Another advantage of this method is that we do not lose observations (in our case five years) because we avoid estimating betas in the first step.

Any asset pricing model can be written in a stochastic discount factor form:

$$E_t[m_{t+1} r_{i,t+1}^e] = 0. \tag{3.9}$$

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[9]As a further robustness check, we regress individual returns on factor loadings and characteristics. In this case, we run the following cross-sectional regressions over all individual stocks for each month: $r_{j,t}^e = \lambda_{0,t} + \lambda_t^{1'} \hat{\beta}_{j,t} + \lambda_t^{2'} Z_{j,t} + \upsilon_{j,t}$, where $Z_{j,t}$ are the characteristics of firm j at month t and λ_t^2 are the respective coefficients.

[10]Although the original Shanken correction is based on time constant betas, Shanken (1992) additionally proposes a correction in the case of time-varying betas. We follow this approach.

The focus in this study is on linear factor models that express the pricing kernel as a linear function of factors: $m_{t+1} = a - F_t'b$. The moment restriction displayed in equation 3.9 does not separately identify the parameters a and b. Thus, we have to consider a normalization. The most popular choice of normalization is to set $a = 1$. As shown by Kan and Robotti (2008) this specification is problematic because the outcome can be affected by an affine transformation of the factors. Moreover, they argue that model comparison is improper if the stochastic discount factors of competing models have different means. In contrast, a version that writes the stochastic discount factor as a linear function of the demeaned factors is free from this problem. We follow this normalization. To achieve identification, we rewrite the stochastic discount factor as $m_{t+1} = \xi[1 - (F_t - \mu_F)'\dot{b}]$, where μ_F is the unconditional mean of F_t, ξ is a scalar $\xi = a - \mu_F'b$ and $\dot{b} = \frac{b}{a - \mu_F'b}$. Then, we divide by ξ. This normalization is also favored by Burnside (2008) finding that demeaned risk factors improve the performance of GMM in the context of consumption based asset pricing. Finally, we are interested if risk factors are priced, though, our GMM coefficients ($\dot{b}$) only tell us if factors are marginally useful in asset pricing tests, given the presence of the other included factors. As proven by Cochrane (2005), p. 106, there exists an equivalent representation between the stochastic discount factor approach and beta pricing models. Risk premia λ are related to $\dot{b}$ by the following formula[11]:

$$\hat{\lambda} = \hat{\sum}_f \hat{\dot{b}}, \tag{3.10}$$

where $\hat{\sum}_f = \frac{1}{T} \sum_{t=1}^{T} (F_t - \mu_F)(F_t - \mu_F)'$. In order to obtain estimates for $\dot{b}$, we consider the following moment condition $g_T(\dot{b})$:

$$g_T(\dot{b}) = \frac{1}{T} \sum_{t=1}^{T} [(1 - (F_t - \mu_F)'\dot{b})r_{i,t+1}^e]. \tag{3.11}$$

$g_T(\dot{b})$ is a $N \times 1$ vector including one moment condition for each portfolio. The GMM optimization problem is as follows:

$$\hat{\dot{b}} = \arg\min_{\dot{b}} g_T(\dot{b})'W_T^{-1}g_T(\dot{b}), \tag{3.12}$$

where W_T is a weighting matrix. Hansen and Jagannathan (1997) argue in favor of the second moment weighting matrix $W_T = \frac{1}{T} \sum_{t=1}^{T} r_t^e r_t^{e\prime}$. r_t^e represents a $N \times 1$ vector of test portfolio excess returns. This estimation procedure is different from the conventional two-stage GMM approach by Hansen (1982) who suggests to use the estimated variance-covariance matrix of moment conditions as the weighting matrix. Using Hansen's weighting matrix results in efficient estimates. Despite the theoretical advantage of this matrix,

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[11]Burnside (2008) provides a detailed derivation of $\hat{\lambda}$ and its asymptotic distribution.

the second moment matrix approach entails a number of other advantages. Since we intend to compare different asset pricing models, it is important that the weighting matrix remains constant. The better model should be better because it improves on the pricing error rather than just blowing up the weighting matrix. Secondly, the GMM objective function evaluated at the estimated parameters has an intuitively appealing interpretation as the squared distance between a candidate discount factor and the space of true discount factors.

However, Kan and Robotti (2008) point out that the traditional HJ distance is inappropriate while imposing a constraint on the mean of the stochastic discount factor. On this account, they suggest a modified version of the HJ distance measure using the inverse of the covariance matrix rather than the inverse of the second moment matrix of the excess returns. The modified HJ distance measure is given by:

$$\tilde{HJ} = \sqrt{g_T(\hat{\hat{b}})' \left(\frac{1}{T}\sum_{t=1}^{T} \tilde{r}_t^e \tilde{r}_t^{e'}\right)^{-1} g_T(\hat{\hat{b}})}, \tag{3.13}$$

where $\tilde{r}_t^e$ are the demeaned excess returns of the test portfolios. This measure can be interpreted as the squared distance between a candidate discount factor and an admissible stochastic discount factor that has unit mean. If the asset pricing model is correctly specified, both measures have the same asymptotic distribution. The statistic $T*\tilde{HJ}^2$ is asymptotically distributed as a weighted sum of $\chi^2_{(1)}$-distributed random variables. We run the simulation suggested by Jagannathan and Wang (1996) 100,000 times in order to determine the p-value for testing the null hypothesis $\tilde{HJ} = 0$. We report the modified version of the HJ distance measure and use the inverse of the covariance matrix as our weighting matrix. For the sake of completeness, we also calculate the classical J-test, which uses the estimated variance-covariance matrix of moment conditions as the weighting matrix.

3.3   Is Idiosyncratic Risk Priced?

3.3.1   Portfolio Returns

Table 3.2 reports the returns and standard deviations for quintile portfolios sorted by idiosyncratic volatility. As we intend to contrast the results of Bali and Cakici (2008), we concentrate on equally-weighted portfolios. Studying the US market Bali and Cakici (2008) find that the negative relation between returns and idiosyncratic risk disappears

for equally-weighted portfolios.[12] Table 3.2 clearly illustrates that portfolio 1 including low idiosyncratic risk stocks has higher average returns than portfolio 5 consisting of high idiosyncratic risk stocks. The average difference between the highest and lowest idiosyncratic risk portfolio is 0.79% per month, which is significant at the 5% level. That means the trading strategy of buying stocks with the lowest idiosyncratic risk and selling stocks with the highest idiosyncratic risk creates a yearly return of over 9% disregarding transaction costs. Although this result provides strong evidence that low idiosyncratic stocks outperform high idiosyncratic risk stocks, we do not find a monotonically decreasing relation between expected returns and idiosyncratic risk at a 10% level. The p-value of the MR test is 0.13. Table 3.2 also shows that the standard deviation increases from portfolio 1 to portfolio 5 which seems plausible as idiosyncratic volatility is a substantial part of the total standard deviation. Table 3.3 provides further insights about the five idiosyncratic risk portfolios. Stocks with lower idiosyncratic risk are bigger firms and firms with a lower book-to-market ratio. Moreover, they tend to have smaller return skewness. Thus, it seems to be important to control for other characteristics, which will be done in the next subsection.

Additionally, we sort portfolios based on their total volatility over the previous month in order to demonstrate the similarity to the results of the idiosyncratic volatility measure. Results are depicted in table 3.2. Low volatility stocks have high returns and high volatility stocks low returns. This finding is not very surprising given the findings in Goyal and Santa-Clara (2003). They document that idiosyncratic risk represents more than 80% of the average individual stock variance and, hence, stocks with high idiosyncratic risk usually have high volatility.

We also document the results of two subperiods depicted in table 3.4. We differentiate between two subperiods. The first one runs from 1974:02-1991:12 and the second one from 1992:01-2006:12. The reason for considering these subperiods is twofold. Firstly, the period from 1992 to 2006 is characterized by a distinctly higher volatility in the market driven by the dotcom bubble and the subsequent slump in stock prices. Amongst others this fact is mirrored by higher standard deviations for the (idiosyncratic) volatility portfolios in the second subperiod. Secondly, the average number of stocks in the first subperiod is 207, whereas in the subperiod after 1992 it is 446. The rise in the number of firms is mainly due to the euphoria about the New Economy and the resultant hot

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[12]Using value-weighted portfolios, our estimation results are very similar throughout.

Table 3.2: Portfolios Sorted by (Idiosyncratic) Volatility

	Idiosyncratic Volatility		Total Volatility	
Portfolio	Mean	Std Dev	Mean	Std Dev
Low	1.06	4.41	1.02	4.15
2	0.94	5.18	1.01	5.23
3	0.94	6.67	0.89	6.56
4	0.55	7.64	0.60	7.84
High	0.27	8.57	0.26	8.71
High-Low	-0.79		-0.76	
t-stat	-2.60**		-2.39**	
MR test	0.133		0.107	

** significant (5-percent level)

* significant (10-percent level)

We form quintile portfolios every month by sorting stocks based on idiosyncratic and total volatility. Idiosyncratic volatility is computed over the previous month using daily residuals relative to the CAPM. Total volatility is the standard deviation of daily returns over the previous month. Portfolio 1 (5) is the portfolio with the lowest (highest) volatility, respectively. The statistics in the columns Mean and standard deviation (Std Dev) are measured in monthly percentage terms. Critical values for the t-statistic are based on robust Newey and West (1987) standard deviations. In the last row, we display the p-value of the Monotonicity test proposed by Patton and Timmermann (2010). The sample period runs from 1974:02-2006:12.

Table 3.3: Characteristics of the Idiosyncratic Volatility Portfolios

Portfolio	Size	BM	Skewness	Kurtosis
Low	20.00	0.64	-0.46	5.18
2	19.34	0.64	-0.54	5.47
3	18.80	0.67	-0.30	5.11
4	18.35	0.71	0.10	6.53
High	17.84	0.76	-0.06	6.42

We form quintile portfolios every month by sorting stocks based on idiosyncratic volatility which is computed over the previous month using daily residuals relative to the CAPM. Portfolio 1 (5) is the portfolio with the lowest (highest) idiosyncratic volatility, respectively. Size reports the average log market capitalization for firms within the portfolio and BM reports the average book-to-market ratio. Skewness and Kurtosis represent the third and fourth central moments of the portfolio. The sample period runs from 1974:02-2006:12.

IPO wave. A distinctly larger cross-section makes the results more robust to outliers. The difference in returns between the lowest and highest idiosyncratic risk portfolio is more pronounced for the second subperiod. The monthly difference is -1.28%, which is an excess return of more than 15% per year. The difference is significant. We also find evidence for a monotonic relation. Contrarily, we do not reject the hypothesis of a flat pattern in expected returns when moving from portfolio 1 to 5 for the first subperiod. The first subperiod running from 1974:02 - 1991:12 produces a lower difference between the high and low idiosyncratic risk portfolio, -0.39% per month, even though it is still significant due to the lower standard deviations of the portfolio returns. The results are very similar for total volatility.

3.3.2 Controlling for Various Cross-Sectional Effects

In order to evaluate if other variables that have proved to be useful in asset pricing drive the results shown so far, we conduct robustness checks by controlling for other potential cross-sectional asset pricing effects like (short-run) momentum, size, book-to-market, coskewness, and illiquidity. Each month, we first sort stocks into five portfolios based on a characteristic (momentum, coskewness, illiquidity, size, and book-to-market) and then, within each quintile we sort stocks into five portfolios based on idiosyncratic risk. We receive 25 portfolios. For each classification of idiosyncratic risk portfolios (low, 2, 3, 4, and high), we average over the five corresponding characteristic portfolios. Hence, we obtain idiosyncratic risk quintile portfolios controlling for one of the characteristics. Table 3.5 reports the results.

Controlling for Momentum

Jegadeesh and Titman (1993) provide evidence that low momentum stocks have lower returns than high momentum stocks. Furthermore, Hong et al. (2000) argue that the momentum effect is asymmetric and has a stronger negative effect on declining stocks than a positive effect on rising stocks. A potential explanation behind the idiosyncratic risk puzzle is that loser stocks are overrepresented in the high idiosyncratic risk portfolio. Momentum is measured as the past 2-12 months return. Additionally, we incorporate short-term reversal, which is based on the previous month return and, thus, it considers the same horizon as our idiosyncratic risk measure. Huang et al. (2010) find that the negative relation between idiosyncratic risk and return is driven by monthly stock

Table 3.4: Portfolios Sorted by (Idiosyncratic) Volatility - Subperiods

	Idiosyncratic Volatility			
	1974:02-1991:12		1992:01-2006:12	
Portfolio	Mean	Std Dev	Mean	Std Dev
Low	1.08	4.61	1.05	4.05
2	1.07	4.67	0.80	5.68
3	1.21	5.05	0.62	7.98
4	0.84	5.06	0.22	9.62
High	0.69	5.25	-0.23	11.18
High-Low	-0.39		-1.28	
t-stat	-2.05**		-2.04**	
MR test	0.609		0.033**	
	Total Volatility			
Low	1.04	4.27	1.00	3.90
2	1.11	4.79	0.89	5.65
3	1.17	4.98	0.55	7.82
4	0.91	5.17	0.23	9.89
High	0.66	5.35	-0.21	11.38
High-Low	-0.38		-1.21	
t-stat	-1.92*		-1.86*	
MR test	0.207		0.074*	

** significant (5-percent level)

* significant (10-percent level)

We form quintile portfolios every month by sorting stocks based on idiosyncratic volatility, the upper part of the table, and total volatility, the lower part of the table. Idiosyncratic volatility is computed over the previous month using daily residuals relative to the CAPM. Total volatility is the standard deviation of daily returns over the previous month. Portfolio 1 (5) is the portfolio with the lowest (highest) volatility, respectively. The statistics in the columns Mean and standard deviation (Std Dev) are measured in monthly percentage terms. Critical values for the t-statistic are based on robust Newey and West (1987) standard deviations. In the last row, we display the p-value of the Monotonicity test proposed by Patton and Timmermann (2010).

return reversals. The results for the two variables are summarized in the first two rows of table 3.5. In comparison to table 3.2 the difference between the high and low portfolio shrinks to -0.67% and -0.76% per month after controlling for momentum and short-term reversal, respectively. For both variables, the difference is significant. The findings clearly demonstrate that neither momentum nor short-term reversal drive the results substantially. Independent of the momentum measure, we do not reject the hypothesis of a flat relation between expected returns and idiosyncratic risk.

Controlling for Coskewness

Harvey and Siddique (2000) find that stocks with more negative coskewness have higher returns. Maybe stocks with high idiosyncratic volatility have positive coskewness, giving them low returns. If this were the case, the difference between the high and low idiosyncratic portfolio would be less significant after controlling for coskewness. Following Harvey and Siddique (2000) we define coskewness as:

$$coskew = \frac{E[\epsilon_{i,t}\epsilon_{m,t}^2]}{\sqrt{E[\epsilon_{i,t}^2]E[\epsilon_{m,t}^2]}},$$

where $\epsilon_{i,t,d} = r_{i,t,d} - r_{f,t,d} - \alpha_{i,t} - \beta_{i,t}^m(r_{m,t,d} - r_{f,t,d})$. Coskewness is estimated using daily data over the last month.

However, as table 3.5 shows, this conjecture is not confirmed. The difference between the high and low idiosyncratic portfolio is still significant and the hypothesis of a flat relation between expected returns and idiosyncratic risk is rejected in favor of a monotonically decreasing relation.

Controlling for Illiquidity

It is generally thought that more illiquid stocks have higher returns. Although we disregard the most illiquid stocks, results can still be driven by an illiquidity effect. In order to examine this conjecture, we construct an illiquidity measure, as in Amihud (2002).[13] It follows Kyle (1985)'s concept of illiquidity - the response of price to order flow. It is defined as the average ratio of the daily absolute return to the daily trading volume over the previous twelve months and, hence, it is only defined if the trading volume is nonzero

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[13]Goyenko et al. (2009) compare different price impact measures and show that the Amihud measure outperforms other price impact measures like the Gamma measure developed by Pastor and Stambaugh (2003) and the Amivest measure.

Table 3.5: Controlling for Other Variables

Controlling for	Low	2	3	4	High	High-Low	t-stat	MR test
Momentum	0.94	0.92	0.97	0.66	0.28	-0.67	-3.57**	0.194
	[4.69]	[5.08]	[5.29]	[5.80]	[6.55]			
Short-term Reversal	0.96	1.08	1.00	0.75	0.20	-0.76	-3.26**	0.504
	[4.22]	[4.86]	[5.64]	[6.47]	[7.39]			
Coskewness	1.10	1.07	0.92	0.63	0.31	-0.76	-2.66**	0.046**
	[3.81]	[4.93]	[5.69]	[6.42]	[7.74]			
Illiquidity	1.10	1.07	0.92	0.63	0.29	-0.81	-2.93**	0.061*
	[3.81]	[4.93]	[5.69]	[6.42]	[7.87]			
Book-to-market	1.11	0.99	0.80	0.56	0.20	-0.91	-4.10**	0.012**
	[4.12]	[4.89]	[5.69]	[6.27]	[6.75]			
Size	1.11	1.03	0.94	0.72	0.59	-0.53	-2.10**	0.010**
	[4.06]	[5.01]	[5.63]	[6.36]	[6.90]			

** significant (5-percent level)

* significant (10-percent level)

In the panels controlling for momentum, coskewness, illiquidity, size, and book-to-market, we perform a double sort. Each month, we first sort into five portfolios based on the first characteristic (momentum coskewness, illiquidity, size, and book-to-market) and then, within each quintile we sort stocks into five portfolios based on idiosyncratic risk. The five idiosyncratic risk portfolios are then averaged over each of the five characteristic portfolios. Hence, they represent idiosyncratic risk portfolios controlling for the characteristic. Coskewness is computed over the previous month. It is calculated in the same way as in Harvey and Siddique (2000). Momentum is measured by the past returns over the previous two to twelve months. Short-term reversal represents the return of the previous month. Illiquidity is the yearly average of the absolute return divided by the trading volume as applied by, e.g., Amihud (2002). Size represents the market capitalization and is measured at the end of the previous month. Book-to-market is the ratio between the book-value and the market capitalization at the end of December. Idiosyncratic risk is the volatility of the residuals relative to the CAPM. Values are measured in monthly percentage terms. Standard deviations are given in square brackets. Critical values for the t-statistic are based on robust Newey and West (1987) standard deviations. In the last column, we display the p-value of the Monotonic Relation test proposed by Patton and Timmermann (2010). The sample period runs from 1975:01-2006:12.

for some days. This is the case for all stocks in the sample. This ratio represents the daily price impact of order flow. However, one problem arises. Measuring trading volume in an appropriate way is not straightforward. Trading volume at the Frankfurter Stock Exchange only reflects the overall trading volume to a small extent after the inception of XETRA in November 1997. Especially, more liquid stocks are predominantly traded on XETRA. On this account, we consider not only the trading volume of the Frankfurter Stock Exchange but also the trading volume of all German Stock Exchanges (Frankfurter Stock Exchange, XETRA, and regional Stock Exchanges).[14] Findings are qualitatively the same. The presented results are based on the total trading volume. If illiquidity is able to explain the idiosyncratic volatility effect, the difference in returns between the high and low idiosyncratic volatility portfolio should be insignificant. However, the row labeled *Illiquidity* shows that the difference is -0.81% and significant after controlling for illiquidity. Moreover, we reject the hypothesis of a flat relation between expected returns and idiosyncratic risk in favor of a monotonically decreasing relation at the 10% level.

Controlling for Book-to-market

There is compelling evidence in the empirical asset pricing literature that value stocks earn higher returns than growth stocks. Therefore, we control for the book-to-market equity ratio next. Table 3.5 illustrates that the difference between low and high idiosyncratic stocks totals -0.91% per month after controlling for the book-to-market effect. We even find a monotonically decreasing relation between idiosyncratic risk and return.

Controlling for Size

One exceptional characteristic of the German stock market is the absence of a size effect. Results rather show that bigger firms tend to have larger returns than smaller firms. To some extent, this could be an explanation for the finding that low idiosyncratic risk stocks earn higher returns since low idiosyncratic stocks tend to be big firms. Indeed, table 3.5 shows that the size effect partly explains the risk premium. The difference shrinks to -0.53%. However, we still find a significant premium for idiosyncratic risk. Further, we find a monotonically decreasing relation between returns and idiosyncratic risk.

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[14]Unfortunately, trading volume is not available for the following three months: 1983:02, 1983:03, and 1983:10. We assume that the illiquidity measures behave in the same way as for the other months in these twelve months windows.

3.3.3   Regressions

Potentially, results are not driven by one specific variable but by an interplay of different asset pricing effects. In this subsection, we estimate the risk premium for idiosyncratic risk while controlling for a multiplicity of different asset pricing variables at the same time. We regress returns on our idiosyncratic factor and, additionally, on other factors such as size, book-to-market, momentum, and short-term reversal. This is done by conducting Fama-MacBeth and GMM regressions as described in subsection 3.2.2. We examine whether the idiosyncratic risk factor is priced when it is added to the CAPM, the Fama-French three-factor or the Carhart four-factor model.

Before presenting the results of the Fama-MacBeth regressions, we summarize the descriptive statistics of the risk factors applied in the analysis. Besides the idiosyncratic risk factor (IR), we include the market factor (MKT), the size factor (SMB), the book-to-market factor (HML), the momentum factor (WML), and the short-term reversal factor (STR). Table 3.6 reports the mean, the standard deviation, the minimum, the maximum, and the correlation between the six factors. The IR factor has a higher mean in absolute terms, a similar standard deviation and a higher maximum and minimum in absolute terms compared to the market excess return. The correlation between both variables is positive indicating that stocks with a high idiosyncratic risk perform better in up than in down markets relative to stocks with a low idiosyncratic risk. Moreover, we find a strong book-to-market effect since the mean of the book-to-market factor is 0.59% per month representing an annual return of 7%. The momentum effect is even stronger creating an excess return of 1.12% per month. Short-term reversal is negative showing that stocks with a bad performance in the previous month tend to outperform stocks with a good performance in the following month. The size effect is absent. Bigger firms tend to earn higher returns than smaller firms. This stands in contrast to the US evidence. Yet, it is in line with other papers analyzing the German stock market like Breig and Elsas (2009) and Schrimpf et al. (2007). The idiosyncratic risk factor is hardly correlated to the size factor, but positively correlated to the book-to-market factor. The largest correlation in absolute terms is the one to the momentum factor.

Fama-MacBeth

In a first step, we estimate risk premia applying the classical Fama-MacBeth test. The estimation period runs from 1980:01-2006:12 since the first five years are needed to es-

Table 3.6: Descriptive Statistics of the Factors

Variable	Mean	Std Dev	Min	Max	Correlations					
					MKT	SMB	HML	WML	STR	IR
MKT	0.42	4.98	-24.12	19.80	1.00	-0.54	-0.02	-0.25	-0.23	0.34
SMB	-0.35	3.19	-12.96	10.61		1.00	-0.02	-0.14	-0.10	0.09
HML	0.59	3.03	-12.24	19.23			1.00	0.19	0.11	-0.39
WML	1.12	3.83	-17.78	17.55				1.00	0.45	-0.50
STR	-0.33	4.07	-30.71	21.43					1.00	-0.29
IR	-0.72	4.97	-28.84	33.64						1.00

This table shows the summary statistics of the factors. MKT is the market excess return, SMB and HML are the size and the book-to-market factor constructed in the same way as in Fama and French (1993). WML is the momentum factor as constructed by Carhart (1997). The short-term reversal factor STR is constructed in the same way but based on the previous month return. IR is the return on the strategy of going long on stocks with the highest idiosyncratic risk and shorting stocks with the lowest idiosyncratic risk. The first four columns show the mean, the standard deviation, the minimum as well as the maximum expressed as monthly percentages. The sample period runs from 1975:01-2006:12.

timate betas. The CAPM is the first model coming under scrutiny. Table 3.7 presents the estimation results. The CAPM produces an average adjusted R^2 of 6%. The market risk premium is insignificant and the constant is significant. Next, we test a two-factor model adding the IR factor to the CAPM. The average adjusted R^2 soars to 16%, the constant is insignifiant, the IR risk premium is significant and the market risk premium is now positive albeit insignificant. The risk premium of the IR factor is -1.01% per month. The inclusion of the IR factor switches the sign and raises the t-statistic of the market factor (MKT). Next, we examine the Fama-French three-factor model. It produces only a slightly higher $\bar{R}^2$ than the two-factor model. As it has been found in other studies covering the German stock market, the size premium is negative. The book-to-market and market premia are insignificant as in all other specifications. Including the IR factor into the three-factor model, we still find a significant IR risk premium, which is -0.92% per month. The average adjusted R^2 increases to 22%. This clearly shows that the IR factor contains an addtional risk relative to the Fama-French three-factor model. Furthermore, the IR factor is also priced in the presence of other factors like the momentum (WML) and short-term reversal (STR) factor. The risk premia are -0.86% and -0.98% per month, respectively. Apart from the significant risk premium for the momentum factor, none of the other risk factors appears to be priced.

Table 3.7: Fama-MacBeth Regressions of 15 Idiosyncratic Risk & Momentum Portfolios

1980:01-2006:12	λ_0	MKT	SMB	HML	WML	STR	IR	$\bar{R}^2$
λ	0.99	-0.74						0.06
t-stat	1.80*	-1.06						
t-stat (adj.)	1.78*	-1.05						
λ	0.18	1.21					-1.01	0.16
t-stat	0.28	1.55					-3.33**	
t-stat (adj.)	0.27	1.45					-3.13**	
λ	1.15	0.07	-1.17	0.32				0.18
t-stat	2.05**	0.10	-3.25**	1.03				
t-stat (adj.)	1.89*	0.09	-3.00**	0.95				
λ	0.41	0.87	-0.79	-0.10			-0.92	0.22
t-stat	0.72	1.34	-2.10**	-0.29			-3.01**	
t-stat (adj.)	0.68	1.27	-1.99**	-0.28			-2.85**	
λ	0.20	0.96	-0.62	-0.08	1.12		-0.86	0.27
t-stat	0.37	1.58	-1.64	-0.19	4.21**		-2.76**	
t-stat (adj.)	0.35	1.47	-1.54	-0.18	3.94**		-2.59**	
λ	0.35	0.85	-0.43	-0.18		0.78	-0.98	0.26
t-stat	0.63	1.36	-1.17	-0.53		1.68*	-3.15**	
t-stat (adj.)	1.27	-1.08	-0.49	-0.18		1.56	-2.93**	

* significant (10-percent level)

** significant (5-percent level)

This table depicts the Fama-MacBeth (1973) factor premia on 15 portfolios sorted by idiosyncratic risk relative to the CAPM and the momentum strategy. MKT is the market excess return, SMB and HML are the size and book-to-market factors as constructed by Fama-French (1993), WML is the momentum factor analogous to Carhart (1997), STR a short-term reversal factor, and IR is the idiosyncratic risk factor. We compute two t-statistics for each estimate. The first one is calculated using the uncorrected Fama-MacBeth standard errors. The second one is calculated using the Shanken (1992) correction. The column $\bar{R}^2$ reports the average of the cross-sectional adjusted R^2. Risk premia are expressed as monthly percentages.

As a robustness check we examine two subperiods running from 1980:01 to 1991:01 and from 1992:01 to 2006:12, respectively. Results are depicted in table 3.8. For the first subperiod we find that idiosyncratic risk is priced for all specifications except in the Carhart four-factor model. The MKT, SMB, and HML factors are not priced in any specification, whereas WML and STR are priced in the four-factor model augmented with the idiosyncratic risk factor. In the second subperiod, as shown in table 3.9, the market risk premium is again not priced except in the two-factor model. The IR risk premium is significant exhibiting a risk premium of -1.35% per month after controlling for market risk. The $\bar{R}^2$ rises from 15% for the CAPM to 21% for the two-factor model. The three-factor model exhibits a negatively priced size premium whereas the book-to-market premium is not priced. The inclusion of the IR factor increases the average adjusted R^2 from 23% to 28%. Again, the IR factor is priced even in the presence of the size and book-to-market factors. Finally, we examine the four-factor model proposed by Carhart (1997) augmented with the IR. Still, the IR remains significant. This is also true when we consider the short-term reversal factor instead of the momentum factor. All in all, we find a negative and significant risk premium for idiosyncratic risk in almost all specifcations, which demonstrates that idiosyncratic risk is negatively priced.

The results of cross-sectional asset pricing depend on the choice of test portfolios. Therefore, as a another robustness check, we employ 16 independently and value-weighted sorted size and book-to-market portfolios as test portfolios. Although the use of size and book-to-market portfolios as test portfolios is prevalent in asset pricing, it decreases the power of asset pricing tests since the variation in returns across size portfolios is rather low on the German stock market. Nonetheless, we conduct it as another robustness check. Results are presented in table 3.10. The IR factor is significant in all models providing additional support for the relevance of the IR factor. The findings for the other factors essentially diverge from the results before. Book-to-market risk is priced in all and size risk only in the three-factor model. Momentum, short-term reversal, and market risk are priced in none of the specifications.

Supplementary, we also consider individual stock returns instead of portfolio returns. Ang et al. (2008) argues in favor of using individual stocks as they discover that the use of test portfolios leads to higher standard errors of risk premia estimates. Besides, we avoid the problem of choosing inadequate test portfolios. Applying an analysis based on individual stock returns, it is common to incorporate firm characteristics instead of risk

Table 3.8: Fama-MacBeth Regressions of 15 Idiosyncratic Risk & Momentum Portfolios - Subperiod 1

1980:01-1991:12	λ_0	MKT	SMB	HML	WML	STR	IR	$\bar{R}^2$
λ	1.26	-0.71						0.02
t-stat	1.63	-1.07						
t-stat (adj.)	1.61	-1.06						
λ	1.67	-0.92					-0.60	0.11
t-stat	2.09**	-1.31					-2.66**	
t-stat (adj.)	1.99**	-1.25					-2.54**	
λ	1.06	-0.23	-0.24	-0.19				0.11
t-stat	1.30	-0.33	-0.66	-0.48				
t-stat (adj.)	1.28	-0.32	-0.65	-0.46				
λ	0.98	-0.26	0.23	0.13			-0.49	0.12
t-stat	1.22	-0.37	0.58	0.31			-2.17**	
t-stat (adj.)	1.17	-0.36	0.56	0.30			-2.09**	
λ	0.54	0.15	0.15	0.22	0.85		-0.35	0.20
t-stat	0.75	0.24	0.36	0.47	3.37**		-1.55	
t-stat (adj.)	0.70	0.23	0.34	0.44	3.17**		-1.45	
λ	0.76	0.05	0.51	0.15		1.30	-0.55	0.16
t-stat	1.04	0.08	1.31	0.34		2.48**	-2.48**	
t-stat (adj.)	0.87	0.07	1.10	0.28		2.08**	-1.99**	

* significant (10-percent level)

** significant (5-percent level)

This table depicts the Fama-MacBeth (1973) factor premia on 15 portfolios sorted by idiosyncratic risk relative to the CAPM and the momentum strategy for subperiod 1. MKT is the market excess return, SMB and HML are the size and book-to-market factors as constructed by Fama-French (1993), WML is the momentum factor constructed by Carhart (1997), STR is the short-term reversal factor, and IR is the idiosyncratic risk factor. We compute two t-statistics for each estimate. The first one is calculated using the uncorrected Fama-MacBeth standard errors. The second one is calculated using Shanken's (1992) correction. The column $\bar{R}^2$ reports the average of the cross-sectional adjusted R^2. Risk premia are expressed as monthly percentages.

Table 3.9: Fama-MacBeth Regressions of 15 Idiosyncratic Risk & Momentum Portfolios - Subperiod 2

1992:01-2006:12	λ_0	MKT	SMB	HML	WML	STR	IR	$\bar{R}^2$
λ	0.77	-0.76						0.15
t-stat	1.00	-0.66						
t-stat (adj.)	0.99	-0.66						
λ	-1.01	2.92					-1.35	0.21
t-stat	-1.07	2.28**					-2.60**	
t-stat (adj.)	-0.88	1.87*					-2.14**	
λ	1.22	0.30	-1.93	0.72				0.23
t-stat	1.59	0.30	-3.32**	1.58				
t-stat (adj.)	1.36	0.25	-2.85**	1.36				
λ	-0.04	1.78	-1.62	-0.28			-1.27	0.28
t-stat	-0.05	1.73*	-2.73**	-0.55			-2.44**	
t-stat (adj.)	-0.04	1.49	-2.35**	-0.47			-2.10**	
λ	-0.06	1.60	-1.24	-0.31	1.34		-1.27	0.32
t-stat	-0.08	1.66	-2.07**	-0.49	3.07**		-2.40**	
t-stat (adj.)	0.07	1.46	-1.81*	-0.43	2.70**		-2.10**	
λ	0.02	1.49	-1.17	-0.44		0.36	-1.33	0.33
t-stat	0.03	1.48	-2.03**	-0.86		0.50	-2.52**	
t-stat (adj.)	0.03	1.30	-1.78*	-0.76		0.44	-2.21**	

* significant (10-percent level)

** significant (5-percent level)

This table depicts the Fama-MacBeth (1973) factor premia on 15 portfolios sorted by idiosyncratic risk relative to the CAPM and the momentum strategy for subperiod 2. MKT is the market excess return, SMB and HML are the size and book-to-market factors as constructed by Fama-French (1993), WML is the momentum factor constructed by Carhart (1997), STR is the short-term reversal factor, and IR is the idiosyncratic risk factor. We compute two t-statistics for each estimate. The first one is calculated using the uncorrected Fama-MacBeth standard errors. The second one is calculated using Shanken's (1992) correction. The column $\bar{R}^2$ reports the average of the cross-sectional adjusted R^2. Risk premia are expressed as monthly percentages.

Table 3.10: Fama-MacBeth Regressions of 16 Size & Book-to-Market Portfolios

1980:01-2006:12	λ_0	MKT	SMB	HML	WML	STR	IR	$\bar{R}^2$
λ	0.03	0.38						0.16
t-stat	0.12	1.13						
t-stat (adj.)	0.12	1.13						
λ	0.28	0.36					-0.83	0.18
t-stat	0.96	1.01					-2.44**	
t-stat (adj.)	0.94	0.99					-2.39**	
λ	0.81	-0.25	-0.33	0.73				0.21
t-stat	2.51**	-0.62	-1.80*	4.09**				
t-stat (adj.)	2.42**	-0.60	-1.74*	3.95**				
λ	0.74	-0.19	-0.30	0.63			-0.79	0.23
t-stat	2.23**	-0.47	-1.60	3.51**			-1.91*	
t-stat (adj.)	2.17**	-0.46	-1.56	3.41**			-1.86*	
λ	0.99	-0.42	-0.31	0.66	0.34		-1.15	0.23
t-stat	2.70**	-0.94	-1.64	3.54**	0.79		-2.59**	
t-stat (adj.)	2.60**	-0.91	-1.57	3.40**	0.76		-2.49**	
λ	0.86	-0.22	-0.27	0.59		0.81	-0.95	0.24
t-stat	2.37**	-0.51	-1.46	3.23**		1.59	-2.06**	
t-stat (adj.)	2.29**	-0.50	-1.41	3.12**		1.54	-2.00**	

* significant (10-percent level)

** significant (5-percent level)

This table depicts the Fama-MacBeth (1973) factor premia on 16 portfolios sorted by size and book-to-market ratio. MKT is the market excess return, SMB and HML are the size and book-to-market factors as constructed by Fama-French (1993), WML is the momentum factor proposed by Carhart (1997), STR is the short-term reversal factor, and IR is the idiosyncratic risk factor. We compute two t-statistics for each estimate. The first one is calculated using the uncorrected Fama-MacBeth standard errors. The second one is calculated using Shanken's (1992) correction. The column $\bar{R}^2$ reports the average of the cross-sectional adjusted R^2. Risk premia are expressed as monthly percentages.

Table 3.11: Fama-MacBeth Regressions for Individual Stocks

	Cons	β^M	β^{SMB}	β^{HML}	Size	BM	Ret(2-12)	Ret(1)	σ	$\bar{R}^2$
λ	2.45	-0.32			-0.07	0.30	0.011		-0.25	0.09
t-stat	3.18**	-1.90*			-1.91*	3.47**	5.00**		-4.25**	
λ	1.62	-0.38			-0.03	0.32	0.011	-0.05	-0.19	0.11
t-stat	2.20**	-2.36**			-0.92	3.69**	4.98**	-7.24**	-3.15**	
λ	1.36	-0.43	0.12	0.11	-0.02	0.32	0.012	-0.05	-0.18	0.13
t-stat	1.74*	-1.88*	0.80	0.77	-0.53	3.87**	5.79**	-7.57**	-3.02**	

* significant (10-percent level)

** significant (5-percent level)

This table depicts the Fama-MacBeth regressions of returns on factor loadings and firm characteristcs for individual stocks. Cons is the average cross-sectional constant, β^M is the market beta, β^{SMB} is the beta of the size factor, and β^{HML} is the beta of the book-to-market factor. All betas are estimated over the previous twelve months using daily returns. Size is the log market capitalization of the previous month, BM is the log ratio of the book-value divided by the market capitalization measured at the end of the year, Ret(2-12) is the past return over the previous two to twelve months, Ret(1) is the previous month return. σ is the idiosyncratic volatility relative to the CAPM measured over the previous month. Returns and idiosyncratic volatility are measured in percentage terms. The column $\bar{R}^2$ reports the average of the cross-sectional adjusted R^2. The sample period runs from 1975:01-2006:12.

factors. This approach has been applied in numerous recent studies like Ang et al. (2009), Fu (2009), Jiang et al. (2009), and Huang et al. (2010). For every month, we regress individual stock returns on firm characteristics based on the Fama-MacBeth procedure. Apart from the market beta, we include firm characteristics like market capitalization, book-to-market equity, previous month return, past 2-12 months return and idiosyncratic volatility. Table 3.11 shows our findings. We find that there is a significant and negative relationship between idiosyncratic risk and returns, which is in line with our findings so far. Even when we include the previous month return, the coefficient remains negative and significant, which stands in contrast to Huang et al. (2010). Huang et al. (2010) find that the coefficient becomes insignificant after controlling for short-term reversal. Moreover, we discover a negative relation between returns and previous month return as well as a positive one to the book-to-market ratio and past returns (2-12). Finally, we also incorporate the size and book-to-market betas as for instance done so in Ang et al. (2009). The results qualitatively remain the same. The size and book-to-market betas are insignificant.[15]

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[15]Additionally, we apply the approach by Litzenberger and Ramaswamy (1979). Results are consistent.

GMM

In the second step, we estimate risk premia using GMM. In contrast to Fama-MacBeth, we do not have to estimate betas in the first step and, hence, the estimation period starts in 1975:01. This has to be kept in mind when comparing the results with those of the Fama-MacBeth test. Results are documented in table 3.12.

We first turn to the CAPM. The market risk premium is significantly positive exhibiting a risk premium of 0.83% per month. Creating a two-factor model by adding the IR factor to the CAPM, we find significant market and IR risk premia. As for the Fama-MacBeth regressions we find IR risk to be negatively priced. In the Fama-French three-factor model we find SMB negatively and MKT positively priced. The HML risk premium is insignficant. For the Fama-French model augmented with the IR risk factor, we find significant values for all factors. The IR risk premium is -0.91% (t-value=-2.83) per month resulting in a risk premium of over 10% per year after controlling for the Fama-French three-factor model. Finally, we consider the Carhart four-factor model augmented with the IR risk factor. Again, we find a significant IR risk premium even in the presence of the other four factors. Even when we replace the momentum by the short-term reversal factor, the premium remains significant. Apart from the short-term reversal factor (STR), all other factors are priced. In order to test if the models are rejected, we compute the $\tilde{HJ}$ measure and the J-Test. Under the null hypothesis that the model is correctly specified $\tilde{HJ}$ and the value for the J-Test should be equal to zero. According to both measures all models are rejected at the 5% level.

Furthermore, we examine two subperiods illustrated in table 3.13 and 3.14. For the first subperiod, we find a priced IR factor for all specifications. SMB is only significant in the three-factor model and HML is not found to be priced in any specification. WML and STR are priced. Again, all models are rejected at the 5% level except the five-factor models. When focusing on the second subperiod, we also obtain significant estimates of the IR risk premium. SMB, MKT, and WML are throughout priced whereas HML and STR are not. We do not reject the three-factor model augmented with the IR factor according to both J-Test and modified HJ distance measure at the 5% level. The same is valid when we also include STR. The Carhart model plus IR is not rejected with respect to the J-Test.

Again, we also estimate risk premia based on the 16 size and book-to-market portfolios. Table 3.15 shows the results. In contrast to the Fama-MacBeth approach, we find that

Table 3.12: GMM Regressions of 15 Idiosyncratic Risk & Momentum Portfolios

1975:01-2006:12	MKT	SMB	HML	WML	STR	IR	$\tilde{H}J$	J-Test
b	3.34						0.47	65.11
t-stat	2.66**						[0.000]	[0.000]
λ	0.83							
t-stat	2.87**							
b	5.33					-5.62	0.40	47.69
t-stat	3.69**					-4.58**	[0.000]	[0.000]
λ	0.89					-0.99		
t-stat	2.97**					-3.74**		
b	-0.20	-16.19	0.63				0.41	42.48
t-stat	-0.11	-3.46**	0.13				[0.000]	[0.000]
λ	1.34	-1.63	0.09					
t-stat	3.91**	-4.17**	0.21					
b	7.84	-4.83	-28.27			-12.74	0.34	20.38
t-stat	2.09**	-0.69	-2.68**			-3.19**	[0.019]	[0.041]
λ	1.47	-1.32	-1.83			-0.91		
t-stat	3.87**	-2.62**	-2.25**			-2.83**		
b	9.02	-0.47	-21.31	4.74		-9.49	0.30	19.32
t-stat	2.62**	-0.07	-2.03**	1.94*		-2.27**	[0.015]	[0.036]
λ	1.28	-1.03	-1.33	1.21		-0.91		
t-stat	3.32**	-2.03**	-1.57	4.41**		-2.94**		
b	7.12	-6.68	-27.40		2.34	-12.84	0.34	20.68
t-stat	1.53	-0.73	-2.66**		0.48	-3.19**	[0.014]	[0.024]
λ	1.47	-1.39	-1.81		0.25	-0.91		
t-stat	3.84**	-2.38**	-2.25**		0.38	-2.83**		

* significant (10-percent level)

** significant (5-percent level)

This table reports GMM estimates based on the stochastic discount factor form using the inverse of the covariance matrix of the test portfolios as the weighting matrix. Portfolios are sorted by idiosyncratic risk relative to the CAPM and the momentum strategy. MKT is the market excess return, SMB and HML are the size and book-to-market factors, WML is the momentum factor, STR a short-term reversal factor and IR is the idiosyncratic risk factor. b represents the coefficients from the stochastic discount factor model and λ the corresponding risk premia. Risk premia are expressed as monthly percentages. The J-Test is Hansen's (1982) χ^2 test statistics on the overidentifying restrictions of the model. $\tilde{H}J$ denotes a modification of the Hansen and Jagannathan (1997) distance measure as proposed by Kan and Robotti (2008), which is defined in equation 3.13. p-values of the J-Test and the $\tilde{H}J$ distance measure are provided in square brackets.

Table 3.13: GMM Regressions of 15 Idiosyncratic Risk & Momentum Portfolios - Subperiod 1

1975:01-1991:12	MKT	SMB	HML	WML	STR	IR	$\tilde{H}J$	J-Test
b	3.83						0.42	32.48
t-stat	2.00**						[0.000]	[0.003]
λ	0.68							
t-stat	2.15**							
b	3.92					-6.87	0.40	25.88
t-stat	1.97*					-2.08**	[0.001]	[0.018]
λ	0.70					-0.38		
t-stat	2.18**					-2.06**		
b	1.43	-5.46	4.38				0.42	28.25
t-stat	0.49	-1.02	0.54				[0.000]	[0.005]
λ	0.71	-0.50	0.26					
t-stat	2.19**	-1.68*	0.61					
b	7.21	12.63	10.86			-13.66	0.38	24.21
t-stat	2.00**	1.38	1.15			-2.30**	[0.002]	[0.012]
λ	0.54	0.04	0.63			-0.41		
t-stat	1.61	0.10	1.28			-2.09**		
b	8.19	-11.12	-0.27	10.83		-8.88	0.29	16.09
t-stat	2.16**	1.27	-0.03	3.10**		-1.53	[0.058]	[0.097]
λ	0.58	-0.04	0.15	0.91		-0.35		
t-stat	1.72*	-0.12	0.28	3.98**		-1.88*		
b	16.63	15.63	-13.29		-40.83	-11.28	0.30	11.12
t-stat	2.75**	1.36	-0.94		-2.61**	-1.51	[0.163]	[0.348]
λ	0.67	0.16	-0.06		-1.98	-0.42		
t-stat	1.83*	0.32	-0.08		-2.63**	-1.92*		

* significant (10-percent level)

** significant (5-percent level)

This table reports GMM estimates based on the stochastic discount factor form using the second moment matrix as the weighting matrix for subperiod 1. Portfolios are sorted by idiosyncratic risk relative to the CAPM and the momentum strategy. MKT is the market excess return, SMB and HML are the size and book-to-market factors, WML is the momentum factor by Carhart (1997), STR is the short-term reversal factor, and IR is the idiosyncratic risk factor. b represents the coefficients from the stochastic discount factor model and λ the corresponding risk premia. Risk premia are expressed as monthly percentages. The J-Test is Hansen's (1982) χ^2 test statistics on the overidentifying restrictions of the model. $\tilde{H}J$ denotes a modification of the Hansen and Jagannathan (1997) distance measure as developed by Kan and Robotti (2008) distance measure which is defined in equation 3.13. p-values of the J-Test and the $\tilde{H}J$ distance measure are provided in square brackets.

Table 3.14: GMM Regressions of 15 Idiosyncratic Risk & Momentum Portfolios - Subperiod 2

1992:01-2006:12	MKT	SMB	HML	WML	STR	IR	$\tilde{H}J$	J-Test
b	3.72						0.61	46.73
t-stat	2.24**						[0.000]	[0.000]
λ	1.22							
t-stat	2.42**							
b	7.12					-5.73	0.52	36.71
t-stat	3.40**					-4.28**	[0.000]	[0.000]
λ	1.39					-1.51		
t-stat	2.61**					-2.83**		
b	0.96	-21.94	3.05				0.52	23.23
t-stat	0.40	-3.22**	0.65				[0.005]	[0.026]
λ	2.48	-3.01	0.46					
t-stat	3.57**	-3.84**	0.79					
b	8.74	-9.83	-21.92			-12.24	0.42	11.90
t-stat	2.25**	-1.26	-2.28**			-3.01**	[0.053]	[0.372]
λ	2.37	-2.16	-1.53			-1.35		
t-stat	3.39**	-2.44**	-1.63			-2.23**		
b	8.94	-8.29	-18.85	1.81		-10.55	0.42	12.23
t-stat	2.29**	-0.93	-1.89*	0.55		-2.24**	[0.031]	[0.304]
λ	2.24	-2.04	-1.20	1.68		-1.34		
t-stat	3.15**	-2.22**	-1.23	3.34**		-2.26**		
b	4.94	-21.22	-24.17		8.72	-13.58	0.37	9.52
t-stat	0.94	-1.69*	-2.24**		1.70*	-2.90**	[0.162]	[0.484]
λ	2.67	-2.95	-1.72		0.87	-1.35		
t-stat	3.34**	-2.43**	-1.68*		0.90	-2.10**		

* significant (10-percent level)

** significant (5-percent level)

This table reports GMM estimates based on the stochastic discount factor form using the second moment matrix as the weighting matrix for subperiod 2. Portfolios are sorted by idiosyncratic risk relative to the CAPM and the momentum strategy. MKT is the market excess return, SMB and HML are the size and book-to-market factors, WML is the momentum factor by Carhart (1997), STR is the short-term reversal factor, and IR is the idiosyncratic risk factor. b represents the coefficients from the stochastic discount factor model and λ the corresponding risk premia. Risk premia are expressed as monthly percentages. The J-Test is Hansen's (1982) χ^2 test statistics on the overidentifying restrictions of the model. $\tilde{H}J$ denotes a modification of the Hansen and Jagannathan (1997) distance measure as developed by Kan and Robotti (2008) distance measure which is defined in equation 3.13. p-values of the J-Test and the $\tilde{H}J$ distance measure are provided in square brackets.

the IR factor is insignificant except in the two-factor model. We do not reject the four and five-factor models at a 5% level according to the J-test. Based on the modified HJ distance measure all models are rejected except the five-factor model incorporating the STR factor.

All in all, our results suggest that the IR factor captures an additional risk, which cannot be captured by other prominent risk factors or characteristics.

3.4   Further Insights

3.4.1   Downside vs. Upside Idiosyncratic Volatility

The use of volatility as a risk measure has an obvious drawback. It weights both upside and downside deviations equally. Virtually all investors are tolerant to sudden upside movements, but tend to avoid corresponding downside movements. In other words, it is not investments that go up that create unease but that go down significantly. A high downside idiosyncratic risk reflects a high probability of a large loss in comparison to what the market model predicts. On this account, loss averse investors should avoid these stocks or demand a greater compensation. By contrast, stocks with a high upside idiosyncratic volatility possess a chance of a large gain relative to what the market model predicts. Bali et al. (2009) show that stocks with the highest maximum daily returns earn the lowest expected returns.[16]Possibly, our results are driven by upside idiosyncratic volatility as investors like stocks with high upside idiosyncratic volatility and, thus, require a compensation for holding low upside idiosyncratic risk stocks. In this subsection we investigate whether a differentiation between upside and downside idiosyncratic volatility resolves the puzzle.

In the following, we choose two months windows in order to have sufficient observations for the upcoming empirical analysis. In particular, this is of relevance since the differentiation between upside and downside idiosyncratic risk halves the number of observations. Statistically, a differentiation between upside and downside risk is only reasonable if the

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[16]Other fields of the Economics' literature support this finding. For example, Golec and Tamarkin (1998) provide evidence that people tend to overbet on the long-shot horse with the chance of winning large returns rather than the favorite with the greatest expected returns. Furthermore, Garrett and Sobel (1999) find that people buying lottery tickets are more concerned with the size of the top prize than the expected value of the lottery. Kumar (2009) shows that individual investors prefer stocks with lottery features.

Table 3.15: GMM Regressions of 16 Size & Book-to-Market Portfolios

1975:01-2006:12	MKT	SMB	HML	WML	STR	IR	$\tilde{H}J$	J-Test
b	1.75						0.34	40.35
t-stat	1.56						[0.000]	[0.000]
λ	0.43							
t-stat	1.63							
b	2.87					-3.36	0.33	37.84
t-stat	2.26**					-2.12**	[0.001]	[0.001]
λ	0.45					-0.62		
t-stat	1.69*					-1.74*		
b	1.21	-2.07	7.85				0.26	23.06
t-stat	0.87	-1.04	4.55**				[0.041]	[0.017]
λ	0.45	-0.33	0.72					
t-stat	1.69*	-1.97*	4.34**					
b	-1.48	-5.29	11.41			4.81	0.24	18.56
t-stat	-0.67	-1.10	3.93**			1.61	[0.028]	[0.10]
λ	0.42	-0.35	0.77			0.30		
t-stat	1.58	-2.10**	4.52**			0.68		
b	-1.36	-4.85	11.42	1.52		5.64	0.24	18.15
t-stat	-0.54	-1.46	4.52**	0.36		1.82*	[0.018]	[0.078]
λ	0.43	-0.35	0.76	0.07		0.35		
t-stat	1.60	-2.10**	4.49**	0.11		0.77		
b	-4.73	-8.89	12.82		6.67	6.05	0.23	14.84
t-stat	-1.50	-2.32**	2.75**		1.18	1.86*	[0.066]	[0.190]
λ	0.37	-0.35	0.77		0.95	0.61		
t-stat	1.38	-2.08**	4.50**		1.14	1.18		

* significant (10-percent level)

** significant (5-percent level)

This table reports GMM estimates based on the stochastic discount factor form using the second moment matrix as the weighting matrix. Portfolios are sorted by size and the book-to-market. MKT is the market excess return, SMB and HML are the size and book-to-market factors, WML is the momentum factor by Carhart (1997), STR the short-term reversal factor, and IR is the idiosyncratic risk factor. b represents the coefficients from the stochastic discount factor model and λ the corresponding risk premia. Risk premia are expressed as monthly percentages. The J-Test is Hansen's (1982) χ^2 test statistics on the overidentifying restrictions of the model. $\tilde{H}J$ denotes the modified Hansen and Jagannathan (1997) distance measure as proposed by Kan and Robotti (2008), which is defined in equation 3.13. p-values of the J-Test and the $\tilde{H}J$ distance measure are provided in square brackets.

distribution of the residuals is asymmetric and the correlation between upside and down-side idiosyncratic risk is not substantial. In the case of a symmetric distribution upside and downside idiosyncratic risk would coincide. One way of testing for symmetry is to check if the sample skewness is significantly different from zero. A classical test for skewness follows Gupta (1967). We calculate the daily residuals relative to the CAPM based on the previous two months. Afterwards, we calculate the sample skewness. The sample skewness is defined as

$$skew = \frac{\frac{1}{m}\sum_{j=1}^{m}(x_i - \bar{x})^3}{(\frac{1}{m}\sum_{j=1}^{m}(x_i - \bar{x})^2)^{\frac{3}{2}}}.$$

$x_1, ..., x_m$ are the observations. Multiplying $skew$ by $\sqrt{\frac{m\mu_2^3}{\mu_6 - 6\mu_2\mu_4 + 9\mu_2^3}}$, where μ_k denotes the kth central moment of the distribution, we obtain a statistic that is asymptotically standard normally distributed. However, this test exhibits two drawbacks. Firstly, the test is not valid for heavy-tailed distributions. Secondly, it does not take into account that there exist asymmetrical distributions for which the third sample moment is zero. On this account, we also apply a test statistic proposed by Ekström and Jammalamadaka (2007). Denoting the order statistic $x_{(1)}, ..., x_{(m)}$ and $D_i = x_{(i+1)} - x_{(i)}$ for $i = 1, ..., m-1$. The sign statistic T is given by $T = \frac{S}{\xi}$, where[17]

$$S = \frac{1}{\sqrt{m}}\sum_{i=1}^{[(m-1)/2]}(\mathbb{1}\{D_i - D_{m-i} \le 0\} - \frac{1}{2})$$

and

$$\xi = \frac{1}{16} + \frac{1}{16(m-2k+1)}\sum_{i=k}^{m-k}(1 - \log\frac{D_{i,l}}{D_{l,k}}).$$

T is asymptotically standard normally distributed.

Using the classical test, we accept the hypothesis that skewness is equal to zero at the 10% level for 88% of the stocks on average over time. The findings for the second test are almost identical. We accept the hypothesis that the distribution is symmetric for 87% of the stocks on average over time.

Supplementary, we calculate the correlation between downside and upside idiosyncratic volatility. Downside idiosyncratic risk is defined as $\sigma^- = \sqrt{E[\epsilon_{i,t,d}^2|\epsilon_{i,t,d} < 0]}$ and differs from the ordinary standard deviation insofar as the expected value is restricted to those

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[17][(m − 1)/2] denotes the greatest integer smaller than or equal to $(m − 1)/2$. Moreover, $D_{i,l} = x_{(i+l)} - x_{(i-l+1)}$ and $l = [m/2]$. Due to the sample size we choose $k = 3$ as proposed by Ekström and Jammalamadaka (2007).

residuals that are less than zero. Therefore, this measure captures the variation in negative deviations relative to the market model. Analogously, upside idiosyncratic risk is defined as $\sigma^+ = \sqrt{E[\epsilon_{i,t,d}^2 | \epsilon_{i,t,d} > 0]}$.[18]

The cross-sectional correlation between downside and upside idiosyncratic risk is 72.7% on average over time. The correlation between idiosyncratic risk and its downside and upside measure is 90.5% and 94.8%, respectively. In order to guarantee the comparability between the measures, idiosyncratic risk is calculated over the previous two months as well.[19] From this analysis we can draw the conclusion that upside and downside idiosyncratic risk do not diverge from each other substantially and, hence, a seperation of upside and downside idiosyncratic risk cannot provide an explanation of why the idiosyncratic risk puzzle exists.

3.4.2 (E)GARCH

Eventually, the puzzle shows up because of the estimation procedure. While most of the literature applies a static OLS model to estimate idiosyncratic risk, we check the robustness of our results by using a dynamic approach. In order to capture the daily variation in volatility, we implement a GARCH(1,1). Additionally, we estimate the EGARCH. The EGARCH model offers the advantage that it imposes no parameter constraints to ensure positive conditional variances. Yet, stationarity constraints are necessary. Since an EGARCH(P,Q) model is treated as an ARMA(P,Q) model for the logarithm of the conditional variance, it requires nonlinear constraints on the coefficients to ensure that the eigenvalues of the characteristic polynomial are inside the unit circle. Furthermore, EGARCH has the advantage that it does not make any assumptions about the conditional distribution, i.e., whether the distribution of $\frac{\epsilon_{i,t,d}}{\sigma_{i,t,d}}$ is Gaussian or Student's t. In contrast to the GARCH model, the EGARCH approach is designed to capture asymmetries between return and volatility by including a leverage term. The functional form of

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[18]An alternative way of measuring downside idiosyncratic risk is $\tilde{\sigma}^- = \sqrt{var(\epsilon_{i,t,d} | \epsilon_{i,t,d} < 0)}$. However, this measure is misleading. Let us consider an extreme case. All negative residuals are -5%. Then, this measure would produce a downside deviation of zero although this stock has a high downside risk. By taking the average of the squared residuals, we take this problem into account.

[19]Results from the previous section also hold when idiosyncratic risk is estimated over a two months horizon.

the EGARCH model follows Bollerslev et al. (1992):

$$
\begin{aligned}
r_{i,t,d} - r_{f,t,d} &= \alpha_{i,t} + \beta_{i,t}^m (r_{m,t,d} - r_{f,t,d}) + \epsilon_{i,t,d} \\
\epsilon_{i,t,d} &= \sigma_{i,t,d} \nu_{i,t,d} \\
\log \sigma_{i,t,d}^2 &= \theta_{0,i} + \theta_{1,i} \log \sigma_{i,t,d-1}^2 + \theta_{2,i}(|\nu_{i,t,d-1}| - E[|\nu_{i,t,d-1}|]) + \theta_{3,i} \nu_{i,t,d-1},
\end{aligned} \tag{3.14}
$$

where $\nu_{i,t,d}$ is an i.i.d. error term with zero mean and unit variance. $\theta_{0,i,t}$, $\theta_{1,i,t}$, $\theta_{2,i,t}$, and $\theta_{3,i,t}$ are coefficients. $\sigma_{i,t,d}$ is the idiosyncratic volatility in month t at day d for stock i. If $\theta_{2,i,t}$ is positive, the deviation of $|\nu_{i,t,d-1}|$ from its expected value increases the variance of $\epsilon_{i,t,d}$. The $\theta_{3,i,t}$ parameter allows this effect to be asymmetric. If $\theta_{3,i,t} = 0$, then a positive surprise will have the same impact on conditional volatility as a negative surprise. If $\theta_{3,i,t}$ is smaller than zero, then a positive surprise has a smaller impact on the conditional volatility than a negative surprise and vice versa.

Since it is not reasonable to estimate an (E)GARCH with daily observations for one month, we select a twelve months window to estimate idiosyncratic volatility. Estimating an (E)GARCH restricts beta to be constant during the estimation period. While the OLS approach conducted in the previous section updates beta every month, we only choose a twelve months horizon in order to make the procedure not to different from the OLS approach with respect to the time variability of beta.[20] For the sake of comparability to the idiosyncratic risk measure introduced in section 3.2.2, we average the daily volatility $\bar{\sigma}_{i,t} = \frac{1}{D} \sum_{d=1}^{D} \sigma_{i,t,d}$ over the last month of the twelve months window. For instance, if our estimation window runs from January to December, we calculate the average daily idiosyncratic volatility in December to create portfolios in January next year. Subsequently, we shift the window one-month ahead. Results are depicted in table 3.16.[21] They clearly document that low volatility stocks earn higher returns and vice versa independent of the estimation procedure we apply. The difference between the high and low volatility portfolios remains significant. The MR test rejects the hypothesis of a flat relation between idiosyncratic risk and return in favor of a monotonically decreasing relation.

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[20]Alternatively, we estimate betas for every month in a first step by OLS and use EGARCH to estimate the idiosyncratic volatility in a second step. Results are qualitatively the same.

[21]Stocks with less than 200 observations over the last twelve months are excluded. We ignore days with missing values and continue with the next observation. As a robustness check, we also remove all stocks for a certain period if there is at least one missing value. Although the sample becomes slightly smaller, we still find similar results.

Table 3.16: Portfolios Sorted by Idiosyncratic Volatility Measured by (E)GARCH

Portfolio	GARCH		EGARCH	
	Mean	Std Dev	Mean	Std Dev
Low	1.06	4.31	1.07	4.34
2	1.04	4.78	1.04	4.75
3	0.83	6.35	0.83	6.43
4	0.63	7.98	0.62	7.95
High	0.19	8.57	0.19	8.53
High-Low	-0.87		-0.88	
t-stat	-2.75**		-2.80**	
MR test	0.079*		0.062*	

** significant (5-percent level)

* significant (10-percent level)

We form quintile portfolios every month by sorting stocks based on idiosyncratic volatility. We model returns by the CAPM and set up GARCH(1,1) and EGARCH(1,1) models for the volatility using daily returns over the previous twelve months. Idiosyncratic volatility is the average estimated volatility over the last month. Portfolio 1 (5) is the portfolio with the lowest (highest) idiosyncratic volatility, respectively. The statistics in the columns Mean and standard deviation (Std Dev) are measured in monthly percentage terms. Critical values for the t-statistic are based on robust Newey and West (1987) standard deviations. In the last row, we display the p-value of the Monotonic Relation test proposed by Patton and Timmermann (2010). The sample period runs from 1975:01-2006:12.

3.4.3 Dimson Betas

When we compute betas based on daily data, we have to be wary of biases induced by infrequent trading. Infrequently traded securities have a beta estimate which is biased downwards while beta estimates for frequently traded securities are upward biased. In order to avoid biases in betas and, hence, in our measure for idiosyncratic risk, we incorporate lags and leads of the market return following the approach by Dimson (1979). We estimate the following equation:

$$r_{i,t,d} - r_{f,t,d} = \alpha_{i,t} + \sum_{j=-k}^{k} \beta_{i,t,j}^{m}(r_{m,t,d+j} - r_{f,t,d+j}) + \epsilon_{i,t,d}. \qquad (3.15)$$

k denotes the number of lags included in the regression and $\epsilon_{i,t,d}$ represents the error terms of the Dimson type market model. Equation 3.1 is a special case of this equation. Both equations coincide when we set $k = 0$. Table 3.17 presents the average returns and standard deviations of five portfolios sorted by idiosyncratic risk relative to the CAPM using Dimson betas. We select k equal to one. Low idiosyncratic risk stocks significantly

outperform high idiosyncratic risk stocks. Yet, we cannot reject the hypothesis of a flat relation between idiosyncratic risk and expected returns. Due to the small number of observations and the necessity to estimate four coefficients, we also use a two months window. Results are qualitatively the same. Findings also remain unchanged when we choose a two months window and set k equal to two.

Table 3.17: Portfolios Sorted by Idiosyncratic Volatility Using Dimson Betas

Portfolio	Mean	Std Dev
Low	1.06	4.42
2	0.88	5.32
3	0.95	6.57
4	0.59	7.54
High	0.29	8.60
High-Low	-0.77	
t-stat	-2.54**	
MR test	0.342	

** significant (5-percent level)

* significant (10-percent level)

We form quintile portfolios every month by sorting stocks based on idiosyncratic volatility. Idiosyncratic volatility is computed over the previous month using daily residuals relative to the CAPM using Dimson betas. Portfolio 1 (5) is the portfolio with the lowest (highest) idiosyncratic volatility, respectively. The statistics in the columns Mean and standard deviation (Std Dev) are measured in monthly percentage terms. Critical values for the t-statistic are based on robust Newey and West (1987) standard deviations. In the last row, we display the p-value of the Monotonic Relation test proposed by Patton and Timmermann (2010). The sample period runs from 1974:02-2006:12.

3.4.4 Idiosyncratic Volatility relative to the Fama-French and Carhart Model

The way of estimating idiosyncratic risk does not only rely on the methodology applied but also on the choice of the model. The analysis conducted so far is based on the assumption that the CAPM is the appropriate model. As further robustness checks, we assume that the Fama-French three-factor and the Carhart four-factor model are the correct models, respectively. We repeat the same exercise as undertaken in section 3.3.1 estimating idiosyncratic risk relative to these two models. Table 3.18 presents the results. Again, we find wide spreads in returns between the high idiosyncratic risk and the low idiosyncratic risk portfolios. The difference rises to -0.85% for the Fama-French model

and to -0.83% for the Carhart model. In both cases, the difference between the high and the low portfolio is significant. Now, we even find a monotonic relation between idiosyncratic risk and return.

Table 3.18: Portfolios Sorted by Idiosyncratic Volatility Relative to the Fama-French and Carhart Model

| | Fama-French Model | | Carhart Model | |
Portfolio	Mean	Std Dev	Mean	Std Dev
Low	1.09	4.42	1.07	4.48
2	0.95	5.25	0.99	5.25
3	0.82	6.69	0.79	6.61
4	0.68	7.50	0.69	7.48
High	0.24	8.58	0.24	8.55
High-Low	-0.85		-0.83	
t-stat	-2.83**		-2.77**	
MR test	0.008**		0.028**	

** significant (5-percent level)

* significant (10-percent level)

We form quintile portfolios every month by sorting stocks based on idiosyncratic volatility. Idiosyncratic volatility is computed over the previous month using daily residuals relative to the Fama-French three-factor and the Carhart four-factor model, respectively. Portfolio 1 (5) is the portfolio with the lowest (highest) idiosyncratic volatility, respectively. The statistics in the columns Mean and standard deviation (Std Dev) are measured in monthly percentage terms. Critical values for the t-statistic are based on robust Newey and West (1987) standard deviations. In the last row, we display the p-value of the Monotonic Relation test proposed by Patton and Timmermann (2010). The sample period runs from 1974:02-2006:12.

3.4.5 Monthly Data

So far, we find evidence that a negative relation between idiosyncratic risk and returns exists estimating idiosyncratic risk from daily data over the last month, which is in line with the results of Ang et al. (2006b) and Ang et al. (2009). However, it stands in sharp contrast to other findings in the US by Malkiel and Xu (2006), Spiegel and Wang (2005), and Fu (2009). They find a positive relation while dealing with monthly data and a wider estimation window. It appears that the use of daily and monthly data produces opposite results. In order to check if this phenomenon also holds on the German stock market, we use monthly data. We sort stocks into five portfolios based on their idiosyncratic risk relative to the CAPM measured over the previous 3, 4, and 5 years. Table 3.19 displays the results. Using a 3 year horizon to estimate idiosyncratic risk we still find

a negative difference between the high idiosyncratic risk portfolio (portfolio 5) and the low idiosyncratic risk portfolio (portfolio 1). By increasing the estimation window the risk premium between portfolio 5 and 1 shrinks and becomes insignificant. Still, high idiosyncratic risk stocks have by far the lowest returns. A monotonically decreasing relation between idiosyncratic risk and return can only be found at a 10% level for the 5 years horizon. Our results indicate that the negative relation between idiosyncratic risk and return is not only a phenomenon that holds for daily data and a one month estimation window, it even exists in the long run using monthly data. However, it is less pronounced. Our findings suggest that the data frequency is not decisive and, thus, stand in sharp contrast to those for the US.

Table 3.19: Portfolios Sorted by Idiosyncratic Volatility Based on Monthly Data

| | 3 years | | 4 years | | 5 years | |
| | 1977:01-2006:12 | | 1978:01-2006:12 | | 1979:01-2006:12 | |
Portfolio	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Low	0.99	4.03	1.02	4.05	1.07	4.08
2	1.06	4.86	1.00	4.85	0.97	4.89
3	1.01	5.48	0.98	5.10	0.99	5.16
4	0.88	6.83	1.01	6.50	0.92	5.91
High	0.55	7.53	0.66	6.95	0.76	6.83
High-Low	-0.44		-0.36		-0.31	
t-stat	-1.68*		-1.45		-1.30	
MR test	0.276		0.101		0.098*	

* significant (10-percent level)

We form quintile portfolios every month by sorting stocks based on idiosyncratic volatility. Idiosyncratic volatility is computed over the previous 3, 4, and 5 years using monthly residuals relative to the CAPM. Portfolio 1 (5) is the portfolio with the lowest (highest) idiosyncratic volatility, respectively. Critical values for the t-statistic are based on robust Newey and West (1987) standard deviations. In the last row, we display the p-value of the Monotonic Relation test proposed by Patton and Timmermann (2010).

3.5   Conclusion

The results of this study endorse the findings by Ang et al. (2006b, 2009) for the German stock market insofar as we document that low idiosyncratic risk stocks earn significantly higher returns than high idiosyncratic risk stocks. In contrast to recent US evidence, these findings even hold when considering equally-weighted portfolios and controlling for a short-term reversal effect.

The portfolio of stocks with the lowest idiosyncratic risk outperforms the portfolio of stocks with the highest idiosyncratic risk by more than 9% per annum over the sample period running from 1974 to 2006. Our findings cannot be explained by other cross-sectional asset pricing effects like momentum, coskewness, liquidity, size, and book-to-market. Applying the MR test recently proposed by Patton and Timmermann (2010), we find mixed evidence for the existence of a monotonically decreasing relation between idiosyncratic risk and return. Results depend on the period, the estimation procedure, (E)GARCH versus OLS, and the choice of the market model.

In contrast to the existing literature, we construct an idiosyncratic risk factor and explicitly estimate the risk premium in the cross-section. We find a significant risk premium of 10% per year even in the presence of the size, book-to-market, and momentum factors. Idiosyncratic risk is negatively significant in almost all specifications, not only for the Fama-MacBeth test, but also for the GMM procedure, for different test portfolios, individual returns, and subperiods. Huang et al. (2010) suggest that the negative relation between returns and idiosyncratic risk is driven by a short-term reversal effect. However, our results clearly show that short-term reversal does not resolve the puzzle in the cross-section of the German stock market.

Furthermore, we address three questions. Firstly, does the differentiation between upside and downside (idiosyncratic) volatility help to explain the puzzle? The answer is no. Upside and downside idiosyncratic volatility are highly correlated such that there exists no systematic difference between these two measures. Secondly, does a dynamic estimation procedure, the use of Dimson betas or the application of the Fama-French and Carhart model resolve the puzzle? Again, the answer is no. The finding that low idiosyncratic risk stocks earn higher returns still prevails. Thirdly, does the use of monthly data lead to different results? This question is motivated by the fact that several US studies find a positive relation between idiosyncratic risk and returns. However, we do not find any evidence that this phenomenon holds on the German stock market as well. We still find a negative relation, even though the relationship is less strong.

Therefore, we draw the conclusion that the idiosyncratic risk puzzle prevails in the German stock market. Finding a reasonable explanation is apparently an interesting avenue for future research.

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