Information and Incentives in Organizations

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Abstract

This thesis asks about the value of information for providing incentives in principal agent models with hidden action and limited liability. The classical literature deals with information in the form of signals that the principal (and the agent) receive *after* the agent's effort choice. It shows that such signals are beneficial if and only if they are informative about the agent's effort. In contrast to these papers, the first three chapters deal with situations where the principal and the agent observe the signal realization *before* the agent's effort choice. The fourth chapter considers ex post information.

Chapter 1 endogenizes the timing of the signal and asks whether the principal prefers the agent to receive an additional signal before or after the agent chooses his effort. For decision problems it is well known that ex ante information is (weakly) beneficial. I show that there is no difference between incentive and decision problems if the signal is uninformative about the agent's effort. In contrast, if the signal is informative ex ante information does strictly worse.

Chapter 2 assumes that the signal the agent observes is the output of a colleague. It identifies a positive effect that acts on the incentives of this colleague and interacts with the effects from Chapter 1. This can make ex ante information optimal even when the colleague's output is informative. I relate these findings to the organizational structure of a firm and its internal transparency.

Chapter 3 keeps the timing fixed and asks whether the principal benefits from an *additional* signal that the agent observes before his effort choice. My finding that such additional information is not always beneficial contrasts with the classical result that more (ex post) information is (weakly) better.

Chapter 4 considers ex post information and asks how the principal's desire to receive more information about the agent's effort leads to inefficient job assignments. A simple trade-off between incentive provision and job assignments explains the Peter Principle and delivers predictions consistent with empirical evidence from personnel economics.

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Introduction

This dissertation studies principal agent models with asymmetric information, and herein the subclass of models with hidden actions (Mirrlees 1999, Holmstrom 1979, Holmstrom 1982, Gjesdal 1982, Grossman and Hart 1983, Jewitt 1988). In these models one party – the agent – can undertake an action (here: provide effort) which is costly to him. Which action he chooses is unobservable to the other party – the principal. The agent's effort influences stochastically the principal's revenue.

As the agent's effort choice is unobservable, the principal designs a compensation scheme which gives incentives for the agent to provide a certain effort. Such a scheme typically conditions on the ex ante uncertain revenue – concentrating higher rewards in states where it is more likely that the agent provided a certain effort. Assuming that the agent is risk averse and the principal risk neutral this creates a trade-off between providing incentives and insuring the agent. The implication is an inefficiently low effort. With a risk neutral agent this trade-off vanishes and the principal implements the efficient effort. Introducing wealth constraints however – which imply that the agent is infinitely risk averse – restores the trade-off (Innes 1990, Kim 1997). In this thesis I consider such risk neutral and wealth constraint agents.
An important question is whether there are ways to alleviate the moral hazard problem by reducing the principal's costs for implementing a certain effort. One possibility identified in the literature is to make use of additional signals that contain information about the agent's effort: the sufficient statistic result of Holmstrom (1979) and Holmstrom (1982). Another is using more informative systems in the spirit of e.g. Blackwell (1953) or Lehmann (1988), where one has to adjust these concepts to the principal agent framework (Grossman and Hart 1983, Gjesdal 1982,

Kim 1995, Jewitt 2006).[1] While most of these papers deal with risk averse agents, I show that Holmstrom's (1982) sufficient statistic result holds in a similar manner[2] for risk neutral and wealth constraints agents – i.e., in the setting used throughout the thesis.

This sufficient statistic result, which many state as "more information is better", is the conceptual starting point of my thesis. My work examines further the circumstances in which it holds. Recent papers showed that more information is not always better when contracts are incomplete (Cremer 1995, Meyer and Vickers 1997, Dewatripont, Jewitt, and Tirole 1999, Prat 2005) or information is asymmetric (Baiman and Evans 1983, Penno 1984, Baiman and Sivaramakrishnan 1991, Amaya 2005). I consider a setting where contracts are complete and contracting is under symmetric information. What I change in the first three chapters is the timing of information: the agent may observe the signal realization before his effort choice.

There are many practical examples where the agent can do so, or where the principal can choose when if he does so. Take two agents working together in a firm. Do they share an office and discuss their results? Then the agent has a good idea about the performance of his colleague before he himself starts working on a (new) project. And this performance can be an informative signal. The principal however can influence the agents' interaction, and thus the knowledge they obtain about each other, e.g. by allocating them to different offices or organizing meetings between different departments of the firm. Hence, it seems to be an important question for the design of organizations to study the impact of such ex ante information on the optimal timing of information and on incentives.

The first three chapters deal with these topics: Chapters 1 and 2 ask whether early information revelation is beneficial. Chapter 3 keeps the timing fixed and asks whether more information – in the sense of an additional informative, ex ante

[1] There is a difference between statistical decision problems and principal agent models: in the former, the decision maker estimates some unobserved variable; in the latter, the unobserved variable is chosen by the agent and the principal wishes to implement a certain level of that variable. In equilibrium the principal knows which effort the agent has chosen.

[2] The information concept differs slightly for risk neutral and wealth constraint agents, but satisfies what Demougin and Fluet (1998) define as "mechanism sufficiency".

observable signal – is better. Chapter 4 shows that the principal's desire to extract more information about the agent can lead to an inefficient assignment of agents to jobs.

Before I describe in more detail the topics of these chapters, I should clarify the terms ex ante, ex post, and intermediate information. Ex ante information refers to cases, where the agent receives information before signing the contract and before his action choice. Intermediate information refers to cases where he receives information between these two stages, and ex post information to those where information is received after these two stages. Because I will show later that intermediate and ex ante information are equivalent in my models, I subsume under "ex ante information" all cases where signals are observable before the effort choice. The case of ex post information is equivalent to a situation where the agent never observes the signal, as long as information is verifiable.

In Chapter 1 (based on Nafziger (2007a)) I endogenize the point in time where an agent observes the realization of an additional signal: before his effort choice or after it. Compared to the ex post information case, ex ante information allows the principal to tailor the agent's effort level to the realized state, which may increase her revenues. This is in contrast to the incentive side: implementation costs are strictly higher if the agent receives ex ante information – given that the signal is informative about effort.

This result shows the distinguishing features between the optimality of ex ante information for incentive and decision problems. First, it emphasises that the relevant information concepts differ for them: for incentive problems it matters whether the signal is informative about the agent's effort; for decision problems it matters whether there are gains from tailoring effort to the state of the world. Second, and this is the contribution of the chapter, it shows that in contrast to decision problems where ex ante information can never harm (the decision maker can simply ignore it) it can harm for incentive problems. Here one cannot ignore the information as it changes the agent's incentives.

Chapter 2 (based on Nafziger and Ludwig (2007)) builds on the analysis of

Chapter 1. The signal realization is now the output of another agent ("the colleague"). This is an important extension to apply the topic of this thesis to the design of organizations. The principal can influence the point in time when the agent receives information about the colleague's output, for example through the allocation of agents to offices. While the analysis for revenues and implementation costs of the agent are the same as in Chapter 1 (where ex ante information was simply a signal), there is now a strategic effect that acts on the incentives of the colleague. This can make ex ante information optimal also from the incentive side – but only if the agent's outputs are informative about each others effort. I argue that such a situation is likely when agents work on similar tasks, as e.g. in a firm structured according to the U-form. Thus, the results of this chapter can shed some light on the consequences of the organizational structure of a firm.

So far I compared ex post with ex ante information, i.e. I asked *when* the principal wants the agent to receive information: ex ante or ex post. The next step is to derive the value of ex ante information, i.e. to ask *whether* the principal likes to acquire an additional signal about the agent's effort – given that she and the agent can observe it before the agent's effort choice. This is the aim of Chapter 3 (based on Nafziger (2007b)). Here I want to see whether Holmstrom's (1979) sufficient statistic result – which shows that an additional (ex post observable) informative signal is beneficial – carries over to the case where the signal is observable before the agent chooses his effort. As I show, observing only the agent's output in some circumstances leads to strictly lower implementation costs than observing in addition an informative signal before the agent's effort choice. That is, there are circumstances where the additional informative signal is harmful. This is in contrast to the classical sufficient statistic result.

Chapter 4 differs from the previous three: it considers ex post information, keeping the timing of the information fixed. I vary, however, the degree of informativeness. The first part of this chapter (based on Koch and Nafziger (2007)) builds on the idea that more (ex post) information reduces implementation costs. However, given that the principal has the choice between two different output systems, the more informative one need not generate higher revenues. I illustrate this with the example where a principal assigns agents with different abilities to one of two jobs. One is a high skill job and one is a low skill one. The model explains why resulting job assignments are often inefficient: if a less talented agent succeeds in a high skill job, it is very likely that he provided high effort. Stated differently, the output of a low ability agent is more informative in a high skill job than in a low skill one. This increase in information reduces implementation costs; and the reduction in implementation costs can outweigh the drop in revenues that a low skilled agent generates by being placed in a high skill job.

The second part of this chapter (based on Nafziger (2006)) also considers job assignments, but in a different setup from the rest of the thesis: the principal has better information about the agent's type than the agent himself. The aim is to keep the extrinsic motivation constant across jobs and show that differences in the intrinsic motivation can also explain inefficient job assignments. Through the job assignment rule the principal signals to the agent what she knows about his type. This in turn influences the intrinsic motivation of the agent. The principal tries to boost this intrinsic motivation, leading to an inefficient assignment.

Chapter 1

Information, Decisions and Incentives

1.1 Introduction

From principal agent theory it is well known that additional information about the agent's effort is (weakly) beneficial: the principal can use it to provide incentives to the agent at lower costs (Holmstrom 1979, Holmstrom 1982). One assumption this literature makes is that the agent observes the signal after he has chosen his effort, and thus leaves open the question about the optimal timing of information. This chapter addresses the issue whether early information revelation – in the sense that a signal is revealed before rather than after the action choice of the agent – is beneficial for the principal.

The optimal timing of information is an important issue for firms: to make optimal decisions, information has to arrive at the right time. But while it is well known for decision problems that receiving information before rather than after the action choice (weakly) increases profits, this need not be the case for incentive problems: as we will show, early information revelation can be detrimental for the provision of incentives. Thus, our model sheds some light such questions as whether or not to conduct midterm evaluations, or when or when not to pass on information to an employee – taking into account not only the decision, but also the incentive side.

For this we endogenize the timing of information in a simple limited liability moral hazard model with two output levels (high and low) and continuous effort. We then ask at what point in time the principal wants an agent (and herself) to observe the signal realization (that can also be high or low): before the agent's effort choice (ex ante information, which in our setting is equivalent to intermediate information) or thereafter (ex post information).[1]

Compared to the ex post information case, ex ante information allows the principal to tailor the agent's effort level to the signal realization. This (weakly) increases her expected revenues. If effort implementation was a simple decision problem where incentives play no role, ex ante information would also enhance expected profits, which are the difference between revenues and the costs of implementing a certain effort level. A change in the timing of information affects implementation costs, however, if effort is unobservable. Our first result is that implementation costs for the same (expected) effort are strictly higher under ex ante information than under ex post information if the signal is informative about the agent's effort (see Section 1.4 for our definition of informativeness).

To understand this, consider first the incentive scheme for the ex post information case. Suppose that the output-signal combination high-high is most informative about the agent's effort. More information about the agent's effort helps to reduce the information rents that the principal has to pay to the agent, as she has a better idea whether or not the agent worked hard. Thus, she exploits this knowledge by concentrating rewards for the agent in the most informative state.[2]

Suppose now that the agent learns the signal realization before he chooses his effort. Then he knows that after a low signal realization his output is not very informative about his effort. This forces the principal to increase the reward in this state. As a consequence, ex ante implementation costs increase: rewards are no longer concentrated in the high-high state, where it is most likely that the agent provided a certain effort. If the signal is uninformative about the agent's effort, ex

[1] If the information is verifiable this is equivalent to a situation where the agent does not observe it at all. We subsume this under ex post information

[2] This is well known from the literature. See Mookherjee (1984), Holmstrom (1979) or Holmstrom (1982) for risk averse agents or the first part of Che and Yoo (2001)'s model for an example with risk neutral agents, who are protected by limited liability.

ante information does not affect incentives and hence implementation costs.

We then analyze the combined effects of ex ante information on expected revenues from tailoring effort and on implementation costs. First, if the signal is uninformative, then providing ex ante information is never worse and sometimes strictly better than ex post information. The reason is the following: if the signal is uninformative about effort, incentives and hence implementation costs are not affected by observing the signal. Thus, the only difference between ex ante and ex post information is the possibility to tailor the agent's effort to the new information. And for such a situation it is well known from decision problems that ex ante information does strictly better if and only if it helps to make better decisions, i.e. if and only if there are gains from tailoring effort. This changes if the signal is informative about the agent's effort – the case our second result deals with. Here we show that in the absence of gains from tailoring effort, ex ante information does strictly worse than ex post information – a situation that can never occur in decision problems. This is driven by the change in incentives which the early observation of the informative signal causes. Thus, the comparison between ex ante and ex post information makes clear that the principal can reduce implementation costs by conditioning an agent's *incentive scheme* on an informative signal, but not by conditioning his implemented *effort* on it.

The chapter is structured as follows. After discussing the related literature, we introduce the model in Section 1.2. In Section 1.3 we solve the first best information scenario. In Section 1.4 we first review the relevant information concepts. Then we derive the optimal wage schemes for the two informational scenarios – ex post versus ex ante information – and show the driving forces behind the differences in implementation costs. In Section 1.5 we derive conditions under which one of the two information scenarios yields higher overall profits. Section 1.6 concludes. All proofs are in the appendix.

Related Literature

The starting point for this chapter is the literature that asks about the value of (ex post) information in principal agent moral hazard models (Holmstrom 1979, Holmstrom 1982, Gjesdal 1982, Grossman and Hart 1983, Kim 1995, Jewitt 2006).[3] Our contribution is to endogenize the point in time at which an agent should receive information – comparing the value of ex ante with that of ex post information. Concerning the consideration of ex ante information, our approach is related to Chapter 3. There, however, I keep the timing fixed at the ex ante stage (i.e. information always reveals early) and ask whether *more* information is better. That is, Chapter 3 compares the value of an additional ex ante observable informative signal with that of no additional information.

Concerning the endogenous timing of information, the chapter is most closely related to Lizzeri, Meyer, and Persico (2002), who consider this topic in a dynamic model: the agent produces in two periods with independent probabilities and may receive feedback about his first period output. If he does not, they show that the optimal incentive scheme rewards the agent when he has a high output in both periods. If he receives feedback, however, the principal has to pay a positive wage both after a low and a high output in the first period. But rewarding him after a low output reduces first period incentives.[4] This makes a feedback policy always worse than no feedback in their setting. Although some of their results are similar to the ones in this chapter, the driving forces behind them are very different (as we explain further in our discussions): our results are caused by the informativeness of the signal about effort and not the dynamic structure of the incentive problem. Furthermore, we allow additionally for gains from tailoring effort to the state world, which are not present in their model. This allows us to show that receiving early information is not always harmful. This matches better the observed behavior in firms, where midterm evaluations sometimes take place and information is passed on in some situations but not in others. In this respect, our model is related to Ederer (2004), who considers a model with such effects for an *exogenously* given wage scheme.

[3]Holmstrom defines more information in the sufficient statistic sense. Others consider more informative output systems using the principal agent analogue of Blackwell's (1953) and Lehmann's (1988) information criteria.

[4]Schmitz (2005) provides a solution for this dilemma taking a sequential move structure of the agents as given: he shows that the principal does sometimes better by employing a different agent in the second period than in the first period.

$f(xz\|e)$	$x=\bar{x}$	$x=\underline{x}$	
$z=\bar{z}$	$p\,h(\bar{x}\|\bar{z},e)$	$p\,h(\underline{x}\|\bar{z},e)$	p
$z=\underline{z}$	$(1-p)\,h(\bar{x}\|\underline{z},e)$	$(1-p)\,h(\underline{x}\|\underline{z},e)$	$1-p$
	$g(\bar{x}\|e)$	$1-g(\bar{x}\|e)$	1

Table 1.1: Joint Distribution.

1.2 The Model

There is one principal, who employs one agent. The agent is risk neutral, has no wealth, and the value of his reservation utility is zero. He produces a verifiable and observable output x (which equals the principal's revenue), which can be either high ($\bar{x}$) or low ($\underline{x}=0$). There is a second – verifiable – signal $z \in \{\underline{z}, \bar{z}\}$, where the probability of $z=\bar{z}$ is p and that of $z=\underline{z}$ is $1-p$. Upon observing z, the agent's posterior probability of producing a high output depends on his effort $e \in \mathcal{E} = [0,b]$ and is denoted by $h(\bar{x}|z,e)$, $h: \mathcal{E} \to (0,1)$, $h \in C^3$ and:

$$\frac{\partial h(\bar{x}|z,e)}{\partial e} \equiv h_e(\bar{x}|z,e) > 0,$$

$$\frac{\partial^2 h(\bar{x}|z,e)}{\partial e^2} \equiv h_{ee}(\bar{x}|z,e) < 0.$$

The joint distribution of the output and the signal, $f(xz|e)$, is then as given in Table 1.1 and the marginal distribution of output is $g(\bar{x}|e) = \sum_z f(\bar{x}z|e)$ and $1-g(\bar{x}|e) = \sum_z f(\underline{x}z|e)$. The agent's cost of effort function is $c(e)$, $c: \mathcal{E} \to I\!R_0^+$, $c \in C^3$, $c(0)=0$, $c_e(e)>0\ \forall e>0$, $c_e(0)=0$, $c_{ee}(e)>0\ \forall e>0$ and $c_{ee}(0)=0$.

The timing is as follows. At date 0 the principal decides when the agent should observe the realization of the signal: before he provides effort (ex ante information) or after (ex post). Furthermore, she offers a wage scheme to the agent, which specifies four wages: $\mathbf{w} = (w(\bar{x}\bar{z}), w(\bar{x}\underline{z}), w(\underline{x}\bar{z}), w(\underline{x}\underline{z}))$.

Under the ex post information scenario, the agent provides effort at date 1 and then the signal z realizes. Effort is unobservable. Under the ex ante information scenario, first the signal z realizes – which is observable to both the principal and the agent – and then the agent provides unobservable effort: $e(\bar{z}) \equiv \bar{e}$ after $z=\bar{z}$

and $e(\underline{z}) \equiv \underline{e}$ after $z = \underline{z}$. After the realization of all outputs, payoffs realize: the principal receives the revenues net of wage payments, while the agent receives his wage minus his effort costs.

Strictly speaking, ex ante information refers to what might more appropriately be called intermediate information as the agent already signed the contract when he receives information. In the appendix, we show that this distinction is unimportant though: there is no difference between the two, because the agent's reservation utility is zero. Hence, we simply refer to the case where the agent observes the signal before the effort choice as ex ante information.

1.3 First Best

As a benchmark, we first consider the informational scenario that the principal would choose if she could observe efforts, i.e. a situation where there is no incentive problem and she only has to decide which effort to implement. Here she compensates the agent exactly for his effort costs and sets $w = c(e)$ if he provides the desired effort of e and pays him zero otherwise.

The problem of the principal for the ex post information scenario is hence to maximize $g(\bar{x}|e)\bar{x} - c(e)$ over e. For the ex ante information scenario – where she can make efforts state contingent – her problem is to maximize $E\left[f(\bar{x}z|e(z))\bar{x} - c(e(z))\right]$ over $(\bar{e}, \underline{e})$. We see that if we impose $\bar{e} = \underline{e}$, the profit functions for the two scenarios coincide. Thus, the maximization problem of the principal is the same for both structures, except that for the ex post information scenario the restriction $\bar{e} = \underline{e}$ applies. But the possibility to let $\bar{e}$ differ from $\underline{e}$ cannot make the principal worse off – if it increases her profits she allows these two effort levels to differ from each other, otherwise she sets them equal:

Proposition 1 *The principal's profits are as least as high under ex ante information as under ex post information. They are strictly higher under ex ante information if and only if there are gains from tailoring effort:*

$$f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\underline{z}|\underline{e}) \neq p\, g(\bar{x}|\bar{e}) + (1-p)\, g(\bar{x}|\underline{e}) \;\forall(\bar{e}, \underline{e}).$$

This is the standard result that ex ante information is strictly beneficial in decision problems if and only if there are "gains from tailoring effort" to the state of the world – which are a property of the underlying distribution function. The principal can exploit these by choosing state contingent effort levels optimally to strictly increase profits under the ex ante compared to the ex post information scenario. In the next section we will explain these gains from tailoring effort in more detail.

1.4 The Wage Scheme

The first best analysis shows that if effort implementation is a simple decision problem, ex ante information can never harm. So what happens if effort implementation is an incentive problem? Here the principal's objective is to maximize for each informational scenario her expected profits over effort and wages, subject to the agent's incentive, limited liability and participation constraints. Call an effort that satisfies these three constraints for a given wage scheme *implementable*.

As usual in a moral hazard setting, we decompose the problem into two parts. In the first step, we fix an implementable effort – e for the ex post and $(\bar{e}, \underline{e})$ for the ex ante information scenario – and minimize expected wage payments $\sum_x [f(x\bar{z}|\bar{e})w(x\bar{z}) + f(x\underline{z}|\underline{e})w(x\underline{z})]$, where $\underline{e} = \bar{e} = e$ for the ex post information scenario. Call the solution to this problem the implementation cost functions $C(\mathbf{xz}|e)$ for the ex post and $C^A(\mathbf{xz}|\bar{e}, \underline{e})$ for the ex ante information scenario. Here $\mathbf{xz}$ is the subset of states $\{\bar{x}\bar{z}, \bar{x}\underline{z}, \underline{x}\bar{z}, \underline{x}\underline{z}\}$ where the principal pays a positive wage. In the second step, we maximize the principal's total profits $\Pi(e) = g(\bar{x}|e)\bar{x} - C(\mathbf{xz}|e)$ and $\Pi^A(\bar{e}, \underline{e}) = [f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\underline{z}|\underline{e})]\,\bar{x} - C^A(\mathbf{xz}|\bar{e}, \underline{e})$ over efforts.

In this section we consider the first step of the incentive problem described above for the two informational scenarios and compare implementation cost functions. To understand the impact of ex ante information on them, we have to clarify what it means for a signal to be "informative". Maximizing solely revenues minus costs (as e.g. in the first best) is a simple decision problem and hence ex ante information is beneficial if and only if there are gains from tailoring effort to the state of the world. For incentive problems ex ante information changes the agent's incentives if the signal is "informative" about the agent's effort.

$f(xz\|e)$	$x = \bar{x}$	$x = \underline{x}$	
$z = \bar{z}$	$a\, g(\bar{x}\|e)\, p - k$	$p\,(1 - a\, g(\bar{x}\|e)) + k$	p
$z = \underline{z}$	$g(\bar{x}\|e)(1 - a\, p) + k$	$1 - g(\bar{x}\|e) - p + a\, g(\bar{x}\|e)\, p - k$	$1 - p$
	$g(\bar{x}\|e)$	$1 - g(\bar{x}\|e)$	1

Table 1.2: Joint Distribution for Definition 1; a and k, such that $f(xz|e) \in (0,1)$ and $\sum\sum f(xz|e) = 1$.

1.4.1 Review of Information Concepts

We review briefly the information concepts relevant to our analysis: information about efforts and gains from tailoring effort. As we want to point out the differences between incentive and decision problems, we are especially interested in examples where the one holds but not the other. Furthermore, we will relate these concepts to the dependence between output and the signal, to see why the latter concept would not suffice for our purpose. To illustrate these ideas, we often use the following examples, which refer to the distribution function in Table 1.2:

Definition 1 *The "times-a model" is defined by:* $k = 0$*. The "plus-k model" is defined by:* $a = 1$*.*

The term "times-a model" refers to the fact that in this model $g(x|e)\,p$ is multiplied by a. The "plus-k model" adds the term k to $g(x|e)p$.[5]

Informativeness about Effort

As mentioned above, we are interested whether the signal is informative about the agent's effort. For this we compare the likelihood ratios $l(\bar{x}\bar{z}|e) \equiv \frac{f_e(\bar{x}\bar{z}|e)}{f(\bar{x}\bar{z}|e)}$, $l(\bar{x}\underline{z}|e) = \frac{f_e(\bar{x}\underline{z}|e)}{f(\bar{x}\underline{z}|e)}$, and $l(\bar{x}|e) = \frac{g_e(\bar{x}|e)}{g(\bar{x}|e)}$. Those tell us how likely it is that the agent provided an effort of e, given a output (output-signal) realization $\bar{x}$ ($\bar{x}z$).

[5]One can generate the plus-k model by defining $\tilde{g}(\bar{x}|e,\sigma) = e+\sigma$ and the times-a one by defining $\tilde{g}(\bar{x}|e,\sigma) = e\sigma$, where σ is a shock that is correlated with the signal. This fits to our notation by setting $g(\bar{x}|e) = E\tilde{g}(\bar{x}|e,\sigma)$, $a = \frac{E\sigma z}{E\sigma Ez}$ (hence $a > 1 \leftrightarrow Cov(\sigma,z) > 0$) and $k = Cov(\sigma,z)$. Thus, the plus-k and times-a model are equivalent to Ederer's (2004) multiplicative and additive models.

Suppose $l(\bar{x}\bar{z}|e) \neq l(\bar{x}\underline{z}|e)$, which implies that those ratios are also unequal to $l(\bar{x}|e)$.[6] Then we say that *the signal is informative about the agent's effort.* We say it is *uninformative* for $l(\bar{x}\underline{z}|e) = l(\bar{x}\bar{z}|e)$, which implies that those ratios are also equal to $l(\bar{x}|e)$. In the following we want to focus on either informative or uninformative signals and hence impose:

Assumption 1 *sign* $(l(\bar{x}\bar{z}|e) - l(\bar{x}\underline{z}|e)) =$ *constant* $\forall e$.

This assumption is for example met by standard models in the literature like the times-a or the plus-k model. Our definition fits what Demougin and Fluet (1998) call "mechanism sufficiency". A statistic is said to be mechanism sufficient if the implementation costs for a certain effort depend only on this statistic. In Section 1.4.2 we will show that if likelihood ratios in $\bar{x}\bar{z}$ and $\bar{x}\underline{z}$ are equal, then implementation costs $C(\mathbf{xz}|e)$ depend only on $x = \bar{x}$ and not on the signal. Hence, the agent's output is mechanism sufficient and for brevity we say that the signal is uninformative. The intuition is simple: if the signal is uninformative it does not help to "estimate" the agent's effort, and thus the principal is indifferent whether or not to condition the wage on it. If the likelihood ratios are unequal implementation costs depend on the signal and we say it is informative.

For risk averse agents the definition differs slightly. Here x is called informative (uninformative) – in the sufficient statistic sense – if likelihood ratios are equal (unequal) also for output signal combinations $\underline{x}z$.[7] Thus, for them the sufficient statistic and the sufficient mechanism coincide.

Turning to our two examples, we see that in the plus-k model the signal is informative about the agent's effort for $k \neq 0$. For $k = 0$ the signal is uninformative. Hence, we can vary the informativeness of the signal by varying k. For the times-a model output is uninformative no matter which value a takes.

[6] Rearranging shows that all these ratios are equal (unequal) depending on whether $g(\bar{x}|e)f_e(\bar{x}\bar{z}|e) - f(\bar{x}\bar{z}|e)g_e(\bar{x}|e)$ is equal (unequal) to zero.

[7] See Holmstrom (1982). Gjesdal (1982), Grossman and Hart (1983), Kim (1995), and Jewitt (2006) use the term informative in the sense of Blackwell (1953) and Lehmann (1988).

Gains from Tailoring Effort

In Section 1.3 (First Best) we mentioned that ex ante information does strictly better than ex post information if there are "gains from tailoring effort", which we said are present for distribution functions satisfying:

$$f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\underline{z}|\underline{e}) \neq p\, g(\bar{x}|\bar{e}) + (1-p)\, g(\bar{x}|\underline{e}).$$

For our model specification with $\underline{x} = 0$ these gains are not simply captured by dependence. Consider for example the plus-k model: here effort and the term that captures dependence (k) are not linked. Thus, the principal cannot increase the expected probability of obtaining output $\bar{x}$ by making effort state contingent, while keeping the sum of efforts constant across states. To see this note that Jensen's Inequality implies that such a policy must decrease the expected probability of state $\bar{x}$ in the absence of gains from tailoring effort:

$$f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\underline{z}|\underline{e}) = p\, g(\bar{x}|\bar{e}) + (1-p)\, g(\bar{x}|\underline{e}) \leq g(\bar{x}|p\,\bar{e} + (1-p)\,\underline{e}).$$

What is the consequence? Take as an example the first best maximization problem. For functions as in the previous equation the principal would set $\bar{e} = \underline{e}$, and ex ante and ex post information do equally well. In contrast, for the times-a model with $a \neq 1$, effort and the term that captures dependence (a) are directly linked. Thus there are gains from tailoring effort and the principal can increase the expected probability by making effort state contingent: choose $\bar{e} > \underline{e}$ ($<$) for $a > 1$ ($a < 1$). This can outweigh the decrease in the value of the concave profit function. Hence, ex ante information would do strictly better than ex post information.

Summary and Relation to Dependence

Why did we introduce two information concepts rather than simply consider dependence between the signal and the output? Dependence can be characterized using copulas. The copula "couples a bivariate distribution function to its one dimensional marginals" (Nelsen 1995): define a copula $K(u, v)$, $K : [0, 1]^2 \to [0, 1]$, $u = g(x|e)$, $v = p(z)$. It lies between the Fréchet bounds: $\max\{u + v - 1, 0\} \leq K(u, v) \leq \min\{u, v\}$, which give bounds for most negatively (positively) dependent random variables. Furthermore, $K(u, v) = uv$ for independent random variables.

	$a = 1$	$a \neq 1$
$k = 0$	NO,NE,NG	O,NE,G
$k \neq 0$	O,E,NG	O,E,G

Table 1.3: Relation between Parameters in Table 1.2 and Information Concepts. O: information about outputs. E: information about effort. G: gains from tailoring effort. NY: no information about $Y \in \{O, E, G\}$.

Consider the plus-k model. By varying k, we shift the copula between the Fréchet bounds: for $k < 0$, the copula lies above uv and one says that the random variables are positively quadrant dependent; for $k > 0$ they are negatively quadrant dependent; and for $k = 0$, they are independent. As explained before, by varying k we affect also the degree of informativeness of the signal about effort. Hence, we can vary informativeness about output and effort simultaneously by shifting k: the plus-k model is an example where the signal is either informative about effort and output or uninformative about both. Dependence, however, fails to capture fully what our two information concepts do: the two random variables are dependent (for $k \neq 0$) but there are no gains from tailoring effort for any value of k.

For the times-a model, $a > 1$ corresponds to positive quadrant dependence, $a < 1$ to negative quadrant dependence, and $a = 1$ to independence. Again dependence does not capture fully what we are interested in: remember that the signal is uninformative about the agent's effort no matter which value a takes, but that there are gains from tailoring effort for $a \neq 1$.

To summarize (see also Table 1.3), dependence between output and the signal does not imply that there are gains from tailoring of effort or that the signal is informative about effort, and none of these latter two concepts implies the other. In the other direction, however gains from tailoring of effort or informativeness about effort imply dependence.

1.4.2 Ex Post Information

In this section we consider the first part of the principal's problem: minimizing the expected wage payments for a given implementable effort level. We start with the ex post information scenario. The problem is to minimize expected wage payments:

$$\min_{\mathbf{w}} \sum_x \sum_z f(xz|\hat{e}) w(xz),$$

subject to the limited liability constraints $w(xz) \geq 0 \ \forall xz$ and the incentive constraint:

$$\hat{e} \in \arg\max_{e \in \mathcal{E}} \quad \sum_x \sum_z f(xz|e) w(xz) - c(e).$$

The participation constraint is satisfied if the other two constraints are, and we therefore omit it in the above problem. Furthermore, higher wages after a low output decrease not only the agent's incentives, but also the principal's profits. Hence, $w(\underline{x}\bar{z}) = w(\bar{x}\underline{z}) = 0$. The reduced maximization problem regarding the wages $w(\bar{x}\bar{z})$ and $w(\bar{x}\underline{z})$ is graphically illustrated in Figure 1.1. Lower wages increase the principal's profits, so her aim is to push the iso-wageline down as far as possible, i.e. as far down as the incentive constraint permits. As this is a linear problem, the optimal wage scheme chooses one of the wages $w(\bar{x}\bar{z})$ or $w(\bar{x}\underline{z})$ to be larger than zero and the other one to be equal zero, depending on the slope of the two lines. For example, if the iso-wageline is strictly steeper than the incentive constraint $\left(-\frac{f(\bar{x}\bar{z}|e)}{f(\bar{x}\underline{z}|e)} > -\frac{f_e(\bar{x}\bar{z}|e)}{f_e(\bar{x}\underline{z}|e)} \Leftrightarrow l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e)\right)$, then $w(\bar{x}\underline{z}) = 0$ and $w(\bar{x}\bar{z})$ is chosen such that the incentive constraint is satisfied: $w(\bar{x}\bar{z}) = c_e(e)/f_e(\bar{x}\bar{z}|e)$. If the slopes (and hence the likelihood ratios) are equal, all wage combinations that satisfy the incentive constraint are optimal.

To understand the intuition, suppose the likelihood ratio in state $\bar{x}\bar{z}$ is larger than the one in state $\bar{x}\underline{z}$. In other words, it is more likely in state $\bar{x}\bar{z}$ than in state $\bar{x}\underline{z}$ that the agent provided a certain effort. Hence, the principal optimally concentrates rewards for the agent in this output state ($w(\bar{x}\bar{z}) > w(\bar{x}\underline{z}) = 0$). If the signal contains no additional information about effort (likelihood ratios are equal), the principal cannot reduce information rents by conditioning the wage scheme on the signal. Doing so, however, does not harm as the agent is risk neutral for all positive wage combinations. Thus, the principal is indifferent whether or not to condition the agent's wage on the signal. The following lemma that we prove formally in the appendix summarizes the discussion:

Lemma 1 *Suppose the principal wants to implement effort e. Then the wages after a low output are zero: $w(\underline{x}\bar{z}) = w(\underline{x}\underline{z}) = 0$. Furthermore:*

(i) If

$$l(\bar{x}\bar{z}|e) \begin{cases} > l(\bar{x}\underline{z}|e) \\ < l(\bar{x}\underline{z}|e) \end{cases} \quad then \quad w(\bar{x}\bar{z}) = \begin{cases} \frac{c_e(e)}{f_e(\bar{x}\bar{z}|e)} \\ 0 \end{cases} \quad and \quad w(\bar{x}\underline{z}) = \begin{cases} 0 \\ \frac{c_e(e)}{f_e(\bar{x}\underline{z}|e)} \end{cases}.$$

(ii) If $l(\bar{x}\bar{z}|e) = l(\bar{x}\underline{z}|e)$, any $w(\bar{x}\bar{z})$ - $w(\bar{x}\underline{z})$ combination that satisfies the incentive constraint is optimal: $\sum_z f_e(\bar{x}z|e)w(\bar{x}z) = c_e(e)$.

In the appendix we derive from Lemma 1 the implementation costs (expected wage) for e for the different wage schemes:

$$C(s|e) \equiv \frac{c_e(e)}{l(s|e)}, \tag{1.1}$$

where $s = \bar{x}\bar{z}$ for $l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e)$, $s = \bar{x}\underline{z}$ for $l(\bar{x}\bar{z}|e) < l(\bar{x}\underline{z}|e)$, and $s = \bar{x}$ for $l(\bar{x}\bar{z}|e) = l(\bar{x}\underline{z}|e)$. Comparing the implementation costs for an informative signal to the ones of an uninformative (which are the same as when no additional signal is available), one sees immediately that the costs for the informative one are strictly lower for a given effort. Hence, an informative signal is strictly beneficial as in Holmstrom (1979).

For our further analysis we make the following assumption:

Assumption 2 *The implementation cost function $\frac{c_e(e)}{l(s|e)}$ is convex in e.*

Strictly speaking this is an assumption on an endogenous function. It is, however, very natural in principal-agent models: it is a sufficient condition for the principal's problem to be strictly concave in effort, and for example met by the times-a or the plus-k model (or any combination of them).

1.4.3 Ex Ante Information

In this section we derive the wage scheme for the agent if he can observe the signal realization before he chooses his effort. This implies that we have to consider two

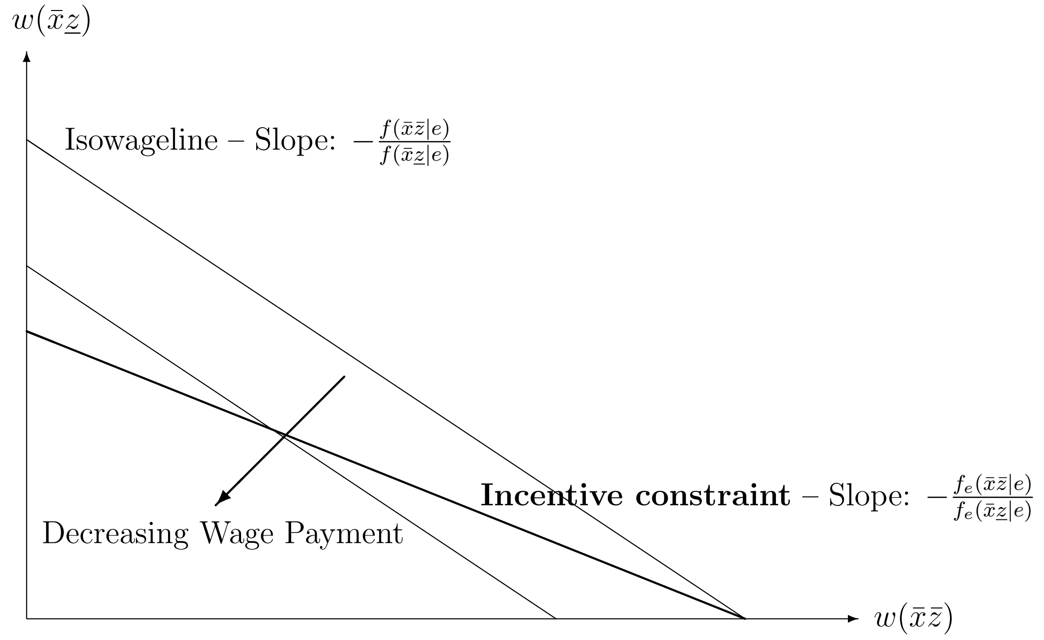

Figure 1.1: Derivation of the Wage Scheme

incentive constraints – one after a high signal and one after a low one:[8]

$$\text{after observing } \bar{z}: \ \bar{e} \in \arg\max_{e\in\mathcal{E}} \ h(\bar{x}|\bar{z}, e)w(\bar{x}\bar{z}) - c(e),$$
$$\text{after observing } \underline{z}: \ \underline{e} \in \arg\max_{e\in\mathcal{E}} \ h(\bar{x}|\underline{z}, e)w(\bar{x}\underline{z}) - c(e).$$

Using the formula for the posterior probabilities we obtain the following wages that are necessary to implement effort levels $(\bar{e}, \underline{e})$: $w(\bar{x}\bar{z}) = \frac{p\, c_e(\bar{e})}{f_e(\bar{x}\bar{z}|\bar{e})}$ and $w(\bar{x}\underline{z}) = \frac{(1-p)\, c_e(\underline{e})}{f_e(\bar{x}\underline{z}|\underline{e})}$. This yields the implementation costs for $(\bar{e}, \underline{e})$ given an informative signal:

$$C^A(\bar{x}\bar{z}, \bar{x}\underline{z}|\bar{e}, \underline{e}) \equiv p\, C(\bar{x}\bar{z}|\bar{e}) + (1-p)\, C(\bar{x}\underline{z}|\underline{e}). \tag{1.2}$$

For an uninformative signal $C(\bar{x}\bar{z}|e) = C(\bar{x}\underline{z}|e) = C(\bar{x}|e)$ and hence this reduces to:

$$C^A(\bar{x}|\bar{e}, \underline{e}) \equiv p\, C(\bar{x}|\bar{e}) + (1-p)\, C(\bar{x}|\underline{e}). \tag{1.3}$$

[8]As aforementioned, it is optimal to set an agent's wage equal to zero in case his output is low (i.e. $w(\underline{x}\bar{z}) = w(\underline{x}\underline{z}) = 0$) – a fact already used in the stated constraints. As the agent has a reservation utility of zero, the participation constraint still does not bind: he receives a positive rent in each state. Using this argument, it is easy to see that there is no difference between ex ante and intermediate information in our setting.

1.4.4 Comparison of Implementation Costs

The purpose of this section is to develop the intuition for why ex ante information may be harmful. For this we compare here only the implementation costs for the two information structures, working with the hypothetical scenario that the principal implements the same expected effort level for both structures. That is, the vector $(\bar{e}, \underline{e})$ for the ex ante information scenario and $e = p\,\bar{e} + (1-p)\,\underline{e}$ for the ex post one. This helps us later – when we compare overall profits – to construct sufficient conditions for one structure to be optimal: $(\bar{e}, \underline{e})$, say, can be *any* effort vector for the ex ante information scenario, including the optimal one. This vector then determines the effort level for the ex post scenario. As a consequence of this restriction, the optimal effort level for this scenario may be excluded though. If, however, in the ex ante information scenario the implementation costs are higher for all effort vectors (one of which is the optimal one) than those in the ex post information scenario with the restricted effort level, then a fortiori they are higher than those in the ex post information with the optimal effort level.

Proposition 2 *Suppose Assumption 2 holds. Then for any $(\bar{e}, \underline{e}) \neq \mathbf{0}$ of the ex ante information scenario the principal can implement at lower costs the same expected effort $e = p\,\bar{e} + (1-p)\,\underline{e}$ under the ex post information scenario. The decrease in implementation costs is strict for:*

(i) the implementation cost function $\frac{c_e(e)}{l(\bar{x}z|e)}$ being strictly convex and $\bar{e} \neq \underline{e}$,

(ii) an informative signal, i.e. $l(\bar{x}\bar{z}|e) \neq l(\bar{x}\underline{z}|e)$.

There are two forces that cause the strict inequality: convex implementation costs (Part (i) of Proposition 2) and a change in the feasible wage scheme (Part (ii)). To illustrate the first force, suppose that the likelihood ratio in state $\bar{x}\bar{z}$ is equal to the one in $\bar{x}\underline{z}$. Hence, $C(\bar{x}\bar{z}|e) = C(\bar{x}\underline{z}|e) = C(\bar{x}|e)$ for $\bar{e} = \underline{e} = e$ and implementation costs for the two informational scenarios are equal: $p\,C(\bar{x}|\bar{e}) + (1-p)\,C(\bar{x}|\underline{e}) = C(\bar{x}|e)$. Intuitively, an uninformative signal does not change the agent's incentives and the principal can simply ignore it. Making effort state contingent ($\bar{e} \neq \underline{e}$) would then imply that the left hand side of the previous equation is a convex combination of the costs to implement efforts $\underline{e}$ and $\bar{e}$, respectively, while the right hand side represents the implementation costs of the convex effort combination $e = p\bar{e} + (1-p)\underline{e}$. Hence, by Jensen's Inequality, implementation costs for the ex post information scenario would be (strictly) lower if and only if the implementation cost function is (strictly) convex given $\underline{e} \neq \bar{e}$.

If the signal is informative about the agent's effort the situation changes. For $\bar{e} = \underline{e} = e$ we now have $C(\bar{x}\bar{z}|e) < C(\bar{x}\underline{z}|e)$, which implies that:

$$p\,C(\bar{x}\bar{z}|e) + (1-p)\underbrace{C(\bar{x}\underline{z}|e)}_{>C(\bar{x}\bar{z}|e)} > C(\bar{x}\bar{z}|e).$$

Hence, the principal cannot simply ignore the information by setting $\bar{e} = \underline{e} = e$, as it changes the agent's incentives.[9] Making effort state contingent again only increases the value of the implementation costs further.

At first glance, it seems surprising that the principal cannot reduce implementation costs by conditioning effort on the informative state of the world. As the agent observes the signal realization, she has to pay a positive wage in both states $\bar{x}\bar{z}$ and $\bar{x}\underline{z}$, and not only in state $\bar{x}\bar{z}$. Paying a positive wage in state $\bar{x}\underline{z}$, where it is less likely that the agent provided effort than in the other state, offsets the informational advantage which the principal can achieve by tailoring effort. Thus, if the signal is informative, the principal can reduce implementation costs by conditioning an agent's *incentive scheme* on the signal, but not by conditioning his implemented *effort* on it.

1.5 Comparing the Structures

In this section we compare overall profits for both information scenarios to make statements about the optimal timing of information. To get necessary and sufficient conditions for optimality, we need to proceed to the second step of the principal's problem and maximize her profits over effort, using the implementation costs derived in the first step. While we do so for the case where the signal is uninformative about

[9]Note, however, that for linear implementation cost functions the strict inequality holds only for $\underline{e} > 0$: if this effort level was zero not only the convexity effect disappears, but also the disadvantage of high implementation costs in the uninformative state. However, under our standard assumptions $\underline{e} > 0$ is optimal.

the agent's effort, we will use the sufficient conditions to compare structures for the case where output is informative: solving the principal's problem in general is not tractable for this case. Nevertheless, the sufficient conditions already allow us to show that ex ante information can do strictly worse in incentive problems, in contrast to decision problems.

1.5.1 Uninformative Signal

If the signal is uninformative about the agent's effort we know from the previous section that setting $\bar{e} = \underline{e}$ makes the implementation cost functions equal for both structures (Proposition 2, Part (i)). This hinges on the likelihood ratios being equal: the information does not change the agent's incentives as it does not help to "estimate" his effort. Furthermore, expected revenues for the ex ante information scenario are equal to the ones for the ex post information scenario if $\bar{e} = \underline{e}$, as then $p\,h(\bar{x}|\bar{z}, \bar{e}) + (1 - p)\,h(\bar{x}|\underline{z}, \underline{e}) = g(\bar{x}|e)$. Therefore, if we require $\bar{e} = \underline{e}$ the ex ante information scenario's profit function coincides with the ex post information scenario's one. Hence, – as for the first best – the principal cannot do worse when maximizing without the restriction $\bar{e} = \underline{e}$, and can do strictly better if and only if there are gains from tailoring of effort:

Proposition 3 *Suppose the signal is uninformative, i.e.* $l(\bar{x}\bar{z}|e) = l(\bar{x}\underline{z}|e)$. *Then the ex ante information scenario yields at least as high profits as the ex post scenario. If and only there are gains from tailoring effort, i.e.:*

$$f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\underline{z}|\underline{e}) \neq p\,g(\bar{x}|\bar{e}) + (1 - p)\,g(\bar{x}|\underline{e})\ \forall(\bar{e}, \underline{e}),$$

the ex ante information scenario yields strictly higher profits than the ex post scenario.

Note the similarity between Propositions 1 and 3. Key for the understanding is that the second best maximization problem parallels the one for the first best: the wage scheme does not change when moving from the ex post to the ex ante information scenario if the signal is uninformative. This implies that the principal can ignore (set $\bar{e} = \underline{e}$) the information if there are no gains from tailoring of effort (which are a property of the distribution function), and exploit these by choosing $\bar{e} \neq \underline{e}$ if there are some. The joint presence of gains from tailoring effort and of an uninformative signal is possible as we showed with the times-a model.

Discussion

Proposition 3 generalizes the result for the "multiplicative model" (which is equivalent to our times-a model) of Ederer (2004). He assumes that the wage scheme is the same for both structures. Our analysis showed that the argument for optimality relies on information not altering the wage scheme. As the latter is driven by the uninformativeness of the signal about effort, we had to point out that there exist cases where this is possible, even though there are gains from tailoring effort.

In the endogenous wage model with one agent of Lizzeri, Meyer, and Persico (2002), $\bar{e} \neq \underline{e}$ is optimal so as to provide first period incentives. This drives up convex implementation costs and makes ex ante information suboptimal because there are no gains from tailoring effort. In comparison, in our model with an uninformative signal, the principal chooses $\bar{e} \neq \underline{e}$ to exploit the gains from tailoring effort and not to influence incentives: ex ante information does not change the wage scheme. This is impossible in their dynamic model. In our setting, providing ex ante information can therefore be strictly better, although wages are endogenous.

1.5.2 Informative Signal

When the signal is informative, the implementation cost functions do not coincide for $\bar{e} = \underline{e}$. Hence, we cannot apply the argument we used to show Proposition 3. To get necessary and sufficient conditions for the present case, we would have to fully determine optimal efforts and compare the value functions. Instead of tackling directly this intractable problem, we build on our previous results to derive sufficient conditions. These already enable us to show that it is possible for ex ante information to do strictly worse than ex post information:

Proposition 4 *Suppose the signal is informative, i.e.* $l(\bar{x}\bar{z}|e) \neq l(\bar{x}\underline{z}|e)$ *and there are no gains from tailoring effort, i.e.* $f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\underline{z}|\underline{e}) = p\, g(\bar{x}|\bar{e}) + (1-p) g\,(\bar{x}|\underline{e})$. *Then the ex ante information scenario leads to strictly lower profits than the ex post information scenario.*

As there are no gains from tailoring effort, expected revenues are lower under ex ante than under ex post information: $p\, g(\bar{x}|\bar{e}) + (1-p)\, g(\bar{x}|\underline{e}) \leq g(\bar{x}|p\,\bar{e} + (1-p)\,\underline{e})$ by Jensen's Inequality. Regarding implementation costs, remember from Proposition 2 that for any $(\bar{e}, \underline{e})$ in the ex ante information scenario, the principal can implement the same expected effort in the ex post information scenario, but with strictly lower implementation costs. Taking both together we see that for any $(\bar{e}, \underline{e}) \neq 0$ (which is optimal for $f_e(\bar{x}z|e) > 0$) we can implement the same expected effort under the ex post scenario leading to strictly higher profits. This profit is lower than the value function: the expected effort from the ex ante information case need not be optimal for the ex post scenario. But the principal cannot do worse with the optimal effort.

Discussion

To arrive at the result of this section we abstracted from gains of tailoring effort in the revenue function – by choosing the joint probability function appropriately and by assuming $\underline{x} = 0$. This already allowed us to show the surprising result that ex ante information can do *strictly* worse than ex post information when effort is unobservable. This can never happen in decision problems.

Adding gains from tailoring of effort would create a trade-off: ex ante information is good because the principal can increase revenues by tailoring effort to the state of the world, but bad because it changes incentives. Depending on the strength of these effects, ex ante information can do strictly better or worse. The following equation gives a sufficient condition for it to do better:

$$\underbrace{[f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\underline{z}|\underline{e}) - g(\bar{x}|e)]\bar{x}}_{\text{possible gain from tailoring effort}} \geq \underbrace{p\,C(\bar{x}\bar{z}|\bar{e}) + (1-p)\,C(\bar{x}\underline{z}|\underline{e}) - C(\bar{x}\bar{z}|e)}_{\text{loss: higher implementation costs}},$$

where $(\bar{e}, \underline{e})$ are the optimal effort levels under the ex ante information scenario and $e = p\,\bar{e} + (1-p)\,\underline{e}$. Analyzing this further is, however, not possible with general functions as one has to determine optimal effort levels. Moreover, it would not lead to fundamentally different insights than those already offered in Propositions 3 and 4.

The suboptimality of ex ante information in Ederer (2004) for his plus-k model has very different causes. The exogenously imposed wages mean that the principal cannot implement the same effort levels across states. Thus, $\bar{e} \neq \underline{e}$ arises – even though there are no gains from tailoring effort. This decreases the value of the concave profit function. In contrast, the suboptimality of ex ante information in our model is driven by a change in the wage scheme, and therefore would even arise if the principal set $\bar{e} = \underline{e}$, or if the profit function was linear.

In Lizzeri, Meyer, and Persico (2002) the suboptimality of ex ante information is also caused by a change in the wage scheme. It is, however, not informativeness about efforts that drives it, but the fact that the agent anticipates that he will receive a positive wage also after a failure in the first period. Thus, the negative effect in their model arises *only* because pre-information incentives change, while in ours the effect comes from changes in post-information incentives. An advantage of our setup is that we can consider more cases than they can: the signal changes or does not change incentives. By this we can point out that a change is needed for information received before the effort choice be harmful.

1.6 Conclusion

In this chapter we considered the impact of ex ante information on incentive and decision problems. We have seen that the optimality of ex ante information relative to ex post information depends on two forces: gains from tailoring effort and whether the signal is informative about the agent's effort. While the relevance of the first for the optimality of ex ante information is well known from decision problems, we showed the importance of the second for incentive problems, and the interplay between the two forces. One of our surprising results is that ex ante information can harm once we consider incentives – but only if it is informative about effort.

Our result sheds some light on the puzzle why information is not always passed on early in organizations or why midterm evaluations are not always conducted. Such policies could help to adjust future actions and thus help to make better decisions and to avoid mistakes. But the model shows that they often affect the agent's incentives in a negative way: if e.g. a positive midterm report indicates that the agent's project will succeed even though he does not work hard, it gets very costly for the principal to provide incentives to him. This negative effect can be so strong that no midterm evaluation takes place – even though it would help to avoid mistakes.

Chapter 2

Transparency and Organizational Structure

2.1 Introduction

In principal agent theory additional information about the effort of the agent is (weakly) beneficial (Holmstrom 1979). An important piece of such information is the performance of colleagues working on closely related projects (Holmstrom 1982, Mookherjee 1984). But while one knows that this information helps to provide incentives, relatively little is known about the optimal timing of information. This, however, is an important question for firms: when and what an agent observes about the outcomes of his colleagues depends on the organizational form. Qian, Roland, and Xu (2006) point out the different impact that a firm's divisional structure can have in this respect. A unitary form (U-form) firm is organized along functional lines. For example, all marketing experts work together in one division and thus can more easily communicate with each other and observe the work progress of their marketing colleagues before making decisions. In contrast, a multi-divisional form (M-form) groups employees according to products or regions. Here the marketing expert responsible for one product is not always up to date about the current status of related marketing projects for other products, but he has information about the outcomes of his research & development colleagues.

But observing a colleague who works on a similar task conveys different information than observing one working on a dissimilar task. The aim of this chapter is to

consider the incentive effects of such different kinds of knowledge about colleagues' performances on the incentive and decision problems in a firm. This sheds some light on the relation between the organizational structure and internal transparency of an organization by addressing questions such as "under which structures does the principal want to take actions that prevent or foster the observation of colleagues' performances?" For example, she can collocate agents under the U-form or M-form in different departments, confine employees to single offices to make observing others harder, place many employees in an open-plan office, or she can make performance evaluations private or public.
To analyze this I extend the model of Chapter 1 to a multiple agent context. There I endogenized the point in time when a principal wants an agent to observe a signal – before or after the effort choice. Based on this, I ask in this chapter when the principal wants an agent to receive information about the performance of a colleague: before the agent's effort choice (agents work e.g. together in a office) or not at all/after this choice (they are separated).[1] To capture the idea that observing colleagues working on a dissimilar task conveys different information than if they work on a similar task, we consider this decision for the cases where the colleague's performance is either uninformative or informative about the agent's effort and/or performance.

It follows from Chapter 1 that providing an agent with information about the colleague's output before rather than after his effort choice increases implementation costs if this output is informative about effort. For example, the agent may learn from observing, say, a low output of his colleague that he is in a situation in which his project is very likely to succeed – even if he does not provide much effort. This knowledge then makes it harder to convince the agent to put in high effort, and therefore increases implementation costs in this state. As a consequence, ex ante implementation costs also increase: rewards are no longer concentrated in the most informative state (here when both the colleague and the agent succeed) as they would be under ex post information. In contrast, observing an uninformative out-

[1] As long as the information is verifiable it does not matter whether the agent receives the information ex post or not at all.

put does not change incentives, and hence implementation costs relative to the ex post information case.

Another factor comes into play in the multiple-agent context though: while information about a colleague's output has a negative impact on the incentives of an agent, we show in this chapter that there is potentially a positive effect on the colleague's incentives if output is informative about effort. To see why this is the case, note that the latter implies that a relative performance incentive scheme is optimal. Suppose the principal rewards the colleague only if the output of both agents is high. Then implementing a low effort for the agent following high output of the colleague demotivates the latter: he expects to receive the reward less often. From the principal's perspective this negative incentive effect can be outweighed, however, by the effect of having to pay the reward less often. Again, this influence is not present if output is uninformative: here the agent's effort does not enter the colleague's implementation cost function because the principal rewards both agents independently from each other.

Overall, we show that providing information before the effort choice of an agent cannot harm if the colleague's output is uninformative about the agent's effort. Such a situation is plausible for the M-form: it is likely that the agents are hit by a common productivity shock as they are working on the same product. However, as they work in different fields, the output of a marketing expert is unlikely to contain information about the effort of the employee from the research & development department. To understand this it is important to keep in mind that those two information concepts do not coincide: we characterize situations where common productivity shocks are present, but nevertheless the agent's output are uninformative about each others effort. Thus, under the M-form the principal can fully exploit the common productivity shocks to tailor the agent's effort to the information that the colleague's output provides. This increases her revenues without distorting incentives. Thus, one expects that such firms are more transparent in the sense that they use more often open-plan offices, department collocation or a system of shared information on performance evaluations and wages.

The situation is more complicated if output is informative about effort. Now ex ante information affects the incentives of the agent and of the colleague whose output is observed. Ex ante information can do strictly better than ex post information – even if there are no revenue gains from tailoring effort to the information (this is not possible in Chapter 1). The driving force behind the result is that ex ante information reduces the colleague's implementation costs. If this effect is small, however, providing feedback about colleagues is strictly worse for the principal than ex post information. The result suggests that within firm transparency varies across organizational structures: U-form firms should be less transparent than M-form firms. For example, the marketing expert in the latter gets to observe the output of colleagues (e.g. R&D and production experts) which may be very informative about the output in his work but not as informative about his effort as the outcomes of other marketing experts that he would observe in the former.

The chapter is structured as follows. After discussing the related literature, we introduce the model in Section 2.2. Section 2.3 considers the optimal wage scheme for the two informational scenarios (ex post versus ex ante information) and show the driving forces behind the differences in implementation costs. In Section 2.4 we show under which circumstances the one or the other scenario yields higher overall profits. Section 2.5 discusses several extensions of our baseline model. Section 2.6 concludes. All proofs are in the appendix.

Related Literature

On the one hand, the chapter is related to the theoretical literature that asks about the optimal timing of information There are three strands in this field. The first considers dynamic tournaments with fixed prizes. Thus, the wage scheme and wages are taken as given. In Lizzeri, Meyer, and Persico (1999) or Aoyagi (2003)[2] agents work for two periods and may or may not observe first period outputs (output is the sum of effort and a shock). There are two effects that arise under ex ante information. First, a strategic effect, defined as the agents' attempts in the first period to influence second period actions. Both papers show that first period effort is the same regardless of whether ex ante information is revealed or not. Second, a "risk aversion in effort effect": effort in the second period depends via first period output

[2]For a related experiment see Ederer and Fehr (2006).

on the realization of the shock, and is hence uncertain from an ex ante perspective. This uncertainty in effort harms the principal if and only if her profit function is concave in effort, i.e. if and only if she is risk averse in effort (as wages are fixed the principal cannot implement the same effort after all output realizations).[3]

In Ederer's (2004) tournament model giving feedback can be optimal despite the risk aversion in effort effect: he assumes that an agent's output not only depends on effort and a shock, but also on (unknown) ability. Feedback on the output difference provides information about the agent's ability. Depending on how effort and ability enter the output function, a feedback policy can be optimal: if they enter in a multiplicative way, the possibility to adjust the agent's effort to his posterior ability and the strategic effect increase overall output, and hence profits, under the feedback policy. If they enter additively, these effects play no role and the optimality of a feedback policy depends again on the concavity of the profit function.

The second strand of the literature endogenizes the wage scheme in single agent models, showing that a feedback policy is bad for the principal (Lizzeri, Meyer, and Persico (2002) or Chapter 1). In comparison, we identify a positive effect on the second agent that can make ex ante information optimal. In our model this effect is present also for cases where it is not in tournament models. This is a consequence of the wage scheme being endogenous.

Finally, there is a third strand of the literature that considers very different models when asking about the impact of feedback on incentives. Several papers from the business accounting literature consider endogenous wage scheme models in the context of double moral hazard (Arya, Glover, and Sivaramakrishnan 1997, Chwolka 2005). In these models the second moving agent is the principal herself, who produces jointly an output with the agent. Hence, there is no incentive problem for her at the second period. From an ex ante perspective, she wants to commit, however, to a certain effort (to better motivate the agent). Both papers show that early information (either before the principal provides effort or before both provide effort) changes the principal's commitment problem and can thus harm. Thus, while some

[3] The model by Jost and Kräkel (2006) considers only the strategic effect: the second agent can observe the effort, but not the output of the first agent. As they analyze changes in expected efforts and not expected profits, results are hard to compare, however.

of the conclusions are similar to those in our setting, the driving forces are very different. Moreover, some papers consider the impact of information about performance and efforts when agents jointly produce an output (Winter 2006, Banerjee and Beggs 1989, Goldfayn 2006, Ludwig and Nafziger 2006). Ertac (2005) asks whether feedback about one's own and a peer's performance should be revealed or not. In contrast to our model, this information is not about a variable on which the incentive scheme conditions.

On the other hand, the chapter is related to the organizational economics literature. First, there are some papers that deal with the firm's choice of its transparency (Mukherjee 2005, Koch and Morgenstern 2005, Koch and Peyrache 2005, Albano and Leaver 2005, Calzolari and Pavan 2006). All of those consider how transparent the firm wants to be toward outsiders (e.g. should it reveal the performance of employees to the market), while we ask about its internal transparency. Second, there are some papers that consider – like ours – the incentive effects of different organizational structures. Aghion and Tirole (1995) argue that the advantage of the M-form is the avoidance of a moral hazard in teams problem, as each agent is only responsible for one product. Thus, if the headquarters suffer from managerial overload they may want to make use of the better (implicit) incentives under the M-form and delegate to subordinates, although the M-form performs worse in terms of specialization. Maskin, Qian, and Xu (2000) study the relation between better incentives and information in U- versus M-form organizations. They do not, however, consider firms but different economic systems like China (M-form structure) and the Soviet Union (U-form structure). This makes our models hard to compare.

2.2 The Model

There are two agents, $i \in \{1, 2\}$ and one principal. Agents are risk neutral and have no wealth. The value of their reservation utility is zero. The observable and verifiable output (which equals the principal's revenue) x of each agent i can be either high ($\bar{x}$) or low ($\underline{x} = 0$). We often refer to a specific output realization as the "state of the world". The probability that an agent produces $\bar{x}$, depends on his effort $e \in \{e_L, e_H\}$: $g(\bar{x}|e)$, $g : \{e_L, e_H\} \to (0, 1)$. We assume that $g(\bar{x}|e_H) > g(\bar{x}|e_L)$.

Upon observing output x of the colleague, the agent's posterior probability of x' given effort own e' and the colleague's effort e is $h(x'|x,e,e')$. From this we obtain the joint probability $f(xx'|e,e') = h(x'|x,e,e')\, g(x|e)$. We assume that agents are symmetric – $f(xx'|e,e') = f(x'x|e',e)$ – and use the convention that the first x (e) is the output (effort) of agent 1 and the second x (e) the output (effort) of agent 2. Finally, the agent's cost of effort are $c = 1$ if he provides high effort and zero else.

The timing is as follows. At date 0 the principal decides about the information structure: should the second agent observe the output of the first one before he himself provides effort (ex ante information scenario)[4] or not (ex post information scenario). At date 1, the principal offers a wage scheme. This specifies four wages for each agent: $\mathbf{w}_i = (w_i(\bar{x}\bar{x}), w_i(\bar{x}\underline{x}), w_i(\underline{x}\bar{x}), w_i(\underline{x}\underline{x}))$. If both agents accept to work for the principal production takes place, otherwise the relationship terminates and the payoff of every player is zero.
Under the ex post information scenario both agents provide effort at date 2. Effort is unobservable. Under the ex ante information scenario only agent one provides effort at date 2. This effort is neither observable to the principal, nor to the second agent. After the first agent's output realized – which is observable to the second agent and the principal – agent 2 provides unobservable effort: $\underline{e}$ after observing $\underline{x}$ and $\bar{e}$ after $\bar{x}$. Once all outputs realized, payoffs incur: the principal receives the revenues net of wage payments, while each agent receives his wage minus his effort costs.

Finally, we have to define what we mean by information about the agent's effort. We just give here the definitions – for a more extensive discussion of them see Chapter 1. Suppose $l(\bar{x}\bar{x}|e) \equiv \frac{f(\bar{x}\bar{x}|e_L,e)}{f(\bar{x}\bar{x}|e_H,e)} < \frac{f(\bar{x}\underline{x}|e_L,e)}{f(\bar{x}\underline{x}|e_H,e)} = \frac{f(\underline{x}\bar{x}|e,e_L)}{f(\underline{x}\bar{x}|e,e_H)} \equiv l(\bar{x}\underline{x}|e)$ (which also implies $l(\bar{x}\bar{x}|e) < l(\bar{x}) < l(\bar{x}\underline{z}|e)$, where $l(\bar{x}) \equiv \frac{g(\bar{x}|e_L)}{g(\bar{x}|e_H)}$). Then we say that *the colleague's output is informative about the agent's effort.* For $l(\bar{x}\bar{x}|e') = l(\bar{x}\underline{x}|e') = l(\bar{x})$ we say that it is *uninformative.*

[4]Strictly speaking it is intermediate information as the agent already signed the contract. It is easy to see that there is no difference between ex ante and intermediate information as the agent's reservation utility is zero. This implies that the ex ante or interim participation constraint is never binding.

$f(xx'\|e,e')$	$\bar{x}$	$\underline{x}$	
$\bar{x}'$	$a\,e\,e' - k$	$e'\,(1 - a\,e) + k$	e'
$\underline{x}'$	$e\,(1 - a\,e') + k$	$1 - e - e' + a\,e\,e - k$	$1 - e'$
	e	$1 - e$	1

Table 2.1: Joint Distribution for the plus-k ($a = 1$) and the times-a ($k = 0$) model, with a and k, such that $f(xx'|e,e') \in (0,1)$ and $\sum\sum f(xx'|e,e') = 1$.

The distribution in Table 2.1 illustrates this: for $k = 0$, the colleagues output is uninformative about the agent's effort as likelihood ratios in states $\bar{x}\bar{x}$ and $\bar{x}\underline{x}$ are equal. For $k \neq 0$ these ratios are unequal and thus the colleague's output is informative. This holds independent of a, i.e. a cannot determine whether or not the colleague's output is informative. Note, however, that for $a = 1$ and any value of k the principal cannot increase her expected revenues $[f(\bar{x}\bar{x}|e,\bar{e}) + f(\bar{x}\underline{x}|e,\underline{e})]\bar{x}$ by making effort state contingent, i.e. set $\bar{e} \neq \underline{e}$ (no "gains from tailoring effort"), but that she can do so for $a \neq 1$. Hence, this example illustrates that output being uninformative about effort does not coincide with the absence of gains from tailoring effort and vice versa.

2.3 The Wage Scheme

The principal's problem is to maximize for each scenario her expected profits over effort and wages, subject to the agents' incentive, limited liability, and participation constraints. As usual in a moral hazard setting we can decompose the problem into two parts. In the first step, we fix an effort that the principal implements – (e,e') for the ex post and $(e,\bar{e},\underline{e})$ for the ex ante information scenario – and maximize with respect to wages. This gives us the optimal wage scheme. The implementation of low effort is trivial: the principal simply sets $w(xx') = 0\,\forall xx'$, which results in implementation costs of $C(e_L) = 0$ and profits $\Pi(e_L) = f(\bar{x}|e_L)\bar{x}$. Hence, in this section we consider only the case where the principal wants to implement high effort. In the second step, one derives – given the wage scheme – the optimal effort. We will do this in Section 2.4, where we also compare the structures.

2.3.1 The Wage Scheme for the Second Agent

Consider the first part of the principal's problem for the second agent under the ex post information scenario, given that she wants him to provide e_H if the first agent provides $e \in \{e_L, e_H\}$:

$$\min_{\mathbf{w}_2} \sum_x \sum_{x'} f(xx'|e, e_H) w_2(xx'),$$
$$\text{s.t. } w_2(xx') \geq 0 \ \forall xx',$$
$$\sum_x \sum_{x'} [f(xx'|e, e_H) - f(xx'|e, e_L)] w_2(xx') \geq 1,$$
$$\sum_x \sum_{x'} f(xx'|e, e_H) w_2(xx') \geq 1.$$

The first constraint is the limited liability, the second the incentive, and the third the participation constraint. Effort e is the (correct) belief of the second agent about the first agent's effort: the agents' efforts constitute a Nash equilibrium of the second stage subgame. A pure strategy equilibrium of this subgame exists as the strategy space is compact and convex and the objective function is quasi-concave. The equilibrium may not be unique though. So either we have to make assumptions on the probability and cost functions such that it is unique; or we have to assume that the principal can implement the desired equilibrium, which is what we will do here.[5]

In the appendix we solve the above problem and show that the optimal wage scheme sets $w_2(\bar{x}\underline{x}) = w_2(\underline{x}\underline{x}) = 0$ and $w_2(\bar{x}\bar{x}) = 1/[f(\bar{x}\bar{x}|e, e_H) - f(\bar{x}\bar{x}|e, e_L)] > w_2(\underline{x}\bar{x}) = 0$ for $l(\bar{x}\bar{x}|e) > l(\bar{x}\underline{x}|e)$. For $l(\bar{x}\bar{x}|e) = l(\bar{x}\underline{x}|e)$ any $w_2(\bar{x}\bar{x})$-$w_2(\underline{x}\bar{x})$ combination that satisfies the incentive constraint is optimal: $\sum_x [f(x\bar{x}|e, e_H) - f(x\bar{x}|e, e_L)] w_2(x\bar{x}) = 1$. This yields the expected wage payment that is necessary to implement an effort of e_H given e:

$$C(s|e, e_H) \equiv \frac{1}{1 - l(s|e)}, \tag{2.1}$$

where $s = \bar{x}\bar{x}$ if the likelihood ratio in state $\bar{x}\bar{x}$ is strictly larger than the one in state $\bar{x}\underline{x}$. If both likelihood ratios are equal $s = \bar{x}$. This implies that $C(\bar{x}|e_H)$ does not depend on the effort and output of the first agent. Comparing the implementation

[5] See Mookherjee (1984) for a discussion and Ma (1988) for other ways how to deal with the multiple equilibria problem.

costs for the case where the colleagues output is informative to the one where it is uninformative, one sees immediately that those for the informative case are strictly lower.

Under the ex ante information scenario we have to consider two incentive constraints - one after a high output in the first period and one after a low one:

$$\text{after observing } \bar{x}: \; [h(\bar{x}|\bar{x}, e, e_H) - h(\bar{x}|\bar{x}, e, e_L)]w_2(\bar{x}\bar{x}) \geq 1,$$
$$\text{after observing } \underline{x}: \; [h(\bar{x}|\underline{x}, e, e_H) - h(\bar{x}|\underline{x}, e, e_L)]w_2(\underline{x}\bar{x}) \geq 1.$$

This gives the following costs to implement the effort vector $(\bar{e}, \underline{e})$ – given an effort level of e for the first agent:

$$C^A(\bar{x}\bar{x}, \underline{x}\bar{x}|e, e_H) \equiv g(\bar{x}|e)\, C(\bar{x}\bar{x}|e, e_H) + (1 - g(\bar{x}|e))\, C(\underline{x}\bar{x}|e, e_H), \tag{2.2}$$

where $C(\underline{x}\bar{x}|e, e_H) = \frac{1}{1 - l(\underline{x}\bar{x}|e)}$ and $C(\underline{x}\bar{x}|e, e_H) = C(\bar{x}\bar{x}|e, e_H) = C(\bar{x}|e_H)$ if $l(\bar{x}\bar{x}|e) = l(\bar{x}\underline{x}|e)$. Comparing this with Equation (2.1), we see immediately that if the principal wants to implement high effort under both informational scenarios, then implementation costs are strictly higher under the ex ante information scenario if $l(\bar{x}\bar{x}|e) \neq l(\bar{x}\underline{x}|e)$.

2.3.2 The Wage Scheme for the First Agent

The derivation of the wage scheme for the first agent under the ex post information scenario is analogous to the one for the second agent. For the ex ante information scenario the incentive constraint changes. It is given by:

$$\sum_x [f(\bar{x}x|e_H, \bar{e}) - f(\bar{x}x|e_L, \bar{e})]w_1(\bar{x}x) + \sum_x [f(\underline{x}x|e_H, \underline{e}) - f(\underline{x}x|e_L, \underline{e})]w_1(\underline{x}x) \geq 1.$$

Consider first the case where the likelihood ratios are unequal. While for the ex post information scenario we have a single effort level, i.e. $\bar{e} = \underline{e} = e$, those two effort levels can differ for the ex ante information scenario. The first agent anticipates correctly that the second agent provides effort $\bar{e}$ after a high first period output and $\underline{e}$ after a low one. By the same argument as for the second agent the principal sets the wages $w_1(\underline{x}\bar{x}) = w_1(\underline{x}\underline{x}) = 0$. This implies that the effort decision of the second agent following low output does not enter the incentive constraint.

But even though $\underline{e}$ does not enter the first agent's incentive constraint, a second effect on implementation costs arises through $\bar{e}$ under ex ante information compared to ex post information. To see this note that the same arguments as in the previous section give the respective implementation costs:

$$C(\bar{x}\bar{x}|e_H, \tilde{e}) = \frac{1}{1 - l(\bar{x}\bar{x}|\tilde{e})}, \tag{2.3}$$

where the second agent's effort level is $\tilde{e} = \bar{e}$ for the ex ante information scenario and $\tilde{e} = e$ for the ex post one. Thus, under which scenario expected wage payments for the first agent are larger depends on the relation between e and $\bar{e}$ and on the influence that this effort level has on the likelihood ratio. While under the ex post information scenario the principal has to implement the same effort in all states of the world, she can implement state contingent effort levels under ex ante information. Thus, if, say, implementation costs for the first agent are lower if the colleague works less, she can set $\bar{e} = e_L$ and $\underline{e} = e_H$. This decreases the implementation costs for the first agent without distorting expected revenues from the second agent too much. To achieve the same cost reduction under the ex post information scenario, the principal has to implement $e = e_L$ in all states of the world, which results in a larger decrease in the revenues from the second agent.

So what is the impact of the second agent's effort on the first agent's implementation costs? Consider as an example the plus-k model (see Table 2.1), where implementation costs are given by:

$$C(\bar{x}\bar{x}|e_H, \tilde{e}) = \frac{\tilde{e}\, e_H - k}{\tilde{e}\,(e_H - e_L)}.$$

From this one can see that a lower effort of the second agent decreases those costs for $k > 0$. In other words, under the ex ante information scenario implementation costs are lower for the first agent than those under the ex post information scenario if the principal implements $\bar{e} = e_L < e = e_H$. The intuition is the following. For $k > 0$ it is optimal for the principal to reward the first agent if he and the second agent both have a high output, independent of the effort to be implemented for the second agent. On the one hand, a higher $\tilde{e}$ increases the first agent's incentives. If the second agent provides more effort after observing $\bar{x}$, the first agent expects to receive the wage more often. Thus, it is easier to motivate him, which decreases

implementation costs. But, on the other hand, the principal then has to pay the wage more often – this increases costs. Overall, the second effect dominates and implementation costs are increasing in the second agent's effort.
We summarize the general result in the following proposition:

Proposition 5 *Suppose the signal is informative, i.e.* $l(\bar{x}\bar{x}|e) \neq l(\bar{x}\underline{x}|e)$*. Implementation costs for the first agent are higher if the second provides* e_H *compared to* e_L *if and only if* $[f(\bar{x}\bar{x}|e_L, e_H)]^2 > f(\bar{x}\bar{x}|e_L, e_L)f(\bar{x}\bar{x}|e_H, e_H)$*.*

For the case where the likelihood ratios are equal it follows immediately from Equation (2.1) that the implementation costs for the first agent do not depend on the second agent's effort:

Proposition 6 *If the signal is uninformative, i.e.* $l(\bar{x}\bar{x}|e) = l(\bar{x}\underline{x}|e)$*, the expected costs to implement* e_H *are equal for the ex post and the ex ante information scenarios.*

As an example consider the times-a model. Here, implementation costs are:

$$C(\bar{x}|e_H) = \frac{g(\bar{x}|e_H)}{g(\bar{x}|e_H) - g(\bar{x}|e_L)} = \frac{e_H}{e_H - e_L},$$

i.e. the expected wage for the first agent depends neither on the output, nor on the effort of the second agent. Hence, whether the second agent provides more or less effort if he receives ex ante information cannot have an impact on the first agent's implementation costs.

2.4 Comparing the Structures

We have seen that switching from the ex post to the ex ante information scenario has two effects: there are potential gains from tailoring effort, and the incentives for the two agents are altered. The interesting case to focus on for the purposes of comparing the two scenarios is the one where both effects can potentially come into play. That is where the high effort is implemented for both agents in the ex post information scenario. This is guaranteed by the following assumption:

Assumption 3

$$[g(\bar{x}|e_H) - g(\bar{x}|e_L)]\bar{x} \quad \geq \quad 2C(\bar{x}\bar{x}|e_H, e_H) - C(\bar{x}\bar{x}|e_H, e_L).$$

For the ex ante information scenario this assumption moreover guarantees that the principal implements the high effort for the first agent (see appendix). Hence, as desired, Assumption 3 allows us to consider solely the effects of ex ante information.

2.4.1 Uninformative Outputs

Consider first the case where the colleague's output is uninformative about the agent's effort. Then the principal's profit under the ex post information scenario is:

$$\Pi^P(e, e') = [g(\bar{x}|e_H)\bar{x} - C(\bar{x}|e_H)] + [g(\bar{x}|e_H)\bar{x} - C(\bar{x}|e_H)].$$

And for the ex ante information scenario it is:

$$\begin{aligned}\Pi^A(e, \bar{e}, \underline{e}) = \quad & [g(\bar{x}|e_H)\bar{x} - C(\bar{x}|e_H)] + [f(\bar{x}\bar{x}|e_H, \bar{e}) + f(\underline{x}\bar{x}|e_H, \underline{e})]\bar{x} \\ - \quad & [g(\bar{x}|e_H)C(\bar{x}|\bar{e}) + (1 - g(\bar{x}|e_H))C(\bar{x}|\underline{e})].\end{aligned}$$

The profit component stemming from the first agent is identical across both information scenarios. The profit component related to the second agent under the ex ante information scenario is a convex combination of the one under the ex post information scenario. Thus, by setting $\bar{e} = \underline{e} = e_H$ the principal can always achieve the same profits under both scenarios. But having the option to set $\bar{e} \neq \underline{e}$ cannot make her worse off and, therefore, the ex ante information scenario performs (weakly) better:

Proposition 7 *Suppose the signal is uninformative, i.e.* $l(\bar{x}\bar{x}|e) = l(\bar{x}\underline{x}|e)$*. Then the ex ante information scenario yields at least as high profits as the ex post one. If and only if there are gains from tailoring effort, i.e.* $h(\bar{x}|x, e_H, e_H) - h(\bar{x}|x, e_H, e_L) < C(\bar{x}|e_H)$ *for some* $x \in \{\bar{x}, \underline{x}\}$*, the ex ante information scenario yields strictly higher profits.*

The condition $h(\bar{x}|x, e_H, e_H) - h(\bar{x}|x, e_H, e_L) < C(\bar{x}|e_H)$ tells us whether the principal can increase profits (via revenues) by making efforts state contingent. To understand this, consider the following example. Suppose that $f(\bar{x}\underline{x}|e_H, e_H) = f(\bar{x}\underline{x}|e_H, e_L)$. Then implementing high effort after a low first period output does not change the second agent's success probability and the principal sets $\underline{e} = e_L$. This saves implementation costs: setting $\underline{e} = e_L$ under the ex ante information scenario reduces implementation costs to $g(\bar{x}|e_H)\,C(\bar{x}|e_H)$. In contrast, under the ex post scenario the principal implements e_H in all states of the world, incurring implementation costs $C(\bar{x}|e_H) > g(\bar{x}|e_H)\,C(\bar{x}|e_H)$.

Can there be a situation where, on the one hand, the first agent's output is uninformative about the second agent's effort, but, on the other hand, there are nevertheless gains from tailoring effort? The times-a model is an example, which has those properties for $a \neq 1$. To see this, note that Condition 3 implies $(e_H - e_L)^2 \geq e_H$. According to Proposition 7, ex ante information can do strictly better for $\frac{a\,e_H}{1-e_H}(e_H - e_L)^2 < e_H$ or $\frac{1-a\,e_H}{1-e_H}(e_H - e_L)^2 < e_H$. The former is consistent with Condition 3 for $a < 1$ and the latter for $a > 1$.

Proposition 7 illustrates that providing employees with information about colleagues can never harm if their outputs are uninformative about the agent's effort. Such a situation is likely between agents with different specializations, for example between a marketing expert and a production worker, who work in the same department of a firm structured according to the M-form. Thus, according to Proposition 7 we expect that such firms have a more transparent internal structure: enabling observations of colleagues can never harm, but it can potentially help the principal increase revenues by tailoring employees' efforts to such information. The same holds for firms, where competition for prizes, promotions, or bonuses play a minor role: the proposition illustrates that a sufficient condition for information about colleagues not to harm is simply that the workers' reward schemes are independent. To facilitate such observations the principal can e.g. allocate agents to open-plan offices or collocate agents in different departments. Moreover, she might not undertake much effort to keep bonuses and wages (that convey information about performances) secret.

2.4.2 Informative Outputs

Assumption 3 puts us in the case we are interested in: the first agent always provides high effort under the ex ante information scenario. Therefore, we can now analyze in detail the different effects that choosing state contingent effort levels $(\bar{e}, \underline{e})$ have on the implementation costs of the two agents. In the following we will focus on the cases where ex ante information can have a positive impact on the first agent's implementation costs. That is, we focus on $[f(\bar{x}\bar{x}|e_L, e_H)]^2 > f(\bar{x}\bar{x}|e_L, e_L) f(\bar{x}\bar{x}|e_H, e_H)$, so that the first agent's implementation costs are lower if the principal implements effort e_L for the second agent. If they were lower with e_H these costs could only increase by making the second agent's effort state contingent. Thus, ex ante information could only do better because of increases in revenues by tailoring effort, which we discussed in Proposition 7.

A Condition for Ex Ante Information to Perform Worse

Proposition 8 *Suppose the signal is informative, i.e.* $l(\bar{x}\bar{x}|e) < l(\bar{x}\underline{x}|e)$ *and it is optimal to implement* $(\bar{e}, \underline{e}) = (e_H, e_H)$ *or* $(\bar{e}, \underline{e}) = (e_L, e_L)$ *under the ex ante information scenario. Then this scenario does strictly worse than the ex post information one.*

To understand the result, note that under these conditions the profit components stemming from the first agent are the same under both structures, as are the revenues from the second agent for (e_H, e_H). It follows, however, from Equations (2.1) and (2.2) that the costs of implementing (e_H, e_H) for the second agent are strictly higher if the principal provides ex ante information. For (e_L, e_L) it follows from Assumption 3 that the profit component due to the second agent is strictly lower under the ex ante information scenario. Therefore, in both cases this scenario performs worse overall.

The result shows that in situations where outputs are informative the principal may want to take actions to prevent workers from observing each other. Such actions of the principal can for example involve the allocation of agents to single offices or a policy of wage secrecy. If there is no positive effect on the first agent because the second agent provides the same effort in all states of the world regardless of the feedback, it is *strictly* better that the second agent receives no ex ante information. The emphasis lies on strictly, because in a simple decision problem ex ante information would perform equally well as ex post information in such a situation.

The U-form is an example for outputs being informative: the output of one marketing expert is likely to be informative about the effort of another marketing expert. Hence, we expected such a firm to be less transparent. Also in a hierarchical firm a policy of less transparency might be optimal: in such an organization promotions play a big role for providing incentives. They can be understood as an interdependent reward scheme. The reason is that otherwise a worker might conclude that it is not worth working hard for a promotion after observing a colleague's performance and recognizing that it is nearly impossible to be better.

A Condition for Ex Ante Information to Perform Better

Can the ex ante information scenario do strictly better? The following proposition shows that the answer is yes – even in situations where there are no gains from tailoring effort:

Proposition 9 *Suppose the signal is informative, i.e.* $l(\bar{x}\bar{x}|e) < l(\bar{x}\underline{x}|e)$, *and the first agent's implementation costs are lower if the second agent provides* e_L, *i.e.* $[f(\bar{x}\bar{x}|e_L, e_H)]^2 > f(\bar{x}\bar{x}|e_L, e_L)f(\bar{x}\bar{x}|e_H, e_H)$. *If it is optimal to implement* $(\bar{e}, \underline{e}) = (e_H, e_L)$ *or* $(\bar{e}, \underline{e}) = (e_L, e_H)$ *under the ex ante information scenario, then there exist parameters such that ex ante information does strictly better than ex post information.*

For the case $(\bar{e}, \underline{e}) = (e_H, e_L)$ informational gains are the only driving force behind the optimality of ex ante information.

The case $(\bar{e}, \underline{e}) = (e_L, e_H)$ is more interesting though: to motivate the first agent the principal implements the high effort for the second agent in the *less* informative state $\underline{x}$. The positive effect on the first agent's motivation comes at the cost of rewarding the second agent in this very uninformative state, which increases implementation costs for the latter. Hence, a trade-off exists. Depending on the strength of the two effects either ex ante or ex post information can be optimal.

This shows that for informative outputs a more transparent firm is only optimal if the feedback about the performance of the colleague changes the agent's actions. And even in this situation it is not always the case that a feedback policy is optimal because of the negative impact on the second agent's incentives. This effect is additionally enforced by implementing e_H for the second agent in the uninformative state and e_L in the informative one.

Discussion

Our model comes to a different conclusion than the tournament models with exogenous wages: given a convex cost function, in the latter ex ante information can never be optimal unless there are informational gains (see e.g. Ederer (2004)). What is the root of this? When wages and the wage scheme are exogenously given, ex ante information cannot have an impact on the incentive scheme. Thus, the positive effect we identify is ruled out by assumption. Determining the wage scheme endogenously implies that the implementation cost function of an agent depends on the effort of the colleague. If desired, the principal can then *implement* a state contingent second period effort that leads to a reduction in first period implementation costs.
This possibility of reducing the first agent's implementation costs is also the reason why in our model ex ante information can be better, while in the single agent settings of Lizzeri, Meyer, and Persico (2002) or Chapter 1 it is always worse. In Lizzeri, Meyer, and Persico (2002) ex ante information reduces first period incentives. With two agents not the agent himself is rewarded after a low output in the first period, but the colleague. Hence, the negative effect on first period incentives disappears – and may even be replaced by a positive effect.

2.5 Extensions

In this section we consider two extensions – taking the ex ante information scenario as given. The first deals with the question which agent should observe which, i.e. in which direction information should optimally flow in a firm. The second asks whether it is always better to hire another agent, given that this agent can observe the outcomes of existing workers. For both we assume that the output of an agent is informative about the colleague's effort.

2.5.1 Heterogeneous Agents

So far we assumed that agents are symmetric and therefore the direction of the information flow was irrelevant under the ex ante information scenario. With heterogeneous agents, however, this is an issue. For example, agents might differ with

respect to their productivity, or more interestingly, with respect to the informativeness of their outputs about effort. For the latter suppose e.g. that the output of agent one is uninformative about agent two's effort, but agent two's effort is informative about agent one's effort. Then the optimal direction of the information flow would indeed be to let the second agent observe the first: the second agent's implementation costs are unchanged by the information, but the principal can reduce the first agent's implementation costs by making second period effort state contingent through an interdependent wage scheme. If she were to change the direction of the information flow, no such positive effect on the first moving agent would be possible: while his wage scheme is independent of the second moving agent's effort, there is a negative effect on the second moving agent's implementation costs.
This provides one explanation for why firms organize information flows to be from the bottom to the top and not the other way round: the outcome of a lower hierarchy worker is unlikely to be informative about the effort of a worker higher up in the hierarchy. But, the other way round, it is likely to be informative: the performance of the higher placed worker might convey firm and industry specific information. This is useful for lower hierarchy workers and might thus change their incentives.

2.5.2 Integration

Is it always optimal to employ two agents given that they can observe each other? Or are there circumstances where not expanding the firm and employing just one agent is better? To examine this question we focus on the case where it is optimal to implement high effort in all states of the world under the ex ante information scenario. Then profits in the single agent firm are:

$$\Pi^S(e_H) = g(\bar{x}|e_H)\bar{x} - C(\bar{x}|e_H).$$

And in the two agent firm they are:

$$\Pi^M(e_H, e_H) = 2g(\bar{x}|e_H)\bar{x} - C(\bar{x}\bar{x}|e_H, e_H) - EC(x\bar{x}|e_H, e_H).$$

The multiple agent firm has not only the advantage that more agents produce, but also that the principal can condition an agent's wage on the output of the colleague. As we have seen before, observing others decreases incentives if the colleague's output is informative about the agent's effort. Hence, a trade-off arises between these two forces and governs whether it is optimal to integrate another agent or not. The principal prefers to expand the firm if and only if:

$$\Pi^M(e_H, e_H) - \Pi^S(e_H) > 0$$
$$\Leftrightarrow$$
$$f(\bar{x}|e_H) - C(\bar{x}\bar{x}|e_H, e_H) \geq EC(x\bar{x}|e_H, e_H) - C(\bar{x}|e_H).$$

As shown in Chapter 3 $EC(x\bar{x}|e_H, e_H) > C(\bar{x}|e_H)$ can hold, because the negative effect of rewarding an agent in a state that is less informative than $\bar{x}$ ($\bar{x}\underline{x}$) cannot outweigh the gain of more information – rewarding him in a state that is more informative than state $\bar{x}$ ($\bar{x}\bar{x}$). Thus, for integration to be optimal the principal's profit from the additional agent has to be higher than the potential increase in implementation costs.
Our model can contribute to the discussion about the boundaries of the firm: the integration of another department or a production spin off changes the available information about the performance of colleagues in a firm and thus the incentives of the agent.[6] Moreover, it can explain why relative performance evaluation schemes are not as common in firms as the standard theory would predict (see e.g. Antle and Smith (1986)): even though employees' outputs are informative the principal may prefer to use an individual performance scheme to prevent the flow of information to others' when this information would harm incentives. Alternatively, the principal may want to select workers in such a way to avoid their output being informative about each others' efforts. This, in turn, would mean that an independent wage scheme is indeed optimal.

2.6 Conclusion

We have shown that information about the performance of colleagues can harm when received before the effort choice: the incentives of the agent who receives information are always adversely affected (it becomes more expensive to induce him to work), while the incentives of the other agent can increase.

[6]Note that our results strengthen the ones of Schmitz (2005): in our setting the integrated firm generates an additional output and more information is available in it.

We discussed the implications of these results for the design of organizations. These include predictions about e.g. the relation between the firm's internal transparency (public or private performance evaluations, office allocation) and its functional form (U versus M). Firms which are structured according to the M-form are predicted to be more transparent. In firms structured according to the U-form the management wants to prevent workers from observing their colleagues' performance under certain circumstances (e.g. via the office allocation, or private performance evaluations). This can lead to inefficiencies because the information might be useful for the agent to tailor his effort to the new information, which helps to avoid wrong decisions.
Furthermore, the result suggests that we should expect to see more transparent firms where internal competition for prizes or promotions does not play a big role. If incentives from promotions or other forms of internal competition are important in a firm, then we should see less transparency.

Chapter 3

Moral Hazard and Ex Ante Observability

3.1 Introduction

Holmstrom's (1979, 1982) sufficient statistic result says that additional signals are valuable in principal agent models with hidden actions if and only if they are informative about the agent's effort: the principal can use the additional information to condition the agent's incentive scheme on it. For example, she can concentrate higher rewards in states where it is more likely that the agent provided a certain effort.

A puzzle therefore is why real world contracts do not condition more often on additional information (see Prendergast (1999) for an overview). In this chapter we provide an explanation for this puzzle based on a simple variation of Holmstrom's (1979) model: the agent (and the principal) might observe the signal before he chooses his effort rather than after. Thus, while we asked in Chapter 1 about the value of *early* information, we consider here the value of *more* information.

For many settings it is more natural to assume that information is available before the effort choice: an agent observes a colleague working on a related project if they are sharing an office, he may have access to internal statistics about e.g. the firm's performance in some fields, the principal may advise him to sample information for his future project, he learns about his ability in a training spell, or if he works a long time on similar projects he has a good idea about his probability of success.

To illustrate the effects of this model, consider as an illustrational example consider the training spell that the agent undergoes before he starts working in his job. The outcome of it provides a signal both to the firm and to the agent about his talent. In other words, when assigned to a job the agent has in hand a signal that is informative about his effort. What impact on incentives does this have? Suppose that the agent (and the firm) learned that he is not particularly talented. Then observing that he succeeds in the job thereafter is a strong indication that he worked very hard. Stated differently, his output is very informative about his effort. In contrast, such a success will be less informative about the effort of a worker who learned that he is very talented for the job: he knows beforehand that he is likely to succeed even when putting in low effort. Comparing the two, incentives need to be much stronger for the worker with the signal that indicated his talent for the job than for the other one. Thus, information observed before effort has two potential effects. On the one hand, the agent may learn that he is very likely to succeed even with low effort, and it becomes harder to provide incentives to him. This implies that effort implementation costs increase. On the other hand, he may discover that his effort influences strongly the work outcome. Then implementation costs decrease. Comparing this to a situation where no additional signal is available, we will show that overall the negative effect of an informative signal may dominate. Thus, observing no additional signal can be better than observing the informative one. In comparison, an uninformative signal never harms: as it is uninformative about effort, it does not change the agent's incentives.
The chapter is structured as follows. After discussing the related literature in the next paragraph, we present in Section 3.2 the model and review briefly the required information concepts. Then we derive the incentive schemes for the different information structures in Section 3.3. In Section 3.4 we ask whether the principal wants an additional signal if the agent observes its realization before he chooses his effort. Section 3.5 discusses robustness and the last section concludes.

Related Literature

"More information" is beneficial in moral hazard problems when contracts are complete. Holmstrom (1979, 1982) showed this for more information in the sense of having additional signals that are informative about the agent's effort. Gjesdal (1982), Grossman and Hart (1983), Kim (1995), or Jewitt (2006) define more information more broadly in the sense of Blackwell (1953) or Lehmann (1988), i.e. they vary the informativeness of the output system. All these papers have in common that they consider only ex post information. In contrast, we use a close analogue of Holmstrom's (1982) information criterion and ask whether more ex ante information is beneficial in a complete contract setting.

Thus, our contribution is to show that the finding from the incomplete contracts literature that more information can harm (Cremer 1995, Meyer and Vickers 1997, Dewatripont, Jewitt, and Tirole 1999, Prat 2005, Amaya 2005) can also obtain in a complete contract setting. From this literature the models by Amaya (2005) and Cremer (1995) are most closely related. Amaya (2005) and similar models from the accounting literature (Baiman and Evans 1983, Penno 1984, Baiman and Sivaramakrishnan 1991) are distinguished from our setting by the assumption that the informative signal is only observable to the agent. Hence, the principal has to pay an extra rent to the agent because she cannot condition the agent's wage on the signal (the authors exclude the possibility to design a mechanism to extract the private information, hence the contract is not complete). Cremer (1995) shows that the principal does not want to have information about the agent's ability if she can only provide incentives to him with the threat of removing him after a bad performance. Her ability to learn that the agent is talented would undermine this threat as it stops her from firing the agent after a bad performance. While the story is related to ours – information changes the incentives of talented agents – the mechanism is very different: acquiring no information serves as a commitment device in Cremer (1995).

Furthermore, the chapter is related to Lizzeri, Meyer, and Persico (2002) and Chapter 1. Both consider the question *when* an agent should receive information: before or after the effort choice. In comparison, in this chapter we keep the timing fixed at the intermediate/ex ante stage and address the issue *whether* an agent should receive a signal or not. Thus, instead of considering the question whether *early* information revelation is beneficial, we ask in this chapter whether *more* information – in the sense of early observing an additional signal that is informative about the agent's

$f(xz\|e)$	$x = \bar{x}$	$x = \underline{x}$	
$z = \bar{z}$	$p\, h(\bar{x}\|\bar{z}, e)$	$p\, h(\underline{x}\|\bar{z}, e)$	p
$z = \underline{z}$	$(1-p)\, h(\bar{x}\|\underline{z}, e)$	$(1-p)\, h(\underline{x}\|\underline{z}, e)$	$1-p$
	$g(\bar{x}\|e)$	$1 - g(\bar{x}\|e)$	1

Table 3.1: Joint Distribution.

effort – is beneficial.

3.2 The Model

3.2.1 Production

There is one principal, who employs one agent. The agent is risk neutral, has no wealth, and the value of his reservation utility is zero. He produces a verifiable and observable output x (which equals the principal's revenue). This can be either high ($\bar{x}$) or low ($\underline{x} = 0$). There is a second – verifiable – signal $z \in \{\underline{z}, \bar{z}\}$, where the probability of $z = \bar{z}$ is p and of $z = \underline{z}$ is $1-p$. Upon observing z, the agent's posterior probability of producing a high output depends on his effort $e \in \mathcal{E} = [0, b]$ and is denoted by $h(\bar{x}|z, e)$, $h : \mathcal{E} \to (0, 1)$, $h \in C^3$. We assume that $\frac{\partial h(\bar{x}|z,e)}{\partial e} \equiv h_e(\bar{x}|z, e) > 0$. Furthermore, $\frac{\partial^2 h(\bar{x}|z,e)}{\partial e^2} \equiv h_{ee}(\bar{x}|z, e) < 0$. From this it follows that the joint distribution of output and signal, $f(xz|e)$ is as given in Table 3.1 and the marginal distribution of output is $g(\bar{x}|e) = \sum_z f(\bar{x}z|e)$ and $\sum_z f(\underline{x}z|e) = 1 - g(\bar{x}|e)$. The agent's cost of effort function is $c(e)$, $c : \mathcal{E} \to I\!R_0^+$, $c \in C^3$, $c(0) = 0$, $c_e(e) > 0\ \forall e > 0$, $c_e(0) = 0$, $c_{ee}(e) > 0\ \forall e > 0$ and $c_{ee}(0) = 0$.

3.2.2 Timing

The timing is as follows. At date 0 the principal can choose whether she and the agent observe at date 2 the realization of the signal z (ex ante information scenario)[1] or not. We assume that if she chooses not to observe the signal, then it

[1]Strictly speaking, ex ante information refers to what might more appropriately be called intermediate information as the agent already signed the contract when he receives information. In the appendix, we show that this distinction is unimportant though: there is no difference between

is also not verifiable (no additional information scenario). At date 1, the principal offers a wage scheme $\mathbf{w}$ to the agent that depends on the realization of the output and if available the signal: if it is observable the scheme specifies four wages: $\mathbf{w} = (w(\bar{x}\bar{z}), w(\bar{x}\underline{z}), w(\underline{x}\bar{z}), w(\underline{x}\underline{z}))$. If not, the scheme specifies two wages: $\mathbf{w} = \{w(\underline{x}), w(\bar{x})\}$. The signal can be either informative or uninformative about the agent's effort (see below for the definition of informativeness). If the principal acquired the signal, it realizes at date 2. The realization is observable to both the principal and the agent. After this observation the agent provides unobservable effort at date $2\frac{1}{2}$: $e(\bar{z}) = \bar{e}$ after observing $\bar{z}$ and $e(\underline{z}) = \underline{e}$ after $\underline{z}$. If no signal is acquired he simply provides unobservable effort e.[2]

As benchmark we also briefly consider the ex post information scenario: here the signal realizes after the agent has chosen his effort. We do not, however, compare the ex post with the ex ante information scenario as in Chapter 1.

3.2.3 Payoffs and the Principal's Problem

In the main part of our analysis we will assume that the principal has to implement an expected effort of E (see e.g. Lizzeri, Meyer, and Persico (2002) for this formulation), i.e. $p\,\bar{e} + (1-p)\,\underline{e} \geq E$ for the ex ante information scenario and $e \geq E$ for the ex post or no additional information scenario (in equilibrium both constraints are binding). Departing in this way from the standard formulation, where the principal receives a revenue proportional to the agent's output, allows us to neatly separate the incentive effects of ex ante information from the effects it has on revenues. We will show in Section 3.5 that our analysis extends naturally to the standard formulation of revenues, where there is an interplay of both effects.

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the two, because the agent's reservation utility is zero. Hence, we simply refer to the case where the agent observes the signal before the effort choice as ex ante information.

[2]There is a subtle difference between Holmstrom (1979) and our model: in the former the signal is in the world and the principal can simply decide whether or not to condition the incentive scheme on it. For this to work one has to assume that the signal is verifiable, but then it is unimportant, whether the agent (ex post) observes it or not. Our model would be uninteresting if the agent would not (ex ante) observe the signal. For the question whether more information is better, we thus have to give the principal an active choice: should she and the agent observe the signal or not. That is, the signal cannot be simply "in the world".

As usual, we can then decompose the principal's problem into two steps. First, taking effort as given, minimize for each information scenario the expected wage payment $\sum_x [f(x\bar{z}|\bar{e})\, w(x\bar{z}) + f(x\underline{z}|\underline{e})\, w(x\underline{z})]$ over wages, subject to the agent's incentive, limited liability, and participation constraints. The solution of this problem defines the implementation cost function $C(\mathbf{xz}|\bar{e}, \underline{e})$, where $\mathbf{xz}$ is the subset of states $\{\bar{x}\bar{z}, \bar{x}\underline{z}, \underline{x}\bar{z}, \underline{xz}\}$ in which the principal pays a positive wage. Similarly, we define $C(\mathbf{x}|e)$ for the no additional information scenario. Second, the principal minimizes $C(\mathbf{xz}|\bar{e}, \underline{e})$ (or $C(\mathbf{x}|e)$) over efforts, subject to $p\,\bar{e}+(1-p)\,\underline{e} \geq E$ (or $e \geq E$).

Our formulation implies that optimal effort levels are both equal to E *in expectation.* It turns out that this simplifies the derivation of *necessary and sufficient* conditions when asking whether the principal prefers the ex ante information or the no additional information scenario. This is, however, not the reason why we adopted the particular revenue formulation. Even if optimal effort levels were not equal in expectation, we would obtain our results on the basis of *sufficient* conditions, using the following argument. Let e be the optimal effort level when no signal is available and $(\bar{e}', \underline{e}')$ those with signal z'. To prove that the value of signal z' exceeds that of no signal it suffices to show that $C(\mathbf{x}|e) > C(\mathbf{xz}'|e)$ because $C(\mathbf{xz}'|e) \geq C(\mathbf{xz}'|\bar{e}', \underline{e}')$. And analogue to show that the value of no signal exceeds that of signal z'.

3.2.4 Definition of an Informative Signal

As mentioned above we, are interested whether the signal is informative about the agent's effort. For this we compare the three likelihood ratios $l(\bar{x}\bar{z}|e) \equiv \frac{f_e(\bar{x}\bar{z}|e)}{f(\bar{x}\bar{z}|e)}$, $l(\bar{x}\underline{z}|e) = \frac{f_e(\bar{x}\underline{z}|e)}{f(\bar{x}\underline{z}|e)}$ and $l(\bar{x}|e) = \frac{g_e(\bar{x}|e)}{g(\bar{x}|e)}$. Those tell us how likely it is that the agent provided an effort of e, given output (output-signal) realization $\bar{x}$ ($\bar{x}z$). Suppose $l(\bar{x}\bar{z}|e) \neq l(\bar{x}\underline{z}|e)$, which implies that those ratios are also unequal to $l(\bar{x}|e)$. Then we say that *the signal is informative about the agent's effort.* We say it is *uninformative* for $l(\bar{x}\underline{z}|e) = l(\bar{x}\bar{z}|e)$, which implies that those ratios are also equal to $l(\bar{x}|e)$. In the following, we want to focus either on informative or on uninformative signals, and therefore impose that $sign\,(l(\bar{x}\bar{z}|e) - l(\bar{x}\underline{z}|e)) = constant\ \forall e$.
Our definition fits what Demougin and Fluet (1998) call "mechanism sufficiency". A

statistic is said to be mechanism sufficient if the implementation costs for a certain effort depend only on this statistic. It turns out that in our case if likelihood ratios in $\bar{x}\bar{z}$ and $\bar{x}\underline{z}$ are equal then implementation costs $C(\mathbf{xz}|e)$ depend only on $x = \bar{x}$ and not on the signal. Hence, the agent's output is mechanism sufficient, and for brevity we say that the signal is uninformative. If the likelihood ratios are unequal implementation costs depend on the signal, and we say that it is informative.
We assume in the following that $l(\bar{x}\bar{z}|e) \geq l(\bar{x}\underline{z}|e)$ $\forall e$ and we will repeatedly use the following lemma in our later analysis. It deals with the ranking of the likelihood ratios of the informative signal:

Lemma 2 *$l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e) \Leftrightarrow l(\bar{x}\bar{z}|e) > l(\bar{x}|e) > l(\bar{x}\underline{z}|e)$.*

From Lemma 2 it is important to remember that if the state $\bar{x}\bar{z}$ is more informative than $\bar{x}\underline{z}$, then observing solely output $\bar{x}$ is less informative than observing $\bar{x}\bar{z}$, but more informative than observing $\bar{x}\underline{z}$.

3.3 The Wage Scheme

In this section we consider the first part of the principal's problem and compare the implementation cost functions for an informative and an uninformative signal.

3.3.1 No Additional Information and a Brief Digression to Ex Post Information

Instead of moving directly to the ex ante information case – which is the main concern of the chapter – we briefly show that conditioning the agent's contract on an additional informative signal leads to strictly lower implementation costs compared to a situation where the principal observes only the agent's output – given that the signal is observable *ex post*. The aim of this is to show that Holmstrom's (1982)'s sufficient statistic result carries over to a model with a risk neutral and wealth constrained agent. This guarantees that our later results are not driven by "abnormalities" that arise only for such an agent.

With ex post information the principal faces the following maximization problem:

$$\begin{aligned}\min_{\mathbf{w}} \quad & \sum_x \sum_z f(xz|\hat{e})w(xz) \\ & \text{s.t. } \hat{e} \in \arg\max_{e \in \mathcal{E}} \sum_x \sum_z f(xz|e)w(xz) - c(e), \\ & \text{s.t. } \sum_x \sum_z f(xz|\hat{e})w(xz) - c(\hat{e}) \geq 0, \\ & \text{s.t. } w(xz) \geq 0 \ \forall xz.\end{aligned}$$

The following lemma summarizes the solution to this problem.

Lemma 3 $w(\underline{x}\bar{z}) = w(\underline{x}\underline{z}) = 0$.

(i) $l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e) \Rightarrow w(\bar{x}\bar{z}) = c_e(e)/f_e(\bar{x}\bar{z}|e)$ *and* $w(\bar{x}\underline{z}) = 0$.

(ii) $l(\bar{x}\bar{z}|e) = l(\bar{x}\underline{z}|e) \Rightarrow$ *all* $w(\bar{x}\bar{z}) - w(\bar{x}\underline{z})$ *combinations that satisfy the incentive constraint are optimal:* $f_e(\bar{x}\bar{z}|e)w(\bar{x}\bar{z}) + f_e(\bar{x}\underline{z}|e)w(\bar{x}\underline{z}) = c_e(e)$.

In Part (i), where the signal is informative, implementation costs (the expected wage) for effort e are given by:

$$C(\bar{x}\bar{z}|e) = \frac{c_e(e)}{l(\bar{x}\bar{z}|e)}. \tag{3.1}$$

We see that implementation costs are inversely related to the likelihood ratio in state $\bar{x}\bar{z}$. Thus, the lower the ratio, i.e. the more likely it is that the agent provided effort e, the lower are these costs.

In Part (ii), where the signal is uninformative, i.e. $l(\bar{x}|e) = l(\bar{x}\bar{z}|e) = l(\bar{x}\underline{z}|e)$, the implementation costs reduce to:

$$C(\bar{x}|e) = \frac{c_e(e)}{l(\bar{x}|e)}. \tag{3.2}$$

The implementation costs for an uninformative signal do not depend on the signal and hence are equal to the ones when no information at all is available. To see this, note that for the latter scenario the incentive constraint is given by:

$$\hat{e} \in \arg\max_{e \in \mathcal{E}} \sum g(x|e)w(x) - c(e). \tag{3.3}$$

Hence, the principal sets $w(\underline{x}) = 0$ and $w(\bar{x}) = c_e(e)/l(\bar{x}|e)$, resulting in implementation costs $C(\bar{x}|e)$.

We next ask about the value of an additional signal:

Lemma 4 *Suppose that the signal is observable to the agent after he chooses his effort (or not at all). Then implementation costs for all e are:*

(i) strictly lower under an incentive scheme that conditions on an additional informative signal than under one which conditions only on output;

(ii) equal under an incentive scheme that conditions on an additional uninformative signal to the ones which conditions only on output.

Lemma 4 extends Holmstrom's (1979) well known sufficient statistic result to the case of a risk neutral and wealth constrained agent. The only difference is that for an uninformative signal the principal is indifferent for a risk neutral and wealth constrained agent whether or not to condition the contract on such a signal, while for risk averse agents she prefers strictly not to do so. Part (ii) holds for both types of agents.

3.3.2 Ex Ante Information

Now we move to the main case of interest: the agent observes the signal before he chooses his effort. Thus, we have to consider two incentive constraints – one after the signal realization $\bar{z}$ and one after $\underline{z}$:

$$\begin{aligned} \text{after observing } \bar{z}: \ & \bar{e} \in \arg\max_{e \in \mathcal{E}} \ h(\bar{x}|\bar{z}, e) w(\bar{x}\bar{z}) - c(e), \\ \text{after observing } \underline{z}: \ & \underline{e} \in \arg\max_{e \in \mathcal{E}} \ h(\bar{x}|\underline{z}, e) w(\bar{x}\underline{z}) - c(e), \end{aligned}$$

where we already used $w(\underline{x}\bar{z}) = w(\underline{x}\underline{z}) = 0$. In the appendix we show that the costs of implementing an effort vector $(\bar{e}, \underline{e})$ given an informative signal are:

$$C(\bar{x}\bar{z}, \bar{x}\underline{z}|\bar{e}, \underline{e}) = p\, C(\bar{x}\bar{z}|\bar{e}) + (1-p)\, C(\bar{x}\underline{z}|\underline{e}), \tag{3.4}$$

where we define $C(\bar{x}\underline{z}|e) = c_e(e)/l(\bar{x}\underline{z}|e)$, with a slight abuse of the definition of the implementation cost function.

The Value of an Uninformative Signal

If the signal is uninformative, implementation costs again do not depend on the signal and are given by:

$$C(\bar{x}|\bar{e}, \underline{e}) = p\, C(\bar{x}|\bar{e}) + (1-p)\, C(\bar{x}|\underline{e}). \tag{3.5}$$

We see that the main difference to the no additional information scenario – where implementation costs are given by $C(\bar{x}|e)$ – is that the principal can make efforts state contingent. Whether or not she can gain anything from doing so depends on whether the implementation cost function is concave or convex. By Jensen's Inequality we have that for implementation costs that are e.g. convex in effort:

$$p\,C(\bar{x}|\bar{e}) + (1-p)\,C(\bar{x}|\underline{e}) \geq C(\bar{x}|p\,\bar{e} + (1-p)\,\underline{e}).$$

Hence, under an uninformative signal and convex implementation costs the principal could implement the effort $e = p\,\bar{e} + (1-p)\,\underline{e}$ more cheaply than the state contingent effort $(\bar{e}, \underline{e})$. Therefore, the principal optimally sets $\bar{e} = \underline{e}$. For concave implementation costs it is exactly the other way round. In the following we assume that these costs are convex in effort. Strictly speaking this is an assumption on an endogenous function. It is, however, very natural in principal-agent models: it is a sufficient condition for the principal's problem to be strictly concave in effort.

Assumption 4 *The implementation cost function* $\frac{c_e(e)}{l(\bar{x}z|e)}$ *is convex in* e*.*

Hence, the principal optimally sets $\bar{e} = \underline{e}$ under an uninformative signal and we denote the implementation costs for the uninformative, ex ante observable signal by:

$$C(\bar{x}|e) = \frac{c_e(e)}{l(\bar{x}|e)}. \tag{3.6}$$

These are equal to the ones of the no additional information scenario:[3]

Proposition 10 *Suppose the signal is uninformative, i.e.* $l(\bar{x}\bar{z}|e) = l(\bar{x}\underline{z}|e)$ *and* $\bar{e} = \underline{e} = e$*. Then implementation costs under the ex ante information scenario with an uninformative signal are equal to those under the no additional information scenario.*

[3]One should note the difference in the wording compared to Lemma 4. We are now not talking about a contract that can condition on the agent's output versus one that conditions on output and on the signal, but about implementation costs under the different information scenarios. Under the former formulation it would not be clear, whether the agent (ex ante) observes the signal or not. The definition of our scenario includes both: (not) observing the signal and (not) having the possibility to condition the contract on it.

Thus, an uninformative signal does not generate additional benefits. The intuition is simple: neither can the principal exploit the additional signal by conditioning the incentive scheme on it, nor does the observation of this uninformative signal change the agent's incentives: the two incentive constraints we have shown above collapse into one.[4] And this one constraint equals the one for the no additional information scenario.

3.4 The Value of Ex Ante Information

3.4.1 General Effects

We have already seen that an ex ante observable uninformative signal leads to the same implementation costs as if the principal observed only the agent's output. We now move to the question whether additional information is beneficial: i.e. does the principal always prefer to have an informative signal to not having it? From Equations (3.4) and (3.6) it follows that an informative signal yields strictly higher implementation costs than no additional information if and only if:

$$p\,C(\bar{x}\bar{z}|\bar{e}) + (1-p)\,C(\bar{x}\underline{z}|\underline{e}) > C(\bar{x}|e), \tag{3.7}$$

for the optimal effort levels $(\bar{e}, \underline{e})$, with $p\,\bar{e} + (1-p)\,\underline{e} = E$, and $e = E$.

In the next section we present specific examples showing that optimal efforts $(\bar{e}, \underline{e})$ can indeed be such that this holds. The purpose of this section is to flesh out the three conflicting forces that underly such a situation. First, there are more incentive constraints under an informative signal than if the principal observed only the agent's output – and more constraints to the principal's problem cannot decrease implementation costs. Second, the principal can use the information content of the informative signal to reduce implementation costs. Third, making effort state contingent ($\bar{e} \neq \underline{e}$) increases the value of the convex implementation cost function. We will now discuss each effect in turn to see more clearly why or why not an additional signal can harm when observed before the effort choice.

[4]The Lagrange multipliers on the incentive constraints in the principal's problem are equal.

More Incentive Constraints

There are more incentive constraints for the ex ante information scenario than for the no additional information scenario, as we have seen in Sections 3.3.1 and 3.3.2. Hence, when asking about the benefits of an additional informative signal we do not only vary the informativeness, but also the number of incentive constraints. To separate the latter effect, we keep the information content constant and compare an informative ex post (one incentive constraint) to an informative ex ante signal (two incentive constraints). The difference in implementation costs for these two scenarios is:

$$p\, C(\bar{x}\bar{z}|\bar{e}) + (1-p)\, C(\bar{x}\underline{z}|\underline{e}) > C(\bar{x}\bar{z}|e), \tag{3.8}$$

where the left hand side gives us the implementation costs for the ex ante information scenario and the right hand side the ones for the ex post information scenario and $E = p\,\bar{e} + (1-p)\,\underline{e}$ and $e = E$.
The intuition for the strict inequality in Equation (3.8) is following (for a formal proof see Chapter 1). Under the ex ante information scenario the principal has to pay a positive wage after both signal realizations ($\bar{z}$ and $\underline{z}$) to induce the agent to work in every state of the world. In comparison, for the ex post information scenario it is optimal to pay only a wage if $\bar{x}\bar{z}$ realizes. An additional reward in $\bar{x}\underline{z}$ – which is the less informative state – therefore increases ex ante implementation costs. Hence, the two incentive constraints under ex ante information indeed lead to an increase in implementation costs.

More Information

Next we ask what happens if we vary simultaneously the number of incentive constraints (first effect) *and* the information content of the signal (second effect), while abstracting from the third factor by imposing the same implemented effort in all states of the world. For this we consider Equations (3.4) and (3.6), setting $\bar{e} = \underline{e} = e$. Compared to the previous paragraph, the wage for the one incentive constraint scenario (the no additional information scenario) conditions now only on $\bar{x}$ and not on $\bar{z}$ anymore. Thus, we have to replace $l(\bar{x}\bar{z}|e)$ on the right hand side of Equation (3.8) with $l(\bar{x}|e)$. The left hand side is unchanged.

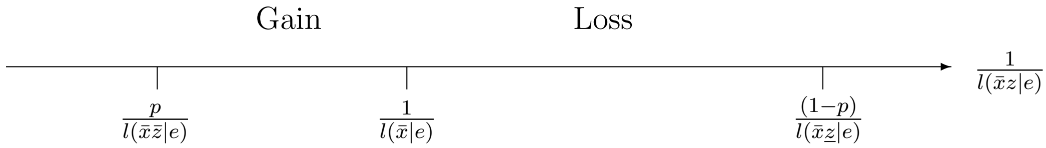

Figure 3.1: Gains and Losses of an Informative Signal

Remember from Lemma 2 that $\bar{x}\bar{z}$ is more informative than $\bar{x}\underline{z}$ about the agent's effort. This implies that observing $\bar{x}\bar{z}$ is more informative than $\bar{x}$. However, observing only the output $\bar{x}$ is more informative than observing $\bar{x}\underline{z}$. Implementation costs are inversely related to the likelihood ratios. Hence, the principal can implement the same effort cheaper in state $\bar{x}\bar{z}$ than in $\bar{x}$, but at higher costs than in $\bar{x}\underline{z}$. Figure 3.1 then illustrates that the loss caused by the informative signal – having to pay the agent in a state, where it is *less* likely than in $\bar{x}$ that he provided a certain effort – can be higher than the gain – having the opportunity to provide incentives in a state where it is *more* likely than in $\bar{x}$ that he provided a certain effort.
The conclusion is reversed for the case where the gain outweighs the loss. This gives us immediately a sufficient condition for an additional informative signal to be beneficial. In the appendix we show:

Proposition 11 *Suppose the signal is informative, i.e.* $l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e)$ *and* $h_e(\bar{x}|\bar{z}, e) < h_e(\bar{x}|\underline{z}, e)$ $\forall e$. *Then implementation costs for the ex ante information scenario with an informative signal are strictly lower than those for the no additional information scenario.*

The condition in the proposition tells us that if the gain is larger than the loss for all efforts e then the informative signal is beneficial: implementation costs are already lower for $\bar{e} = \underline{e} = e = E$, which need not be optimal. Hence, the principal cannot do worse by having the opportunity to set $\bar{e} \neq \underline{e}$.

Convex Implementation Costs and State Contingent Effort

Observing an informative signal changes the agent's incentives, as we have seen in the last two sections. Thus, payoffs are altered even if the principal ignores the information. We are now interested whether the principal prefers sometimes to have no additional information for cases where the loss outweighs the gain (in the context of the previous section with equal efforts $\bar{e} = \underline{e} = e$). That is, can the principal always choose efforts $\bar{e} \neq \underline{e}$ in such a way to reduce the costs, or increase the benefit sufficiently for an informative signal to do better? The following is a sufficient condition that she cannot, using the fact that $p\,C(\bar{x}|\bar{e}) + (1-p)\,C(\bar{x}|\underline{e}) \geq C(\bar{x}|e)$:

$$p\left[\frac{1}{l(\bar{x}\bar{z}|\bar{e})} - \frac{1}{l(\bar{x}|\bar{e})}\right] c_e(\bar{e}) + (1-p)\left[\frac{1}{l(\bar{x}\underline{z}|\underline{e})} - \frac{1}{l(\bar{x}|\underline{e})}\right] c_e(\underline{e}) > 0. \qquad (3.9)$$

For more information to harm the equation can either hold for all $(\bar{e}, \underline{e})$ or at least for the optimal effort vector. Again, the first term in brackets gives us the gain and the second the loss – now evaluated at state contingent effort levels.

As we used an upper bound for $C(\bar{x}|e)$, the violation of Equation (3.9), however, does not tell us that an informative signal is beneficial. Efforts under the ex ante information scenario are state contingent, in contrast to the no additional information scenario under which they are not. But introducing a spread between efforts increases the value of the convex implementation cost function under the ex ante relative to the no information scenario (by Jensen's Inequality). If the gain is much smaller than the loss (evaluated at equal efforts), the spread between $\bar{e}$ and $\underline{e}$ required to outweigh this might drive up convex implementation costs too much. As the examples in the following show, it might not be possible to outweigh the loss.

3.4.2 Examples

Optimal efforts are hard to characterize generally and thus we present in the following three examples that show that more information can harm. The first example gives a rather general condition for the convexity effect to dominate. The second one draws on the discrete effort model that is used in many applications. The third one employs a probability function that is linear in effort, making the computation of optimal efforts tractable.

Example 1

Let $p = \frac{1}{2}$. Suppose that implementation costs in state $\bar{x}\underline{z}$ are so high that it is optimal to set $\underline{e} = 0$. This is the case for $C_e(\bar{x}\underline{z}|0) > C_e(\bar{x}\bar{z}|2E)$.[5] Under

[5] To see this, note that the first order conditions of the principal's problem with respect to effort are: $C_e(\bar{x}\underline{z}|\underline{e}) = (1-p)\,\lambda$ and $C_e(\bar{x}\bar{z}|\bar{e}) = p\,\lambda$, where λ is the Lagrange multiplier.

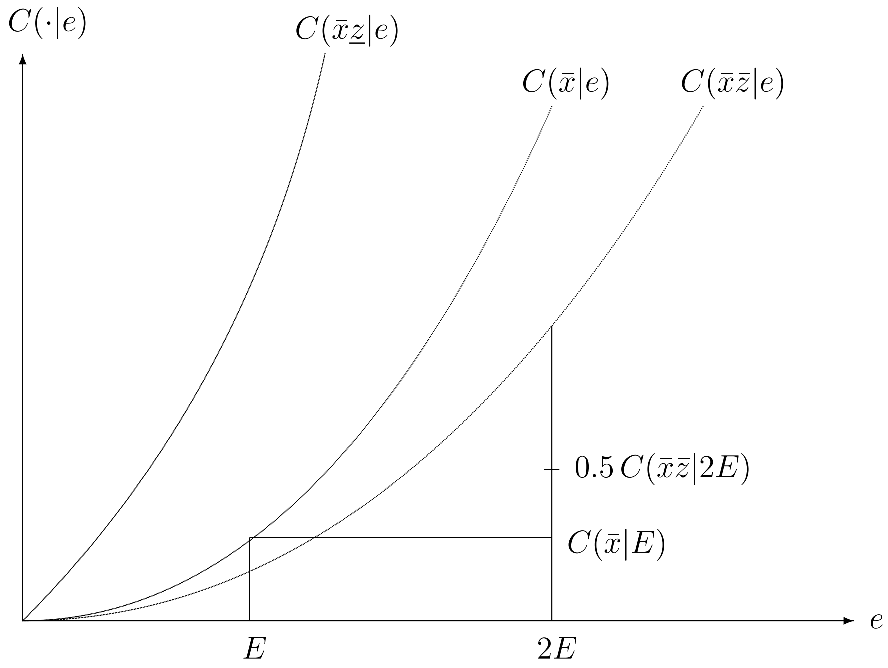

Figure 3.2: Illustration of Example 1

this condition, increasing $\underline{e}$ away from zero raises implementation costs more than decreasing $\bar{e}$ away from the highest effort level $2E$ would reduce them. But then, although we have that implementation costs in state $\bar{x}\bar{z}$ are lower than those when one observes only output ($C(\bar{x}\bar{z}|e) < C(\bar{x}|e)$), the state contingent effort level can drive up the strictly convex implementation costs so much that implementation costs are lower when observing only the agent's output: $C(\bar{x}|E) < 0.5\,C(\bar{x}\bar{z}|2E)$ can happen as we illustrate in Figure 3.2. Hence, an additional informative signal can harm.

Example 2

Let effort be discrete $e \in \{e_L, e_H\}$, $c(e_H) = 1$, $c(e_L) = 0$ and $p = \frac{1}{2}$. Here the likelihood ratio is $l(s) = \frac{f(s|e_L)}{f(s|e_H)}$ and hence $C(s|e_H) = \frac{c}{1-l(s)}$, $s \in \{\bar{x}, \bar{x}\bar{z}\}$. Suppose that it is optimal (or required) to implement high effort in all states of the world. Then implementation costs are lower without the additional information if and only

$p(xz\|e)$	$\bar{x}$	$\underline{x}$	
$\bar{z}$	$a\,e\,p + k$	$p\,(1 - a\,e) - k$	p
$\underline{z}$	$e\,(1 - a\,p) - k$	$1 - e - p + a\,e\,p + k$	$1 - p$
	e	$1 - e$	1

Table 3.2: Joint Distribution. Let $f(xz|e)$ be $\max\{0, p(xz|e)\}$ or $\min\{p(xz|e), 1\}$.

if the loss is larger than the gain, or:

$$\begin{aligned} & C(\bar{x}\bar{z}|e_H) + C(\bar{x}\underline{z}|e_H) > 2\,C(\bar{x}|e_H) \\ \Leftrightarrow \quad & [l(\bar{x}\bar{z}) + l(\bar{x}\underline{z})]\,[1 - l(\bar{x})] + l(\bar{x}\bar{z})\,l(\bar{x}\underline{z}) > 2\,[1 - l(\bar{x})]\,. \end{aligned}$$

Example 3

Let the joint distribution be as in Table 3.2. Furthermore, let $c(e) = \frac{e^2}{2}$. Plugging this into Equations (3.1) and (3.4), using $E = p\,\bar{e} + (1-p)\,\underline{e}$ and subtracting, we obtain:

$$\Delta(\bar{e}, \underline{e}) = p\,(1-p)\,(\bar{e} - \underline{e})^2 + k\left(\frac{1}{a}\,\bar{e} - \frac{1-p}{1-a\,p}\,\underline{e}\right). \tag{3.10}$$

If and only if $\Delta(\bar{e}, \underline{e}) > 0$ for the optimal effort vector $(\bar{e}, \underline{e})$ implementation costs for the informative signal scenario are strictly higher than for the no additional information scenario. The first part of Equation (3.10) shows how trying to outweigh the loss by setting $\bar{e} \neq \underline{e}$ increases convex implementation costs. The second term gives us the possible loss ($\bar{x}\underline{z}$ is less informative about effort than $\bar{x}$) and gain ($\bar{x}\bar{z}$ is more informative) caused by an informative signal. Hence, depending on the optimal effort levels, this second term can be positive or negative.

Lemma 5 *The optimal efforts are* $\bar{e} = E + p\,\mu$ *and* $\underline{e} = E - (1-p)\,\mu$*, with* $\mu = -\frac{k}{2\,a\,p\,(1-a\,p)}$*. Hence,* $\bar{e} > \underline{e}$ *if and only if* $l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e)$*.*

The principal chooses effort levels in a way that mirrors exactly the informativeness of a state of the world: as (by assumption) under $\bar{x}\bar{z}$ it is more likely that the agent provided a certain effort than under $\bar{x}\underline{z}$, the principal implements a larger effort following $\bar{x}\bar{z}$ than following $\bar{x}\underline{z}$. Plugging these optimal effort levels into Equation (3.10), we obtain the following proposition, which shows how the optimality of additional information depends on the parameters:

Proposition 12

(i) for $k < 0$ and $a \leq 1$, or for $k > 0$ and $a \in \left[1, \frac{1}{p}\right)$ implementation costs for the informative signal are strictly lower than the ones for the no additional information scenario;

(ii) For $k > 0$ and $a < 1$, or for $k < 0$ and $a \in \left(1, \frac{1}{p}\right)$ there exist values for (a, p, k, E), such that implementation costs for the informative signal are strictly higher than the ones for the no additional information scenario.

To understand better Part (i) of Proposition , suppose the principal ignores the information and simply sets $\bar{e} = \underline{e}$. Then the first term of Equation (3.10) vanishes and the sign of the second one is determined by $k\,(1-a)$. For $k = 0$ or $a = 1$ this is zero, and obviously, more information is never worse. For $k > 0$ and $a \in \left(1, \frac{1}{p}\right)$, or for $k < 0$ and $a < 1$, the term is negative, and more information has to do strictly better by Proposition 11. Finally, for $k < 0$ and $a \in \left(1, \frac{1}{p}\right)$, or for $k > 0$ and $a < 1$ the term is strictly positive, and an additional ex ante signal may harm as Part (ii) shows.

The intuition for the result is the following. By varying k and a we can vary the informativeness of the signal. For $k = 0$ the signal is uninformative about the agent's effort – no matter which value a takes. For $k \neq 0$ the signal is informative, where $k < 0$ corresponds to $l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e)$. The parameter a cannot influence whether the signal is informative about effort, but how informative it is: it affects the marginal posterior $h(\bar{x}|z, e)$ and by this the relation between the gain and the loss (compare Proposition 11).[6] Combining these two parameters, we see that for $k < 0$ it is more likely in $\bar{x}\bar{z}$ than in $\bar{x}\underline{z}$ that the agent provided a certain effort. As long as $a < 1$, it is also *relatively* more likely in $\bar{x}\bar{z}$ compared to $\bar{x}$ than in $\bar{x}$ compared to $\bar{x}\underline{z}$. But for $a > 1$ it is relatively more likely in $\bar{x}$ compared to $\bar{x}\underline{z}$ than in $\bar{x}\bar{z}$ compared to $\bar{x}$. For $a = 1$ these distances are equal.

So far we discussed for which parameters the gain can be smaller than the loss. What still needs to be explained is why optimal effort levels are not always such that the

[6]Both parameters affect how informative the signal is about the agent's output. See Chapter 1 for a more extensive discussion of the different information concepts and how the informativeness about outputs and effort varies with the parameters of this joint probability function.

loss is outweighed. As we discussed in Section 3.4.1, and as we see from Lemma 5, the principal has to introduce a spread between efforts $\bar{e}$ and $\underline{e}$ to outweigh it. This increases the value of the convex implementation cost, which can be seen from the first part of Equation (3.10). If for $\bar{e} = \underline{e}$ the loss of the informative signal is much larger than the gain, one cannot find an effort combination $\bar{e} \neq \underline{e}$ to outweigh the loss without driving up the convex implementation costs too much.[7]

3.5 Discussion of Robustness

So far we assumed that the principal has to implement an expected effort of E. The standard formulation would be to give her a revenue $\bar{x}$ if the agent produces a high output and a revenue $\underline{x} = 0$ if he produces a low one – resulting in expected revenues $[f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\underline{z}|\underline{e})]\,\bar{x}$.

As already mentioned, the reason for our modelling choice was to separate the incentive effects of ex ante information completely from effects on expected revenues. For example, we have seen that making effort state contingent can never reduce implementation costs when the signal is uninformative about effort. But there are circumstances where this can pay to increase revenues minus implementation costs when the signal is uninformative about effort, but yet there are informational gains. We discussed a situation like this in Chapter 1 (the times-a model). Similarly, gains from tailoring effort in the revenue function can lead to different predictions for the comparison of the informative signal with the no additional information scenario. However, it is not clear what is driven by the incentive effect and what by the revenue effect. As it is well known that ex ante information is (weakly) beneficial for decision problems as is maximizing revenues one, we decided to focus here solely on the incentive side.

So what changes if the principal receives revenues that are coupled to output? Of

[7] One should note the difference to simple decision problems. Here this convexity effect can never harm: for $\bar{e} = \underline{e}$ profits would be equal for an informative signal and for no additional signal. Hence, the principal chooses to set $\bar{e} \neq \underline{e}$ only if the decrease in profits due to the convexity effect is smaller than the gains from tailoring effort. In contrast, for our case, profits are strictly lower for $\bar{e} = \underline{e}$.

course the derivation of the wage schemes for any information scenario is not affected (Section 3.3), nor is any result for the ex post information case (Lemma 4 and the discussion afterwards). What changes is the profit function because we add revenues and by this, the choice of the optimal effort levels in Section 3.4. Furthermore, any necessary and sufficient condition would only be sufficient.

To see that our results are robust to this model reformulation, reconsider Examples 1 and 2. Given that it is optimal to implement high effort in all states of the world[8] – expected revenues in Example 2 are equal for both scenarios: $f(\bar{x}\bar{z}|e_H)+f(\bar{x}\underline{z}|e_H) = g(\bar{x}|e_H)$. Hence, the analysis and the results are exactly the same as before.

For Example 1 we can also find parameter values such that $\underline{e} = 0$ is optimal when the signal is informative: implementation costs can be too high in this less informative state (e.g. $f(\bar{x}\underline{z}|e)\bar{x} \leq (1-p)\, C(\bar{x}\underline{z}|e)$).[9] Letting $\bar{e}$ be the optimal effort level under the informative signal, a sufficient condition for this scenario to lead to lower profits is: $[f(\bar{x}\underline{z}|p\,\bar{e}) - f(\bar{x}\underline{z}|\bar{e})]\bar{x} > C(\bar{x}|p\,\bar{e}) - p\, C(\bar{x}\bar{z}|\bar{e})$. This is consistent with the above condition for e.g. $p\, C(\bar{x}\bar{z}|e) + (1-p)\, C(\bar{x}\underline{z}|e) > C(\bar{x}|e)$, which we have seen can hold.

In sum, our results are robust to adding standard revenues to the principal's profit function. More general examples with optimal effort levels are, however, harder to derive than under our model specification.

3.6 Conclusion

The sufficient statistic result says that conditioning an agent's incentive scheme on an ex post observable and informative signal is beneficial. In this chapter we showed that if the agent observes the signal before he chooses his effort, implementation costs can be strictly lower than if he and the principal observed no additional signal. This implies that information about an agent's effort can be harmful. While there are many papers that showed this for incomplete contracts or asymmetric information, the contribution of our result is that it relies on neither of these two restrictions:

[8] This is optimal if and only if $[f(s|e_H) - f(s|e_L)]\bar{x} > C(s|e_H)$: it is easy to see that there always exist values of $\bar{x}$ for which this is satisfied.

[9] One can construct such a situation with e.g. a linear probability of success function.

the principal as well as the agent observe the signal and they can write a complete contract.

Chapter 4

The Peter Principle Revisited

4.1 Introduction

According to the *Peter Principle* formulated by Peter and Hull (1969) "in a hierarchy every employee tends to rise to his level of incompetence." This captures two stylized facts about hierarchies: first, promotions often place employees into jobs for which they are less well suited than for the previously held position. Second, demotions are extremely rare (Baker, Gibbs, and Holmstrom 1994a, Baker, Gibbs, and Holmstrom 1994b, Gibbons and Waldman 1999). While the former is not surprising – mistakes can happen – the latter is puzzling; why do organizations not correct "wrong" promotion decisions? Perhaps institutional constraints prevent demotions, but surely an organization that perfectly knew its employees' abilities would make sure that each individual's effort leads to the highest output in his or her current job. While compelling, this intuition is flawed. This chapter shows that a simple trade-off between incentive provision and efficient job assignment leads organizations to promote some employees to a job at which they produce less than they would for the same effort in the job held at the previous level.
For this we employ a very simple moral hazard model: a risk neutral and wealth constrained agent can work in one of two jobs (a higher and a lower hierarchy one), where the probability that he succeeds in producing a high output depends in each job on his effort (high or low) and his ability.

In our model, the promotion decision depends on the strength of two effects. On

the one hand, promoting the worker reduces the cost of implementing a given effort level if and only if output in the higher level job is more informative about his effort. Empirically, this is the relevant case: jobs further up in the hierarchy of an organization require higher levels of ability, and their output tends to be more informative about the employee's effort (e.g., Maskin, Qian, and Xu (2000), or Ortega (2003)). This is intuitive: if higher hierarchy jobs require a higher ability, seeing a low ability worker succeeding in such a job – which is difficult form him – indicates that he worked really hard. On the other hand, for a given effort level the worker's ability may be such that his success probability is higher in the current job compared to the higher hierarchy one.
Overall, for some ability levels the reduction in the cost of incentives outweighs the negative effect of a reduced success probability, thus creating an upward distortion in job assignment decisions: the promotion threshold for ability is lower than the full information (first best) benchmark. As a consequence, some workers are promoted even though they would produce more in their current job if they exerted the same level of effort (the Peter Principle). Note that this is not a result of the standard effort distortion arising in a moral hazard model, but driven by the differences across the two jobs in informativeness about effort.

Our model delivers empirical predictions which are consistent with the evidence from the personnel economics literature. Along with a rise in the hierarchy an employee often finds that he works more rather than less, and he is not always happier than before. The reason is that promotions to the more informative higher level job allow the principal to keep employees on toes and to extract a high effort from them at lower costs. This delivers an explanation of Peter and Hull's (1969) observation that some individuals are "promoted to their level of incompetence". Our model links to the Peter Principle the empirical pattern that wages at the top of a job exceed those at the bottom of the wage distribution in the next higher level job (Baker, Gibbs, and Holmstrom 1994a, Baker, Gibbs, and Holmstrom 1994b).

This chapter is structured as follows. In the next paragraph we discuss the related literature. Section 4.2 describes the model and Section 4.3 presents the results. Section 4.4 concludes. Section 4.5 extends the model to capture the fact that agents are not only extrinsically motivated, but also extrinsically.

Related Literature

There is a large literature on careers in organizations and the role of job assignments. One explanation for the use of promotions is learning about the ability of workers. Firms initially place employees in jobs where they can do little harm. Those who prove to be able are then promoted to positions that require a higher level of ability.[1] A puzzle is why demotions are rare even though many workers are less productive in their job than they would be in a position lower in the hierarchy. Lazear (2004) offers an explanation based on temporary shocks to productivity. A promotion means that a worker has delivered a high measurable output, which is the sum of permanent and transitory components. Because of regression to the mean of the temporary productivity shocks, measured output is expected to decline after a promotion. Our approach is complementary to this. We show that a principal who is perfectly informed about an agent's ability would promote some workers to a higher level task at which they are permanently less productive.

The literature offers two other explanations for the Peter Principle. Fairburn and Malcomson (2001) assume that workers can sway the performance evaluation of their supervisors with direct bribes. This makes incentive pay ineffective for the principal: supervisors and their workers would simply collude to extract high wages without any high effort in return. Making workers' pay contingent on their job, and the managers' pay contingent on the firm's profits, aligns managers' interests more closely with those of the firm's owners: promoting a very untalented worker reduces profits, and thus hurts the supervisor's pay, more than the the supervisor would gain from extracting a bribe. Still, some workers close to the efficient promotion threshold are promoted even though they should not be. Faria (2000) formalizes an idea from Peter and Hull (1969). He argues that promotion is based on the performance in one task but that the skills required in the new task may be quite different. So, inevitably, some workers turn out to be less competent in the new job than they were in the old one. All these papers, however, do not address why

[1] See Prescott and Visscher (1980) and the survey of the subsequent literature in Gibbons and Waldman (1999).

"wrong" promotion decisions are not reversed.

From a theoretical point of view, our model is most closely related to Robbins and Sarath (1998). They show that the principal may not always want to choose the output system that generates the highest revenues. Different output systems vary in how informative they are about the agent's effort, where a more informative system generates lower implementation costs. But such a more informative system need not be the one that also generates higher revenues. Output systems in their paper correspond in our setting to agents with different abilities in the two jobs. Our application allows us to look at a continuum of output systems rather than only two systems as in Robbins and Sarath (1998). This enables us to explain the Peter Principle.

4.2 The Model

A risk neutral principal can employ a risk neutral agent for two work periods. The agent is protected by limited liability, and has a reservation utility of zero. The principal can assign the agent to one of two jobs, $j \in \{1,2\}$. In each job the agent produces a verifiable and observable output x, which equals the principal's revenue. Output can either be high ($\bar{x} = 1$) or low ($\underline{x} = 0$). The probability of a high output, $f_j(e,\theta)$, depends on the agent's ability $\theta \in [\theta_L, \theta_H]$, the job $j \in \{1,2\}$ and effort $e \in \{e_L, e_H\}$, with $f_j : [\theta_L, \theta_H] \times \{e_L, e_H\} \rightarrow (0,1)$ and $f_j(\cdot,\theta) \in C^2$. We assume that higher effort leads to a higher success probability, i.e. $f_j(e_H,\theta) > f_j(e_L,\theta)\ \forall\, \theta$. The agent's cost of effort is c if he provides high effort and zero otherwise.

We say that job 1 is the *high ability* job and job 2 the *low ability* one. This is captured by the following *sensitivity to ability* (SA) and *single crossing* (SC) assumptions, where subscripts denote partial derivatives with respect to θ:

Assumption 5

(SA) $0 = f_{2\theta}(e_L,\theta) = f_{2\theta}(e_H,\theta) < f_{1\theta}(e_L,\theta) \leq f_{1\theta}(e_H,\theta)$.

(SC) $f_1(e,\theta_L) < f_2(e,\theta_L)$ *and* $f_2(e,\theta_H) < f_1(e,\theta_H)$ *for* $e \in \{e_L, e_H\}$.

Assumption SA consists of two components. First, it says that expected output in job 1 increases with higher ability, and (weakly) more so if higher effort is provided.

Thus, effort and ability are weak complements. Second, it says that job 2 entails "safe" tasks, where a low ability agent can do no harm as the success probability is insensitive to ability. Hence, we simplify notation to $f_2(e) \equiv f_2(e, \theta)$, hereafter.
Assumption SC states that for the highest ability level θ_H the success probability in job 1 exceeds the one in job 2, given the same effort level. In contrast, an agent with the lowest ability level θ_L is more productive in job 2 than in job 1, given the same effort level. Together with Assumption SA it implies that for each given pair of equal efforts across jobs, (e, e), there exists a unique type $\hat{\theta}_e$, such that all types with $\theta \geq \hat{\theta}_e$ have a higher success probability in job 1 than in job 2, and all types with $\theta < \hat{\theta}_e$ a lower success probability. This is illustrated in Figure 4.1. The figure also shows that the probability functions in the two jobs need not cross for unequal efforts. Thus, our single crossing assumption is weaker than the standard one where the two functions cross once for every effort pair.
The Peter Principle refers to cases where the principal chooses a cutoff different from $\hat{\theta}_e$ and the agent produces less in the higher hierarchy job, say, job j (we define the higher hierarchy job below) than if he were reassigned to the former job given he exerts the same effort level as before:

Definition 2 *The Peter Principle holds for an agent with ability θ who is assigned to job j, in which he exerts effort e, if*

$$f_j(e, \theta) < f_i(e, \theta), \quad i \neq j.$$

There is another way to define the Peter Principle: compare expected outputs. This would be misleading, however, as output could simply rise because the agent works much more – hiding the fact that he is untalented for his job. Thus, we compare expected outputs keeping the effort level fixed across jobs.

The timing is as follows. At date 1 the principal offers a contract $[J(\theta), \mathbf{w}]$, consisting of a job assignment rule $J : [\theta_L, \theta_H] \rightarrow \{1, 2\}$, as well as an output contingent wage schemes for each job $\mathbf{w} = [(w_1(\underline{x}), w_1(\bar{x})), (w_2(\underline{x}), w_2(\bar{x}))]$. At this time, the principal and the agent do not know the agent's ability (see below), but it is common knowledge that types are distributed according to the function $\Phi(\theta)$, with $\Phi : [\theta_L, \theta_H] \rightarrow [0, 1]$ and density $\phi(\theta)$, $\phi : [\theta_L, \theta_H] \rightarrow I\!R$. At date 2, after the principal and agent observed θ, the agent is assigned to job $J(\theta)$ and provides unobservable effort. At the last date, output and payoffs realize. The principal receives the revenue and pays the wage to the agent; the agent receives utility equal to the wage minus his effort costs.

Extension of the Model to Capture Promotion Decisions

So far our model captures only job assignments. The framework extends, however, to promotion decisions without affecting the analysis if we add a second work period and modify the timing as follows. At date 1, when the agent is hired, his ability is unknown to all parties and he receives training at date $1\frac{1}{2}$ (the first work period). Training produces an observable and verifiable perfect signal about the agent's ability θ at the end of the period. In the second period, the agent has experience (tenure=1), and everything else is as above.

Let productivity in a job increase with experience (*learning by doing*). That is, we assume that the agent's success probability depends also on his tenure with the principal ($t = \{0, 1\}$): $f_j(e, \theta, t)$, where $f_j(e, \theta, 1) = f_j(e, \theta)$ as described above. A newly hired agent (i.e. with $t = 0$) has a lower success probability: $f_j(e, \theta, 0) = f_j(e_L, \theta)$ for $e \in \{e_L, e_H\}$. That is, the trainee's success probability in task j equals that of an experienced worker who puts in low effort. Hence, it follows that the principal sets wages $W(\underline{x}) = W(\bar{x}) = 0$ for the first work period. We assume that the expected ability of a trainee is such that at date $1\frac{1}{2}$ it is efficient to assign the agent to the low ability job 2.[2]

Assumption 6 $f_2(e_L) > E[f_1(e_L, \theta)]$.

Thus, a worker will in the second work period either stay in the entry level job 2 or be promoted to job 1. This captures the notion that jobs further up in the hierarchy of an organization require higher levels of ability.

[2]For example, $E[\theta] < \hat{\theta}_L$ and $f_{1\theta\theta} \leq 0$ would be sufficient for this.

4.3 Analysis

4.3.1 Observable Effort (First Best)

As a benchmark, suppose first that the principal can observe the agent's effort choice. Thus, to implement high effort in the second work period in task j, she pays $w_j(\underline{x}) = w_j(\bar{x}) = c$ if the agent provides the desired effort and zero otherwise. The resulting profits from assigning an agent of ability θ to job j are $\Pi_j^{FB}(e_H, \theta) = f_j(e_H, \theta) - c$. In contrast, to implement low effort, she sets $w_j(\underline{x}) = w_j(\bar{x}) = 0$ and her profits are $\Pi_j^{FB}(e_L, \theta) = f_j(e_L, \theta)$. For the second best problem to be interesting, we assume that the principal always wants to implement high effort in the first best:

Assumption 7 $\Delta f_j(\theta) \equiv f_j(e_H, \theta) - f_j(e_L, \theta) > c$, $j = 1, 2$.

The first-best promotion threshold to job 1 is efficient: the principal's job assignment rule is $J^{FB}(\theta) = 2$ for all $\theta < \theta^{FB}$ and $J^{FB}(\theta) = 1$ for all $\theta \geq \theta^{FB}$, where $\theta^{FB} = \hat{\theta}_H$. This follows from $\Pi_1^{FB}(e_H, \theta^{FB}) = \Pi_2^{FB}(e_H, \theta^{FB})$ if and only if $f_1(e_H) = f_2(e_H, \theta^{FB})$. Thus, the agent is always assigned to the job where he is more productive and the Peter Principle is not present. The reason is the following: implementation costs are equal to c in both jobs and thus the principal only looks at the expected outputs when deciding to which job she should assign the agent.

4.3.2 Unobservable Effort (Second Best)

Suppose now that the principal cannot observe the agent's effort. We solve the principal's problem for the second work period with the usual two-step procedure. First, for a given effort level e, we find the cost minimizing wage scheme that implements e. Second, given these wage schemes we maximize profits with respect to effort.

Consider the first part of the problem. If the principal wants to implement low effort in job j she simply sets $w_j(\underline{x}) = w_j(\bar{x}) = 0$, resulting in profits $\Pi_j^{SB}(e_L, \theta) = f_j(e_L, \theta)$. If she wants to implement high effort, her problem is to minimize the expected wage bill,

$$f_j(e_H, \theta)\, w_j(\bar{x}) + [1 - f_j(e_H, \theta)]\; w_j(\underline{x}),$$

subject to the agent's incentive constraint,

$$\Delta f_j(\theta)\; [w_j(\bar{x}) - w_j(\underline{x})] \geq c,$$

where $\Delta f_j(\theta) \equiv f_j(e_H, \theta) - f_j(e_L, \theta)$, limited liability constraint, $w_j(x) \geq 0 \, \forall x$ and participation constraint. The latter is always satisfied if the other two constraints are, as the agent's reservation utility is zero. The solution to this problem is to set the wage after failure equal to zero, i.e. $w_j(\underline{x}) = 0$ and choose the wage after a success such that the incentive constraint holds with equality, i.e. $w_j(\bar{x}) = \frac{c}{\Delta f_j(\theta)}$. Hence, the profits from assigning an agent with type θ to job j and implementing high effort are:

$$\Pi_j^{SB}(e_H, \theta) = f_j(e_H, \theta) - C_j(e_H, \theta), \quad C_j(e_H, \theta) = \frac{f_j(e_H, \theta)}{\Delta f_j(\theta)} \, c, \tag{4.1}$$

where $C_j(e_H, \theta)$ is the expected wage payment for high effort, to which we will refer in the following as the *implementation costs.*

It is optimal to implement high effort in job j if and only if the profits from doing so are higher than the profits from implementing low effort, i.e. if and only if: $\Pi_j^{SB}(e_H, \theta) \geq \Pi_j^{SB}(e_L, \theta)$. Rearranging gives the following condition:

$$[\Delta \, f_j(\theta)]^2 \geq f_j(e_H, \theta) \, c. \tag{Condition 1}$$

If this condition is satisfied then profits in job 1 and the difference in profits across jobs increase with the agent's type:

Lemma 6 *Suppose Condition 1 holds, then:*

(a) $\Pi_{1\theta}^{SB}\,(e_H, \theta) > 0$*,*

(b) $\frac{d}{d\theta}\,\left[\Pi_1^{SB}\,(e_H, \theta) - \Pi_2^{SB}(e, \theta)\right] > 0$ *for* $e \in \{e_L, e_H\}$*.*

We first analyze job assignments for the case where the principal implements high effort in both jobs, and thereafter consider the remaining cases.

High Effort Optimal in Both Jobs

Suppose that Condition 1 is satisfied for both jobs, i.e. the principal induces the agent to provide high effort in both jobs. To determine the cutoff for the principal's job assignment rule, set $\Pi_1^{SB}\left(e_H, \theta^{SB}\right) = \Pi_2^{SB}\left(e_H, \theta^{SB}\right)$. This leads to the following proposition:

Proposition 13 *Suppose Condition 1 holds for $j = 1, 2$. Then, if and only if*

$$f_1\left(e_L, \hat{\theta}_H\right) < f_2(e_L), \qquad \text{(Condition 2)}$$

(a) the promotion threshold to the ability sensitive task 1 is lower than the first-best threshold: $\theta^{SB} < \theta^{FB}$;

(b) the Peter Principle is valid for those at the bottom of the ability range in job 1: for $\theta \in [\theta^{SB}, \theta^{FB})$, $J^{SB}(\theta) = 1$ even though $f_1(e_H, \theta) < f_2(e_H)$.

Before explaining this proposition in more detail, we state the next result which describes empirical predictions related to the agent.

Proposition 14 *The expected wage payment and expected utility after a promotion are:*

(a) lower than that in job 2 for $\theta^{SB} \leq \theta \leq \theta'$ and is higher than that in job 2 for $\theta' > \theta$, where $\theta^{SB} < \theta' < \theta_H$ if and only if

$$\frac{f_2(e_H)}{f_2(e_L)} > \frac{f_1(e_H, \theta_H)}{f_1(e_L, \theta_H)}; \qquad \text{(Condition 3)}$$

(b) everywhere increasing in θ if and only if

$$\frac{f_{1\theta}(e_L, \theta)}{f_1(e_L, \theta)} > \frac{f_{1\theta}(e_H, \theta)}{f_1(e_H, \theta)}. \qquad \text{(Condition 4)}$$

To understand the driving forces behind Proposition 13, consider first the effect of a promotion on the revenues generated by the agent. Given that the principal implements high effort in both jobs, output in job 1 is higher than in job 2 for $\theta \geq \theta^{FB} = \hat{\theta}_H$, as illustrated for a numerical example in Figure 4.1. But for the principal's profits it also matters what the effect is on the cost of implementing high effort. Hence, the promotion threshold θ^{SB} depends on the combination of both effects. To determine whether θ^{SB} is lower or higher than θ^{FB} we exploit Lemma 6. With Condition 1 the profit difference across jobs is increasing in θ and thus $\theta^{SB} < \theta^{FB}$ if and only if it is more profitable to place an agent with ability level $\theta = \theta^{FB} = \hat{\theta}_H$ in job 1 than in job 2. But note that at this ability level the job allocation does not change the expected revenues because the agent is equally productive in both jobs if he works hard. Therefore, the profit difference is equal to the difference in implementation costs. These costs go down when promoting to job 1 an agent with $\theta = \theta^{FB} = \hat{\theta}_H$ if and only if $f_1\left(e_L, \theta^{FB}\right) < f_2(e_L)$ (Condition 2) holds.

Before we discuss the implications of Propositions 13 and 14 we want to ask about the relevant cases to consider. Empirical evidence suggests that positions further up in the hierarchy tend to be more informative about an employee's effort (Maskin, Qian, and Xu 2000, Ortega 2003). This obtains in our model if Condition 2 and Condition 4 hold. Given that higher hierarchy jobs require a higher level of ability, it is intuitive that these conditions imply that our model matches the empirical observation. The latter condition states that if you are relatively talented you succeed with a high probability even if you put in low effort. In contrast, if you have a low ability you have to compensate for the lack in talent with high effort or risk failure with a high probability. Together with Condition 2 this implies that a relatively low ability agent who wanted to shirk would have a harder time hiding this in job 1 than in the lower level job 2 where he is more productive. Stated differently: if a low ability agent succeeded in a high ability job, he must have worked really hard. Therefore, it takes a higher wage following success to prevent shirking in the lower level job 2, or put differently, implementation costs are lower for the higher hierarchy level job 1.

To see this more formally, note that Condition 2 in conjunction with $f_1\left(e_H, \theta^{FB}\right) = f_2(e_H)$ implies the following relation of likelihood ratios:

$$\frac{f_1\left(e_L, \theta^{FB}\right)}{f_1\left(e_H, \theta^{FB}\right)} < \frac{f_2(e_L)}{f_2(e_H)}. \tag{2}$$

This says that at $\theta = \theta^{FB} = \hat{\theta}_H$ output in the high ability job 1 is *more informative* about the agent's effort in the sense of Demougin and Fluet (1998) than that in the lower ability job 2.[3] Condition 4 then extends Condition 2: it implies that this information relation holds for all $\theta < \theta'$, where $\theta' > \theta^{FB}$ is defined as the ability cutoff at which likelihood ratios are equal.

[3]Note that this definition of informativeness differs slightly from those proposed for the case of risk averse agents with unlimited liability, where an information system i is locally more informative than j if the likelihood ratio distribution of i is a mean preserving spread of that of j (Grossman and Hart 1983, Kim 1995, Jewitt 2006).

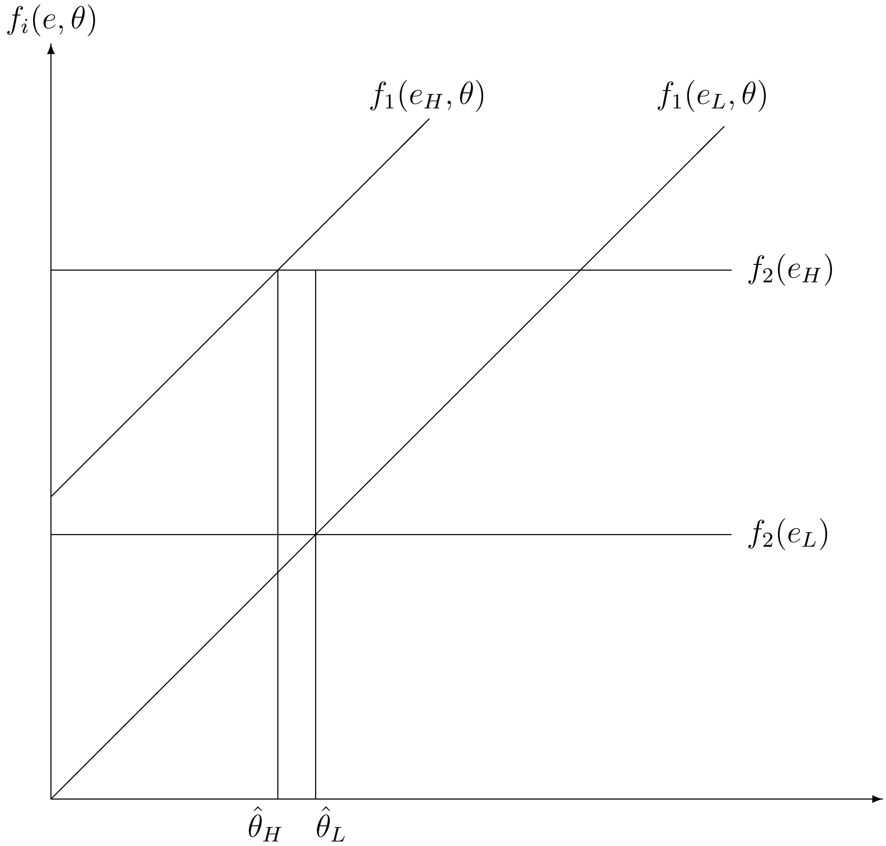

Figure 4.1: Single Crossing for Success Probabilities with e_L and e_H in both Jobs. $f_1(e,\theta) = g_1(e) + \theta$, with $g_1(e_H) = 0.4$, $g_1(e_L) = 0$ and $f_2(e) = g_2(e)$, with $g_2(e_H) = 0.7$ and $g_2(e_L) = 0.35$.

The existence of such a θ' is guaranteed by Condition 3. It implies that at the top of the ability distribution it is likely that talent matters more for output than effort in higher level positions (e.g. Rosen (1982)). Intuitively, Condition 3 captures the notion of increasing returns to talent in higher level jobs in a hierarchy, stating that at the top talent has such a big impact on the success probability that output in the high ability job 1 is less informative about effort than output in lower level job 2. In other words, if you are extremely talented, you succeed easily even in a demanding high skill job, and this makes it hard for the principal to detect whether your success is due to effort or talent. Thus, she has to compensate you more to make you work hard. Imposing these conditions leads to predictions summarized in Propositions 13 and 14 which are broadly consistent with empirical findings as we argue below.

Proposition 13 shows that it is optimal to promote some individuals “to their level of incompetence” because this reduces the cost of getting them to work hard. The promotion threshold is below the first best level, $\theta^{SB} < \theta^{FB} = \hat{\theta}_H$, and thus the

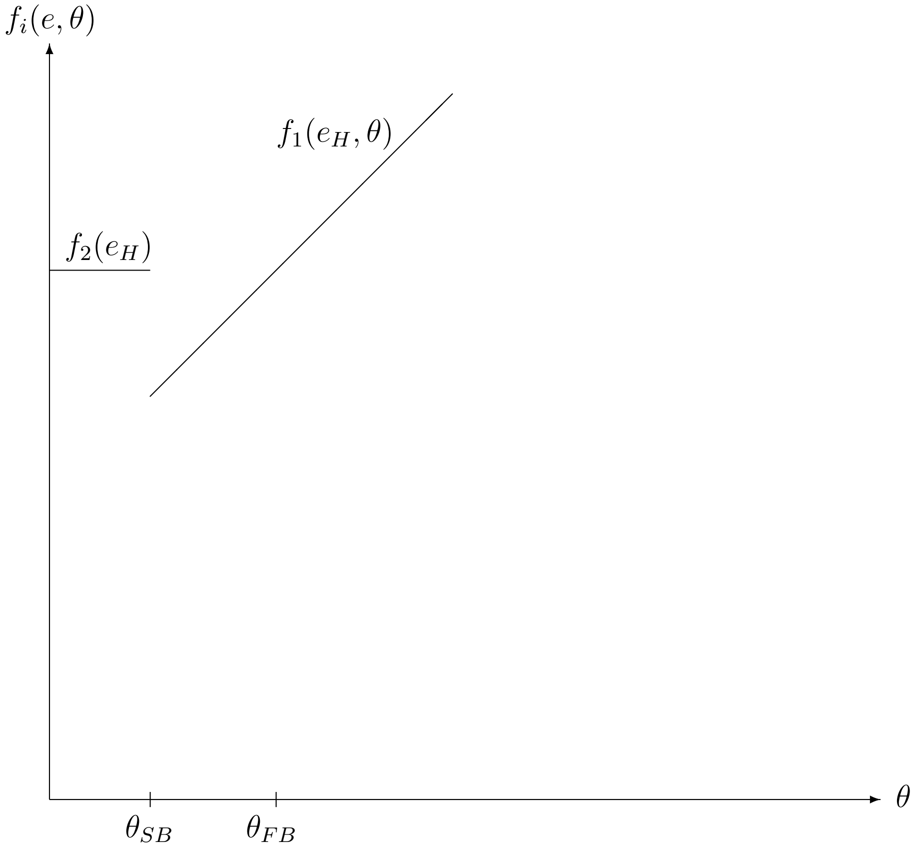

Figure 4.2: Expected Output for e_H in both Jobs. $f_1(e,\theta) = g_1(e)+\theta$, with $g_1(e_H) = 0.4$, $g_1(e_L) = 0$ and $f_2(e) = g_2(e)$, with $g_2(e_H) = 0.7$ and $g_2(e_L) = 0.35$, $c = 0.25$.

Peter Principle holds for an agent with $\theta^{SB} \leq \theta < \theta^{FB}$: he is promoted to job 1 even though he would produce more in job 2 with the same effort level. Figure 4.2 illustrates for a numerical example the pattern of expected output predicted by the model. The inefficient promotion threshold – and hence the discontinuity in expected output – is due to differences across the two jobs in informativeness of output about effort, and not caused by the unobservability of effort per se.

Proposition 14 summarizes the implications for the wage policy of the firm. At the top of the ability distribution, success may be driven mostly by talent, and effort may have a lesser impact. This notion is captured by Condition 3. If it holds, we predict the pattern observed in Baker, Gibbs and Holmstrom's (1994b) study: the wages for the bottom tier of the higher level job are lower than those at top tier of the preceding hierarchy level. Thus, our model offers an intuitive explanation that links this "earnings gap" to the Peter Principle. Agents at the bottom tier of the higher level job are less productive than they would be in their old job: their slightly less able colleagues, who remain in the lower level job, simply produce and

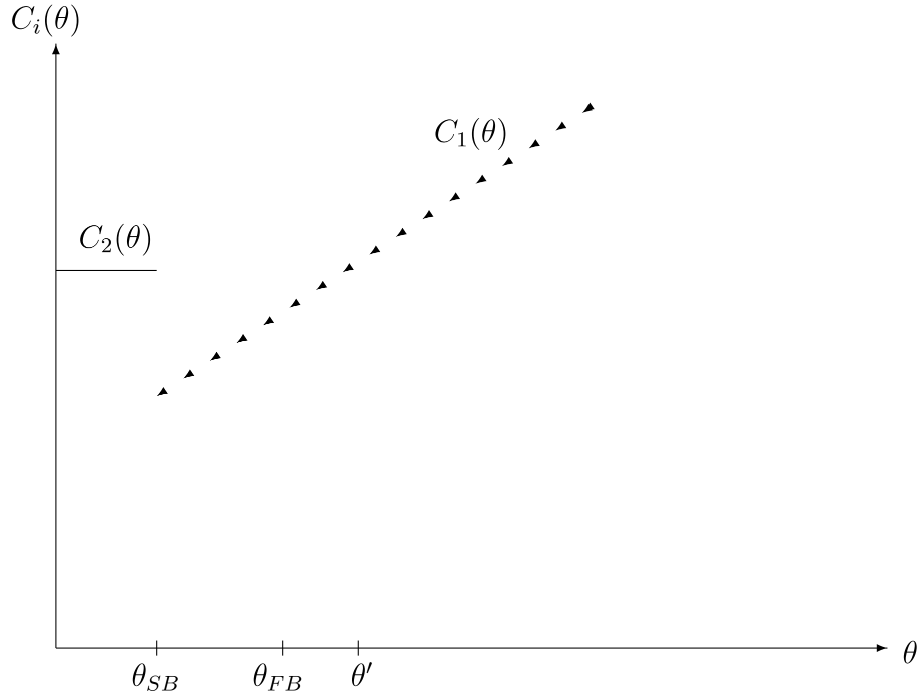

Figure 4.3: Wage Pattern for e_H in both Jobs. $f_1(e, \theta) = g_1(e) + \theta$, with $g_1(e_H) = 0.4$, $g_1(e_L) = 0$ and $f_2(e) = g_2(e)$, with $g_2(e_H) = 0.7$ and $g_2(e_L) = 0.35$, $c = 0.25$.

earn more than they do.

Condition 4 guarantees that expected wages after a promotion increase with the agent's ability. Figure 4.3 shows that one can easily satisfy these conditions using an additively separable production function $f_1(e, \theta) = g(e) + h(\theta)$, which is often employed in the literature (Fairburn and Malcomson 2001).

The expected utility follows the same pattern as the expected wage: it is given by the expected wages less the cost of effort c. Thus, our model offers an explanation for why individuals often are less happy after a promotion: they are kept on toes by being put into a job for which they are not sufficiently talented.

High Effort not Optimal in Both Jobs

Consider now the case where it is not always optimal for the principal to induce the agent to provide high effort. We consider the empirically relevant setting where output in job 1 is more informative about the agent's effort. Therefore, the moral hazard problem is less severe than that in job 2. For this reason it is more likely that the principal implements high effort in job 1 than in job 2. Put differently, when moving up the career ladder the agent is going to work more rather than less and is rewarded more often with a performance related incentive scheme. This is what one typically observes in firms: lower hierarchy workers simply receive a wage, while a manager's compensation often includes performance related elements such as bonuses or stocks. In our model, an implication of this is that the promotion threshold is less distorted than in the case where agents are made to provide high effort in both jobs. The reason is that the principal is less tempted to assign an agent to the higher level job because the agent always earns more than in the lower level job. But, as we will see later, the Peter Principle can still hold. Before discussing this further, we summarize our results on implemented effort levels and the promotion threshold in the following lemma:

Lemma 7

(a) The implemented effort in job 1 is higher than in job 2, i.e. $e_1^{SB}(\theta) \geq e_2^{SB}$, $\forall\,\theta$*;*

(b) The promotion threshold to the ability sensitive task 1, θ^{SB}*, is higher than when* $e_1^{SB}(\theta) = e_2^{SB} = e_H\ \forall\,\theta$*.*

In contrast to the case where high effort was always optimal, the optimal effort level can now stay at e_L or increase from e_L to e_H after a promotion. The possible scenarios are summarized in the following result – showing when the Peter Principle emerges and when not:

Proposition 15 *Suppose that Condition 1 fails to hold for job 2 (i.e.* $e_2^{SB} = e_L$*).*

(a) $\theta^{SB} < \theta^{FB}$ *and the Peter Principle holds for* $\theta \in [\theta^{SB}, \theta^{FB})$ *if and only if Condition 1 is satisfied for job 1 (i.e.* $e_1^{SB}(\theta) = e_H$*) and* $C_1\left(e_H, \theta^{FB}\right) > f_1\left(e_H, \theta^{FB}\right) - f_2(e_L)$*.*

(b) $\theta^{FB} < \theta^{SB} = \hat{\theta}_L$ *and the Peter Principle holds for any ability level if and only if Condition 1 is satisfied for job 1 (i.e.* $e_1^{SB}(\theta) = e_L$*).*

In Part (b), no incentives are required in any job and the promotion threshold depends solely on the impact of the job assignment on output. That is, the promotion threshold is $\hat{\theta}_L$ and the agent is assigned to the job in which he is most productive.

Part (a) shows that at the first-best promotion threshold θ^{FB} assigning the agent to the higher level job leads to an increase in output given by $f_1\left(e_H, \theta^{FB}\right) - f_2(e_L)$. If and only if this is sufficient to compensate for the cost of implementing high effort, $C_1\left(e_H, \theta^{FB}\right)$, we have that $\theta^{SB} < \theta^{FB}$ and the Peter Principle holds. It is interesting to note that an inefficiently low promotion threshold can arise even in the case where the agent always earns strictly more when he moves up the career ladder. But even though the wage is higher, the principal can gain from promoting the agent: as the output of a low ability agent is more informative in a high ability job, the principal can induce him to work hard in job 1 when she would only like to implement low effort in job 2.

If, however, the agent is so untalented that even the lower implementation costs for high effort in job 1 cannot outweigh his lack in talent, then $\theta^{SB} \geq \theta^{FB}$ and the Peter Principle does not hold: even though too few workers are promoted relative to the first best scenario, a worker is never more productive in the other job holding effort constant. The reason is that in both jobs this low talented agent would be made to exert low effort. Thus, the distortion in the promotion threshold cannot be driven by the desire to exploit lower effort implementation costs in job 1.

For both of these cases in Proposition 15, the expected wage and expected utility of the agent increases after a promotion. Thus, our model predicts an immediate wage rise for those cases where the agent has to work harder after a promotion than in the previously held lower hierarchy level job.

4.4 Conclusion

This model provides a simple explanation for why organizations only rarely correct what appears to be at first glance a botched promotion. Jobs higher up in the hierarchy of an organization tend to be more informative about the effort of an employee. For this reason, an organization may find it optimal to promote an employee even if this leads to a reduction in output (the Peter Principle). This decrease in expected output can be outweighed by the reduction in implementation costs caused by the higher level job being more informative about effort.

4.5 Extension – Intrinsic Motivation

In the last sections we saw how differences in extrinsic motivation – i.e. the motivation to provide effort induced by wage payments – can lead to inefficient job assignments. Often, however, agents are not only extrinsically motivated, but also intrinsically, as psychologists pointed out.[4] And job assignments are one important factor that influence this intrinsic motivation. An agent's self esteem – which influences his intrinsic motivation[5] – depends on the assignment: e.g. not getting assigned to a better job may cause frustration and doubts about the own ability. But even though the job assignment rule influences the intrinsic motivation – which may make distortions optimal – it is a priori not clear why the principal does not simply outweigh this with an appropriate wage scheme. This would allow him to implement the efficient rule. We will show that the job assignments rule (influencing the intrinsic motivation) and the wage (influencing in our model only the extrinsic motivation) are not simple substitutes and hence the principal assigns the agent more often than efficient to the "high motivation job in the separating equilibrium we identify.

We adopt the formalization of the concepts of self esteem and intrinsic motivation from Bénabou and Tirole (2003). In their model there is one agent who has imperfect knowledge about his type. He will undertake a certain task only if he has a high self confidence that he will succeed, where self confidence refers to the agent's belief about his probability of success. The principal knows the agent's type. Since effort and ability are complements, the principal wants to enhance the agent's self efficacy by choosing her instrument, a bonus. The bonus thus not only influences the

[4]There are many experiments which confirm this starting with Deci (1971). Ryan, Deci, and Koestner (1999) provide a meta-analysis of existing experiments. For an economic experiment see Gneezy and Rustichini (2000). An overview of the psychologists' definitions of intrinsic and extrinsic motivation (doing something because it leads to a separable outcome) or self efficacy, and how they work together, can be found in Ryan and Deci (2000) or in Leonhard, Beauvais, and Scholl (1995).

[5]Some authors define intrinsic motivation explicitly as the belief about the probability of success (self efficacy) times the value of obtaining a goal.

motivation of the agent directly via the payoff, but also indirectly from the inference process. Bénabou and Tirole (2003) show that rewards are bad news for the agent: the principal offers an equal or lower bonus to a more able agent. Hence, a higher bonus reduces the agent's intrinsic motivation. In equilibrium, the principal either pools the agent on the lowest possible bonus or mixes for the low type between the lowest possible and a higher bonus, while paying the high type the lowest one.

One of the main differences to Bénabou and Tirole (2003) is that a separation of agents by type occurs in our model. The job assignment not only serves as a signalling device, as does the wage in their model, but also directly affects the principal's payoff: an inefficient assignment leads to a lower success probability. In this respect, our model is related to Crutzen, Swank, and Visser (2006), who ask whether differentiation by rank in the workplace can be optimal if agents are intrinsically motivated. Technically, they achieve a separating equilibrium by introducing a term in the production function that captures complementaries between the agents: for example, the agent profits from harder working or more talented colleagues.

Ishida (2006) applies the Bénabou and Tirole (2003) framework to promotion policies, and is therefore most closely related to our model. He, however, does not derive the wage scheme endogenously, but assumes that the agent gets a fixed share from the output. In comparison, we show how job assignments – which in our model affect purely the intrinsic motivation – and the wage – which affect only the extrinsic motivation – interact.

4.5.1 The Model

We adjust the model from Section 4.2 as follows. Following Bénabou and Tirole (2003) we assume that effort (where $e_L = 0$ and $e_H = 1$) and the type θ are complements in the probability function. Without providing effort, the agent will fail, regardless of his talent for the job: $f_j(\theta, e) = ef_i(\theta)$. The multiplicative form implies that the agent's type is not informative about his effort, and hence both jobs are equally informative about effort. Furthermore, we assume $\frac{df_1(\theta)}{d\theta} < 0$. This captures the idea that the two tasks are on the same level of the hierarchy, but require different talents: "low" types can be as good in their job as "high" types in theirs. Thus, compared to Section 4.2, a high or low type should not refer to an agent with a high or low ability, but with a certain ability for a job. Therefore, the model captures optimal job assignment rather than promotion decisions. To achieve single crossing for high effort (let the cutoff be $\hat{\theta}$) we assume that:

Assumption 8 $\frac{df_2(\theta)}{d\theta} > 0$ *and* $\frac{df_1(\theta)}{d\theta} < 0$ *and* $f_2(\theta_H) > f_1(\theta_H) = f_2(\theta_L)$.

The timing and information structure are as follows. At date 1 the type of the agent is neither known to the principal nor to the agent. However, it is common knowledge that types are distributed according to the function $\Phi(\theta)$, with $\Phi : [\theta_L, \theta_H] \to [0, 1]$, $\Phi \in C^1$ and density $\phi(\theta)$, $\phi : [\theta_L, \theta_H] \to I\!R$. We assume that $\Phi(\hat{\theta}) = \frac{1}{2}$. At this date the principal offers a contract. We will restrict our attention to bonus payments that reward the agent if his output is high (we let the wage be w_i) and pay him zero otherwise.[6] We assume that the relationship between the agent and the principal is ongoing, so that there is no participation constraint. This allows us to fully concentrate of the incentive effects of job assignments. At this date the principal also specifies a job assignment rule $\mathcal{J}(\theta)$, $\mathcal{J} : [\theta_L, \theta_H] \to \{1, 2\}$. At date 2 the principal privately learns the agent's type. For example, we can think of this as the outcome of a training phase that the agent undergoes. The agent cannot judge whether he performed well in this training or not. After learning the agent's type, the principal implements the specified job assignment rule at date 3. The agent can observe to which job he is assigned and tries to infer from this something about his talent. Then he provides unobservable effort in his assigned job at date 4. At the last date output and payoffs realize. As before, the principal receives a revenue of $\bar{x}$ if the agent has a high output, and zero otherwise. He also has to pay the specified wage to the agent. The agent receives this wage minus his effort costs. In addition he gets some non-monetary value v ($\bar{x} > v$, $v < c$) out of a high output. This value v stands for the agent's intrinsic gain if he succeeds in completing his task: e.g. he feels proud or happy if he sees that he did a good job. We assume that the total profit is maximized when the agent provides high effort, i.e.:

Assumption 9 $f_j(\theta)(\bar{x} + v) - c > 0 \ \forall \theta, j$.

[6] In the appendix we show that our model implies that the wage does not condition on the type given the contract is in the form of a bonus payment.

Intrinsic Motivation

We adopt the formulation of intrinsic motivation from Bénabou and Tirole (2003).[7] The agent does not know his type when he produces. Hence, he believes that his success probability is $E[f_j(\theta)|\mathcal{I}]$, where $\mathcal{I}$ is the information the agent has about his type at this time. We call this belief about the success probability self esteem or self confidence. Then we can capture by the product $E[f_j(\theta)|\mathcal{I}]v$ the intrinsic motivation of an agent - it is composed of the value of the goal and the self esteem, as proposed by psychologists. It is higher, the higher the self esteem of the agent. Thus, the agent has a higher motivation to provide high effort in a task if he feels more competent.

4.5.2 Analysis

To better understand how imperfect self knowledge matters for the job assignment rule, we first consider a situation, where both the agent and the principal know the agent's type.

The Agent Knows his Type

Suppose that the principal and the agent both know the agent's type when signing the contract. In other words, the self esteem and the intrinsic motivation are predetermined and the principal cannot influence them. We are hence dealing with a standard moral hazard problem. Suppose the principal wants to implement high effort. This is optimal for the agent if the following incentive constraint holds: $f_j(\theta)(v + w_j) - c \geq 0$. As this constraint is binding in equilibrium the principal sets $w_j = \frac{c}{f_j(\theta)} - v$ and her expected profit is: $f_j(\theta)(\bar{x} + v) - c$. Assumption 9 guarantees that this is greater than zero. Hence, she wants indeed to implement high effort. Expected profits are maximized if she assigns an agent with $\theta \geq \hat{\theta}$ to job 1 and one with $\theta < \hat{\theta}$ to job 2.

In contrast to Section 4.3, moral hazard does not cause any inefficiencies in the assignment because the likelihood ratio does not depend on the type. Hence, the job

[7] Note that in contrast to Bénabou and Tirole (2003) we do not assume that $f_j(\theta)$ is linear in θ. We can capture, however, with our more general function also this case.

assignment does not improve the estimate of effort since the likelihood ratio is the same for both jobs. Furthermore, there is no distortion in effort because the agent receives ex ante no limited liability rent: his expected wage is equal to his effort costs c.

The Agent Does not Know his Type

We solve by backward induction. First we consider the incentives of an agent to provide effort given a wage scheme and job assignment, before we derive the optimal job assignment rule and the wage scheme.

Incentives to Provide Effort

The agent does not know his success probability when making his effort choice at date 4, but holds a posterior belief $E[f_j(\theta)|\mathcal{I}]$ about it. $\mathcal{I}$ summarizes the agent's information at this date. This information consists solely of the job assignment he observed at date 3: the wage offered at stage 1 conveys no further information about the type. First, the wage does not condition on the type (which the principal might announce to the agent), but only on the observable job assignment. Second, every wage pair induces a subgame. Hence, also for out-of-equilibrium wages the beliefs are pinned down by the observed job assignment.
Therefore, for any w_j (on or off the equilibrium path) the agent provides high effort in his job if and only if:

$$E[f_j(\theta)|\mathcal{I}](w_j + v) - c \geq 0.$$

Assumption 9 implies that it is optimal to induce an agent to provide high effort.

Pooling Equilibria

At date 3 – after observing the agent's type at date 2 – the principal implements the job assignment rule she specified at date 1: the agent can observe the implemented rule and thus she has no incentive to deviate from her announcement. She can either assign all agents to the same job j (pooling equilibria); or some to job 1 and some to job 2 (separating equilibrium). In this section we consider pooling equilibria. In the appendix we show that an equilibrium always exists in the subgame following date 2 and discuss also how to deal with multiple of equilibria.

If the principal pools on job j, the agent holds the belief that his success probability is $Ef_j(\theta)$: the assignment rule conveys no further information. Can we support such a pooling equilibrium? Suppose w_j is such that the agent provides high effort and $\bar{x} \geq w_j$, i.e. profits from pooling on job j are positive. If we can find for a given w_i an out-of-equilibrium belief $\tilde{\theta}$ such that $f_i(\tilde{\theta})(w_i + v) < c$ or if for the pair (w_i, w_j) we have $f_j(\theta)(\bar{x} - w_j) \geq f_i(\theta)(\bar{x} - w_i)\ \forall\theta$, then the principal has no incentive to deviate and assign the agent to job i instead of j. In the former case the agent does not provide high effort in job i – resulting in zero profits. In the latter profits are higher in job j than in job i for all types of agents.

Given that the principal indeed wants to pool the agent on job j, she then selects at date 1 w_i in such a way that one can support the pooling equilibrium (see above). The wage w_j is chosen such that profits are maximized, i.e. such that the incentive constraint is just binding: $w_j = \frac{c}{Ef_j(\theta)} - v$. For wages that are higher (up to $\bar{x}$) or lower than this one, pooling on j can still be an equilibrium (depending on w_i), but such equilibria would lead to lower profits than this one. Finally, whether the principal wants to pool on job 1 or job 2 (assuming that pooling is optimal) depends on whether $Ef_1(\theta)$ is smaller or larger than $Ef_2(\theta)$. She will announce the rule that maximizes her profits.

Separating Equilibria

We now turn to separating equilibria. Here the principal selects wages (w_1, w_2) and specifies the job assignment at date 1 (a cutoff θ_S) that induce the highest profits to her.[8] That is, the principal maximizes her expected profits under the constraints that a) the agent provides the specified effort given his belief (which is $E[f_j(\theta)|\theta_S, j]$ when assigned to job j given cutoff θ_S) and b) a separating equilibrium for these wages indeed exists. The latter requires that the principal is indifferent between

[8] As for the pooling equilibrium, first the principal has no incentive to deviate at date 3 from the job assignment rule she announced at date 1. Second, for other wages than the ones characterized in the following a separating equilibrium can exist – but the following lead to the highest profits and will thus be selected in equilibrium.

assigning an agent with type θ_S to job 1 or job 2: $f_1(\theta_S)\,(\bar{x}-w_1) = f_2(\theta_S)\,(\bar{x}-w_2)$:

$$\begin{aligned} \max_{w_1,w_2,\theta_S} \quad & \int_{\theta_S}^{\theta_H} f_1(\theta)\phi(\theta)d\theta\,(\bar{x}-w_1) + \int_{\theta_L}^{\theta_S} f_2(\theta)\phi(\theta)d\theta\,(\bar{x}-w_2), \\ \text{s.t.} \quad & E[f_1(\theta)|\theta_S,1]\,(w_1+v) - c \geq 0, \\ & E[f_2(\theta)|\theta_S,2]\,(w_2+v) - c \geq 0, \\ & f_1(\theta_S)\,(\bar{x}-w_1) - f_2(\theta_S)\,(\bar{x}-w_2) = 0. \end{aligned}$$

The following proposition shows that there exists a unique solution to this problem:

Proposition 16 *Suppose it is optimal for the principal to separate agents, i.e. she assigns an agent with $\theta < \theta_S$ to job 2 and one with $\theta \geq \theta_S$ to job 1. Then:*

(i) There exists a unique cutoff θ_S, call it θ_M, satisfying $f_1(\theta_M)(\bar{x}-w_1(\theta_M)) = f_2(\theta_M)(\bar{x}-w_2(\theta_M))$.

(ii) Wages satisfy: $w_1 = w_1(\theta_M)$ and $w_2 = w_2(\theta_M)$, with $w_i(\theta_M) = \frac{c}{E[f_i(\theta)|\theta_M,i]} - v$, and $w_1 > w_2$ if and only if $E[f_1(\theta)|\hat{\theta},1] < E[f_2(\theta)|\hat{\theta},2]$.

(iii) If and only if $E[f_1(\theta)|\hat{\theta},1] < E[f_2(\theta)|\hat{\theta},2]$ (>), $\hat{\theta} < \theta_M$ (>), i.e. fewer agents than efficient are assigned to job 1.

(iv) If and only if $E[f_1(\theta)|\hat{\theta},1] < E[f_2(\theta)|\hat{\theta},2]$ (>), the agent has a higher intrinsic motivation in job 2 than in job 1: $E[f_1(\theta)|\theta_M,1] < E[f_2(\theta)|\theta_M,2]$.

To interpret the proposition we define the "high motivation job" in the sense that under an *efficient* assignment rule the intrinsic motivation is higher in this job and assume in the following that this is job 2. Following Part (iv) this implies that in equilibrium the motivation is still higher in job 2. Part (iii) of Proposition 16 shows that if job 2 is the high motivation job, the principal assigns the agent more often than efficient to job 2. This is because the intrinsic motivation in job 2 at $\hat{\theta}$ is higher than in 1. Thus, assigning the agent more often to job 2 increases his motivation. Since a higher motivation leads to a lower wage, this higher motivation of the agent compensates for his lower success probability in job 2.

Part (ii) of the proposition characterizes the wages – they are such that the incentive constraint is just binding for a given cutoff. Combining Part (ii) with Part (iv) we see that the wage is lower in the job in which the intrinsic motivation is higher. When choosing the wages, the principal faces the following trade-off. She can choose the wages in a way that makes the incentive constraint just binding: $w_1(\theta_M)$ and $w_2(\theta_M)$. However, this implies a large distortion $\theta_M < \hat{\theta}$. Or, she can increase w_2 away from $w_2(\theta_M)$, which leads to a smaller distortion, but to higher wages. To achieve the minimum distortion she would set $w_1 = w_2 = w_2(\hat{\theta})$. As Part (i) of the proposition shows, the principal distorts the cutoff, because the gain – lower wages due to a a higher intrinsic motivation of the agent – is larger than the loss – a reduced success probability for an agent with $\theta \in (\hat{\theta}, \theta_M]$. Thus, intrinsic and extrinsic motivation are not simply substitutes.

Separating versus Pooling

Lastly, we have to consider whether the principal would like to choose the profit maximizing pooling equilibrium or the separating equilibrium:

Proposition 17 *The principal always prefers to separate the agents as described above to an assignment rule that pools all agents on one of the two jobs.*

4.5.3 Conclusion

The second part of this chapter showed that not only differences in the extrinsic motivation – as in the first part – but also in the intrinsic motivation lead to inefficient job assignments. The job assignment rule trades off the increased motivation in the high motivation job with a lower success probability. Although the principal can outweigh the lower motivation with a higher wage and reduce the distortion in the cutoff, she chooses not to do so. This shows that intrinsic and extrinsic motivation are not simply substitutes.

One direction for future research is to let agents not only be intrinsically motivated, but also care for their "ego utility". Köszegi (2006) provides such a formulation where utility depends directly on the believed self, i.e. $u(E[\theta|\mathcal{I}])$. The agent has a higher utility the better he thinks of himself. Note that this would generalize our model, as here u was linear in $E[\theta|\mathcal{I}]$. Moreover, one can assume that the ego utility does not only depend on the perceived self, but also on the job. For example, agents may feel pride if they work in a higher hierarchy job.

Conclusion

The leitmotif of this thesis is the value of information in incentive problems: is more or early information always better? The first three chapters deal with the optimal timing of information and the value of ex ante information. The last chapter shows how the principal's desire to extract more information may lead to inefficient job assignments.

Chapter 1 endogenizes the point in time when an agent observes the realization of a signal – before or after his effort choice. From decision problems it is well known that if there are informational gains, the decision maker can strictly increase revenues by conditioning effort to the state of the world. Otherwise, she simply ignores the information which implies that revenues are unaltered. I show that for incentive problems she cannot simply ignore the information as an informative signal changes the agent's incentives. I discuss in this chapter the possible constellations for the signal to lead to informational gains, but to be uninformative about effort, or for it to be informative about effort, but not to lead to informational gains. Based on this I show that there is no difference between incentive and decision problems if the signal is uninformative about the agent's effort: ex ante information is never worse and – if there are gains from tailoring effort to the new information – strictly better than ex post information. If the signal is informative about the agent's effort and there are no informational gains, however, the surprising result is that ex ante information harms.

Chapter 2 builds on the analysis of Chapter 1, but the signal the agent observes now is the output of a colleague. While this does not change the analysis from Chapter 1 for the agent, there might be a positive effect on the colleague: I show that this effect can make ex ante information optimal only if the colleague's output is informative about effort. The interesting point is that ex ante information can be optimal even though there are no informational gains, which are the only source that can make ex ante information strictly better in decision problems. If it is uninformative, then the two agent incentive problem behaves again like a decision problem. These findings connect a firm's organizational structure and its internal transparency – showing that M-form firms are more likely to be transparent than U-form firms.

Chapter 3 asks whether the sufficient statistic result – which shows that additional signals that are informative about the agent's effort are valuable in principal agent models with hidden actions – is valid if the agent and the principal observe the signal realization before he chooses his effort. I give a condition on the shape of the likelihood ratios under which the value of such an additional signal is negative.

Chapter 4 differs from the first three chapters in the sense that I consider only ex post information. Still it deals with information in moral hazard problems: it shows that more information may not be beneficial if spread asymmetrically over jobs. More precisely, the first part of this chapter shows how the principal's desire to extract more informative signals about the agent's effort leads to inefficient job assignments. If an agent is not very talented for a job, a success by him indicates that he worked really hard. Stated differently, a success of an untalented agent in a high skill job is very informative about effort. Relative to placing the agent in a low skill job, assigning him to a high skill job thus reduces implementation costs. This can outweigh the drop in the success probability. Thus, this chapter provides a simple explanation for the Peter Principle showing that it is optimal for the firm to promote some employees to their level of incompetence.
The second part of Chapter 4 identifies another source of inefficient job assignments (assuming that there are no inefficiencies from moral hazard as in the first part) – workers are not only extrinsically motivated, but also intrinsically. The agent has imperfect self knowledge, while the principal knows the agent's type. Thus, the job assignment rule the principal chooses, provides information to the agent about his type. This belief about the type influences the agent's intrinsic motivation. Again, there is a trade-off between motivating the agent and assigning him to the job that suits him best. I show that she chooses an inefficient assignment in equilibrium, although she can motivate the agent additionally with monetary payments.

Overall, this thesis demonstrates the importance of information in principal agent problems with hidden actions when agents are risk neutral and protected by limited liability. The first three chapters make a contribution to the understanding of the benefits of information in moral hazard problems, pointing out that the point in time when an agent receives information matters. Since there are many examples in firms when the agent receives information before he chooses his effort or where the principal can choose the time where he receives it, these results have important implications for the design of organizations. To apply them further to such examples seems an important direction for future research. The last chapter is a first attempt to do so: it applies the role of information in incentive problems to an organizational context. It provides a simple explanation for the Peter Principle and provides predictions in line with the empirical evidence.

Appendix

Appendix to Chapter 1

Proof Proposition 1.

Let (e) be any effort vector of the ex post information scenario. In the following we show first that we can find $(\bar{e}, \underline{e})$, satisfying $e = p\bar{e} + (1-p)\underline{e}$ that yield strictly higher profits if the condition in the proposition is satisfied. Note that this vector $(\bar{e}, \underline{e})$ need not to be optimal for the ex ante information scenario. However, – as profits are already higher with these non-optimal ones, profits have to be higher with the optimal ones.

Consider first the cost function. By a Taylor Approximation we have (assume that $\bar{e} \geq \underline{e}$, the case $\bar{e} \leq \underline{e}$ is analogous):

$$c(\bar{e}) = c(\underline{e}) + c_e(\underline{e})\,(\bar{e} - \underline{e}) + \sum_k \frac{c_k(\underline{e})\,(\bar{e} - \underline{e})^k}{k!},$$

and

$$c(e) = c(\underline{e}) + p\,c_e(\underline{e})\,(\bar{e} - \underline{e}) + \sum_k \frac{c_k(\underline{e})\,p^k\,(\bar{e} - \underline{e})^k}{k!}.$$

Hence, we can we write the implementation costs for the ex ante information scenario as:

$$p\,c(\bar{e}) + (1-p)\,c(\underline{e}) = c(\underline{e}) + p\,c_e(\underline{e})\,(\bar{e} - \underline{e}) + p\sum_k \frac{c_k(\underline{e})\,(\bar{e} - \underline{e})^k}{k!}.$$

Subtracting implementation costs for the ex post information scenario from the ones for the ex ante information scenario we get the difference:

$$g(\bar{e} - \underline{e}) \equiv \sum_k \frac{c_k(\underline{e})\,(p - p^k)\,(\bar{e} - \underline{e})^k}{k!}.$$

Note that $g(0) = 0$ and $\frac{\partial g(\bar{e} - \underline{e})}{\partial(\bar{e} - \underline{e})}|_{\bar{e} = \underline{e}} = 0$. Thus, costs do not increase by introducing a small spread under the ex ante information scenario.

The difference in expected revenues is given by:

$$f(\bar{x}\bar{z}|\bar{e}) - f(\bar{x}\bar{z}|e) + f(\bar{x}\underline{z}|\underline{e}) - f(\bar{x}\underline{z}|e).$$

Again by a Taylor Approximation:

$$f(\bar{x}\bar{z}|e) = f(\bar{x}\bar{z}|\bar{e}) - (1-p)\, f_e(\bar{x}\bar{z}|\bar{e})\, (\bar{e} - \underline{e}) - \sum_k \frac{(1-p)^k\, f_k(\bar{x}\bar{z}|\bar{e})(\bar{e} - \underline{e})}{k!}$$

and:

$$f(\bar{x}\underline{z}|e) = f(\bar{x}\underline{z}|\underline{e}) + p\, f_e(\bar{x}\underline{z}|\underline{e})\, (\bar{e} - \underline{e}) + \sum_k \frac{p^k\, f_k(\bar{x}\underline{z}|e)\, (\bar{e} - \underline{e})}{k!}.$$

Hence, we can write the difference in revenues as:

$$\begin{aligned} r(\bar{e} - \underline{e}) \;\; &\equiv \;\; [(1-p) f_e(\bar{x}\bar{z}|\bar{e}) - p f_e(\bar{x}\underline{z}|\underline{e})](\bar{e} - \underline{e}) \\ &+ \;\; \sum_k \frac{(1-p)^k\, f_k(\bar{x}\bar{z}|\bar{e})\, (\bar{e} - \underline{e})}{k!} - \sum_k \frac{p^k\, f_k(\bar{x}\underline{z}|\underline{e})(\bar{e} - \underline{e})}{k!}. \end{aligned}$$

Note that $r(0) = 0$ and $\frac{\partial r(\bar{e}-\underline{e})}{\partial(\bar{e}-\underline{e})}|_{\bar{e}=\underline{e}} = (1-p)\, f_e(\bar{x}\bar{z}|\bar{e}) - p\, f_e(\bar{x}\underline{z}|\underline{e})$. Dividing by p and $1-p$ shows that the sign of this derivative is given by $\text{sign}[h_e(\bar{x}|\bar{z}, \bar{e}) - h_e(\bar{x}|\underline{z}, \underline{e})]$. Note then that $f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\underline{z}|\underline{e}) \neq p\, g(\bar{x}|\bar{e}) + (1-p)\, g(\bar{x}|\underline{e}) \Rightarrow h_e(\bar{x}|\bar{z}, e) \neq h_e(\bar{x}|\underline{z}, e)$. Hence, if $f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\underline{z}|\underline{e}) \neq p\, g(\bar{x}|\bar{e}) + (1-p)\, g(\bar{x}|\underline{e})$ we can find for any e a vector $(\bar{e}, \underline{e})$, satisfying $e = p\, \bar{e} + (1-p)\, \underline{e}$, making ex ante information scenario strictly better than ex post information.

To see the necessary part suppose to the contrary that not, i.e. profits are strictly higher under the ex ante information scenario, but $f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\bar{z}|\underline{e}) = p\, g(\bar{x}|\bar{e}) + (1-p)\, g(\bar{x}|\underline{e})$. Take the optimal $(\bar{e}, \underline{e})$ for the ex ante information scenario. By Jensen's Inequality we have that the profit function for the ex ante information scenario $g(\bar{x}|\bar{e}) + (1-p)\, g(\bar{x}|\underline{e}) - p\, c(\bar{e}) - (1-p)\, c(\underline{e})$ lies below the one for the ex post information scenario $g(\bar{x}|e) - c(e)$ given $e = p\, \bar{e} + (1-p)\, \underline{e}$ – contradicting that profits are strictly higher. ∎

Proof Lemma 1.

Before we can solve the principal's problem we have to check that we can indeed omit the participation constraint and that the incentive constraint yields a global maximum of the agent's problem for the equilibrium wage scheme.

Omit the Participation Constraint:

This constraint is given by:

$$f(\bar{x}\bar{z}|\hat{e}) w(\bar{x}\bar{z}) + f(\bar{x}\underline{z}|\hat{e}) w(\bar{x}\underline{z}) + f(\underline{x}\bar{z}|\hat{e}) w(\underline{x}\bar{z}) + f(\underline{x}\underline{z}|\hat{e}) w(\underline{x}\underline{z}) - c(\hat{e}) \geq 0.$$

Since the agent's reservation utility is zero, the participation constraint is automatically satisfied if the limited liability and the incentive constraint are: by choosing an effort level of zero, the agent can achieve at least an expected utility of zero as wages are larger than zero by the limited liability constraint. Hence, by maximizing his expected utility over effort (the incentive constraint) he can never do worse then zero.

Incentive Constraint Yields Unique Global Maximum:

Concerning the incentive constraint note that a global maximum of the agent's problem exists, since the second order conditions are satisfied for all wage schemes, satisfying $w(\bar{x}z) \geq w(\underline{x}z)$ (which we show below holds in equilibrium): the cost function is strictly convex and the probability function concave for all output combinations. For out of equilibrium wages $w(\bar{x}z) < w(\underline{x}z)$ the agent provides an effort of zero.

The Principal's Problem:

We can now solve the principal's problem (as stated in the text). This is a linear optimization problem, with a convex feasible set and a continuous and concave objective function. Since for such problems any local maximum is a global maximum, the Kuhn-Tucker first order conditions are necessary and sufficient for an optimum.

Note that $w(\underline{x}\bar{z}) > 0$ (analogue $w(\underline{x}\underline{z}) > 0$) decreases incentives, compared to setting $w(\underline{x}\bar{z}) = 0$ (analogue $w(\underline{x}\underline{z}) = 0$), since $f_e(\underline{x}z|e) < 0$. This cannot be profit maximizing for the principal since effort is smaller and the principal pays the agent more by setting $w(\underline{x}\bar{z}) > 0$ (analogue $w(\underline{x}\underline{z}) > 0$), which reduces her profit. Therefore, $w(\underline{x}\bar{z}) = 0$ (analogue $w(\underline{x}\underline{z}) = 0$).

Denoting by λ the Lagrange multiplier and by $\mathcal{L}$ the Lagrange function for the principal's problem, the first order conditions with respect to $w(\bar{x}\bar{z})$ and $w(\bar{x}\underline{z})$ are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(\bar{x}\bar{z})} &= -f(\bar{x}\bar{z}|e) + \lambda\, f_e(\bar{x}\bar{z}|e) \leq 0, \quad w(\bar{x}\bar{z})\,\frac{\partial \mathcal{L}}{\partial w(\bar{x}\bar{z})} = 0,\\
\frac{\partial \mathcal{L}}{\partial w(\bar{x}\underline{z})} &= -f(\bar{x}\underline{z}|e) + \lambda\, f_e(\bar{x}\underline{z}|e) \leq 0, \quad w(\bar{x}\underline{z})\,\frac{\partial \mathcal{L}}{\partial w(\bar{x}\underline{z})} = 0.
\end{aligned}$$

There exists a λ such that both equations hold with equality if and only if $l(\bar{x}\bar{z}|\hat{e}) = l(\bar{x}\underline{z}|e)$. In this case wages are determined by the incentive constraint: $[f_e(\bar{x}\bar{z}|e)w(\bar{x}\bar{z}) + f_e(\bar{x}\underline{z}|e)w(\bar{x}\underline{z})] = c_e(e)$.

So let these ratios be unequal and assume first $l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e)$. Suppose that $\frac{\partial \mathcal{L}}{\partial w(\bar{x}\bar{z})} = 0$ and hence $w(\bar{x}\bar{z}) \geq 0$. Rearranging gives us $\lambda = 1/l(\bar{x}\bar{z}|e)$. Plugging this in $\frac{\partial \mathcal{L}}{\partial w(\bar{x}\underline{z})}$ and rearranging again shows that this holds with $<$ given the assumption that $l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e)$.

Hence, $w(\bar{x}\underline{z}) = 0$. From the incentive constraint it then follows that the principal sets $w(\bar{x}\bar{z}) > 0$ to implement a strictly positive effort. The case $l(\bar{x}\bar{z}|e) < l(\bar{x}\underline{z}|e)$ follows analogue. ■

Proof Equation (1.1).

As described in the Proof of Lemma 1 if likelihood ratios are equal, wages for a given effort vector e are determined by:

$$f_e(\bar{x}\bar{z}|e)w(\bar{x}\bar{z}) + f_e(\bar{x}\underline{z}|e)w(\bar{x}\underline{z}) = c_e(e).$$

Substituting for $f_e(\bar{x}\bar{z}|e)$ from the likelihood ratios yields:

$$f_e(\bar{x}\underline{z}|e)\left[\frac{f(\bar{x}\bar{z}|e)}{f(\bar{x}\underline{z}|e)}w(\bar{x}\bar{z}) + w(\bar{x}\underline{z})\right] = c_e(e).$$

Rearranging gives us the expected wage payment:

$$C(\bar{x}|e) = f(\bar{x}\bar{z}|e)w(\bar{x}\bar{z}) + f(\bar{x}\underline{z}|e)w(\bar{x}\underline{z}) = \frac{f(\bar{x}\underline{z}|e)}{f_e(\bar{x}\underline{z}|e)}c_e(e) = \frac{g(\bar{x}|e)}{g_e(\bar{x}|e)}c_e(e).$$

The last equality follows as $l(\bar{x}\bar{z}|e) = l(\bar{x}\underline{z}|e) \Leftrightarrow f_e(\bar{x}|e)f(xz|e) = g(\bar{x}|e)f_e(xz|e)$. Also $l(\bar{x}\bar{z}|e) = l(\bar{x}|e) \leftrightarrow f_e(\bar{x}|e)f(xz|e) = g(\bar{x}|e)f_e(xz|e)$.

For $l(\bar{x}\bar{z}|\hat{e}) \neq l(\bar{x}\underline{z}|\hat{e})$ note that we can write the incentive constraint as:

$$f_e(xz|e)w(xz) = c_e(e),$$

where $xz \in \{\bar{x}\bar{z}, \bar{x}\underline{z}\}$, depending on the likelihood ratios as described in the Proof of Lemma 1. Hence, to implement effort levels e the wage has to satisfy:

$$w(xz) = \frac{c_e(e)}{f_e(xz|e)}. \tag{3}$$

To calculate then the expected wage, $C(xz|e)$, multiply $w(xz)$ by $f(xz|e)$. ■

Proof Equation (1.2).

To obtain Equation (1.2) multiply $w(\bar{x}\bar{z})$ by $f(\bar{x}\bar{z}|e)$, which gives $p\,\frac{f(\bar{x}\bar{z}|e)}{f_e(\bar{x}\bar{z}|e)}$ and $w(\bar{x}\underline{z})$ by $f(\bar{x}\underline{z}|e)$, which gives $(1-p)\,\frac{f(\bar{x}\underline{z}|e)}{f_e(\bar{x}\underline{z}|e)}$. Adding up yields:

$$C^A(\bar{x}\bar{z}, \bar{x}\underline{z}|\bar{e}, \underline{e}) = p\,\frac{c_e(\bar{e})}{l(\bar{x}\bar{z}|\bar{e})} + (1-p)\,\frac{c_e(\underline{e})}{l(\bar{x}\underline{z}|\underline{e})}$$

Then use the definition of $C(s|e) = \frac{c_e(e)}{f_e(s|e)}c_e(e)$. ■

Ex Ante versus Intermediate Information.

What differs under these two informational scenarios is the agent's participation constraint(s). For the intermediate information scenario it is:

$$\sum_x \sum_z f(xz|e(z))w(xz) - p(z)c(e(z)) \geq 0,$$

and for the ex ante information scenario:

$$\sum_x f(x\bar{z}|e(\bar{z}))w(x\bar{z}) - p\, c(e(\bar{z})) \geq 0,$$

$$\sum_x f(xz|e(\underline{z}))w(x\underline{z}) - (1-p)\, c(e(\underline{z})) \geq 0.$$

Using the two incentive constraint we see that the agent receives in every of these two states a strictly positive rent. Hence, they are satisfied. The ex ante participation constraint (for the intermediate information scenario) then also holds. ∎

Proof Proposition 2.

Suppose that likelihood ratios are equal, i.e. $l(\bar{x}\bar{z}|e) = l(\bar{x}\underline{z}|e) = l(\bar{x}|e)$. Using this in Equation (1.2), we obtain for the expected wage of the ex ante information scenario:

$$E\left[\frac{c_e(e(z))}{l(\bar{x}|e(z))}\right].$$

For the ex post information scenario the expected wage is:

$$\left[\frac{c_e(Ee(z))}{l(\bar{x}|Ee(z))})\right].$$

Suppose $\bar{e} \neq \underline{e}$. Defining $g(e) = \frac{c_e(e)}{l(\bar{x}|e)}$ and applying Jensen's Inequality shows that the expected wage under the ex ante information scenario is larger if and only if $Eg(e(z)) \geq g(Ee(z))$, i.e. if and only if $g(e)$ is convex.

For $l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e)$ (the other case is analogue), it follows that:

$$\begin{aligned} C^A(\bar{x}\bar{z}, \bar{x}\underline{z}|\bar{e}, \underline{e}) &= p\,\frac{c_e(\bar{e})}{l(\bar{x}\bar{z}|\bar{e})} + (1-p)\,\frac{c_e(\underline{e})}{l(\bar{x}\underline{z}|\underline{e})} \\ &> p\,\frac{c_e(\bar{e})}{l(\bar{x}\bar{z}|\bar{e})} + (1-p)\,\frac{c_e(\underline{e})}{l(\bar{x}\bar{z}|\underline{e})} \\ &\geq \frac{c_e(e)}{l(\bar{x}\bar{z}|e)}. \end{aligned}$$

∎

Proof Proposition 3.

The principal's expected profit if the agent receives information ex post is:

$$\begin{aligned} \Pi(e) &= g(\bar{x}|e)\bar{x} - C(\bar{x}|e) \\ &= \{p\, h(\bar{x}|\bar{z}, \bar{e}) + (1-p)\, h(\bar{x}|\underline{z}, \underline{e})\}\, \bar{x} - \{p\, C(\bar{x}|\bar{e}) + (1-p)\, C(\bar{x}|\underline{e})\}, \end{aligned}$$

with $\bar{e} = \underline{e}$. This is just the profit function of the ex ante information scenario, where we, however, do not require $\bar{e} = \underline{e}$. Hence, the maximization problem of the principal is identical for both structures up to this restriction. Maximizing without it cannot make the principal worse off.

The proof that ex ante information can do strictly better if and only if there are gains from tailoring effort follows exactly the same lines as the Proof of Proposition 1 ■

Proof Proposition 4.

We first argue that for any $(\bar{e}, \underline{e})$ of the ex ante information scenario we can implement the same expected effort for the ex post information scenario yielding to strictly higher profits.

For the implementation cost part remember from Proposition 2 that those are strictly higher for the same expected effort under an informative signal. Furthermore, expected revenues are lower for the ex ante information scenario, since: 1) we excluded gains from tailoring effort, i.e. $f(\bar{x}\bar{z}|\bar{e}) + f(\bar{x}\underline{z}|\underline{e}) = p\, g(\bar{x}|\bar{e}) + (1-p)g(\bar{x}|\underline{e})$ and 2) $g(\bar{x}|e)$ is concave in e.

Taking implementation costs and expected revenues together we see that for any $(\bar{e}, \underline{e}) \neq 0$ (which is optimal for $f_e(\bar{x}z|e) > 0$) for the ex ante information scenario, we can implement the same expected profit for the ex post scenario, leading to strictly higher profits. This effort need not to be optimal. But the principal cannot do worse with the optimal one. ■

Appendix to Chapter 2

Proof Equation (2.1).

Note that we can omit the participation constraint: it is automatically satisfied if the limited liability (which requires $w(xx') \geq 0$) and the incentive constraint are. Furthermore, wages after a failure are zero, as they would decrease the principal's profits and agent's incentives.

The principal's problem is a linear optimization problem, with a convex feasible set and a continuous and concave objective function. Since for such problems any local maximum is a global maximum, the Kuhn-Tucker first order conditions are necessary and sufficient for an optimum. Denoting by λ the Lagrange multiplier and by $\mathcal{L}$ the Lagrange function, the first order conditions with respect to $w(\bar{x}\bar{z})$ and $w(\bar{x}\underline{z})$ are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w(\bar{x}\bar{x})} &= -f(\bar{x}\bar{x}|e, e_H) + \lambda[f(\bar{x}\bar{x}|e, e_H) - f(\bar{x}\bar{x}|e, e_L)] \leq 0, \quad w(\bar{x}\bar{x})\,\frac{\partial \mathcal{L}}{\partial w(\bar{x}\bar{x})} = 0, \\
\frac{\partial \mathcal{L}}{\partial w(\bar{x}\underline{x})} &= -f(\underline{x}\bar{x}|e, e_H) + \lambda[f(\underline{x}\bar{x}|e, e_H) - f(\underline{x}\bar{x}|e, e_L)] \leq 0, \quad w(\bar{x}\underline{x})\,\frac{\partial \mathcal{L}}{\partial w(\bar{x}\underline{x})} = 0.
\end{aligned}$$

There exists a λ such that both equations hold with equality if and only if $l(\bar{x}\bar{x}|e) = l(\underline{x}\bar{x}|e) = l(\bar{x}|e)$. In this case wages are determined by the incentive constraint. Substituting for $\frac{f(x\bar{x}|e,e_H)-f(x\bar{x}|e,e_L)}{f(x\bar{x}|e,e_H)} = \frac{g(\bar{x}|e_H)-g(\bar{x}|e_L)}{g(\bar{x}|e_H)}$ from the likelihood ratios and plugging into the incentive constraint gives the expected wage payment (implementation costs) $\sum_x f(x\bar{x}|e, e_H)w(x\bar{x}) = \frac{c}{1-l(\bar{x}|e)}$.

So suppose that these ratios are unequal. For $l(\bar{x}\bar{x}|e) > l(\underline{x}\bar{x}|e)$ it is easy to see that this implies $w(\bar{x}\bar{x}) > w(\underline{x}\bar{x}) = 0$. Hence, we can write the incentive constraint as:

$$[f(\bar{x}\bar{x}|e, e_H) - f(\bar{x}\bar{x}|e, e_H)]w(\bar{x}\bar{x}) = c,$$

which gives us $w(\bar{x}\bar{x})$ as stated in the text. Multiplying by $f(\bar{x}\bar{x}|e, e_H)$ gives us the implementation costs. ■

Proof Equation (2.2).

Derive $w(\bar{x}\bar{x})$ and $w(\underline{x}\bar{x})$ from the incentive constraints and take expectations. ■

Proof Proposition 5.

$$\begin{aligned}
C(\bar{x}\bar{x}|e_H, e_H) - C(\bar{x}\bar{x}|e_H, e_L) &= \frac{1}{1 - l(\bar{x}\bar{x}|e_H)} - \frac{1}{1 - l(\bar{x}\bar{x}|e_L)} \\
&= \frac{l(\bar{x}\bar{x}|e_H) - l(\bar{x}\bar{x}|e_L)}{(1 - l(\bar{x}\bar{x}|e_H))(1 - l(\bar{x}\bar{x}|e_L))}.
\end{aligned}$$

This is larger than zero for:

$$l(\bar{x}\bar{x}|e_H) > l(\bar{x}\bar{x}|e_L) \Leftrightarrow [f(\bar{x}\bar{x}|e_L, e_H)]^2 > f(\bar{x}\bar{x}|e_L, e_L)f(\bar{x}\bar{x}|e_H, e_H).$$

■

Proof Assumption 3.

To see that Assumption 3 implies that both agents provide high effort note that this is optimal if and only if:

$$2g(\bar{x}|e_H)\bar{x} - 2C(\bar{x}\bar{x}|e_H, e_H) \geq [g(\bar{x}|e_H) + g(\bar{x}|e_L)]\bar{x} - C(\bar{x}\bar{x}|e_H, e_L).$$

Rearranging gives the assumption.

Next we want to show that Assumption 3 implies that the first agent under the ex ante information scenario provides high effort. Suppose first that $\bar{e} = \underline{e} = e_H$. Then implementing $e_1 = e_H$ is optimal if and only if:

$$\begin{aligned}
& g(\bar{x}|e_H)\bar{x} - C(\bar{x}\bar{x}|e_H, e_H) + g(\bar{x}|e_H)\bar{x} \\
- \; & \underbrace{[g(\bar{x}|e_H)C(\bar{x}\bar{x}|e_H, e_H) + (1 - g(\bar{x}|e_H))C(\underline{x}\bar{x}|e_H, e_H)]}_{\leq g(\bar{x}|e_L)C(\bar{x}\bar{x}|e_H,e_H)+(1-g(\bar{x}|e_L))C(\underline{x}\bar{x}|e_H,e_H)} \\
\geq \; & g(\bar{x}|e_L)\bar{x} + g(\bar{x}|e_H)\bar{x} - g(\bar{x}|e_L)C(\bar{x}\bar{x}|e_L, e_H) - (1 - g(\bar{x}|e_L))C(\underline{x}\bar{x}|e_L, e_H),
\end{aligned}$$

or:

$$[g(\bar{x}|e_H) - g(\bar{x}|e_L)]\bar{x} \geq C(\bar{x}\bar{x}|e_H, e_H) + \underbrace{EC(x\bar{x}|e_H, e_H) - EC(x\bar{x}|e_L, e_H)}_{\leq C(\bar{x}\bar{x}|e_H,e_H)-C(\bar{x}\bar{x}|e_H,e_L)},$$

where the inequality in the lower bracket is shown by showing $C(\underline{x}\bar{x}|e_H, e_H) - C(\underline{x}\bar{x}|e_L, e_H)] < C(\bar{x}\bar{x}|e_H, e_H) - C(\bar{x}\bar{x}|e_H, e_H)$. Hence, this always holds given Assumption 3.

Suppose now $\bar{e} = e_H$ and $\underline{e} = e_L$. Then implementing $e_1 = e_H$ is optimal if and only if:

$$\begin{aligned}
& g(\bar{x}|e_H)\bar{x} - C(\bar{x}\bar{x}|e_H, e_H) + f(\bar{x}\bar{x}|e_H, e_H)\bar{x} + f(\underline{x}\bar{x}|e_H, e_L)\bar{x} - g(\bar{x}|e_H)C(\bar{x}\bar{x}|e_H, e_H) \\
\geq \; & g(\bar{x}|e_L)\bar{x} + f(\bar{x}\bar{x}|e_L, e_H)\bar{x} + f(\underline{x}\bar{x}|e_L, e_L)\bar{x} - g(\bar{x}|e_L)C(\bar{x}\bar{x}|e_L, e_H),
\end{aligned}$$

or:

$$\begin{aligned}
& \{g(\bar{x}|e_H) - g(\bar{x}|e_L) + f(\bar{x}\bar{x}|e_H, e_H) - f(\bar{x}\bar{x}|e_L, e_H) + f(\underline{x}\bar{x}|e_H, e_L) - f(\underline{x}\bar{x}|e_L, e_L)\} \\
\geq \; & C(\bar{x}\bar{x}|e_H, e_H) + g(\bar{x}|e_H)C(\bar{x}\bar{x}|e_H, e_H) - g(\bar{x}|e_L)C(\bar{x}\bar{x}|e_L, e_H),
\end{aligned}$$

because Assumption 3 implies $g(\bar{x}|e_H) - g(\bar{x}|e_L) > C(\bar{x}\bar{x}|e_H, e_H)$. As furthermore, $h(\bar{x}|\bar{x}, e_H, e_H) - h(\bar{x}|\bar{x}, e_L, e_H) > C(\bar{x}\bar{x}|e_H, e_H) - C(\bar{x}\bar{x}|e_L, e_H)$ this is always satisfied. Suppose now $\bar{e} = e_L$ and $\underline{e} = e_H$. Then implementing $e_1 = e_H$ is optimal if and only if:

$$\begin{aligned} & g(\bar{x}|e_H)\bar{x} - C(\bar{x}\bar{x}|e_H, e_H) + f(\bar{x}\bar{x}|e_H, e_L)\bar{x} + f(\underline{x}\bar{x}|e_H, e_H)\bar{x} \\ & -(1 - g(\bar{x}|e_H))C(\underline{x}\bar{x}|e_H, e_H) \\ \geq \; & g(\bar{x}|e_L)\bar{x} + f(\bar{x}\bar{x}|e_L, e_L)\bar{x} + f(\underline{x}\bar{x}|e_L, e_H)\bar{x} - (1 - g(\bar{x}|e_L))C(\underline{x}\bar{x}|e_L, e_H), \end{aligned}$$

which holds because by Assumption 3 $g(\bar{x}|e_H) - g(\bar{x}|e_L) > C(\bar{x}\bar{x}|e_H, e_H)$. And furthermore, $(1 - g(\bar{x}|e_L))[C(\underline{x}\bar{x}|e_H, e_H) - C(\underline{x}\bar{x}|e_L, e_H)] < C(\bar{x}\bar{x}|e_H, e_H) - C(\bar{x}\bar{x}|e_H, e_H)$. ■

Proof Proposition 8.

Note first that $(\bar{e}, \underline{e}) = (e_H, e_H)$ can arise under Assumption 3. The condition for $\bar{e} = e_H$ to be optimal is :

$$\begin{aligned} & [g(\bar{x}|e_H) + f(\bar{x}\bar{x}|e_H, e_H)]\bar{x} - (1 + g(\bar{x}|e_H))C(\bar{x}\bar{x}|e_H, e_H) \\ \geq \; & [g(\bar{x}|e_H) + f(\bar{x}\bar{x}|e_H, e_L)]\bar{x} - C(\bar{x}\bar{x}|e_H, e_H), \end{aligned}$$

or:

$$[f(\bar{x}\bar{x}|e_H, e_H) - f(\bar{x}\bar{x}|e_H, e_L)]\bar{x} \geq g(\bar{x}|e_H)C(\bar{x}\bar{x}|e_H, e_H).$$

Note that if $h(\bar{x}|\bar{x}, e_H, e_H) - h(\bar{x}|\bar{x}, e_H, e_L) = g(\bar{x}|e_H) - g(\bar{x}|e_L)$ this is always satisfied by Assumption 3.

Condition for $\underline{e} = e_H$ is:

$$\begin{aligned} & [g(\bar{x}|e_H) + f(\bar{x}\underline{x}|e_H, e_H)]\bar{x} - C(\bar{x}\bar{x}|e_H, e_H) - (1 - g(\bar{x}|e_H))C(\underline{x}\bar{x}|e_H, e_H) \\ \geq \; & [g(\bar{x}|e_H) + f(\bar{x}\underline{x}|e_H, e_H)]\bar{x} - C(\bar{x}\bar{x}|e_H, e_H), \end{aligned}$$

or:

$$[f(\underline{x}\bar{x}|e_H, e_H) - f(\underline{x}\bar{x}|e_H, e_L)]\bar{x} \geq (1 - g(\bar{x}|e_H))C(\underline{x}\bar{x}|e_H, e_H).$$

$C(\underline{x}\bar{x}|e_H, e_H)$ does not appear in Assumption 3 and hence this can be satisfied.

For $(\bar{e}, \underline{e}) = (e_H, e_H)$ profits for the first agent are the same under both structures. Revenues for the second agent are also the same. However it follows from Equation (2.1) and (2.2) that implementation costs are strictly higher for the second agent if the principal provides ex ante information.

That ex ante information does strictly worse for $(\bar{e}, \underline{e}) = (e_L, e_L)$ follows directly from Assumption 3. ■

Proof Proposition 9.

As we have shown in the Proof of Proposition 8 $(\bar{e}, \underline{e}) = (e_H, e_L)$ or $(\bar{e}, \underline{e}) = (e_L, e_H)$ can both arise under Assumption 3. The former only if $h(\bar{x}|\bar{x}, e_H, e_H) - h(\bar{x}|\bar{x}, e_H, e_L) \neq g(\bar{x}|e_H) - g(\bar{x}|e_L)$.

Consider first the case, where $(\bar{e}, \underline{e}) = (e_L, e_H)$. Here ex ante information does strictly better than ex post information if and only if:

$$\begin{aligned} & 2[g(\bar{x}|e_H)\bar{x} - C(\bar{x}\bar{x}|e_H, e_H)] \\ \leq \;& g(\bar{x}|e_H)\bar{x} - C(\bar{x}\bar{x}|e_H, e_L) + [f(\bar{x}\bar{x}|e_H, e_L) + f(\underline{x}\bar{x}|e_H, e_H)]\bar{x}\bar{x} \\ - \;& (1 - g(\bar{x}|e_H))C(\underline{x}\bar{x}|e_H, e_H), \end{aligned}$$

or:

$$[f(\bar{x}\bar{x}|e_H, e_H) - f(\bar{x}\bar{x}|e_H, e_L)]\bar{x} \leq 2C(\bar{x}\bar{x}|e_H, e_H) - C(\bar{x}\bar{x}|e_H, e_L) - (1 - g(\bar{x}|e_H))C(\underline{x}\bar{x}|e_H, e_H).$$

Consider next the case, where $(\bar{e}, \underline{e}) = (e_H, e_L)$. Here ex ante information does strictly better than ex post information if and only if:

$$\begin{aligned} & 2(g(\bar{x}|e_H)\bar{x} - C(\bar{x}\bar{x}|e_H, e_H)) \\ \leq \;& g(\bar{x}|e_H)\bar{x} - C(\bar{x}\bar{x}|e_H, e_H) + [f(\bar{x}\bar{x}|e_H, e_H) + f(\underline{x}\bar{x}|e_H, e_L)]\bar{x} - g(\bar{x}|e_H)C(\bar{x}\bar{x}|e_H, e_H), \end{aligned}$$

or:

$$[f(\underline{x}\bar{x}|e_H, e_H) - f(\underline{x}\bar{x}|e_H, e_L)]\bar{x} \leq (1 - g(\bar{x}|e_H))C(\bar{x}\bar{x}|e_H, e_H).$$

This cannot for example hold in the plus-k model. ■

Appendix to Chapter 3

Proof Lemma 2.

$$\frac{f_e(\bar{x}\bar{z}|e)}{f(\bar{x}\bar{z}|e)} > \frac{f_e(\bar{x}\underline{z}|e)}{f(\bar{x}\underline{z}|e)} \quad \Leftrightarrow \quad \frac{f_e(\bar{x}\bar{z}|e)}{f(\bar{x}\bar{z}|e)} > \frac{g_e(\bar{x}|e) - f_e(\bar{x}\bar{z}|e)}{g(\bar{x}|e) - f(\bar{x}\bar{z}|e)}$$
$$\Leftrightarrow \quad \frac{g(\bar{x}|e)f_e(\bar{x}\bar{z}|e) - f(\bar{x}\bar{z}|e)g_e(\bar{x}|e)}{f(\bar{x}\bar{z}|e)[g(\bar{x}|e) - f(\bar{x}\bar{z}|e)]} > 0.$$

And:

$$\frac{g_e(\bar{x}|e)}{g(\bar{x}|e)} > \frac{f_e(\bar{x}\underline{z}|e)}{f(\bar{x}\underline{z}|e)} \Leftrightarrow \frac{g(\bar{x}|e)f_e(\bar{x}\bar{z}|e) - f(\bar{x}\bar{z}|e)g_e(\bar{x}|e)}{f(\bar{x}\bar{z}|e)[g(\bar{x}|e) - f(\bar{x}\bar{z}|e)]} > 0.$$

And:

$$\frac{f_e(\bar{x}\bar{z}|e)}{f(\bar{x}\bar{z}|e)} > \frac{g_e(\bar{x}|e)}{g(\bar{x}|e)} \Leftrightarrow \frac{g(\bar{x}|e)f_e(\bar{x}\bar{z}|e) - f(\bar{x}\bar{z}|e)g_e(\bar{x}|e)}{f(\bar{x}\bar{z}|e)g(\bar{x}|e)} > 0.$$

■

Proof Lemma 3.

Before we can solve the principal's problem we have to check that we can indeed omit the participation constraint and that the incentive constraint yields a global maximum of the agent's problem for the equilibrium wage scheme.

Omit the Participation Constraint:

This constraint is given by:

$$f(\bar{x}\bar{z}|\hat{e})w(\bar{x}\bar{z}) + f(\bar{x}\underline{z}|\hat{e})w(\bar{x}\underline{z}) + f(\underline{x}\bar{z}|\hat{e})w(\underline{x}\bar{z}) + f(\underline{x}\underline{z}|\hat{e})w(\underline{x}\underline{z}) - c(\hat{e}) \geq 0.$$

Since the agent's reservation utility is zero, the participation constraint is automatically satisfied if the limited liability and the incentive constraint are: by choosing an effort level of zero, the agent can achieve at least an expected utility of zero as wages are larger than zero by the limited liability constraint. Hence, by maximizing his expected utility over effort (the incentive constraint) he can never do worse then zero.

Incentive Constraint Yields Unique Global Maximum:

Concerning the incentive constraint note that a global maximum of the agent's problem exists, since the second order conditions are satisfied for all wage schemes, satisfying $w(\bar{x}z) \geq w(\underline{x}z)$ (which we show below holds in equilibrium): the cost function is strictly convex and the probability function concave for all output combinations. For out of equilibrium wages $w(\bar{x}z) < w(\underline{x}z)$ the agent provides an effort of zero.

The Principal's Problem:

We can now solve the principal's problem (as stated in the text). This is a linear optimization problem, with a convex feasible set and a continuous and concave objective function. Since for such problems any local maximum is a global maximum, the Kuhn-Tucker first order conditions are necessary and sufficient for an optimum.

Note that $w(\underline{x}\bar{z}) > 0$ (analogue $w(\underline{x}\underline{z}) > 0$) decreases incentives, compared to setting $w(\underline{x}\bar{z}) = 0$ (analogue $w(\underline{x}\underline{z}) = 0$), since $f_e(\underline{x}z|e) < 0$. This cannot be profit maximizing for the principal since effort is smaller and the principal pays the agent more by setting $w(\underline{x}\bar{z}) > 0$ (analogue $w(\underline{x}\underline{z}) > 0$), which reduces her profit. Therefore, $w(\underline{x}\bar{z}) = 0$ (analogue $w(\underline{x}\underline{z}) = 0$).

Denoting by λ the Lagrange multiplier and by $\mathcal{L}$ the Lagrange function for the principal's problem, the first order conditions with respect to $w(\bar{x}\bar{z})$ and $w(\bar{x}\underline{z})$ are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial w(\bar{x}\bar{z})} &= -f(\bar{x}\bar{z}|e) + \lambda\, f_e(\bar{x}\bar{z}|e) \leq 0, \quad w(\bar{x}\bar{z})\,\frac{\partial \mathcal{L}}{\partial w(\bar{x}\bar{z})} = 0, \\ \frac{\partial \mathcal{L}}{\partial w(\bar{x}\underline{z})} &= -f(\bar{x}\underline{z}|e) + \lambda\, f_e(\bar{x}\underline{z}|e) \leq 0, \quad w(\bar{x}\underline{z})\,\frac{\partial \mathcal{L}}{\partial w(\bar{x}\underline{z})} = 0. \end{aligned}$$

There exists a λ such that both equations hold with equality if and only if $l(\bar{x}\bar{z}|e) = l(\bar{x}\underline{z}|e)$. In this case wages are determined by the incentive constraint: $[f_e(\bar{x}\bar{z}|e)w(\bar{x}\bar{z}) + f_e(\bar{x}\underline{z}|e)w(\bar{x}\underline{z})] = c_e(e)$.

So let these ratios be unequal and assume first $l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e)$. Suppose that $\frac{\partial \mathcal{L}}{\partial w(\bar{x}\bar{z})} = 0$ and hence $w(\bar{x}\bar{z}) \geq 0$. Rearranging gives us $\lambda = 1/l(\bar{x}\bar{z}|e)$. Plugging this in $\frac{\partial \mathcal{L}}{\partial w(\bar{x}\underline{z})}$ and rearranging again shows that this holds with $<$ given the assumption that $l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e)$. Hence, $w(\bar{x}\underline{z}) = 0$. From the incentive constraint it then follows that the principal sets $w(\bar{x}\bar{z}) > 0$ to implement a strictly positive effort. The case $l(\bar{x}\bar{z}|e) < l(\bar{x}\underline{z}|e)$ follows analogue. ■

Proof Equation (3.1) and (3.2).

As described in the Proof of Lemma 3 if likelihood ratios are equal, wages for a given effort vector e are determined by:

$$f_e(\bar{x}\bar{z}|e)w(\bar{x}\bar{z}) + f_e(\bar{x}\underline{z}|e)w(\bar{x}\underline{z}) = c_e(e).$$

Substituting for $f_e(\bar{x}\bar{z}|e)$ from the likelihood ratios yields:

$$f_e(\bar{x}\underline{z}|e)\left[\frac{f(\bar{x}\bar{z}|e)}{f(\bar{x}\underline{z}|e)}w(\bar{x}\bar{z})+w(\bar{x}\underline{z})\right]=c_e(e).$$

Rearranging gives us the expected wage payment:

$$C(\bar{x}|e)=f(\bar{x}\bar{z}|e)w(\bar{x}\bar{z})+f(\bar{x}\underline{z}|e)w(\bar{x}\underline{z})=\frac{f(\bar{x}\underline{z}|e)}{f_e(\bar{x}\underline{z}|e)}c_e(e)=\frac{g(\bar{x}|e)}{g_e(\bar{x}|e)}c_e(e).$$

The last equality follows as $l(\bar{x}\bar{z}|e)=l(\bar{x}\underline{z}|e)\Leftrightarrow f_e(\bar{x}|e)f(xz|e)=g(\bar{x}|e)f_e(xz|e)$. Also $l(\bar{x}\bar{z}|e)=l(\bar{x}|e)\leftrightarrow f_e(\bar{x}|e)f(xz|e)=g(\bar{x}|e)f_e(xz|e)$.

For $l(\bar{x}\bar{z}|\hat{e})\neq l(\bar{x}\underline{z}|\hat{e})$ note that we can write the incentive constraint as:

$$f_e(xz|e)w(xz)=c_e(e),$$

where $xz\in\{\bar{x}\bar{z},\bar{x}\underline{z}\}$, depending on the likelihood ratios as described in the Proof of Lemma 3. Hence, to implement effort levels e the wage has to satisfy:

$$w(xz)=\frac{c_e(e)}{f_e(xz|e)}. \tag{4}$$

To calculate then the expected wage, $C(xz|e)$, multiply $w(xz)$ by $f(xz|e)$. ■

Proof Lemma 4.

To see this subtract implementation costs:

$$\begin{aligned}(C(\bar{x}\bar{z}|e)-C(\bar{x}|e))/c_e(e) &= \frac{f(\bar{x}\bar{z}|e)}{f_e(\bar{x}\bar{z}|e)}-\frac{g(\bar{x}|e)}{g_e(\bar{x}|e)}\\ &= \frac{f(\bar{x}\bar{z}|e)g_e(\bar{x}|e)-g(\bar{x}|e)f_e(\bar{x}\bar{z}|e)}{f_e(\bar{x}\bar{z}|e)g_e(\bar{x}|e)}<0,\end{aligned}$$

which holds as $C(\bar{x}\bar{z}|e)$ are the relevant implementation costs if $l(\bar{x}\bar{z}|e)>l(\bar{x}\underline{z}|e)$, which we have shown in the Proof of Lemma 2 is satisfied for $f(\bar{x}\bar{z}|e)g_e(\bar{x}|e)-g(\bar{x}|e)f_e(\bar{x}\bar{z}|e)<0$. Hence, the profit function under the informative signal lies above the one for no additional information scenario for all effort levels. Thus, also the maximized value of implementation costs has to lie above. ■

Proof Equation (3.4).

The first order conditions characterize a global maximum as the second order conditions are satisfied: the probability and cost function are concave/convex (one strictly) by assumption.

The posterior probabilities are:

$$h(\bar{z}|\bar{x},e)=\frac{f(\bar{x}\bar{z}|e)}{p}\quad\text{and}\quad h(\bar{z}|\underline{x},e)=\frac{f(\bar{x}\underline{z}|e)}{1-p}.$$

Plugging them in the incentive constraints, we receive the following wages that are necessary to implement effort levels $(\bar{e}, \underline{e})$: $w(\bar{x}\bar{z}) = \frac{p}{f_e(\bar{x}\bar{z}|e)} c_e(\bar{e})$, and $w(\bar{x}\underline{z}) = \frac{1-p}{f_e(\bar{x}\underline{z}|e)} c_e(\bar{e})$. To obtain Equation (3.6) multiply $w(\bar{x}\bar{z})$ by $f(\bar{x}\bar{z}|e)$, which gives $p \frac{f(\bar{x}\bar{z}|e)}{f_e(\bar{x}\bar{z}|e)}$ and $w(\bar{x}\underline{z})$ by $f(\bar{x}\bar{z}|e)$, which gives $(1-p) \frac{f(\bar{x}\underline{z}|e)}{f(\bar{x}\underline{z}|e)}$. Adding up yields:

$$C^A(\bar{x}\bar{z}, \bar{x}\underline{z}|\bar{e}, \underline{e}) = p \, \frac{c_e(\bar{e})}{l(\bar{x}\bar{z}|\bar{e})} + (1-p) \, \frac{c_e(\underline{e})}{l(\bar{x}\underline{z}|\underline{e})}$$

Then use the definition of $C(s|e) = \frac{c_e(e)}{f_e(s|e)} c_e(e)$. ■

Proof Proposition 11.

To obtain the condition in the corollary rearrange Equation (3.7) assuming that the principal implements in all states and scenarios an effort of e:

$$\frac{f(\bar{x}\bar{z}|e) g_e(\bar{x}|e) - g(\bar{x}|e) f_e(\bar{x}\bar{z}|e)}{f_e(\bar{x}\bar{z}|e) g_e(\bar{x}|e)} \left(p - (1-p) \frac{f_e(\bar{x}\bar{z}|e)}{f_e(\bar{x}\underline{z}|e)} \right) c_e(e).$$

The first term gives us the difference in the inverse likelihood ratios in state $\bar{x}\bar{z}$ and $\bar{x}$, which is – as we assumed $l(\bar{x}\bar{z}|e) > l(\bar{x}\underline{z}|e)$ – smaller than zero. The second term is smaller than zero if and only if the marginal posterior success probability in state $\bar{x}\bar{z}$ is larger than the one in state $\bar{x}\underline{z}$. If these terms have opposite signs, then an informative signal leads to lower implementation costs for the same effort (say the optimal one for the uninformative one). But as implementation costs are already lower for $\bar{e} = \underline{e} = e$, which need not to be optimal, the principal cannot do worse by having the opportunity to choose $(\bar{e}, \underline{e})$ optimally. ■

Proof Equation (3.10).

Implementation costs for the informative signal are:

$$\begin{aligned} & p \frac{a\,\bar{e}\,p + k}{a\,p} \bar{e} + (1-p) \, \frac{\underline{e}\,(1 - a\,p) - k}{1 - a\,p} \underline{e} \\ = \quad & \underbrace{p\,(\bar{e})^2 + (1-p)\,(\underline{e})^2}_{A} + \underbrace{k \left(\frac{1}{a} \bar{e} - \frac{1-p}{1-a\,p} \underline{e} \right)}_{B}. \end{aligned}$$

Implementation costs in case no signal is available are:

$$\underbrace{(p\,\bar{e} + (1-p)\,\underline{e})^2}_{C}.$$

Subtracting C from A:

$$\begin{aligned} & p\,(\bar{e})^2 + (1-p)\,(\underline{e})^2 - (p\,\bar{e} + (1-p)\,\underline{e})^2) \\ = \;& p\,(1-p)\,(\bar{e})^2 + p\,(1-p)\,(\underline{e})^2 - 2\,p\,(1-p)\,\bar{e}\,\underline{e} \\ = \;& p\,(1-p)\,(\bar{e}-\underline{e})^2. \end{aligned}$$

Adding term B gives us Equation (3.10). ■

Proof Lemma 5.

We first ignore the possibilities of corner solutions. Those can arise because the probabilities have to lie between zero and one. We check below under which conditions the equilibrium is indeed interior for the parameter range we are interested in. The maximization problem of the principal for the informative signal is then:

$$\begin{aligned} \max_{\underline{e},\bar{e}} \quad & -p\frac{a\,p\,\bar{e}+k}{a\,p}\bar{e} - (1-p)\frac{(1-a\,p)\,\underline{e}-k}{1-a\,p}\underline{e} \\ & \text{s.t. } p\bar{e} + (1-p)\underline{e} \geq E. \end{aligned}$$

The first order conditions are (where λ is the Lagrange multiplier):

$$\begin{aligned} \bar{e}: \quad & -2\,p\,\bar{e} - \frac{k}{a} + \lambda\,p = 0, \\ \underline{e}: \quad & -2(1-p)\,\underline{e} + k\frac{(1-p)}{(1-a\,p)} + \lambda\,(1-p) = 0. \end{aligned}$$

The second order conditions are satisfied: the constraint set defines a convex set and the objective function is strictly concave. From these two equations we obtain:

$$(\bar{e}-\underline{e}) = -\frac{k}{2}\left(\frac{1}{1-a\,p} + \frac{1}{a\,p}\right).$$

Define:

$$\mu \equiv -\frac{k}{2}\left(\frac{1}{1-a\,p} + \frac{1}{a\,p}\right).$$

From the restriction that the principal has to implement at least an expected effort of E: $p(\bar{e}-\underline{e}) + \underline{e} = E$. Hence, $\underline{e} = E - p\mu$ and $\bar{e} = E + (1-p)\mu$. Below in the Proof of Proposition 3.4.2 we consider the restrictions from the probability distribution. ■

Proof Proposition 3.4.2.

Plugging optimal efforts from Lemma 5 in Equation (5):

$$\begin{aligned}
\Delta &= p\,(1-p)\,(\bar{e}-\underline{e})^2 + k\left(\frac{1}{a}\bar{e} - \frac{1-p}{1-a\,p}\underline{e}\right) \\
&= p\,(1-p)\,\mu^2 + k\left(\frac{E+(1-p)\,\mu}{a} - \frac{(1-p)\,(E-p\,\mu)}{1-a\,p}\right) \\
&= p\,(1-p)\,\mu^2 + k\left(\frac{E\,(1-a)+(1-p)\,\mu}{a\,(1-a\,p)}\right) \\
&= k\,E\,\frac{(1-a)}{a\,(1-a\,p)} - \frac{k\,(1-p)}{2\,a\,p\,(1-a\,p)}\left(\frac{k}{a\,(1-a\,p)} - \frac{k}{2\,a\,(1-a\,p)}\right) \\
&= k\,E\,\frac{(1-a)}{a\,(1-a\,p)} - \frac{k^2\,(1-p)}{4\,a^2\,p\,(1-a\,p)^2} \\
&= \frac{k}{a\,(1-a\,p)}\left(E\,(1-a) - \frac{k\,(1-p)}{4\,a\,p\,(1-a\,p)}\right).
\end{aligned}$$

We define:

$$\Delta(a,p,k,E) = \frac{k}{a\,(1-a\,p)}\left(E\,(1-c) - \frac{k\,(1-p)}{4\,a\,p\,(1-a\,p)}\right). \tag{5}$$

A sufficient condition for $\Delta(a,p,k,E) > 0$ is $\frac{E\,k\,(1-a)}{a\,(1-a\,p)} > 0$. For our leading case $k < 0$ we hence need for $\Delta > 0$ that $E\,(a-1) > -\frac{k\,(1-p)}{4\,a\,p\,(1-a\,p)}$ holds.

Assume first that $\underline{e} > 0 \leftrightarrow E > -\frac{(1-p)\,k}{2\,a\,p\,(1-a\,p)}$. Hence, for $a \geq 3/2$, $\Delta > 0$ is automatically satisfied once we assume an interior equilibrium. We have to check that those values are actually compatible with a proper joint probability function (what we do here for the second set of parameters in Proposition 3.4.2). For this we first have to check whether none of the four joint probabilities is smaller than zero or larger than one (i.e. whether we have determined an interior equilibrium). It is easy to see that those eight conditions reduce to the three following.

(a) $a\,p\,\bar{e} + k \geq 0$.

(b) $p\,(1-a\,\bar{e}) - k \geq 0$.

(c) $1-p-\underline{e}\,(1-a\,p) + k \geq 0$.

The other five conditions are:

(d) $a\,p\,\bar{e} + k \leq 1$. Holds under $p\,(1-a\,\bar{e}) - k \geq 0$.

(e) $p\,(1-a\,\bar{e}) - k \leq 1$. Holds under $a\,\bar{e}\,p + k \geq 0$.

(f) $\underline{e}\,(1-a\,p) - k \geq 0$. Holds as $k \leq 0$ and $a \leq \frac{1}{p}$.

(g) $\underline{e}\,(1-a\,p) - k \leq 1$. Holds under $1-p-\underline{e}\,(1-a\,p) + k \geq 0$.

(h) $1 - p - \underline{e}\,(1 - a\,p) + k \geq 0$. Holds as $k \leq 0$ and $a \leq \frac{1}{p}$.

We choose $p = 1/2$. Hence, $a \in (1, 2)$. Then $\bar{e} = E - \frac{k}{a\,(2-a)}$ and $\underline{e} = E + \frac{k}{a\,(2-a)}$. Condition (a) implies that $\bar{e} > -\frac{2\,k}{a}$. Hence, if $a \geq 3/2$ this holds. Condition (b) requires that $\bar{e} \geq \frac{1}{a}(1 - 2\,k)$. Given that (a) is satisfied we have that if $E \leq \frac{1}{a}$, then (b) holds, too. As $a < 2$ this implies $E \leq \frac{1}{2}$. Finally, Condition (c) implies that $\underline{e} \leq \frac{1+2\,k}{2-a}$ (for this to be positive we need $-\frac{1}{2} \leq k$). Hence, we need $\frac{1+2\,k}{2-a} \geq E + \frac{k}{a\,(2-a)}$. This holds under Condition (b).

To summarize, we get for $p = 1/2$ the following conditions $a \in \left[\frac{3}{2}, 2\right)$, $-\frac{1}{2} \leq k$ and $E \leq \frac{1}{2}$ (note that these are sufficient conditions for the three conditions to hold). Following our above discussion we know that for $a \geq \frac{3}{2}$, $\Delta > 0$ is then automatically satisfied.

Assume next that $\underline{e} = 0$ and hence $\bar{e} = E/p$. Plugging in Equation (3.10) we obtain: $\Delta = \frac{1-p}{p}\,E^2 + \frac{k}{a\,p}E = E\left(\frac{1-p}{p}\,E + \frac{k}{a\,p}\right)$, which is larger than zero for $(1-p)E\,a > -k$. From the three conditions above it follows that $1 - p \geq -k$, $p - k \geq a\,E$ and $a\,E \geq -k$. Hence, one can find values for (p, a, E, k) such that $\Delta > 0$ and the three conditions hold.

Finally, we have to check, whether the participation constraint is indeed satisfied: compared to our assumptions note that for $\tilde{e}$, where $f(xz|\tilde{e}) = 0$ holds, we do not have $c(\tilde{e}) = 0$ or $c'(\tilde{e}) = 0$. After a signal realization $\underline{z}$ this is not a problem, as $\underline{e}(1 - a\,p) - k > 0$ for $\underline{e} = 0$, where hence $c(\underline{e} = 0) = 0$. So we have to check, whether $a\,p\,\bar{e} + k - \frac{(\bar{e})^2}{2} > 0$ at the optimal $\bar{e}$: $\bar{e}\left(a\,p - \frac{E+(1-p)\,\mu}{2}\right) + k > 0$. For $p = 1/2$ this is satisfied if and only if $\bar{e}\left(1/2\,a\, - 1/2\,E + \frac{k}{4\,a(1-1/2\,a)}\right) + k > 0$. Choose $a = 3/2$ and $E = 1/2$. Then this holds if $8/9\,k^2 + 2\,k + 1/4 > 0$. This is positive for values $-0.13 < k$. ■

Appendix to Chapter 4

Proof Lemma 6.

$$\begin{aligned}
\Pi_{1\theta}^{SB}(e_H,\theta) &= f_{1\theta}(e_H,\theta) - C_{1\theta}(\theta) \\
&= f_{1\theta}(e_H,\theta) - \frac{f_1(e_H,\theta)\, f_{1\theta}(e_L,\theta) - f_1(e_L,\theta)\, f_{1\theta}(e_H,\theta)}{[\Delta f_1(\theta)]^2}\, c \\
&> f_{1\theta}(e_H,\theta) - \frac{f_1(e_H,\theta)\, f_{1\theta}(e_L,\theta)}{[\Delta f_1(\theta)]^2}\, c > 0,
\end{aligned}$$

as $f_{1\theta}(e_H,\theta) \geq f_{1\theta}(e_L,\theta)$ (Assumption 5 SA) and $[\Delta f_1(\theta)]^2 > f_1(e_H,\theta)\, c$ (Condition 1). This shows Part (a). Part (b) follows immediately. ■

Proof Proposition 13.

$$\begin{aligned}
\Pi_1^{SB}(e_H,\theta^{FB}) - \Pi_2^{SB}(e_H) &> 0 \\
\Leftrightarrow\quad f_1(e_H,\theta^{FB}) - C_1(e_H,\theta^{FB}) &> f_2(e_H) - C_2(e_H) \\
\Leftrightarrow\quad C_1(e_H,\theta^{FB}) &< C_2(e_H) \\
\Leftrightarrow\quad f_1(e_H,\theta^{FB}) - f_1(e_L,\theta^{FB}) &> f_2(e_H) - f_2(e_L) \\
\Leftrightarrow\quad f_1(e_L,\theta^{FB}) &< f_2(e_L),
\end{aligned}$$

using repeatedly $f_1(e_H,\theta^{FB}) = f_2(e_H)$. The resulting inequality $f_2(e_L) > f_1(e_L,\theta^{FB})$ implies that $\hat{\theta}_H < \hat{\theta}_L$. The rest of Part (a) follows because the profit difference is strictly increasing in θ (Lemma 6). Part (b) follows directly from $\theta^{SB} < \theta^{FB}$ and because high effort is implemented in both jobs. ■

Proof Proposition 14.

For Part (a) note that the difference between the expected wage payment in job 1 and job 2 for an agent with type θ is:

$$\frac{f_1(e_H,\theta)}{f_1(e_H,\theta) - f_1(e_L,\theta)}\, c - \frac{f_2(e_H)}{f_2(e_H) - f_2(e_L)}\, c = \frac{f_1(e_L,\theta)\, f_2(e_H) - f_1(e_H,\theta)\, f_2(e_L)}{[f_1(e_H,\theta) - f_1(e_L,\theta)]\,[f_2(e_H) - f_2(e_L)]}\, c.$$

This is negative at θ^{SB} because $\theta^{SB} < \hat{\theta}_L$:

$$f_1\left(e_L,\theta^{SB}\right)\, f_2(e_H) - f_1\left(e_H,\theta^{SB}\right)\, f_2(e_L) = f_2(e_H)\, \left[f_1\left(e_L,\theta^{SB}\right) - f_2(e_L)\right] < 0,$$

and positive at θ^{FB} if and only if:

$$f_1(e_L,\theta_H)\, f_2(e_H) - f_1(e_H,\theta_H)\, f_2(e_L) > 0 \Leftrightarrow \frac{f_2(e_H)}{f_2(e_L)} > \frac{f_1(e_H,\theta_H)}{f_1(e_L,\theta_H)},$$

which gives us – using the intermediate value theorem – the condition in the text.

For Part (b) note that $C_{1\theta}(\theta) = \frac{f_1(e_H,\theta)\,f_{1\theta}(e_L,\theta) - f_1(e_L,\theta)\,f_{1\theta}(e_H,\theta)}{[\Delta f_1(\theta)]^2} c$ is positive if and only if the numerator is positive which gives Condition 4 when rearranged.

Part (c) follows directly because the expected utility is the expected wage minus cost of effort c. ■

Proof Lemma 7 and Proposition 15.

Consider the second best profits in jobs 1 and 2. Suppose that *high* effort is implemented in both. From the Proof of Proposition 13 we know that the threshold where both these profits (which need not be the optimal second best levels - low effort implementation can be optimal) are equal is less than $\theta^{FB} = \hat{\theta}_H < \hat{\theta}_L$. Denote the threshold that makes them equal by $\tilde{\theta}$. Thus, $f_1\left(e_L, \tilde{\theta}\right) < f_2(e_L)$ so:

$$\begin{aligned} &\Pi_1^{SB}\left(e_H, \tilde{\theta}\right) - \Pi_1^{SB}\left(e_L, \tilde{\theta}\right) = \Pi_2^{SB}(e_H) - f_1\left(e_L, \tilde{\theta}\right) \\ > \ &\Pi_2^{SB}(e_H) - f_2(e_L) = \Pi_2^{SB}(e_H) - \Pi_2^{SB}(e_L). \end{aligned}$$

The second best promotion threshold is strictly higher than $\tilde{\theta}$. Suppose first that Condition 1 holds for job 1. Then $\Pi_2^{SB}(e_L) = f_2(e_L) > f_1\left(e_L, \tilde{\theta}\right) - C_1\left(e_L, \tilde{\theta}\right) = \Pi_1^{SB}\left(e_H, \tilde{\theta}\right) = \Pi_2^{SB}(e_H)$. As $\Pi_1^{SB}(e_H, \theta)$ is increasing in θ, we have that $\theta^{SB} > \tilde{\theta}$. Suppose now that Condition 1 does not hold for job 1. Then it follows immediately that $\theta^{SB} = \hat{\theta}_L > \tilde{\theta}$. ■

Appendix to Chapter 4 - Extensions

The Wage Scheme

The wage scheme can condition on the following variables: the job (which is observable and verifiable), the agent's output (which is observable and verifiable), and the type the principal announces to the agent. In the text we assumed first that the wage scheme is a bonus payment (i.e. conditions only on the output realization $\bar{x}$) and second does not condition on the type the principal announces to the agent. While the former is an assumption that is usually made in the literature (see Bénabou and Tirole (2003)), we can derive the latter endogenously:

Lemma 8 *Suppose the wage scheme is of the form of a bonus payment. Then the wage does not condition on type the principal announces to the agent.*

Proof Lemma 8.

We have to show that wages are attached to jobs and output and not to announced types. Suppose instead that the principal offers a wage scheme of the form - "if I learn your type is $\tilde{\theta}$, I will pay you $w(\tilde{\theta})$", where $\tilde{\theta}$ is the type the principal announces to an agent.

Suppose to the contrary of the claim in Lemma 8 that the wage scheme is such that for some $\tilde{\theta}_1$ and $\tilde{\theta}_2$ assigned to the same job we have $w(\tilde{\theta}_1) \neq w(\tilde{\theta}_2)$. Suppose further that $w(\tilde{\theta}_i)$ is such that both types provide effort. But then the principal has an incentive to announce to the type that receives the higher wage, say $\tilde{\theta}_1$, to be of type $\tilde{\theta}_2$. If one type does not work under the specified wage and the other does, the principal again has an incentive to deviate and give the type that does not work the wage of the type that works. Also she would like to offer a larger wage if none of the individuals worked, also if they held the worst beliefs, since by assumption it is always profitable to induce all types to work. ■

Proofs

Proof (Existence of Equilibrium in Subgame following Date 2).

To see that an equilibrium exists note that given our continuity and monotonicity assumptions we have that for any (w_j, w_i) either $f_j(\theta)(\bar{x} - w_j) > f_i(\theta)(\bar{x} - w_i)$ $\forall\theta$, $f_j(\theta)(\bar{x} - w_j) < f_i(\theta)(\bar{x} - w_i)$ $\forall\theta$, or there exists a unique $\tilde{\theta}$, such that

$f_j(\theta)(\bar{x} - w_j) = f_i(\theta)(\bar{x} - w_i)$. In the first case a pooling equilibrium on job j can be supported ($\theta_S = \theta_H$), in the second pooling on job i ($\theta_S = \theta_L$) and in the third an interior separating equilibrium exists.

Multiple equilibria can arise in the subgame following date 2. The principal, however, announces at date 1 the job assignment rule. She has no incentive to deviate from this announcement at date 3 (as the agent can observe the assignment). She announces at date 1 the rule that maximizes his profits. Thus, this announcement helps to coordinate on one equilibrium in the subgame at date 3. ■

Proof Proposition 16.

In the following we consider only the case $\frac{1}{1-\Phi(\hat{\theta})}\int_{\hat{\theta}}^{\theta_H} f_1(\theta)\phi(\theta)d\theta < \frac{1}{\Phi(\hat{\theta})}\int_{\theta_L}^{\hat{\theta}} f_2(\theta)\phi(\theta)d\theta$. The other case follows analogue. The principal's maximization problem is as stated in the text. We structure the proof in several steps to determine the equilibrium wage and cutoff. Like this we do not have to make further assumptions about the concavity of the problem.

Some preliminary facts. Define $\underline{w}_i(\theta_S) = \frac{c}{E[f_i(\theta)|\theta_S,i]} - v$. Note that $\frac{\partial \underline{w}_1(\theta_S)}{\partial \theta_S} < 0$ and $\frac{\partial \underline{w}_2(\theta_S)}{\partial \theta_S} > 0$. To see this note that $\frac{1}{\Phi(\theta)}\int_{\theta_L}^{\theta} f_2(\theta')\phi(\theta')d\theta' = \frac{1}{\Phi(\theta)} f_2(\tilde{\theta})\int_{\theta_L}^{\theta}\phi(\theta')d\theta' = \frac{\Phi(\theta)}{\Phi(\theta)} f_2(\tilde{\theta})$, with $\tilde{\theta} \in (\theta_L, \theta)$ and thus $f_2(\hat{\theta}) > f_2(\theta)$. Define the function $\underline{w}(\theta_S) = \underline{w}_1(\theta_S) - \underline{w}_2(\theta_S)$. This is a decreasing and continuous function in θ_S. Furthermore, $\underline{w}(\hat{\theta}) > 0$ and $\underline{w}(\theta_L) < 0$.[9] Hence, by the Intermediate Value Theorem, there exists a unique $\theta \in (\tilde{\theta}, \theta_H)$, call it θ_W such that $\underline{w}(\theta_W) = 0$.

Part 1: $w_i > \underline{w}_i(\theta_M)\ \forall i$ can never be optimal

Suppose first that $w_1 > w_2$. For $f_1(\theta_S)(W - w_1) - f_2(\theta_S)(W - w_2) = 0$ (which we call in the following the indifference condition) to hold we must have $\theta_S > \hat{\theta}$. Reduce instead w_1 to $w_1 = w_2$. Note that then $w_1 = w_2 > \underline{w}_2(\hat{\theta}) > \underline{w}_2(\theta_S)$ and $\underline{w}_2(\hat{\theta}) > \underline{w}_1(\hat{\theta})$. Hence, this induces higher profits, since the incentive constraint is still satisfied, the cutoff is the efficient one and w_1 is smaller. Hence $w_i > \underline{w}_i(\theta_M)$ and $w_1 > w_2$ cannot be optimal.

Suppose now that $w_2 > w_1$. For the indifference condition to hold we must have $\theta_S < \hat{\theta}$. Reduce instead w_2 marginally, leaving w_1 unchanged. This increases θ_S and hence $\underline{w}_2(\theta_S)$, but still $w_2 > \underline{w}_2(\theta_S)$. Hence, profits increase.

So suppose $w_1 = w_2$. This implies $\hat{\theta} = \theta_S$. Then $\underline{w}_1(\hat{\theta}) < \underline{w}_2(\hat{\theta}) < w_1 = w_2$. But setting

[9] Since $f_1(\theta_L) = f_2(\theta_H) > \int_{\theta_L}^{\theta_H} f_2(\theta)\phi(\theta)d\theta$.

$w_1 = w_2 = \underline{w}_2(\hat{\theta})$ leads to higher profits.

Part 2: iff $w_1 = \underline{w}_1$ and $w_2 \geq \underline{w}_2$, then $\theta_S = \theta_M$ is profit maximizing.

From the indifference condition we know that $w_2(\theta_S) = \frac{f_2(\theta_S)-f_1(\theta_S)}{f_2(\theta_S)}\bar{x} + \frac{f_1(\theta_S)}{f_2(\theta_S)}\underline{w}_1(\theta_S)$. We have to check in which range $w_2(\theta_S) \geq \underline{w}_2(\theta_S)$ can hold. Define:

$$g(\theta_S) = \frac{f_2(\theta_S) - f_1(\theta_S)}{f_2(\theta_S)}\bar{x} + \frac{f_1(\theta_S)}{f_2(\theta_S)}\underline{w}_1(\theta_S) - \underline{w}_2(\theta_S)$$

Note that $g(\theta_W) < 0$, since $f_2(\theta_W) < f_1(\theta_W)$ and $\underline{w}_1(\theta_W) = \underline{w}_2(\theta_W)$ and $g(\hat{\theta}) > 0$, since $\underline{w}_1(\hat{\theta}) > \underline{w}_2(\hat{\theta})$ and $f_2(\hat{\theta}) = f_1(\hat{\theta})$. Furthermore, $\frac{\partial g(\theta_S)}{\partial \theta_S} < 0$ as $\underline{w}_1(\theta_S)$ is decreasing in θ_S, $\underline{w}_2(\theta_S)$ is increasing and:

$$\frac{f_2'(\theta_S)f_1(\theta_S) - f_1'(\theta_S)f_2(\theta_S)}{(f_2(\theta_S))^2}\left(\bar{x} + v - \frac{c}{\frac{1}{\Phi(\theta_S)}\int_{\theta_L}^{\theta_S} f_2(\theta)\phi(\theta)d\theta}\right) < 0.$$

Hence, there exists a unique θ_S (θ_M) such that $g(\theta_M) = 0$. Thus, we can conclude that whenever the incentive constraint in job 1 is binding and in job 2 (weakly) not that $\theta_S \in [\theta_L, \theta_M]$.

Consider now the profit function with these wages plugged in:

$$\begin{aligned}\pi(\theta_S) &= \int_{\theta_S}^{\theta_H} f_1(\theta)\phi(\theta)d\theta(\bar{x} + v - \underline{w}_1(\theta_S)) \\ &+ \int_{\theta_L}^{\theta_S} f_2(\theta)\phi(\theta)d\theta\left(\bar{x} - \frac{f_2(\theta_S) - f_1(\theta_S)}{f_2(\theta_S)}\bar{x} - \frac{f_1(\theta_S)}{f_2(\theta_S)}\underline{w}_1(\theta_S)\right).\end{aligned}$$

This is an increasing function in θ_S:

$$\begin{aligned}\frac{\partial\pi(\theta_S)}{\partial\theta_S} &= \left[(\bar{x} - \underline{w}_1(\theta_S))\frac{f_1'(\theta_S)f_2(\theta_S) - f_2'(\theta_S)f_1(\theta_S)}{(f_2(\theta_S))^2} - \underline{w}_1'(\theta_S)\frac{f_1(\theta_S)}{f_2(\theta_S)}\right]\int_{\theta_L}^{\theta_S} f_2(\theta)\phi(\theta)d\theta \\ &- \underline{w}_1'(\theta_S)\int_{\theta_S}^{\theta_H} f_1(\theta)\phi(\theta)d\theta \\ &+ \underbrace{(\bar{x} - \underline{w}_1(\theta_S))f_1(\theta_S)\phi(\theta_S) - (\bar{x} - w_2(\theta_S))f_2(\theta_S)\phi(\theta_S)}_{0} > 0.\end{aligned}$$

Hence, the principal sets $\theta_S = \theta_M$, which is highest possible cutoff consistent with these wages. Thus, $w_2 = w_2(\theta_M)$.

Part 3: iff $w_1 \geq \underline{w}_1$ and $w_2 = \underline{w}_2$, then $\theta_S = \theta_M$ is profit maximizing.

Analogue to Part 2.

Part 4: iff $w_i = \underline{w}_i$, then $\theta_S = \theta_M < \hat{\theta}$

See Part 2 and 3. That equilibrium wages satisfy $w_1 > w_2$ follows then from the indifference condition and from $\theta_M < \hat{\theta}$. ■

Proof Proposition 17.

Assume $\theta_M > \hat{\theta}$ (the other case is analogue). The principal prefers the separating job assignment rule to one that pools the agents iff:

$$(\bar{x}+v)\left(\int_{\theta_S}^{\theta_H} f_1(\theta)\phi(\theta)d\theta + \int_{\theta_L}^{\theta_S} f_2(\theta)\phi(\theta)d\theta\right) - c \geq (\bar{x}+v)\int_{\theta_L}^{\theta_H} f_i(\theta)\phi(\theta)d\theta - c,$$

or iff:

$$\int_{\theta_S}^{\theta_H} f_1(\theta)\phi(\theta)d\theta + \int_{\theta_L}^{\theta_S} f_2(\theta)\phi(\theta)d\theta \geq \int_{\theta_L}^{\theta_H} f_i(\theta)\phi(\theta)d\theta. \tag{PS}$$

Suppose first that $\int_{\theta_L}^{\theta_H} f_1(\theta)\phi(\theta)d\theta > \int_{\theta_L}^{\theta_H} f_2(\theta)\phi(\theta)d\theta$. Then Condition (PS) holds iff: $\int_{\theta_L}^{\theta_S}(f_2(\theta)-f_1(\theta))\phi(\theta)d\theta \geq 0$. Note that as $\theta_M > \hat{\theta}$ we have $E[f_2(\theta)|\theta \leq \theta_S] > E[f_1(\theta)|\theta \geq \theta_S]$. Furthermore, $E[f_1(\theta)|\theta \geq \theta_S] \geq E[f_1(\theta)|\theta \leq \theta_S]$ and hence $E[f_2(\theta)|\theta \leq \theta_S] > E[f_1(\theta)|\theta \leq \theta_S]$ and the condition is satisfied for $\Phi(\hat{\theta}) = 1/2$ (hence $\Phi(\theta_S) > 1/2$).

Suppose now that $\int_{\theta_L}^{\theta_H} f_1(\theta)\phi(\theta)d\theta < \int_{\theta_L}^{\theta_H} f_2(\theta)\phi(\theta)d\theta$, then Condition (PS) can hold only iff: $\int_{\theta_S}^{\theta_H}(f_1(\theta)-f_2(\theta))\phi(\theta)d\theta \geq 0$. Note that $\theta_S > \hat{\theta}$. Hence, we have $f_2(\theta) < f_1(\theta)$ $\forall \theta \in [\theta_S, \theta_H]$ and this is always satisfied. Hence, the principal always would like to separate the agents. ■

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