Essays on Auctions, Tournaments, and Imperfect Competition

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1. Introduction

This thesis is a collection of essays in applied microeconomic theory that deal with issues of market design, incomplete information, and imperfect competition.

It is composed of three parts:

- *Part I:* Procurement of innovation: tournaments vs auctions
- *Part II:* Strategic information transmission in sequential auctions
- *Part III:* Signaling in market games with downstream interaction.

Part I deals with the procurement of innovations and compares the performance of two prominent procurement mechanisms: scoring auctions and fixed-prize tournaments, two widely used and well documented methods to procure innovations.

In the past, when intellectual property rights were not well established and royalty licensing was not feasible, fixed-prize tournaments were employed to procure major innovations. In 1795, Napoléon Bonaparte offered a prize of FF12.000 for a method of food preservation that was in high need to serve his military excursions across Europe. The winner of that tournament was Nicolas Appert, who invented the method of food canning, which is still widely used today. And in 1714 the British Parliament offered a prize of £20.000 for a method to determine longitude at sea, following a series of maritime disasters. That prize was won by John Harrison who invented the first mechanical chronometer.[1]

More recent examples range from the procurement of weapon systems, energy efficient refrigerators, and pharmaceutical innovations, to the awarding of academic grants and fellowships, to name just a few.

Similarly, scoring auctions are widely used in the procurement of

[1] However, the prize committee was dominated by astronomers who pursued their own agenda, sabotaged the work of the clockmaker, and tried to withhold the prize from him (see Sobel, 1996).

goods and services that involve innovative activity. For example, when the clients of the World Bank procure the design or construction of a power plant or of a national health care system, contractors compete not only with price, but also with technical proposals that lay out innovative solutions to problems of technical or institution design. Price and quality are then mapped into a unique score, based on a fixed scoring rule, and the contract is awarded to whoever reaches the highest score.

Inspired by these and other examples, R&D contests were analyzed extensively in the theoretical literature, in particular by Taylor (1995), who introduced the base model employed by the subsequent literature, and Fullerton and McAfee (1999), Fullerton, Linster, McKee, and Slate (2002), Che and Gale (2003), and Schöttner (2008) who compared the relative performance of auctions and fixed-prize tournaments under different assumptions concerning the innovation technology and feasibility of entry fees.

A key (implicit) assumption of the literature on R&D contests, with which we take issue here, is that contestants submit their best innovation regardless of its value. This ignores that innovators may withhold innovations that are worth considerably more than the prize, so that only the "lemons", i.e., the inferior innovations are submitted.

If there is only one potential user of the innovation, i.e., if the procurer is a monopsonist, the procurer can in principle prevent this adverse selection problem by committing himself to never bargain with innovators who bypass the contest. However, such a commitment is difficult to achieve.

There are many cases where innovations were inspired by a contest, but innovators ultimately decided to bypass the contest when they felt that their innovation had a substantially higher commercial value than the prize offered by the contest, and then successfully negotiated more profitable license agreements after bypassing the contest.

A case in point is the invention of celluloid. Its inventor, John Wesley Hyatt, was encouraged to develop a new substance after he saw an advertisement by Phelan & Collander, offering $10,000 to the person who invented a usable substitute for ivory in billiard balls. He eventually succeeded by inventing celluloid, which seemed to be a perfect substitute for ivory in billiard balls, but finally decided to patent his innovation instead of submitting it to the tournament and collecting the prize. This allowed him to license his innovation not only for use in billiard balls, but also in a variety of other products, ranging from film and ping-pong balls to dental plates.

Motivated by this and other examples, our essay analyzes the procurement of innovations when the procurer is unable to commit himself to never bargain with innovators and innovators consider to bypass a contest and engage in bargaining after the contest game has been played.

As another important departure from the bulk of the literature, we allow the procurer to collect entry fees from those who register for the contest. Entry fees are important because without them contests lack an important tool for surplus extraction. This is particularly important in auctions which are seriously handicapped relative to fixed-prize contests if no entry fees are used.[2]

Altogether, we show that if bypass is possible and entry fees may be collected, the optimal fixed-prize tournament outperforms the optimal auction. Essentially, this result is due to the distinct role that entry fees play in the two mechanisms: In the optimal auction, entry fees are necessary, even if bypass is not an issue. If bypass is possible,

[2] As we show in our base model on pages 37-40, in the absence of a bypass problem, if one does not use entry fees the auction is generically inferior to the fixed-prize contest, because the auction can neither assure participation nor full surplus extraction without using entry fees, while the fixed prize contest can assure both without entry fees.

high entry fees induce bypass, and bypass can only be prevented by lowering the entry fee, which cuts into the procurer's expected profit. Whereas in fixed-prize tournaments, entry fees are less important. There, the procurer can easily deter bypass by setting a sufficiently high prize and then (partially) offset the resulting reduction in his expected profit by raising the entry fee.

Part II deals with issues of strategic information transmission that occur if bidders participate in a sequence of auctions and bidders' valuations are correlated across auctions. In such a framework, the outcome of the early auction may reveal information concerning bidders' valuations which may adversely affect their expected payoffs in later auctions.

Many market transactions have the structure of an auction, and many such auctions are recurring events. For example, price competition between retailers is essentially a (reverse) auction. And this auction is typically a recurring event in which the relevant valuations (unit costs) are stable, at least for some time. Assuming stable or perfectly correlated valuations is appropriate for analyzing wine, stamp, and real estate auctions where different lots of (almost) identical goods are usually auctioned within minutes to the same group of potential buyers.

Evidently, in these cases bidders must pay attention to the information they reveal about their valuations through their bids. This gives rise to a problem of strategic information transmission.

In a first-price auction with publicly observed bids (which is analyzed in Chapter 4) bidders can infer the underlying valuations from observed bids, if equilibrium strategies are monotone. Of course, bidders take into account that their bids affects others' beliefs, and adjust their bidding behavior in such a way that the inference from observed bids to underlying valuations is somewhat blurred, to which we refer as a case of "signal jamming".

As a result, partial pooling occurs in the sense that bidders with a high valuation imitate the bidders with a low valuation, with positive probability, in order to keep the rivals in the dark about their true type, until the last auction is played.

Part III focuses on the interaction between strategic behavior and subsequent downstream interaction in an oligopoly aftermarket, using the examples of wage bargaining and takeover bidding. The common feature in both applications is that the negotiated wages resp. the observed winning bid in a takeover contest serve as a signal of the respective players' type, which affects the interaction in the downstream oligopoly market. Players take this signaling aspect into account, which may give rise to inflated equilibrium wages respectively takeover bids.

Decentralized Union-Oligopoly Wage Bargaining The wage bargaining application is covered in Chapter 5 (based on Ding (2010)). It is motivated by the classical labor literature which claimed that decentralized wage bargaining leads to significantly lower wages than industry wide bargaining.

In the literature on collective wage bargaining one finds two fundamentally distinct approaches: the so-called "right-to-manage" and the "complete labor contracts" model.[3] Whereas the complete contract model assumes that unions and employers negotiate complete contracts that stipulate wages *and* employment in each firm, the right-to-manage model assumes that unions and employers negotiate

[3]The model of complete labor contracts was introduced by Leontief (1946) and later extended McDonald and Solow (1981); the right-to-manage model was introduced by Oswald (1982) and subsequently used by Dowrick (1989), by Dixon (1988) and others.

wages but leave the choice of employment to the discretion of each firm.

The protagonists of the complete contract model emphasize that complete contracts assure efficiency, while incomplete contracts give rise to inefficient combinations of wages and employment, because the wage-employment combinations on the labor demand function are off the contract curve. Essentially, this is due to the fact that if wages exceed the competitive wage, firms reduce employment below the efficient level. This distortion of efficiency, which resembles the well-known welfare loss of monopoly, can be remedied by writing complete labor contracts that control wages and employment, just like a perfectly price discriminating monopoly that controls both quantity *and* price can extract the full surplus.

The protagonists of the right-to-manage model of wage bargaining emphasize that complete contracts are impractical already because the event space may be too large and, more fundamentally, because firms are subject to unpredictable events which cannot be handled by contingent contracts that can only condition on predictable events. Moreover, contingent contracts may not be feasible due to lack of verifiability. As Aghion and Holden (2011, p.190) put it in their recent survey of the incomplete contracts approach:[4]

> *"Perhaps the central issue is that economic actors ... cannot anticipate all possible contingencies. It might well be that certain states of nature or actions cannot be verified by third parties after they arise, ..., and thus cannot be written into an enforceable contract. When contracts are incomplete, ..., any contract negotiated in advance must leave some discretion ... [to] the "owner" of the firm ... "*

Apart from these theoretical arguments, the matter must ultimately be settled by looking at the facts, which were studied and reviewed in Hall and Lilien (1979) and McCurdy and Pencavel (1986). They found that the typical labor contract is incomplete and typically prescribes wages (and sometimes labor time) but allows firms to choose the number of employees. This suggests that the right-to-manage model is the more appropriate approach to study wage bargaining.

Assuming the right-to-manage model, a classical theme of the labor

[4] However, Maskin and Tirole (1999) showed that, by using an intelligent revelation mechanism, one can make observable information verifiable to a third party. This suggests that lack of verifiability may not preclude state contingent contracts, contrary to what is commonly presumed.

literature is the comparison of centralized and decentralized wage bargaining in oligopolistic industries. Centralized bargaining is predominant in countries like Germany, where industry wide unions bargain with centralized associations of employers, whereas decentralized bargaining is predominant in countries like the U.S., where employees of a firm typically chose a particular union that represents them in collective bargaining with their employer, if they choose collective bargaining at all.

The main result of that literature is that centralized collective bargaining in an oligopoly gives rise to higher wages, because if unions negotiate with employers at the industry rather than the firm level, the negotiations capture the benefits of implicit collusion in the product market. Essentially, by raising wages not only for the own firm but also for the rival firm, the wage increase is less costly to employers, because it also increases the rival's cost, which partially compensates the increased cost in the form of a reduced output of rival firms.

In our essay on wage bargaining we reconsider the analysis of decentralized wage bargaining and modify the usual analysis in several directions. First of all, we assume that the union does not engage in an ultimatum game and sets a take-it-or-leave it wage (as in the most of the literature that considers a "monopoly union"), but instead assume that union and employer engage in cooperative bargaining, maximizing the Nash product of their gains from trade. Secondly, we introduce incomplete information by assuming that firm-union coalitions have private information concerning their productivity, i.e., they know their own productivity but not that of other firm-union coalitions. Third, we assume that firms observe each others' wage settlements before they choose employment and play the downstream oligopoly game.

The presence of private information concerning productivity and the observability of wage settlements gives rise to a signalling issue. When firm-union coalitions negotiate a wage per worker, they must take into account how their own wage settlement alters the beliefs of rival firms concerning each other's cost. In particular, a higher wage settlement may signal a higher productivity, which confers a strategic advantage in the subsequent downstream oligopoly game. Of course, in equilibrium, no misleading signalling occurs, which suggests that the potential for signalling gives rise to a pointwise higher equilibrium wage schedule. Equilibrium wages are thus inflated to such an extent that exaggerated signalling is deterred because it becomes too costly.

This result is not only interesting in itself, it has also implications for the comparison between centralized and decentralized wage bargaining. Of course, we do not negate that centralized wage bargaining contributes to wage inflations, because it captures the benefits of implicit collusion extracted from consumers. However, we show that this effect is at least partially compensated by the wage inflation induced in decentralized bargaining by the firms' attempts to signal their strength which is geared to gain a strategic advantage in the subsequent oligopoly game.

Mergers and takeover bidding The takeover application is covered in Chapter 6 (based on Ding, Fan, and Wolfstetter (2010)). It is motivated by the analysis of mergers and the merger paradox in an oligopoly framework, when mergers are subject to synergies in the form of cost reductions. There, we analyze takeover bidding between oligopoly firms that have private information concerning the synergy effect due to merging their firm with a takeover target, employing a somewhat unusual but highly profitable auction rule.

The classical merger paradox has been introduced by Salant, Switzer, and Reynolds (1983) who observed that, in a simple Cournot oligopoly, mergers are not profitable to the firms that merge unless almost all firms (the rule of thumb is "at least 80% of all firms") merge. Of course, large mergers are typically not approved by antitrust authorities who, as a rule, prohibit mergers that achieve a position of market dominance. This finding is somewhat paradoxical because one might argue that the merged firm can always maintain the output strategy that its members played prior to the merger. Therefore, one may think that a merger should never lower the profits of those who merge.

However, this reasoning is flawed, essentially because it ignores that the merged firm is just one player of the game it plays with those who are not part of the merger. In the absence of synergies, the firms that are not part of the merger are happy to learn about the merger, and respond to the thus reduced competition by raising their outputs. Similarly, the merged firm will also reduce its output below the level of the pre-merger aggregate output of its members in order to take advantage of reduced competition. As a result, equilibrium profits of all firms, merged and not merged alike, increase, but the increased profit of the merger does not compensate the loss of profits of those firms that have vanished due to the merger.[5]

[5] Note that a merger would always be profitable if it could be kept secret. However, mergers are always publicly observed, because they have to be approved by antitrust authorities who in turn must publish their decisions.

In the subsequent literature it has been observed that the picture does change if a merger entails synergies. Such synergies may take the form of cost reductions that may be realized by retaining the most efficient departments of the merged firms and closing the inefficient ones. Synergies may also take the form of streamlining product lines or of taking advantage of complementarities between products in multi-product firms.

Cost reducing synergies were introduced into the analysis of mergers in oligopoly by Farrell and Shapiro (1990), and product differentiation and Bertrand competition were introduced by Deneckere and Davidson (1985). Altogether, these contributions showed that even small mergers may be profitable.

In our essay on mergers we also allow for synergies but introduce private information, assuming each firm knows which level of cost reduction it may realize by merging with a given takeover target, but does not know the cost reductions that may be realized by other mergers with the takeover target.

Another ingredient of our analysis is that mergers take place through takeover bidding. The presence of private information makes auctions appealing in mergers and acquisitions. And indeed, auctions are widely used in practice (see the empirical study by Boone and Mulherin, 2007). Under the predominant corporate law in the U.S. (which is the *Delaware* corporate law), auctions are even compelling, because under that law,

> *" ... once a takeover offer has been made, the board of directors is actually obliged to act like an auctioneer, and get the best price for the stockholder of the company, which is one of the reasons why a takeover offer must remain open for at least 20 business days" (Cramton, 1998).*

The fact that bidders are competitors in a downstream oligopoly implies that the takeover auction is a somewhat unusual auction game in which bidding is subject to externalities. In particular, since non-merged firms benefit from a merger if synergies are low, bidders are subject to a positive externality with positive probability. Whereas if synergies are sufficiently high, bidders are subject to a negative externality.

Another particular feature of takeovers is the fact that ownership stakes in the merged firm make post-merger profits verifiable to all co-owners. Therefore, it becomes feasible to make the price to be paid by the winner of the takeover contest conditional on the post-merger profitability. This is achieved by adopting a *share auction* in lieu of a standard *cash auction*. In such a share auction the winner of the auction awards the original owners of the takeover target an ownership stake in the newly formed merged firm. This ownership stake entitles the original owners to earn a share of the profit of the merged firm.

As we show, share auctions are more profitable than cash auctions. The bidder who offers the highest share offers the highest overall payment, and that overall payment is pointwise higher than the bid in a corresponding cash auction. Altogether, this surprising result can be viewed as an implication of the “linkage principle”. According to that principle, linking the price to a variable that is correlated with bidders’ private information lowers bidders’ information rent (Milgrom, 1987).

Share auctions are not only interesting in theory, they are also widely used in real world takeover contests. A prominent recent example is the takeover of *GE Insurance Solutions* (a major reinsurer) by *Swiss Re*, which made *Swiss Re* the world’s largest player in the oligopolistic reinsurance market. Several bidders participated in that takeover contest, including the famous investor *Warren Buffett*.

Interestingly, the winning bid offered *GE* a significant ownership stake which made *GE* a major shareholder of *Swiss Re* (see Boyle, 2005).

Our essay combines these unique features of takeover auctions: the presence of significant externalities due to the downstream interaction among bidders, the possible use of share auctions in lieu of standard cash auctions, and the potential to signal strength through bids.

Our main results are that the bidding games have a separating equilibrium even though firms may be subject to a positive externality and that profit-share auctions are more profitable than standard cash auctions, regardless of whether firms observe the merged firm's synergy parameter or only an imperfect signal of it.

Part I.

Procurement of Innovation: Tournaments vs Auctions

2. Prizes and Lemons: Procurement of Innovation under Imperfect Commitment[1]

2.1. Introduction

Contests are a widely used and well documented method to procure innovations. In the past, fixed-prize tournaments were employed to procure major bottleneck innovations. For example, in 1795 Napoleon Bonaparte offered a prize of FF12.000 for a method of food preservation that was in high need to serve his military excursions across Europe. The winner of that tournament was Nicolas Appert, who invented the method of food canning, which is still widely used

[1] This chapter is based on Ding and Wolfstetter (2011a).

today.[2] In 1714 the British Parliament offered a prize of £20.000 for a method of determining longitude at sea, following a series of maritime disasters. That tournament was won by John Harrison who invented the first mechanical chronometer to provide reliable time measurement service at sea.[3] More recent examples range from the procurement of weapon systems, energy efficient refrigerators,[4] and pharmaceutical innovations[5] to the awarding of academic grants and fellowships, to name just a few.

Contests in the form of a scoring auction are also widely used in the procurement of goods and services that involve innovative activity. For example, when the World Bank procures the design or

[2] In-container sterilization is known as "appertisation" in francophone regions, in memory of Nicolas Appert.

[3] However, the prize committee was dominated by astronomers who pursued their own agenda, sabotaged the work of the clockmaker, and tried to withhold the prize from him. For a vivid account of these incidents see Sobel (1996).

[4] In 1991 a $10 million prize was sponsored. Whirlpool won the tournament but never collected the prize because it failed to sell the 250.000 units required within the first five years after the tournament (see Langreth, 1994).

[5] Recently, the U.S. Congress discussed setting up a "Medical Innovation Prize Fund" with an annual budget of $8 billion (see Stiglitz, 2006) and the NSF set up a huge Experimental Innovation Inducement Prize Program (see National Research Council, 1982).

construction of a power plant or a national health care system, potential contractors compete not only with price, but also with technical proposals that lay out innovative solutions to problems of technical or institution design (see The World Bank, 2004a,b).

Elements of a research contest are also present in architecture competitions where designers and contractors are typically asked to present pilot proposals that are rewarded with fixed cash prizes and that play a crucial role in the final selection.

Inspired by these and other examples, R&D contests were analyzed extensively in the recent theoretical literature. In his seminal paper Taylor (1995) introduced a model of innovation activity that has been widely used and adapted in the subsequent literature. There, innovations are measured by their value added (the increment in wealth that their application would induce), and innovation activities are viewed as costly draws from a given *i.i.d.* probability distribution of innovations, similar to the independent private-values model in auction theory. The intensity of innovation activity is described by an optimal stop-rule that prescribes continued draws until either a threshold value of the innovation is reached or the deadline of submission has been reached.

Fullerton and McAfee (1999) simplify this analysis by letting innovators chose a fixed number of draws in lieu of the optimal stop rule, extend the analysis by introducing asymmetric innovators whose cost of innovation differ, and propose that the procurer should induce the best selection of contestants through auctioning the right to participate in the contest to a fixed number of innovators.

Fullerton, Linster, McKee, and Slate (2002) and Che and Gale (2003) compare the profitability of procuring innovations by fixed-prize tournaments and auctions, and show that auctions are more profitable for the procurer. Two major restrictions of the analysis by Che and Gale (2003) are a deterministic innovation technology and the exclusion of entry fees.

Schöttner (2008) reconsidered their finding and shows that a fixed-prize tournament may be more profitable for the procurer than an auction. Like Che and Gale (2003) her analysis assumes that entry fees cannot be employed by the procurer; however, unlike Che and Gale (2003) she assumes a stochastic innovation technology and replaces their simultaneous moves mechanism game by a sequential game in which innovators observe each others' innovations before they engage in bidding. She finds a sufficient condition for the superiority of fixed-prize tournaments. That condition requires that the probability distribution of the quality difference between firms' innovations either dominates an exponential distribution or exhibits log-convexity.

The literature on R&D contests implicitly assumes that contestants submit their best innovation regardless of its value. This assumption ignores that innovators may withhold innovations that are worth considerably more than the prize, so that only the lemons, i.e., the inferior innovations are submitted. If there is only one potential user of the innovation, i.e., if the procurer is a monopsonist, the procurer can in principle prevent this adverse selection problem by committing himself to never bargain with innovators who bypass the contest. However, such a commitment is difficult to achieve.[6]

There are many cases where innovations were inspired by a contest, but innovators ultimately decided to bypass the contest when they

[6] In principle, the procurer could write a contract with a third party that stipulates a high penalty whenever he procures from someone who did not participate in the contest. However, a procurer must always consider the possibility that superior innovations are forthcoming from outside innovators, for example because they did not know about the contest or did not expect to contribute to this particular application. No procurer would want to forego such potential trades. Moreover, such arrangements invite either renegotiation or may induce collusion between innovators and the third party, with the intention to collect the penalty.

felt that their innovation had a substantially higher commercial value than the prize offered by the contest, and then successfully negotiated more profitable license agreements after bypassing the contest.[7]

For example, the inventor John Wesley Hyatt was encouraged to develop a new substance after he saw an advertisement by Phelan & Collander, offering $10,000 to the person who invented a usable substitute for ivory in billiard balls. Hyatt eventually succeeded by inventing celluloid, which seemed to be a perfect substitute for ivory in billiard balls, but finally decided to patent his innovation instead of submitting it to the tournament and collecting the prize.[8] This bypass of the fixed-prize tournament allowed him to license his innovation not only for use in billiard balls, but also in a variety of other products, ranging from film and ping-pong balls to dental plates.[9]

[7] See the excellent survey by Cabral, Cozzi, Denicoló, Spagnolo, and Zanza (2006).

[8] As reported in *Wikipedia* "the English inventor Daniel Spill developed the same product which he patented in England as 'Xylonite', and later pursued Hyatt in a number of costly court cases between 1877 and 1884. The eventual outcome found that the true inventor of celluloid was Alexander Parkes, and that all manufacturing of celluloid could continue, including Hyatt's."

[9] Despite its initial success, the popularity of celluloid billiard balls diminished rapidly after a number of incidents caused by the high flammability of celluloid.

Motivated by this and other examples, the present chapter analyzes the procurement of innovations when the procurer is unable to commit to never bargain with innovators and innovators consider to bypass a contest in the event that they draw a high value innovation, when the value of innovation is not verifiable to third parties, and when the benefits of innovation accrue exclusively to the procurer. We compare two different methods to procure innovations: fixed-prize tournaments and (scoring) auctions, and determine which of these mechanisms is more profitable for the procurer if both mechanisms are potentially subject to a bypass and subsequent lemons problem.[10]

Our main finding is that this imperfect commitment generally affects the profitability of both mechanisms, but in substantially different ways, and depending on whether one employs a simple fixed-prize tournament or amends it by requiring advance registration and entry fees, just like in the optimal auction.

Altogether, we show that the optimal fixed-prize tournament outperforms the optimal auction. Specifically, we construct a fixed-prize tournament that prevents bypass (just like the optimal auction) and matches the profitability of the optimal auction. However, we also

[10] For a survey of alternative methods to procure innovations see Scotchmer (2005).

identify cases in which the optimal fixed-prize tournament is strictly more profitable than the optimal auction. Interestingly, a simple fixed-prize tournament that does not employ entry fees can be more profitable than the optimal auction that employs entry fees.

The intuition for this result is as follows. In the auction, entry fees are essential for surplus extraction by the procurer, yet high entry fees induce bypass. Therefore, in order to deter bypass, the procurer needs to significantly lower the entry fee, which reduces his expected profit. In fixed-prize tournaments, entry fees are not essential for surplus extraction, and the procurer must be primarily concerned with setting a sufficiently high prize to deter that innovators bypass *ex post*, after they have drawn their innovations. Of course, a high prize also reduces the procurer's expected profit, but this may be offset by charging an entry fee, as long as this does not induce *ex ante* bypass.

The plan of the chapter is as follows: sections 2.2 and 2.3 introduce the model and show that the optimal auction and fixed-prize tournament are revenue equivalent under perfect commitment. In section 2.4 we compare the profitability of the two mechanisms under imperfect commitment when innovators may bypass the mechanism and construct a fixed-prize tournament that is equally profitable as the optimal auction. In section 2.5 we state sufficient conditions for the strict superiority of the optimal fixed-prize tournament and offer some intuition. And in section 2.6 we show that the auction cannot be improved by applying an *ex post* minimum score requirement in addition to *ex ante* entry fees. The chapter closes with a discussion in section 2.7. Several proofs are in the appendix.

2.2. The model

A risk neutral procurer wishes to buy an innovation from one of two short-listed innovators, using either a fixed-prize tournament or a scoring auction. The procurer can commit to employ one of these mechanisms, but is unable to commit to never trade with an innovator who bypassed it.

Innovation technology: Innovation is modeled as an *i.i.d.* random variable, X, drawn from the c.d.f. $G : [\underline{x}, \bar{x}] \to [0, 1]$ with positive density g everywhere, at cost $c > 0$. X measures the increment in wealth that result if the procurer adopts it. The innovation has no value for anyone other than the procurer. G is such that $H(x) :=$

$\int_0^x G(y)dy$ is log-concave for all x.[11] For convenience, the support of G is normalized to $[0, 1]$. Order statistics of the sample of two random draws are denoted by $X_{(1)} \geq X_{(2)}$, and, as a rule, random variables are denoted by capital and realizations by lowercase letters.

Information: At the time when the contest is played, innovations are innovators' private information. That information becomes known to the procurer only after the contest game has been played or a bypass has occurred. Innovations are not verifiable to third parties, which restricts the set of feasible auction rules and rules out the use of bilateral contracts.

Contests: The procurer adopts either a fixed-prize tournament or a scoring auction. In the fixed-prize tournament the procurer sets a prize p to be paid to the best submitted innovation and possibly an entry fee, f, to be paid by innovators who register for the tournament.[12] If an innovator registers, innovates, and submits his innovation, he earns $p - f - c$ if he wins and $-f - c$ if he loses. In the auction,

[11] Log-concavity is frequently assumed in information economics (see the survey by Bagnoli and Bergstrom, 2005). The assumed log-concavity of H is obviously weaker than log-concavity of G.

[12] In the language of contest design, this is a "best-in-class" rather than a "first-past-to-post" contests.

the procurer selects a scoring rule, a pricing rule, and an entry fee f. The scoring rule maps an innovation, x, and a financial proposal, b (the smallest price requested for the innovation), into a score, S, and then selects the highest score as winner. The pricing rule maps the winner's score into the price that the procurer shall pay.

Since innovations are not verifiable by third parties, the only incentive-compatible scoring rule is the non-discriminating rule, $S(x,b) = x - b$, that scores bids by the net surplus that they promise to deliver to the procurer; and the only incentive-compatible auction is the first-score auction that requires the procurer to pay the winner the price he requested, b (see Che and Gale, 2003). This does, however, not rule out using the second-score auction as a proxy of the first-score auction, as explained later by Lemma 2.1.

In the following we denote the procurer's payoff by π_p and innovators' payoff by π; occasionally we add a superscript to identify the kind of contest, and write either a for auction or t for tournament.

Since innovators may bypass the contest prescribed by the procurer, the contest rules do not fully describe the game played between the procurer and innovators. In addition, we need to consider the bargaining game between innovator and procurer that applies if one or both innovators bypass the contest.

Bargaining in the event of bypass: The procurer engages in bargaining if either both innovators or only the owner of the superior innovation bypassed. For simplicity, we indicate the (owners of the) superior and inferior innovation by subscripts 1 and 2, and characterize the outcome of the bargaining game by the Nash (1950) bargaining solution.

Suppose only innovator 1 bypassed. Then, the procurer already has the inferior innovation, x_2, for which he is obliged to pay a transfer, t_2, determined by the rules of the mechanism in which 2 had participated. Hence, the procurer's default payoff is equal to $x_2 - t_2$, while that of innovator 1 is equal to zero since innovations have no alternative use.

Denote the price negotiated between the two parties by P and note that the payoff of the innovator is equal to P. The bargaining parties maximize the Nash product of their payoffs, subject to the budget constraint,

$$\max_{\pi_p, P} \left(\pi_p - (x_2 - t_2)\right) P, \quad \text{s.t.} \quad \pi_p + P + t_2 \leq x_1. \tag{2.1}$$

Evidently, the maximizer is $P = \frac{1}{2}(x_1 - x_2)$, $\pi_p = \frac{1}{2}(x_1 + x_2) - t_2$. Hence, in the event of one bypass, the equilibrium price of the superior innovation is equal to

$$P = \frac{1}{2}(x_1 - x_2), \tag{2.2}$$

and the total price paid by the procurer is $P + t_2$.

Now suppose both innovators bypassed. In that case we assume that the procurer bargains first with innovator 1, and only if that negotiation fails, bargains with innovator 2.[13] This bargaining problem is solved by backward induction. First, we solve the bargaining problem between procurer and innovator 2 (denoting the negotiation price by P_2 and the procurer's profit by π_{p_2}), which maximizes $\pi_{p_2} P_2$, subject to the budget constraint $\pi_{p_2} + P_2 \leq x_2$, which yields $\pi_{p_2} = \frac{1}{2} x_2$.

[13] The assumed sequence is not essential for the payoff of the procurer. If the procurer would first bargain with innovator 2, he would acquire the option to buy the inferior innovation and pay "liquidated damage fees" when that right is not exercised (as in Diamond and Maskin, 1979), that share the increment in profit due to having acquired a bargaining chip in dealing with the owner of superior innovation. As one can easily confirm, the resulting total price paid by the procurer (including the damage fee) is the same as the above price P' in (2.4).

Therefore, in the preceding bargaining between procurer and innovator 1, the parties maximize the Nash product (where $\frac{1}{2}x_2$ is the procurer's default payoff),

$$\max_{\pi_p, P'} \left(\pi_p - \frac{1}{2}x_2\right) P', \quad \text{s.t.} \quad \pi_p + P' \leq x_1 \tag{2.3}$$

$$\text{which gives} \quad P' = \frac{1}{2}\left(x_1 - \frac{1}{2}x_2\right). \tag{2.4}$$

Hence, when both innovators bypassed, the procurer buys the superior innovation at price P', while innovator 2 receives nothing. Note that the possibility to trade with innovator 2 confers an advantage to the procurer as it allows him to reduce the price of the superior innovation.

As an alternative to bargaining in the event when both innovators bypass, the procurer may also consider to let the two innovators compete in Bertrand fashion, by running a Vickrey auction in which bids are profits promised to the procurer. Evidently, if both innovators participate in that auction, the procurer obtains the superior innovation at a price equal to $x_1 - x_2$, which gives him a payoff equal to x_2. Yet, if only one innovator participates, the procurer obtains that innovator's innovation for a price equal to its full value, X.

However, consistent with our assumptions, one must take into account that the procurer cannot commit to never bargain with innovators who bypass this Bertrand competition game. But this implies, as we show now, that the Bertrand is never profitable for the procurer and shall not be considered by him.

Note, Bertrand competition is never profitable for the procurer if one innovator bypasses it; and if no innovator bypasses, Bertrand competition is more profitable for the procurer than bargaining if and only if $x_2 > x_1 - P'$. However, innovator 1 participates in the Bertrand competition if and only if he earns more than from bargaining, i.e., $x_1 - x_2 > P'$. Hence, Bertrand competition is never a profitable alternative.

Timeline: At date 0, the procurer announces the contest rule. At date 1, innovators simultaneously register for the contest and pay an entry fee or do not register (bypass may already occur at this point), if registration is required. At date 2, innovators simultaneously draw an innovation (or one or both do not innovate), not knowing whether their rival has registered for the contest. At date 3, innovators privately observe their innovation and either submit it to the contest or bypass. If the contest is an auction, submission requires a financial bid. If at least one innovation was submitted, at date 4 the mechanism game is executed, the winner/loser is selected and the winner is paid.

If an innovator has bypassed the mechanism, at date 5, this innovator proposes bargaining, and the procurer bargains with him if and only if he has drawn the superior innovation.

Parameter restrictions: We assume that the procurer is subject to a sufficiently high loss in the event that no procurement takes place. This is the case when the support of the random innovation is sufficiently bounded away from zero, so that $\underline{x} > 0$.[14] Normalizing that support of X to $[0,1]$ then means that a zero profit from no procurement is transformed into a negative profit of no procurement, equal to $-\underline{x}$. That loss is taken to be so high that one considers only mechanisms that assure procurement with probability one.

We also assume that the expected social surplus from two innovation draws exceeds that from one draw, that the surplus from one draw is positive, $E[X_{(1)}] - 2c > E[X] - c > 0$ (or equivalently $c \leq {}^1\!/_2 E\left[X_{(1)} - X_{(2)}\right]$), and that procuring an innovations does not require subsidies.

[14] This is typically the case when the procurement concerns some bottleneck innovation.

2.3. Baseline optimal mechanisms

As a baseline, we first assume that the procurer can commit to never negotiate with innovators who bypass the mechanism. We consider a standard fixed-prize tournament where the procurer offers to pay a fixed prize p for the best submitted innovation, and a scoring auction described by a scoring rule, a pricing rule, and entry fee f. As it will become clear, under full commitment entry fees are essential for surplus extraction in auctions but play no role in fixed-prize tournaments.

Baseline fixed-prize tournament The tournament game has a unique equilibrium outcome: the procurer sets the smallest prize that assures that both innovators innovate and submit their innovation (see Taylor, 1995). The equilibrium prize, p^*, and equilibrium payoffs of procurer, π_p^*, and innovators, π^*, are

$$p^* = 2c, \quad \pi_p^* = E[X_{(1)}] - p^* = E[X_{(1)}] - 2c, \quad \pi^* = \frac{1}{2}p^* - c = 0. \tag{2.5}$$

Note, if the procurer would set a smaller prize, $p \in [c, 2c)$, innovators would play mixed strategies. Given our assumption concerning the cost of "no procurement", the procurer will never set a price that induces mixed strategies since this would involve that procurement fails with positive probability.

Also, note that the optimal tournament is efficient and allows the procurer to extract the entire surplus.

Baseline Scoring Auction In a scoring auction, bids are two-dimensional, (x_i, b_i); where x_i is the value of the innovation, and b_i the minimum price requested. Bids are scored by a non-discriminatory scoring rule $S_i(x_i, b_i) := x_i - b_i$ that ranks innovations by their value-added for the procurer. The highest scoring bid wins. In the first-score auction the winner receives the price he requested; in the second-score auction the winner receives the price that makes his score match the second highest score.

The second-score auction is not incentive compatible if the value of innovations is not verifiable to third parties. Therefore, we assume that the procurer adopts a first-score auction. However, as we show in appendix 2.A.1):

Lemma 2.1. *First- and second-score auctions are payoff equivalent.*

Therefore, we can view the second-score auction as a proxy for the first-score auction. This allows us to highlight the performance of auctions without getting entangled in unnecessarily complex bidding strategies.[15]

At the time of bidding, the cost of innovation, c, is already sunk. Therefore, in the second-score auction, it is an equilibrium in dominant strategies to bid a score equal to the value of the innovation, x_i.[16] The associated equilibrium price, P, then solves the equation $X_{(1)} - P = S_{(2)} = X_{(2)}$, which gives $P = X_{(1)} - X_{(2)}$.

In the optimal second-score auction, the procurer levies an entry fee, f, which bidders have to pay in advance to register for the auction, before they draw their innovation. Only registered bidders can participate in the auction.[17]

The equilibrium expected price, entry fee, and payoffs are

$$E[P] = E\left[X_{(1)} - X_{(2)}\right] \quad \text{(expected price)} \tag{2.6}$$

[15] We mention that the payoff-equivalence of first- and second-score auctions does not apply to models in which bidders choose effort and price as in Che and Gale (2003), Schöttner (2008).

[16] To prove this, note that bidding a higher score can only change something if $x_i < x_j$, in which case the price becomes negative. Similarly, one can show that it never pays to bid a lower score.

[17] In procurement this corresponds to the commonly employed short-listing procedure.

$$f^* = \frac{1}{2}E[X_{(1)} - X_{(2)}] - c \quad \text{(entry fee)} \tag{2.7}$$

$$\pi_p = E[X_{(1)}] - E[P] + 2f = E[X_{(1)}] - 2c = \pi_p^* \tag{2.8}$$

$$\pi = \frac{1}{2}E[X_{(1)} - X_{(2)}] - c - f = 0 = \pi^*. \tag{2.9}$$

Obviously,

Proposition 2.1. *Under perfect commitment the two mechanisms are payoff equivalent and allow the procurer to extract the entire surplus.*

Whereas the optimal auction achieves full surplus extraction only by charging entry fees, the optimal fixed-prize tournament requires no entry fees. This fact plays a pivotal role in our later analysis.

2.4. Auction vs. fixed-prize tournament under imperfect commitment

Now assume the procurer cannot commit to never trade with an innovator who did not participate in the contest. In that case, innovators may bypass the contest and engage in bargaining after the contest game has been played.

A bypass can occur in two ways: either the innovator does not register for the contest, yet innovates and engages in bargaining after the contest game has been played, or he registers for the contest, draws an innovation but then abstains from bidding and engages in bargaining. Of course, an innovator can always not register and not innovate, to which we refer as *quit*.

The auction is characterized by the entry fee f and the fixed prize contest by the fixed prize and the entry fee, (p, f). Since tournaments do not necessarily require registration and entry fees (a case in point is the above benchmark optimal tournament), a "simple fixed-prize tournament" will be referred to as a tournament without registration requirement, which is defined by p (without f).

2.4.1. Optimal auction

Innovators' play a simultaneous moves game where they choose among the following action profiles: 1) register, innovate, bid, in short: register, bid, 2) register, innovate, not bid (bargain), in short: register, bargain, 3) not register, innovate, bargain, in short: not register, bargain, and 4) quit.

We show, in a sequence of lemmas, that the optimal entry fee induces both innovators to register, innovate, and bid, so that in equilibrium no bypass ever occurs. The only effect of imperfect commitment is that the procurer lowers the entry fee below the rate that is optimal under full commitment,[18] and in this way suffers from the lack of commitment.[19]

In a first step we show that bypass in the form of not submitting a bid after having registered, which is the above action profile 2), can be ruled out by elimination of dominated strategies:

Lemma 2.2. *If an innovator has registered for the auction, participation in the auction strictly dominates bypass.*

Proof. Consider an innovator who registered for the auction and drew the innovation $x > 0$. Suppose the other innovator also registered for the auction and bids. Then, the innovator's payoff from bidding, π, is greater than that from bargaining (bypass), π', since

$$\pi(x) = \int_0^x (x-y)g(y)dy > \int_0^x \frac{1}{2}(x-y)\,g(y)dy = \pi'(x).$$

[18] The restriction imposed on entry fees by the possibility of bypass is similar to the restriction based on listing fees due to search costs in auction hosting site pricing (see Deltas and Jeitschko, 2007).

[19] As we show later, in section 2.6, the profitability of the auction cannot be increased by adopting a minimum score requirement.

Next, suppose the other innovator does not bid (either because he registered and did not bid or did not register). Then, participation is even more profitable than bypass, since being the only bidder yields a price for the innovation equal to the full value of the innovation x. Therefore, conditional upon registration, participation in the auction is the dominant strategy. □

We can thus reduce innovators' strategies to: *register* (r) (short for register and innovate and bid), *not register* (n) (short for not register and innovate and bargain), and *quit* (q) (short for not register and not innovate). Table 2.1 summarizes the payoffs of innovator 1 for all combinations of innovators' strategies.

		Innovator 2		
		register (r)	not register (n)	quit (q)
Innovator 1	r	$\frac{1}{2}E[X_{(1)}-X_{(2)}]-c-f$	$E[X]-c-f$	$E[X]-c-f$
	n	$\frac{1}{2}\frac{1}{2}E[X_{(1)}-X_{(2)}]-c$	$\frac{1}{2}\frac{1}{2}E[X_{(1)}-\frac{1}{2}X_{(2)}]-c$	$\frac{1}{2}E[X]-c$
	q	0	0	0

Table 2.1.: (Reduced form) entry game in the auction

Denote innovators' payoff function by $\pi(\cdot,\cdot)$ where the first entry in the strategy profile refers to the own strategy and the second to that of the rival innovator. And define

$$f^{**} := \sup\{f \mid \pi(r,r) \geq \pi(n,r)\} = \frac{1}{4}E\left[X_{(1)} - X_{(2)}\right] \tag{2.10}$$

$$f^{*} := \sup\{f \mid \pi(r,r) \geq \pi(q,r)\} = \frac{1}{2}E\left[X_{(1)} - X_{(2)}\right] - c. \tag{2.11}$$

Lemma 2.3. (r,r) *is the unique equilibrium of the entry game that survives elimination of dominated strategies if and only if* $f \leq \min\{f^{**}, f^{*}\}$.

Proof. 1) The proof of necessity is trivial.

2) To prove sufficiency, suppose $f \leq \min\{f^{**}, f^{*}\}$. Then, obviously, “register” is a best reply to “register” and, as we show in Appendix 2.A.2, “register” is the unique best reply to “not register” and to “quit”. Therefore, “register” is a dominant strategy. □

Lemma 2.4. *The procurer’s expected profit is maximized if he sets* $f = \min\{f^{**}, f^{*}\}$.

Proof. If $f > \min\{f^{**}, f^{*}\}$, the strategy profile (r,r) is no longer an equilibrium since either q or n is the best reply to r (see Table 2.1). In that case, the game played between innovators has either one of the following equilibrium strategy profiles: (q,q), (r,n), (n,n), (n,q), (r,q) (plus the asymmetric equilibria obtained by renaming players) or, for that matter, no (pure strategy) equilibrium. In either case, the procurer's expected profit is lower than π_p^a, as we show in Appendix 2.A.3. Of course, setting $f < \min\{f^{**}, f^*\}$ is never profitable for the procurer. □

Proposition 2.2 (Optimal auction). *The optimal auction involves the entry fee $f = \min\{f^{**}, f^*\}$. It induces all innovators to register, innovate, and bid, and earns the procurer a positive expected profit equal to:*

$$\pi_p^a = \begin{cases} E[X] & \text{if} \quad c \leq \bar{c} \\ E[X_{(1)}] - 2c & \text{if} \quad c \geq \bar{c}. \end{cases} \tag{2.12}$$

where $\bar{c}$ is the cost level at which f^* is equal to f^{**}:

$$\bar{c} := \frac{1}{4} E[X_{(1)} - X_{(2)}]. \tag{2.13}$$

Proof. The optimality of $f = \min\{f^{**}, f^*\}$ follows from the above lemmas. To compute the associated expected profit of the procurer, note that if $c \leq \bar{c}$, one has $f = f^{**}$ and $\pi_p = E[X_{(1)}] - E[X_{(1)} - X_{(2)}] + 2f = \frac{1}{2}E[X_{(1)} + X_{(2)}] = E[X]$. Whereas if $c \geq \bar{c}$, one has

$f = f^*$ and $\pi_p = E[X_{(1)}] - E[X_{(1)} - X_{(2)}] + 2f = E[X_{(1)}] - 2c.$

$\square$

Corollary 2.1. *Pure bargaining (obtained by inducing (n,n) as equilibrium of the entry game) is strictly less profitable for the procurer than the optimal auction.*

Corollary 2.2. *Imperfect commitment does not affect payoffs if $c \geq \bar{c}$, yet reduces the procurer's payoff from $E[X_{(1)}] - 2c$ to $E[X]$, leaving the surplus $1/4 E[X_{(1)} - X_{(2)}] - c > 0$ to each innovator, if $c < \bar{c}$.*

Altogether, the procurer who adopts an auction faces a dilemma: in order to extract surplus he must rely on high entry fees, even if bypass is not an issue (see (2.7)); however, high entry fees make bypass profitable. As a rule, the optimal auction under imperfect commitment requires the procurer to set a sufficiently low entry fee that deters bypass. The difference between the surplus extracting and the bypass preventing entry fee measures the cost of imperfect commitment to the procurer.

There is one exception to this rule: If the cost of innovation is sufficiently high, specifically iff $c \geq \bar{c}$, the surplus extracting entry fee is already so low that innovators gain more from participating in the auction than from avoiding the entry fee by bypassing. In that case, the optimal baseline auction is also optimal under imperfect commitment.

2.4.2. Fixed-prize tournament

Innovators play a simultaneous moves game where they choose among the same action profiles as in the above auction, provided one substitutes bid by submit.

Innovators' equilibrium play is determined by the procurer's choice of prize and, if registration is required, an entry fee, (p, f). The prize p determines whether an innovator who registered either submits his innovation regardless of its value or submits it only if its value is below a certain threshold level, and the registration fee influences the registration decision.

In a first step we analyze the equilibrium play assuming both innovators have registered or registration is not required.

In the following, "simple fixed-prize tournaments", in which contestants are not asked to register for the contest (and thus no registration fee is required) will play a particular role.

Lemma 2.5. *Suppose registration is not required or, if it is required, both innovators have registered. Then, innovators play cutoff strategies* $\gamma_1, \gamma_2 \in (0, 1]$ *and submit their innovation* x_i *if and only if* $x_i < \gamma_i$.

Proof. Suppose innovator 2 plays cutoff strategy γ_2, and innovator 1 has drawn innovation x_1. We need to show that 1) if "not submit" (ns) is the best reply of innovator 1 to γ_2, then ns is also his best reply for all innovation values greater than x_1; and 2) if "submit" (s) is the best reply of innovator 1, then s is also his best reply for all innovation values smaller than x_1.

To prove 1), suppose $x_1 < \gamma_2$. Then, the assumption $\pi(ns, \gamma_2) \geq \pi(s, \gamma_2)$ is equivalent to

$$\frac{1}{2}\int_0^{x_1}(x_1 - y)dG(y) \geq p\,(G(x_1) + (1 - G(\gamma_2)))$$
$$\frac{1}{2}\int_0^{x_1} G(y)dy \geq p\,(G(x_1) + (1 - G(\gamma_2)))$$
$$\frac{1}{2}\frac{H(x_1)}{H'(x_1)} \geq p\left(1 + \frac{1 - G(\gamma_2)}{G(x_1)}\right).$$

Since H is log-concave, the LHS of the last inequality is increasing and the RHS is decreasing in x_1. Therefore, this inequality holds also for all innovations valued higher than x_1.

Suppose $x_1 > \gamma_2$. Then, the assumption $\pi(ns, \gamma_2) \geq \pi(s, \gamma_2)$ is equivalent to

$$\frac{1}{2}\int_0^{\gamma_2}(x_1 - y)dG(y) + \frac{1}{2}\int_{\gamma_2}^{x_1}\left(x_1 - \frac{1}{2}y\right)dG(y) \geq p.$$

The LHS of the last inequality is increasing in x_1 and the RHS is constant. Therefore, this inequality holds also for all innovations valued higher than x_1.

The proof of 2) is similar and hence omitted. □

Lemma 2.6. *Suppose registration is not required or, if it is required, both innovators have registered. Then, the game has a unique symmetric equilibrium cutoff strategy $\gamma(p) \in (0,1]$. $\gamma(p)$ is monotone increasing in p and exhibits $\gamma = 1$ (always submit) if and only if $p \geq \max\{\bar{p}, 2c\}$, where*

$$\bar{p} := \frac{1}{2}\int_0^1 G(x)dx = \frac{1}{2}(1 - E[X]). \qquad (2.14)$$

Proof. 1) $\gamma(p) = 1$ is a symmetric equilibrium if and only if p is such that for all x:

$$\pi(\mathrm{s}, \mathrm{s}) \geq \pi(ns, \mathrm{s})$$

$$G(x)p \geq G(x)E[{}^1\!/\!{}_2(x - Y) \mid Y < x]$$

$$p \geq \frac{1}{2}\int_0^x \frac{G(y)}{G(x)}dy = \frac{1}{2}\frac{H(x)}{H'(x)}.$$

Recall that $H(x) := \int_0^x G(y)dy$ is log-concave; hence, $H(x)/H'(x)$ is increasing, and thus,

$$\bar{p} := \inf\left\{p \mid p \geq \frac{1}{2}\frac{H(x)}{H'(x)}, \forall x\right\} = \frac{1}{2}\frac{H(1)}{G(1)} = \frac{1}{2}(1 - E[X]).$$

Therefore, $\gamma = 1$ for all $p \geq \max\{\bar{p}, 2c\}$, which is the smallest prize that induces $\gamma(p) = 1$ ($2c$ enters to assure that $\pi(s,s) \geq 0$, for all x).

2) If $p < \bar{p}$, one obtains $\gamma(p) \in (0,1)$ and $p' > p \Rightarrow \gamma(p') > \gamma(p)$. Since this property is not essential for our analysis, we relegate the proof to Chapter 3. □

This result has an intuitive interpretation. Suppose an innovator has drawn the best possible innovation $X = 1$. Then, he expects to be paid $\frac{1}{2}(1 - E[Y \mid Y < 1]) = \frac{1}{2}(1 - E[X])$ if he chooses to bypass the tournament. Therefore, any $p \geq \frac{1}{2}(1 - E[X]) = \bar{p}$ will keep him in the tournament if the other innovator is in, and, due to the monotonicity of $\frac{1}{2}(x - E[Y < x])$ in x, this extends to all innovations x. It follows that if the conditional expected spread $E[X_{(1)} - X_{(2)} \mid X_{(1)} = 1] = 1 - E[X]$ is small, bypass is deterred in the tournament already at a relatively low prize.

We now construct a fixed-prize tournament $(\hat{p}, \hat{f})$ that achieves the same expected profit of the procurer as the optimal auction. That particular tournament involves a prize $\hat{p}$ that is higher than $\bar{p}$. While $\bar{p}$ is the smallest prize that makes $\gamma = 1$ (submit all innovations) an equilibrium if both innovators registered (or no registration is required), $\hat{p}$ (together with $\hat{f}$) makes $\gamma = 1$ an equilibrium in dominant strategy, independent of whether the rival innovator registered.

Proposition 2.3. *The fixed-prize tournament* $(\hat{p}, \hat{f})$*,*

$$\hat{p} = \frac{1}{2} - \frac{1}{4}E[X] \tag{2.15}$$

$$\hat{f} = \begin{cases} \frac{1}{4} - \frac{1}{16}\left(5E[X_{(1)}] - 3E[X_{(2)}]\right) & \text{if } c \leq \bar{c} \\ \frac{1}{4} - \frac{1}{8}E[X] - c & \text{if } c \geq \bar{c}. \end{cases} \tag{2.16}$$

achieves the same expected profit for the procurer as the optimal auction. Therefore, the optimal fixed-prize tournament is at least as profitable for the procurer as the optimal auction.

The following Lemmas prove the above interpretation of $\hat{p}$ and prepare the proof of proposition 2.3.

Lemma 2.7. *At the fixed-prize contest* $(\hat{p}, \hat{f})$ *"quit" is a strictly dominated strategy.*

Proof. "Quit" yields a payoff equal to zero. Consider the simple alternative strategy "register (if required) and always submit". If this strategy is played, the worst that can happen is that the rival plays the same strategy, in which case one collects the prize $\hat{p}$ if and only if one has the best innovation, yielding a payoff equal to $1/2\hat{p} - \hat{f} - c$, which is positive regardless of whether $c < \bar{c}$ or $c > \bar{c}$. This is the worst case, because if the rival plays any other strategy and does not submit all his innovations, the own payoff can only be higher. It follows that the equilibrium payoff is bounded away from zero; hence, the strategy "quit" that yields a payoff equal to zero is strictly dominated. This allows us to eliminate "quit" from further consideration. □

Lemma 2.8. *Consider an innovator who has registered or who is not required to register.* $\gamma = 1$ *is that innovators dominant strategy if and only if* $p \geq \hat{p}$.

Proof. We have already eliminated "quit"; therefore, the innovator will either "submit" (s) or "not submit" (ns). First, suppose that the rival innovator did not register. Then, our innovator submits if and only if $\Delta(p,x) \geq 0$, for all x:

$$\begin{aligned}
\Delta(p,x) :&= \pi(s,n) - \pi(ns,n) \\
&= p - G(x)E\left[\frac{1}{2}(x - \frac{1}{2}Y) \mid Y < x\right] \\
&= p - \frac{1}{2}\int_0^x (x - \frac{1}{2}y)g(y)dy \\
&= p - \frac{1}{4}xG(x) - \frac{1}{4}\int_0^x G(y)dy.
\end{aligned} \tag{2.17}$$

Evidently, $\Delta(p,x)$ is strictly increasing in p and decreasing in x, and one has

$$\Delta(p,x)|_{p=\hat{p},x=1} = \hat{p} - \frac{1}{4} - \frac{1}{4}\int_0^1 G(y)dy = \hat{p} - \frac{1}{2} + \frac{1}{4}E[X] = 0.$$

$$\Delta(p,x)|_{p=\bar{p},x=1} = \frac{1}{4}\left(\int_0^1 G(y)dy - 1\right) < \frac{1}{4}\left(\int_0^1 dy - 1\right) = 0$$

Therefore, $\hat{p}$ is the smallest prize that assures $\Delta(p,x) \geq 0$, for all x, and $\bar{p} < \hat{p}$, since $\Delta(\bar{p},x) < 0$ for high values of x.

Second, suppose the rival innovator is also registered. Recall that $p = \bar{p}$ is the smallest prize that assures $\Delta(p,x) \geq 0$ for all x, if both innovators registered. Since $\hat{p} > \bar{p}$ it follows that $p = \hat{p}$ also assures that the innovator submits. $\square$

We are now ready to prove Proposition 2.3.

Proof. By construction of $\hat{p}$, an innovator who registered plays $\gamma = 1$ and "quit" is a dominated strategy. Therefore, the simultaneous entry game can be described by the following reduced payoff matrix (since the game is symmetric we list only the payoffs of innovator 1):[20]

		Innovator 2	
		register (r)	not register (n)
Innovator 1	r	$\frac{1}{2}\hat{p} - f - c$	$\hat{p} - c - f$
	n	$\frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)} - X_{(2)}\right]\right) - c$	$\frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)} - \frac{1}{2}X_{(2)}\right]\right) - c$

Table 2.2.: (Reduced form) entry game in the fixed-prize tournament with entry fees

Suppose $c \leq \bar{c} = {}^1\!/_4 E[X_{(1)} - X_{(2)}]$. Then, for $f = \hat{f}$ one has

$$\pi(r,r) = \frac{1}{2}\hat{p} - \hat{f} - c = \frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)} - X_{(2)}\right]\right) - c = \pi(n,r)$$

[20] Suppose innovator 1 plays r; if innovator 2 plays n, 1 wins the prize for sure, whereas if 2 plays also r, 1 wins the prize with probability 1/2. Suppose innovator 1 plays n and thus speculates on bargaining; he wins only if he has the better innovation, which occurs with probability 1/2, and in that event the procurer's threat point depends on whether innovator 2 played r or n, as explained before.

$$\begin{aligned}\pi(r,n) &= \hat{p} - \hat{f} - c \\ &> \frac{1}{2}\hat{p} - \hat{f} - c \\ &= \frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)} - \frac{1}{2}X_{(2)}\right]\right) - c = \pi(n,n),\end{aligned}$$

Therefore, r dominates n. Hence, (r,r) is the unique equilibrium of the entry game, by iterated elimination of dominated strategies, and the procurer's equilibrium expected profit is $\pi_p^t = E[X_{(1)}] - \hat{p} + 2\hat{f} = E[X] = \pi_p^a$.

Similarly, in case $c \geq \bar{c}$ one has

$$\pi(r,r) = \frac{1}{2}\hat{p} - \hat{f} - c = 0 \geq \frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)} - X_{(2)}\right]\right) - c = \pi(n,r)$$

$$\pi(r,n) = \hat{p} - \hat{f} - c > \frac{1}{2}\left(\frac{1}{2}E\left[X_{(1)} - \frac{1}{2}X_{(2)}\right]\right) - c = \pi(n,n).$$

Again, (r,r) is the unique equilibrium and the procurer's equilibrium expected profit is $\pi_p^t = E[X_{(1)}] - \hat{p} + 2\hat{f} = E[X_{(1)}] - 2c = \pi_p^a$ in this case. □

2.5. Superiority of the fixed-prize tournament

Having shown that the optimal fixed-prize tournament is at least as profitable as the optimal auction, we now provide sufficient conditions for the strict superiority of the fixed-prize tournament. Interestingly, in these cases a "simple fixed-prize tournament", that does not require registration and entry fees, is more profitable than the optimal auction. Of course, a simple fixed-prize tournament is strategically simpler since it poses no entry decision and innovators maintain the option to submit until they have drawn their innovation.

Proposition 2.4. *The optimal fixed-prize tournament is strictly more profitable than the optimal auction if $c < \bar{c}$ and*

$$\eta := E\left[\frac{3}{2}X_{(1)} - \frac{1}{2}X_{(2)}\right] > 1. \tag{2.18}$$

Proof. We show that the "simple fixed-prize tournament" $p = \max\{\bar{p}, 2c\}$, that does not require registration and includes no entry fee f, is more profitable than the optimal auction. By lemma 2.6, $\max\{\bar{p}, 2c\}$ is the smallest prize for which $(\gamma_1, \gamma_2) = (1, 1)$ as an equilibrium.

The assumption $c < \bar{c}$ implies $\pi_p^a = E[X]$ (by proposition 2.2). The assumption $\eta > 1$ implies that $\bar{p} < 2\bar{c}$. We distinguish between the cases when $c \in (\frac{\bar{p}}{2}, \bar{c})$ and $c < \frac{\bar{p}}{2}$.

1) Suppose $c \in (\frac{\bar{p}}{2}, \bar{c})$. Then, $\max\{\bar{p}, 2c\} = 2c$, and one has,

$$\begin{aligned}
\pi_p^t - \pi_p^a \geq \pi_p^t\big|_{p=2c} - \pi_p^a \\
&= E[X_{(1)}] - 2c - E[X] \\
&> E[X_{(1)}] - 2\bar{c} - E[X] \\
&= E[X_{(1)}] - \frac{1}{2}\big(E[X_{(1)}] - E[X_{(2)}]\big) - E[X] = 0.
\end{aligned} \tag{2.19}$$

2) Suppose $c < \frac{1}{2}\bar{p}$. Then, $\max\{\bar{p}, 2c\} = \bar{p}$, and one has,

$$\begin{aligned}
\pi_p^t - \pi_p^a \geq \pi_p^t\big|_{p=\bar{p}} - \pi_p^a \\
&= E[X_{(1)}] - \bar{p} - E[X] \\
&= \frac{1}{2}\left(\frac{3}{2}E[X_{(1)}] - \frac{1}{2}E[X_{(2)}] - 1\right) \\
&= \frac{1}{2}\Big(\eta - 1\Big) > 0.
\end{aligned} \tag{2.20}$$

□

Combining propositions 2.4 and 2.3 it follows immediately:

Corollary 2.3 (Optimality of the fixed-prize tournament). *The optimal fixed-prize tournament is never less and sometimes more profitable than the optimal auction.*

These results indicate that the optimal fixed-prize tournament is strictly superior to the optimal auction if the spread between the order statistics $X_{(1)}, X_{(2)}$ is relatively large (so that $\eta > 1$), and the cost of innovation c is lower than $\bar{c}$.

In the auction, a large spread between the order statistics induces a high equilibrium price for the best innovation. In that case, surplus extraction requires high entry fees. However, high entry fees make bypass attractive. Therefore, the procurer can only deter bypass by significantly reducing the entry fees, which reduces his expected profit. This reflects in the "loss of imperfect commitment":

$$\Delta_a = \pi_p^* - E[X] = \frac{1}{2}E[X_{(1)} - X_{(2)}] - 2c. \qquad (2.21)$$

In other words, the optimal auction performs well only if the expected spread $E[X_{(1)} - X_{(2)}]$ is small.

Whereas in the auction the bypass decision is made at the entry stage, in the tournament the key bypass decision is made after innovators have drawn their innovations.

In the tournament, the benefit to bypass depends on innovators' marginal contribution to the surplus, conditional on having the best innovation. If the conditional expected spread $E[X_{(1)} - X_{(2)} \mid X_{(1)} = 1] = 1 - E[X]$ is small, bypass is deterred in the tournament already at a relatively low prize, and thus full surplus extraction can be possible, even if it is impossible in the optimal auction. This property reflects in the "loss of imperfect commitment" in the optimal tournament, defined as (in the case $c \leq \frac{\bar{p}}{2}$),

$$\Delta_t \leq \pi_p^* - \pi_p^t\big|_{p=\bar{p}} = \frac{1}{2}(1 - E[X]) - 2c. \qquad (2.22)$$

As one can see from (2.22), the loss of imperfect commitment in the tournament, Δ_t, is small if the probability distribution exhibits a concentration on high values, so that $E[X]$ is relatively large. And the loss of imperfect commitment in the auction, Δ_a, is large if the unconditional expected spread between the order statistics is large.

Example 2.1. *Consider the Kumaraswamy (1980) distribution* $G(x) = 1 - (1 - (x/\gamma)^{\alpha})^{\beta}$*, with parameter values* $\alpha = 1, \beta = 1/5$*, which exhibits a concentration on high values.*[21] *Then,* $\eta = 15/14 > 1$*,* $\bar{c} = 5/84$*,* $\bar{p} = 1/12$*, and for all* $c \leq \frac{1}{24}$*:* $\pi_p^t \geq \pi_p^t\big|_{p=\bar{p}} = 73/84 > 5/6 = \pi_p^a$*. As one can easily confirm, this distribution satisfies the log-concavity of* $H(x) = \int_0^x G(y)dy$*, for all* x*, which we assumed in the present chapter. Therefore, this is an example for the strict superiority of the optimal fixed-prize tournament.*

[21] This distribution has been introduced as a more convenient alternative to the Beta distribution.

2.6. The auction cannot be improved by a minimum score

One may think that the profitability of the auction can be improved by adding a minimum score requirement, similar to the standard optimal auction problem where the seller benefits from strategically setting a binding reserve price above his own valuation.[22]

However, as we show formally in the technical supplement (Chapter 3), the auction cannot be improved by adding a minimum score.

Proposition 2.5. *The optimal minimum score is equal to zero; hence, the procurer cannot raise his expected profit by employing a minimum score requirement in addition to charging entry fees.*

The proof is in the technical supplement contained in Chapter 3.

Intuitively this is due to the fact that the optimal auction already uses an entry fee. And using an entry fee is more profitable than a minimum score requirement because the entry fee is collected at the time when innovators are still behind a “veil of ignorance”.

[22] Myerson (1981) showed that a binding reserve price that excludes participation of bidders with low values is optimal for the seller except if buyers’ valuations are considerably larger than the seller’s own valuation.

2.7. Discussion

In the present chapter we analyze the procurement of innovations, assuming that the procurer cannot commit himself to never bargain with innovators who did not participate in the mechanism of his choice. We compare two methods of procurement: auctions and fixed-prize tournaments, and show that the optimal fixed-prize tournament is more profitable than the optimal auction. While auctions are never attractive without positive entry fees, a standard fixed-prize contest that does not require registration and thus does not include entry fees can be more profitable than the optimal auction.

Our analysis assumes that the value of an innovation is not verifiable to third parties. This excludes the use of contracts to procure innovations. It also precludes the use of a discriminatory scoring rule as well as a second-score auction in which the price paid by the procurer depends on the difference between the values of the two best innovations. Nevertheless, since in the present framework first- and second-score auctions are revenue equivalent, we are able to highlight the performance of auctions without getting entangled in unnecessarily complex bidding strategies, by using the second-score auction as a proxy for the first-score auction, Our analysis has been carried out in a simple framework, with only two contestants and a stylized innovation technology. In further research one may wish to extend the analysis to cases in which it is optimal to short-list more than two contestants, introduce asymmetries between innovators, either with respect to their cost functions or with respect to the probability distributions from which they draw their innovations, and extend the analysis to a common (or affiliated) value framework.

2.A. Appendix

2.A.1. Proof of Lemma 2.1

We show that first- and second-score auctions yield the same expected prices, p^1, p^2.

Denote the bidding strategy of the first-score auction by the score function $s(x)$. The highest score wins and the winner is paid $b(x) := x - s(x)$. By an argument similar to the solution of a standard first-price auction, one finds the equilibrium score function is $s(x) = E[Y \mid Y \leq x]$.

Since the innovator with the highest x has also the highest score, the winning score is $s(X_{(1)})$, and the equilibrium price paid by the procurer to the winner is $X_{(1)} - s(X_{(1)})$. Therefore, one finds, using the density of the order statistic $X_{(1)}$, $g_{(1)}(x) = 2G(x)g(x)$, $p^1 = E[X_{(1)} - s(X_{(1)})] = E[X_{(1)}] - \int_0^1 \int_0^x 2yg(y)dyg(x)dx$.

Rearrange p^2 in (2.6), using the joint density of the two order statistics $g_{(1,2)}(x,y) = 2g(x)g(y)$ for $x > y$, and one concludes by the law of iterated expectations:

$$\begin{aligned}
p^2 &= E[X_{(1)}] - E[X_{(2)}] \\
&= E[X_{(1)}] - E[E[X_{(2)} \mid X_{(1)}]] \\
&= E[X_{(1)}] - \int_0^1 \int_0^x y \frac{g_{(1,2)}(x,y)}{g_{(1)}(x)} dyg_{(1)}(x)dx \\
&= E[X_{(1)}] - \int_0^1 \int_0^x 2yg(y)dyg(x)dx = p^1.
\end{aligned}$$

2.A.2. Supplement to the proof of Lemma 2.3

Here we show that “register” (r) is the unique best reply to “not register” (n) and to “quit” (q). We denote the payoff function of innovator 1 by $\pi(\cdot,\cdot)$ where the first entry in the strategy profile refers to strategy of innovator 1 and the second to that of innovator 2.

1) Let $c \leq \bar{c}$. Then, $f \leq \min\{f^{**}, f^*\} = f^{**}$, and one finds, using

Table 2.1,

$$
\begin{aligned}
\pi(r,n) &= E[X] - f - c \\
&\geq \frac{1}{2}E\left[X_{(1)} + X_{(2)}\right] - \frac{1}{4}E\left[X_{(1)} - X_{(2)}\right] - c \\
&= \frac{1}{4}E[X_{(1)}] + \frac{3}{4}E[X_{(2)}] - c > 0 = \pi(q,n) \\
\pi(r,n) &> \frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}] - c = \pi(n,n).
\end{aligned}
$$

Therefore, r is the unique best reply to n in this case.

2) Let $c \geq \bar{c}$, then $f \leq \min\{f^{**}, f^{*}\} = f^{*}$, and hence

$$
\begin{aligned}
\pi(r,n) &\geq E[X] - c - \left(\frac{1}{2}E[X_{(1)} - X_{(2)}] - c\right) \\
&= \frac{1}{2}E[X_{(1)} + X_{(2)}] = E[X] > 0 = \pi(q,n), \\
\pi(n,n) &= \frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}] - c \\
&\leq \frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}] - \frac{1}{4}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}] \\
&= \frac{1}{8}E[X_{(2)}] < \pi(r,n).
\end{aligned}
$$

Therefore, r is the unique best reply to n also in this case.

3) Similarly one can show that r is the unique best reply to q.

2.A.3. Proof of Lemma 2.4

1) Inducing an equilibrium (q,q), where no one innovates, yields zero profit to the procurer, whereas inducing (r,r) yields $\pi_p^a > 0$ (see (2.12)).

2) If (r,n) is induced, one has $\pi(r,n) \geq \pi(n,n)$, which is equivalent to

$$f \leq \frac{1}{4}E[X_{(1)}] + \frac{5}{8}E[X_{(2)}]. \tag{2.23}$$

In such an equilibrium the procurer's expected profit is

$$\begin{aligned}\pi_p &= E[X_{(1)}] - E[X] + f - \frac{1}{2}\left(\frac{1}{2}E[X_{(1)} - \frac{1}{2}X_{(2)}]\right) \\ &= \frac{1}{4}E[X_{(1)}] - \frac{3}{8}E[X_{(2)}] + f.\end{aligned}$$

If $c \leq \bar{c}$, it follows by (2.23) and (2.12) that

$$\begin{aligned}\pi_p &\leq \frac{1}{4}E[X_{(1)}] - \frac{3}{8}E[X_{(2)}] + \frac{1}{4}E[X_{(1)}] + \frac{5}{8}E[X_{(2)}] \\ &= \frac{1}{2}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}] < E[X] = \pi_p^a.\end{aligned} \tag{2.24}$$

And if $\bar{c} \leq c \leq \frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}]$, it follows by (2.23) and (2.12) that

$$\begin{aligned}
\pi_p^a &= E[X_{(1)}] - 2c \\
&\geq E[X_{(1)}] - 2\left(\frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}]\right) \\
&= \frac{1}{2}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}] \\
&\geq \frac{1}{4}E[X_{(1)}] - \frac{3}{8}E[X_{(2)}] + f = \pi_p.
\end{aligned} \tag{2.25}$$

3) If (n,n) is induced, the payoffs of innovators' and the procurer are

$$\pi(n,n) = \frac{1}{2}\left(\frac{1}{2}E[X_{(1)} - \frac{1}{2}X_{(2)}]\right) - c = \frac{1}{4}E[X_{(1)}] - \frac{1}{8}E[X_{(2)}] - c \geq 0$$

$$\pi_p = E[X_{(1)}] - \frac{1}{2}E[X_{(1)} - \frac{1}{2}X_{(2)}] = \frac{1}{2}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}].$$

Hence, one must have $c \leq {}^1\!/_4 E[X_{(1)}] - {}^1\!/_8 E[X_{(2)}]$.

If $c \leq \bar{c}$, one finds $\pi_p^a = E[X] = \frac{1}{2}E[X_{(1)} + X_{(2)}] > \pi_p$.

And if $\bar{c} \leq c \leq {}^1\!/_4 E[X_{(1)}] - {}^1\!/_8 E[X_{(2)}]$, one has

$$\begin{aligned}
\pi_p^a &= E[X_{(1)}] - 2c \geq E[X_{(1)}] - \frac{1}{2}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}] \\
&= \frac{1}{2}E[X_{(1)}] + \frac{1}{4}E[X_{(2)}] = \pi_p.
\end{aligned}$$

4) Inducing (n,q) is less profitable for the procurer than inducing (n,n), since bargaining with two innovators is obviously better than

dealing with one, and this, in turn, is less profitable than inducing (r,r), as already shown in step 3).

5) (r,q) can only be induced if $E[X]-c-f \geq 0$ (see Table 2.1). But this implies, together with the assumed efficiency of two draws, that inducing (r,q) is less profitable for the procurer than inducing (r,r):

$$\begin{aligned}\pi_p^a - \pi_p &= \big(E[X_{(1)}]-2c\big) - f \\ &\geq \big(E[X_{(1)}]-2c\big) - (E[X]-c) > 0.\end{aligned}$$

3. Technical Supplement[1]

3.1. Introduction

In this technical supplement to the previous chapter we provide some more detailed proofs and add material for the interested reader. In particular, we provide a more detailed characterization of payoff functions in fixed-prize tournaments, a characterization of optimal fixed-prize tournaments, and more results concerning the ranking of the optimal auction relative to the optimal simple fixed-prize tournament.

[1] This chapter is based on Ding and Wolfstetter (2011b).

3.2. Supplement to the proof of Lemma 2.6

Suppose both innovators have registered or no registration is required. In Lemmas 2.5 and 2.6 in the previous chapter we showed already that the game played between innovators has an equilibrium in cutoff strategies and that innovators submit all innovations if and only if $p \geq \bar{p}$. However, there we did not prove that the cutoff strategy γ is strictly monotone increasing in p for all $p < \bar{p}$. Here we we fill in this gap.

To prepare the proof and solve the symmetric equilibrium cutoff strategy γ, consider one player, say player 1, who contemplates the deviating strategy $\gamma_1 \geq \gamma$, while his rival, player 2, plays the equilibrium strategy γ. To compute the payoff function of player 1, $\pi_1(\gamma_1, \gamma)$, take a look at the state space representation of that innovator's payoffs in Figure 3.1. Using the joint density $g_{12}(x_1, x_2) = g(x_1)g(x_2)$, one can then compute the payoff function by integrating over the relevant subsets of the state space $[0,1] \times [0,1]$:

$$\pi_1(\gamma_1,\gamma) = p\left(\int_0^{\gamma}\int_0^{x_1} dG(x_2)dG(x_1) + \int_{\gamma}^{\gamma_1}\int_0^{\gamma} dG(x_2)dG(x_1) + \int_0^{\gamma_1}\int_{\gamma}^{1} dG(x_2)dG(x_1)\right) + \frac{1}{2}\int_{\gamma_1}^{1}\int_0^{\gamma}(x_1 - x_2)\, dG(x_2)dG(x_1) + \frac{1}{2}\int_{\gamma_1}^{1}\int_{\gamma}^{x_1}\left(x_1 - \frac{1}{2}x_2\right) dG(x_2)dG(x_1) - c \tag{3.1}$$

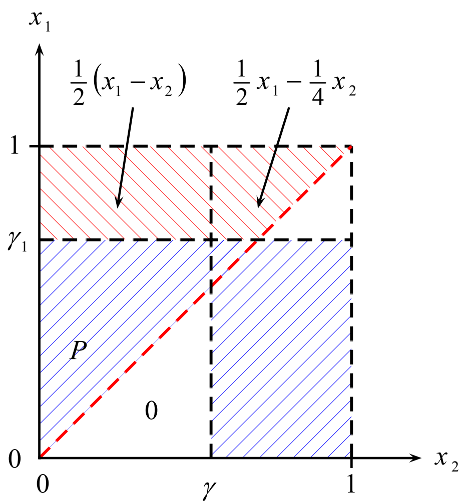

Figure 3.1.: Payoffs of innovator 1 for $\gamma_1 \geq \gamma$ in the state space $[0,1] \times [0,1]$

Proposition 3.1. *In a tournament with fixed price p, suppose both innovators have registered or no registration is required and assume $p < \bar{p}$. The game played between innovators has a unique symmetric equilibrium strategy $\gamma \in (0,1]$, which is implicitly defined as the solution of*

$$p = \frac{1}{2}\int_0^{\gamma} G(x)dx. \tag{3.2}$$

γ is strictly increasing in p for all $0 < p < \bar{p}$.

Proof. Consider one innovator, say innovator 1. We need to show that for each given p, the γ implicitly defined in Proposition 3.1 satisfies the equilibrium requirement

$$\gamma = \underset{0 \le \gamma_1 \le 1}{\arg\max}\, \pi_1(\gamma_1, \gamma). \tag{3.3}$$

For this purpose, first consider "upward" deviations from the equilibrium, $\gamma_1 \ge \gamma$, as in (3.1). Computing the partial derivative of π_1 w.r.t. γ_1 gives

$$\begin{aligned} \frac{\partial \pi_1}{\partial \gamma_1} &= p\,(G(\gamma)g(\gamma_1) + (1 - G(\gamma))g(\gamma_1)) \\ &\quad - \frac{1}{2}g(\gamma_1)\int_0^{\gamma}(\gamma_1 - x_2)dG(x_2) \\ &\quad - g(\gamma_1)\int_{\gamma}^{\gamma_1}\left(\frac{1}{2}\gamma_1 - \frac{1}{4}x_2\right)dG(x_2) \end{aligned} \tag{3.4}$$

$$\left.\frac{\partial \pi_1}{\partial \gamma_1}\right|_{\gamma_1=\gamma} = \left(p - \frac{1}{2}\int_0^{\gamma} G(x)dx\right) g(\gamma) =: \xi(p,\gamma)g(\gamma). \qquad (3.5)$$

Using the Lagrange function $\mathcal{L} := \pi_1 + \lambda(1-\gamma_1)$, with the Lagrangian λ, and invoking the equilibrium requirement that γ must be such that the best response of innovator 1 to γ is $\gamma_1 = \gamma$ (see (3.3)), the equilibrium strategy γ must solve the Kuhn-Tucker (KT) conditions

$$\begin{aligned} &\left.\frac{\partial \mathcal{L}}{\partial \gamma_1}\right|_{\gamma_1=\gamma} = \left.\frac{\partial \pi_1}{\partial \gamma_1}\right|_{\gamma_1=\gamma} - \lambda = 0 \\ &\left.\frac{\partial \mathcal{L}}{\partial \lambda}\right|_{\gamma_1=\gamma} = 1-\gamma \geq 0 \quad \text{and} \quad \left.\frac{\partial \mathcal{L}}{\partial \lambda}\right|_{\gamma_1=\gamma} \lambda = 0. \end{aligned} \qquad (3.6)$$

For $p \geq \bar{p}$ one finds (omitting the subscript 1) $\partial\pi/\partial\gamma|_{\gamma=1} \geq 0$; hence, the KT conditions are solved by $(\gamma = 1, \lambda = \partial\pi/\partial\gamma|_{\gamma=1})$. This confirms Lemma 2.6.

For $0 < p < \bar{p}$ one finds (omitting the subscript 1) $\partial\pi/\partial\gamma|_{\gamma=1} < 0$; hence, the KT conditions are solved by $(0 < \gamma < 1, \lambda = 0)$, where γ is implicitly defined as the unique solution of equation (3.2).

A similar argument deals with "downward" deviations, $\gamma_1 \leq \gamma$; it yields the same results.

Uniqueness of the solution for $p < \bar{p}$ follows from the fact that $\xi(p,\gamma)$ is strictly decreasing in γ and that $\gamma = 0 \Rightarrow \xi(p,\gamma) = p > 0$

and $\gamma = 1 \Rightarrow \xi(p,\gamma) = p - 1/2(1 - E[X]) < 0$. Monotonicity of $\gamma(p)$ follows easily.

Finally, we show that the unique solution of the condition

$$\left.\frac{\partial \pi_1}{\partial \gamma_1}\right|_{\gamma_1=\gamma} = \xi(p,\gamma)g(\gamma) = 0$$

is indeed a maximizer of the payoff of innovator 1 (assuming innovator 2 also plays the strategy γ). We prove this by showing that the function $\pi_1(\gamma_1,\gamma)$ is pseudoconcave in γ_1.[2] For this purpose, compute the cross derivative, using (3.4):

$$\frac{\partial^2}{\partial \gamma_1 \partial \gamma}\pi_1 = \frac{1}{4}\gamma g(\gamma_1)g(\gamma) \geq 0.$$

Together with the monotonicity of $\xi(p,\gamma)$ in γ, it follows that

$$\begin{aligned}
\gamma_1 < \gamma \Rightarrow \frac{\partial}{\partial \gamma_1}\pi_1(\gamma_1,\gamma) &\geq \frac{\partial}{\partial \gamma_1}\pi_1(\gamma_1,\gamma_1) = \xi(p,\gamma_1)g(\gamma_1) \\
&> \xi(p,\gamma)g(\gamma_1) = 0 \\
\gamma_1 > \gamma \Rightarrow \frac{\partial}{\partial \gamma_1}\pi_1(\gamma_1,\gamma) &\leq \frac{\partial}{\partial \gamma_1}\pi_1(\gamma_1,\gamma_1) = \xi(p,\gamma_1)g(\gamma_1) \\
&< \xi(p,\gamma)g(\gamma_1) = 0.
\end{aligned}$$

Therefore, $\pi_1(\gamma_1,\gamma)$ is increasing to the left of its stationary point (for $\gamma_1 < \gamma$) and decreasing to the right of its stationary point (for $\gamma_1 > \gamma$). Hence, the stationary point is a global maximum. □

[2] On pseudoconcavity see Avriel, Diewert, Schaible, and Zang (1988, p. 93 ff.).

3.3. Optimal fixed-prize tournament

In this section we show how one can compute the optimal simple fixed-prize tournament and illustrate it with an example.

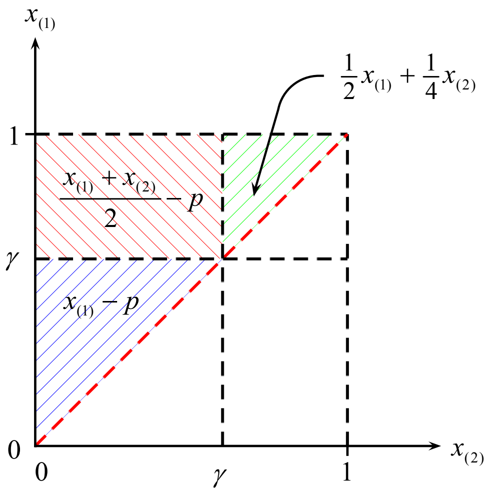

Figure 3.2.: Payoffs of the procurer in the order statistics space

For this purpose, we compute the procurer's payoff as a function of γ, eliminating the variable p. For this task, take a look at Figure 3.2, where the procurer's profits are represented in the order statistics space. The joint p.d.f. of $X_{(1)}, X_{(2)}$ is $g_{(1,2)}(x, y) = 2g(x)g(y)$. Therefore, one obtains, after a bit of rearranging, for all $\gamma \leq 1$ (resp.

$p \leq \bar{p}$):

$$\begin{aligned}\pi_p(\gamma) = 2&\int_0^\gamma \int_0^x (x-p)g(x)g(y)dydx \\ &+2\int_\gamma^1 \int_0^\gamma \left(\frac{x+y}{2} - p\right) g(y)g(x)dydx \\ &+2\int_\gamma^1 \int_\gamma^x \left(\frac{1}{2}x + \frac{1}{4}y\right) g(y)g(x)dydx.\end{aligned} \tag{3.7}$$

Whereas for $p \geq \bar{p}$ (resp. $\gamma = 1$) one has

$$\pi_p = E[X_{(1)}] - p. \tag{3.8}$$

The optimal fixed-prize tournament maximizes the procurer's expected profit over γ, resp. p, subject to the constraint that innovators' equilibrium expected payoff is nonnegative.

Example 3.1. *Suppose $G(x) \equiv x$ (uniform distribution) and $c = \frac{1}{15}$. Then, $\bar{p} = \frac{1}{2}\int_0^1 ydy = \frac{1}{4}$, innovators' equilibrium strategy is*

$$\gamma(p) = \begin{cases} 2\sqrt{p} & \text{if} \quad p < 1/4 \\ 1 & \text{if} \quad p \geq 1/4 \end{cases}$$

the procurer's payoff function, as a function of p, is $\pi_p(\gamma) = 5/12 + \gamma^2/4 - \gamma^3/2 + \gamma^4/4$, the optimal fixed-prize tournament without registration is

$$\gamma^* = \arg\max_\gamma \pi_p(\gamma) = 1/2, \quad \textit{resp.} \quad p^* = 1/16,$$

and innovators' equilibrium payoff is $\pi^* = {}^{15}\!/_{128} - c \geq 0$ *(see Figure 3.3).*

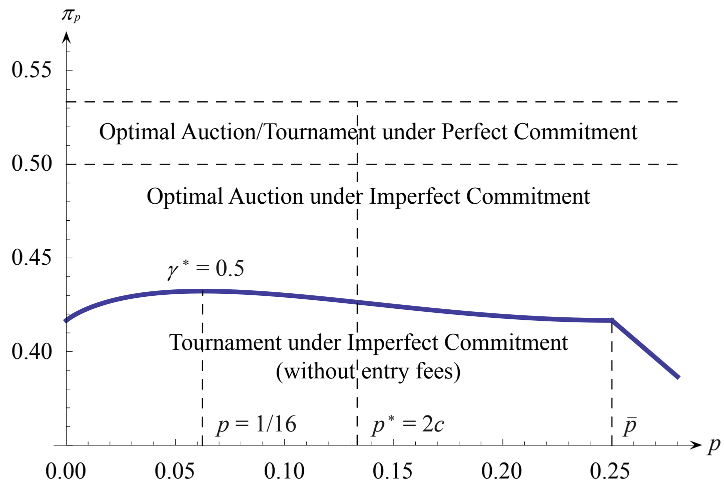

Figure 3.3.: Comparing the optimal auction with the optimal fixed-prize tournament (without entry fees), assuming a uniform distribution

This example for a uniform distribution (see Example 3.1) is illustrated in Figure 3.3. There, the solid curve plots the procurer's expected profit in the fixed-prize tournament, as a function of the prize p. It has a kink at $p = \bar{p} = {}^{1}\!/_{4}$ (which is the smallest prize that prevents bypass). The optimal prize is equal to $p = {}^{1}\!/_{16}$, which is substantially lower than the optimal prize under perfect commitment ($p^* = 2/15$), and the optimal γ is equal to $\gamma^* = 1/2$. Therefore, in the optimal fixed-prize tournament, all innovations $X > 1/2$ bypass the tournament. Evidently, the lack of perfect commitment hurts the procurer in the fixed-prize tournament as well as in the auction.

Not surprisingly, the optimal auction performs better in this example, since a uniform distribution does not satisfy the requirement that $\eta > 1$ for superiority of tournaments stated in Proposition 2.4 from the last chapter.

3.4. Why the auction cannot be improved by requiring a minimum score

It has been shown in the theory of auctions that an optimal auction usually involves *either* an entry fee *or* a reserve price, and there is no benefit of employing both. However, unlike in the standard optimal auction problem, in our analysis the entry fee is levied before potential bidders draw their value. Thus, one may think that the profitability of an auction can be improved by adding a reserve bid requirement, following an entry fee.

In a first step we assure that the minimum score does not affect the existence of an equilibrium in cutoff strategies.

To ensure that the auction has a symmetric equilibrium in cutoff strategies, we assume that $K := -G$ is star-shaped, which is weaker than concavity but stronger than subadditivity of G. The function K is star-shaped if for each $\alpha \in [0, 1]$, and all x: $K(\alpha x) \leq \alpha K(x)$ (see Bruckner and Ostrow, 1962). star-shapedness implies that $K(x)/x$ is increasing, resp. $G(x)/x$ is decreasing. This property is used in the proof below.

Suppose the procurer accepts only bids that match or exceed a stated minimum score, which is denoted by R. This changes the auction as follows: if exactly one bidder, say bidder 1, submits a score $S_1 = x_1 - b_1 \geq R$, that bidder wins the auction and is paid a price equal to $x_1 - R$ (instead of a price equal to $x_1 - x_2$); if no bidder submits a score equal to R or more, no trade occurs in the auction; and if both bidders submit a score $S \geq R$, the minimum score does not bind, and the auction proceeds as before. Of course, if a bidder does not submit a valid bid, he will try to engage in bargaining, after the auction.

In the presence of a minimum score requirement, bidders play cutoff strategies and bid if and only if the value of their innovation is equal or greater than a threshold value, which is denoted by r. We look for a symmetric equilibrium. In such an equilibrium, a bidder with value $x = r$ must be indifferent between submitting a score $S = R$ and not bidding, and bidding must be profitable for all $x > r$, and unprofitable for all $x < r$.

Due to symmetry, a bidder with value $x = r$ wins anything only if the rival's value is less than r. In other words, this bidder has zero gain as long as the rival submits (has a value higher than r). Therefore, indifference between bidding and not bidding for $x = r$ means that

$$G(r)(r - R) = \frac{1}{2}\int_0^r (r - \frac{1}{2}y)g(y)dy.$$

This implies the following unique and strictly increasing relationship between the minimum score R and the threshold value r

$$R = \frac{1}{2}r + \frac{1}{4}E[X \mid X \leq r], \tag{3.9}$$

which in turn allows us to eliminate the variable R, compute the procurer's expected profit as a function of r, and then maximize that payoff over r.

Next we prove that the procurer cannot increase his expected profit by adding a minimum score requirement.

Proof. Denote the difference between bidders' payoff when bidding and not bidding by Δ. Assume $x > r$. Then, using the relationship between R and r

$$\begin{aligned}
\Delta &= G(r)(x-R) + \int_r^x (x-y)g(y)dy - \frac{1}{2}\int_0^r (x-\frac{1}{2}y)g(y)dy \\
&\quad - \frac{1}{2}\int_r^x (x-y)g(y)dy \\
&= G(r)(x-R) + \frac{1}{2}\int_r^x (x-y)g(y)dy - \frac{1}{2}\int_0^r (x-\frac{1}{2}y)g(y)dy \\
&= G(r)x - G(r)r + \frac{1}{2}\int_0^r (r-\frac{1}{2}y)g(y)dy + \frac{1}{2}\int_r^x (x-y)g(y)dy \\
&\quad - \frac{1}{2}\int_0^r (x-\frac{1}{2}y)g(y)dy \\
&= G(r)(x-r) + \frac{1}{2}\int_0^r (r-x)g(y)dy + \frac{1}{2}\int_r^x (x-y)g(y)dy \\
&= G(r)(x-r) + \frac{1}{2}(r-x)G(r) + \frac{1}{2}\int_r^x (x-y)g(y)dy \\
&= \frac{1}{2}G(r)(x-r) + \frac{1}{2}\int_r^x (x-y)g(y)dy \\
&> 0.
\end{aligned}$$

Similarly, one obtains for $x \leq r$:[3]

$$\Delta = G(r)(x-R) - \frac{1}{2}\int_0^x (x-\frac{1}{2}y)g(y)dy$$

[3]Note, for some interval of x values below r one has nevertheless $x > R$.

$$
\begin{aligned}
&= G(r)x - G(r)r + \frac{1}{2}\int_0^r (r - \frac{1}{2}y)g(y)dy - \frac{1}{2}\int_0^x (x - \frac{1}{2}y)g(y)dy \\
&= G(r)(x-r) + \frac{1}{2}rG(r) - \frac{1}{2}xG(x) - \frac{1}{4}\int_x^r yg(y)dy \\
&\leq G(r)(x-r) + \frac{1}{2}rG(r) - \frac{1}{2}xG(x) - \frac{1}{4}\int_x^r xg(y)dy \\
&= G(r)(x-r) + \frac{1}{2}rG(r) - \frac{1}{2}xG(x) - \frac{1}{4}x(G(r) - G(x)) \\
&= \frac{3}{4}xG(r) - \frac{1}{4}xG(x) - \frac{1}{2}rG(r) \\
&\leq \frac{1}{2}xG(r) - \frac{1}{4}xG(x) - \frac{1}{2}rG(r) + \frac{1}{4}rG(x) \\
&= \left(G(r) - \frac{1}{2}G(x)\right)\left(\frac{1}{2}x - \frac{1}{2}r\right) < 0,
\end{aligned}
$$

where we used the fact that $-G$ is star-shaped and thus $\frac{G(x)}{x} > \frac{G(r)}{r}$.

The addition of a minimum score implies restrictions on the entry fee. For a given r the procurer sets the highest entry fee that ensures that both innovators register. Consider an innovator whose rival registers. Denote his payoff if he also registers by π^r and if he does not register by π^n. Then, the procurer sets the highest fee that ensures $\pi^r \geq \pi^n$ and $\pi^r \geq 0$.

After some rearranging and changing the order of integration one finds

$$\pi^r = \frac{1}{2}\int_0^r \int_0^x (x - \frac{1}{2}y)g(x)g(y)dydx$$

$$
\begin{aligned}
&+\int_r^1\int_r^x (x-y)g(x)g(y)dydx \\
&+\int_r^1\int_0^r (x-R)g(x)g(y)dydx - f - c \\
\pi^n = &\frac{1}{2}\int_0^r\int_y^1 (x-\frac{1}{2}y)g(x)g(y)dxdy \\
&+\frac{1}{2}\int_r^1\int_y^1 (x-y)g(x)g(y)dxdy - c.
\end{aligned}
$$

The highest entry fee, f^{**}, that ensures $\pi^r \geq \pi^n$ is

$$
\begin{aligned}
f^{**} := &\int_r^1\int_r^x (\frac{1}{2}x-\frac{1}{2}y)g(x)g(y)dydx \\
&+\int_r^1\int_0^r (\frac{1}{2}x-\frac{1}{4}y-R)g(x)g(y)dydx.
\end{aligned}
$$

And the highest entry fee, f^*, that ensures $\pi^r \geq 0$ is

$$
\begin{aligned}
f^* := &\frac{1}{2}\int_0^r\int_0^x (x-\frac{1}{2}y)g(x)g(y)dydx \\
&+\int_r^1\int_r^x (x-y)g(x)g(y)dydx \\
&+\int_r^1\int_0^r (x-R)g(x)g(y)dydx - c.
\end{aligned}
$$

Therefore, the optimal entry fee is $f = \min\{f^{**}, f^*\}$.

Finally, compute the procurer's expected profit, using the optimal registration fee and the relationship between R and r, writing π_p as a function of r. If $f^{**} \leq f^*$, one finds

$$\begin{aligned}
\pi_p &= 2\int_0^r \int_0^x (\frac{1}{2}x + \frac{1}{4}y)g(x)g(y)dydx + 2\int_r^1 \int_r^x yg(x)g(y)dydx \\
&\quad + 2\int_r^1 \int_0^r Rg(x)g(y)dydx + 2f^{**} \\
&= \int_0^1 \int_0^x (x + \frac{1}{2}y)g(x)g(y)dydx + \int_r^1 \int_r^x \frac{1}{2}yg(x)g(y)dydx,
\end{aligned}$$

which is decreasing in r and thus reaches the maximum at $r = 0$, associated with $R = 0$. Thus, in this case, the procurer cannot benefit from including a minimum score requirement.

Similarly, if $f^* \leq f^{**}$,

$$\begin{aligned}
\pi_p &= 2\int_0^r \int_0^x (\frac{1}{2}x + \frac{1}{4}y)g(x)g(y)dydx + 2\int_r^1 \int_r^x yg(x)g(y)dydx \\
&\quad + 2\int_r^1 \int_0^r Rg(x)g(y)dydx + 2f^* \\
&= \int_0^1 \int_0^x 2xg(x)g(y)dydx - 2c \\
&= E[X_{(1)}] - 2c.
\end{aligned}$$

Since $E[X_{(1)}] - 2c$ is the procurer's expected profit in the auction without minimum score, it follows also in this case that the procurer cannot benefit from a minimum score requirement. □

Part II.

Strategic Information Transmission in Sequential Auctions

4. Signal Jamming in a Sequence of First-Price Auctions[1]

4.1. Introduction

Many market transactions have the structure of an auction, and many such auctions are recurring events. For example, price competition between retailers is essentially a (reverse) auction. And this auction is typically a recurring event in which the relevant valuations (unit costs) are stable, at least for some time. Evidently, in these cases bidders must pay attention to the information they reveal about their valuations through their bids. This gives rise to a problem of strategic

[1] This chapter is based on Ding, Jeitschko, and Wolfstetter (2010).

information transmission.

The present chapter analyzes this problem in a stylized model of a recurring auction, where two bidders meet in a sequence of two first-price auctions. Bidders valuations are *iid* random variables (private-values model). Bidders draw their own valuation prior to the first auction, which is their private information. That valuation applies also to the second auction. Bidders want to win each auction, but they are also concerned with concealing their valuation in order to reduce the intensity of price competition.

If bidders were to play a strictly monotone increasing strategy in the first auction, which they would do if they played myopically and ignored how their early bid affects bidding in the later auction, they would reveal their private information, and the second auction would be one under complete information, resulting in fierce competition that wipes out profits. Bidders may thus attempt to keep their rival unsure about their valuation by sending an ambiguous, non-revealing signal. This may be achieved by mimicking the bidder with a low valuation, with positive probability.

In communication theory, the introduction of artificial noise into communication is called signal-jamming.[2] In this sense, a sequential

[2]In finance, the notion of signal-jamming has been used to describe the behavior

auction gives rise to signal-jamming if bidders keep their rivals "in the dark" about their type, with positive probability, until the last auction is played.

Similar partial pooling results (non-monotone bidding strategies) have been observed in other branches of the auctions literature. For example, Haile (2000) finds that the possibility of resale, triggered by more precise information acquired after the auction, induces pooling for a range of low signals at a bid equal to the reserve price.

The fact that an equilibrium in strictly monotone strategies may cease to exist has been noted elsewhere in the literature for similar settings. For example, Waehrer (1999) considers the sequential procurement of a primary and secondary good that are subject to closely related cost. The primary good is procured in an auction, and the the procurer uses the information revealed in the auction to negotiate the terms of procuring the secondary good. His main result is the impossibility result that the auction cannot have a strict monotone i.e., fully revealing equilibrium, and hence the auction cannot assure efficiency.

Kannan (2010) compares the impact of different information revela-

of investors with insider information who buy or sell securities in such a way that their information cannot be fully detected.

tion policies in a sequential auction on bidding behavior and social welfare. In particular, he compares the policy of publishing all bids vs publishing only the winning bid and shows by examples that either policy can be optimal for the procurer.

Münster (2009) adapts the sequential auction model to a repeated contest (which is essentially a repeated all-pay auction), and also identifies a partial pooling equilibrium. There, pooling also serves the purpose to blur the information revealed through bids to prevent that the procurer resp. other contestants take(s) advantage of it.

The present chapter can also be viewed as a follow-up to Jeitschko and Wolfstetter (2002). However, that paper assumes that bidders draw new valuations before each auction, which may however be stochastically dependent, due to stochastic scale effects; whereas the present chapter assumes that valuations remain stable across auctions. Both models are suitable to analyze different applications in economics. For example, assuming stable or perfectly correlated valuations is appropriate for analyzing wine, stamp, and real estate auctions where different lots of (almost) identical goods are usually auctioned within minutes. Whereas assuming imperfectly correlated valuations is compelling when there is long time lag between auctions, like in many licence auctions or procurements of military hardware.

After presenting the model (Section 4.2) and its equilibrium (Section 4.3), we consider signal-jamming in greater detail in Section 4.4. This is followed by an examination on how signal-jamming affects the distribution of prices across the two auctions (Section 4.5).

4.2. The model

Consider a sequence of first-price auctions for two identical objects, and two ex ante symmetric bidders, named 1 and 2. Bidders draw their valuation before the first auction and keep that valuation to the second.

Valuations V are *iid* random variables which assume either a low value 0 (normalized) or a high value $v > 0$, i.e., $V \in \{0, v\}$, with probability $\rho := \Pr\{V = v\} \in (0, 1)$.

The auction is sequential. Before the second object is auctioned, bidders observe both bids of the first auction, and use this information to update their beliefs concerning their rival's valuation.

If two bidders tie in the auction, the winner is selected by the flip of a fair coin, with one exception: If bidders tie in the second auction, the winner in the first auction is selected as the winner also for the second auction. In particular, if bidders tie in the second auction and exactly one bid was positive in the first auction, the one who made a positive bid in the first auction is declared as winner of the second.

Without this assumption the existence of equilibrium fails.[3] However, this nonexistence problem is artificial since existence can be restored by allowing positive but infinitesimally small bids. Therefore, it is appropriate to bypass this problem by a convenient tie rule, as proposed here.

We denote bidder i's bid in the j-th auction by b_i^j, and continuation payoffs by $\pi(h)$, where h denotes the history of the game prior to the second auction.

4.3. Equilibrium strategies

A bidder with valuation $V = 0$ obviously bids zero with certainty in both auctions (which will not be repeated from here onwards). This does, however, not imply that a zero bid can only come from a bidder with valuation $V = 0$. Indeed, in a signal-jamming equilibrium a

[3] The proof is in appendix 4.A.1.

high value bidder, with $V = v$, may also bid zero in the first auction in order to keep his rival in doubt about his valuation.

We now solve the equilibrium strategies of a bidder with valuation $V = v$ in both auctions, for all possible histories of the game.

The possible histories, h, of the game are described by the past bids which have been observed by both players. The following histories must be distinguished; there, only the sign of observed bids matters.

1. The history at the beginning of the game, $h_\emptyset$.
2. The histories with equal bids, either both zero or both positive: $h_{00} := \{b_1^1 = 0, b_2^1 = 0\}$, and $h_{11} := \{b_1^1 > 0, b_2^1 > 0\}$.
3. The histories with one positive bid and one bid equal to zero: $h_{10} := \{b_1^1 > 0, b_2^1 = 0\}$, and $h_{01} := \{b_1^1 = 0, b_2^1 > 0\}$.

We solve the game recursively, using the equilibrium concept of a sequential equilibrium with observable moves. Unless stated otherwise, we represent strategies as cumulative distribution functions.

As a working hypothesis, suppose $F : [0, \bar{b}] \to [0, 1]$ is the symmetric equilibrium mixed strategy of a bidder with $V = v$ in the first auction (history $h_\emptyset$), and suppose that he may also bid zero in the first auction with positive probability in order to keep his rival in doubt about his valuation, i.e., $F(0)$ may be positive. This allows us to characterize the equilibrium play in the second auction, and then confirm the working hypothesis concerning the equilibrium play in the first auction. Altogether, this procedure confirms that the game has a unique symmetric equilibrium.

To avoid unnecessary duplication we will state only the equilibrium strategies and beliefs of one player, named player 1.

4.3.1. Equilibrium in the second auction

After the first auction, bidders observe their bids, process this information to update their beliefs about the rival bidder's valuation, and then play the second auction. Updated beliefs must be consistent with the equilibrium strategy of the first auction and observed bids. Hence,

Lemma 4.1 (Consistent beliefs)**.** *Suppose player 1 observed the bid b_2^1. Consistency of his beliefs with the equilibrium strategy of the first auction requires:*

$$\Pr\{V_2 = v \mid b_2^1\} = \begin{cases} 1 & \text{if} \quad b_2^1 > 0 \\ \frac{F(0)\rho}{F(0)\rho + (1-\rho)} =: q & \text{if} \quad b_2^1 = 0. \end{cases} \tag{4.1}$$

Proof. Using Bayes' rule and the predicted strategy of bidder 2 one finds

$$
\begin{aligned}
q :&= \Pr\{V_2 = v | b_2^1 = 0\} \\
&= \frac{\Pr\{b_2^1 = 0 | V_2 = v\}\rho}{\Pr\{b_2^1 = 0 | V_2 = v\}\rho + \Pr\{b_2^1 = 0 | V_2 = 0\}(1-\rho)} \\
&= \frac{F(0)\rho}{F(0)\rho + (1-\rho)}
\end{aligned}
\tag{4.2}
$$

Using a similar procedure one can confirm that $\Pr\{V_2 = v \mid b_2^1 > 0\} = 1$. □

Note, this belief system involves only a fairly innocent prescription of "off-equilibrium path" beliefs by stipulating that $\Pr\{V_2 = v \mid b_2^1 > 0\} = 1$ also for bids that are higher than "predicted", i.e. for $b_2^1 > \bar{b}$.

Using these consistent beliefs, we now characterize the equilibrium strategies of the second auction, depending on the relevant history of the game.

Proposition 4.1. *Consider the second auction. The equilibrium strategy of player 1 (with $V_1 = v$) depends on the history as follows: $h_{11} \Rightarrow b_1^2 = v$, and*

$$
G : [0, qv] \to [0,1], G(b) = \frac{b(1-q)}{(v-b)q} = \frac{b(1-\rho)}{(v-b)\rho F(0)} \tag{4.3}
$$

$$\textit{if } h \in \{h_{00}, h_{01}\}$$

$$H : [0, qv] \to [0,1], H(b) = \frac{v(1-q)}{v-b} = \frac{v(1-\rho)}{(v-b)\,(1-\rho+\rho F(0))} \tag{4.4}$$

$$\textit{if } h = h_{10}.$$

H has one mass point at $b = 0$, $H(0) = {}^{(1-\rho)}/_{(1-\rho+\rho F(0))} = 1-q > 0$.

The associated equilibrium continuation payoffs are $\pi(h_{11}) = 0$ and $\pi(h_{00}) = \pi(h_{10}) = \pi(h_{01}) = \frac{v(1-\rho)}{F(0)\rho+(1-\rho)} = v(1-q)$.

Proof. The strategies and payoffs are self-evident for h_{11}, since both believe that the rival has high value with certainty and both will bid v in the second round. Whoever wins will have payoff 0.

A bidder with $V = v$ who observed history h_{00} must be indifferent between all bids from the support of his strategy, $[0, qv]$. If he bids $b = qv$, he wins for sure and earns a payoff equal to $v - qv$; whereas if he bids $b \in [0, qv)$ his payoff is equal to $(v-b)\,(qG(b) + (1-q))$, i.e. either in case the rival has high value, makes positive but lower bid, or in case the rival has low value and makes zero bid for sure. Therefore, one must have, for all $b \in [0, qv)$

$$(v-b)\,(qG(b) + (1-q)) = v(1-q), \tag{4.5}$$

which is obviously satisfied for the function G stated in (4.3).

The continuation payoff of that player is equal to the payoff he earns if he plays $b = qv$ (by the above indifference property), which is

$$\pi(h_{00}) = v(1-q) = \frac{v(1-\rho)}{F(0)\rho + (1-\rho)}. \tag{4.6}$$

Histories h_{01} and h_{10} result in an asymmetric auction in which it must be shown that the asserted equilibrium strategies keep both bidders indifferent between all bids from the support of their strategies, $[0, qv]$. Choose $b \in [0, qv)$ and $b' = qv$. Then, for history h_{01}, the indifference condition for bidder 2 (the bidder who had made a positive bid) is

$$(v-b)(qG(b) + (1-q)) = v(1-q). \tag{4.7}$$

Again, the left hand side is the expected payoff by bidding b, while the right hand side is the expected payoff by bidding 0. Our tie rule favors the bidder who made a positive bid, such that bidder 2 would win by bidding zero if bidder 1 makes a zero bid, which happens with probability $(1-q)$. Note that bidder 1's strategy is G that has no mass point. This implies bidder 1 who made a zero bid would bid zero again if and only if he has low value. Inserting G and q shows that condition is satisfied for all $b \in [0, qv)$.

Similarly, bidder 1 who had made a zero bid is indifferent between bidding b and bidding qv

$$(v-b)H(b) = v(1-q), \tag{4.8}$$

taking into account that bidder 1 believes that bidder 2 has high value for sure. Inserting q and H shows that this condition is satisfied for all $b \in [0, qv)$.

Since all bids from the support of the mixed strategies G, H are best replies to the rival's strategy, the equilibrium continuation payoffs are obtained by evaluating payoffs at $b = qv$, which gives, for both players,

$$\pi(h_{10}) = \pi(h_{01}) = v(1-q) = \frac{v(1-\rho)}{F(0)\rho + (1-\rho)}. \tag{4.9}$$

□

Figure 4.1 depicts the bid-distribution of high types in the second auction when there is no complete information revelation after the first auction/for the histories h_{10} resp. h_{01}.

Note, the mass point $H(0) > 0$ is due to the fact that in histories h_{01}, h_{10} our tie rule favors the bidder who made a positive bid in the first auction and lets him win with a zero bid in the second auction

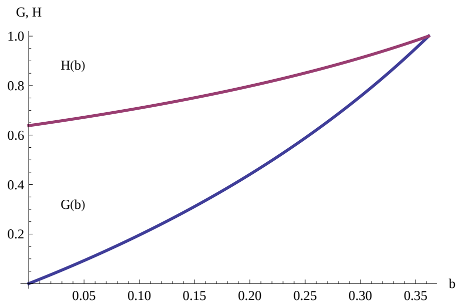

Figure 4.1.: Second-Auction Bid Distributions G and H ($v = 1$ and $\rho = .85$).

if both bidders bid zero in the second auction. This tie rule makes it profitable to bid zero with positive probability in history h_{10} resp. h_{01}. Also note that $H(0)$ is strictly decreasing in ρ since speculating on $V_2 = 0$ is obviously less attractive as ρ increases.

A bidder benefits from signal jamming in the first auction if his rival happens to have a high valuation. Signal jamming makes the rival change his belief from ρ to $q < \rho$. If the rival made a positive bid in the first auction, that change in his beliefs makes the rival bid less aggressively, $G(b) < H(b), \forall b \in [0, qv)$, to the benefit of the bidder who engaged in signal jamming. And if the rival also engaged in signal jamming, both bidders bid less aggressively and preserve a positive expected profit in the second auction, while profits would have been completely wiped out if they had both abstained from signal jamming.

4.3.2. Equilibrium in the first auction

In the first auction, a bidder with high valuation may wish to invest in signal jamming and keep his rival uninformed about his valuation. This pays off in the event when the rival has also a high valuation. Because if a bidder can influence his high value rival to believe with higher probability that he has a low valuation, the rival bidder will be induced to bid low, to the advantage of the high value bidder who concealed his valuation. However, signal jamming is costly, and its benefit outweighs the cost only if it is sufficiently likely to compete with high value bidder.

Proposition 4.2. *Suppose $\rho > 1/3$. The equilibrium strategy in the first auction (conditional on $V = v$) is $F : [0, \bar{b}] \to [0, 1]$:*

$$F(b) := \frac{b - b\rho + 2\rho v - 2v + v\sqrt{3 - 4\rho + \rho^2}}{(v - b)\rho} \tag{4.10}$$

$$\bar{b} := v\left(2 - \rho - \sqrt{3 - 4\rho + \rho^2}\right). \tag{4.11}$$

F has a mass point at zero: $F(0) = \sqrt{(1/\rho - 1)(3/\rho - 1)} - 2(1/\rho - 1) > 0$ which has maximum at $\rho = 3/4$ and approaches zero as $\rho \to 1$.

Proof. Consider one bidder, say bidder 1 with $V = v$, and history $h_\emptyset$ (first auction). To confirm the asserted equilibrium mixed strategy F, stated in (4.10), we must show that this bidder is indifferent between all bids from the support of F, which is $[0, \bar{b}]$ where $\bar{b}$ is stated in (4.11).

If bidder 1 with $V = v$ makes a bid $b \in (0, \bar{b}]$ his payoff is equal to

$$(v - b + \pi(h_{11}))\,\rho\,(F(b) - F(0)) \\ + (v - b + \pi(h_{10}))\,((1-\rho) + \rho F(0))\,. \tag{4.12}$$

The first summand is the overall payoff for the case that bidder 2 has high value, makes positive but lower bid. The second is the overall payoff for the case that bidder 2 makes a zero bid.

If bidder 1 bids zero, his payoff is

$$\left(\frac{v}{2} + \pi(h_{00})\right)(\rho F(0) + (1-\rho)) + \pi(h_{01})\rho(1 - F(0)). \tag{4.13}$$

By inserting F it follows immediately that (4.3.2) and (4.13) are identical for all $b \in [0, \bar{b}]$, which proves that F is the equilibrium strategy for history $h_\emptyset$ (first auction). □

Having derived the signal-jamming equilibrium that obtains for $\rho > 1/3$, we now briefly analyze the case when the event $V = v$ is not very likely ($\rho \leq 1/3$).

Proposition 4.3. *Suppose $\rho \leq 1/3$. The equilibrium strategies in the first auction are as follows: if $V = 0$, bid zero with certainty, and if $V = v$ play the "myopic" mixed strategy $K : [0, \rho v] \to [0, 1]$, which has no mass point*[4]

$$K(b) := \frac{1-\rho}{\rho} \frac{b}{v-b}. \tag{4.14}$$

(In the second auction, the bidder with $V = v$ bids v if he observed a positive first-auction bid from his rival, and otherwise bids 0.)

Proof. The high valued bidder's overall (i.e., two-period) equilibrium expected payoff is equal to $2v(1-\rho)$. Bidding outside of the support of the equilibrium strategy is clearly dominated. When placing positive mass on the lower end of the support, the first auction instantaneous payoff is positive only when winning a tie against the low-type rival, $\frac{1}{2}(1-\rho)v$, and the second auction payoff is bounded above by v. The overall payoff is thus bounded above by

[4] The "myopic" strategy K is the equilibrium of the associated one-shot-game. It is a strictly monotone (separating) strategy.

$\frac{1}{2}(1-\rho)v+v$. This bound is below the equilibrium expected payoff of $2v(1-\rho)$ whenever $\rho \leq \frac{1}{3}$. □

Remark 4.1. This equilibrium obtained for the case $\rho \leq 1/3$ is the unique symmetric Bayesian Nash equilibrium, but not a sequential equilibrium. In order to see this suppose one bidder, say bidder 1, has valuation $V_1 = v$ and observed history h_{00}. Consistency of beliefs with first round strategies implies that bidder should believe with probability one that $V_2 = 0$ and therefore that bidder 2 will bid zero. But then bidding zero is not the best response of bidder 1 (in fact, no best response exists in that case, since the payoff functions are not continuous).

4.4. Signal-jamming

Signal-jamming occurs if a bidder with a high valuation bids zero in the first auction with positive probability, $F(0) > 0$, and thus sometimes mimics a bidder with a low valuation in order to keep the rival uninformed.

Evidently, F has a mass-point at $b = 0$ if and only if $\rho \in (1/3, 1)$. Specifically, given the equilibrium strategy in Proposition 4.2, one obtains

$$F(0) = \sqrt{\left(\frac{1}{\rho}-1\right)\left(\frac{3}{\rho}-1\right)} - 2\left(\frac{1}{\rho}-1\right) > 0 \qquad \Longleftrightarrow \rho \in (1/3, 1). \tag{4.15}$$

This probability of signal jamming $F(0)$, as a function of ρ, is depicted in Figure 4.2.

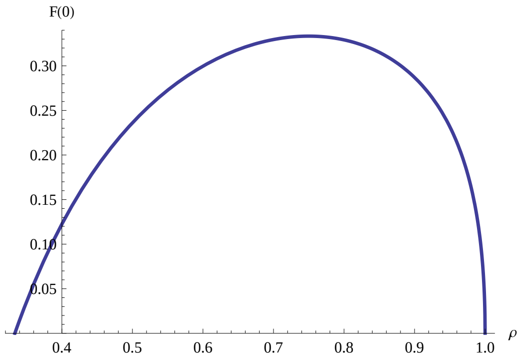

Figure 4.2.: Probability of Signal-Jamming, $F(0)$, as a function of ρ.

Altogether, bidder 1 benefits from signal jamming if and only if bidder 2 also has a high valuation. Signal jamming leads bidder 2 to update his belief from ρ to $\Pr\{V_1 = v \mid b_1^1 = 0\} = q < \rho$ instead of revealing bidder 1's type, which in turn induces him to bid stochastically lower, no matter how he bid in the first auction. If bidder 2 made a positive bid in the first auction, he plays the stochastically lower mixed strategy $H(b) > G(b), \forall b \in [0, qv]$, and if he also engaged in signal jamming, both bidders play the mixed strategy G which preserves a positive expected profit in the second auction.

However, signal jamming is also costly since it entails the risk of losing the first auction. It follows that it pays to "invest" in signal-jamming only if it is sufficiently likely that the rival has a high valuation. Interestingly, this relationship is not monotone, and $F(0)$ has a global maximum at $\rho = 3/4$.

To see the effect of signal-jamming, it is also useful to compare the equilibrium strategy in the first auction with that of a myopic bidder, who does not take into account that his bidding may affect the play of his rival in the second auction.

Proposition 4.4. *Signal-jamming induces pointwise less aggressive bidding in the first auction, in the sense that the myopic strategy K first-order stochastically dominates the strategy $\bar{F}$*

$$\bar{F}(b) \geq K(b), \quad \forall b \in [0, \rho v]. \tag{4.16}$$

Proof. This is due to the fact that $F(b) \geq K(b)$, $\forall \rho \in [0, 1]$, and

$\forall b \in [0, \bar{b}]$:

$$F(b) - K(b) = \frac{b - b\rho + 2\rho v - 2v + v\sqrt{3 - 4\rho + \rho^2}}{(v-b)\rho} - \frac{b(1-\rho)}{\rho(v-b)}$$
$$= \frac{v\sqrt{3-4\rho+\rho^2} - 2v(1-\rho)}{\rho(v-b)} \geq 0 \tag{4.17}$$

since the factor $\sqrt{3-4\rho+\rho^2} - 2(1-\rho)$ is always non-negative. And $\bar{F}$ is the continuously extended strategy for the enlarged domain $[0, \rho v]$. □

We now consider the effect of signal-jamming on bidding in the second auction. A useful benchmark is how bidders bid in auctions in which there is no signal jamming, *viz.* in which bidders used the myopic strategies in the first auction. One obtains,

Proposition 4.5. *Signal-jamming in the first auction leads to lower average bidding in the second auction when compared to the benchmark in which first-auction bidding is myopic.*

Proof. Under myopic bidding the bidder's types are fully revealed. In the second auction a bidder with a high value bids zero unless his rival was also revealed to be high, in which case he bids v. Therefore his expected payoff is $(1-\rho)v+\rho\cdot 0=(1-\rho)v$ (which is also the expected payoff when there is no learning). In contrast, in the equilibrium of the second auction after signal-jamming in the first auction, by Proposition 4.1 the expectation of a high-valued bidder's payoff is given by

$$
\begin{aligned}
E\left[\pi^2\right] &= F(0)\left[(1-\rho+\rho F(0))\pi(h_{00})+\rho(1-F(0))\pi(h_{01})\right] \\
&\quad + (1-F(0))\left[(1-\rho+\rho F(0))\pi(h_{10})+\rho(1-F(0))\pi(h_{11})\right] \\
&= \left[F(0)+(1-F(0))(1-\rho+\rho F(0))\right](1-q)v+\rho(1-F(0))\cdot 0 \\
&= \left[F(0)+(1-F(0))\frac{1-\rho}{1-q}\right](1-q)v \\
&= \left[F(0)(1-q)+(1-F(0))(1-\rho)\right]v. \qquad (4.18)
\end{aligned}
$$

Since $(1-q)$ is greater than $(1-\rho)$, the second-auction expected payoff in the equilibrium is higher compared to the benchmark myopic bidding. Since both auctions (both myopic bidding and the second round auction under signal-jamming) are efficient, high-valued bidder always gets the good. A higher expected payoff implies lower expected bids in the equilibrium when compared to myopic bidding. □

While the main purpose of signal-jamming in the first auction is to lessen the competition in the second auction, the intuition for the result is not actually directly tied to this. Indeed, since signal-jamming is anticipated in equilibrium, it cannot be effective: While signal-jamming has the effect of reducing the probability in which all bidder-surplus is wiped out when both types are revealed to be high, it also implies that bidders types are not fully revealed when they have low values, inducing their rivals to continue to place positive bids in instances where otherwise prices would be lower.

An immediate corollary to Propositions 4.4 and 4.5 is that because signal-jamming lowers bids in both the first and the second auction compared to a benchmark of myopic bidding, the auctioneer is strictly better off when auctioning the good off simultaneously. In sum,

Proposition 4.6. *Whenever* $\rho \in (1/3, 1)$ *an auctioneer strictly prefers the simultaneous sale of the goods compared to the sequential sale in which bids are depressed in both auctions due to signal-jamming.*

4.5. Dynamics of equilibrium prices

In some sequential auctions it has been observed that prices tend to decline from first to second auction (see, for example Ashenfelter, 1989, Ashenfelter and Genovese, 1992). Various authors have attempted to explain this "declining price anomaly" (see, for example, McAfee and Vincent, 1993, Gale and Hausch, 1994, Jeitschko and Wolfstetter, 2002). In the present context, one might expect that prices are stochastically increasing since signal jamming involves bidding low and signal jamming pays only in the first auction. However, as we now show, this occurs only if the prior probability ρ is larger than some threshold level that is greater than $1/3$.

In the following assume $\rho > 1/3$ and denote the continuously extended strategies for the enlarged domain $[0, v]$ by $\bar{G}(b) := \min\{G(b), 1\}$, $\bar{H}(b) := \min\{H(b), 1\}$. One can easily confirm that $\bar{G}(b) = G(b)$, $\bar{H}(b) = H(b)$ for all $b \in \left[0, \frac{F(0)\rho}{F(0)\rho+(1-\rho)} v\right] =: [0, \bar{\bar{b}}]$.

It is straightforward to compute the probability distributions of the equilibrium price in the first auction P^1:

Lemma 4.2. *Let $\rho > 1/3$. Then the probability distribution of the* *equilibrium price in the first auction,* $F_{P^1} : [0, v] \to [0, 1]$*, is*

$$\begin{aligned} F_{P^1}(p) :&= \Pr\{P^1 \leq p\} \\ &= \Pr\{\tilde{b}_1^1 \leq p \quad \textit{and} \quad \tilde{b}_2^1 \leq p\} \\ &= \begin{cases} (1-\rho+\rho F(p))^2 & \textit{if} \quad p \leq \bar{b} \\ 1 & \textit{if} \quad p \geq \bar{b}. \end{cases} \end{aligned} \tag{4.19}$$

F_{P^1} *has exactly one mass point,* $F_{P^1}(0) = \left((1-\rho) - \sqrt{3-4\rho+\rho^2}\right)^2 > 0$*, which is strictly decreasing in* ρ *with* $\lim_{\rho \to 1} F_{P^1}(0) = 0$*.*

Computing the probability distribution of the equilibrium price in the second auction P^2 is a bit more involved. We find:

Lemma 4.3. *Let* $\rho > 1/3$*. Then the probability distribution of the equilibrium price in the second auction,* $F_{P^2} : [0, v] \to [0, 1]$*, is for* $0 \leq p < v$*:*

$$\begin{aligned} &F_{P^2}(p) = \Pr\{P^2 \leq p\} \\ =&\rho^2 \left((1-F(0))^2 \cdot 0 + F(0)^2 \bar{G}(p)^2 + 2F(0)(1-F(0))\bar{G}(p)\bar{H}(p)\right) \\ &+ 2\rho(1-\rho)\left(F(0)\bar{G}(p) + (1-F(0))\bar{H}(p)\right) + (1-\rho)^2. \end{aligned} \tag{4.20}$$

F_{P^2} *has mass points at* $p = 0$ *and at* $p = v$*:*

$$F_{P^2}(0) = (1-\rho)\sqrt{3-4\rho+\rho^2} > 0 \tag{4.21}$$

$$\Pr\{P^2 = v\} = \left(2-\rho-\sqrt{3-4\rho+\rho^2}\right)^2 > 0, \tag{4.22}$$

$\Pr\{P^2 = v\}$ *is strictly increasing in* ρ*, and* $F_{P^2}(0)$ *is strictly decreasing with* $\lim_{\rho\to 1} F_{P^2}(0) = 0$.

To compute the probability distribution, consider the following events. Event 1, both bidders have high value which takes place with probability ρ^2. Both bid exactly v with certainty and thus the price is v in the second auction. This implies that for a given p that is strictly smaller that v, the probability that the price is lower or equal to p is zero. Other terms in this event are self-evident. Event 2, one bidder has high value and the other has low value. Recall that the bidder who made a zero bid in the first auction plays strategy G, whereas the bidder who made a positive bid in the first round plays strategy H. And one bidder (the one with low value) makes zero bid for sure. Event 3, both bidders have low value and both bid zero, which happens with probability $(1-\rho)^2$. The price is also zero.

We find:

Proposition 4.7. *Let* $\rho > 1/3$*. Then equilibrium prices are increasing in the sense of first-order stochastic dominance,* $F_{P^2}(p) \le F_{P^1}(p)$ *(with strict inequality except for* $p = v$*), if and only if the prior probability* ρ *is sufficiently large, i.e.* $\rho > \rho^* := 2 - 3/5\sqrt{5}$.

If $\rho \in (1/3, \rho^*)$, P^2 has more probability mass on low prices than P^1 in the lower price range, and more mass on high prices in the upper price range; therefore, prices are not stochastically increasing in the sense of first-order stochastic dominance. But the expected price in the second auction is higher than the expected price in the first one.

Proof. 1) Suppose $\rho > \rho^*$. Let $p \in [0, v)$ and define (with slight abuse of notation apply the functions G and H to the enlarged domain $[0, v)$):

$$\begin{aligned}&\tilde{F}_{P^2}(p) :\\ =&\rho^2 \left((1-F(0))^2 \cdot 0 + F(0)^2 G(p)^2 + 2F(0)(1-F(0))G(p)H(p)\right)\\ &\quad + 2\rho(1-\rho)\left(F(0)G(p) + (1-F(0))H(p)\right) + (1-\rho)^2.\end{aligned}$$

By definition, $G(p) \geq \bar{G}(p), H(p) \geq \bar{H}(p)$. Therefore, it follows immediately that $\tilde{F}_{P^2}(p)$ is a pointwise upper bound of $F_{P^2}(p)$, i.e., $\tilde{F}_{P^2}(p) \geq F_{P^2}(p)$, for all $p \in [0, v)$, and hence, in particular, for all $p \in [0, \bar{b}]$.

As one can easily confirm, $F_{P^1} > \tilde{F}_{P^2}(p), \forall p \in [0, \bar{b}] \iff \rho > \rho^*$. Since $F_{P^1}(p) = 1, \forall b \geq \bar{b}$ and $F_{P^2} < 1, \forall b < v$ (since it has a mass point at $p = v$), we conclude that $\rho > \rho^* \Rightarrow F_{P^2}(p) \leq F_{P^1}(p)$, with strict inequality everywhere except at $p = v$, as asserted.

2) Suppose $\rho < \rho^*$. Then, as one can easily confirm, $F_{P^2}(0) > F_{P^1}(0)$. Moreover, $F_{P^1}(p) = 1 > F_{P^2}(p), \forall p \in [\bar{b}, v)$ (since $F_{P^1}(\bar{b}) = 1$ and F_{P^2} has a mass point at $p = v$). Therefore, $F_{P^2}(p)$ and $F_{P^1}(p)$ must intersect at least once; hence no first-order stochastic dominance relationship applies to P^2 and P^1.

Despite a ranking in the sense of first-order stochastic dominance not being possible, the ranking of expected prices is unambiguous.

For expected price in the first auction, use has been made of Proposition 4.2:

$$\begin{aligned}
E[P^1] &= \int_0^v p dF_{P^1}(p) \\
&= \int_0^{\bar{b}} 2p[1-\rho+\rho F(p)]\rho \frac{dF(p)}{dp} dp \\
&= \int_0^{\bar{b}} -\frac{2pv^2\left(-1+\rho+\sqrt{3-4\rho+\rho^2}\right)^2}{(p-v)^3} dp \\
&= v\left(7+2\rho(\rho-4)+2(\rho-2)\sqrt{3-4\rho+\rho^2}\right)
\end{aligned} \tag{4.23}$$

Similarly, the expected price in the second auction auction (note that

F_{P^2} has a mass point at $p = v$), using equations 4.20 and 4.22, is:

$$
\begin{aligned}
E[P^2] &= \int_0^v p dF_{P^2}(p) \\
&= \int_0^{\bar{\bar{b}}} p \frac{dF_{P^2}(p)}{dp} dp + v \Pr\{P^2 = v\} \\
&= \int_0^{\bar{\bar{b}}} \frac{2pv^2(-1+\rho)^2 \left(-3+\rho+\sqrt{3-4\rho+\rho^2}\right)}{(p-v)^3 \left(-1+\rho+\sqrt{3-4\rho+\rho^2}\right)} dp \\
&\quad + v\left(2-\rho-\sqrt{3-4\rho+\rho^2}\right)^2 \\
&= \frac{v\left(14-5\rho^3+25\rho^2-34\rho-(8+5\rho^2-14\rho)\sqrt{3-4\rho+\rho^2}\right)}{-1+\rho+\sqrt{3-4\rho+\rho^2}}
\end{aligned}
\tag{4.24}
$$

The difference between the prices is, after some arrangements

$$
E[P^1] - E[P^2] = v\sqrt{3-4\rho+\rho^2}\left(-7+5\rho+4\sqrt{3-4\rho+\rho^2}\right), \tag{4.25}
$$

where the term in the parenthesis is non-positive for all $\rho \in [\frac{1}{3}, 1)$, zero for $\rho = \frac{1}{3}$ and strictly negative for $\rho \in ({}^1/{}_3, \rho^*)$. □

This result is illustrated in Fig 4.3 which plots the probability distributions of F_{P^1}, F_{P^2} for $\rho = {}^1/{}_2 < \rho^*$ (figure on the left), and for $\rho = {}^3/{}_4 > \rho^*$ (figure on the right).

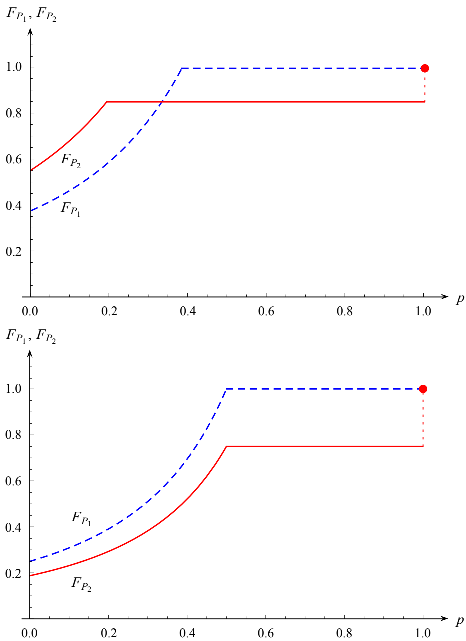

Figure 4.3.: Comparison F_{P^1} (dashed) and F_{P^2} (solid) for $v = 1$ and $\rho = 1/2 < \rho^*$ (left) resp. $\rho = 3/4 > \rho^*$ (right)

A ranking in the sense of first-order stochastic dominance is not possible (neither in the sense of second-order stochastic dominance) for the case of $\rho \in (1/3, \rho^*)$, because for smaller values of ρ (yet large enough to induce signal-jamming) the probability of both bidders having a high value and this being revealed in the auction becomes very small. As a result, a lot of mass is placed on low bids, yet, because the event of full information leakage (both bidders are revealed to have high values) nonetheless has positive probability the mass-point at the upper end of the value-support implies a crossing of the price distributions for low values of ρ.

4.6. Discussion

The information leakage that can take place between auctions leads bidders with high values to signal-jam in the first auction—thereby depressing prices. Moreover, in the signal-jamming equilibrium prices are also depressed compared to the myopic benchmark. An immediate implication of this strategic manipulation and use of information is that the auctioneer is harmed by signal-jamming in both auctions and would benefit from the prevention of such strategic information manipulation (e.g., by conducting a simultaneous auction). Similarly, the implication for markets in which firms compete in prices is that consumer surplus decreases due to signal-jamming. Since signal-jamming is directly tied to the information structure of the game, some policy implications readily follow: in procurement auctions and Bertrand competition it may be undesirable from a revenue/consumer surplus standpoint to increase the amount of information that is generated by the auction mechanism—as in trying to circumvent this, bidders become less aggressive; moreover, information may be better protected in order to circumvent such strategic manipulation of information, as this induces inefficiencies in the first auction, whenever there is a tie between a high and a low type and the low types wins. In this vein it is important to better understand other informational structures, such as bidding when only the winning bid is revealed, or when only the identity (but not the bid) of the winner is made public.

In the present chapter we assumed that bidders have stable valuations. An alternative framework would be to assume that valuations are subject to stochastic scale effects, as in Jeitschko and Wolfstetter (2002).

We also assumed that bidders observe all first auction bids before they bid in the second auction. If instead bidders could only learn whether they either won or lost the first auction, in some subgames bidders would know the rank order of valuations, as in Landsberger, Rubinstein, Wolfstetter, and Zamir (2001) and Février (2003).

Moreover, we assumed a passive auctioneer. Therefore, signal jamming served exclusively the purpose of misleading the rival bidder. The scope of signal jamming is further increased if the auctioneer employs reserve prices and is able to adjust them by taking advantage of information acquired during the first auction.

This angle of the signal jamming issue has been addressed in Caillaud and Mezzetti (2004) who consider a sequence of English clock auctions, and assume that the auctioneer employs reserve prices in each auction, but is unable to commit himself to a sequence of reserve prices prior to the auctions.

Unlike in our model, bidders have no interest in influencing other bidders' beliefs, since in an English clock auction equilibrium bidding is not affected by bidders' beliefs concerning each others' private values. However, the auctioneer's belief about bidders valuations affects his choice of optimal reserve price. Therefore, bidders take into account how their bids in the first auction may affect the auctioneer's belief, updated after observing the outcome of the first auction, and thus the reserve price in the second auction.

This link is briefly explained as follows.

Following Myerson (1981), the optimal selling method is to sell both objects in one bundle and to set the usual static optimal reserve price for that bundle, or equivalently, to sell one good each in a sequence of two auctions and set the same reserve price in both auctions, exactly equal to one half of the reserve price of the optimal bundle auction.

However, this optimal sequence of auctions is not time consistent. Since the auctioneer is free to adjust the reserve price after he observed the outcome of the first auction, he will always reset the reserve price in such a way that it reflects his updated beliefs about bidders' valuations. Of course, bidders anticipate that the auctioneer will use information revealed during the first auction to their disadvantage, and thus attempt to engage in some form of signal jamming.

The main result of Caillaud and Mezzetti (2004) is that in this framework signal jamming takes a simple form: Like in a static setting, bidders bid truthfully if they bid; however, some bidder types strategically refrain from participation in the first auction, i.e., they do not bid even though their valuation exceeds the reserve price. This strategic nonparticipation has the purpose to affect the beliefs of the auctioneer in such a way that he keeps the second auction reserve price lower. As Caillaud and Mezzetti (2004, p.78) put it: "Some buyers who would profitably buy at the reserve price refrain from participating in order to decrease the second auction reserve price."

4.A. Appendix

4.A.1. Some notes on the tie rule

Here we show that no equilibrium exists if one uses the tie rule to select the winner by flipping a fair coin.

Proof. Consider history h_{10}. Denote the second auction strategy of player 1 by $F_1^2 : [0, \bar{b}] \to [0, 1]$ and that of player 2 by $F_2^2 : [0, \bar{b}] \to [0, 1]$, both conditional on $V_i = v$. Using the tie rule of flipping a fair coin instead of tie rule assumed in the chapter (that in the event of a tie player 1 wins the second auction), bidding zero in the second auction is a strictly dominated strategy for bidders with $V_i = v$. This is because whenever player i wins by bidding zero (the case of a tie at 0 and wins expected $\frac{1}{2}v$), there is a profitable deviation by bidding a little bit ϵ more than zero, winning for sure and getting $v - \epsilon > \frac{1}{2}v$. Therefore, one has $F_1^2(0) = F_2^2(0) = 0$.

First, suppose there is no mass on $\bar{b}$. Then, in equilibrium, the indifference conditions require for all $b_1^2, b_2^2 \in [0, \bar{b}]$:

$$(1 - q + qF_2^2(b_1^2))(v - b_1^2) = v - \bar{b} \tag{4.26}$$

$$F_1^2(b_2^2)(v - b_2^2) = v - \bar{b}. \tag{4.27}$$

The left-hand side of these equations is player 1's (resp. player 2's) expected payoff when bidding $b_1^2 \in [0, \bar{b})$ (resp. $b_2^2 \in [0, \bar{b})$), and the right-hand side is his expected payoff by bidding $\bar{b}$. Evidently, these two equations cannot both hold for bids close to zero, which means in turn that no such equilibrium strategies F_1^2, F_2^2 exist.

Second, we show that there cannot be mass on $\bar{b}$. If there were mass on $\bar{b}$, the second indifference condition (4.27) changes to

$$\text{LHS} := F_1^2(b_2^2)(v - b_2^2) = (1 - m_1)(v - \bar{b}) + m_1 \frac{1}{2}(v - \bar{b}) =: \text{RHS}$$

where $m_1 > 0$ denotes the probability with which player 1 bids $\bar{b}$. Note that this holds for all $b_2^2 \in [0, \bar{b})$ and that

$$\lim_{b_2^2 \to \bar{b}} \text{LHS} = (1 - m_1)(v - \bar{b}) \neq (1 - \frac{1}{2}m_1)(v - \bar{b}) = \text{RHS}.$$

Therefore, this indifference condition does not hold for those bids close to $\bar{b}$. $\square$

Part III.

Signaling in Market Games with Downstream Interaction

5. Decentralized Union-Oligopoly Bargaining[1]

5.1. Introduction

The present chapter analyzes decentralized wage bargaining in a unionized oligopoly industry when firms are subject to incomplete information concerning their cost. The novel feature of the proposed model is that wages may signal firms' private information. This potential for signaling has a significant effect on the equilibrium wage profile negotiated by unions and firms.

When firms interact in a downstream oligopoly market, signaling strength confers a strategic advantage since the rival firm tends to

[1] This chapter is based on Ding (2010).

be a less aggressive player if he is made to believe that one's own cost is low. Of course, in equilibrium no misleading signaling occurs. Nevertheless, the potential for signaling shapes the equilibrium wage profile and, as we show, introduces an upward push on wages.

Following the bulk of the literature on wage bargaining, we assume that unions and firms negotiate wage rates but leave the firm free to choose employment. This assumption is commonly known as the "right-to-manage model" (Oswald, 1982). The alternative would be to assume complete contracts that stipulate both wages and employment as in Leontief (1946) and McDonald and Solow (1981). The latter has the advantage that the bargaining outcome is on the contract curve and thus assures efficiency. However, empirical evidence suggests that firms set employment unilaterally (see Hall and Lilien, 1979), which is why the labor literature usually prefers the "right-to-manage model".

The analysis is closely related to the literature on union-firm bargaining in an oligopoly industry, initiated by Davidson (1988), Horn and Wolinski (1988), Dowrick (1989). That literature generally assumes a framework of complete information, and focuses on a comparison between the outcome of collective bargaining when a single wage is bargained at the industry-level, which is typically the case in countries like Germany (see Haucap, Pauly, and Wey, 2007), and when wages are bargained at the firm-level which is typically the case in the U.S. The main finding of that literature is that bargaining at the firm level leads to lower wages if firms' products are substitutes, because if a firm agrees to a higher wage, its competitive position in the aftermarket is weakened, and the competitor takes advantage of it by raising his output and employment. Whereas industry wide bargaining internalizes that externality, and thus leads to higher equilibrium wages.[2]

Compared to this literature, the distinct feature of the present chapter is that we introduce incomplete information and allow for signaling. As we show, the potential for signaling exerts an upward pressure on the wages negotiated at the firm level which contributes to reverse the ranking of wages negotiated by unions at the firm level relative to the wages negotiated by an industry wide union.

A related paper by Vannetelbosch (1997) also covers wage-bargaining under incomplete information. However, he assumes that the bargaining parties are subject to incomplete information concerning

[2] This case of substitutes was covered by Davidson (1988); Horn and Wolinski (1988) showed that the reverse is true if products are complements.

their respective discount rates rather than with respect to their cost.[3] Another recent paper by Mukherjee and Suetrong (2010) considers the impact of the union structure on firms' foreign direct investment, and shows that decentralized unions may give rise to higher wages.

Methodologically, the present analysis is also related to the literature on auctions with externalities where bidders interact after the auction in an oligopoly game, and bids may reveal bidders' private information (Goeree, 2003, Das Varma, 2003, Ding, Fan, and Wolfstetter, 2010). The crucial difference between these and our contribution is that auctions give rise to a winner-takes-all situation, which involves considerably different issues and solution procedures.

The plan of the chapter is as follows: In Section 5.2 we state the model. Section 5.3 introduces two benchmark models that cover two possible interpretations of the model without signaling. In the first interpretation signaling is excluded because firms' cost parameters become common knowledge before firms play the oligopoly game (similar to the auctions with externalities analysis by Jehiel and Moldovanu (2000)), whereas in the second interpretation signaling is

[3] A strong point of Vannetelbosch (1997) is that he considers an alternating offer bargaining game in the spirit of Rubinstein (1982) whereas we employ the Nash-bargaining solution based on Nash (1950).

excluded because firms do not observe each others' wages and play a simultaneous moves game in the spirit of the complete contracts model of Leontief (1946) and McDonald and Solow (1981). In Section 5.4 we analyze our model in which signaling is possible and compare the equilibrium wage profile to those of the benchmark models. The chapter concludes in Section 5.5 with a discussion. Some technical proofs are in the Appendix.

5.2. The model

Consider a duopoly industry where firms engage in decentralized wage bargaining with union. Each firm draws its employees from a "large" pool of union members. Unions are firm specific, and they are able to require a "union shop" that employs only union members.[4]

Each firm and its union negotiates a wage rate, w, to be paid per worker (labor time is fixed), but allows firms to freely choose employment as long as they pay the stipulated wage. Therefore, the set-up

[4] In the U.S. the Taft-Hartley Act outlawed the "closed shop" in 1947, but permits the "union shop", except in those states that have passed right-to-work laws. In a "union shop" unions may require that those who are employed become members of the union. This is the case when the union is sufficiently strong.

is that of the so called “right-to-manage” model which is frequently applied in the labor literature (see, for example, Oswald, 1982).

After wages have been negotiated, they are publicly observed, and the two firms, named 1 and 2, play a Cournot market game with (perfect) substitutes. For simplicity, inverse market demand P is linear in outputs L_1, L_2

$$P(Q) := \max\{1-Q, 0\}, \quad Q := L_1 + L_2. \tag{5.1}$$

Firms use capital and labor with fixed input coefficient (Leontief-technology). Firms have the same labor input coefficient, normalized to 1, so that L_i stands for output as well as employment of firm i, whereas capital input coefficients, denoted by θ_i, may differ.

Each firm knows its own capital input coefficient but not that of the other. Firms view their rival’s capital input coefficients as *iid* random variable, drawn from the continuously differentiable c.d.f. $F : [0, \alpha] \rightarrow [0, 1]$ (private values assumption), with expected value $\bar{\theta} := \int_0^\alpha y dF(y)$.

Workers and firms are risk neutral and their default payoffs are taken to be equal to zero. Workers’ utility function is additively separable in income and employment.

The union represents its members. Without loss of generality, the number of union members is taken to be equal to 1. Members are drawn at random into employment. Therefore, L_i does not only measure employment and output, but also the probability that a union member is drawn into employment.

The union's payoff is equal to (the expectation is taken over the unknown cost parameter of the rival firm)[5]

$$U_u = \int_0^{\alpha} L_i\,(w_i - \delta)\,dF(\theta_j), \tag{5.2}$$

where δ represent the cost of foregone leisure and is exogenously given. Firms' expected profit is

$$U_f = \int_0^{\alpha} \left(1 - L_i - L_j - w_i - \theta_i\right) L_i dF(\theta_j). \tag{5.3}$$

And the payoff of union-firm coalitions is their Nash product

$$N := U_f U_u. \tag{5.4}$$

The game is as follows:

1. Firms simultaneously negotiate wage rates with their respective union, maximizing their Nash product N. There, strategies

[5] For simplicity of exposition, we ignore severance payments to unemployment members.

are wage schedules $(w_1(\theta), w_2(\theta))$ that prescribe wage rates contingent on each firm's cost parameter θ.

2. The negotiated wages are paid and publicly observed, and firms update their beliefs concerning the unknown cost parameter of their rival.

3. Firms play a simultaneous moves Cournot duopoly game, based on the observed wages and updated beliefs. There, firms maximize profits. Strategies are outputs resp. employment (L_1, L_2).

Throughout the chapter we assume for the two parameters α, δ that

$$\alpha < \frac{(1-\delta)}{2}. \tag{5.5}$$

This condition, which is exclusively used in Proposition 5.3, assures that, in a particular sense, no profile of opportunity costs of the two firm-union coalitions propels monopoly.[6]

[6] Suppose firm/union coalitions base their outputs decision on their respective opportunity costs. These opportunity costs are $(c_1, c_2) := (\theta_1 + \delta, \theta_2 + \delta)$. Then, the most favorable case for monopoly is the profile $\theta_1 = \alpha, \theta_2 = 0$, or *vice versa*. The above condition assures that even in that most favorable case for monopoly, both equilibrium outputs are positive.

5.3. Benchmark model(s) without signaling

As a benchmark we first consider two variations of our model that characterize the equilibrium wage profile when signaling is not possible. These models offer alternative interpretations of union-oligopoly bargaining in the absence of signaling.

In benchmark model A, the profiles of wages and cost parameters become common knowledge before firms play the oligopoly game. That model is a stage game in which firm-union coalitions simultaneously choose their wage profile, without knowing their rival's cost parameter, and firms then continue to play a duopoly game under complete information.

In benchmark model B, firms neither observe the wage profile nor the profile of unit costs when they play the oligopoly game. That model is a simultaneous moves game, where firm-union coalitions simultaneously choose their wage profiles and their output resp. employment, without knowing their rival's cost parameter.

5.3.1. Benchmark model A: Stage game without signaling

Suppose signaling is not possible because firms observe the profile of wages and cost parameters before they play the oligopoly game, similar to the analysis of license auctions with externalities by Jehiel and Moldovanu (2000)) where firms observe each others' private information before they play the oligopoly game.

In that interpretation, our game without signaling is a simple two-stage game. In the first stage the two firm-union coalitions simultaneously chose their wage schedules, $w(\cdot)$, as a function of own cost parameter θ_i, without knowing their rival's cost parameter θ_j. In the second stage, firms observe the profile of wages, w_1, w_2, and cost parameter, θ_1, θ_2, and then play a Cournot oligopoly game in output resp. employment strategies under complete information.

The equilibrium strategies of all conceivable oligopoly subgames in stage 2 are, of course,

$$L_i = \frac{1 - 2(w_i + \theta_i) + w_j + \theta_j}{3}, \quad i \neq j, i, j = 1, 2. \tag{5.6}$$

The associated equilibrium profits are equal to L_i^2.

We adopt the following methodology to solve the stage 1 wage bargaining game:

As a working hypothesis assume that the game has a symmetric equilibrium wage schedule, $w_A(\theta)$.

Consider one firm-union coalition, say coalition 1. Let this coalition adopt a wage w that may deviate from the equilibrium wage $w_A(\theta)$, while the rival firm-union coalition 2 plays the equilibrium strategy $w_A(\theta_2)$. Then, since $w_A(\theta)$ is an equilibrium strategy, the following conditions must hold.

$$w_A(\theta) = \arg\max_w N_A, \quad N_A := U_f U_u \tag{5.7}$$

$$U_f = E(L_A^2) := \int_0^\alpha L_A^2 dF(\theta_2) \tag{5.8}$$

$$U_u = E(L_A)(w-\delta) := (w-\delta)\int_0^\alpha L_A dF(\theta_2) \tag{5.9}$$

$$L_A = \frac{1-2(w+\theta)+w_A(\theta_2)+\theta_2}{3}. \tag{5.10}$$

Here, as elsewhere, the expectation is taken over the random variable θ_2.

Evidently, the equilibrium strategy $w_A(\theta)$ must satisfy the condition that $\partial_w N_A|_{w=w_A(\theta)} = 0, \forall\theta$, where

$$\begin{aligned}\partial_w N_A &= -\frac{4}{3}E(L_A)^2(w-\delta) + E(L_A^2)\left(E(L_A) - \frac{2}{3}(w-\delta)\right)\\ &= -\frac{2}{3}(w-\delta)\left(2E(L_A)^2 + E(L_A^2)\right) + E(L_A^2)E(L_A)\end{aligned} \tag{5.11}$$

$$= (w-\delta)E(L_A^2)E(L_A)\left(\frac{1}{w-\delta} - \frac{2}{3}\left(\frac{2E(L_A)}{E(L_A^2)} + \frac{1}{E(L_A)}\right)\right). \tag{5.12}$$

Since L_A is decreasing in the own wage w, by (5.10), it follows immediately from (5.11) that $\partial_{ww} N_A < 0$. Therefore, N_A is strictly concave in w and the wage schedule w_A is a mutual best reply.

We shall use these facts concerning $w_A(\theta)$ in Proposition 5.2 below, where we delineate how the possibility to signal through wages affects the equilibrium wage schedule.

5.3.2. Benchmark model B: Simultaneous moves game without signaling

Alternatively, suppose signaling is not possible because firms neither observe the profile of cost parameters nor the profile of wages before they play the oligopoly game.

In that interpretation, our game without signaling is a simultaneous moves game under incomplete information without subgames. There, firm-union coalitions simultaneously choose their wage profile and output resp. employment without knowing their rival's cost parameter. Essentially, this benchmark model is the complete contract model in the spirit of Leontief (1946) and McDonald and Solow

(1981); the only difference is that we analyze it in a framework of incomplete information and embed it into an oligopoly framework.

As a working hypothesis we stipulate that this game has a symmetric and strictly monotone equilibrium, described by the output resp. employment schedule $L_B(\theta)$ and the wage schedule $w_B(\theta)$, which will be confirmed below.

The symmetric equilibrium, $(L_B(\theta), w_B(\theta))$, solves the following requirements, for all θ:

$$\begin{aligned} L_B(\theta) = \arg\max_{\{\tilde{L}(\cdot)\}} \int_0^\alpha \int_0^\alpha & \Big(1 - \tilde{L}(\theta) - L_B(y) - w_B(\theta) - \theta\Big) \\ & \tilde{L}(\theta) dF(y) dF(\theta) \end{aligned} \tag{5.13}$$

$$\begin{aligned} w_B(\theta) = \arg\max_{\{\tilde{w}(\cdot)\}} \int_0^\alpha \int_0^\alpha & (1 - L_B(\theta) - L_B(y) - \tilde{w}(\theta) - \theta) \\ & L_B(\theta) dF(y) \left(\int_0^\alpha L_B(\theta) (\tilde{w}(\theta) - \delta) dF(y) \right) dF(\theta). \end{aligned} \tag{5.14}$$

The Euler equations of these variational problems, combined with the equilibrium requirements that the best-response $\tilde{L}(\theta)$ shall be equal to $L_B(\theta)$ and the best-response to $\tilde{w}(\theta)$ be equal to $w_B(\theta)$, are

$$1 - 2L_B(\theta) - w_B(\theta) - \theta = \int_0^\alpha L_B(y) dF(y) \tag{5.15}$$

$$1 + \delta - L_B(\theta) - 2w_B(\theta) - \theta = \int_0^\alpha L_B(y) dF(y). \tag{5.16}$$

Solving these Euler equations gives the unique equilibrium (recall $\bar{\theta} := E(\theta)$):

$$w_B(\theta) = \frac{1}{4}\left(1 + 3\delta + \frac{1}{3}\bar{\theta}\right) - \frac{1}{3}\theta \tag{5.17}$$

$$L_B(\theta) = \frac{1}{4}\left(1 - \delta + \frac{1}{3}\bar{\theta}\right) - \frac{1}{3}\theta. \tag{5.18}$$

Evidently, both w_B and L_B are strictly monoton decreasing in the own cost parameter, which confirms the assumed working hypothesis.

5.4. How signaling affects the equilibrium wage profile

Now assume that firms only observe the wage profile but not each others cost parameters before they play the oligopoly game. In that case, each firm-union coalition may use the wage as a signal of its cost. Specifically, if the equilibrium wage schedule is strictly monotone, firms can infer the cost parameter θ that underlies the observed wage. This gives rise to a signaling issue that affects the equilibrium schedule.

In particular, each firm-union coalition may use the bargained wage to signal strength to their rival, by making the rival believe that one's cost parameter θ is lower than it happens to be. As in other oligopoly contexts, signaling strength confers a strategic advantage in the oligopoly game.

Of course, in equilibrium no such "misleading" signaling occurs. However, this requires that the wage schedule is modified in such a way that the benefit of signaling strength is exactly matched by an equally high wage cost, at all possible values of the cost parameter θ.

In the following we characterize the resulting equilibrium wage schedule. The main purpose of the analysis is to find out how the possibility to signal one's cost parameter affects the equilibrium wage schedule.

The following methodology is employed to characterize the equilibrium wage schedules: As a working hypothesis we stipulate that the wage-bargaining games have a symmetric and strictly monotone equilibrium wage schedule $w(\theta)$, either strictly increasing or decreasing. Each firm-union coalition can then perfectly infer the cost parameter of its rival firm-union coalition from the observed wage.

We consider one firm-union coalition, say coalition 1, and without loss of generality, restrict attention to those unilateral deviations from equilibrium play, where coalition 1 applies the wage $w(z)$, as if its cost parameter where equal to $z \in [0, \alpha]$ rather than θ_1, while coalition 2 plays the equilibrium strategy w. By construction, both players believe that their rival plays equilibrium, and therefore both take the signal revealed as the true signal. This implies that if $z \neq \theta_1$, firm 2 is mislead to believe that the cost parameter of coalition 1 is z, after observing $w(z)$.

We solve the payoff function of coalition 1, denoted by $N(z, \theta)$, by solving the duopoly subgames that may occur if coalition 1 unilaterally deviates from the equilibrium wage schedule, and then invoke the equilibrium requirement that no deviation from the equilibrium wage schedule w shall pay. Of course, in the end, we confirm the strict monotonicity of the equilibrium wage schedule.

5.4.1. Solution of the duopoly subgames

Each duopoly subgame Γ is fully characterized by the perceived profile of unit costs (c_1, c_2). However, the profile of unit costs perceived by firm 2 may differ from the profile perceived by firm 1. This occurs when firm-union coalition 1 unilaterally deviates from the equilibrium wage profile and thus induces “wrong” beliefs in firm 2.

Suppose coalition 1 unilaterally deviates from the equilibrium wage schedule, $w(\theta_1)$ and sets a wage $w(z)$ rather than $w(\theta_1)$. Then, firm 2 is mislead to believe that the profile of unit costs is equal to $(c_1, c_2) = (w(z)+z, w(\theta_2)+\theta_2)$. In that case, firm 1 understands that firm 2 believes to play the duopoly subgame $\Gamma(w(z)+z, w(\theta_2)+\theta_2)$. And firm 1 predicts that firm 2 plays the equilibrium strategy:

$$L_2 = \frac{1-2c_2+c_1}{3} = \frac{1-2(w(\theta_2)+\theta_2)+(w(z)+z)}{3} \tag{5.19}$$

At the same time, firm 1 privately knows that its true unit cost is equal to $c_1 = w(\theta_1)+\theta_1$. Therefore, firm 1 plays its best reply to L_2, which gives (for ease of notation we suppress the subscript 1 whenever we refer to firm 1):

$$L = \arg\max_{L_1} (1-L_1-L_2-w(z)-\theta_1)\, L_1, \tag{5.20}$$

which is

$$L = \frac{1}{3}\left(1-\left(\frac{3}{2}\theta_1+\frac{1}{2}z+2w(z)\right)+(w(\theta_2)+\theta_2)\right). \tag{5.21}$$

The associated equilibrium profit of firm 1 is equal to $\pi = L^2$.

Therefore, for all (z, θ_1, θ_2), the solution of the subsequent duopoly subgame is (L, L_2).

5.4.2. Equilibrium wage bargaining

Using the above equilibria of all possible oligopoly subgames, we now characterize the equilibrium wage profile, and analyze the wage bargaining problems solved by the two firm-union coalitions. The wage bargaining game is a simultaneous moves game, where firm-union coalitions simultaneously choose their wage profiles, maximizing its Nash product, N, (here and elsewhere expected values are taken over the random variable $\tilde{\theta}_2$ and the number of union members normalized to be equal to 1):

$$N := U_f U_u \tag{5.22}$$

$$U_f = E(L^2) \quad \text{(payoff of firm 1)} \tag{5.23}$$

$$U_u = E(L)(w(z) - \delta) \quad \text{(payoff of union 1).} \tag{5.24}$$

There, employment levels are determined by the equilibrium of the continuation game, characterized in the previous section.

The equilibrium wage schedule is that function w that satisfies the following equilibrium requirement

$$w(\theta) = \arg \max_{z \in (0,\alpha)} N. \tag{5.25}$$

Since the maximizer of N is the same as that of every concave transformation of N, for convenience we now consider maximization of $\log N$ in lieu of maximization of N.

Take the partial derivative of $\log N$ w.r.t. z, the first order condition requires

$$\partial_z \log N = \frac{w'(z)}{w(z)-\delta} + \frac{\partial_z E(L^2)}{E(L^2)} + \frac{\partial_z E(L)}{E(L)}, \tag{5.26}$$

where

$$\partial_z E(L) = -\frac{1}{6}\left(1+4w'(z)\right) \tag{5.27}$$

$$\partial_z E(L^2) = 2E(L\partial_z L) = -\frac{1}{3}\left(1+4w'(z)\right)E(L). \tag{5.28}$$

Therefore, the equilibrium requirement $\partial_z \log N|_{z=\theta} = 0$ can be written as:

$$\frac{w'(\theta)}{w(\theta)-\delta} - \frac{(1+4w'(\theta))E(L)}{3E(L^2)} - \frac{1+4w'(\theta)}{6E(L)} = 0. \tag{5.29}$$

If $w(\theta)$ is strictly monotone decreasing, it follows immediately that $w'(\theta) < -\frac{1}{4}$ for all θ.

Unfortunately, the equilibrium condition (5.29) is not a regular differential equation, since variables cannot be separated. Hence, $w(\theta)$ cannot be explicitly solved. However, some properties of the solution can be characterized.

The condition (5.29) is of course only necessary for an equilibrium. In addition, the equilibrium strategy $w(\theta)$ needs to satisfy second-order conditions for global maxima. In the following we use the second order conditions to show that the equilibrium wage schedule cannot be monotone increasing.

Lemma 5.1. *$\partial_{z\theta} \log N \geq 0$, for all z, θ, if $w'(\theta) < 0$ everywhere; and $\partial_{z\theta} \log N \leq 0$, for all z, θ, if $w'(\theta) > 0$ everywhere.*

Proof. Note first that $\frac{\partial E(L)}{\partial \theta} = -\frac{1}{2}$ and $\frac{\partial E(L^2)}{\partial \theta} = -E(L)$. Differentiating (5.26) with respect to θ gives

$$\begin{aligned}
\partial_{z\theta} \log N &= \frac{\partial_\theta \partial_z E(L^2) \cdot E(L^2) - \partial_z E(L^2) \cdot \partial_\theta E(L^2)}{(E(L^2)^2)} \\
&\quad + \frac{\partial_\theta \partial_z E(L) \cdot E(L) - \partial_z E(L) \cdot \partial_\theta E(L)}{E(L)^2} \\
&= \frac{-\frac{1}{3}(1+4w'(z))(-\frac{1}{2})E(L^2) + \frac{1}{3}(1+4w'(z))E(L) \cdot (-E(L))}{(E(L^2)^2)} \\
&\quad + \frac{\frac{1}{6}(1+4w'(z))(-\frac{1}{2})}{E(L)^2} \\
&= -\frac{1}{12}(1+4w'(z)) \left(\frac{-2}{E(L^2)} + \frac{4E(L)^2}{(E(L^2))^2} + \frac{1}{E(L)^2} \right) \\
&= -\frac{1}{12}(1+4w'(z)) \left(\left(\frac{2E(L)}{E(L^2)} - \frac{1}{E(L)} \right)^2 + \frac{2}{E(L^2)} \right)
\end{aligned}$$

We have already shown that $w' < -\frac{1}{4}$ if $w' < 0$. Hence, it follows immediately that $\partial_{z\theta} \log N$ has the opposite sign as w', as asserted.

□

Lemma 5.2. *If* $\partial_{z\theta} \log N \geq 0$*, for all* z, θ*, then the solution of first-order conditions* (5.29) *yields a global maximum of* $\log N$*; and if* $\partial_{z\theta} \log N \leq 0$*, the solution of first-order conditions* (5.29) *yields a global minimum of* $\log N$.

Proof. Suppose $\partial_{z\theta} \log N \geq 0$, for all z, θ. Then for a given θ with $\partial_z \log N(z,\theta)|_{z=\theta} = 0$ we have

$$z < \theta \Rightarrow \partial_z \log N(z,\theta) \geq \partial_z \log N(z,z) = 0 \tag{5.30}$$

$$z > \theta \Rightarrow \partial_z \log N(z,\theta) \leq \partial_z \log N(z,z) = 0. \tag{5.31}$$

Therefore, $\log N$ is increasing in z to the left and decreasing to the right of its stationary point θ, which implies that the stationary point is a global maximum of $\log N$.

Similarly, if $\partial_{z\theta} \log N \leq 0$, for all z, θ, it follows that $\log N$ is decreasing in z to the left and increasing to the right of its stationary point, which implies that the stationary point is a global minimum of $\log N$. □

Proposition 5.1. *In a separating equilibrium the wage schedule $w(\theta)$ is strictly monotone decreasing. In particular, $w'(\theta) < -1/4$.*

Proof. In a separating equilibrium, one has either $w'(\theta) > 0$, or $w'(\theta) < 0$ for all θ. We prove that $w'(\theta) > 0$ cannot occur in equilibrium. The proof is by contradiction.

Suppose $w'(\theta) > 0$ for all θ. Then by Lemma 5.1 one has $\partial_{z\theta} \log N \leq 0$ for all z, θ. By lemma 5.2 this implies that $\log N$ has a global minimum at $z = \theta$. A global minimum is also a local minimum. Therefore, $\log N$ is convex in z at $z = \theta$, i.e. $\partial_{zz} \log N|_{z=\theta} \geq 0$. Since

$$\partial_{zz} \log N = \frac{\partial_{zz} N \cdot N - (\partial_z N)^2}{N^2}, \tag{5.32}$$

the local convexity of $\log N$ implies $\partial_{zz} N > 0$.

Therefore, N has a local minimum at $z = \theta$, for all θ, and hence N cannot has a global maximum at $z = \theta$, which contradicts that w is an equilibrium strategy.

As we have already observed above, if $w'(\theta) < 0$, (5.29) implies that $w'(\theta) < -\frac{1}{4}$ for all θ. □

Proposition 5.2. *The potential for signaling gives rise to a pointwise higher equilibrium wage schedule compared to the benchmark* *model A, i.e.,* $w(\theta) > w_A(\theta)$.

Proof. We prove this by showing that the partial derivative of the Nash product in benchmark model A, $\partial_w N_A$, evaluated at $w = w(\theta)$ and $w_2(\theta_2) = w(\theta_2)$, is negative for all θ. This indicates that the function N_A is declining at $w(\theta)$; in other words, since N_A has been shown to be strictly concave in the own wage, $w(\theta)$ must be larger than the maximizer of N_A, which is w_A, as asserted.

By the equlibrium condition (5.29) of the game with signaling one has

$$\frac{1}{w(\theta)-\delta} = \frac{1+4w'(\theta)}{6w'(\theta)}\left(\frac{1}{E(L)}+\frac{2E(L)}{E(L^2)}\right). \tag{5.33}$$

Therefore, by (5.12) it follows that at $w = w(\theta)$, $w_2(\theta_2) = w(\theta_2)$ and hence $L_A = L$,

$$\partial_w N_A = (w(\theta)-\delta)E(L^2)E(L)\left(\frac{1}{w(\theta)-\delta} - \frac{2}{3}\left(\frac{2E(L)}{E(L^2)}+\frac{1}{E(L)}\right)\right)$$
$$= (w(\theta)-\delta)E(L^2)E(L)\left(\frac{1+4w'(\theta)}{6w'(\theta)} - \frac{2}{3}\right)\left(\frac{2E(L)}{E(L^2)}+\frac{1}{E(L)}\right)$$
$$< 0,$$

where the last inequality is due to the fact that $w'(\theta) < 0$ (in particular $w'(\theta) < -1/4$), by Proposition 5.1. □

Finally, we compare the equilibrium wage schedule $w(\theta)$ to the wage profile in the alternative benchmark model B, where the absence of signaling occurs because firms neither observe the profile of cost parameters nor the profile of wages before they play the oligopoly game. For that purpose we assume specifically that the expected value of the cost parameter is "not too small": $\bar{\theta} := E(\theta) \geq \frac{2}{11}\alpha$.[7]

Proposition 5.3. *Suppose $\bar{\theta} \geq \frac{2}{11}\alpha$. The potential for signaling gives rise to a pointwise higher equilibrium wage schedule also compared to the benchmark model B, i.e., $w(\theta) > w_B(\theta)$.*

Proof. We show that $\partial_z N$, evaluated at $w(\theta) = w_B(\theta)$ and $z = \theta$, is negative at all θ. This shows that the wage schedule $w_B(\theta)$ induces signaling strength by playing as if the cost parameter were lower than the true θ. To remove that incentive to signal strength, the wage schedule must be shifted upwards for a range of cost parameters below θ in the neighborhood of θ. Since this applies to all θ, it

[7] This excludes probability distribution that exhibit a high concentration on low values. It is satisfied for most typically employed distributions (including the uniform distribution for which $\bar{\theta} = \alpha/2$), and it holds for the family of truncations of F, $G(\theta) : [d, \alpha] \to [0, 1]$, $G_d(\theta) := (F(\theta)-F(d))/(\alpha-d)$, provided $0 < d < \alpha$ is sufficiently large.

follows that the equilibrium wage schedule must be pointwise higher than $w_B(\theta)$ everywhere.

The remainder of the proof is in Appendix 5.A.1. $\square$

The intuition for these results is as follows: Consider one firm-union coalition and suppose its rival plays the wage strategy that is an equilibrium in the model without signaling. Then, the coalition benefits from signaling strength by setting an inflated wage that mislead the rival that its capital input coefficient is lower than it is. Of course, in equilibrium no misleading signaling can occur. Therefore, in order to establish an equilibrium, the wage schedule must be adjusted in such a way that signaling strength becomes sufficiently costly. In other words, the potential for signaling exerts an upward pressure on equilibrium wage schedules, even though in equilibrium no misleading signals are observed.

5.5. Discussion

The present chapter considered decentralized union-firm wage bargaining, assuming firms interact in an oligopoly market, and assuming firms are subject to incomplete information concerning their unit cost. The novel feature of the analysis is that wage bargaining involves a signaling problem. There, firms may have an incentive to inflate their wage in order to signal strength, with the intention to gain a strategic advantage in the subsequent oligopoly game.

While no misleading signaling occurs in equilibrium, the potential for signaling exerts an upward pressure on wages that counterbalances the externality that is generally seen as weakening the bargaining power in decentralized wage bargaining.

In further research one might wish to extend the analysis to also cover the case of centralized wage bargaining by an industry wide union in order to assess whether the signaling effect may be so strong as to reverse the rank order of wages in centralized and decentralized bargaining. However, this involves a complex multilateral bargaining problem under incomplete information. One may also wish to consider other specifications of the market game where firms' products are complements or where firms play a Bertrand rather than a Cournot market game.

5.A. Appendix

5.A.1. Supplement to the proof of Proposition 5.3

Here we show that, evaluated at $z = \theta$, $w = w_B$, one has $\partial_z \log N < 0$, and thus $\partial_z N < 0$, for allθ, assuming that $\bar{\theta} \geq \frac{2}{11}\alpha$.

Observe that, evaluated at $z = \theta$, $w = w_B$,

$$\begin{aligned}
\partial_z E(L) &= \frac{1}{6}\int\left(-1+\frac{4}{3}\right)dF(\theta_2) = \frac{1}{18} \\
\partial_z E(L^2) &= \int 2L\partial_z L dF(\theta_2) = \frac{1}{9}E(L) \\
E(L) &= \frac{1}{36}\big(9(1-\delta)+7\bar{\theta}-16\theta_1\big)
\end{aligned}$$

Therefore, evaluated at $z = \theta$, $w = w_B$,

$$\begin{aligned}
2E(L)-(w_B-\delta) &= \frac{1}{36}\big(9(1-\delta)+11\bar{\theta}-20\theta_1\big) \\
&> \frac{1}{36}\big(18\alpha+11\bar{\theta}-20\theta_1\big) \quad \text{(since} \quad \alpha < {}^{1-\delta}/2) \\
&\geq \frac{1}{36}\big(18\alpha+11\bar{\theta}-20\alpha\big) \quad \text{(since} \quad \theta_1 \leq \alpha) \\
&= \frac{1}{36}\big(11\bar{\theta}-2\alpha\big) \\
&\geq 0 \quad \text{(since } \bar{\theta} \geq \frac{2}{11}\alpha).
\end{aligned}$$

Hence, evaluated at $z = \theta, w = w_B$,

$$\begin{aligned}
\partial_z \log N &= \frac{w'(\theta)}{w(\theta)-\delta} + \frac{\partial_z E(L^2)}{E(L^2)} + \frac{\partial_z E(L)}{E(L)} \\
&= -\frac{1}{3(w_B(\theta)-\delta)} + \frac{E(L)}{9E(L^2)} + \frac{1}{18E(L)} \\
&< -\frac{1}{3(w_B(\theta)-\delta)} + \frac{E(L)}{9E(L)^2} + \frac{1}{18E(L)} \\
&= -\frac{1}{3(w_B(\theta)-\delta)} + \frac{1}{6E(L)} \\
&= -\frac{1}{3}\left(\frac{1}{(w_B(\theta)-\delta)} - \frac{1}{2E(L)}\right) < 0.
\end{aligned}$$

There, the first inequality is based on Jensen's Inequality concerning a continuous variation of the random variable θ_2 for the convex function $(\cdot)^2$, and the last inequality follows from the fact that $w_B(\theta) - \delta < 2E(L)$, as shown above.

6. Horizontal mergers in oligopoly: first-price vs. profit-share auction[1]

6.1. Introduction

In the present chapter we consider horizontal mergers, assuming that a takeover target is auctioned among competing firms, and firms have private information concerning their synergy benefits of a merger. Our analysis has several distinct features:

- bidders are competitors in a downstream Cournot market game and synergies take the form of cost reductions,

[1] This chapter is based on Ding, Fan, and Wolfstetter (2010).

- bidders have private information concerning the synergy effect of merging their firm with the takeover target,
- before firms play the oligopoly game they observe either the merged firm's synergy parameter or the winning bid,
- bidders may influence their rivals' beliefs through their bid,
- the merger target is auctioned to the highest bidder, either in a standard first-price (cash auction) or a profit-share auction.

The presence of synergies assures that mergers are potentially profitable for the coalition of merged and merging firm, and the presence of private information makes auctions an appealing mechanism for matching the takeover target with another firm.

Under the predominant corporate law in the U.S., once a takeover offer has been made, the board of directors is actually obliged to act like an auctioneer, and get the best price for the stockholder of the company, which is one of the reasons why a takeover offer must remain open for at least 20 business days (see Cramton, 1998).[2] And indeed, auctions are not only advised but also widely used in takeovers (see the empirical study by Boone and Mulherin, 2007).

[2] As ruled in Revlon (1986).

The fact that bidders are competitors in a downstream oligopoly implies that the takeover bidding is a somewhat peculiar auction game where bidding is subject to externalities. In particular, since non-merged firms benefit from a merger if synergies are low, bidders are subject to a positive externality with positive probability. Whereas if synergies are sufficiently high, bidders are subject to a negative externality.

A second peculiar feature of takeover auctions is the fact that they can use a somewhat unusual but highly profitable auction format. Ownership stakes in the merged firm make post-merger profits verifiable to all co-owners. This makes it feasible to make the price to be paid by the winner of the auction conditional on the post-merger profitability, simply by adopting a share auction in lieu of a standard "cash auction". In such a share auction the winner of the auction awards the owners of the takeover target with an ownership stake in the merged firm, which entitles them to share of its profits.

Share auctions are more profitable than cash auctions, as we will show below. And they are widely used in takeover bidding. A case in point is the takeover of "GE Insurance Solutions" (a major reinsurer) by "Swiss Re", which made Swiss Re the world's largest player in the oligopolistic reinsurance market. Several bidders participated in that takeover bidding, including the famous investor Warren Buffett who was however outbid by Swiss Re. Interestingly, the winning bid offered GE a significant ownership stake in the form of common stock, which made GE a major shareholder of Swiss Re (see Boyle, 2005).

In the present chapter we combine the two unique features of takeover auctions: the presence of significant externalities, due to the downstream interaction among bidders, and the possible use of share auctions in lieu of standard cash auctions.

We consider two specifications of our model: before the oligopoly game is played, firms can either observe the merged firm's synergy parameter or only the winning bid. When firms can only observe bids (in particular the winning bid), the bidding games involve a signaling element, which exerts an upward pressure on equilibrium bids.

The chapter is related to the ongoing debate on horizontal merger. A starting point of that literature is the "merger paradox" which observes that "small" mergers are not profitable if firms compete in a Cournot market game with substitutes and mergers do not involve synergy benefits (see Salant, Switzer, and Reynolds, 1983).

However, small mergers become profitable for the coalition of merged firms if synergies are sufficiently high (see Farrell and Shapiro, 1990) or firms produce differentiated goods in a Bertrand market game (Deneckere and Davidson, 1985), or, to some extent, if market demand is sufficiently concave (see Faulý-Oller, 1997).

Mergers can also be profitable if firms are uncertain about their post-merger synergy benefit (Choné and Linnemer, 2008, Amir, Diamantoudi, and Xue, 2009). Indeed, mergers can be profitable even if, in expectation, there are no synergy benefits, provided the variance of the unknown synergy benefit is sufficiently high (see Hamada, 2011).

The use of auctions in horizontal mergers was considered for example by Jehiel and Moldovanu (2000) for whom takeover bidding in a Cournot oligopoly is a prime example of an auction that is subject to positive externalities, if synergies are sufficiently low. Auctions with positive externalities are viewed as interesting outliers where pooling occurs if bidders are subject to a minimum bid requirement.

Brusco, Lopomo, Robinson, and Viswanathan (2007) and Gärtner and Schmutzler (2009) consider mergers when firms are subject to double private information, because the takeover target does not know the synergy benefit brought about by a partner, and prospective partners do not know each other's pre-merger unit costs. While the former adopt an optimal mechanism design perspective, the latter focus on bargaining issues and aspire to resolve the puzzle why many horizontal mergers happen to flop, as observed in the empirical literature (Ravenscraft and Scherer, 1989, Moeller, Schlingemann, and Stulz, 2005). Both incorporate a rich information structure; however, neither includes a full analysis of the interrelationship with the downstream oligopoly game.

Similar to Jehiel and Moldovanu (2000), the present chapter adopts an auction perspective and assumes that firms have private information concerning their synergy parameter while firms' pre-merger unit costs are common knowledge. However, unlike Jehiel and Moldovanu (2000), we consider profit-share auctions in addition to standard cash auctions,[3] allow for nonlinear demand, more than three firms, and assume that firms may observe only an imperfect signal of the merged firm's synergy parameter before the oligopoly

[3]Contingent-payment auctions like profit-share auctions were introduced by Hansen (1985). Crémer (1987) pointed out that if the post-auction valuation is verifiable, the auctioneer can, in principle, extract the full surplus. Samuelson (1987) discusses limitations of full surplus extraction. Apart from takeover bidding, share auctions are widely used in book publishing.

game is played, as it is the case when firms observe only bids, in particular the winning bid (like in the analysis of patent licensing by Das Varma, 2003, Goeree, 2003, Fan, Jun, and Wolfstetter, 2011).

Our main results are as follows: we show that the bidding games have a separating equilibrium even though firms may be subject to a positive externality; and we show that a profit-share auction is more profitable than a first-price auction, regardless of whether firms observe the merged firm's synergy parameter or only an imperfect signal of it.

The plan of the chapter is as follows. Section 6.2 introduces the framework and assumptions. Section 6.3 considers the benchmark model in which firms perfectly observe the synergy parameter of the merged firm before they play the oligopoly game. This assumption is then replaced in Section 6.4 where bidders observe only the winning bid, which introduces a signaling issue. The chapter concludes with a discussion. Some proofs are in the Appendix.

6.2. Model

Consider a Cournot oligopoly composed of $N+1 \geq 3$ firms among which one, say firm $N+1$, is willing to be merged with either one of the firms $\{1,2,\ldots,N\}$. The owners of the takeover target auction their firm either in a standard first-price or in a profit-share auction, supplemented by an entry or participation fee. Entry fees may be necessary to assure that the takeover target does not suffer losses in some states.

In a profit-share auction bids are shares in the equilibrium profit of the merged firm that bidders offer conditional on being merged with firm $N+1$. The takeover target selects the bidder who offers the highest share as winner. Profit-share auctions are feasible in the takeover context because the parties that become co-owners of the merged firm can naturally verify the post-merger profit of that firm.

If a merger occurs, the merged firm enjoys a synergy benefit in the form of a lower unit cost. Firms that are not part of the merger have the same unit cost c, whereas the merged firm has the unit cost $c-\theta$. Large mergers of more than two firms are not on the agenda or not approved by the Antitrust Authority.

Prior to the auction, firms $\{1,\ldots,N\}$ have private information concerning their synergy parameter θ. From the point of view of other firms, firms' synergy parameters are *iid* random variables, drawn from the log-concave distribution $F : [0, c] \to [0, 1]$, with positive density, F', everywhere. We denote the c.d.f. of the largest synergy parameter of a sample of $N-1$ firms by $G(\theta) := F(\theta)^{N-1}$ and note that log-concavity of F implies log-concavity of G.

After the bidding game has been played firms play a Cournot oligopoly game. Two models are distinguished: In the first model, the synergy parameter of the merged firm becomes known to all firms before the oligopoly game is played. In the second model firms only observe the winning bid from which they draw inferences concerning the synergy parameter of the merged firm.

In the first model the downstream oligopoly game is one of complete information, which is fully determined by the cost parameter c and the synergy parameter of the merged firm, θ. In the second model the oligopoly game is one of incomplete information. There, firms update their beliefs concerning the synergy parameter of the merged firm, after they observe the winning bid. In turn bids may be used to influence the beliefs of rival bidders, which introduces a signaling aspect into the bidding game.

In the following we denote the equilibrium profit of the merged firm by $\pi_m(\theta)$, the equilibrium profit of the firms that have not been merged by $\pi_n(\theta)$, and the (default) equilibrium profits if no merger has taken place by π_0. Both π_m and π_n are functions of the synergy parameter of the firm that has been merged with firm $N+1$, and π_m is strictly monotone increasing and π_n is strictly monotone decreasing in θ. Obviously, $\forall\theta : \pi_m(\theta) > \pi_0$, and $\forall\theta > 0 : \pi_m(\theta) > \pi_n(\theta)$, whereas for some $\hat{\theta} \in (0, c)$,

$$\pi_n(\theta) \gtreqqless \pi_0 \iff \theta \lesseqqgtr \hat{\theta} \quad \text{(positive/negative externality)}. \qquad (6.1)$$

In other words, for low θ the firm that has *not* been merged benefits from reduced competition due to the merger. However, if the synergy is sufficiently large, that *positive externality* turns into a *negative externality*, because then the disadvantage of facing a competitor whose cost has been reduced outweighs the benefit of reduced competition due to the merger.

We assume that c is sufficiently high to assure existence of $\hat{\theta} < c$, and sufficiently low to assure that mergers do not propel monopoly at all possible synergies. We also assume that firms are risk neutral and inverse market demand P is a decreasing and concave function of aggregate output.[4]

Of course, "small" mergers are not profitable for the merger coalition if synergies are absent: $\pi_m(0) < 2\pi_0$ ("merger paradox").

6.3. Takeover bidding without signaling

Following Jehiel and Moldovanu (2000), we first consider a highly stylized model in which the synergy parameter becomes common knowledge after the auction and before the oligopoly game is played. There, the profits of the merged firm and the non-merged firms are fully described by the functions $\pi_m(\theta), \pi_n(\theta)$, which are exclusively functions of the merged firm's synergy parameter θ. Of course, if no firm bids, all firms earn the default equilibrium payoff π_0.

Bid functions are denoted by the Roman letters b (first-price) and s (profit-share auction).

[4] This assures existence of a unique pure strategy equilibrium of the oligopoly game (see Szidarovszky and Yakowitz, 1977).

6.3.1. First-price auction

The bidder who makes the highest bid wins the auction, the winner pays his bid, and all those who choose to bid must pay the entry or participation fee R.

As a working hypothesis suppose b is strictly increasing, and the entry fee induces a cutoff value of θ, denoted by r, such that $b(r) = 0$, and a bidder bids only if his synergy parameter is $\theta \geq r$ and otherwise abstains from bidding.

Consider a marginal bidder with $\theta = r$. That bidder must be indifferent between bidding and not bidding:

$$\begin{aligned} & G(r)\,(\pi_m(r) - b(r)) + \int_r^c \pi_n(z)dG(z) - R \\ & = G(r)\pi_0 + \int_r^c \pi_n(z)dG(z). \end{aligned}$$

Since $\pi_m(\theta)$ is strictly increasing and $b(r) = 0$ it follows that for all $R \in [0, \pi_m(c) - \pi_0)$, the entry fee R induces a unique critical valuation r, which is implicitly defined as the solution of the equation

$$R = G(r)\,(\pi_m(r) - \pi_0)\,. \tag{6.2}$$

Proposition 6.1 (First-price auction)**.** *The equilibrium strategy of* *the first-price auction is*

$$b(\theta) = \int_r^\theta \frac{G'(x)}{G(\theta)} (\pi_m(x) - \pi_n(x))\, dx. \tag{6.3}$$

Proof. By the assumed monotonicity of b, the equilibrium bidding problem for a bidder with θ, $y \geq r$ can be stated in the form:

$$\theta = \arg\max_{y \geq r} G(y)\,(\pi_m(\theta) - b(y)) + \int_y^c \pi_n(z) dG(z) - R. \tag{6.4}$$

Therefore, b has to solve the differential equation,

$$(G(\theta) b(\theta))' = G'(\theta)\,(\pi_m(\theta) - \pi_n(\theta)). \tag{6.5}$$

Integrating and using the initial condition $b(r) = 0$ yields (6.3). To confirm the assumed strict monotonicity of b, note that $\pi_m(\theta) - \pi_n(\theta)$ is positive and strictly increasing for all θ. Using these facts and applying integration by parts gives:

$$\begin{aligned} & b'(\theta) \\ & = \frac{G'(\theta)}{G(\theta)} \left(\pi_m(\theta) - \pi_n(\theta) - \frac{1}{G(\theta)} \int_r^\theta G'(x)\,(\pi_m(x) - \pi_n(x))\, dx \right) \\ & = \frac{G'(\theta)}{G(\theta)^2} \left(G(r)\,(\pi_m(r) - \pi_n(r)) + \int_r^\theta \partial_x\,(\pi_m(x) - \pi_n(x))\, G(x) dx \right) \\ & > 0. \end{aligned} \tag{6.6}$$

Finally, we need to confirm that bidding is more profitable than not bidding if and only if $\theta \geq r$. That proof is in Appendix 6.A.2. $\square$

In order to pin down the role of the externality implied by mergers, let $\hat{b}$ denote the hypothetical equilibrium bid function that would apply if the loser of the auction were not affected by the merger, i.e, if $\pi_n(\theta)$ were equal to π_0. Then,[5]

$$\hat{b}(\theta) - b(\theta) \gtreqless E\left(\pi_n(\tilde{\theta}) - \pi_0 \mid \tilde{\theta} \leq \theta\right) \gtreqless 0. \tag{6.7}$$

In other words, bidding becomes less aggressive when the conditional expected value of the externality is positive (which occurs if θ is sufficiently "small") and more aggressive when that conditional expected value is negative. Of course, a positive externality makes it less attractive to win the auction, which makes bidders less eager to win, and *vice versa*. Therefore, this relationship is intuitively plausible.

Figure 6.1 illustrates this relationship for the example of linear demand, $c = 0.49$, $r = 0.001$, $F(\theta) = \theta/c$ (uniform distribution), and $N = 2$. There, the vertical, dotted line separates the range of positive externalities ($\pi_n(\theta) > \pi_0$) from negative externalities ($\pi_n(\theta) < \pi_0$).

[5] As one can confirm easily, that hypothetical equilibrium bid function is $\hat{b}(\theta) = \int_r^{\theta} (\pi_m(x) - \pi_0)\, G'(x)/G(\theta) dx$.

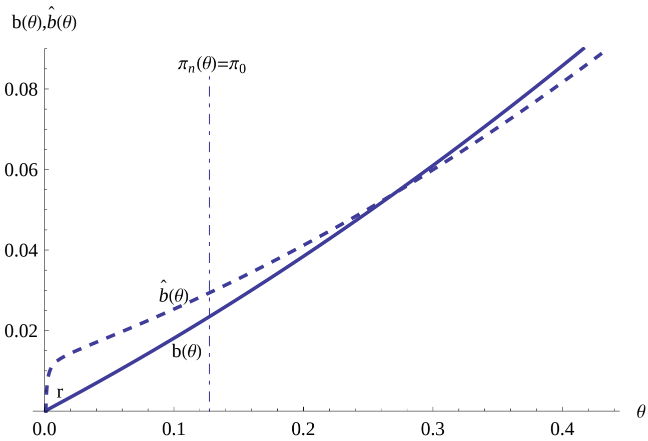

Figure 6.1.: Equilibrium first-price auction with (solid) and without (dashed) externality

We mention that if we would employ a minimum bid in lieu of an entry fee requirement, the equilibrium bid function would exhibit pooling at the reserve price. Jehiel and Moldovanu already observed that “entry fees and reserve prices are not equivalent in the positive externality case” (Jehiel and Moldovanu, 2000, p. 782).

6.3.2. Profit-share auction

Now consider the profit-share auction with entry fee R. There, bidders must pay the entry fee, regardless of winning or losing, the bidder who offers the highest share wins the auction, the winner has to grant the promised share $s(\theta)$ of the profit of the merged firm, and losers pay nothing.

As one can easily confirm, R induces the same critical valuation r as the first-price auction, see (6.2).

Using the same solution procedure as the above, the equilibrium bidding problem of a bidder with θ, $y \geq r$ can be stated in the form of the equilibrium requirement:

$$\theta = \arg\max_{y \geq r} G(y)\pi_m(\theta)(1-s(y)) + \int_y^c \pi_n(z)dG(z) - R. \quad (6.8)$$

Therefore, the equilibrium strategy has to solve the first order differential equation

$$(G(\theta)s(\theta))' = G'(\theta)\frac{\pi_m(\theta)-\pi_n(\theta)}{\pi_m(\theta)}. \tag{6.9}$$

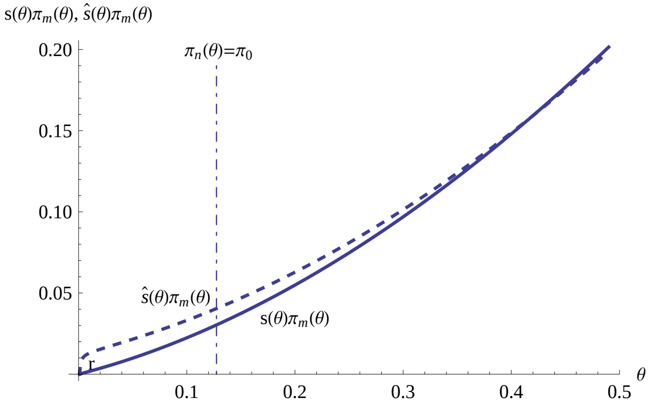

Figure 6.2.: Equilibrium profit-share auction with (solid) and without (dashed) externality

Proposition 6.2 (Profit-share auction)**.** *The equilibrium strategy of the profit-share auction is*

$$s(\theta) = \int_r^\theta \frac{G'(x)}{G(\theta)}\frac{\pi_m(x)-\pi_n(x)}{\pi_m(x)}dx. \tag{6.10}$$

Proof. Integrating the differential equation (6.9), using the initial condition $s(r)=0$, gives the equilibrium strategy (6.10). To confirm that s is strictly increasing, as assumed, note that $(\pi_m(\theta)-\pi_n(\theta))/\pi_m(\theta)$ is positive and strictly increasing. Using these facts and applying integration by parts gives:

$$s'(\theta) = \frac{G'(\theta)}{G(\theta)}\left(\frac{\pi_m(\theta)-\pi_n(\theta)}{\pi_m(\theta)} - \frac{1}{G(\theta)}\int_r^\theta G'(x)\frac{\pi_m(x)-\pi_n(x)}{\pi_m(x)}dx\right)$$
$$= \frac{G'(\theta)}{G(\theta)^2}\left(G(r)\frac{\pi_m(r)-\pi_n(r)}{\pi_m(r)} + \int_r^\theta \partial_x\left(\frac{\pi_m(x)-\pi_n(x)}{\pi_m(x)}\right)G(x)dx\right)$$
$$> 0.$$

Finally, we need to confirm the assumed cutoff participation strategy. The proof is in Appendix 6.A.3. □

In Figure 6.2 we plot the payments, $s(\theta)\pi_m(\theta)$, that are implicitly offered in equilibrium by bidders provided $\theta \geq r$. We also plot the hypothetical payments based on the share function, $\hat{s}(\theta)$, that would apply if externalities were absent. These plots assume the same linear example that underlies Figure 6.1. Again, the presence of externalities exerts a downward pressure on equilibrium bids, except for high synergy parameters.

Remark 6.1. *We mention that in standard auction problems in which the losers' payoff is equal to zero and no entry fees are charged,* (6.10) *implies* $s(\theta) \equiv 1$. *Hence, in this case, the share auction has* *no strictly monoton equilibrium. This can be remedied by adding entry fees or cash prices, as it is typically done in book publishing, where share auctions are frequently used.*

6.3.3. Superiority of the profit-share auction

Proposition 6.3. *The profit-share auction is more profitable for the owners of the merger target than the first-price auction, for all R.*

Proof. Let θ be the highest of the sample of N synergy parameters. Then, the difference in equilibrium profits of firm $N+1$ in the profit-share and the first-price auction is equal to:

$$\begin{aligned}
\Delta_U(\theta) &:= s(\theta)\pi_m(\theta) - b(\theta) \\
&= \pi_m(\theta)\int_r^\theta \frac{G'(x)}{G(\theta)}\frac{\pi_m(x)-\pi_n(x)}{\pi_m(x)}dx - b(\theta) \\
&> \int_r^\theta \frac{G'(x)}{G(\theta)}(\pi_m(x)-\pi_n(x))\,dx - b(\theta) \quad \text{(as } \pi_m'(\theta)>0) \\
&\equiv 0 \quad \text{(by (6.3))}.
\end{aligned}$$

Therefore, the expected profit in the profit-share auction, $U_s(r) = \int_r^c \pi_m(\theta)s(\theta)dF_{(1)}(\theta) + \mu(r) + \pi_0 F(r)^N$, is higher than that of the first-price auction, $U_c(r) = \int_r^c b(\theta)dF_{(1)}(\theta) + \mu(r) + \pi_0 F(r)^N$, for all $r \in [0,c)$ (where $\mu(r)$ denotes the expected value of collected entry fees and $F_{(1)}(\theta)$ denotes the c.d.f. of the order statistic of the highest synergy parameter). □

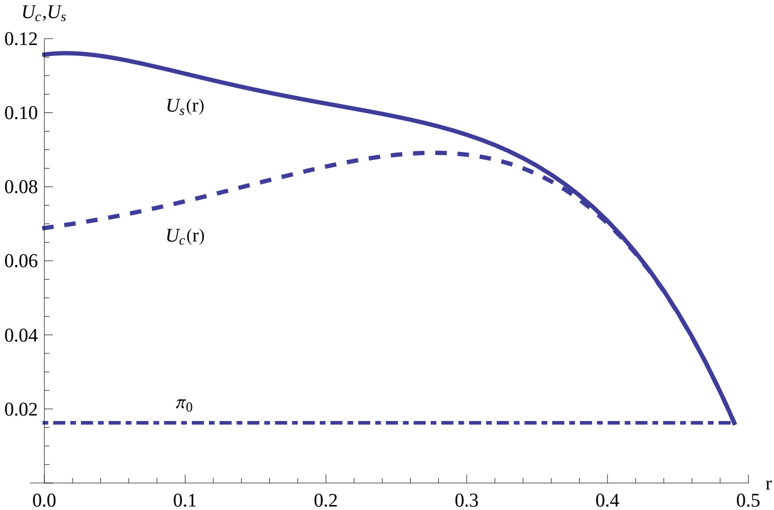

Figure 6.3.: Expected profit: profit-share (solid) vs. first-price (dashed) auction

In Figure 6.3 we plot U_c, U_s, π_0 as functions of the critical valuations r induced by the entry fee R for the example of linear demand, $c = 0.49$, $N = 2$, and $F(\theta) = \theta/c$ (uniform distribution). Evidently, the maximum of U_s far exceeds that of U_c and U_c far exceeds π_0.

This indicates that the profit-share auction is considerably more profitable than the first-price auction and than the *status quo* prior to the merger. The figure also suggests that the entry fee plays no significant role in the share auction, unlike in the first-price auction.

We mention that one may interpret the superiority of the profit-share auction as an example of the "linkage principle". According to that well-known principle, linking the price to a variable that is correlated with bidders' private information lowers bidders' information rent (Milgrom, 1987).

6.4. Takeover bidding with signaling

An implausible feature of the above model is that the synergy parameter of the merged firm becomes known before the oligopoly game is played. We now switch to the more plausible model in which firms only observe the winning bid, and then update their beliefs concerning the synergy parameter of the merged firm before playing the oligopoly game.

This modification introduces a signaling aspect into the bidding game. Firms are no longer exclusively concerned with winning or losing the auction, but also with how their bid impacts rivals' beliefs. In particular, firms may wish to inflate their bids in order to signal high synergy, with the intention to gain a strategic advantage in the subsequent oligopoly game.[6]

In order to visibly distinguish between the two models, equilibrium bid functions are now denoted by the Greek letters β (first-price) and σ (profit-share auction).

We employ the following solution procedure: As a working hypothesis suppose the bidding game has a symmetric, strictly monotone increasing equilibrium that allows the losers of the auction to draw a perfect inference from the observed winning bid to the underlying synergy parameter of the merged firm. We consider one bidder, say bidder 1 with synergy parameter θ, who assumes that his rivals play the strictly increasing equilibrium strategy β, resp. σ but considers to make a deviating bid.

Without loss of generality all relevant deviating bids are captured by bids from the interval $[\beta(r), \beta(c)]$, resp. $[\sigma(r), \sigma(c)]$, because bidding outside that interval is obviously dominated. In other words,

[6]Signaling in auctions with downstream interaction has been analyzed in the context of patent licensing by Das Varma (2003), Goeree (2003), and Fan, Jun, and Wolfstetter (2011).

bidding according to the equilibrium strategy β, resp. σ *as if* the synergy parameter were equal to $y \in (r, c)$ captures all relevant deviating bids.

We first characterize all oligopoly subgames that may occur if bidder 1 unilaterally deviates from the equilibrium bid while everyone believes that all rival firms play the equilibrium bidding strategy β resp. σ.

6.4.1. Downstream oligopoly "subgames"

Suppose $y \geq \theta$; then two classes of oligopoly subgames must be distinguished:[7]

Case a): $y > x := \{\theta_2, \ldots, \theta_n\}$ In this case firm 1 wins the auction. All other firms believe that the synergy parameter of the merged firm is equal to y. Therefore, they believe to play an N player oligopoly game that is characterized by the profile of unit costs $(c - y, c, \ldots, c)$. Denote the equilibrium strategy of players $2, \ldots, N$ by $q_n(y)$ and their equilibrium profit by $\pi_n(y)$. (The full characterization of the

[7] The case of $y \leq \theta$ is similar, and requiring that no "downward" deviating bids should be profitable, yields the same differential equation (6.12), resp. (6.15).

equilibrium of this game which players $(2, 3, \ldots, N)$ believe to play, is contained in Appendix 6.6.)

However, firm 1 privately knows that the merged firm's synergy parameter is equal to θ rather than the pretended y. Therefore, firm 1 plays its best response strategy

$$q_m(\theta, y) := \arg\max_q \pi(q, q_n(y), \ldots, q_n(y), \theta)$$

and earns the equilibrium payoff

$$\bar{\pi}_m(\theta, y) := \pi(q_m(\theta, y), q_n(y), \ldots, q_n(y), \theta).$$

Case b): $y < x := \max\{\theta_2, \ldots, \theta_N\}$ In this case, firm 1 loses the auction and the synergy parameter realized by the merger is equal to x. The subsequent oligopoly subgame is characterized by the profile of unit costs $(c, c, \ldots, c, c - x, c, \ldots, c)$ and the associated equilibrium profit of firm 1 is denoted by $\pi_n(x)$.

Note that $\bar{\pi}_m(\theta, y)|_{y=\theta} = \pi_m(\theta)$, and $\partial_y \bar{\pi}_m(\theta, y)|_{y=\theta} > 0$, as we show in equations (6.29), (6.30) in Appendix 6.A.6.

6.4.2. First-price auction with signaling

By a procedure similar to that used in the model without signaling, the equilibrium requirement concerning β can be stated as follows, for $\theta \geq r$

$$\theta = \arg\max_{y \geq r} G(y)\,(\bar{\pi}_m(\theta, y) - \beta(y)) + \int_y^c \pi_n(x) dG(x) - R. \tag{6.11}$$

The relationship between r and R is the same as in the model without signaling.

Therefore, β must solve the differential equation for all $\theta \geq r$:

$$(\beta(\theta)G(\theta))' = G'(\theta)\,(\bar{\pi}_m(\theta,\theta) - \pi_n(\theta)) + G(\theta)\partial_y\, \bar{\pi}_m(\theta, y)|_{y=\theta}\,. \tag{6.12}$$

And we find:

Proposition 6.4 (First-price auction)**.** *In the first-price auction, the potential for signaling induces more aggressive equilibrium bidding, for all $\theta > r$ and for all r:*

$$\beta(\theta) = b(\theta) + \int_r^\theta \frac{G(x)}{G(\theta)} \partial_y\, \bar{\pi}_m(x, y)|_{y=x}\, dx > b(\theta). \tag{6.13}$$

Proof. Using the fact that $\bar{\pi}_m(\theta, y)|_{y=\theta} = \pi_m(\theta)$ and the initial condition, $\beta(r) = 0$, it is easy to confirm that β solves the differential equation (6.12). The assertion that $\beta(\theta) > b(\theta), \forall\theta > r$ follows from the fact that $\partial_y \bar{\pi}_m(x, y)|_{y=x} > 0$. To complete the proof one needs to confirm the assumed strict monotonicity of β, which we confirm in Appendix 6.A.4. □

The intuition for this result is straightforward. If rival bidders would play the strategy b (which is the equilibrium without signaling), every bidder would benefit from signaling strength by bidding as if the own synergy parameter where higher than it is. Of course, in equilibrium no such misleading signaling can occur. Therefore, the bid function must be adjusted in such a way that signaling strength is made sufficiently costly, which is achieved by raising bids pointwise; hence, $\beta(\theta) > b(\theta), \forall\theta > r$. In other words, the potential for signaling exerts an upward pressure on equilibrium bids, to the benefit of the owners of the takeover target.

6.4.3. Profit-share auction with signaling

Denote the equilibrium bid function in the signaling model by σ. Similar to the above, the equilibrium requirement takes the form, for all $\theta \geq r$,

$$\theta = \arg\max_{y \geq r} G(y)\bar{\pi}_m(\theta, y)\,(1-\sigma(y)) + \int_y^c \pi_n(z)dG(z) - R. \tag{6.14}$$

The relationship between r and R is the same as in the model without signaling.

Therefore, σ must solve the differential equation for all $\theta \geq r$:

$$\sigma'(\theta) + \alpha(\theta)\sigma(\theta) - (\alpha(\theta) - \gamma(\theta)) = 0 \tag{6.15}$$

$$\alpha(\theta) := \partial_\theta \ln G(\theta) + \partial_y \ln \bar{\pi}_m(\theta, y)|_{y=\theta} \tag{6.16}$$

$$\gamma(\theta) := \frac{G'(\theta)}{G(\theta)} \frac{\pi_n(\theta)}{\bar{\pi}_m(\theta,\theta)}. \tag{6.17}$$

And we find:

Proposition 6.5 (Profit-share auction)**.** *The equilibrium strategy of the profit-share auction in the model with signaling is, for all $\theta \geq r$:*

$$\sigma(\theta) = \int_r^\theta (\alpha(x) - \gamma(x))\,\varphi(x,\theta)dx \tag{6.18}$$

$$\varphi(x,\theta) := \exp\left(-\int_x^\theta \alpha(z)dz\right). \tag{6.19}$$

Proof. It is straightforward to confirm that the asserted equilibrium bid function (6.18) solves the differential equation (6.15).

For a constructive proof, multiply the differential equation with the positive valued $\mu(\theta) := \exp\left(\int_r^\theta \alpha(z)dz\right)$. Then, one can rewrite the differential equation (6.15) as

$$(\mu(\theta)\sigma(\theta))' = \mu(\theta)\left(\alpha(\theta) - \gamma(\theta)\right).$$

Integrating and using the initial condition $\sigma(r) = 0$ yields (6.18).

The assumed strict monotonicity of σ is confirmed in Appendix 6.A.5. □

Finally, we show that the revenue ranking of the two auction formats extends to the signaling model:

Proposition 6.6. *The profit-share auction is more profitable than the first-price auction, for all R.*

Proof. Using the definitions of α and γ, rewrite the bid function β as:

$$\beta(\theta) = \frac{1}{G(\theta)} \int_r^\theta \left(\alpha(x) - \gamma(x)\right) G(x)\bar{\pi}_m(x,x)dx.$$

Let θ be the highest of the sample of N synergy parameters. Then, the difference between the equilibrium profits of the takeover target firm $N+1$ in the profit-share and the first-price auction is:

$$\Delta_U(\theta) := \sigma(\theta)\bar{\pi}_m(\theta,\theta) - \beta(\theta)$$

$$= \int_r^\theta (\alpha(x) - \gamma(x)) \left(\varphi(x,\theta)\bar{\pi}_m(\theta,\theta) - \frac{G(x)}{G(\theta)}\bar{\pi}_m(x,x) \right) dx.$$

We will show that $\varphi(x,\theta)\bar{\pi}_m(\theta,\theta) - \frac{G(x)}{G(\theta)}\bar{\pi}_m(x,x) > 0$, which together with the fact that $\alpha(x) - \gamma(x) > 0$ proves $\Delta_U(\theta) > 0, \forall\theta$.

A bit of rearranging gives:

$$\begin{aligned}
&\varphi(x,\theta)\bar{\pi}_m(\theta,\theta) - \frac{G(x)}{G(\theta)}\bar{\pi}_m(x,x) \\
&= \exp\left(-\int_x^\theta \alpha(z)dz \right)\bar{\pi}_m(\theta,\theta) - \frac{G(x)}{G(\theta)}\bar{\pi}_m(x,x) \\
&= \exp\left(-\int_x^\theta \left(\partial_z \ln G(z) + \partial_y \ln \bar{\pi}_m(z,y)|_{y=z} \right) dz \right)\bar{\pi}_m(\theta,\theta) \\
&\quad - \frac{G(x)}{G(\theta)}\bar{\pi}_m(x,x) \\
&= \frac{G(x)}{G(\theta)} \left(\frac{\bar{\pi}_m(\theta,\theta)}{\exp\left(\int_x^\theta \partial_y \ln \bar{\pi}_m(z,y)|_{y=z} dz \right)} - \bar{\pi}_m(x,x) \right).
\end{aligned}$$

The latter is positive if $\exp\left(\int_x^\theta \partial_y \ln(\bar{\pi}_m(z,y)|_{y=z}) dz \right) < \frac{\bar{\pi}_m(\theta,\theta)}{\bar{\pi}_m(x,x)}$, or equivalently if

$$\begin{aligned}
\int_x^\theta \partial_y \ln \bar{\pi}_m(z,y)|_{y=z} dz &< \ln \frac{\bar{\pi}_m(\theta,\theta)}{\bar{\pi}_m(x,x)} \\
&\equiv \int_x^\theta \partial_z \ln \bar{\pi}_m(z,y)|_{y=z} dz.
\end{aligned}$$

Evidently, the marginal impact of a truthfully revealed cost reduction on the merged firm's profit is greater than that of an equally sized purely pretended cost reduction; therefore,

$$\partial_z \bar{\pi}_m(z, y)|_{y=z} > \partial_y \bar{\pi}_m(z, y)|_{y=z}, \tag{6.20}$$

(for a formal proof see Appendix 6.A.6). Hence, $\Delta_U(\theta) > 0, \forall \theta$, as asserted. □

6.5. Discussion

One limitation of the present chapter is that we consider only takeovers that are motivated by synergies. This excludes takeovers that serve the purpose to reorganize firms that are subject to organization slack. If such reorganization is the issue, bidders may be willing to pay a premium for acquiring full residual claimant status, and thus avoid diluted incentives. This, in turn, tilts the balance in favor of cash auctions, and it may even make auctions undesirable altogether.

Another issue is that takeovers are typically prompted by individual bidders or investment banks rather than by the seller. This makes takeovers particularly prone to preemptive bidding or the prior acquisition of toe-holds that affects bidding.[8]

[8]Bulow, Huang, and Klemperer (1999) showed that if a bidder has acquired a

Last, but not least, our analysis ignores both drastic innovations and the possibility of building larger coalitions through a sequence of mergers between two firms (analyzed in Kamien and Zang, 1991, Bloch, 1995, 1996), and all related regulatory issues (see Nocke and Whinston, 2010).

6.A. Appendix

6.A.1. Linear example

Here we sketch the linear example that underlies the plots in Figures 6.1-6.3.

There we set $N = 2$ and assume linear demand $P(Q) := \max\{1 - Q, 0\}$, $Q := q_1 + q_2$, which gives $\pi_0 = {}^{(1-c)^2}/_{16}$, $\pi_m(\theta) = {}^{(1-c+2\theta)^2}/_9$, $\pi_n(\theta) = {}^{(1-c-\theta)^2}/_9$. Hence, for all $\theta : \pi_m(\theta) > \pi_0$, and for all $\theta > 0 :$ $\pi_m(\theta) > \pi_n(\theta)$, and

$$\pi_n(\theta) \gtreqless \pi_0 \iff \theta \lesseqgtr \hat{\theta} := {}^{(1-c)}/_4 \quad \text{(positive/negative externality).}$$

toehold prior to bidding, and the auction is an open-ascending auction, the equilibrium tends to be highly asymmetric and the equilibrium price may be "low".

To compute the bid functions, b, s and the expected profits U_c, U_s plotted in Figs. 6.1, 6.3, we also assume $F(\theta) = \theta/c$ (uniform distribution). The computations are in a *Mathematica* file available upon request from the authors.

6.A.2. Supplement to the proof of Proposition 6.1

We show that participation in the first-price auction is more profitable than non-participation if and only if $\theta \geq r$. Denote the expected payoff from bidding by Π_p, from non-bidding by Π_n, and $\Delta := \Pi_p - \Pi_n$.

1) Let $\theta > r$, then

$$\Delta = \left(G(\theta)\left(\pi_m(\theta) - b(\theta)\right) + \int_\theta^c \pi_n(z) dG(z) - R \right) - \left(G(r)\pi_0 + \int_r^c \pi_n(z) dG(z) \right).$$

Evidently, $\Delta(r) = 0$, by definition of r, and Π_n is independent of θ. Therefore, using (6.5), $\partial_\theta \Delta = \partial_\theta \Pi_p = G(\theta)\partial_\theta \pi_m(\theta) > 0$. Hence, $\Delta > 0, \forall \theta > r$.

2) Let $\theta < r$ and suppose a bidder participates and makes a bid $b(y)$, as if his synergy parameter were equal to $y \geq r > \theta$. Then, using

(6.2), (6.3), and applying integration by parts,

$$
\begin{aligned}
\Delta &= \left(G(y)\left(\pi_m(\theta) - b(y)\right) + \int_y^c \pi_n(z) dG(z) - R \right) \\
&\quad - \left(G(r)\pi_0 + \int_r^c \pi_n(z) dG(z) \right) \\
&= G(y)\left(\pi_m(\theta) - b(y)\right) - \int_r^y \pi_n(z) dG(z) - G(r)\pi_m(r) \\
&= G(y)\left(\pi_m(\theta) - \pi_m(y)\right) + \int_r^y \partial_z \pi_m(z) G(z) dz \\
&= G(y)\pi_m(\theta) - G(r)\pi_m(r) - \int_r^y \pi_m(z) dG(z) \\
&< G(y)\pi_m(\theta) - G(r)\pi_m(r) - \pi_m(r)\left(G(y) - G(r)\right) \\
&= G(y)\left(\pi_m(\theta) - \pi_m(r)\right) < 0 \quad \text{(since } \theta < r\text{)}.
\end{aligned}
$$

6.A.3. Supplement to the proof of Proposition 6.2

Like in Appendix 6.A.2 we show that participation in the profit-share auction is more profitable than non-participation if and only if $\theta \geq r$.

1) Let $\theta > r$, then

$$
\begin{aligned}
\Delta = &\left(G(\theta)\pi_m(\theta)\left(1 - s(\theta)\right) + \int_\theta^c \pi_n(z) dG(z) - R \right) \\
&- \left(G(r)\pi_0 + \int_r^c \pi_n(z) dG(z) \right).
\end{aligned}
$$

Evidently, $\Delta(r) = 0$, by definition of r, and Π_n is independent of θ. Therefore, using (6.9), $\partial_\theta \Delta = \partial_\theta \Pi_p = G(\theta)(1-s(\theta))\partial_\theta \pi_m(\theta) > 0$. Hence, $\Delta > 0, \forall \theta > r$.

2) Let $\theta < r$ and suppose a bidder participates and makes a bid $s(y)$, as if his synergy parameter were equal to $y \geq r > \theta$. Then, using (6.2), (6.10), one has,

$$\begin{aligned}
\Delta &= \left(G(y)\pi_m(\theta)(1-s(y)) + \int_y^c \pi_n(z)dG(z) - R \right) \\
&\quad - \left(G(r)\pi_0 + \int_r^c \pi_n(z)dG(z) \right) \\
&= G(y)\pi_m(\theta) - G(r)\pi_m(r) \\
&\quad - \int_r^y \left(\pi_n(z) + \pi_m(\theta)\left(1 - \frac{\pi_n(z)}{\pi_m(z)}\right)\right) dG(z) \\
&= G(r)\left(\pi_m(\theta) - \pi_m(r)\right) - \int_r^y \frac{\pi_n(z)\left(\pi_m(z) - \pi_m(\theta)\right)}{\pi_m(z)} dG(z) \\
&< 0 \quad (\text{by } y \geq r > \theta).
\end{aligned}$$

6.A.4. Supplement to the proof of Proposition 6.4

$$\beta'(\theta) = b'(\theta) - \frac{G'(\theta)}{G(\theta)^2} \int_r^\theta \partial_y\, \bar{\pi}_m(z,y)\big|_{y=z} G(z)dz + \partial_y\, \bar{\pi}_m(\theta,y)\big|_{y=\theta}$$

$$= \frac{G'(\theta)}{G(\theta)^2} \left(G(r)\left(\pi_m(r) - \pi_n(r)\right) + \int_r^\theta \frac{d}{dz}\left(\bar{\pi}_m(z,z) - \pi_n(z)\right) G(z)dz \right)$$

$$
\begin{aligned}
&-\frac{G'(\theta)}{G(\theta)^2}\int_r^\theta \partial_y\,\bar{\pi}_m(z,y)|_{y=z}\,G(z)dz+\partial_y\,\bar{\pi}_m(\theta,y)|_{y=\theta} \quad \text{(by (6.6))}\\
&=\frac{G'(\theta)}{G(\theta)^2}\left(\int_r^\theta\left(\frac{d}{dz}\left(\bar{\pi}_m(z,z)-\pi_n(z)\right)-\partial_y\,\bar{\pi}_m(z,y)|_{y=z}\right)G(z)dz\right.\\
&\quad\left.+G(r)\left(\bar{\pi}_m(r,r)-\pi_n(r)\right)\right)+\partial_y\,\bar{\pi}_m(\theta,y)|_{y=\theta}\\
&=\frac{G'(\theta)}{G(\theta)^2}\left(\int_r^\theta\left(\partial_z\left(\bar{\pi}_m(z,y)|_{y=z}-\pi_n(z)\right)\right)G(z)dz\right.\\
&\quad\left.+G(r)\left(\bar{\pi}_m(r,r)-\pi_n(r)\right)\right)+\partial_y\,\bar{\pi}_m(\theta,y)|_{y=\theta}\\
&>0.
\end{aligned}
$$

6.A.5. Supplement to the proof of Proposition 6.5

Note that $\partial_\theta\varphi(x,\theta)=-\partial_x\varphi(x,\theta)\frac{\alpha(\theta)}{\varphi(x)}$. We have then, by differentiating (6.18),

$$\sigma'(\theta)=\alpha(\theta)-\gamma(\theta)-\alpha(\theta)\int_r^\theta(\alpha(x)-\gamma(x))\varphi(x,\theta)dx \tag{6.21}$$

$$=(\alpha(\theta)-\gamma(\theta))-\alpha(\theta)\int_r^\theta\left(1-\frac{\gamma(x)}{\alpha(x)}\right)\partial_x\varphi(x,\theta)dx \tag{6.22}$$

$$=\frac{\alpha(\theta)}{\alpha(r)}(\alpha(r)-\gamma(r))\varphi(r,\theta)$$
$$+\alpha(\theta)\int_r^\theta\varphi(x,\theta)\partial_x\left(1-\frac{\gamma(x)}{\alpha(x)}\right)dx, \tag{6.23}$$

which is positive if $\frac{\gamma(x)}{\alpha(x)}$ is decreasing.

A bit of rearranging gives,

$$\frac{\gamma(x)}{\alpha(x)} = \frac{\frac{G'(x)}{G(x)} \frac{\bar{\pi}_n(x)}{\bar{\pi}_m(x,x)}}{\frac{G'(x)}{G(x)} + \frac{\partial_y \bar{\pi}_m(x,y)|_{y=x}}{\bar{\pi}_m(x,x)}} \tag{6.24}$$

$$= \frac{\pi_n(x)}{\bar{\pi}_m(x, x) + \partial_y \, \pi_m(x, y)|_{y=x} \frac{G(x)}{G'(x)}}. \tag{6.25}$$

This is strictly monotone decreasing, since $\bar{\pi}_m$ is increasing in x and y, by equation (6.28); $\partial_y \bar{\pi}_m(x, y)$ is increasing in x, since

$$\partial_{yx}\bar{\pi}_m(x, y) = (N-1)q_n'(y)\left(P'(\cdot) + q_m(x, y)P''(\cdot)\right)\partial_x q_m(x, y) > 0; \tag{6.26}$$

π_n is decreasing in x, and G is log-concave. Hence, it follows immediately that $\sigma'(x) > 0$, as asserted.

6.A.6. Supplement to the proof of Proposition 6.6

Here we prove inequality (6.20). Note that (here z is the true and y the pretended cost reduction of firm 1)

$$\bar{\pi}_m(z, y) := (P(q_m(z, y) + (N-1)q_n(y)) - c + z)\, q_m(z, y).$$

By the envelope theorem,

$$\partial_z \bar{\pi}_m(z, y) = q_m(z, y) \tag{6.27}$$

$$\partial_y \bar{\pi}_m(z,y) = P'(\cdot)(N-1)q_n'(y)q_m(z,y). \tag{6.28}$$

Therefore,

$$\partial_z \bar{\pi}_m(z,y) - \partial_y \bar{\pi}_m(z,y) = \left(1 - P'(\cdot)(N-1)q_n'(y)\right) q_m(z,y). \tag{6.29}$$

By construction, if firm 1 wins the auction, the $N-1$ other firms believe that they play an oligopoly game with the profile of unit cost $(c-y, c, \ldots, c)$, which has the equilibrium solution:[9]

$$\begin{aligned} q_m^*(y) &= \arg\max_q \left(P(q + (N-1)q_n(y)) - c + y\right) q, \\ q_n(y) &= \arg\max_q \left(P(q_m^*(y) + (N-2)q_n(y) + q) - c\right) q. \end{aligned}$$

The associated first–order conditions are:

$$\begin{gathered} P'(q_m^*(y) + (N-1)q_n(y))q_m^*(y) \\ + P(q_m^*(y) + (N-1)q_n(y)) - c + y = 0 \\ P'(q_m^*(y) + (N-1)q_n(y))q_n(y) \\ + P(q_m^*(y) + (N-1)q_n(y)) - c = 0. \end{gathered}$$

[9]Note, $q_m^*(y)$ is the strategy that firms $(2,3,\ldots N)$ believe firm 1 to play; it is not the strategy that firm 1 actually plays, since firm 1 has private information about its cost reduction.

Differentiating these w.r.t. y, and solving the equation system for $q_n'(y)$ one finds:

$$q_n'(y) = \frac{P'(\cdot) + P''(\cdot) q_n(y)}{P'(\cdot)\big((N+1)P'(\cdot) + P''(\cdot)\big(q_m^*(y) + (N-1)q_n(y)\big)\big)}. \tag{6.30}$$

Finally, substituting (6.30) into (6.29), confirms (6.20):

$$\begin{aligned}
\partial_z \bar{\pi}_m(z,y) - \bar{\partial}_y \pi_m(z,y) &= \big(1 - P'(\cdot)(N-1)q_n'(y)\big)\, q_m(z,y) \\
&= \frac{2P'(\cdot) + P''(\cdot) q_m^*(y)}{(N+1)P'(\cdot) + P''(\cdot)\big(q_m^*(y) + (N-1)q_n(y)\big)} q_m(z,y) > 0.
\end{aligned}$$

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