

# Essays in Empirical Macroeconomics

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# Introduction

This thesis consists of three self-contained chapters in empirical macroeconomics and analyzes important issues related to monetary policy and labor supply. The first chapter deals with the problem of weak identification in the estimation of inflation dynamics and a monetary policy rule. We suggest constructing additional instruments by estimating factors from a large macroeconomic data set. The rationale underlying the use of the factor-augmented instrument set is that a central banker relies on a large information set in his forecasts of important macroeconomic variables. In the final two parts of this thesis, the focus shifts to the aggregation and estimation of labor supply elasticities. The second chapter develops an aggregation procedure for the Frisch elasticity of labor supply. The exact size of this particular elasticity matters a lot when macroeconomists try to assess the quantitative implications of certain types of policies on employment and hours worked. A particular emphasis is laid on worker heterogeneity in observables and unobservables and applicability to an individual labor supply function with non-employment as a possible outcome. The third chapter treats the estimation of micro elasticities of labor supply for the entire conditional hours distribution. Not only economists but also policy-makers are interested in estimates of the Marshallian and the Hicks elasticities: the responsiveness of hours worked to changes in the wage rate induced by tax increases determines the amount of tax revenue raised.

CHAPTER 1.<sup>1</sup> Recently, the problem of weak identification or weak instruments has attracted attention in the analysis of structural macroeconomic models. Using

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<sup>1</sup>This chapter is based on joint work with Harun Mirza. Our paper is forthcoming in the Journal of Money, Credit and Banking (see Mirza and Storjohann, 2014).

weak-identification robust methods can result in large confidence sets making precise inference difficult. We overcome this problem in the analysis of a forward-looking Taylor rule and the hybrid New Keynesian Phillips Curve (NKPC) by employing stronger instruments. Rather than relying solely on typical instruments such as own lags of variables in the model that can result in uninformatively large robust confidence sets, we suggest exploiting information from a comprehensive macroeconomic data set by generating factors and using them as additional instruments. Our empirical results illustrate that the use of factors in Generalized Method of Moments (GMM) estimation substantially reduces the size of weak-identification robust confidence sets, as the factor-augmented instrument set is stronger. This allows us to conclude first that there has been a shift towards more active monetary policy from the pre-Volcker regime to the Volcker-Greenspan tenure. Second, this leads to evidence of dominant forward-looking dynamics in the estimation of a hybrid NKPC.

CHAPTER 2.<sup>2</sup> The aggregate Frisch elasticity of labor supply has played a key role in modern business cycle analysis for many years. It is of interest from a theoretical as well as from an empirical perspective and measures the reaction of total hours worked to a small change in the mean wage when wealth is held constant. This chapter develops an aggregation procedure for the Frisch wage-elasticity that requires neither specific assumptions about model parameters nor distributions of explanatory variables. The procedure offers worker heterogeneity in observables and unobservables and also allows us to simultaneously study the role that workers' participation (extensive margin) and hours decisions (intensive margin) play for the size of the aggregate Frisch elasticity. We derive an analytical expression for the aggregate elasticity and illustrate its main components: (i) the intensive and extensive adjustment of hours worked, (ii) the extensive adjustment of wages, and (iii) the aggregate employment ratio. We illustrate the importance of aggregation by empirically implementing the aggregation approach using individual-specific data from the German Socio-Economic Panel (SOEP) for males at working age in former West Germany. The data base provides indirect evidence on non-employed workers' reservation wages. We use this variable in conjunction with a two-step conditional

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<sup>2</sup>This chapter is based on joint work with Prof. Dr. Alois Kneip and Prof. Monika Gehrig-Merz, Ph. D.

density estimator to retrieve the extensive adjustment of hours worked and wages paid. The intensive hours' adjustment follows from estimating a conventional panel data model of individual hours worked. Our estimation results yield an average individual Frisch wage-elasticity of 0.29 – a value that stands in sharp contrast to our estimated aggregate values which vary between 0.63 and 0.70 over the period ranging from 2000 to 2008.

CHAPTER 3. The last chapter covers the semiparametric estimation of micro elasticities of labor supply by quantile regression methods. We analyze labor supply decisions of working women with respect to the intensive margin (hour's decision) and their reaction to wage changes for the entire conditional hours distribution. We use micro level data from the German Socio-Economic Panel (SOEP) for females at working age in former West Germany to estimate the Marshallian, the Hicks, and the income elasticity of labor supply. In order to account for the endogeneity of wages, and to analyze the hours' reaction to changes in wage, non-labor income, or other covariates for the entire conditional hours distribution we estimate the individual labor supply function by instrumental variable (IV) quantile regressions. Our estimation results yield an average of 0.62 and 0.63 for the Marshallian and the Hicks elasticity, respectively. Using quantile regression methods suggests the conclusion that females at the low end of the conditional hours distribution are more sensitive to changes in their wages than females at the upper end.

# Making Weak Instrument Sets Stronger: Factor-Based Estimation of Inflation Dynamics and a Monetary Policy Rule

## 1.1 Introduction

This paper combines the insights from the literature on factor models and from studies on the weak-identification problem in the estimation of single-equation time-series models. We show that adding factors, generated from a large macroeconomic data set, as additional instruments in Generalized Method of Moments (GMM) estimation yields more precise results for a forward-looking Taylor rule and the hybrid New Keynesian Phillips Curve (NKPC).

In a recent paper, Mavroeidis (2010) reassesses the seminal work by Clarida, Galí, and Gertler (2000). Given that their analysis of monetary policy rules in the US might suffer from weak instrumental variables (IV),<sup>1</sup> which can lead to biased estimators and inference, he evaluates their model using methods that are robust against weak IVs. In constructing joint confidence sets for the parameters on expected future inflation and the output gap, he empirically confirms the conclusion that pre-Volcker monetary policy was accommodative to inflation. In contrast to Clarida, Galí, and Gertler (2000) though, he claims that with the use of robust

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<sup>1</sup>Note that for ease of reference we denote the case of weak identification also as a problem of weak instruments.

methods it cannot be shown whether monetary policy during the Volcker-Greenspan tenure was adherent to the Taylor principle or not due to inconclusive confidence sets. Similarly, Kleibergen and Mavroeidis (2009) estimate the hybrid NKPC, as introduced by Galí and Gertler (1999), using weak-identification robust methods. They find confidence sets that are so large as to be consistent with both dominant forward- and backward-looking inflation dynamics.

We follow a different route in this paper. Rather than relying solely on typical instruments such as own lags of variables in the model that can result in uninformatively large robust confidence sets, we construct additional instruments by estimating factors from a comprehensive macroeconomic data set (Stock and Watson, 2008). We employ these factors in the first stage of the respective estimation, an approach applied to point estimates of the NKPC by Beyer, Farmer, Henry, and Marcellino (2008) and Kapetanios and Marcellino (2010) and to Taylor rules by Bernanke and Boivin (2003) and Favero, Marcellino, and Neglia (2005). In contrast to these studies, we consider confidence sets of the parameters in order to derive conclusions with respect to the Taylor principle and the joint behavior of the parameters of the NKPC. In addition, we rely on the weak-identification robust statistic suggested by Kleibergen (2005) given that it is not known a priori whether factors will be strong instruments.

The literature on factor analysis has shown that dimension-reduction techniques can be successful in summarizing a vast amount of information in few variables (e. g. Stock and Watson, 2002, 2008). These variables, i. e., the factors, can perform well as additional instruments in IV and GMM estimation as has been shown in formal evaluations by Bai and Ng (2010) and Kapetanios and Marcellino (2010), respectively. Kapetanios, Khalaf, and Marcellino (2011) analyze factor-based weak-IV robust statistics for linear IV estimation.

Our empirical results illustrate that the use of factors substantially reduces the size of the two-dimensional weak-IV robust confidence sets, as the factor-augmented instrument set is stronger in the estimation procedure. First, this leads to evidence of dominant forward-looking dynamics in the NKPC, while the coefficient on the marginal cost measure is not significantly different from zero. Second, the results with respect to the Taylor rule allow us to conclude that in the Volker-Greenspan period, monetary policy satisfied the Taylor principle. For this period, we also evaluate

the usefulness of survey-based expectations as instruments and find that they can somewhat improve precision of the Taylor rule estimates if added to the factor-based instrument set or to the variable set of the factor model.

The structure of the paper is as follows. In Section 1.2, we introduce the hybrid NKPC, as well as the assumed Taylor rule and the corresponding transmission mechanism. Section 1.3 presents our approach, and Section 1.4 corresponding results. Section 1.5 concludes.

## 1.2 Model

### 1.2.1 The Hybrid New Keynesian Phillips Curve

We analyze the hybrid version of the NKPC as used by Galí and Gertler (1999) and Kleibergen and Mavroeidis (2009), among others. This version of inflation dynamics includes both forward- and backward-looking elements:

$$\pi_t = \delta mc_t + \gamma_f \mathbb{E}_t \pi_{t+1} + \gamma_b \pi_{t-1} + u_t, \quad (1.1)$$

where  $\pi_t$  and  $mc_t$  are the inflation rate and a measure of marginal costs, respectively, and  $\mathbb{E}_t$  is the expectation operator with respect to information up to time  $t$ . The parameter  $\delta$  is the slope, and  $\gamma_f$  and  $\gamma_b$  can be interpreted as the respective weights on forward- versus backward-looking dynamics in the economy. The variable  $u_t$  is an unobserved cost-push shock with  $\mathbb{E}_{t-1} u_t = 0$ . The estimation equation is obtained by replacing expected future inflation by its realization:

$$\pi_t = \delta mc_t + \gamma_f \pi_{t+1} + \gamma_b \pi_{t-1} + e_t^{(1)}, \quad (1.2)$$

where the resulting error  $e_t^{(1)} = u_t - \gamma_f(\pi_{t+1} - \mathbb{E}_t \pi_{t+1})$  may be autocorrelated at lag 1.

### 1.2.2 A Model of Monetary Policy

#### *A Forward-Looking Taylor Rule*

The conduct of monetary policy we assume is the Clarida, Galí, and Gertler (2000) version of a forward-looking Taylor rule with a certain degree of interest rate smooth-

ing, which is also used in Mavroeidis (2010):

$$r_t = \alpha + \rho(L) r_{t-1} + (1 - \rho)(\psi_\pi \mathbb{E}_t \pi_{t+1} + \psi_x \mathbb{E}_t x_t) + \varepsilon_t, \quad (1.3)$$

where the variables  $r_t$ ,  $\pi_{t+1}$ , and  $x_t$  are the policy interest rate, the one-period-ahead inflation rate, and the output gap, respectively.<sup>2</sup> The monetary policy shock is an i. i. d. innovation such that  $\mathbb{E}_{t-1} \varepsilon_t = 0$ . The intercept  $\alpha$  is a linear combination of the inflation and the resulting interest rate target, and  $(\psi_\pi, \psi_x)$  are the feedback coefficients of the policy rule.  $\rho(L) = \rho_1 + \rho_2 L + \dots + \rho_n L^{n-1}$  displays the degree of policy smoothing, where  $L$  is the lag operator, and  $\rho = \rho_1 + \rho_2 + \dots + \rho_n$ .

The estimation equation is once more obtained by replacing the expected values by their realizations:

$$r_t = \alpha + \rho(L) r_{t-1} + (1 - \rho)(\psi_\pi \pi_{t+1} + \psi_x x_t) + e_t^{(2)}, \quad (1.4)$$

where the resulting error  $e_t^{(2)} = \varepsilon_t - (1 - \rho)[\psi_\pi(\pi_{t+1} - \mathbb{E}_t \pi_{t+1}) + \psi_x(x_t - \mathbb{E}_t x_t)]$  may exhibit first-order autocorrelation.

### *Transmission Mechanism*

The transmission mechanism used to interpret the results is fully characterized by two equilibrium conditions which are derived from a standard New Keynesian sticky-price model by log-linearization around the steady state (see e. g. Clarida, Galí, and Gertler, 2000; Lubik and Schorfheide, 2004). Together with equation (1.3) these two conditions, namely an Euler equation for output,  $y_t = \mathbb{E}_t y_{t+1} - \sigma(r_t - \mathbb{E}_t \pi_{t+1}) + g_t$ , and a version of the New Keynesian Phillips Curve,  $\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda(y_t - z_t)$ , capture the dynamics of the model. The output elasticity of inflation  $\lambda > 0$  reflects the degree of nominal rigidities,  $0 < \beta < 1$  is the discount factor,  $y_t$  stands for output, and  $z_t = y_t - x_t$  captures variation in the marginal cost of production. In the Euler equation  $\sigma$  is the intertemporal elasticity of substitution and  $g_t$  represents exogenous shifts in preferences and government spending.

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<sup>2</sup>As the output gap  $x_t$  is not known at the time the interest rate is set in period  $t$ , we use its expected value.

As highlighted in Woodford (2003, ch. 4), determinacy in this model requires:

$$\psi_\pi + \frac{1 - \beta}{\lambda} \psi_x - 1 \geq 0. \quad (1.5)$$

Further, the interest rate response should not be too strong – a condition that is not binding for the empirical results in this paper.<sup>3</sup>

Equation (1.5) is a generalized version of Taylor’s principle that the policy rate should be raised more than one for one with inflation to guarantee macroeconomic stability and can be seen as a benchmark to evaluate monetary policy (see Taylor (1999) for a qualitative and Clarida, Galí, and Gertler (2000) for a more quantitative perspective on this principle).

## 1.3 Factor-GMM Methodology

### 1.3.1 Benchmark Specifications

As the realizations of future inflation and the output gap are unknown at time  $t$ , we estimate both models with GMM assuming rational expectations, where the moment conditions are  $\mathbb{E}Z_t^{(i)}e_t^{(i)} = 0$  for any predetermined instrument set  $Z_t^{(i)}$  and  $i = 1, 2$ . For both models we use an estimation sample consisting of quarterly data from 1961:I to 2006:I (see the data appendix for details). This corresponds exactly to the specifications in Mavroeidis (2010) and is similar to that in Kleibergen and Mavroeidis (2009).<sup>4</sup>

#### *New Keynesian Phillips Curve*

In accordance with the paper by Kleibergen and Mavroeidis (2009) we estimate the NKPC with the labor share as a proxy for marginal costs and a benchmark instrument set that comprises three lags of inflation and the labor share.<sup>5</sup>

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<sup>3</sup>Recent studies show that other factors might also be important in guaranteeing determinacy (see e.g. Davig and Leeper, 2007; Coibion and Gorodnichenko, 2011). Cochrane (2011) argues that the existence of a unique equilibrium in a New Keynesian model with a Taylor rule requires imposing strong assumptions. Further, he shows analytically that the forward-looking version we analyze in this paper can be identified.

<sup>4</sup>The data set in the latter study goes until 2007:4 which, however, would not be possible in our context given limited data availability for the factor model.

<sup>5</sup>In order to guarantee comparability with the study by Kleibergen and Mavroeidis (2009) we



Point estimates by Galí and Gertler (1999) indicate a dominance of forward-over backward-looking dynamics and further that the coefficient on the labor share is positive and significantly different from zero.<sup>6</sup> Recent criticism of such an approach emphasizes that the parameters of the NKPC could be weakly identified, and thus researchers should rely on weak-instrument robust inference (see e. g. Ma, 2002; Mavroeidis, 2004, 2005). It has been shown that conventional GMM methods can be biased in the single-equation context, when the expected Jacobian of the moment equation is not of full rank as the instruments are insufficiently correlated with the relevant first-order conditions (see Stock and Wright, 2000; Mavroeidis, 2004, among others).

Hence, Kleibergen and Mavroeidis (2009) base their interpretations on one- and two-dimensional confidence sets that are found by inverting weak-identification robust statistics such as Stock and Wright's S or Moreira's MQLR, which are applications to GMM of the Anderson-Rubin and Morereira's CLR statistic, respectively, as well as the K-LM and the JKLM statistic from Kleibergen (2005).<sup>7</sup> In our analysis we rely on the combined K-LM test discussed in Kleibergen (2005) and also used in Mavroeidis (2010) that is a combination of a 9 percent level K-LM test and a 1 percent level JKLM test, which improves the power of the former test against irrelevant alternatives.<sup>8</sup> Further, Newey and Windmeijer (2009) show that this version of the K-LM test and the test based on the MQLR statistic are asymptotically valid even under many weak moment conditions. These results, however, do not apply to the finite sample case if many moments are arbitrarily weak (e. g. if the instruments are irrelevant).

Kleibergen and Mavroeidis (2009) find confidence intervals that are so wide as to accommodate both dominant backward- and dominant forward-looking dynamics, i. e., values of  $\gamma_f$  both larger and smaller than 0.5, respectively. Further, they provide evidence that the coefficient on labor share is statistically indistinguishable from zero.

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treat the labor share as endogenous.

<sup>6</sup>Note that the estimation sample in the study by Galí and Gertler (1999) only goes until 1997:IV and that their instrument set also contains lags of the long-short interest rate spread, output gap, wage inflation, and commodity price inflation.

<sup>7</sup>For a discussion of the behavior of these statistics see the latter paper.

<sup>8</sup>To have more reliable results, we actually use a combination of a 4.5 percent level K-LM test and a 0.5 percent level JKLM test. Henceforth, whenever we mention the K-LM test we refer to this combined version.

### *Taylor Rule*

For the Taylor rule the benchmark instrument set consists of four lags of each the Federal Funds rate, inflation, and the output gap. The estimation sample is split such that the pre-Volcker and Volcker-Greenspan periods run from 1961:I to 1979:II and 1979:III to 1997:IV, respectively. We also briefly consider a third period from 1987:III to 2006:I which corresponds to the mandate of Alan Greenspan. Mavroeidis (2010) uses the same instrument set and time periods, and in order to guarantee comparability of our results, we stick with the additional assumption that  $n = 2$  for the first and  $n = 1$  for the following time periods, i. e.,  $\rho(L) = \rho_1 + \rho_2 L$  and  $\rho(L) = \rho_1$ , respectively.<sup>9</sup>

Clarida, Galí, and Gertler (2000) find evidence that in the pre-Volcker period monetary policy was accommodative to inflation and therefore might have allowed for sunspot fluctuations in inflation, while in the second era it satisfied the Taylor principle, as depicted by inequality (1.5).

It has been pointed out, however, that estimation of DSGE models may be subject to the weak-identification problem (see e. g. Lubik and Schorfheide, 2004; Canova and Sala, 2009). Therefore, Mavroeidis (2010) reconsiders the empirical evidence of Clarida, Galí, and Gertler (2000) by testing different joint parameter specifications for the feedback coefficients of the Taylor rule using the K-LM test that is weak-instrument robust and for a high degree of overidentification more powerful than a test based on Stock and Wright's S statistic (see Kleibergen, 2005).

For the pre-Volcker period Mavroeidis' results support the previous finding that monetary policy did not satisfy the Taylor principle. For the second subsample, on the other hand, he shows that there is inconclusive evidence whether a determinate equilibrium exists or not due to uninformative confidence sets.

### **1.3.2 A Factor Model**

The size of the weak-IV robust confidence sets by Kleibergen and Mavroeidis (2009) and Mavroeidis (2010) suggests that in both models instruments are indeed weak,

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<sup>9</sup>Clarida, Galí, and Gertler (2000) use four lags of commodity price inflation, M2 growth, and the spread between the long-term bond rate and the three-month Treasury bill rate as additional instruments and consider slightly different time periods, where the first period spans 1960:I to 1979:II and the second 1979:III to 1996:IV.

and therefore stronger instruments are called for. Thus, we follow the approach of generating factors from a large macroeconomic data set and using them in the first stage of the estimation as discussed for the NKPC by Beyer et al. (2008) and Kapetanios and Marcellino (2010) and for Taylor rules in Bernanke and Boivin (2003) and Favero, Marcellino, and Neglia (2005). In contrast to these authors, who consider only point estimates, we also analyze joint confidence sets of the parameter estimates. This enables us to make inference with respect to the Taylor principle. Further, we provide a discussion on the comparison of forward- and backward-looking dynamics in the NKPC jointly with an analysis of the coefficient on the labor share. The rationale underlying the use of Factor GMM is that a central banker relies on a large information set in his forecasts of important macroeconomic variables. While each individual variable in this data set is only weakly correlated with future inflation, the output gap, or the labor share and therefore contains only little information, the factors serve as a summary of that information and are thus better predictors for our variables of interest (Bernanke and Boivin, 2003).

The results by Stock and Watson (2002, 2008) indicate that the factors derived from their data sets contain important information with respect to inflation and output. Consequently, they have the potential to make the benchmark instrument set stronger. In order for the factors to be appropriate instruments, we need to make sure that they are uncorrelated with the error terms in equations (1.2) and (1.4). Therefore, the validity of the overidentifying restrictions is discussed in Section 1.4.

The properties of Factor-IV and Factor-GMM estimation are analyzed with Monte-Carlo simulations by Bai and Ng (2010) and Kapetanios and Marcellino (2010), respectively. Kapetanios, Khalaf, and Marcellino (2011) evaluate factor-based weak-IV robust statistics. Favero, Marcellino, and Neglia (2005) compare two different ways to construct factors in a dynamic factor model: dynamic and static principal components (for the two approaches see Forni, Hallin, Lippi, and Reichlin, 2000 and Stock and Watson, 2002, respectively). The authors report that the results for the two methods are comparable. Overall the static factors perform slightly better in their applications, while the dynamic factors seem to provide a better summary of information as fewer factors explain as much variation in the variables from the data set. For simplicity we rely on static principle components, given that the performance of

both methods seems comparable.

Principal component analysis relies on the assumption that the set of variables is driven by a small set of factors and some idiosyncratic shocks. We assume the data-generating process underlying the variables to admit a factor representation:

$$X_t = \Lambda F_t + \nu_t, \quad (1.6)$$

where  $X_t$  is an  $N \times 1$  vector of zero-mean,  $I(0)$  variables,  $\Lambda$  is an  $N \times k$  matrix of factor loadings,  $F_t$  is an  $k \times 1$  vector of the factors, and  $\nu_t$  is an  $N \times 1$  vector of idiosyncratic shocks, where  $N$ , the number of variables, is much larger than the number of factors  $k$ . Static factors can be estimated by minimizing the following objective function:

$$V_{N,T}(F, \Lambda) = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T (X_{it} - \Lambda'_i F_t)^2, \quad (1.7)$$

where  $F = (F_1, F_2, \dots, F_T)'$ ,  $\Lambda'_i$  is the  $i$ -th row of  $\Lambda$ ,  $X_{it}$  is the  $i$ -th component of  $X_t$ , and  $T$  is the number of time periods.

### 1.3.3 Data Set

To construct the factors we employ the data set by Stock and Watson (2008), which is an updated version of the data they use for former papers, e. g. Stock and Watson (2002). The subset of this data set relevant for the estimation of factors includes 109 quarterly time series that have strong information content with respect to inflation and output, consisting of disaggregated price and production data, as well as indices, among others. The time series span 1959:III to 2006:IV with  $T = 190$  observations. We use principal component analysis to extract the factors from the transformed data series, where we carried out the same transformations as indicated in Stock and Watson (2008) to guarantee stationarity of both the time series and the resulting factors (see the data appendix for details).

Stock and Watson (2008) use the factors for forecasting and provide evidence that if potential changes in the factor model are sufficiently small there is a particular benefit in calculating the factors for the whole data set by principal components,

even if there exists a structural break in the forecasting equation.<sup>10</sup> Moreover, in the construction of the factors having more observations increases the signal-to-noise ratio.

So far there is no general consensus on how to determine the number of factors  $k$ . We rely on the criteria that are recommended by Bai and Ng (2002) in this context ( $PC_1$ ,  $PC_2$ ,  $IC_1$ ,  $IC_2$ ) and are frequently used in the literature on factor models as they seem to perform well for large  $N$ . The PC criteria, which are shown to rather overestimate the true number of factors, are consistent with five or six factors, whereas the IC criteria are consistent with two or four factors for the whole data set. Based on these results and the canonical correlations between subsample and full-sample estimates of the factors, Stock and Watson (2008) make a case for using four factors, and we follow their suggestion. Using more factors does not improve our estimation results significantly, while it introduces even more instruments, and with fewer factors the results are somewhat less accurate; in either case the main conclusions would persist.<sup>11</sup>

## 1.4 Results

### 1.4.1 New Keynesian Phillips Curve

We estimate equation (1.2) as described in Subsection 1.3.1 and employ the same data set as Kleibergen and Mavroeidis (2009) for the benchmark results. However, in order to have more information with respect to the two endogenous variables and thus more precise estimation results, we expand the benchmark instrument set by the four factors we generated from the Stock and Watson (2008) data set. As the contemporaneous values of the factors may be correlated with the error term  $e_t^{(1)}$ , we

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<sup>10</sup>If one interprets the factor model as a set of policy functions, where the factors can be seen as states, a structural break in the Taylor rule has the potential to cause a break in the factor model. However, as Stock and Watson (2008) show, the factor model is relatively stable such that any potential regime change in monetary policy conduct would have only affected the dynamics of the benchmark instruments while the factor model implied policy functions are relatively unchanged.

<sup>11</sup>More recently proposed criteria like those by Onatski (2009) or Ahn and Horenstein (2013) are in line with our choice. The criterion by Onatski as well as the two criteria by Ahn and Horenstein predict two factors. Simulations by the respective authors have shown that these criteria tend to rather underestimate the true number of factors. As underestimation of the number of factors is more severe than overestimation in this context, the use of four factors seems a reasonable choice.

Table 1.1: Point Estimates for the Parameters of the NKPC

|            | Time period (in quarters) |                   |
|------------|---------------------------|-------------------|
|            | 1961:I-2006:I             |                   |
|            | BM                        | Factor GMM        |
| $\delta$   | 0.02<br>(0.03)            | 0.03*<br>(0.02)   |
| $\gamma_f$ | 0.73***<br>(0.08)         | 0.65***<br>(0.02) |
| $\gamma_b$ | 0.27***<br>(0.08)         | 0.34***<br>(0.02) |

*Notes:* \*\*\*, \*\*, and \* denote significance at the 1, 5, and 10 percent level, respectively. Standard errors are in brackets. Estimation of the NKPC, equation (1.2), is conducted by GMM using Newey-West weight matrix. BM refers to the results based on the benchmark instrument set comprising three lags of each inflation and the labor share. The Factor-GMM results are generated extending the instrument set by lags one to four of the factors derived before.

use only their first four lags as instruments. To investigate whether the overidentifying restrictions are satisfied, we calculate the weak-identification robust S sets for both instrument sets considered. These confidence sets are based on the S statistic that equals the value of the GMM objective function at the parameter values of the null hypothesis. They contain all parameter values, where one cannot jointly reject the null hypothesis and the validity of the overidentifying restrictions. The fact that the S sets are indeed not empty provides evidence that our identifying assumptions are reasonable (see Stock and Wright, 2000).

Point estimates are presented in Table 1.1. As discussed in Kleibergen and Mavroeidis (2009), results based on the benchmark instrument set indicate a dominance of forward- over backward-looking dynamics with parameter values of  $(\gamma_f, \gamma_b) = (0.73, 0.27)$  both being significant at the 1 percent level. This is in line with the findings by Galí and Gertler (1999). The coefficient on the labor share is positive and – unlike in the latter study – insignificant. Including the factors in the instrument set yields more precise estimates of the parameters with all standard errors reduced

substantially. In the Factor-GMM model the labor share is positive and significant at the 10 percent level. However, one needs to keep in mind that in the case of weak instruments point estimates are unreliable. Further, it needs to be taken into account that using conventional two-step procedures after pretesting for identification is not recommended, as the size of such methods cannot be controlled (see e. g. Andrews, Moreira, and Stock, 2006). Similar to Kleibergen and Mavroeidis (2009) we thus rely on two-dimensional confidence sets found by inverting the weak-IV robust K-LM statistic (see Subsection 1.3.1), which does not seem to display a serious power loss in the case of strong instruments (Kleibergen, 2005). The fact that the factor-based confidence sets are smaller than the benchmark results provides evidence that our point estimates are more likely to be reliable.

Galí and Gertler (1999) and Kleibergen and Mavroeidis (2009) emphasize that a restricted model, where  $\gamma_f + \gamma_b = 1$ , performs well. Given that our point estimates support these findings we follow the approach by Kleibergen and Mavroeidis (2009) and from here on focus on the restricted model.<sup>12</sup>

Figure 1.1 shows the joint confidence sets at 95 percent significance for both the benchmark and the factor-based instrument set.<sup>13</sup> These sets contain all values of  $(\gamma_f, \delta)$  that cannot be rejected by the K-LM test. The shape of the K-LM sets may seem unconventional. However, note that confidence sets can be non-convex and unbounded if based on the K-LM statistic as explained by Kleibergen (2005).

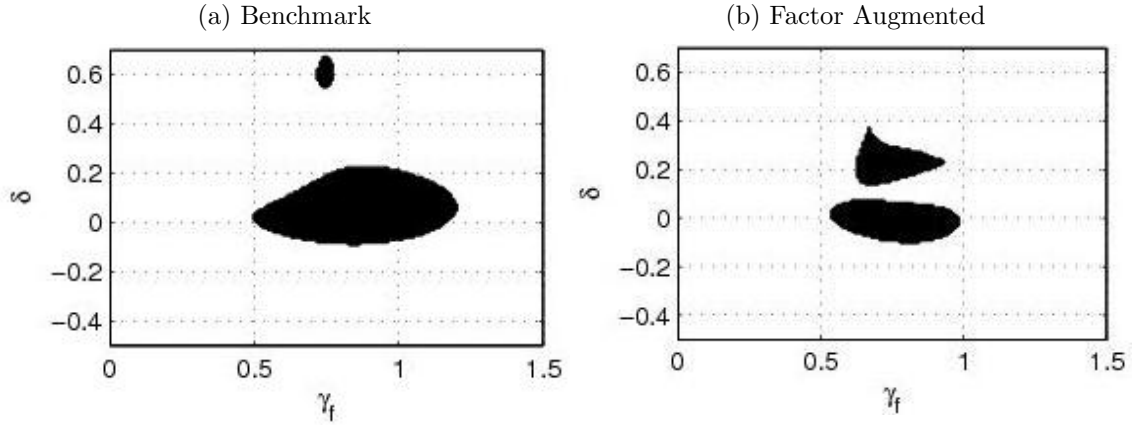
The robust confidence set based on the benchmark instrument as shown in Figure 1.1(a) is so large as to be in line with both dominant forward- and backward-looking dynamics. Further, the K-LM test cannot reject parameter values of  $1 < \gamma_f \leq 1.2$  which would imply a negative backward-looking coefficient. The largest part of the confidence set lies around a value of zero for the coefficient on the labor share  $\delta$ , indicating that the NKPC is relatively flat and that identification problems are present as explained in Kleibergen and Mavroeidis (2009). A small outlier part of the K-LM set lies around a value of  $\delta = 0.6$ .

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<sup>12</sup>Kleibergen and Mavroeidis (2009) argue that inflation can be non-stationary and hence for the restricted model the use of lags of  $\pi_t$  as instruments may violate the conditions necessary for asymptotic theory to apply. In order to control for this possibility we instead use lags of  $\Delta\pi_t$  in the restricted model as suggested by the authors.

<sup>13</sup>Figure 1.1 is constructed using MATLAB and the code by Kleibergen and Mavroeidis (2009). The factors are added as additional instruments.

Figure 1.1: 95 Percent Weak-Identification Robust Confidence Sets for the Coefficients of the NKPC



*Notes:* The figure shows weak-identification robust confidence sets for the coefficients  $(\gamma_f, \delta)$  of the NKPC, as specified in equation (1.2) under the restriction that  $\gamma_f + \gamma_b = 1$  for the period 1961:I to 2006:I using quarterly data. The left part shows the K-LM set using the benchmark instrument set comprising two lags of the first difference in inflation and three lags of the labor share. The right part depicts the K-LM set with lags one to four of the factors as additional instruments.

Figure 1.1(b) provides evidence that adding factors to the instrument set can improve on the estimation as the resulting confidence set is smaller than in the benchmark case. Containing only values of  $\gamma_f$  between 0.54 and 0.98 it provides evidence for dominant forward-looking dynamics. Further, the outlier region has vanished from the confidence set such that the range of values for  $\delta$  not rejected by the K-LM test is greatly reduced. However, as before a value of  $\delta = 0$  cannot be rejected at 95 percent significance. This finding highlights that the NKPC is relatively flat resulting in identification problems for the coefficient on the marginal cost measure, as stressed in the previous literature: e. g. Woodford (2003); Kleibergen and Mavroeidis (2009); Kapetanios and Marcellino (2010).

### 1.4.2 Taylor Rule

We estimate equation (1.4) using the same time periods and methods as Mavroeidis (2010), i. e., GMM with Newey-West weight matrix, and expand the benchmark instrument set by lags of the factors in order to achieve more precise estimation results.<sup>14</sup> The S sets are non-empty for both instrument sets and both periods

<sup>14</sup>Note that there are papers stressing the importance of using real-time rather than final revised data, e. g. Orphanides (2001). This is not a concern for our study, as we are interested in the actual



Table 1.2: Point Estimates for the Parameters of the Taylor Rule

|            | Time period (in quarters) |                   |                   |                   |                   |                   |
|------------|---------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|            | 1961:I-1979:II            |                   | 1979:III-1997:IV  |                   | 1987:III-2006:I   |                   |
|            | BM                        | Factor GMM        | BM                | Factor GMM        | BM                | Factor GMM        |
| $\alpha$   | 0.54***<br>(0.18)         | 0.76***<br>(0.08) | 0.16<br>(0.19)    | 0.36***<br>(0.13) | -0.18<br>(0.18)   | -0.07<br>(0.12)   |
| $\psi_\pi$ | 0.86***<br>(0.07)         | 0.83***<br>(0.03) | 2.24***<br>(0.32) | 1.91***<br>(0.18) | 2.80***<br>(0.65) | 2.80***<br>(0.68) |
| $\psi_x$   | 0.29***<br>(0.10)         | 0.19***<br>(0.04) | 0.82*<br>(0.43)   | 0.84***<br>(0.20) | 1.43***<br>(0.28) | 1.54***<br>(0.26) |
| $\rho$     | 0.68***<br>(0.10)         | 0.57***<br>(0.04) | 0.83***<br>(0.05) | 0.83***<br>(0.03) | 0.89***<br>(0.02) | 0.92***<br>(0.01) |

*Notes:* \*\*\*, \*\*, and \* denote significance at the 1, 5, and 10 percent level, respectively. Standard errors are in brackets. Estimation of the Taylor rule, equation (1.4), is conducted by GMM using Newey-West weight matrix. BM refers to the results based on the benchmark instrument set comprising four lags of each inflation, the interest rate, and the output gap. The Factor-GMM results are generated extending the instrument set by lags one to four of the factors derived before.

considered providing evidence for the validity of the overidentifying restrictions.

For illustrative purposes point estimates for our specification are presented in Table 1.2. Note, that the Factor-GMM results closely resemble the evidence by Favero, Marcellino, and Neglia (2005).<sup>15</sup> The results based on the benchmark instrument set are similar in spirit to Clarida, Galí, and Gertler (2000).<sup>16</sup> The confidence sets based on the K-LM statistic discussed below provide evidence that the new instrument set is stronger, and hence factor-based point estimates are more likely to be reliable. One should keep in mind, though, that in the presence of weak instruments point estimates are inconsistent and standard errors are not reliable. What stands out from the results is the substantial reduction in standard errors by roughly 50 percent for the first and second period and all coefficients. Consequently, in our specification

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feedback coefficients rather than the intended ones.

<sup>15</sup>Favero, Marcellino, and Neglia (2005) estimate a forward-looking Taylor rule for the US from 1979:I to 1998:IV. In contrast to them, however, we use a different benchmark instrument set, a different data set for generating the factors, and also consider the pre-Volcker and Greenspan period.

<sup>16</sup>In contrast to Clarida, Galí, and Gertler (2000), though, we leave out the three additional instruments commodity price inflation, M2 growth, and the spread between the long-term bond rate and the three-month Treasury Bill rate, as Mavroeidis (2010) does in his analysis. We verify that this does not influence the main results significantly.

all estimated coefficients (but  $\alpha$ ) are significant at the 1 percent level. The point estimates indicate that there is a shift in the conduct of monetary policy from the first period to the second. While the feedback coefficients  $(\psi_\pi, \psi_x)$  in the pre-Volcker regime are estimated to be  $(0.83, 0.19)$ , their estimates increase to  $(1.91, 0.84)$  in the Volcker-Greenspan regime. These results already point to a more aggressive response of monetary policy to inflation and the output gap in the second period. To get information about the more recent stance of monetary policy, we also include a third period, which coincides with the Greenspan regime, 1987:III to 2006:I. Monetary policy under Greenspan seems to be characterized by a high degree of smoothing ( $\rho = 0.92$ ), as also noted by Mavroeidis (2010), and an even stronger response to inflation and the output gap. The standard errors of the feedback coefficients are larger for this period, which is probably a result of the increased persistence of the policy rate (see Mavroeidis, 2010).

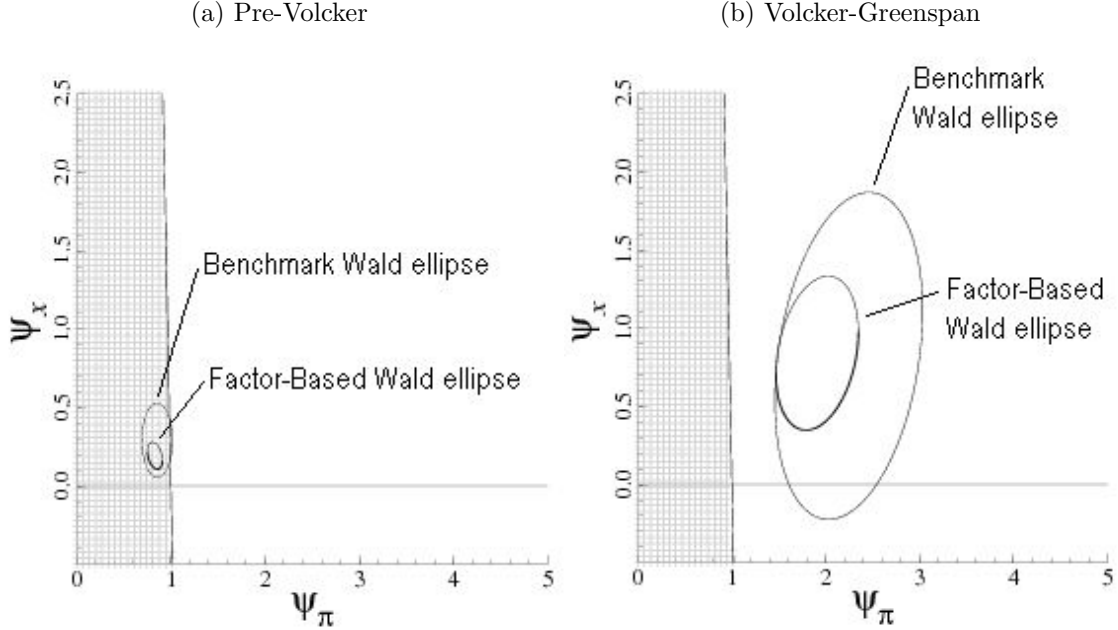
In order to be able to draw conclusions with respect to the Taylor principle, however, we consider joint estimates of the feedback coefficients. Figure 1.2 shows the Wald ellipses for the two parameters of interest, i. e.,  $\psi_x$  and  $\psi_\pi$ , based on the point estimates presented before.<sup>17</sup> Interpreting their results Clarida, Galí, and Gertler (2000) and Mavroeidis (2010) assume that the degree of nominal rigidities  $\lambda$  and the discount factor  $\beta$  are equal to 0.3 and 0.99, respectively. They argue that these assumptions are in line with empirical evidence, and we stick to them for comparability, verifying that they do not influence our main conclusions. The almost vertical line represents equation (1.5), i. e., the Taylor principle, under these assumptions, and is thus the boundary between indeterminacy (to the left) and determinacy (to the right).

For both periods discussed the factor-based Wald ellipse lies firmly within the ellipse based on the original instrument set. As presented in Figure 1.2(a), the pre-Volcker regime Wald ellipses are both located in the indeterminacy region. In contrast to that, the ellipses for the Volcker-Greenspan period have shifted to the determinacy region, as shown in Figure 1.2(b). These results provide evidence that the Taylor principle is satisfied under Volcker-Greenspan, while it has been violated

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<sup>17</sup>Figures 1.2 and 1.3 are constructed using the programming language Ox, see Doornik (2007), and the code by Mavroeidis (2010). The factors are added as additional instruments.

Figure 1.2: 95 Percent Wald Ellipses for the Feedback Coefficients of the Taylor Rule



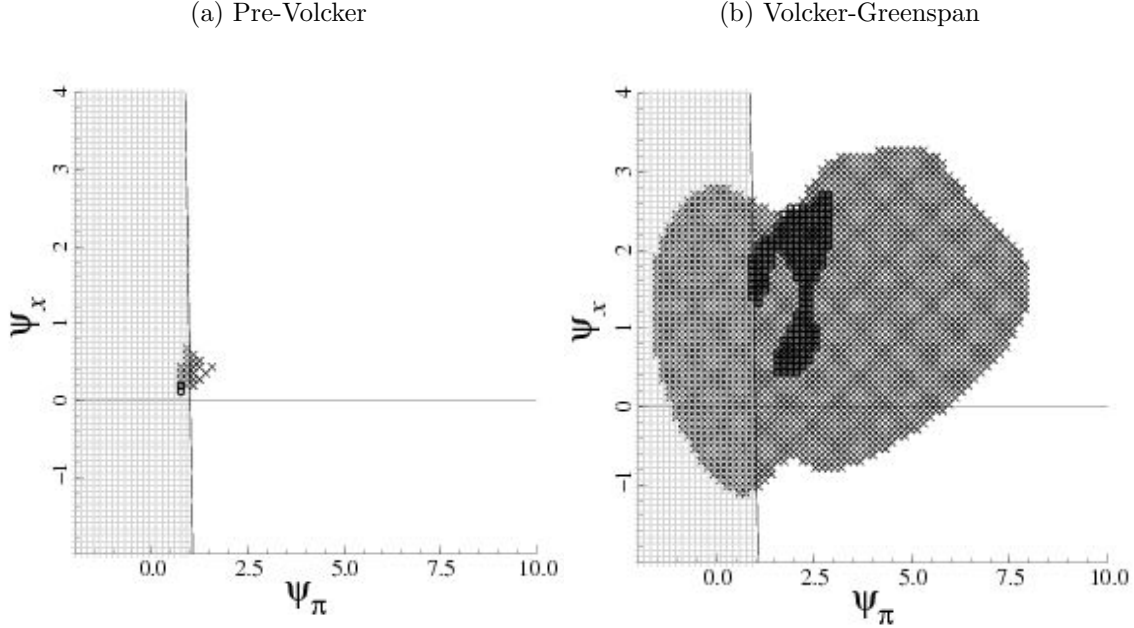
*Notes:* The Wald ellipses for the feedback coefficients  $(\psi_\pi, \psi_x)$  of the Taylor rule, as specified in equation (1.4), are constructed using GMM with four lags of the instruments and Newey-West weight matrix. The benchmark Wald ellipses are based on the point estimates similar to those by Clarida, Galí, and Gertler (2000), where the instrument set comprises four lags of each inflation, the interest rate, and the output gap. The factor-based results are generated extending the instrument set by lags one to four of the factors derived before. The almost vertical line represents equation (1.5), i.e., the Taylor principle with  $\lambda = 0.3$  and  $\beta = 0.99$ , being the boundary between indeterminacy (to the left) and determinacy (to the right).

before.

However, in the presence of weak instruments point estimates are inconsistent resulting in unreliable Wald ellipses. Therefore, we rely on the weak-IV robust K-LM test which guarantees comparability with the results of Mavroeidis (2010). Figure 1.3 shows the factor-based joint confidence sets at 95 percent significance for both subsamples (dark grey areas). For comparison we include the results from Mavroeidis (2010), namely the weak-IV robust confidence sets, constructed with the benchmark instrument set (light grey areas). These sets contain all values of  $(\psi_\pi, \psi_x)$  that cannot be rejected by the K-LM test.

Figure 1.3(a) provides further evidence that pre-Volcker monetary policy was not adherent to the Taylor principle, as the Factor-GMM confidence set also lies within the indeterminacy region. The large reduction in the size of the confidence

Figure 1.3: 95 Percent Weak-Identification Robust Confidence Sets for the Feedback Coefficients of the Taylor Rule



*Notes:* The figure shows weak-identification robust confidence sets for the feedback coefficients  $(\psi_\pi, \psi_x)$  of the Taylor rule, as specified in equation (1.4). The light grey areas (crosses) represent the K-LM sets as estimated by Mavroeidis (2010) using the benchmark instrument set comprising four lags of each inflation, the interest rate, and the output gap. The dark grey areas (circles) are the K-LM sets with lags one to four of the factors as additional instruments. The almost vertical line represents equation (1.5), i. e., the Taylor principle with  $\lambda = 0.3$  and  $\beta = 0.99$ , being the boundary between indeterminacy (to the left) and determinacy (to the right).

set for the second period corroborates our finding that the factors contain relevant information for the estimation. Most importantly, our confidence set clearly lies outside the indeterminacy region, while in contrast to that, Mavroeidis' confidence set for this time period has a considerable part in this very area, and his results are even consistent with negative values for both parameters. A substantial part of our confidence set is located around the point estimate of  $(\hat{\psi}_\pi, \hat{\psi}_x) = (1.91, 0.84)$ , whereas another part lies above it, showing that there is some remaining uncertainty with respect to the feedback coefficients of the Taylor rule. Our findings highlight that with the inclusion of additional important information it can be empirically shown that monetary policy conduct under Volcker and Greenspan was more aggressive towards fighting inflation than pre-Volcker and thus satisfied the Taylor principle.<sup>18</sup>

<sup>18</sup>A decrease in  $\lambda$  or  $\beta$  would rotate the boundary of the indeterminacy region counterclockwise around the intersection with the horizontal axis as explained by Mavroeidis (2010). For all admis-

The results with fewer factors or lags are less precise, but go in the same direction, i. e., a shift outwards from the indeterminacy region, while with more factors the results are comparable. Results using the weak-IV robust MQLR statistic (see Subsection 1.3.1) rather than the K-LM statistic are very similar providing evidence for the robustness of our findings. With the use of more recent data, i. e., until 2006:I, the confidence sets shift more towards the indeterminacy region, suggesting that there might have been some time variation in the conduct of monetary policy under Alan Greenspan.<sup>19</sup>

Our results corroborate the empirical evidence by Lubik and Schorfheide (2004), Coibion and Gorodnichenko (2011), Boivin and Giannoni (2006), or Inoue and Rossi (2011), among others. Using Bayesian methods, Lubik and Schorfheide (2004) estimate the parameters of the whole model that underlies our single-equation estimation, whereas Coibion and Gorodnichenko (2011) analyze a similar model under the assumption of a positive and time-varying inflation trend. Boivin and Giannoni (2006) examine the monetary transmission mechanism using a vector autoregressive framework. Albeit the different approaches, these studies find a move of the US economy from indeterminacy to determinacy as a result of a more aggressive monetary policy regime. Inoue and Rossi (2011) use both DSGE models and vector autoregressions allowing for structural breaks in all parameters and show that changes in monetary policy parameters have, among other factors, led to the Great Moderation.

### 1.4.3 The Number of Instruments

Comparing results based on the benchmark instrument set with those using a larger factor-based instrument set raises the question whether it is the information from the factors or just the increased number of instruments that causes the extra precision in the estimation of the Taylor rule for the Volcker-Greenspan period (Figure 1.3(b)).<sup>20</sup> In order to demonstrate that it is the former rather than the latter, we fix the number of instruments to be equal to the benchmark case for the comparison. These

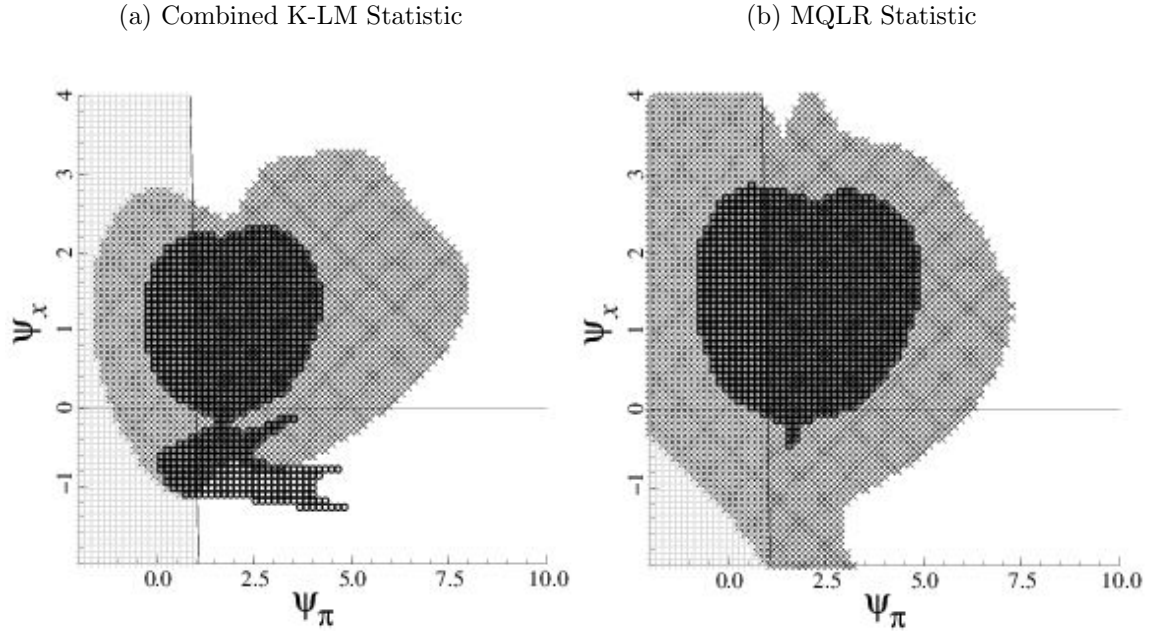
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sible values a change in either parameter would not alter our conclusion of determinacy for the second period as our confidence sets are already to the right of the boundary. Similarly, given our estimation results, for the first period  $\lambda$  would have to be smaller than 0.01 to change our finding of indeterminacy.

<sup>19</sup>The results for these alternative specifications are available from the authors upon request.

<sup>20</sup>We thank an anonymous referee for pointing this out.

Figure 1.4: 95 Percent Weak-Identification Robust Confidence Sets for the Feedback Coefficients of the Taylor Rule With Selected Instruments



*Notes:* The figure shows weak-identification robust confidence sets for the feedback coefficients  $(\psi_\pi, \psi_x)$  of the Taylor rule, as specified in equation (1.4), for the Volcker-Greenspan period. The light grey areas (crosses) represent the confidence sets as estimated by Mavroudis (2010) using the benchmark instrument set comprising four lags of each inflation, the interest rate, and the output gap. The dark grey areas (circles) are the confidence sets with the instruments selected by means of hard thresholding, namely the exogenous first lag of the interest rate, the first four lags of each inflation, and the output gap, and the second lag of factor one and two and the fourth lag of factor two. Figure 1.4(a) and (b) show results based on the combined K-LM statistic and the MQLR statistic, respectively. The almost vertical line represents equation (1.5), i.e., the Taylor principle with  $\lambda = 0.3$  and  $\beta = 0.99$ , being the boundary between indeterminacy (to the left) and determinacy (to the right).

instruments are selected by means of hard thresholding as suggested by Bai and Ng (2008) which amounts to ranking all instruments by their explanatory power for the endogenous variables (see Appendix B for more details). In the following analysis, the twelve highest-ranked instruments from the factor-based set are used, leading to an instrument set of the same size as in the benchmark case. This procedure yields the following instruments: Apart from the exogenous first lag of the interest rate, the first four lags of inflation and the output gap are included which does not come as a surprise given the relative persistence in either variable. Further, the second lag of the first two factors and the fourth lag of factor four are selected.

Confidence sets for the combined K-LM statistic and the MQLR statistic based

on these twelve instruments are presented in Figure 1.4. The results are more precise than the results using the benchmark instrument set of the same size where a higher relative precision is even clearer for the confidence set based on the MQLR statistic. This highlights that the factors contain relevant information for inflation and the output gap, and thus it is not just the increased number of instruments which drives the results in Figure 1.3(b).<sup>21</sup>

#### 1.4.4 Using Survey Expectations as Instruments

Results for the Taylor rule estimates during the Volcker-Greenspan period indicate that parameters are still somewhat imprecisely estimated. Given that expectations of future inflation are available from surveys these should have explanatory power for actual realizations. Ang, Bekaert, and Wei (2007) show that inflation surveys are successful in forecasting inflation out-of-sample over the next year. Moreover, Coibion (2010) and Adam and Padula (2011) estimate different versions of the Phillips Curve, where they replace expected future inflation by expectations from the Survey of Professional Forecasters (SPF) arguing that this approach yields plausible estimates. Similarly, Orphanides (2004) estimates Taylor rules where he replaces expected future inflation by Greenbook forecasts for the specific horizons.

In order to further improve results, we use survey expectations in two different ways in our estimation procedure. On the one hand, we expand the factor-augmented instrument set by one lag of the mean of expected inflation two-periods ahead, i. e.,  $\mathbb{S}_{t-1}\pi_{t+1}$ , and one lag of the mean of expected output growth one-period ahead from the SPF, i. e.,  $\mathbb{S}_{t-1}g_{y,t}$  (see the data appendix for details).<sup>22</sup> On the other hand, we expand the variable set in the factor model by the two survey variables from the SPF. We estimate four factors from the survey-augmented data set and add their first four lags to the benchmark instrument set.

Figure 1.5 shows the results for these two specifications which are rather similar.

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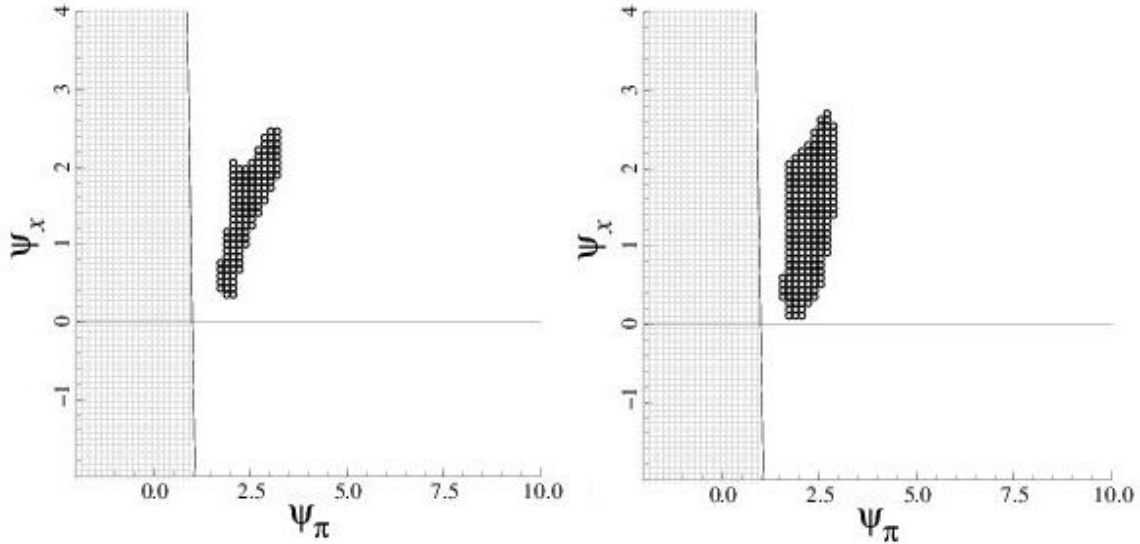
<sup>21</sup>The oddly-shaped lower part of the confidence region based on the K-LM statistic below the x-axis is related to the fact that the behavior of the K statistic is spurious around inflection points and extrema. Increasing the weight on the J statistic ensures that this region vanishes.

<sup>22</sup>Given that expected output gaps are not provided we also construct expected output gap estimates by using the one-sided Christiano-Fitzgerald filter (2003), however, this does not change the main results. We also use median values rather than means, however, this does not seem to have substantial influence either.

Figure 1.5: 95 Percent Weak-Identification Robust Confidence Sets for the Feedback Coefficients of the Taylor Rule With Survey Data

(a) Adding Survey Expectations

(b) Survey-augmented Factors



*Notes:* The figure shows weak-identification robust confidence sets for the feedback coefficients  $(\psi_\pi, \psi_x)$  of the Taylor rule, as specified in equation (1.4), for the Volcker-Greenspan period. The left graph shows the K-LM set estimated using the factor-based instrument set (see notes of Figure 1.3) expanded by  $S_{t-1}\pi_{t+1}$  and  $S_{t-1}g_{y,t}$  taken from SPF (mean values). The right graph depicts the results, where the variable set in the factor model has been expanded by the variables mentioned before. The almost vertical line represents equation (1.5), i. e., the Taylor principle with  $\lambda = 0.3$  and  $\beta = 0.99$ , being the boundary between indeterminacy (to the left) and determinacy (to the right).

In comparison to the factor-based results, the estimated output gap coefficient  $\psi_x$  is essentially unaffected. The estimate of the parameter on expected future inflation  $\psi_\pi$  is more precise resulting in confidence sets that are more clearly located in the determinacy region. We also use the Greenbook forecasts provided by the Federal Reserve for the variables discussed before instead of those from the SPF. Given that the results are very similar, we omit them here.<sup>23</sup>

<sup>23</sup>We also estimate a version, where we extend the benchmark instrument set by lags of the survey variables rather than the factors. However, it turns out that the factors yield much more precise estimates. This finding could be explained by the evidence of Nunes (2010), who shows that rational expectations play a more dominant role in inflation dynamics than do survey expectations. Also, Coibion (2010) shows that surveys consistently overestimated inflation in the 1980's and 1990's. A different reason could relate to the fact that we use revised data, whereas the surveys contain real-time expectations. It may thus be the case that surveys are more informative in predicting variables in real-time. Finally, expected future output growth does not seem to be very informative with respect to future output gaps.



## 1.5 Conclusion

In this paper, we conduct factor-based inference of the hybrid New Keynesian Phillips Curve and a forward-looking version of the Taylor rule, as analyzed by Kleibergen and Mavroeidis (2009) and Mavroeidis (2010), respectively. These authors evaluate the models by using weak-identification robust methods. However, both studies find large confidence sets such that reliable interpretation of the estimated parameters is impaired. Therefore, we propose to employ factors generated from a large macroeconomic data set as additional instruments. The inclusion of these factors in the estimation procedure reduces the size of weak-identification robust confidence sets substantially. On the one hand, we show that forward-looking dominate backward-looking dynamics in the NKPC, while the curve is so flat that we cannot exclude a coefficient of zero on the marginal cost measure. On the other hand, our results with respect to the Taylor rule allow us to conclude that monetary policy in the after-1979 Volcker-Greenspan period satisfied the Taylor principle and thus contributed to containing inflation dynamics from there on. Our paper highlights that Factor GMM can be a useful tool to overcome the weak-identification problem common to many macroeconomic applications.

# **A1 Appendix to Chapter 1**

## **A1.1 Data Appendix**

### **New Keynesian Phillips Curve**

For the estimation of the NKPC we use quarterly US data for the GDP deflator and the labor share from 1960:I to 2006:II from Kleibergen and Mavroeidis (2009).

Website:

[http://www.econ.brown.edu/fac/Frank\\_Kleibergen/](http://www.econ.brown.edu/fac/Frank_Kleibergen/)

### **Taylor Rule**

For the estimation of the Taylor rule we use the same data set as Mavroeidis (2010). It consists of the federal funds rate, the annualized quarter-on-quarter inflation rate based on the seasonally adjusted GDP deflator, and the CBO output gap for the US. Data is of quarterly frequency from 1960:I to 2006:II.

Website:

[http://www.aeaweb.org/aer/data/mar2010/20071447\\_data.zip](http://www.aeaweb.org/aer/data/mar2010/20071447_data.zip)

### **Factor Data**

For generating the factors we use quarterly data for the US from 1959:III to 2006:IV by Stock and Watson (2008), which is an updated version of the data they use for former papers, e. g. Stock and Watson (2002). Details for the 109 quarterly time series that have strong information content with respect to inflation and output, as well as the transformations needed to guarantee stationarity are provided by Stock and Watson (2008) in the data appendix of their paper.

Website:

[http://www.princeton.edu/~mwatson/papers/hendryfestschrift\\_stockwatson\\_April282008.pdf](http://www.princeton.edu/~mwatson/papers/hendryfestschrift_stockwatson_April282008.pdf)

## Survey Data

The survey data can be downloaded from the Philadelphia FED. From the SPF we use mean two-quarter ahead expectations of the growth rate of the GDP deflator (dpgdp4) and mean one-quarter ahead expectations of GDP growth (rgdp3). The same variables are used from the Greenbook forecasts (i. e., PGDPdot4 and RGDPdot3).

Websites:

[http://www.phil.frb.org/research-and-data/real-time-center/  
survey-of-professional-forecasters/](http://www.phil.frb.org/research-and-data/real-time-center/survey-of-professional-forecasters/)

[http://www.philadelphiafed.org/research-and-data/real-time-center/  
greenbook-data/philadelphia-data-set.cfm](http://www.philadelphiafed.org/research-and-data/real-time-center/greenbook-data/philadelphia-data-set.cfm)

## A1.2 Hard Thresholding

To order the instruments for the Taylor rule we conduct hard thresholding as suggested by Bai and Ng (2008). Hard thresholding amounts to ranking the instruments by their explanatory power for the endogenous variables. The estimation equation for this is:

$$X_{end,t} = \gamma_0 + \gamma_1 X_{exo,t} + \gamma_{2,i} Z_{i,t} + \eta_{i,t}. \quad (1.8)$$

The endogenous variable  $X_{end,t}$  is regressed on a constant, the exogenous variables  $X_{exo,t}$  (the lagged policy rate in our case), and an instrument  $Z_{i,t}$ . The error term  $\eta_{i,t}$  is assumed to be i.i.d. This equation is estimated for both endogenous variables  $\pi_{t+1}$  and  $x_t$  and for all instruments  $i = 1, \dots, 27$ . For both endogenous variables we develop a ranking of all instruments according to the  $t$  statistic for their respective coefficients  $\gamma_{2,i}$ . For the instrument set in the estimation of the Taylor rule we always include the exogenous variable and first add the highest ranked variable of the regression on  $\pi_{t+1}$  followed by the highest ranked from the regression on  $x_t$  that is not yet included. We proceed in this way until we have the number of instruments desired. We start with an instrument from the regression on  $\pi_{t+1}$  given that from the first stage  $R^2$  it seems that it is more difficult to predict inflation than the output gap (see Table 1.3 for the resulting ranking of the instruments).

Table 1.3: Hard Thresholding for the Taylor Rule

| No. of instruments | Instrument name | Variable | Ranking   |
|--------------------|-----------------|----------|-----------|
| 1                  | fyff_l1         |          | exogenous |
| 2                  | infl_l1         | infl     | 1         |
| 3                  | gap_l1          | gap      | 1         |
| 4                  | infl_l2         | infl     | 2         |
| 5                  | gap_l2          | gap      | 2         |
| 6                  | infl_l3         | infl     | 3         |
| 7                  | gap_l3          | gap      | 3         |
| 8                  | infl_l4         | infl     | 4         |
| 9                  | gap_l4          | gap      | 4         |
| 10                 | fac2_l2         | infl     | 5         |
| 11                 | fac1_l2         | gap      | 5         |
| 12                 | fac2_l4         | infl     | 6         |
| 13                 | fyff_l4         | gap      | 6         |
| 14                 | fac2_l1         | infl     | 7         |
| 15                 | fac1_l1         | gap      | 7         |
| 16                 | fac2_l3         | infl     | 8         |
| 17                 | fac1_l3         | gap      | 8         |
| 18                 | fac4_l2         | infl     | 13        |
| 19                 | fac1_l4         | gap      | 9         |
| 20                 | fac3_l1         | infl     | 17        |
| 21                 | fyff_l3         | gap      | 10        |
| 22                 | fac4_l4         | infl     | 18        |
| 23                 | fyff_l2         | gap      | 11        |
| 24                 | fac4_l1         | infl     | 19        |
| 25                 | fac4_l3         | gap      | 14        |
| 26                 | fac3_l2         | infl     | 22        |
| 27                 | fac3_l4         | gap      | 22        |
| 28                 | fac3_l3         | infl     | 27        |

*Notes:* Abbreviations: infl=inflation, gap=output gap, fyff=interest rate, var.li= $i$ -th lag of var, fac*i*= $i$ -th factor. This table shows the ranking from hard thresholding of the instruments for the Taylor rule. The first column presents the final ranking, the second gives the name of the variable, and the last two show the ranking of it for either inflation or the output gap.

# Aggregation and Labor Supply Elasticities

## 2.1 Introduction

The aggregate Frisch elasticity of labor supply has been at center stage in modern business cycle analysis for many years. It was first introduced into the literature by Ragnar Frisch and continues to be of interest from a theoretical as well as from an empirical perspective. At any point in time, it measures the reaction of total hours worked to a small change in the mean wage when wealth is held constant. The exact size of this particular elasticity matters a lot when macroeconomists try to assess the quantitative implications of certain types of policies on employment and hours worked. For example, changes in monetary or fiscal policy parameters which directly or indirectly impact a worker's net wage rate typically lead to a change in total labor supply. In spite of its relevance, the size of this aggregate change cannot easily be determined when worker heterogeneity is taken seriously. That is because the reaction of total labor supply is a highly complex object whose various components need to be accounted for. This object not only depends on the distribution of wage rates across employed workers and that of reservation wage rates across non-employed workers. It also depends on the hours' adjustment of existing workers (intensive margin) as well as of those who move between employment and non-employment following a wage change (extensive margin). Lastly, the overall reaction depends on the exact implementation of the underlying policy change.

In this paper, we develop a unified framework which allows us to simultaneously study the role that workers' participation and hours decisions play for the size of the

aggregate Frisch elasticity. We depart from MaCurdy's (1985) standard intertemporal labor supply model that features complete markets, uncertainty, and worker heterogeneity in observable and unobservable characteristics. We then modify the aggregation approach developed by Paluch, Kneip, and Hildenbrand (2012) to allow for a corner solution in a worker's labor supply decision. This procedure has the distinct advantage of being widely applicable, because it requires neither a particular preference structure nor specific distributional assumptions for explanatory variables. We use it to aggregate our individual labor supply functions and wage rates. In order to derive the aggregate Frisch elasticity of labor supply, we subject all offered or paid wages to an unanticipated temporary increase. By eliminating wealth effects and taking account of the implied adjustment of labor supply, we derive an analytical expression for the aggregate elasticity and illustrate its components: (i) the intensive and extensive adjustment of hours worked, (ii) the extensive adjustment of wages, and (iii) the aggregate employment ratio.

To empirically implement our aggregation approach, we rely on specific econometric models and estimate them using micro-level data from the German Socio-Economic Panel (SOEP). The SOEP is unique in that it provides evidence on non-employed workers' reservation wage rates. This variable is essential for estimating the adjustment of hours worked and wages paid of workers who change their participation decision – so-called movers. We estimate the adjustment of hours worked along the intensive margin, i. e., of stayers, with the help of a standard panel model. Our sample comprises German males who are between 25 and 64 years old and live in former West Germany, because their labor supply behavior is well captured by the intertemporal model. Our estimation results yield an average individual Frisch wage-elasticity of 0.29 – a value that stands in sharp contrast to our estimated aggregate values which vary between 0.63 and 0.70 over the period ranging from 2000 to 2008.

We are not the first ones to study the aggregate Frisch elasticity of labor supply in an environment with heterogeneous workers. Our work is related to two main strands of the literature. First, it relates to the many contributions in modern business cycle analysis where the aggregate Frisch elasticity enters as key entity that affects the reaction of total labor supply to a change in wages induced by policy

or exogenous disturbances. The basic idea goes back to Lucas and Rapping (1969) which is considered as the origin of intertemporal labor supply in modern macroeconomics. Employment lotteries as introduced into the literature by Hansen (1985) and Rogerson (1988) have illustrated the importance of the extensive margin adjustment for the aggregate Frisch wage-elasticity, but except for the *ex post* status of a worker in the labor force it ignores worker heterogeneity. More closely related to our work are the papers by Chang and Kim (2005; 2006) who allow for worker heterogeneity and explore how the size of the aggregate Frisch elasticity of hours worked varies with incomplete markets. They focus on the intensive margin only. The work by Gourio and Noyal (2006) is also relatively closely related to ours. They use a complete market setup to explore the role of ‘marginal workers’ who by definition are indifferent between working and not working for adjustment along the extensive margin when wages change. All these contributions commonly use a parameterized version of a structural utility function which makes it possible to derive a functional relationship between the aggregate labor supply and aggregate wages. They differ with respect to the type and degree of worker heterogeneity, the assumed market structure, and whether they focus on the intensive or the extensive margin of adjusting labor supply. Another related piece is by Fiorito and Zanella (2012). They use the Panel Study of Income Dynamics (PSID) to empirically explore the link between the micro and the macro Frisch wage-elasticity without deriving an exact analytical relationship between them. They nicely illustrate how the difference between the individual and the aggregate Frisch elasticity changes for various subpopulations, but they cannot measure the extensive margin. Second, our work relates to the growing micro literature that has produced estimates of the individual wealth-compensated wage elasticity of hours worked since the early work by MaCurdy (1981; 1985) and Altonji (1986). Their estimates for males range from 0.10 to 0.45, and from 0 to 0.35, respectively. The recent study by Chetty, Guren, Manoli, and Weber (2012) provides quasi-experimental evidence on individual wage-elasticities. Its conclusion that the intensive margin of 0.5 is twice as large as the extensive one is juxtaposed to a central finding of the labor supply literature summarized in Blundell and MaCurdy (1999) that the extensive margin matters most for explaining variation in total person hours over the business cycle.



Our contribution to this literature is twofold. First, we develop an aggregation approach which does not require specific assumptions about model parameters or distributions of explanatory variables. It is comprehensive enough to simultaneously capture adjustment along the intensive and the extensive margin when wage rates change unexpectedly in an environment where workers are heterogeneous. Secondly, we illustrate the importance of aggregation by empirically implementing it using the German SOEP which contains as special feature information on reservation wage rates for non-employed workers.

This paper is organized as follows. Section 2.2 presents a dynamic model of individual labor supply under uncertainty. Section 2.3 develops a general aggregation procedure that features labor supply adjustment along the intensive and the extensive margin and is used to derive an analytical expression for the aggregate Frisch wage-elasticity of labor supply. Section 2.4 specifies the two econometric models used for empirical estimation, a panel data model on hours worked and a two-stage procedure to estimate conditional densities. Section 2.5 presents our database and introduces the main variables used for estimation. Section 2.6 reports all estimation results. Section 2.7 concludes.

## 2.2 A Dynamic Labor Supply Model

Underlying our aggregation exercise is an individual-specific labor supply function which relates the amount of labor that an individual supplies to the market in any given period  $t$  to a set of determinants. We view this function as the outcome of an intertemporal optimization problem under uncertainty.<sup>1</sup> In what follows we sketch this problem including the preferences, the constraints, and the informational setting for each individual. For the sake of notational simplicity, we abstain from introducing a person-specific index until Section 2.4.

Consider an infinitely-lived consumer. Her preferences are captured by a momentary utility function  $U$  which depends on private consumption  $c$ , leisure  $l$ , a vector of observable individual characteristics  $X$ , and a vector of unobservable individual variables  $Z$ , including tastes and talents.  $U$  is assumed to be twice differentiable,

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<sup>1</sup>Our model exposition closely follows that in MaCurdy (1985).

separable over time and also in consumption  $c$  and market hours worked  $h$ . Furthermore,  $U$  is strictly increasing and concave in  $c$  and  $h$ . When choosing sequences of leisure, consumption, and future asset holdings to maximize her expected lifetime utility, the consumer takes the real wage rate  $w$  and the real market return on assets  $r$  as given and respects the following two constraints: First, the per-period time-constraint

$$\bar{T}_t \geq l_t + h_t \tag{2.1}$$

which equates the available time  $\bar{T}$  to the sum of leisure and market hours worked  $h_t$  in each period  $t$ . Second, the budget constraint

$$c_t + a_{t+1} \leq w_t h_t + (1 + r_t) a_t \tag{2.2}$$

that sets the sum of consumption expenditures and the change in asset holdings  $a_{t+1} - a_t$  equal to total earnings plus interest income from current period asset holdings  $a_t$ . A consumer starts life with initial assets  $a_0$ .

Denoting by  $\mathbb{E}_t$  the mathematical expectation conditional on information known at the beginning of time  $t$  and by  $0 < \tilde{\beta} < 1$  the discount rate, the consumer's choice problem can be summarized as follows:

$$\max_{\{c_t, l_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t U(c_t, l_t; X_t, Z_t) \tag{2.3}$$

subject to equations (2.1) and (2.2), the non-negativity constraints  $c_t > 0$ ,  $l_t \geq 0$ , and the initial condition  $a_0 > 0$ .<sup>2</sup> For any differentiable function  $f(x_1, \dots, x_n)$  let  $\partial_{x_i} f(x_1, \dots, x_n)$  denote the partial derivative with respect to the  $i$ -th component. Then, letting  $\lambda_t$  denote the Lagrange multiplier associated with the period  $t$  budget constraint, the first-order necessary conditions for utility maximization are given by:

---

<sup>2</sup>A complete formulation of the consumer's dynamic decision problem also requires a transversality condition for wealth:  $\lim_{\bar{T} \rightarrow \infty} \lambda_{\bar{T}} a_{\bar{T}} = 0$ .

$$\partial_c U(\cdot) - \lambda_t = 0, \tag{2.4a}$$

$$\partial_l U(\cdot) - \lambda_t w_t = 0, \tag{2.4b}$$

$$\lambda_t = \tilde{\beta} \mathbb{E}_t[(1 + r_{t+1})\lambda_{t+1}]. \tag{2.4c}$$

With the help of the implicit function theorem equations (2.4a) and (2.4b) can be solved for individual consumption and labor supply as functions of the form

$$c_t = c(w_t, \lambda_t, X_t, Z_t), \tag{2.5}$$

$$h_t = h(w_t, \lambda_t, X_t, Z_t). \tag{2.6}$$

The time-invariant functions  $c(\cdot)$  and  $h(\cdot)$  depend only on the specifics of the utility function  $U(\cdot)$  and on whether corner solutions are optimal for hours worked in period  $t$ . These functions contain two types of arguments, namely those that capture what is going on in the current period –  $w_t$ ,  $X_t$ , and  $Z_t$  – and  $\lambda_t$  which is a sufficient statistic for past and future information relevant for the individual's current choices. If we further assume consumption and leisure to be normal goods, the concavity of the utility function implies

$$\partial_\lambda c < 0, \partial_w h \geq 0, \partial_\lambda h \geq 0. \tag{2.7}$$

Equation (2.4c) summarizes the stochastic process governing  $\lambda_t$ . Assuming interest rates do not vary stochastically, this process can alternatively be expressed as an expectational difference equation:

$$\lambda_t = \tilde{\beta}(1 + r_{t+1})\mathbb{E}_t \lambda_{t+1}.$$

Recall that any variable can be rewritten as the sum of what was expected and an expectational error  $\varepsilon_t$ :

$$\lambda_t = \mathbb{E}_{t-1} \lambda_t + \varepsilon_t.$$

Combining the last two expressions and solving backward yields

$$\lambda_t = \tilde{\beta}^{-t} R_t \lambda_0 + \sum_{j=0}^{t-1} \varepsilon_{t-j} \equiv \tilde{\beta}^{-t} R_t \lambda_0 + \eta_t, \quad (2.8)$$

where  $R_t \equiv 1/[(1+r_1)(1+r_2)\cdots(1+r_t)]$  is the common discount rate. Equation (2.8) nicely illustrates that apart from the sum of past expectational errors,  $\eta_t$ , the time-varying individual marginal utility of wealth consists of a fixed individual component  $\lambda_0$  and a common time-varying component. When inserting this expression together with the consumption and labor supply function (2.5) and (2.6) into the individual life-time budget constraint which results from solving equation (2.2) forward we get

$$a_0 \geq \sum_{t=0}^{\infty} R_t [c(w_t, \lambda_t, X_t, Z_t) - w_t h(w_t, \lambda_t, X_t, Z_t)]. \quad (2.9)$$

Equation (2.9) implicitly defines  $\lambda_t$ . It shows that the marginal utility of consumption is a highly complex variable that depends on the initial assets, life-time wages, the market interest rate, observable and unobservable individual characteristics, and preferences. For the purpose of our analysis it matters that the assumed concavity of preferences implies

$$\frac{\partial \lambda_t}{\partial a_0} < 0, \quad \frac{\partial \lambda_t}{\partial w_t} \leq 0. \quad (2.10)$$

Taken together the inequalities in (2.7) and (2.10) indicate that there exists a direct and an indirect effect of wages on hours worked. A rise in the current period's wage rate directly leads to an increase in hours worked. The indirect link exists, because a rising wage rate contributes to a rise in wealth which tends to reduce labor supply. Hence, in the intertemporal framework laid out the net effect of a change in wages on individual labor supply is unclear from a theoretical point of view.

Summing up, we can express the individual labor supply function as follows:

$$\begin{aligned}
 h_t &= \begin{cases} h(w_t, \lambda(w_t, \eta_t), X_t, Z_t) > 0 & \text{if } w_t \geq w_t^R \\ 0 & \text{if } w_t < w_t^R \end{cases} \\
 &= h(w_t, \lambda(w_t, \eta_t), Y_t) I(w_t \geq w_t^R),
 \end{aligned} \tag{2.11}$$

where  $I(\cdot)$  denotes the indicator function, the vectors  $X_t$  and  $Z_t$  are combined into  $Y_t = (X_t, Z_t)$ , and  $\lambda_t = \lambda(w_t, \eta_t)$ . The individual reservation wage rate in period  $t$  is derived from expression (2.4b):

$$w_t^R = \frac{\partial_t U[c_t, T; Y_t]}{\partial_c U[c_t, T; Y_t]}$$

with  $(1 + r_t)a_t \geq a_{t+1}$ . Equation (2.11) implies that the individual wage rate  $w_t$  is observed only if it is greater than or equal to the individual's reservation wage  $w_t^R$ . In general, we can think of  $w_t$  as the maximal wage rate *offered*. We introduce the wage rate as a possibly hypothetical quantity so that we can later define a suitable population model.

We use the labor supply function to define the individual Frisch wage-elasticity:

$$\epsilon_t = \left. \frac{\partial \log h(w, \lambda_t, Y_t)}{\partial \log w} \right|_{w=w_t} \tag{2.12a}$$

$$= \lim_{\Delta \rightarrow 0} \frac{\log h(w_t + \Delta, \lambda_t, Y_t) - \log h(w_t, \lambda_t, Y_t)}{\log(w_t + \Delta) - \log(w_t)}, \tag{2.12b}$$

where the last equality simply follows from the definition of a derivative. This definition will prove useful in our aggregation exercise.

Frisch requires us to only consider the **direct** effects of a wage change. We compensate indirect effects due to a rise in wealth by keeping  $\lambda_t = \lambda(w_t, \eta_t)$  fixed at their individual levels, instead of allowing  $\lambda_t$  to change with changes in  $w_t$ .<sup>3</sup> Given that this elasticity abstracts from the wealth effect of a wage change, by definition it cannot become negative. In fact,  $\epsilon_t$  is non-negative for continuing workers and zero for anyone whose offered wage falls short of the reservation wage rate. There

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<sup>3</sup>If we allowed  $\lambda_t$  to change to  $\lambda(w_t + \Delta, \eta_t)$ , our approach could generate a Marshallian wage-elasticity of labor supply.

may be individual workers whose incremental wage change makes them change their employment status. We call those workers marginal, and for them  $\epsilon_t$  is not defined.

## 2.3 Aggregation and the Frisch Elasticity

The derivation of the individual Frisch wage-elasticity lends itself to aggregation in a straightforward way: we replace individual working hours  $h_t$  and individual wages  $w_t$  in equation (2.12b) by their respective population means  $\bar{H}_t$  and  $\bar{W}_t$ .<sup>4</sup>

For each period  $t$ , individual working hours  $h_t$ , wage rates  $w_t$ , reservation wage rates  $w_t^R$ , as well as  $\lambda_t$  and  $Y_t$  are random variables with means depending on the corresponding distributions within the respective population. The mean labor supply as well as the mean wage rate received by all working individuals are given by the following two expressions:

$$\bar{H}_t = \mathbb{E}(h_t) = \int h(w, \lambda, Y) I(w \geq w^R) d\pi_{w, w^R, \lambda, Y}^t, \quad (2.13a)$$

$$\bar{W}_t = \mathbb{E}(w_t) = \int w I(w \geq w^R) d\pi_{w, w^R}^t, \quad (2.13b)$$

where  $\pi_{w, w^R, \lambda, Y}^t$  denotes the joint distribution of the variables  $(w_t, w_t^R, \lambda_t, Y_t)$  over the population, and  $\pi_{w, w^R}^t$  stands for the marginal distribution of  $(w_t, w_t^R)$ . All other marginal distributions are written analogously. The new mean wage,  $\bar{W}_t(\Delta)$ , and the new mean working hours,  $\bar{H}_t(\Delta)$ , corresponding to the incremental wage changes are given by:

$$\begin{aligned} \bar{H}_t(\Delta) &= \mathbb{E} \left( h(w_t + \Delta, \lambda_t, Y_t) I(w_t + \Delta \geq w_t^R) \right) \\ &= \int h(w + \Delta, \lambda, Y) I(w + \Delta \geq w^R) d\pi_{w, w^R, \lambda, Y}^t, \end{aligned} \quad (2.14a)$$

$$\begin{aligned} \bar{W}_t(\Delta) &= \mathbb{E} \left( (w_t + \Delta) I(w_t + \Delta \geq w_t^R) \right) \\ &= \int (w + \Delta) I(w + \Delta \geq w^R) d\pi_{w, w^R}^t. \end{aligned} \quad (2.14b)$$

Inserting the various aggregates into equation (2.12b) yields the aggregate Frisch

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<sup>4</sup>Of course, we could alternatively compute the population mean of  $\log h$  and  $\log w$ . This would slightly modify the subsequent formulae without substantially changing the analysis.

wage-elasticity

$$\begin{aligned}
e_t &= \lim_{\Delta \rightarrow 0} \frac{\log \bar{H}_t(\Delta) - \log \bar{H}_t}{\log \bar{W}_t(\Delta) - \log \bar{W}_t} \\
&= \frac{\frac{\partial}{\partial \Delta} \log \bar{H}_t(\Delta)|_{\Delta=0}}{\frac{\partial}{\partial \Delta} \log \bar{W}_t(\Delta)|_{\Delta=0}} = \frac{\bar{W}_t \frac{\partial}{\partial \Delta} \bar{H}_t(\Delta)|_{\Delta=0}}{\bar{H}_t \frac{\partial}{\partial \Delta} \bar{W}_t(\Delta)|_{\Delta=0}}.
\end{aligned} \tag{2.15}$$

This equation nicely illustrates that the aggregate Frisch elasticity measures changes in mean working hours in reaction to a small change of the mean wage rate. There exists an alternative interpretation of the above definition. Mean hours worked depend among others on the distribution of wages across individuals,  $\pi_w^t$ . Any specific change in individual wages affects the shape of the wage distribution and therefore also the new mean hours worked and the new mean wage. One can think of many different ways in which individual wages change. Here, we consider the simplest possible wage transformation by letting the wage distribution shift by a constant  $\Delta > 0$  while holding everything else constant. This corresponds to each individual facing an unanticipated temporary fixed change of her wage rate  $w_t$ , so that  $w_t$  is transformed into  $w_t + \Delta$  for some  $\Delta$  close to zero.

In equation (2.15), the aggregate quantities  $\bar{W}_t$  and  $\bar{H}_t$  can be determined from observed data so that we only have to analyze the expressions  $\frac{\partial}{\partial \Delta} \bar{H}_t(\Delta)|_{\Delta=0}$  and  $\frac{\partial}{\partial \Delta} \bar{W}_t(\Delta)|_{\Delta=0}$ . For the subsequent analysis, we denote the conditional distribution of some random variable  $V$  given a random variable  $W$  by  $\pi_{V|W}^t$  and its density, if existent, by  $f_{V|W}^t(\cdot)$ . In particular, we will assume that the conditional distribution  $\pi_{w^R|w}^t$  of  $w_t^R$  given  $w_t = w$  has a continuous density  $f_{w^R|w}^t(\cdot)$ . We require that the marginal distribution  $\pi_w^t$  of  $w_t$  also possesses a continuous density  $f_w^t(\cdot)$ .

Let us first consider the simpler term  $\bar{W}_t(\Delta)$  which, for  $\Delta > 0$ , quantifies the new mean wage rate paid by employers. Note that for a working individual her new wage rate simply is  $w_t + \Delta$ , and hence  $\frac{\partial}{\partial \Delta}(w_t + \Delta)|_{\Delta=0} = 1$ . This is not generally true at the aggregate level. The point is that for  $\Delta > 0$  we consider the increase in the mean wage rate for the entire labor force and not only for the subpopulation of employed workers. The transformation implies that a wage rate  $w_t + \Delta$  is offered to an unemployed individual, but the actual wage rate paid will remain zero if  $w_t + \Delta < w_t^R$ . On the other hand, there exist marginal workers who do not work at a wage rate  $w_t$ , but may decide to work at a higher wage rate  $w_t + \Delta$ .

More precisely, by equation (2.14b) we have

$$\begin{aligned}\bar{W}_t(\Delta) &= \int (w + \Delta)I(w \geq w^R)d\pi_{w,w^R}^t + \int (w + \Delta)I(w^R \in [w, w + \Delta])d\pi_{w,w^R}^t \\ &= \int (w + \Delta)I(w \geq w^R)d\pi_{w,w^R}^t + \int (\nu + \Delta) \left( \int_{\nu}^{\nu+\Delta} f_{w^R|\nu}^t(\tilde{\nu})d\tilde{\nu} \right) f_w^t(\nu)d\nu.\end{aligned}\tag{2.16}$$

Taking derivatives yields

$$\frac{\partial}{\partial \Delta} \bar{W}_t(\Delta)|_{\Delta=0} = \underbrace{\int I(w \geq w^R)d\pi_{w,w^R}^t}_{EPR_t} + \underbrace{\int \nu f_{w^R|\nu}^t(\nu) f_w^t(\nu)d\nu}_{\tau_{w,t}^{ext}}.\tag{2.17}$$

The first term  $EPR_t$  corresponds to the employment ratio in period  $t$ , i. e., the fraction of the population employed.  $EPR_t$  enters here because the wage change relates to all employees whereas the change in the mean wage is computed by summing over the entire population. The second term is due to changes in mean earnings with respect to employment adjustment along the extensive margin. For a given wage rate  $w$  the term  $w f_{w^R|w}^t(w)$  quantifies the rate of increase of wages to be paid to marginal workers if  $w$  increases by  $\Delta > 0$ .  $\tau_{w,t}^{ext}$  is the mean of these rates over all wages,  $\tau_{w,t}^{ext} = \mathbb{E}(w_t f_{w^R|w_t}^t(w_t))$ .

Necessarily  $\tau_{w,t}^{ext} \geq 0$ , and one typically expects that  $\tau_{w,t}^{ext} > 0$ . To simplify the argument consider the case that  $w_t^R$  and  $w_t$  are independent such that  $f_{w^R|w}^t \equiv f_{w^R}^t$  does not depend on  $w$  and is equal to the marginal density of reservation wages.<sup>5</sup> Then,  $\tau_{w,t}^{ext} > 0$  if for some wage rate  $\nu$  with  $f_w^t(\nu) > 0$  we also have  $f_{w^R}^t(\nu) > 0$ . In other words,  $\tau_{w,t}^{ext} > 0$  if there exists some overlap between the support of the distribution of wages  $w_t$  and the support of the distribution of reservation wages  $w_t^R$ . This will typically be fulfilled for any real economy.

Let us now analyze the term  $\bar{H}_t(\Delta)$  which, for  $\Delta > 0$ , quantifies the new mean

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<sup>5</sup>The micro model implies that reservation wages are variables which do not depend on actual wages paid or offered. Therefore, it does not seem implausible to assume that the random variables  $w_t^R$  and  $w_t$  are independent. However, there may exist an indirect link due to correlations with common explanatory variables such as education, for example. Highly educated individuals tend to have higher reservation wages than others, and they are likely to receive higher wage offers. This may introduce a correlation between  $w_t^R$  and  $w_t$  over the population. Our procedure for estimating  $\tau_{w,t}^{ext}$  described in Section 2.4 takes such effects into account.



working hours. Similar to equation (2.16) we obtain

$$\begin{aligned}\bar{H}_t(\Delta) &= \int h(w + \Delta, \lambda, Y) I(w \geq w^R) d\pi_{w, w^R, \lambda, Y}^t \\ &\quad + \int h(w + \Delta, \lambda, Y) I(w^R \in [w, w + \Delta]) d\pi_{w, w^R, \lambda, Y}^t,\end{aligned}\tag{2.18}$$

where the second term quantifies the part of the change of  $\bar{H}_t$  which is due to the fact that if wage rates rise from  $w_t$  to  $w_t + \Delta$ , then the subpopulation of all individuals with reservation wage rates  $w_t^R \in [w_t, w_t + \Delta]$  will contribute non-zero working hours. Using  $\partial_w h(w, \lambda, Y)$  to denote the partial derivative of  $h$  with respect to  $w$ , the derivative of the first term simply is  $\mathbb{E}(\partial_w h(w_t, \lambda_t, Y_t))$ . Calculating the derivative of the second term is slightly more complicated. A rigorous analysis can be found in Appendix A2.1. We then arrive at the following expression:

$$\begin{aligned}\left. \frac{\partial \bar{H}_t(\Delta)}{\partial \Delta} \right|_{\Delta=0} &= \underbrace{\int \partial_w h(w, \lambda, Y) I(w \geq w^R) d\pi_{w, w^R, \lambda, Y}^t}_{\tau_{h,t}^{int}} \\ &\quad + \underbrace{\int \mathbb{E}\left(h_t \mid w_t^R = w_t = \nu\right) f_{w^R|\nu}^t(\nu) f_w^t(\nu) d\nu}_{\tau_{h,t}^{ext}}.\end{aligned}\tag{2.19}$$

The first term  $\tau_{h,t}^{int}$  quantifies the average derivatives of the individual functions  $h$  for the subpopulation  $\mathcal{E}_t$  of all individuals already working at wage rate  $w_t$ . Put differently,  $\tau_{h,t}^{int}$  measures the total labor supply adjustment along the intensive margin. It can also be interpreted as a weighted mean of individual Frisch elasticities for the subpopulation  $\mathcal{E}_t$ . Recall that individual Frisch elasticities are given by

$$\epsilon_t = \left. \frac{\partial \log h(w, \lambda_t, Y_t)}{\partial \log w} \right|_{w=w_t} = \partial_w h(w_t, \lambda, Y) \frac{w_t}{h_t}. \text{ Therefore,}$$

$$\tau_{h,t}^{int} = \int_{\mathcal{E}_t} \partial_w h(w, \lambda, Y) d\pi_{w, w^R, \lambda, Y}^t = \mathbb{E}_{\mathcal{E}_t}(\partial_w h(w_t, \lambda_t, Y_t)) = \mathbb{E}_{\mathcal{E}_t} \left( \epsilon_t \frac{h_t}{w_t} \right), \tag{2.20}$$

where  $\mathbb{E}_{\mathcal{E}_t}(\cdot)$  is used to denote expected values over all individuals in  $\mathcal{E}_t$ . Note that usually  $\mathbb{E}_{\mathcal{E}_t}(\epsilon_t \frac{h_t}{w_t}) \neq \mathbb{E}_{\mathcal{E}_t}(\epsilon_t) \frac{\bar{H}_t}{\bar{W}_t}$  which means that even  $\frac{\bar{W}_t}{\bar{H}_t} \tau_{h,t}^{int}$  does not correspond to a simple mean of individual elasticities over  $\mathcal{E}_t$ .

The second term  $\tau_{h,t}^{ext} \geq 0$  captures all adjustments of working hours along the extensive margin, i. e., all changes due to transitions between non-employment and

employment. Its interpretation is analogous to that of  $\tau_{w,t}^{ext}$  already discussed above. Note that  $\mathbb{E}(h_t | w_t^R = w_t = w)$  is the average number of hours a marginal worker with reservation wage rate  $w_t^R = w$  intends to work if she is offered the wage rate  $w_t = w$ . For a given wage rate  $w$  the term  $\mathbb{E}(h_t | w_t^R = w_t = w) f_{w^R|w}^t(w)$  quantifies the rate of change of hours worked by marginal workers if  $w$  changes.

Summarizing our discussion, the aggregate Frisch wage-elasticity is given by<sup>6</sup>

$$e_t = \frac{\overline{W}_t}{\overline{H}_t} \left( \frac{\tau_{h,t}^{int} + \tau_{h,t}^{ext}}{EPR_t + \tau_{w,t}^{ext}} \right). \quad (2.21)$$

The quantities  $\overline{W}_t$ ,  $\overline{H}_t$ , and  $EPR_t$  can be determined directly from real-world data. Contrary to what we observed for the individual wage elasticity, the aggregate Frisch wage-elasticity explicitly takes into account the behavior of marginal workers. In fact, the size of the extensive margins of adjustment crucially depends on the relative size of this group of workers. We will capture their behavior using reservation wage data for unemployed workers who are willing to work at a given wage. We measure the total adjustment along the intensive margin by looking at employed workers who change hours on the job in reaction to a wage shock.

## 2.4 Econometric Modeling

In what follows, we will describe an econometric approach to estimate the total labor supply adjustment along the intensive margin as well as the adjustments along the extensive margin in our general effort to quantify the aggregate Frisch wage-elasticity  $e_t$ .

For a given period  $t$ , the expression for the total labor supply adjustment along the intensive margin from equation (2.19) can be estimated via its sample equivalent

$$\hat{\tau}_{h,t}^{int} = \frac{1}{N_t^w} \sum_{i: h_{it} > 0} \partial_w \hat{h}(w_{it}, \lambda_{it}, Y_{it}), \quad (2.22)$$

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<sup>6</sup>Most existing work in business cycle analysis is based on models which assume time-invariant wage elasticities of labor supply. At a first glance it may come as a surprise that aggregate elasticities determined by equation (2.21) explicitly depend on time. Time dependence of  $e_t$  is an inevitable consequence of the fact that all major determinants vary over time, albeit at a high degree of persistence.

where  $N_t^w$  denotes the employed workers in period  $t$  in our sample. The determinants of the individual labor supply  $h_{it} = h(w_{it}, \lambda_{it}, Y_{it}) I(w_{it} \geq w_{it}^R)$  are given by the wage rate  $w_{it}$ , the marginal utility of wealth  $\lambda_{it}$ , observable individual characteristics  $X_{it}$ , and unobservable random factors  $Z_{it}$  with  $Y_{it} = (X_{it}, Z_{it})$ . We closely follow the empirical literature on male labor supply analysis where hours worked are treated as a continuous variable. Assuming that all determinants have a linear effect on the individual labor supply we get the following panel data model:<sup>7</sup>

$$\log h_{it} = \gamma_0 + \gamma_1 \log w_{it} + (X_{it})' \beta + \lambda_{it} + z_{it}, \quad (2.23)$$

where  $X_{it}$  is a vector of  $p$  different observable attributes and the  $p$ -dimensional parameter vector  $\beta$  captures their influence on individual labor supply. The term  $z_{it}$  measures the influence of unobservable individual characteristics. For the sake of our aggregation exercise we need to measure the hours' reaction of employed workers to a surprise wage change. Standard labor supply analysis typically is interested in statements on individual labor supply in the context of the entire labor force, and hence selection may matter. Selection plays no role in our analysis, because we focus on changes in aggregate labor supply: only the employed workers matter for the intensive margin in the aggregate. Even if we estimated the panel data model on the entire labor force,  $\gamma_1$  would not correspond to the aggregate Frisch wage-elasticity, since  $\gamma_1$  is relevant for employed workers only. The respective wage elasticity for those who remain unemployed is always zero, and the group of marginal workers serves to determine the extensive margins of adjustment in the aggregate.

In order to retrieve the individual fixed components of  $\lambda_{it}$  and  $z_{it}$  we decompose

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<sup>7</sup>Note that if we assumed the utility function to be separable between leisure and consumption, linearity would follow directly. Let  $U = f(c_{it}, Z_{it}) - \exp(-X_{it}'\beta^* - z_{it}^*)(T - l_{it})^\sigma$  as in MaCurdy (1985) where  $\beta^*$  is a vector of parameters associated with the observable individual characteristics  $X_{it}$ ,  $z_{it}^*$  is the contribution of the unmeasured characteristics, and  $\sigma > 1$  is a preference parameter common to all individuals. Then, the first-order condition (2.4b) reads as follows and can be reformulated further:

$$\begin{aligned} \lambda_{it} w_{it} &= \exp(-X_{it}'\beta^* - z_{it}^*) \sigma h_{it}^{\sigma-1} \\ \log \lambda_{it} + \log w_{it} &= -X_{it}'\beta^* - z_{it}^* + \log \sigma + (\sigma - 1) \log h_{it} \\ \log h_{it} &= (\sigma - 1)^{-1} (-\log \sigma + \log w_{it}) + X_{it}'\beta + \tilde{\lambda}_{it} + \tilde{z}_{it}, \end{aligned}$$

with  $\beta = (\sigma - 1)^{-1} \beta^*$ ,  $\tilde{\lambda}_{it} = (\sigma - 1)^{-1} \log \lambda_{it}$ , and  $\tilde{z}_{it} = (\sigma - 1)^{-1} z_{it}^*$ .

their sum into their respective time averages and a time-varying residual:

$$\lambda_{it} + z_{it} = \underbrace{\lambda_i + z_i}_{\mu_i} + \underbrace{\lambda_{it} - \lambda_i + z_{it} - z_i}_{\xi_{it}}. \quad (2.24)$$

This yields

$$\log h_{it} = \gamma_0 + \gamma_1 \log w_{it} + (X_{it})' \beta + \mu_i + \xi_{it}, \quad (2.25)$$

where we now assume  $\xi_{it}$  to be i. i. d. idiosyncratic errors with zero mean and common variance. Since the individual wage rate is correlated with the marginal utility of wealth  $\lambda_{it}$  which enters the error term, we instrument for wage rates. The structure of the panel model above as well as the instrumental variable (IV) approach are in accordance with the setup commonly used in the literature estimating the individual labor supply of males (cf. for example Blundell and MaCurdy (1999), Fiorito and Zanella (2012)). The instruments must be uncorrelated with the time-varying wealth and preference component of the error, i. e.,  $\lambda_{it} - \lambda_i$  and  $z_{it} - z_i$ . However, they may correlate with the individual fixed effects. We estimate equation (2.25) using a fixed-effect estimator. In order to guarantee identification of  $\beta$ , there may not be a constant in  $X$ , and none of the observable attributes may be determined by the wage rate, so that the matrix  $\mathbb{E}\{[X - \mathbb{E}[X | \log w]][X - \mathbb{E}[X | \log w]]'\}$  be positive definite. As is common in this literature, the sum over all individual effects is standardized to equal zero.

The panel data model implies that an estimate of the derivative of the individual labor supply function with respect to the wage rate is given by

$$\partial_w \hat{h}(w_{it}, \lambda_{it}, Y_{it}) = \frac{h_{it}}{w_{it}} \hat{\gamma}_1,$$

so that for each period  $t$  the total labor supply adjustment along the intensive margin can be estimated by

$$\hat{\tau}_{h,t}^{int} = \frac{1}{N_t^w} \sum_{i: h_{it} > 0} \frac{h_{it}}{w_{it}} \hat{\gamma}_1. \quad (2.26)$$

Let us now consider the adjustments along the extensive margin. To maintain a

high degree of generality, we take a nonparametric estimation approach. Recall from equations (2.17) and (2.19) that  $\tau_{w,t}^{ext}$  and  $\tau_{h,t}^{ext}$  are given by

$$\tau_{w,t}^{ext} = \int \nu f_{w^R|\nu}^t(\nu) f_w^t(\nu) d\nu \quad (2.27)$$

and

$$\tau_{h,t}^{ext} = \int \mathbb{E}\left(h_t \mid w_t^R = w_t = \nu\right) f_{w^R|\nu}^t(\nu) f_w^t(\nu) d\nu, \quad (2.28)$$

respectively. Therefore, for given  $\nu$  we have to find estimates for the product of densities  $f_{w^R|\nu}^t(\nu) f_w^t(\nu) = f_{w^R,w}^t(\nu, \nu)$  and the conditional expectation  $\mathbb{E}(h_t \mid w_t^R = w_t = \nu)$ . As the joint distribution of reservation wages and hourly wage rates is unknown, we condition on observable individual characteristics,  $X$ , to estimate the product of densities

$$\begin{aligned} f_{w^R,w}^t(w_1, w_2) &= \int f_{w^R,w|X}^t(w_1, w_2) d\pi_X^t \\ &= \int f_{w^R|X}^t(w_1) f_{w|X}^t(w_2) d\pi_X^t \end{aligned} \quad (2.29)$$

and assume independence of the wage and the reservation wage conditional on individual characteristics. This implies that the joint density of the wage and the reservation wage can be factorized conditional on individual characteristics.<sup>8</sup> Both densities as well as the conditional expectation are estimated nonparametrically, resulting in  $\hat{f}_{w^R|X}^t(\cdot)$ ,  $\hat{f}_{w|X}^t(\cdot)$ , and  $\hat{\mathbb{E}}(h_t \mid w_t^R = w_t = \cdot)$ , respectively. We employ a two-step conditional density estimator and consider first two simple regression models, followed by a nonparametric kernel density estimator to determine an estimate from the residuals of the regression models. For the estimation of the conditional expectation we employ a local constant kernel estimator, also referred to as the Nadaraya-Watson kernel estimator.<sup>9</sup> For each period  $t$ ,  $\tau_{w,t}^{ext}$  and  $\tau_{h,t}^{ext}$  can then be

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<sup>8</sup>This assumption is comparable to what Hall (2013) calls proportionality hypothesis which states that individual reservation wage rates and actual wage rates are proportional to the individual productivity.

<sup>9</sup>The nonparametric estimation procedure for  $\hat{f}_{w^R|X}^t(\cdot)$ ,  $\hat{f}_{w|X}^t(\cdot)$ , and  $\hat{\mathbb{E}}(h_t \mid w_t^R = w_t = \cdot)$  is described in Appendix A2.2 (see e. g. Li and Racine (2006)).

approximated by

$$\hat{\tau}_{w,t}^{ext} = \int \nu \left( \frac{1}{N_t} \sum_i \hat{f}_{w^R|X=X_{it}}^t(\nu) \hat{f}_{w|X=X_{it}}^t(\nu) \right) d\nu \quad (2.30)$$

and

$$\hat{\tau}_{h,t}^{ext} = \int \hat{\mathbb{E}} \left( h_t \mid w_t^R = w_t = \nu \right) \left( \frac{1}{N_t} \sum_i \hat{f}_{w^R|X=X_{it}}^t(\nu) \hat{f}_{w|X=X_{it}}^t(\nu) \right) d\nu, \quad (2.31)$$

where  $N_t$  denotes the sum of working and non-working individuals in period  $t$  in our sample. This allows us to estimate the aggregate Frisch wage-elasticity as specified in equation (2.21) for any period  $t$ .

## 2.5 Data

Our empirical work is based on data from the German Socio-Economic Panel (SOEP), a representative sample of private households and individuals living in Germany. The panel was started in 1984 (wave A) and has been updated annually through 2011 (wave BB). The panel design closely follows that of the Panel Study of Income Dynamics (PSID) – a representative sample of US households and individuals – but also takes idiosyncrasies of the German legal and socio-economic framework into account.<sup>10</sup> Since 2000, the SOEP covers on average 12,000 households and 20,000 individuals per year. A set of core questions is asked every year, including questions on education and training, labor market behavior, earnings, taxes, and social security, etc.

We use the SOEP, because we consider it particularly well suited for the purpose of our analysis. To our knowledge it is the only micro panel currently available that contains indirect information on reservation wage rates of non-employed workers. This variable is essential for our effort to quantify changes in a worker’s participation decision. Apart from detailed information on individual characteristics, the SOEP also reports an employed individual’s market hours worked and earnings. We can thus compute an individual’s hourly wage rate.

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<sup>10</sup>A detailed description of the panel’s design, its coverage, the main questions asked, etc. is contained in the *Desktop Companion* to the SOEP, which is accessible online at [www.diw.de](http://www.diw.de).

### 2.5.1 Sample

For the sake of our empirical analysis we need consistent data on individual labor market behavior over a rather long time horizon. Therefore, we focus on the working age population of German males living in former West Germany who are between 25 and 64 years old. We do so, because we are neither interested in the peculiarities of women's working behavior nor in the institutional differences between former East and West Germany. Including females in a relatively long panel study would be problematic because in Germany, unlike in many other countries, females have undergone severe changes in their labor market behavior during the past decades and are less attached to the workforce than elsewhere. Since we want to focus on those who actively participate in the labor market, we exclude retirees, individuals in military service under conscription or in community service which can serve as substitute for compulsory military service, and individuals currently undergoing education. We also exclude individuals with missing information on unemployment experience or the amount of education or training. A maximum of 56 individuals is affected. Our sample ranges from 2000 to 2009. That is because in 2000 a refreshment sample was added to the SOEP which effectively doubled the number of observations.

Moreover, our fixed-effect estimation procedure requires the time index  $t$  to converge to infinity to ensure consistent estimates of the individual fixed effects. Therefore, we create a balanced panel from our sample which includes those working males who are continuously employed over the sample period. Our balanced panel comprises 1,296 individuals. We use these individuals whenever we compute measures related to employed workers. For all questions related to non-employment we consider individuals who are not employed and have answered the question on reservation wages. This leaves us with 91 to 140 individuals between 2000 and 2009.<sup>11</sup>

### 2.5.2 Variables

Our key variables of interest are the hourly wage rate and actual working hours for the employed, the reservation wage rate for the unemployed, and individual

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<sup>11</sup>A detailed description of our sample is given in Appendix A2.3. In particular, Table 2.6 shows summary statistics, and we list all refinements to the original data.

characteristics.<sup>12</sup> A person's total hours worked,  $h_{it}$ , are given by the average actual weekly working hours. There is a wide range of answers to the question "And how much on average does your actual working week amount to, with possible overtime?" – answers range from 5.5 to 80 hours per week. In fact, the distribution of  $h_{it}$  is not discrete in nature, but quite dispersed, in particular during the last 15 to 20 years. It seems that the traditional 40 hours workweek gradually loses its prevalence as there are increasing possibilities of part-time work, higher skilled workers are asked to work more, and more flexible work options have become available.<sup>13</sup>

The hourly wage rate is calculated by dividing the current net monthly earnings by the product of 4.3 and contractual weekly working hours. We use net earnings, since information on the reservation wage is only available in net terms, and we need the wage rate,  $w_{it}$ , and the reservation wage rate to be comparable. We convert all nominal values into real ones by dividing all nominal expressions by the consumer price index which uses 2005 as base year.

The reservation wage is generated from answers to the question "How much would the net pay have to be for you to consider taking the job?" which is posed to all individuals who are not in gainful employment or in military service and who intend to take up a job in the future. The associated working hours are deduced from the variable "Interest in full- or part-time work". We assume persons answering the question "Are you interested in full- or part-time employment?" with "Full-time employment", "Either", or "Don't know" to be interested in 40 hours of work per week. We assign 20 hours of work per week to those who indicate an interest in "Part-time employment". The reservation wage rate corresponds to the ratio of the monthly net reservation earnings to the product of 4.3 and desired weekly working hours. Since 2007 the SOEP contains detailed information on desired weekly working hours. If available we use the answer to the question "In your opinion how many hours a week would you have to work to earn this net income?" to calculate the reservation wage. In fact, we can use this more detailed information to check whether attributing 20 and 40 hours work per week is reasonable. Table 2.1 shows that for

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<sup>12</sup>A list of all SOEP variables with respective names as well as a list of all generated variables with description is given in Appendix A2.3.

<sup>13</sup>Histograms of actual hours worked for the years 2000, 2005, and 2009 are available in Appendix A2.3.



Table 2.1: Preferred Working Hours Linked to Reservation Net Income [%]

| Wave | Full-time, Either, Don't know |        |         |         | Part-time |        |         |         |
|------|-------------------------------|--------|---------|---------|-----------|--------|---------|---------|
|      | Obs.                          | [0,35) | [35,45] | (45,70] | Obs.      | [0,15) | [15,25] | (25,40] |
| 2007 | 107                           | 0.05   | 0.88    | 0.07    | 11        | 0.00   | 0.64    | 0.36    |
| 2008 | 86                            | 0.08   | 0.87    | 0.05    | 5         | 0.00   | 0.60    | 0.40    |
| 2009 | 112                           | 0.05   | 0.88    | 0.06    | 7         | 0.00   | 0.43    | 0.57    |

*Notes:* Obs. denotes the number of observations for West German males aged 25 to 64 with answers “Full-time”, “Either”, “Don't know”, and “Part-time”, respectively, to the question “Are you interested in full- or part-time employment?”.

individuals who are indifferent or those interested in full-time work the assumed 40 hours of work per week for the years 1984 to 2006 are a reasonable choice. For the years 2007, 2008, and 2009, around 88 % of those individuals believe that they would have to work between 35 and 45 hours to earn the desired reservation net income. For individuals interested in part-time work the picture is not as clear. Part-time work is usually any work with less than 30 to 35 hours per week, but in a legal sense is defined as employment with fewer hours than a comparable full-time job. This vague definition is reflected in the relative frequencies of the number of working hours associated with the reservation net earnings in Table 2.1. However, note that for all years few individuals fall into this category, in fact at most 11 individuals. Therefore, we stick to the assumption of 20 working hours per week for individuals interested in part-time work.

We use different individual characteristics for the employed and the non-employed. For the sake of estimating our panel model, we consider as individual characteristics of the employed a dummy for the family status (1 if married or currently living in dwelling with steady partner, 0 otherwise), work experience in full-time employment, and three dummy variables on the occupational group. Each working individual belongs to one out of the following four occupational groups. The first group comprises employees in agriculture, animal husbandry, forestry, horticulture, or in mining. The second group comprises employees in manufacturing or technical occupations (e.g. engineers, chemists, technicians). All employees in the service industry belong to the third group. The fourth group comprises all other workers, in particular persons who do not report an established profession or workers without any further specification of their professional activity.

As mentioned in Section 2.4 we use an IV approach to account for the possible endogeneity of wages. Following the ideas of Mincer (1974) who viewed wages as predominantly determined by accumulated human capital, we include as instruments schooling, work experience in full-time employment, and work experience squared. The schooling variable is based on the number of years of education or training undergone and exhibits some variation over time. It includes secondary vocational education and ranges from 7 to 18 years.<sup>14</sup>

The determinants of the reservation wage which are needed for the estimation of the conditional density  $f_{w^R|X}^t(\cdot)$  are given by unemployment experience in years, a dummy on whether or not information for unemployment benefits is provided, the size of unemployment benefits, and a dummy for highly qualified individuals. The latter group has obtained a college or university degree.<sup>15</sup> Note that in each year individuals are asked about the size of the unemployment benefits in the previous year so that the information about unemployment benefits is not available for the last wave, i. e., 2009. For estimating  $f_{w|X}^t(\cdot)$  we use schooling, work experience in full-time employment, and work experience squared.

## 2.6 Results

We start this section by presenting results from the panel, density, and conditional expectation estimation needed for the determination of the total adjustments along the intensive and extensive margin, respectively. Then, we provide results for the aggregate Frisch wage-elasticity of labor supply.

### 2.6.1 Panel Model Estimation

For calculating the total labor supply adjustment along the intensive margin  $\tau_{h,t}^{int}$ , we first have to estimate the panel data model for the working population. Results for the first stage of the panel model estimation are given in Table 2.7 in Appendix A2.4.

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<sup>14</sup>There exist alternative instruments, e. g. a regionally varying unemployment rate which is available from IAB (German Bureau of Labor Statistics), Nuremberg.

<sup>15</sup>These determinants of the reservation wage rate are in line with the literature as Prasad (2004) and Addison, Centeno, and Portugal (2009), among others, find that duration of joblessness, availability and level of unemployment compensation, and observables of education or skill level are the most important determinants of reservation wages.

Table 2.2: Results for the Panel Model Estimation

| $\log h$ | With IVs<br>(Benchmark) | Without IVs |
|----------|-------------------------|-------------|
| $\log w$ | 0.29***                 | -0.12***    |
| FAMILY   | -0.02**                 | 0.01        |
| EXPFT    | -6.55E-04               | 1.90E-03*** |
| O1       | 0.02                    | 0.03        |
| O3       | 0.01*                   | 0.01*       |
| O4       | 0.03                    | 0.04**      |
| CONST    | 3.07***                 | 4.03***     |

*Notes:* \*\*\*, \*\*, and \* denote significance at the 1, 5, and 10 percent level, respectively. FAMILY, EXPFT, O1, O3, O4, and CONST represent the family status dummy variable, work experience in years, dummy variables on occupational group, and a constant, respectively. The sample underlying the estimation is described in Section 2.5.

All instruments and the constant are highly significant. Wage rates rise in the years of schooling and in work experience gathered. However, the coefficient on work experience squared is negative, so that each further increase in experience conveys a progressively smaller increase in the wage rate.

Table 2.2 shows results for the panel model estimation, equation (2.25). For the benchmark specification, i.e., the IV approach, the coefficient on the family status dummy variable is significantly negative at the 5 percent level showing that married or cohabiting individuals have a lower engagement in the workforce. The constant and the coefficient on the logarithm of the wage rate are highly significantly positive. The parameter estimate of the latter variable equals 0.29. This estimate corresponds to the wealth-compensated individual wage elasticity of labor supply which has received a lot of attention in the empirical labor literature. Our estimate for working age males in Germany is in line with what is commonly reported in that literature. The third column of Table 2.2 shows that neglecting the endogeneity of wage rates leads to negative point estimates on the logarithm of the hourly wage rate as is also discussed in Reynaga and Rendon (2012).

An important issue when using an IV approach is the strength of instruments. The first stage F statistic which is equivalent to the Cragg-Donald statistic in a linear IV regression in the case of one endogenous regressor is 52.09 (cf. Cragg and Donald (1993)). In the case of an IV regression with a single endogenous regressor and i. i. d. errors, instruments are considered to be strong, if the first stage F statistic

exceeds 10 (cf. rule of thumb by Staiger and Stock (1997)). For linear IV regressions Stock and Yogo (2005) provide critical values to test for weak instruments based on the maximum Wald test size distortion. The critical value for one endogenous regressor and two instruments at the 10% significance level is 19.93. Whenever the Cragg-Donald statistic exceeds the critical values, one can reject the null hypothesis of weak instruments. We consider this as evidence of strong instruments.

## 2.6.2 Conditional Density and Expectation Estimation

As described in Section 2.4 and in Appendix A2.2 we have to first estimate the wage and reservation wage regressions, equations (2.32) and (2.33), to get the conditional densities  $\hat{f}_{w|X}^t(\cdot)$  and  $\hat{f}_{w^R|X}^t(\cdot)$ , respectively. Regression results are shown in Table 2.3 and 2.4.

As is the case for the first stage of the panel model estimation, for all years except for 2001 the coefficients on the individual characteristics as well as the constant are highly significant. Wage rates rise in the years of schooling and in work experience gathered. However, the coefficient on work experience squared is negative, so that each further increase in experience conveys a progressively smaller increase in the wage rate.

For the estimation of equation (2.33) we have between 91 and 140 observations and the constant is highly significant between 8.35 and 10.44. The coefficient on the unemployment duration is mostly negative and not significant. The predominant sign of the coefficient is in line with predictions from theoretical models and empirical evidence that the reservation wage decreases with waiting time for a new job. The reservation wage rate significantly decreases if non-employed individuals receive unemployment benefits, but they increase in the level of those benefits. Being a highly qualified individual, i. e., having obtained a college or university degree, increases the reservation wage, in most cases (highly) significantly.

The resulting conditional densities  $f_{w|X}^t(\cdot)$  and  $f_{w^R|X}^t(\cdot)$  vary with individual characteristics  $X = X_{it}$ . Therefore, we restrict our analysis to the densities conditional on mean individual characteristics, i. e.,  $X_{it} = \bar{X}_t$ . Note that this choice is rather arbitrary. One could also consider results for median or prespecified individual char-

Table 2.3: Results for the Wage Regression, Equation (2.32)

| Wave | CONST        | SCHOOL       | EXPFT        | EXPFT2        |
|------|--------------|--------------|--------------|---------------|
| 2000 | -5.356504*** | 0.9735708*** | 0.6026383*** | -0.0122329*** |
| 2001 | -4.24767     | 1.260115***  | 0.1101167    | -0.0004187    |
| 2002 | -4.408469*** | 0.9711685*** | 0.5200087*** | -0.010236***  |
| 2003 | -5.467164*** | 1.062251***  | 0.4847722*** | -0.0083184*** |
| 2004 | -4.966449*** | 1.027526***  | 0.4441191*** | -0.0072727*** |
| 2005 | -7.751894*** | 1.152485***  | 0.5540573*** | -0.0092175*** |
| 2006 | -7.050421*** | 1.070592***  | 0.5755334*** | -0.0098765*** |
| 2007 | -8.159443*** | 1.134918***  | 0.5571551*** | -0.0089508*** |
| 2008 | -5.125946*** | 1.05965***   | 0.3536003*** | -0.0050957*** |
| 2009 | -6.227629*** | 1.047241***  | 0.4771336*** | -0.0074261*** |

*Notes:* See Table 2.2. CONST, SCHOOL, EXPFT, and EXPFT2 denote a constant, the schooling variable, work experience, and work experience squared, respectively.

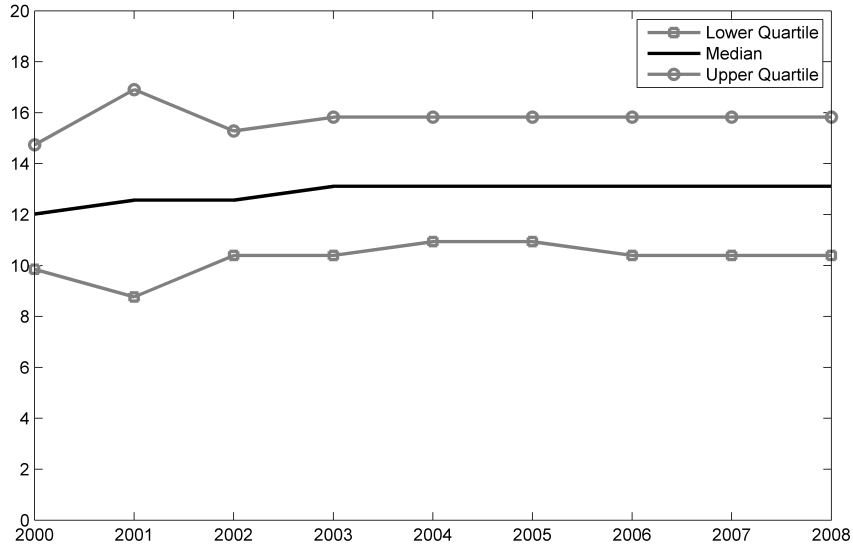
Table 2.4: Results for the Reservation Wage Regression, Equation (2.33)

| Wave | CONST       | EXPUE       | UEBEN        | HQD         | UEBEND       |
|------|-------------|-------------|--------------|-------------|--------------|
| 2000 | 9.287464*** | -0.074731   | 0.0018317    | 1.579977*   | -2.46782**   |
| 2001 | 8.632496*** | -0.170788** | 0.003354***  | 2.905584*** | -2.710164*** |
| 2002 | 9.934094*** | -0.188054** | 0.0029888*** | 0.8428682   | -3.273079*** |
| 2003 | 9.488645*** | -0.05987    | 0.0035565*** | 3.540291*** | -4.117936*** |
| 2004 | 10.4398***  | -0.191361** | 0.0019663*** | 1.274672    | -3.644075*** |
| 2005 | 8.584524*** | -0.11929    | 0.005742***  | 3.807951*** | -5.233218*** |
| 2006 | 8.935665*** | -0.235793** | 0.0051221*** | 4.686461*** | -4.434229*** |
| 2007 | 8.3453***   | -0.097721   | 0.0037337*** | 3.437334*** | -4.102274*** |
| 2008 | 8.747728*** | 0.098416    | 0.0053015    | 0.9004927   | -5.430932    |

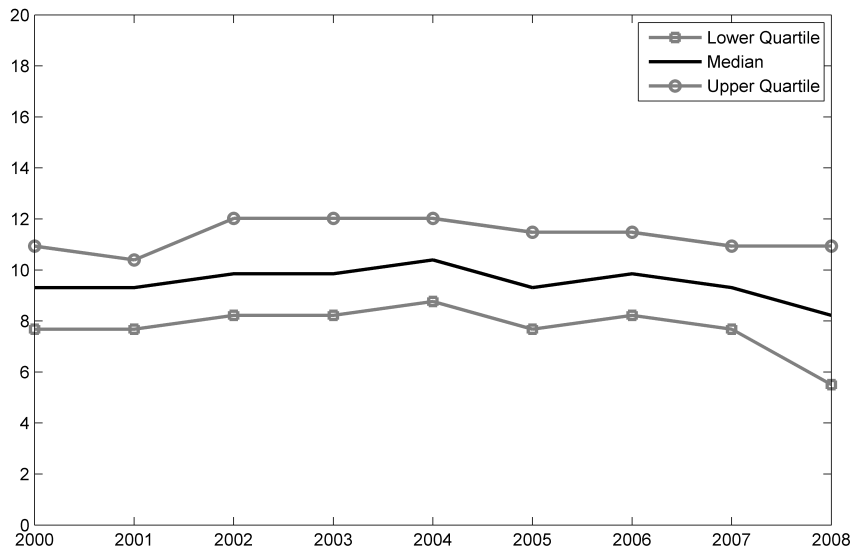
*Notes:* See Table 2.2. CONST, EXPUE, UEBEN, HQD, and UEBEND denote a constant, unemployment experience in years, unemployment benefits in 100 euros, a dummy for highly qualified individuals, and one on whether information on unemployment benefits is provided, respectively. We do not provide results for the year 2009 as data for the size of unemployment benefits are not available for this year.

Figure 2.1: Quartiles of the Densities Conditional on  $X = \bar{X}$

(a)  $\hat{f}_{w|\bar{X}}^t(\cdot)$



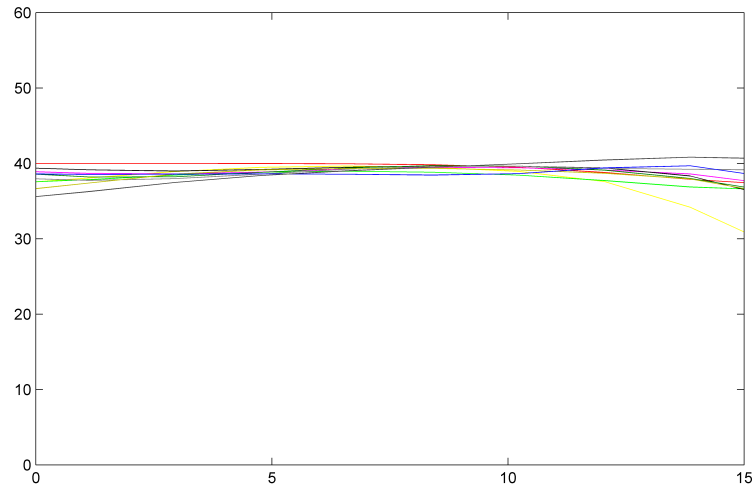
(b)  $\hat{f}_{w^R|\bar{X}}^t(\cdot)$



*Notes:* The horizontal axes measure years, and the vertical axes represent the wage rate (a) and the reservation wage rate (b), respectively. This figure shows the lower quartile, the median, and the upper quartile of the conditional densities  $\hat{f}_{w|\bar{X}}^t(\cdot)$  and  $\hat{f}_{w^R|\bar{X}}^t(\cdot)$ , respectively.

acteristics. Figure 2.1 shows the lower quartile, the median, and the upper quartile for the wage as well as the reservation wage distribution conditional on mean individual characteristics. It does not come as a surprise that the distribution of the reservation wage is left of the wage distribution for all years as individuals are only working if the offered wage exceeds the reservation wage. For the wage distribution, the lower quartile, the median, and the upper quartile vary around 10.3, 12.9, and 15.8, respectively. For 2001 the distribution is more dispersed which is possibly also one reason for the less accurate regression results in this year. On the other hand, for the reservation wage distribution, the lower quartile, the median, and the upper quartile vary around 7.7, 9.5, and 11.4, respectively. In 2008, the distribution shifts slightly to the left which probably is the result of a decrease in the mean size of unemployment benefits which have a negative influence on reservation wage rates.

Figure 2.2: Expectation of Weekly Working Hours Conditional on  $w = w^R$



*Notes:* The horizontal axis measures the real hourly wage rate, and the vertical axis represents working hours. This figure shows the regression functions for the conditional expectation  $\hat{\mathbb{E}}(h_t | w_t^R = w_t)$  for the years 2000 to 2009.

Next, we consider results from the conditional expectation estimation generated by considering the reservation wage and associated hours data for each year. Figure 2.2 shows nonparametric regression results for all years. The expectation corresponds to the hours a marginal worker would work at her reservation wage. Therefore, the estimated values of around 40 working hours per week seem plausible.

### 2.6.3 The Aggregate Frisch Wage-Elasticity of Labor Supply

For the calculation of the aggregate Frisch elasticity we determine the employment ratio  $EPR_t$ , the mean labor supply  $\bar{H}_t$  as well as the mean wage rate  $\bar{W}_t$  received by all working individuals directly from observed data (see Table 2.8 in Appendix A2.4). Results for the estimated determinants of the aggregate Frisch wage-elasticity, i. e.,  $\hat{\tau}_{h,t}^{int}$ ,  $\hat{\tau}_{h,t}^{ext}$ , and  $\hat{\tau}_{w,t}^{ext}$  are shown in Table 2.9 in Appendix A2.4 whereas results for the aggregate Frisch wage-elasticity

$$\begin{aligned}\hat{e}_t &= \frac{\bar{W}_t}{\bar{H}_t} \left( \frac{\hat{\tau}_{h,t}^{int} + \hat{\tau}_{h,t}^{ext}}{EPR_t + \hat{\tau}_{w,t}^{ext}} \right) \\ &= \underbrace{\frac{\bar{W}_t}{\bar{H}_t} \frac{1}{EPR_t + \hat{\tau}_{w,t}^{ext}} \cdot \hat{\tau}_{h,t}^{int}}_{\tilde{\tau}_{h,t}^{int}} + \underbrace{\frac{\bar{W}_t}{\bar{H}_t} \frac{1}{EPR_t + \hat{\tau}_{w,t}^{ext}} \cdot \hat{\tau}_{h,t}^{ext}}_{\tilde{\tau}_{h,t}^{ext}}.\end{aligned}$$

and its weighted components  $\tilde{\tau}_{h,t}^{int}$  and  $\tilde{\tau}_{h,t}^{ext}$  are shown in Table 2.5. The aggregate Frisch elasticity ranges from 0.63 in 2008 to 0.70 in 2003 and 2004. Considering only the first eight years from 2000 to 2007, the aggregate Frisch elasticity varies very little between 0.68 and 0.70. The slightly lower value of 0.63 in 2008 is caused by the lower hours' adjustment along the extensive margin, i. e., a lower value of  $\hat{\tau}_{h,t}^{ext}$  in 2008 compared to the other years. Our estimates of the aggregate elasticity are very close to what Fiorito and Zanella (2012) report for continuously employed men in the US. Table 2.5 also shows that about one third of the aggregate adjustment is due to hours' adjustment of stayers and the remaining two-thirds are due to hours worked by new entrants into the labor market.

## 2.7 Conclusion

This paper illustrates the power and the importance of taking aggregation seriously when thinking about possible links between individual and aggregate Frisch wage-elasticities of labor supply in an environment where workers are heterogeneous. Aggregation introduces non-linearities which drive a wedge between the mean of individual elasticities and their aggregate counterpart. Moreover, it allows for simultaneous treatment of hours' adjustment along the intensive and the extensive margin. When



Table 2.5: The Aggregate Frisch Wage-Elasticity and Weighted Components

| Wave | $\hat{\epsilon}_t$ | $\tilde{\gamma}_{h,t}^{int}$ | $\tilde{\gamma}_{h,t}^{ext}$ |
|------|--------------------|------------------------------|------------------------------|
| 2000 | 0.69               | 0.22                         | 0.47                         |
| 2001 | 0.68               | 0.27                         | 0.42                         |
| 2002 | 0.68               | 0.22                         | 0.46                         |
| 2003 | 0.70               | 0.22                         | 0.48                         |
| 2004 | 0.70               | 0.21                         | 0.49                         |
| 2005 | 0.69               | 0.24                         | 0.45                         |
| 2006 | 0.69               | 0.23                         | 0.46                         |
| 2007 | 0.68               | 0.24                         | 0.45                         |
| 2008 | 0.63               | 0.25                         | 0.38                         |

*Notes:* For the determination of the aggregate Frisch wage-elasticity  $\hat{\epsilon}_t$  we consider the sample as described in Section 2.5.

illustrating the method's quantitative implications using information on males at working age living in former West-Germany we find that adjustment along the extensive margin is twice as important as that along the intensive margin for the total variation in hours work. We thus corroborate a key result from the literature which reports that ca. two-thirds of all person hours' variation is due to workers moving in or out of employment.

The aggregation method developed in this paper is very general and flexible. Adapting it to alternative models of labor supply or using it to compute static elasticities such as Marshallian or Hicks is a straightforward exercise.

## A2 Appendix to Chapter 2

### A2.1 Formal Derivation of the Derivative of Equation (2.18), Second Term

We obtain

$$\begin{aligned}
& \int h(w + \Delta, \lambda, Y) I(w^R \in [w, w + \Delta]) d\pi_{w, w^R, \lambda, Y}^t \\
&= \int \int \int h(w + \Delta, \lambda, Y) d\pi_{(\lambda, Y)|(w^R, w)}^t I(w^R \in [w, w + \Delta]) d\pi_{w^R|w}^t d\pi_w^t \\
&= \int \left( \int_{\nu}^{\nu + \Delta} \mathbb{E} \left( h(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \tilde{\nu}, w_t = \nu \right) f_{w^R|\nu}^t(\tilde{\nu}) d\tilde{\nu} \right) f_w^t(\nu) d\nu.
\end{aligned}$$

In what follows we assume the conditional expectation  $\mathbb{E} \left( h(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \tilde{\nu}, w_t = \nu \right)$  as well as  $f_{w^R|\nu}^t(\tilde{\nu})$  to be continuous functions of  $\nu$  and  $\tilde{\nu}$ . Also note that  $\mathbb{E} \left( h(w_t, \lambda_t, Y_t) \mid w_t^R = w_t = \nu \right) = \mathbb{E} \left( h_t \mid w_t^R = w_t = \nu \right)$ . The mean value theorem then implies that for all  $\nu$  there exists a  $\xi_\nu \in [\nu, \nu + \Delta]$  such that

$$\begin{aligned}
& \int \left( \int_{\nu}^{\nu + \Delta} \mathbb{E} \left( h(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \tilde{\nu}, w_t = \nu \right) f_{w^R|\nu}^t(\tilde{\nu}) d\tilde{\nu} \right) f_w^t(\nu) d\nu \\
&= \int \Delta \mathbb{E} \left( h(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \xi_\nu, w_t = \nu \right) f_{w^R|\nu}^t(\xi_\nu) f_w^t(\nu) d\nu \\
&= \Delta \int \mathbb{E} \left( h_t \mid w_t^R = w_t = \nu \right) f_{w^R|\nu}^t(\nu) f_w^t(\nu) d\nu \\
&\quad + \Delta \int \left( \mathbb{E} \left( h(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \xi_\nu, w_t = \nu \right) f_{w^R|\nu}^t(\xi_\nu) \right. \\
&\quad \quad \left. - \mathbb{E} \left( h(w_t, \lambda_t, Y_t) \mid w_t^R = w_t = \nu \right) f_{w^R|\nu}^t(\nu) \right) f_w^t(\nu) d\nu.
\end{aligned}$$

Obviously, for all  $\nu$ ,

$$\left| \mathbb{E} \left( h(w_t + \Delta, \lambda_t, Y_t) \mid w_t^R = \xi_\nu, w_t = \nu \right) f_{w^R|\nu}^t(\xi_\nu) - \mathbb{E} \left( h(w_t, \lambda_t, Y_t) \mid w_t^R = w_t = \nu \right) f_{w^R|\nu}^t(\nu) \right| \rightarrow 0$$

as  $\Delta \rightarrow 0$ . Therefore,

$$\begin{aligned}
& \frac{\partial}{\partial \Delta} \int h(w + \Delta, \lambda_t, Y) I(w^R \in [w, w + \Delta]) d\pi_{w, w^R, \lambda, Y}^t \Big|_{\Delta=0} \\
&= \lim_{\Delta \rightarrow 0} \frac{\int h(w + \Delta, \lambda_t, Y) I(w^R \in [w, w + \Delta]) d\pi_{w, w^R, \lambda, Y}^t}{\Delta} \\
&= \int \mathbb{E} \left( h_t \mid w_t^R = w_t = \nu \right) f_{w^R | \nu}^t(\nu) f_w^t(\nu) d\nu.
\end{aligned}$$

## A2.2 Conditional Density and Expected Hours Estimation

In order to approximate  $\tau_{h,t}^{ext}$  and  $\tau_{w,t}^{ext}$  we need to first estimate the conditional densities  $f_{w|X}^t(\cdot)$  and  $f_{w^R|X}^t(\cdot)$  as well as the conditional expectation  $\mathbb{E}(h_t \mid w_t^R = w_t = \cdot)$ .

For the density estimation, we employ a two-step conditional density estimator and consider first the following two simple regression models for each period  $t$  and individuals  $i$  with positive (reservation) wage rate

$$w_{it} = \alpha_{t0} + \sum_{j=1}^p \alpha_{tj} X_{it,j} + \delta_{it}, \quad i = 1, \dots, N_t^w, \quad (2.32)$$

$$w_{it}^R = \alpha_{t0}^R + \sum_{j=1}^p \alpha_{tj}^R X_{it,j} + \delta_{it}^R, \quad i = 1, \dots, N_t^R, \quad (2.33)$$

where  $N_t^w$  denotes the number of wage observations in period  $t$ ,  $N_t^R$  denotes the number of reservation wage observations in period  $t$ ,  $\alpha_t = (\alpha_{t0}, \dots, \alpha_{tp})'$  and  $\alpha_t^R = (\alpha_{t0}^R, \dots, \alpha_{tp}^R)'$  are of dimension  $(p+1 \times 1)$  and  $X_{it}$  is a vector of  $p$  different observable attributes. We assume that the distributions of the random terms  $\delta_{it}$  and  $\delta_{it}^R$  are independent of  $X_{it}$  and calculate estimates  $\hat{\alpha}_t$  as well as residuals  $\hat{\delta}_{it} = w_{it} - \hat{\alpha}_{t0} - \sum_{j=1}^p \hat{\alpha}_{tj} X_{it,j}$  and  $\hat{\alpha}_t^R$  as well as  $\hat{\delta}_{it}^R = w_{it}^R - \hat{\alpha}_{t0}^R - \sum_{j=1}^p \hat{\alpha}_{tj}^R X_{it,j}$ , respectively.

Let  $f_{\delta}^t$  ( $f_{\delta^R}^t$ ) denote the density of the error terms  $\delta_{it}$  ( $\delta_{it}^R$ ) over the population. Then, on the one hand  $f_{w|X=X_{it}}^t(w_2) = f_{\delta}^t(w_2 - \alpha_{t0} - \sum_{j=1}^p \alpha_{tj} X_{it,j})$ , and we use a nonparametric kernel density estimator to determine an estimate  $\hat{f}_{\delta}$  from the residuals  $\{\hat{\delta}_{it}\}_{i=1}^{N_t^w}$  of regression model (2.33), on the other hand  $f_{w^R|X=X_{it}}^t(w_1) = f_{\delta^R}^t(w_1 - \alpha_{t0}^R - \sum_{j=1}^p \alpha_{tj}^R X_{it,j})$ , and we use a nonparametric kernel density estimator

to determine an estimate  $\hat{f}_{\delta^R}$  from the residuals  $\{\hat{\delta}_{it}^R\}_{i=1}^{N_t^R}$  of regression model (2.32):

$$\begin{aligned}\hat{f}_{w|X=X_{it}}^t(\cdot) &= \frac{1}{N_t^w b w_t^w} \sum_{j=1}^{N_t^w} k\left(\frac{\hat{\delta}_{jt} - (\cdot - \hat{\alpha}_{t0} - \sum_{l=1}^p \hat{\alpha}_{tl} X_{it,l})}{b w_t^w}\right), \\ \hat{f}_{w^R|X=X_{it}}^t(\cdot) &= \frac{1}{N_t^R b w_t^{w^R}} \sum_{j=1}^{N_t^R} k\left(\frac{\hat{\delta}_{jt}^R - (\cdot - \hat{\alpha}_{t0}^R - \sum_{l=1}^p \hat{\alpha}_{tl}^R X_{it,l})}{b w_t^{w^R}}\right),\end{aligned}$$

where  $k(\cdot)$  is a standard normal kernel and the bandwidths  $b w_t^{w^R}$  and  $b w_t^w$  are chosen according to the normal reference rule of thumb, i. e.,

$$\begin{aligned}k(v) &= \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}v^2\right), \\ b w_t^w &= 1.06 \cdot \sigma_{\delta_t} \cdot (N_t^w)^{-1/5}, \quad \text{and} \quad b w_t^{w^R} = 1.06 \cdot \sigma_{\delta_t^R} \cdot (N_t^R)^{-1/5},\end{aligned}$$

with  $\sigma_{\delta_t}$  ( $\sigma_{\delta_t^R}$ ) being the standard deviation of the error terms  $\delta_{it}$  ( $\delta_{it}^R$ ) in period  $t$ .

For the estimation of the conditional expectation  $\mathbb{E}(h_t | w_t^R = w_t = \cdot)$  we employ a local constant kernel estimator, also referred to as the Nadaraya-Watson kernel estimator (cf. Nadaraya (1964) and Watson (1964)). We use the reservation wage  $w^R$  as explanatory variable and associated desired working hours  $h^R$  as dependent variable to account for the condition  $w_t^R = w_t$ . This leads to

$$\hat{\mathbb{E}}\left(h_t | w_t^R = w_t = \nu\right) = \frac{\int h^R \hat{f}_{h^R, w^R}^t(\nu, h^R) dh^R}{\hat{f}^t(\nu)} = \frac{\sum_{i=1}^{N_t^R} h_{it}^R \cdot k\left(\frac{w_{it}^R - \nu}{b w^{\mathbb{E}}}\right)}{\sum_{i=1}^{N_t^R} k\left(\frac{w_{it}^R - \nu}{b w^{\mathbb{E}}}\right)}, \quad (2.34)$$

where  $b w^{\mathbb{E}}$  denotes the bandwidth and is calculated as follows. We use local constant least squares cross-validation with leave-one-out kernel estimator to calculate the smoothing parameter for each year. Then, the bandwidth  $b w^{\mathbb{E}}$  is the average over all smoothing parameters.

## A2.3 Data

### SOEP Samples

Each household and thereby each individual in the SOEP is part of one of the following samples:

- Sample A: ‘Residents in the FRG’, started 1984
- Sample B: ‘Foreigners in the FRG’, started 1984
- Sample C: ‘German Residents in the GDR’, started 1990
- Sample D: ‘Immigrants’, started 1994/95
- Sample E: ‘Refreshment’, started 1998
- Sample F: ‘Innovation’, started 2000
- Sample G: ‘Oversampling of High Income’, started 2002
- Sample H: ‘Extension’, started 2006
- Sample I: ‘Incentivation’, started 2009

## SOEP Variables

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| Variable Name | Variable Label                             |
|---------------|--|
| \$SAMREG      | Current wave sample region                 |
| PSAMPLE       | Sample member                              |
| SEX           | Gender                                     |
| GEBJAHR       | Year of birth                              |
| \$POP         | Sample membership                          |
| \$NETTO       | Current wave survey status                 |
| LABNET\$\$    | Monthly net labor income                   |
| \$TATZEIT     | Actual weekly working hours                |
| \$VEBZEIT     | Agreed weekly working hours                |
| \$UEBSTD      | Overtime per week                          |
| STIB\$\$      | Occupational position                      |
| Y11101\$\$    | Consumer price index                       |
| e. g. DP170   | Amount of necessary net income             |
| e. g. AP20    | Interest in full- or part-time work        |
| e. g. XP19    | Number of hours for net income             |
| EXPFT\$\$     | Working experience full-time employment    |
| EXPUE\$\$     | Unemployment experience                    |
| KLAS\$\$      | StaBuA 1992 Job Classification             |
| ISCED\$\$     | Highest degree/diploma attained            |
| \$FAMSTD      | Marital status in survey year              |
| e. g. DP9201  | Currently have steady partner              |
| e. g. HP10202 | Partner lives in household                 |
| \$BILZEIT     | Amount of education or training (in years) |
| \$P2F03       | Amount of monthly unemployment insurance   |
| \$P2G03       | Amount of monthly unemployment assistance  |

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## SOEP Variable Refinements

- Actual weekly working hours: When the value for the variable actual weekly working hours is missing, we use instead, if available, agreed weekly working hours and, if available, add overtime per week.
- Agreed weekly working hours: When the value for the variable agreed weekly working hours is missing, we use instead, if available, actual weekly working hours and, if available, subtract overtime per week.
- Amount of necessary net income: For the years 1984 to 2001 DM-values are converted to euros by dividing the respective DM-values by 1.95583.

## Sample

| Sample Definition   | Condition   |
|---|---|
| Only private households   | keep if $POP=1 \vee POP=2$                              |
| Only successful interviews  | keep if $NETTO \in \{10, 12, 13, 14, 15, 16, 18, 19\}$  |
| No first time interviewed persons aged 17   | drop if $NETTO=16$                                      |
| Male population   | drop if $SEX=2$   |
| West Germany  | drop if $SAMPREG=2$                                     |
| Age   | drop if $AGE < 25 \vee AGE > 64$                        |
| Exclusion of retirees   | drop if $STIB=13$                                       |
| Exclusion of individuals in military service under conscription or in community service as substitute for compulsory military service | drop if $STIB=15$                                       |
| Exclusion of individuals that are currently in education  | drop if $STIB=11$                                       |
| Individuals from sample A, E, F, H, I   | drop if $PSAMPLE \in \{2, 3, 4, 7\}$                    |
| No individuals with missing information   | drop if $BILZEIT < 0$ , drop if $EXPUE < 0$ and $h = 0$ |

## Descriptive Statistics

Table 2.6: Summary Statistics of Our Sample

|                                       | Employees |       |       | Non-Employees |       |       |
|---------------------------------------|-----------|-------|-------|---------------|-------|-------|
|                                       | 2000      | 2005  | 2009  | 2000          | 2005  | 2009  |
| Wave                                  |           |       |       |               |       |       |
| Observations                          | 1,296     | 1,296 | 1,296 | 121           | 126   | 119   |
| Age [yrs.]                            | 39.35     | 44.35 | 48.35 | 41.73         | 42.09 | 42.29 |
| Schooling completed [yrs.]            | 12.54     | 12.57 | 12.59 | 11.15         | 11.11 | 10.83 |
| Work experience [yrs.]                | 16.60     | 21.51 | 25.43 | 16.62         | 16.51 | 15.60 |
| Married or cohabiting [%]             | 0.81      | 0.84  | 0.84  | 0.68          | 0.78  | 0.66  |
| High-skilled [%]                      | 0.24      | 0.24  | 0.25  | 0.12          | 0.10  | 0.08  |
| Employed in O1                        | 0.01      | 0.02  | 0.01  | -             | -     | -     |
| Employed in O2                        | 0.45      | 0.44  | 0.41  | -             | -     | -     |
| Employed in O3                        | 0.54      | 0.54  | 0.57  | -             | -     | -     |
| Employed in O4                        | 0.00      | 0.00  | 0.01  | -             | -     | -     |
| Duration of non-employment [yrs.]     | -         | -     | -     | 2.70          | 3.34  | 3.88  |
| Entitled to unemployment benefits [%] | -         | -     | -     | 0.60          | 0.29  | -     |

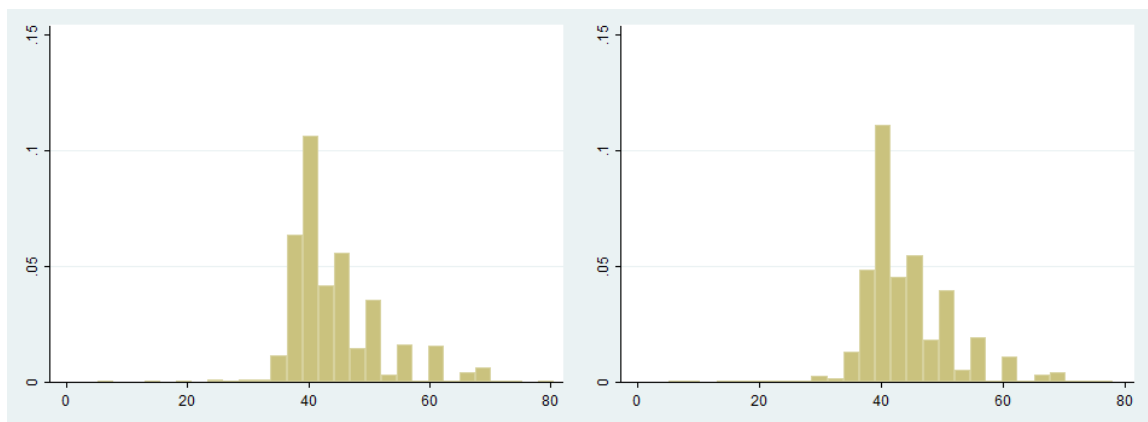
*Notes:* O1 represents workers employed in agriculture and related fields. O2 stands for employment in manufacture or technical occupations. O3 measures employment in services. O4 comprises all other workers. A detailed description of all variables is given in Section 2.5.



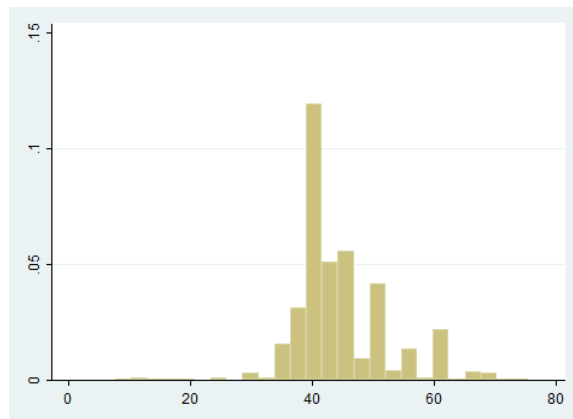
Figure 2.3: Histograms of Actual Weekly Hours Worked

(a) 2000

(b) 2005



(c) 2009



## A2.4 Results

Table 2.7: Results for the First Stage of the Panel Model Estimation

| $\log w$ | Coef.        |
|----------|--------------|
| SCHOOL   | 0.07***      |
| EXPFT    | 0.03***      |
| EXPFT2   | -4.52E-04*** |
| FAMILY   | 0.06***      |
| O1       | 0.01         |
| O3       | -3.96E-03    |
| O4       | 0.02         |
| CONST    | 1.30***      |

*Notes:* See Table 2.2. SCHOOL, EXPFT, EXPFT2, FAMILY, O1, O3, O4, and CONST denote the instruments for the wage rate, i. e., the schooling variable, work experience, and work experience squared, as well as the exogenous variables such as the family status, the three occupational groups, and a constant, respectively.

Table 2.8: Means of Hours Worked, Wages, and Employment Ratios

| Wave | $\bar{H}_t$ | $\bar{W}_t$ | $EPR_t$ |
|------|-------------|-------------|---------|
| 2000 | 44.19       | 12.61       | 0.91    |
| 2001 | 44.01       | 13.35       | 0.92    |
| 2002 | 44.05       | 13.18       | 0.91    |
| 2003 | 43.88       | 13.57       | 0.90    |
| 2004 | 43.93       | 13.47       | 0.91    |
| 2005 | 43.92       | 13.72       | 0.91    |
| 2006 | 44.16       | 13.64       | 0.91    |
| 2007 | 44.66       | 13.61       | 0.92    |
| 2008 | 44.48       | 13.44       | 0.93    |
| 2009 | 44.25       | 13.74       | 0.92    |

*Notes:* The employment ratio  $EPR_t$  is computed by dividing the number of working individuals by the total sample size in each period  $t$ .

Table 2.9: Estimated Components of the Aggregate Frisch Elasticity

| Wave | $\hat{\gamma}_{h,t}^{int}$ | $\hat{\gamma}_{h,t}^{ext}$ | $\hat{\gamma}_{w,t}^{ext}$ |
|------|----------------------------|----------------------------|----------------------------|
| 2000 | 1.23                       | 2.56                       | 0.64                       |
| 2001 | 1.21                       | 1.88                       | 0.45                       |
| 2002 | 1.14                       | 2.43                       | 0.66                       |
| 2003 | 1.12                       | 2.41                       | 0.65                       |
| 2004 | 1.12                       | 2.58                       | 0.71                       |
| 2005 | 1.12                       | 2.12                       | 0.55                       |
| 2006 | 1.12                       | 2.25                       | 0.60                       |
| 2007 | 1.13                       | 2.13                       | 0.54                       |
| 2008 | 1.15                       | 1.74                       | 0.44                       |

# Female Labor Supply in Germany: Evidence from Instrumental Variable Quantile Regression

## 3.1 Introduction

Labor supply elasticities have been of interest for economists since the sixties. The Marshallian and the Hicks elasticities help to understand labor supply decisions and to evaluate the influence of permanent tax changes on labor supply. Not only economists but also policy-makers are interested in estimates of labor supply elasticities: the responsiveness of labor supply to changes in the after-tax wage rate induced by tax increases determines the amount of tax revenue raised. The effects and benefits of gender-based taxation with lower marginal tax rates for women have been analyzed by Alesina, Ichino, and Karabarbounis (2011). It is of special importance to understand the reaction to tax changes of women who are often second earners and according to the German tax schedule therefore face higher tax rates than men.<sup>1</sup>

Each individual faces a participation decision of whether to work or not (extensive margin) and – if working – the decision of how much hours of work to supply (intensive margin). At the beginning of the modern labor supply literature in the 1960s to

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<sup>1</sup>Steiner and Wrohlich (2008) analyze labor supply effects of alternatives to the current system of joint taxation of married couples in Germany. The alternatives are a French-type family splitting and two full family splitting proposals. They show that under all three alternatives families with children in the upper part of the income distribution would gain most from the reforms.

1980s the participation rates of women in the labor market were low and far below the one of males (see e. g. Killingsworth and Heckman (1987), Heim (2007)). Therefore, the importance of the extensive margin for women has been stressed (Kimmel and Kniesner (1998)). However, in recent years the participation of women in the labor market has increased steadily. Female labor supply behavior has become more like the one of males, and elasticities have declined recently to values that are closer to male elasticities (Leibowitz and Klerman (1995), Juhn and Murphy (1997), Heim (2007), Blau and Kahn (2007), Bishop, Heim, and Mihaly (2009)).

As the importance of the extensive margin has declined, we analyze labor supply decisions of working women with respect to the intensive margin and their reaction to tax changes. We consider a static, within-period labor supply model as described in Blundell and MaCurdy (1999) and analyze micro elasticities of labor supply. We use micro level cross-section data from the German Socio-Economic Panel (SOEP) to analyze the Marshallian, the Hicks, and the income elasticity of labor supply. To evaluate the income effect, we need a measure for non-labor income which is defined as the total net household income not earned by the respective women. We use the earnings of the spouse as measure and assume that the spouse's labor supply decision and hence its earnings are exogenously determined and not influenced by the wife's decision on how much to work.

To analyze micro elasticities of labor supply, we estimate the individual labor supply function by parametric and semiparametric estimation methods. The semi-parametric instrumental variable (IV) quantile approach is more flexible and at the same time more easily tractable for applied researchers than fully nonparametric methods. Besides, this approach allows us to estimate the derivative of the individual labor supply function with respect to the individual wage rate which is needed for the calculation of micro elasticities of labor supply. The quantile approach ensures that we can analyze changes in the entire conditional hours distribution. This is of importance because the hours' reaction to changes in wage, non-labor income, or other covariates is possibly different at the lower than at the upper tail of the hours distribution.

We are not the first to use quantile regression methods in the estimation of labor supply functions. These methods have been used in the labor literature to analyze the

earnings or wage structure as well as labor supply. For example Buchinsky (1994, 1995) analyzes changes in the US wage structure and in the returns to education and experience at different points of the wage distribution as well as the shape of the entire conditional wage distribution by quantile regression methods. Abadie (1997) analyzes changes in the labor income structure as well as changes in the lower and upper tail of the conditional distribution of labor income in Spain. Dostie and Kromann (2012) estimate income and substitution labor supply and participation elasticities for Canadian married women by quantile regression methods and compare labor supply elasticities at different points of the conditional hours distribution. A brief review of recent empirical applications of quantile regression estimation in labor economics can be found in Koenker and Hallock (2001).

In the estimation of the individual labor supply function we need to account for the endogeneity of individual wage rates resulting from unobservable characteristics simultaneously affecting hours of work and wages. We do so by instrumenting for the individual hourly wage rate and employ the instrumental variable quantile regression estimator developed in Chernozhukov and Hansen (2005, 2006, 2008). This approach has been used in empirical studies by Chernozhukov and Hansen (2004), Hausman and Sidak (2004), Melly (2005), and Maynard and Qiu (2009), among others, however, to our knowledge there is no application with respect to labor supply.

For the instrumental variable quantile regression models, we find estimates of 0.62 and 0.63 on average for the Marshallian and the Hicks elasticity, respectively. The income elasticity is in general small and negative. Using quantile regression methods sheds further light on the labor supply function: females at the low end of the conditional hours distribution are more sensitive to changes in their wages. This implies higher labor supply elasticities in magnitude for these women and lower elasticities for women already working a lot.

The estimation of micro labor supply elasticities for females goes back to Mincer (1962). Killingsworth and Heckman (1987) provide a survey of female labor supply and consider static models of family labor supply, of the allocation of time, and of labor supply with heterogeneous jobs. Another strand of the literature considers dynamic models: Smith (1977), Heckman and MaCurdy (1980, 1982), Attanasio, Low, and Sanchez-Marcos (2008), amongst others, analyze life-cycle models of female

labor supply whereas Heckman (1976), MaCurdy (1981), and Browning, Deaton, and Irish (1985) analyze life-cycle models for males. A survey of male labor supply is done by Pencavel (1987), a recent survey of the male and female labor supply literature is provided by Keane (2011).

This paper is organized as follows. Section 3.2 lays out the static model of individual labor supply and introduces the different elasticities. Section 3.3 presents an econometric approach to empirically estimate the statistical objects of interest. Section 3.4 describes the data used for the empirical analysis, and Section 3.5 presents the results. Section 3.6 concludes.

## 3.2 A Labor Supply Model and Labor Supply Elasticities

We consider a static labor supply model as described in Blundell and MaCurdy (1999). For each period  $t$ , a person's utility depends on consumption  $c_t$  and leisure  $l_t$ . The utility function  $U$  is assumed to be twice differentiable, strictly increasing, and concave in consumption and leisure. The time-constraint in each period is given by

$$l_t + h_t = T \tag{3.1}$$

whereas the budget constraint is given by

$$c_t = w_t h_t + Y_t. \tag{3.2}$$

The variables  $h_t$ ,  $T$ ,  $w_t$ , and  $Y_t$  denote working hours, total time available, the after-tax wage rate, and non-labor income, respectively. An individual's optimization problem is given by

$$\max_{c_t, l_t} U(c_t, l_t) \quad \text{subject to} \quad 0 \leq l_t = T - h_t \tag{3.3}$$

$$0 \leq c_t = w_t h_t + Y_t. \tag{3.4}$$

The constraints state, that an individual splits her total available time between working and leisure and that for each period consumption is given by the sum of after-tax labor income and non-labor income. This implies that individuals do not borrow or save. One can interpret this as extreme short-sighted behavior of individuals that consider only the current period in their optimization decision.

Using both constraints the utility function can be rewritten as  $U(c_t, l_t) = U(w_t h_t + Y_t, T - h_t)$  such that the first-order condition for optimization with respect to working hours is given as follows:<sup>2</sup>

$$\partial_h U = \partial_c U w_t - \partial_l U = 0. \quad (3.5)$$

This can be reformulated such that the ratio of the marginal utility of leisure to the marginal utility of consumption equals the wage rate. Applying the implicit function theorem, equation (3.5) can be used to derive an expression for the individual labor supply:

$$\log h_t = g(\log w_t, Y_t). \quad (3.6)$$

We are interested in both the Marshallian and the Hicks labor supply elasticities.<sup>3</sup> The Marshallian elasticity  $e_t^M$  is sometimes also referred to as the uncompensated elasticity and describes the percentage change in labor supply for a one percent change in the wage:

$$e_t^M = \frac{\partial \log h_t}{\partial \log w_t} = \frac{w_t}{h_t} \frac{\partial h_t}{\partial w_t} = \partial_{\log w} g(\log w_t, Y_t). \quad (3.7)$$

An increase in the wage rate has two opposing effects. On the one hand, individuals can maintain a given consumption level with less work. Due to diminishing marginal utility of consumption this leads to a reduction of working hours (negative income effect). On the other hand, leisure is more costly relative to before and individuals tend to increase working hours by substituting work for leisure (positive substitution

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<sup>2</sup>For any differentiable function  $f(x_1, \dots, x_n)$  let  $\partial_{x_i} f(x_1, \dots, x_n)$  denote the partial derivative with respect to the  $i$ -th component.

<sup>3</sup>Keane (2011) not only provides a survey of the labor supply literature, he also devotes an entire section to the connection between labor supply and optimal taxation and elaborates on the different elasticity concepts.



effect). The Hicks elasticity  $e_t^H$  corresponds to the substitution effect and is also referred to as the compensated labor supply elasticity. The elasticity is given by the percentage change in labor supply for a one percent change in the wage given that the income is adjusted to keep utility constant. The income effect describes the percentage reduction in labor supply for a one percent increase in non-labor income:

$$e_t^Y = \frac{\partial \log h_t}{\partial \log Y_t} = \frac{Y_t}{h_t} \frac{\partial h_t}{\partial Y_t} = Y_t \partial_y g(\log w_t, Y_t). \quad (3.8)$$

The Slutsky equation links the Marshallian elasticity and the income and substitution effect:

$$e_t^M = e_t^H + \frac{w_t h_t}{Y_t} e_t^Y. \quad (3.9)$$

The income effect is negative, and the substitution effect is positive. Therefore, the Marshallian elasticity is smaller than the Hicks elasticity and can be positive or negative depending on the magnitudes of income and substitution effect. The Marshallian elasticity is negative if and only if the income effect exceeds the Hicks elasticity. The Hicks elasticity is given by the difference between the Marshallian elasticity and the income effect:

$$e_t^H = e_t^M - \frac{w_t h_t}{Y_t} e_t^Y. \quad (3.10)$$

Therefore, the Hicks elasticity can be thought of as measuring the effect of an increase in the marginal tax rate which is redistributed as a lump sum tax. The sole influence of an increased tax rate and hence lower after-tax wage rate on working hours is measured by the Marshallian elasticity. The redistribution as non-labor income is captured by the income effect.

### 3.3 Estimation Procedure

Recall that the individual labor supply is given by  $\log h_t = g(\log w_t, Y_t)$ . As is common in the labor supply literature, we assume a linear dependence of the wage

rate and non-labor income on working hours for each individual  $i$ :<sup>4</sup>

$$\log(h_t) = \text{const} + \alpha \log(w_t) + \beta Y_t + X_t' \gamma + \varepsilon_t, \quad (3.11)$$

where  $X_t$  is a vector of different observable individual characteristics and the parameter vector  $\gamma$  captures their influence on individual labor supply. The normally distributed zero mean error term  $\varepsilon_t$  accounts for the existence of unobservable individual characteristics.<sup>5</sup>

Recall that given the covariates the linear regression model, equation (3.11), specifies a conditional mean function:

$$\mathbb{E}[\log(h_t) | \log(w_t), Y_t, X_t] = \text{const} + \alpha \log(w_t) + \beta Y_t + X_t' \gamma. \quad (3.12)$$

We further estimate the individual labor supply model by conditional median – or in more general by quantile – rather than by ordinary conditional mean regressions. This allows us to estimate differential effects of the explanatory variables on quantiles in the conditional distribution of  $\log(h_t)$ :

$$\log(h_t) = \text{const}^{(q)} + \alpha^{(q)} \log(w_t) + \beta^{(q)} Y_t + X_t' \gamma^{(q)} + \varepsilon_t^{(q)}, \quad 0 < q < 1, \quad (3.13)$$

where we require the  $q$ -th quantile of the error process to be equal to zero.<sup>6</sup> Therefore,

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<sup>4</sup>Note that it is not uncommon in static labor supply models to state the labor supply function, i. e., equation (3.11), directly without considering preferences or an underlying utility function. However, it is possible to generate equation (3.11) by the indirect utility function  $v(w_t, Y_t) = \frac{w_t^{\alpha+1}}{\alpha+1} - \frac{\exp(-\beta Y_t)}{\beta \exp(\text{const}) \exp(X_t' \gamma) \exp(\varepsilon_t)}$  where by Roy's identity  $h_t = \frac{\partial v(w_t, Y_t)}{\partial w_t} / \frac{\partial v(w_t, Y_t)}{\partial Y_t}$ .

<sup>5</sup>As we consider female labor supply, one might expect sample selection to be an issue discussed in this section. However, as mentioned in the introduction and elaborated in Section 3.4 labor force participation of women has increased strongly and become more like the one of males. Besides, empirical tests on the significance of the coefficient of the estimated inverse Mills ratio show that the hypothesis of no sample selection bias cannot be rejected in almost all years considered (see Heckman (1974) and Melino (1982)). Results are available from the author upon request. Therefore, we refrain from considering sample selection explicitly in the estimation procedure.

<sup>6</sup>The estimation of quantile regressions goes back to Koenker and Bassett Jr. (1978). An introduction is given in Koenker and Hallock (2001), whereas Koenker (2005) describes the estimation and related issues in more detail. It amounts to estimating the values of  $\widehat{\text{const}}^{(q)}$ ,  $\hat{\alpha}^{(q)}$ ,  $\hat{\beta}^{(q)}$ , and  $\hat{\gamma}^{(q)}$  that minimize the following term:

$$\sum_{i=1}^n \rho_q \left( \log(h_{it}) - \text{const}^{(q)} - \alpha^{(q)} \log(w_{it}) - \beta^{(q)} Y_{it} - X_{it}' \gamma^{(q)} \right), \quad (3.14)$$

where  $n$  stands for sample size,  $\rho_q(x) = (1 - q)|x|I(x < 0) + qxI(x \geq 0)$  is a check function and

the  $q$ -th conditional quantile for the dependent variable given the covariates is:

$$Q^{(q)}(\log(h_t) | \log(w_t), Y_t, X_t) = \text{const}^{(q)} + \alpha^{(q)} \log(w_t) + \beta^{(q)} Y_t + X_t' \gamma^{(q)}. \quad (3.15)$$

The intercept  $\text{const}^{(q)}$  corresponds to the  $q$ -th quantile of the hours distribution given that all covariates are equal to zero. This approach has several advantages compared to linear regression analysis. First, it is more robust to outliers and non-normal errors. It is semiparametric in the sense that no assumption about the parametric distribution of the error process is needed. Second, in contrast to linear regression models which only consider the conditional mean, it provides a richer characterization of the conditional distribution of a variable of interest and allows us to detect various forms of shape shifts. Thereby, it considers the impact of explanatory variables on the entire distribution and allows us to analyze heterogeneity that is associated with the covariates. We estimate nine quantile regressions for  $q = 0.1, 0.2, \dots, 0.8, 0.9$  to get a comprehensive picture of the labor supply function and provide corresponding results for the labor supply elasticities.

In the estimation of the individual labor supply function we need to account for the endogeneity of individual wage rates resulting from unobservable characteristics simultaneously affecting hours of work and wages. We do so by instrumenting for the individual hourly wage rate and employ the instrumental variable quantile regression estimator described in Chernozhukov and Hansen (2008).<sup>7</sup> This paper advances

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$I(\cdot)$  denotes the indicator function.

<sup>7</sup>The estimation procedure is as follows: For a given value of  $\alpha$  and  $q \in (0, 1)$  estimate the quantile regression as described in footnote 6 of  $\log(h_t) - \alpha \log(w_t)$  on the constant,  $Y_t$ ,  $X_t$ , and the excluded exogenous instruments  $Z_t$ , i. e., find values of  $\widehat{\text{const}}^{(q)}(\alpha)$ ,  $\widehat{\beta}^{(q)}(\alpha)$ ,  $\widehat{\gamma}^{(q)}(\alpha)$ , and  $\widehat{\delta}^{(q)}(\alpha)$  that minimize

$$\sum_{i=1}^n \rho_q \left( \log(h_{it}) - \alpha \log(w_{it}) - \text{const}^{(q)} - \beta^{(q)} Y_{it} - X_{it}' \gamma^{(q)} - \delta^{(q)} Z_{it} \right). \quad (3.16)$$

This ordinary quantile regression is performed for a series of different candidate values of  $\alpha$ . Next, among those values find  $\widehat{\alpha}^{(q)}$  that makes the coefficients on the instrumental variables,  $\widehat{\delta}^{(q)}(\alpha)$ , as close to zero as possible, i. e., find  $\widehat{\alpha}^{(q)}$  that minimizes

$$n[\widehat{\delta}^{(q)}(\alpha)]' \widehat{A}^{(q)}(\alpha) [\widehat{\delta}^{(q)}(\alpha)], \quad (3.17)$$

where  $\widehat{A}^{(q)}(\alpha)$  is the inverse of the variance-covariance matrix of  $\widehat{\delta}^{(q)}(\alpha)$  from the ordinary quantile regression estimation. Then, coefficient estimates are given by  $\widehat{\alpha}^{(q)}$ ,  $\widehat{\text{const}}^{(q)}(\widehat{\alpha}^{(q)})$ ,  $\widehat{\beta}^{(q)}(\widehat{\alpha}^{(q)})$ ,  $\widehat{\gamma}^{(q)}(\widehat{\alpha}^{(q)})$ , and  $\widehat{\delta}^{(q)}(\widehat{\alpha}^{(q)})$ .

previous work by Chernozhukov and Hansen (2006) in two ways. First, the estimator and inference procedure accommodate overidentified models. Second, the inference procedure is robust to weak, partial, and non-identification.

The IV quantile regression procedure amounts to estimating a series of ordinary quantile regressions and requires the excluded exogenous instruments  $Z_t$  to be independent of the error term  $\varepsilon_t^{(q)}$ . It is based on the observation that zero is the  $q$ -th quantile of

$$\log(h_t) - \text{const}^{(q)} - \alpha^{(q)} \log(w_t) - \beta^{(q)} Y_t - X_t' \gamma^{(q)} \quad (3.18)$$

conditional on  $(Y_t, X_t, Z_t)$  and shown to be consistent and asymptotically normal under appropriate conditions.

Finally, depending on whether we consider equation (3.11) or the instrumental variable quantile model, we estimate the Marshallian elasticity by

$$\hat{e}_t^M = \hat{\alpha} \quad \text{and} \quad \hat{e}_t^M = \hat{\alpha}^{(q)}, \quad (3.19)$$

respectively. The income effect is given by

$$\hat{e}_t^Y = Y_t \hat{\beta} \quad \text{and} \quad \hat{e}_t^Y = Y_t \hat{\beta}^{(q)}, \quad (3.20)$$

whereas the resulting Hicks elasticity is calculated as

$$\hat{e}_t^H = \hat{\alpha} - w_t h_t \hat{\beta} \quad \text{and} \quad \hat{e}_t^H = \hat{\alpha}^{(q)} - w_t h_t \hat{\beta}^{(q)}, \quad (3.21)$$

respectively.

### 3.4 Data

We use micro level cross-section data from the German Socio-Economic Panel (SOEP) to analyze the Marshallian, the Hicks, and the income elasticities of labor supply. The SOEP is a longitudinal panel dataset of the German population that spans up to now the time from 1984 (wave A) to 2011 (wave BB). It is a representative sample of

private households as well as on the individuals belonging to those households. The SOEP covers about 12,000 households and more than 20,000 adult persons. Each household and thereby each individual in the SOEP is part of one of the samples A to I. Household members are re-interviewed annually and are asked a set of (partly changing) questions belonging to different topics, such as labor market and employment, income, taxes and social security, living and household, health, education and qualification, etc.<sup>8</sup>

In our analysis, we focus on the years 2003, 2006, and 2009 to get an overview over recent developments and consider individuals from sample A, E, F, H, and I to get information with respect to German residents. In particular, we focus on the female population of former West Germany because we are not interested in the effects of the different labor market behavior in former East and West Germany but rather want to investigate consistent data. We consider the working age population of females aged between 25 and 64. Therefore, we exclude retirees and individuals that are currently in education. Furthermore, we focus on employed females that have a spouse or partner relationship in a SOEP household.<sup>9</sup>

We use the following variables in our estimation procedure.<sup>10</sup> Person  $i$ 's total hours worked  $h_{it}$  is given by average actual weekly working hours, including potential overtime. There is a wide range of answers to the question "And how much on average does your actual working week amount to, with possible overtime?" – answers range from 1 to 77 hours per week. In fact, the distribution of  $h_{it}$  is not discrete in nature, but quite dispersed.<sup>11</sup> The hourly wage rate is calculated by dividing the current labor income by 4.3 times the contractual weekly working hours. We use net earnings, as the model requires information on after-tax wage rates. Non-labor income is defined as the total net household income not earned by women  $i$  and given by the spouse's labor income. We assume that the spouse's labor supply decision and hence its labor income is exogenously determined and not influenced by the

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<sup>8</sup>For a detailed description of the dataset see Haisken-DeNew and Frick (2005).

<sup>9</sup>A detailed description of the estimation sample is given in Appendix A3.1. In particular, we list all refinements to the original data.

<sup>10</sup>A list of all SOEP variables with respective names as well as a list of all generated variables with description is given in Appendix A3.1.

<sup>11</sup>Histograms of actual hours worked for the years 2003, 2006, and 2009 are available in Appendix A3.1.

wife's decision on how much to work. The use of other sources of non-labor income is limited by data quality and availability. In order to make estimation results for the different years comparable the consumer price index provided by the SOEP is used to convert nominal values of the wage rate and non-labor income to real values using 2005 as base income year.

Table 3.1: Summary Statistics of Our Sample

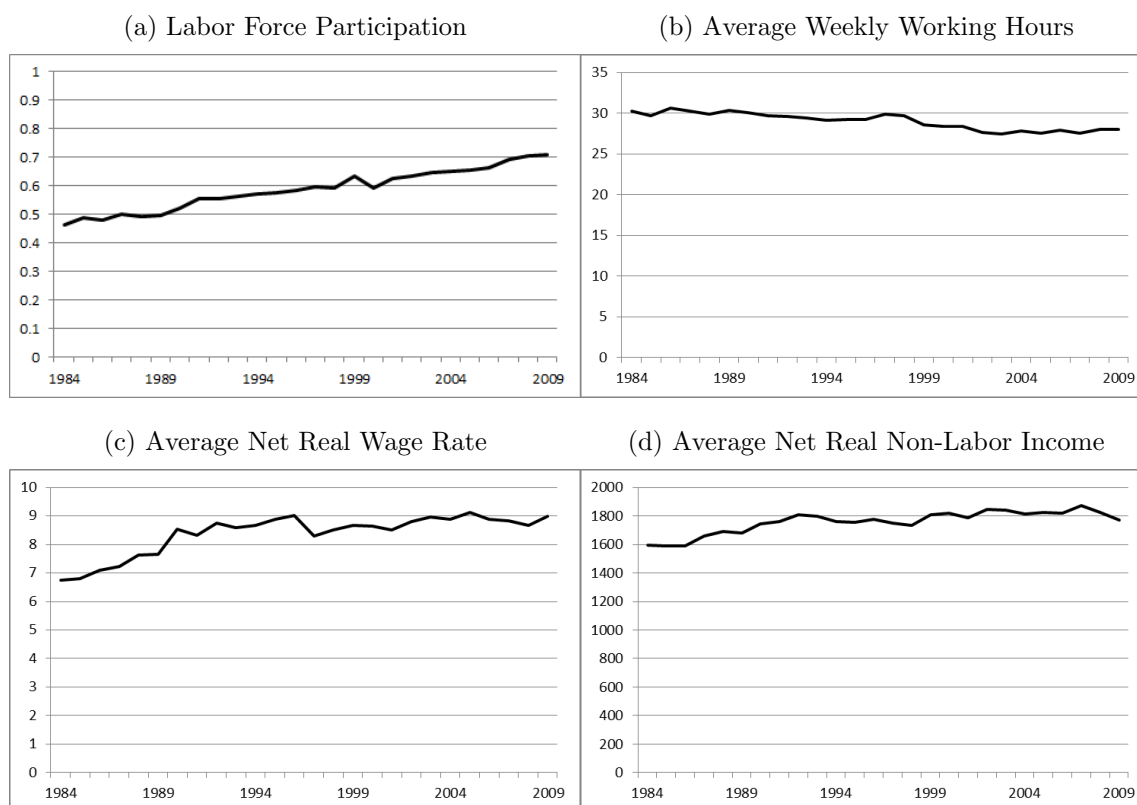
|                            | Employees        |                  |                  |
|----------------------------|------------------|------------------|------------------|
|                            | 2003             | 2006             | 2009             |
| Wave                       | 2003             | 2006             | 2009             |
| Observations               | 1,974            | 1,982            | 1,914            |
| Age [yrs.]                 | 42.97<br>(8.98)  | 44.06<br>(8.96)  | 44.99<br>(9.09)  |
| Real wage rate [EUR]       | 8.94<br>(5.04)   | 8.87<br>(4.83)   | 8.99<br>(5.10)   |
| Actual hours worked        | 27.50<br>(13.27) | 27.93<br>(13.32) | 27.97<br>(13.37) |
| Spouse's earnings [EUR]    | 1,843<br>(1,117) | 1,818<br>(1,139) | 1,772<br>(1,108) |
| Work experience [yrs.]     | 10.68<br>(8.95)  | 11.18<br>(9.07)  | 11.08<br>(9.01)  |
| Schooling completed [yrs.] | 12.07<br>(2.45)  | 12.28<br>(2.49)  | 12.43<br>(2.53)  |
| Number of children         | 1.67<br>(1.07)   | 1.58<br>(1.10)   | 1.49<br>(1.12)   |
| Young children [%]         | 0.08             | 0.07             | 0.09             |
| Medium-aged children [%]   | 0.11             | 0.10             | 0.09             |
| Older children [%]         | 0.43             | 0.40             | 0.40             |
| Married [%]                | 0.87             | 0.86             | 0.86             |

*Notes:* Standard deviations are given in parentheses. A detailed description of all variables is given in the text.

The individual characteristics and family structure variables are given by work experience in full-time employment, a dummy for the family status (1 if married, 0 otherwise), the number of children, a dummy variable for young children (1 if at least one child is younger than three years, 0 otherwise), a dummy variable for medium-aged children (1 if at least one child is between four and six years old, 0 otherwise), and a dummy variable for older children (1 if at least one child is between seven and 18 years old, 0 otherwise).

As mentioned in Section 3.3 we use an IV approach to account for the possible endogeneity of wages. Typical instruments used in the estimation of labor supply functions include age, education, interactions between age and education, regional

Figure 3.1: Married Female Sample Statistics



*Notes:* The sample is described in Section 3.4. For the labor force participation we not only consider employed females but all females with the before-mentioned characteristics.

dummies, experience, years of schooling, health dummies, and parental education. Following the ideas of Mincer (1974) who viewed wages as predominantly determined by accumulated human capital, we include as instruments schooling and work experience squared. The schooling variable is based on the number of years of education or training undergone. It includes secondary vocational education and ranges from 7 to 18 years.

Summary statistics of our sample are shown in Table 3.1. Compared to a similar sample of male employees, females have a lower real hourly wage rate. In average their work experience in full-time employment and their working hours are lower, as more women work part-time.

As mentioned in the introduction, the labor supply behavior of women has changed over time. Killingsworth and Heckman (1987) consider labor force participation rates by age and marital status of females in Germany during the twentieth century (1895 to 1981). They find substantial increases in the labor force participa-

tion among those aged between 25 and 64 where most of the increase is due to an increase in the participation rate of married women. Recent empirical trends (1984 to 1995) in Germany are discussed by Blundell and MaCurdy (1999). They consider the employment to population ratio, annual hours worked, and real average hourly earnings by education and age for males and for females: female labor force participation has increased slowly until 1992 for all groups, the gradual decrease of working hours has been slow, and the real wage has increased for all education groups.

Figure 3.1 shows the labor force participation rate, working hours, the wage rate and non-labor income for our sample and the years 1984 to 2009. The labor force participation of women increases steadily from 46 % in 1984 to 71 % in 2009. Working hours decrease slightly and range from 27 to 31 hours a week whereas the wage rate and non-labor income increase slightly from 1984 to 1990 and stay more or less constant from then on.

### 3.5 Results

In this section we present results for the estimation of the individual labor supply models and the three different labor supply elasticities. As is common in this literature we expect modest positive Marshallian elasticities and a small income effect (see Table 2.26 in Killingsworth and Heckman (1987) and Table 7 in Keane (2011)).

Table 3.2 shows results for the estimation of the linear benchmark model, equation (3.11).<sup>12</sup> Almost all coefficients are highly significant. The wage coefficients are positive and have increased from 0.60 in 2003 to 0.71 in 2009 whereas the coefficients on non-labor income are negative and very small in absolute value. The coefficients on work experience in full-time employment are highly significantly positive for all years and lie between 0.01 and 0.02. The coefficients on the number of children are

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<sup>12</sup>Results for the first stage of the IV approach are presented in Appendix A3.2. An important issue when using an IV approach is the strength of instruments. The first stage F statistic is equivalent to the Cragg-Donald statistic in the linear IV regression in the case of one endogenous regressor (see Cragg and Donald (1993)). In the case of an IV regression with a single endogenous regressor and i.i.d. errors, instruments are considered to be strong, if the first stage F statistic exceeds 10 (see the rule of thumb by Staiger and Stock (1997)). For linear IV regressions Stock and Yogo (2005) provide critical values to test for weak instruments based on the maximum Wald test size distortion. The critical value for one endogenous regressor and two instruments at the 10% significance level is 19.93. Whenever the Cragg-Donald statistic exceeds the critical values, one can reject the null hypothesis of weak instruments. We consider this as evidence of strong instruments.



also highly significant and negative, i. e., working hours of female employees decrease in the number of children. All family status coefficients are negative, implying lower working hours by married women. This is also the case for the dummy variables for young, medium-aged, and older children: estimated coefficients lie between -0.44 and -0.24 for young, -0.23 and -0.16 for medium-aged, and -0.11 and -0.06 for older children.

Table 3.2: Benchmark Estimates for Equation (3.11)

|       | 2003                    | 2006                    | 2009                    |
|-------|-------------------------|-------------------------|-------------------------|
| $w$   | 0.60<br>(0.08)          | 0.68<br>(0.09)          | 0.71<br>(0.10)          |
| $Y$   | -5.99E-05<br>(1.37E-05) | -7.04E-05<br>(1.36E-05) | -3.96E-05<br>(1.56E-05) |
| EXP   | 0.01<br>(0.00)          | 0.01<br>(0.00)          | 0.02<br>(0.00)          |
| MAR   | -0.19<br>(0.05)         | -0.25<br>(0.05)         | -0.23<br>(0.05)         |
| SK    | -0.05<br>(0.02)         | -0.08<br>(0.02)         | -0.06<br>(0.02)         |
| K1    | -0.35<br>(0.06)         | -0.24<br>(0.06)         | -0.44<br>(0.08)         |
| K2    | -0.23<br>(0.05)         | -0.16<br>(0.05)         | -0.22<br>(0.06)         |
| K3    | -0.11<br>(0.03)         | -0.06<br>(0.03)         | -0.10<br>(0.04)         |
| CONST | 2.24<br>(0.17)          | 2.14<br>(0.18)          | 2.01<br>(0.20)          |

*Notes:* Standard errors are in parentheses. CONST, EXP, MAR, SK, K1, K2, and K3 represent a constant, work experience in full-time employment, the family status dummy variable, the number of children, and dummy variables for young, medium-aged, and older children, respectively.

Table 3.3: Elasticity Estimates for the Benchmark Case

| Wave                   | 2003  | 2006  | 2009  |
|------------------------|-------|-------|-------|
| Marshallian elasticity | 0.60  | 0.68  | 0.71  |
| Income elasticity      | -0.11 | -0.13 | -0.07 |
| Hicks elasticity       | 0.62  | 0.70  | 0.72  |

*Notes:* This table shows estimates for the Marshallian, income, and Hicks elasticities as described in equations (3.7), (3.8), and (3.10), respectively, for the years 2003, 2006, and 2009.

Estimation results for the Marshallian, the Hicks, and the income elasticity for the years 2003, 2006, and 2009 are shown in Table 3.3. For the Marshallian elasticity, all results are positive and range from 0.60 to 0.71. The Hicks elasticity is quite similar

to the Marshallian elasticity as the derivative of working hours with respect to non-labor income is small. The income elasticities for the three years are in general small and negative.

Our estimates are in line with what other authors report. Both Keane (2011) and Killingsworth and Heckman (1987) summarize elasticity estimates of selected studies of female labor supply. For the Marshallian elasticity the former author considers three studies and finds a range of -0.2 to 0.89 whereas the latter authors report 89 estimates which range from -0.89 to the extreme value of 15.24. However, values between the lower and upper quartile of the elasticity estimates only range from 0 to 1.14. For the income elasticity, Killingsworth and Heckman (1987) report values from -0.89 to 0.48 and -0.195 to -0.02 considering only the range between the lower and upper quartile.

The use of quantile regression methods allows us to investigate the labor supply function and the entire conditional hours distribution in more detail. Instrumental variable quantile regression estimates are presented in Table 3.4 and in Figures 3.5 to 3.13 in Appendix A3.2. The constant as well as the coefficients on the hourly wage rate, non-labor income, the marital status, the number of children, and the dummies for young, medium-aged, and older children are significant or highly significant in most of the cases, in particular for the middle part of the hours distribution. In contrast, work experience in full-time employment has in none of the specifications a highly significant influence on labor supply of female employees.

In general, individuals at the upper tail of the conditional hours distribution show less reaction in response to changes in covariates than individuals at the middle and lower tail, i. e., coefficients are closer to zero for a high  $q$ . On the one hand, employees already working a lot have less possibility to increase working hours in response to changes in individual characteristics. On the other hand, individuals with high working hours might have not as flexible work options: those individuals work possibly full-time and long hours and are not allowed or do not allow themselves to work part-time or reduce overtime. Besides, females working full-time and their families, if any, might depend more heavily on earnings than females working part-time who are even more often second earners.

For the lower tail and the middle part of the conditional hours distribution the

Table 3.4: IV Quantile Regression Estimates

| 2003     | 0.1                     | 0.2                     | 0.3                     | 0.4                     | 0.5                     | 0.6                     | 0.7                     | 0.8                     | 0.9                     |
|----------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| <i>w</i> | 0.70<br>(0.13)          | 0.61<br>(0.09)          | 0.57<br>(0.09)          | 0.58<br>(0.09)          | 0.64<br>(0.08)          | 0.59<br>(0.07)          | 0.54<br>(0.07)          | 0.48<br>(0.07)          | 0.47<br>(0.09)          |
| Y        | -5.31E-05<br>(2.98E-05) | -3.39E-05<br>(1.98E-05) | -4.30E-05<br>(1.50E-05) | -4.93E-05<br>(1.36E-05) | -5.96E-05<br>(1.27E-05) | -6.09E-05<br>(1.23E-05) | -5.62E-05<br>(1.33E-05) | -6.32E-05<br>(1.39E-05) | -5.81E-05<br>(1.69E-05) |
| EXP      | 0.02<br>(0.01)          | 0.02<br>(0.01)          | 0.01<br>(0.01)          | 0.01<br>(0.01)          | 0.01<br>(0.00)          | 0.01<br>(0.00)          | 0.01<br>(0.00)          | 0.01<br>(0.00)          | 0.00<br>(0.00)          |
| MAR      | -0.22<br>(0.11)         | -0.31<br>(0.06)         | -0.27<br>(0.04)         | -0.22<br>(0.04)         | -0.17<br>(0.04)         | -0.13<br>(0.03)         | -0.10<br>(0.03)         | -0.07<br>(0.03)         | -0.04<br>(0.04)         |
| SK       | -0.09<br>(0.04)         | -0.07<br>(0.02)         | -0.07<br>(0.02)         | -0.07<br>(0.02)         | -0.06<br>(0.02)         | -0.06<br>(0.01)         | -0.05<br>(0.02)         | -0.03<br>(0.01)         | -0.01<br>(0.01)         |
| K1       | -0.60<br>(0.15)         | -0.63<br>(0.12)         | -0.48<br>(0.10)         | -0.40<br>(0.11)         | -0.32<br>(0.10)         | -0.23<br>(0.08)         | -0.14<br>(0.07)         | -0.11<br>(0.07)         | -0.01<br>(0.07)         |
| K2       | -0.29<br>(0.11)         | -0.26<br>(0.07)         | -0.42<br>(0.07)         | -0.30<br>(0.07)         | -0.28<br>(0.07)         | -0.25<br>(0.06)         | -0.24<br>(0.05)         | -0.25<br>(0.05)         | -0.19<br>(0.06)         |
| K3       | -0.15<br>(0.08)         | -0.22<br>(0.06)         | -0.16<br>(0.04)         | -0.17<br>(0.04)         | -0.17<br>(0.03)         | -0.16<br>(0.03)         | -0.08<br>(0.03)         | -0.07<br>(0.03)         | -0.03<br>(0.04)         |
| CONST    | 1.29<br>(0.28)          | 1.91<br>(0.21)          | 2.24<br>(0.20)          | 2.35<br>(0.19)          | 2.31<br>(0.18)          | 2.49<br>(0.16)          | 2.65<br>(0.16)          | 2.83<br>(0.15)          | 2.96<br>(0.18)          |
| 2006     | 0.1                     | 0.2                     | 0.3                     | 0.4                     | 0.5                     | 0.6                     | 0.7                     | 0.8                     | 0.9                     |
| <i>w</i> | 0.72<br>(0.13)          | 0.68<br>(0.10)          | 0.66<br>(0.09)          | 0.68<br>(0.09)          | 0.63<br>(0.08)          | 0.59<br>(0.08)          | 0.60<br>(0.08)          | 0.53<br>(0.08)          | 0.44<br>(0.08)          |
| Y        | -6.74E-05<br>(2.35E-05) | -8.22E-05<br>(2.03E-05) | -6.76E-05<br>(1.46E-05) | -6.90E-05<br>(1.36E-05) | -6.79E-05<br>(1.31E-05) | -5.91E-05<br>(1.30E-05) | -5.21E-05<br>(1.31E-05) | -4.24E-05<br>(1.31E-05) | -3.87E-05<br>(1.29E-05) |
| EXP      | 0.02<br>(0.01)          | 0.02<br>(0.01)          | 0.01<br>(0.01)          | 0.01<br>(0.01)          | 0.01<br>(0.00)          | 0.01<br>(0.00)          | 0.01<br>(0.00)          | 0.01<br>(0.00)          | 0.01<br>(0.00)          |
| MAR      | -0.42<br>(0.08)         | -0.38<br>(0.05)         | -0.34<br>(0.04)         | -0.27<br>(0.04)         | -0.21<br>(0.04)         | -0.19<br>(0.04)         | -0.16<br>(0.03)         | -0.11<br>(0.03)         | -0.07<br>(0.03)         |
| SK       | -0.16<br>(0.04)         | -0.12<br>(0.03)         | -0.09<br>(0.02)         | -0.08<br>(0.02)         | -0.07<br>(0.02)         | -0.06<br>(0.02)         | -0.04<br>(0.02)         | -0.04<br>(0.02)         | -0.02<br>(0.01)         |
| K1       | -0.85<br>(0.25)         | -0.35<br>(0.23)         | -0.09<br>(0.09)         | -0.08<br>(0.07)         | -0.08<br>(0.07)         | -0.06<br>(0.07)         | -0.02<br>(0.07)         | -0.03<br>(0.06)         | -0.05<br>(0.06)         |
| K2       | -0.15<br>(0.10)         | -0.22<br>(0.08)         | -0.28<br>(0.07)         | -0.23<br>(0.07)         | -0.21<br>(0.06)         | -0.20<br>(0.06)         | -0.21<br>(0.06)         | -0.15<br>(0.06)         | -0.10<br>(0.06)         |
| K3       | 0.04<br>(0.07)          | -0.06<br>(0.05)         | -0.08<br>(0.04)         | -0.08<br>(0.04)         | -0.12<br>(0.04)         | -0.11<br>(0.04)         | -0.14<br>(0.04)         | -0.10<br>(0.04)         | -0.07<br>(0.04)         |
| CONST    | 1.42<br>(0.29)          | 1.90<br>(0.22)          | 2.09<br>(0.19)          | 2.14<br>(0.20)          | 2.34<br>(0.18)          | 2.48<br>(0.17)          | 2.54<br>(0.17)          | 2.77<br>(0.16)          | 2.99<br>(0.15)          |
| 2009     | 0.1                     | 0.2                     | 0.3                     | 0.4                     | 0.5                     | 0.6                     | 0.7                     | 0.8                     | 0.9                     |
| <i>w</i> | 0.84<br>(0.12)          | 0.81<br>(0.11)          | 0.73<br>(0.09)          | 0.64<br>(0.08)          | 0.56<br>(0.08)          | 0.55<br>(0.08)          | 0.49<br>(0.08)          | 0.56<br>(0.09)          | 0.72<br>(0.18)          |
| Y        | -1.52E-05<br>(2.11E-05) | -3.64E-05<br>(1.83E-05) | -3.28E-05<br>(1.56E-05) | -3.99E-05<br>(1.42E-05) | -4.23E-05<br>(1.35E-05) | -4.44E-05<br>(1.45E-05) | -5.01E-05<br>(1.52E-05) | -5.31E-05<br>(1.84E-05) | -7.77E-05<br>(2.97E-05) |
| EXP      | 0.02<br>(0.01)          | 0.02<br>(0.01)          | 0.01<br>(0.01)          | 0.01<br>(0.01)          | 0.01<br>(0.01)          | 0.01<br>(0.01)          | 0.01<br>(0.00)          | 0.01<br>(0.01)          | 0.01<br>(0.01)          |
| MAR      | -0.41<br>(0.07)         | -0.33<br>(0.06)         | -0.23<br>(0.05)         | -0.22<br>(0.04)         | -0.19<br>(0.04)         | -0.20<br>(0.04)         | -0.15<br>(0.04)         | -0.13<br>(0.04)         | -0.12<br>(0.06)         |
| SK       | -0.15<br>(0.04)         | -0.12<br>(0.03)         | -0.09<br>(0.02)         | -0.08<br>(0.02)         | -0.07<br>(0.02)         | -0.06<br>(0.02)         | -0.03<br>(0.02)         | -0.02<br>(0.01)         | -0.01<br>(0.02)         |
| K1       | -0.83<br>(0.25)         | -0.58<br>(0.12)         | -0.66<br>(0.11)         | -0.54<br>(0.13)         | -0.39<br>(0.11)         | -0.39<br>(0.10)         | -0.26<br>(0.11)         | -0.21<br>(0.11)         | -0.09<br>(0.15)         |
| K2       | -0.26<br>(0.11)         | -0.26<br>(0.09)         | -0.27<br>(0.07)         | -0.31<br>(0.07)         | -0.25<br>(0.07)         | -0.19<br>(0.07)         | -0.18<br>(0.06)         | -0.16<br>(0.07)         | -0.12<br>(0.08)         |
| K3       | -0.08<br>(0.08)         | -0.11<br>(0.06)         | -0.13<br>(0.05)         | -0.13<br>(0.04)         | -0.12<br>(0.04)         | -0.13<br>(0.04)         | -0.10<br>(0.04)         | -0.10<br>(0.04)         | -0.08<br>(0.05)         |
| CONST    | 1.16<br>(0.28)          | 1.51<br>(0.25)          | 1.83<br>(0.20)          | 2.14<br>(0.18)          | 2.40<br>(0.16)          | 2.51<br>(0.16)          | 2.69<br>(0.15)          | 2.68<br>(0.17)          | 2.56<br>(0.31)          |

*Notes:* This table shows instrumental variable quantile regression estimates for equation (3.13) for the years 2003, 2006, and 2009 and  $q = 0.1, 0.2, \dots, 0.8, 0.9$ . Standard errors are in parentheses. CONST, EXP, MAR, SK, K1, K2, and K3 represent a constant, work experience in full-time employment, the family status dummy variable, the number of children, and dummy variables for young, medium-aged, and older children, respectively. This table is constructed using code by Chernozhukov and Hansen (2006, 2008).

covariates can be split into two groups. For the first group, the reaction of working hours in response to changes in the respective covariate does more or less not depend on the location of the individual on the hours distribution. This is the case for non-labor income and the dummy on older children. For the second group, i. e., wage, work experience in full-time employment, the marital status, the number of children, and the dummies on young and medium-aged children, there are different reactions in response to changes in the respective covariates at different points of the hours distribution.

Considering the first group, especially for the middle part of the conditional hours distribution the coefficients on non-labor income and the dummy for older children do not depend on how much an individual works, but are more or less the same for different  $q$ . The coefficients on non-labor income are all negative, leading to an income elasticity of the same sign. The coefficients on the dummy for older children are in general negative and different for the lower quantiles depending on the year considered. In general, results for the three years are less similar for the lower quantiles. This is possibly due to the presence of outliers which can influence results for the very low and very high quantiles.

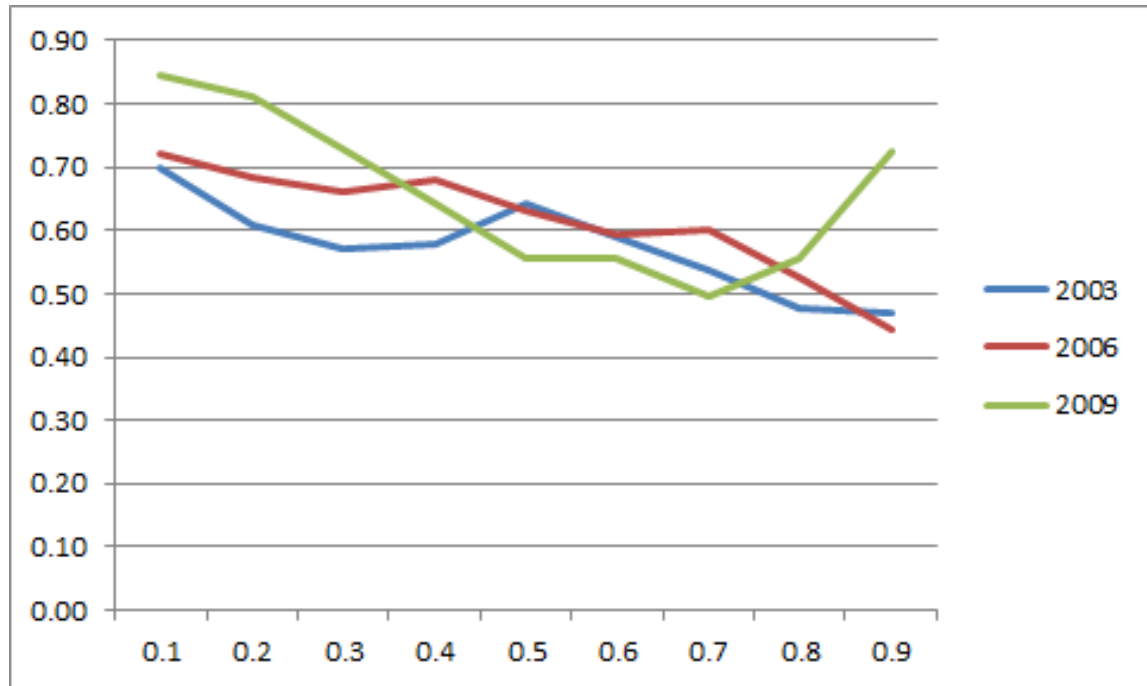
The coefficients on the individual characteristics for the second group vary over the hours distribution. The wage rate has a positive influence on labor supply leading to a positive Marshallian elasticity of labor supply. The influence of the wage rate is higher the lower the quantile considered. This is also true for the influence of work experience in full-time employment, the marital status, the number of children, and the presence of young and medium-aged children. However, the coefficients on the latter four covariates are negative. This means that being married, having more children, or having young or medium-aged children has a negative influence on labor supply and in general the more so for women who are situated at the lower tail of the conditional hours distribution: females which already work not so much reduce their hours more in the presence of young or medium-aged children than females which are working a lot.

Results for the Marshallian elasticity for the years 2003, 2006, and 2009 and  $q = 0.10, 0.20, \dots, 0.80, 0.90$  are shown in Figure 3.2.<sup>13</sup> All elasticities are positive.

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<sup>13</sup>The Hicks elasticity is quite similar to the Marshallian elasticity as the derivative of working

Figure 3.2: Marshallian Elasticity



Notes: This figures shows instrumental variable quantile regression estimates for the Marshallian elasticity as described in equation (3.7) for the years 2003, 2006, and 2009 and  $q = 0.1, 0.2, \dots, 0.8, 0.9$  (horizontal axis).

Elasticities for the year 2006 are in general higher than elasticities for the year 2003. Marshallian elasticities are higher for low  $q$  and vice versa. On the one hand, this means that individuals with low working hours exhibit a higher sensitivity with respect to wage changes as they can probably adjust working hours more easily than individuals working a lot. On the other hand, we observe lower sensitivity with respect to wage changes for individuals at the upper end of the conditional hours distribution. Considering only the lower quantiles, elasticities increased from 2003 to 2009. This is possibly due to the Hartz 4 reform which came into effect on January 1, 2005 and restructured the German labor market and unemployment benefit system. For the middle part of the conditional hours distribution there are less differences between the elasticities for the different years than for the lower and upper part. This is due to the different behavior of elasticities in 2009. First, the decrease in elasticities for the quantiles  $q = 0.1$  to  $q = 0.7$  is more pronounced for this year. Second, for 2009 elasticities for the upper two quantiles show an opposing trend compared to before. In 2009 individuals at the upper part of the conditional hours

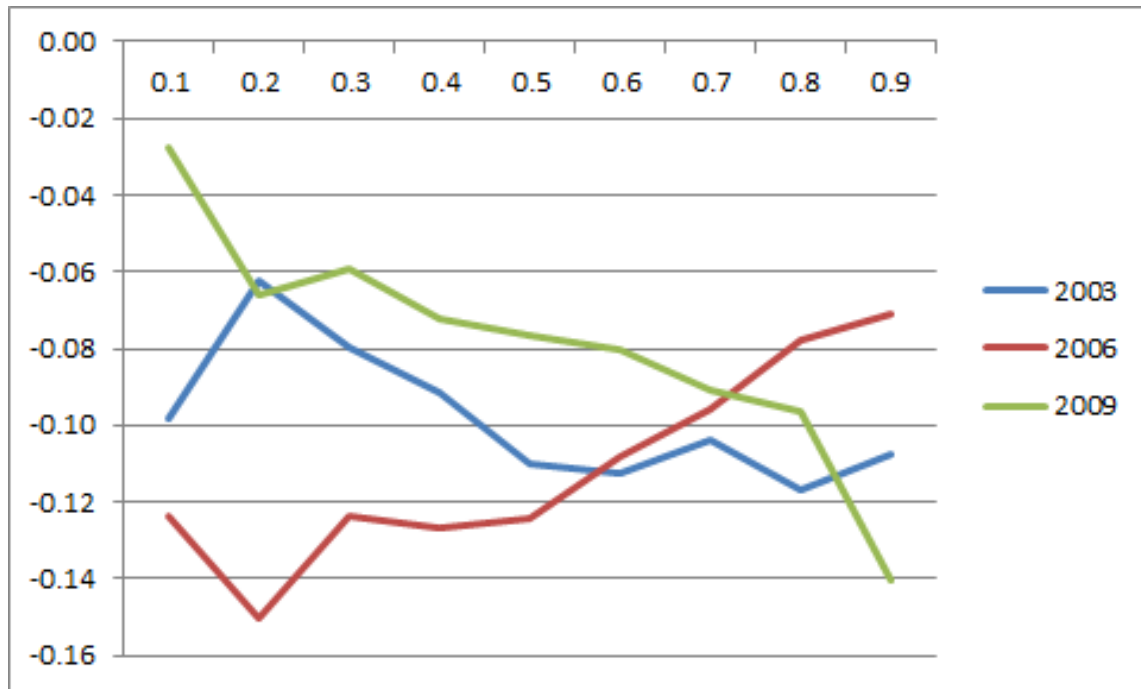
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hours with respect to non-labor income is small. Therefore, results for the Hicks elasticity are presented in Appendix A3.2.

distribution react more to wage changes than individuals at the middle part.

Figure 3.3 shows income elasticities for 2003, 2006, and 2009 and the before-mentioned nine values of  $q$ . Income elasticities are small and negative as individuals can maintain consumption with less work when non-labor income increases.

Figure 3.3: Income Elasticity



*Notes:* This figures shows instrumental variable quantile regression estimates for the income elasticity as described in equation (3.8) for the years 2003, 2006, and 2009 and  $q = 0.1, 0.2, \dots, 0.8, 0.9$  (horizontal axis).

Our results corroborate findings by Dostie and Kromann (2012) who estimate income and substitution labor supply and participation elasticities for Canadian married women by quantile regression methods. They find that wives with fewer working hours per week are more sensitive to changes in their own or spouse’s wages than wives with more weekly working hours. This is not a common phenomenon across countries and gender: Ribeiro (2001) estimates conditional quantile labor supply functions for prime age urban male employees in Brazil. He finds that wage and non-labor income influence working hours only for employees working more than the standard workweek at the upper tail of the conditional hours distribution.

Additionally we include a linear interaction effect between the logarithm of the hourly wage rate and non-labor income in the estimation of equations (3.11) and (3.15). In almost all cases the interaction effect is not significant. Interestingly,

although regression coefficients for the different specifications change, we observe hardly any difference for elasticity estimates.

### **3.6 Conclusion**

In this paper, we estimate micro elasticities of labor supply by instrumental variable quantile regression methods. In particular, we consider the Marshallian, the Hicks, and the income elasticity of labor supply for female employees living in former West Germany. Using quantile regression methods allows us to analyze labor supply elasticities for the entire conditional hours distribution. We can show that females at the low end of the conditional hours distribution are more sensitive to changes in their wages than females at the upper end. Part of this can be explained by the Hartz 4 reform which restructured the German labor market system.

Our paper highlights that quantile regression methods give a more comprehensive picture of labor supply than conventional conditional mean regressions. These methods can be a useful tool in the analysis of many other macroeconomic applications.

## A3 Appendix to Chapter 3

### A3.1 Data

#### SOEP Samples

Each household and thereby each individual in the SOEP is part of one of the following samples:

- Sample A: ‘Residents in the FRG’, started 1984
- Sample B: ‘Foreigners in the FRG’, started 1984
- Sample C: ‘German Residents in the GDR’, started 1990
- Sample D: ‘Immigrants’, started 1994/95
- Sample E: ‘Refreshment’, started 1998
- Sample F: ‘Innovation’, started 2000
- Sample G: ‘Oversampling of High Income’, started 2002
- Sample H: ‘Extension’, started 2006
- Sample I: ‘Incentivation’, started 2009



## SOEP Variables

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| Variable Name | Variable Label                             |
|---------------|--|
| \$SAMREG      | Current wave sample region                 |
| PSAMPLE       | Sample member                              |
| SEX           | Gender                                     |
| GEBJAHR       | Year of birth                              |
| \$POP         | Sample membership                          |
| \$NETTO       | Current wave survey status                 |
| LABNET\$\$    | Monthly net labor income                   |
| \$STATZEIT    | Actual weekly working hours                |
| \$VEBZEIT     | Agreed weekly working hours                |
| \$UEBSTD      | Overtime per week                          |
| Y11101\$\$    | Consumer price index                       |
| PARTZ\$\$     | Partner indicator                          |
| PARTNR\$\$    | Person ID number of partner                |
| \$FAMSTD      | Marital status in survey year              |
| SUMKIDS       | Total number of children born              |
| KIDGEB01      | Year of birth of 1st child                 |
| :             | :  |
| KIDGEB15      | Year of birth of 15th child                |
| \$BILZEIT     | Amount of education or training (in years) |
| EXPFT\$\$     | Working experience full-time employment    |

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## SOEP Variable Refinements

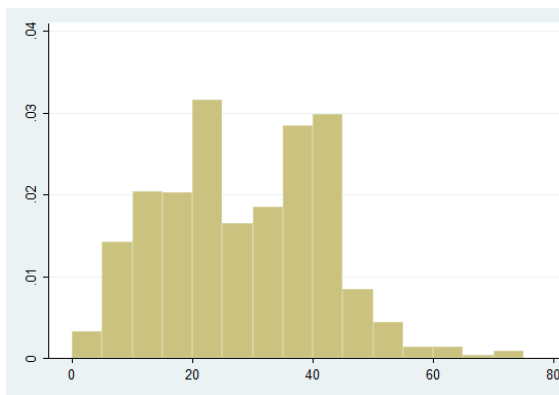
- Actual weekly working hours: When the value for the variable actual weekly working hours is missing, we use instead, if available, agreed weekly working hours and, if available, add overtime per week.
- Agreed weekly working hours: When the value for the variable agreed weekly working hours is missing, we use instead, if available, actual weekly working hours and, if available, subtract overtime per week.
- Monthly net labor income: When the value for the monthly net labor income of the spouse is missing because it does not apply, we set the value to zero. We exclude spouses with monthly net labor income above 5000 euros.

## Sample

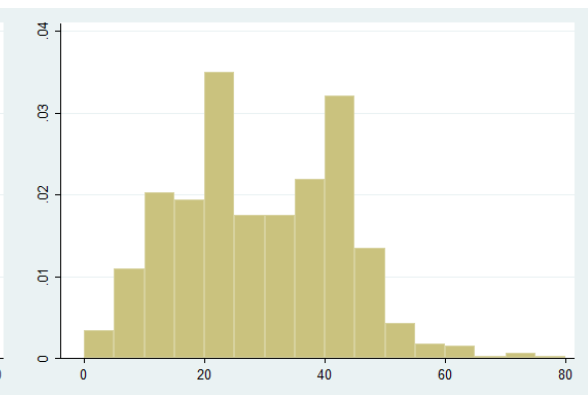
| Sample Definition  | Condition  |
|--|--|
| Only private households                                  | keep if $POP=1 \vee POP=2$   |
| Only successful interviews                               | keep if $NETTO \in \{10, 12, 13, 14, 15, 16, 18, 19\}$             |
| No first time interviewed persons aged 17                | drop if $NETTO=16$   |
| Female population  | drop if $SEX=1$  |
| West Germany   | drop if $SAMPREG=2$  |
| Age  | drop if $AGE < 25 \vee AGE > 64$                                   |
| Exclusion of retirees                                    | drop if $STIB=13$  |
| Exclusion of individuals that are currently in education | drop if $STIB=11$  |
| Individuals from sample A, E, F, H, I                    | drop if $PSAMPLE \in \{2, 3, 4, 7\}$                               |
| Working individuals only                                 | drop if $TATZEIT \leq 0$ , drop if $LABNET \leq 0$                 |
| No individuals with missing information                  | drop if $FAMSTD < 0$ , drop if $BILZEIT < 0$ , drop if $EXPFT < 0$ |

Figure 3.4: Histograms of Actual Weekly Hours Worked

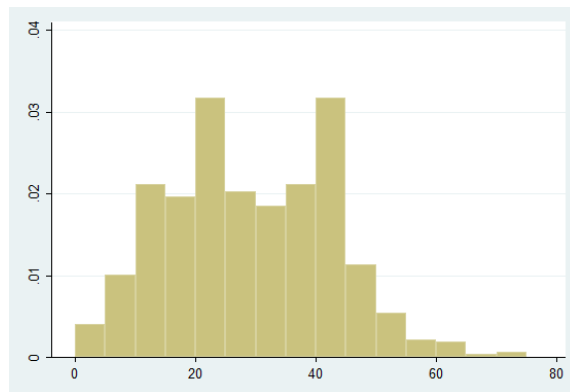
(a) 2003



(b) 2006



(c) 2009



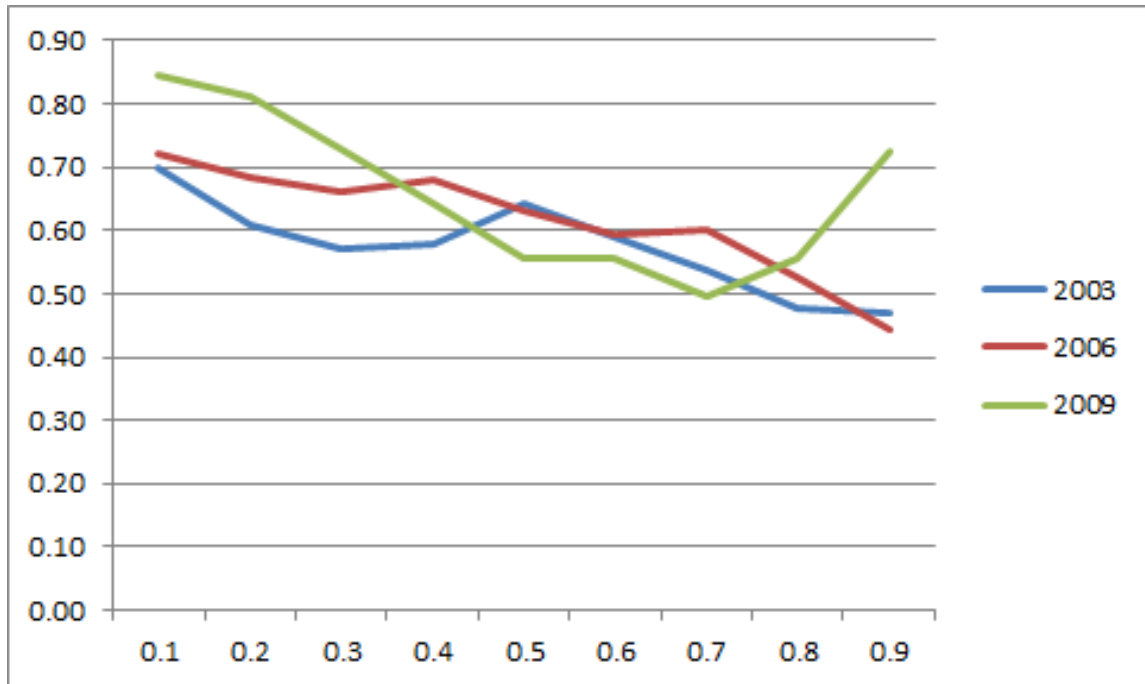
## A3.2 Results

Table 3.5: Results for the First Stage of the Benchmark Estimation for Equation (3.11)

|             | 2003                    | 2006                    | 2009                    |
|-------------|-------------------------|-------------------------|-------------------------|
| SCHOOL      | 0.08<br>(0.00)          | 0.07<br>(0.00)          | 0.07<br>(0.00)          |
| EXPSQ       | -2.46E-04<br>(9.87E-05) | -2.60E-04<br>(1.03E-04) | -6.21E-05<br>(1.13E-04) |
| Y           | 5.27E-06<br>(9.35E-06)  | -3.65E-06<br>(9.40E-06) | -1.41E-06<br>(1.08E-05) |
| EXP         | 0.02<br>(0.00)          | 0.02<br>(0.00)          | 0.01<br>(0.00)          |
| MAR         | -0.06<br>(0.03)         | -0.02<br>(0.03)         | -0.04<br>(0.04)         |
| SK          | 0.01<br>(0.01)          | 0.02<br>(0.01)          | 0.01<br>(0.01)          |
| K1          | 0.14<br>(0.04)          | 0.15<br>(0.04)          | 0.18<br>(0.05)          |
| K2          | -0.01<br>(0.03)         | 0.03<br>(0.04)          | 0.06<br>(0.04)          |
| K3          | 0.01<br>(0.02)          | 0.00<br>(0.02)          | -0.01<br>(0.03)         |
| CONST       | 0.99<br>(0.07)          | 1.02<br>(0.07)          | 1.06<br>(0.08)          |
| F statistic | 52.56                   | 41.71                   | 35.63                   |

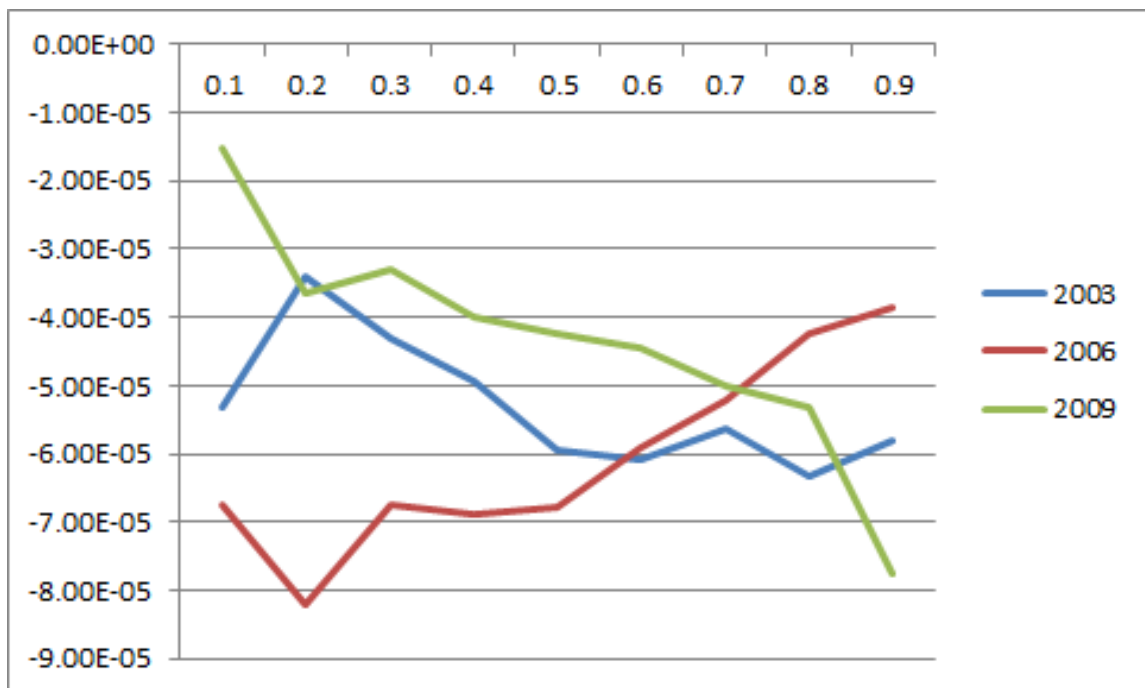
*Notes:* Standard errors are in parentheses. CONST, SCHOOL, EXP, EXPSQ, MAR, SK, K1, K2, and K3 represent a constant, schooling, work experience in full-time employment, work experience squared, the family status dummy variable, the number of children, and dummy variables for young, medium-aged, and older children, respectively.

Figure 3.5: IV Quantile Regression Coefficients for the Wage Rate



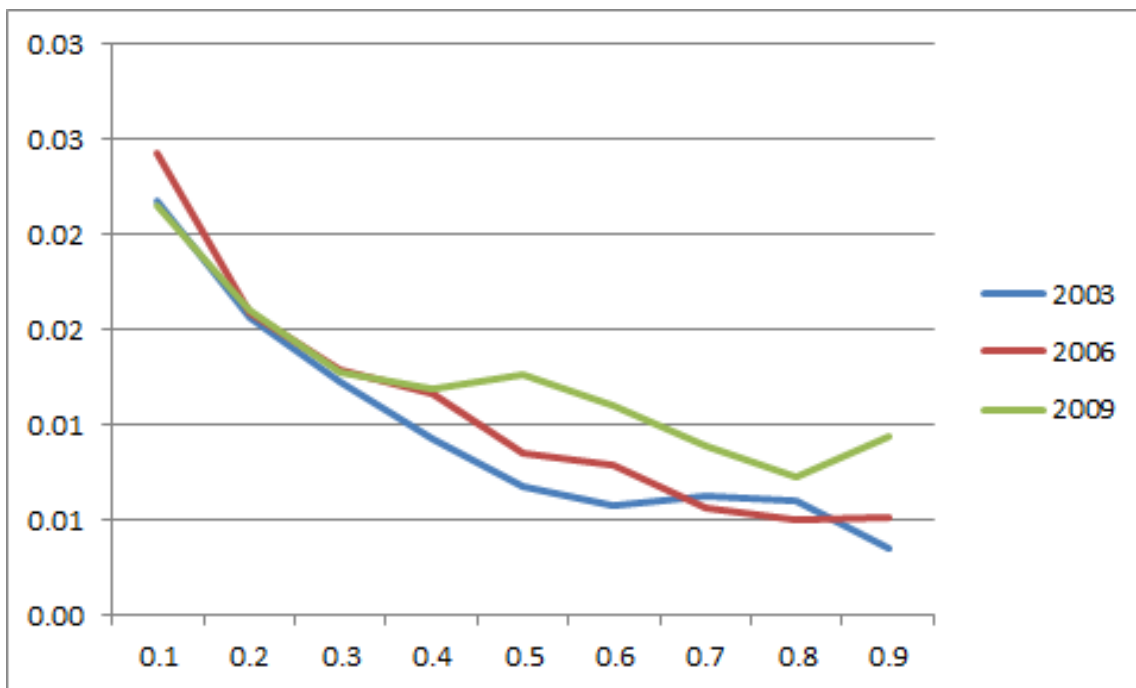
Notes: This figures shows instrumental variable quantile regression estimates of the wage rate coefficients as described in equation (3.13) for the years 2003, 2006, and 2009 and  $q = 0.1, 0.2, \dots, 0.8, 0.9$  (horizontal axis).

Figure 3.6: IV Quantile Regression Coefficients for Non-Labor Income



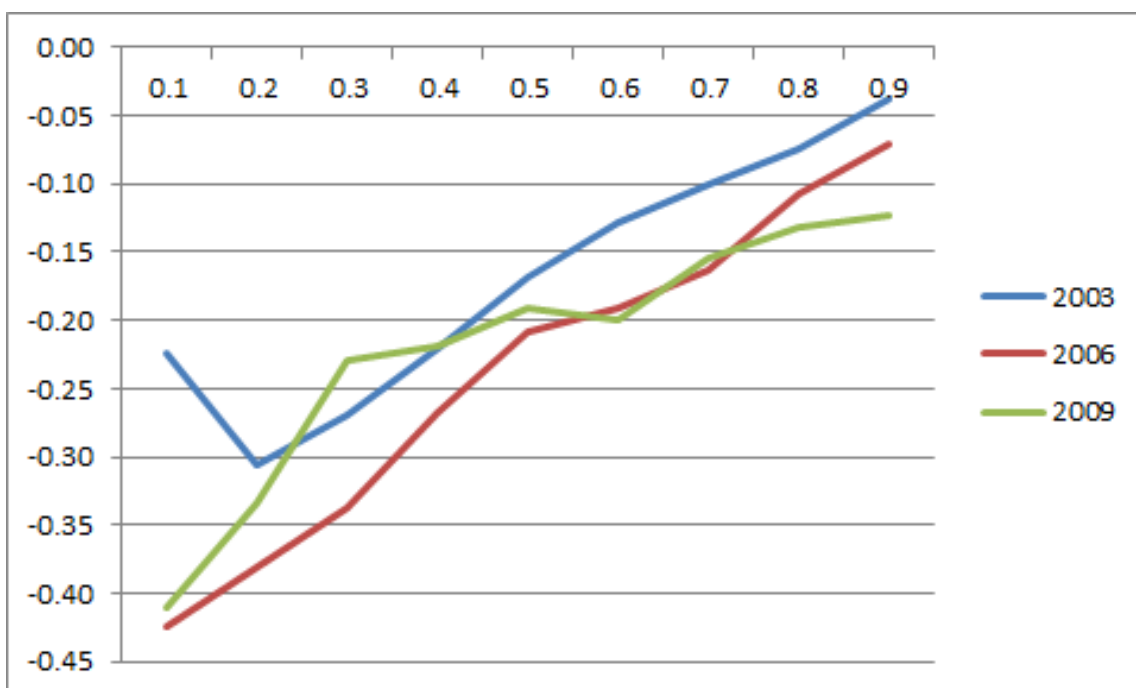
Notes: See Figure 3.5.

Figure 3.7: IV Quantile Regression Coefficients for Work Experience



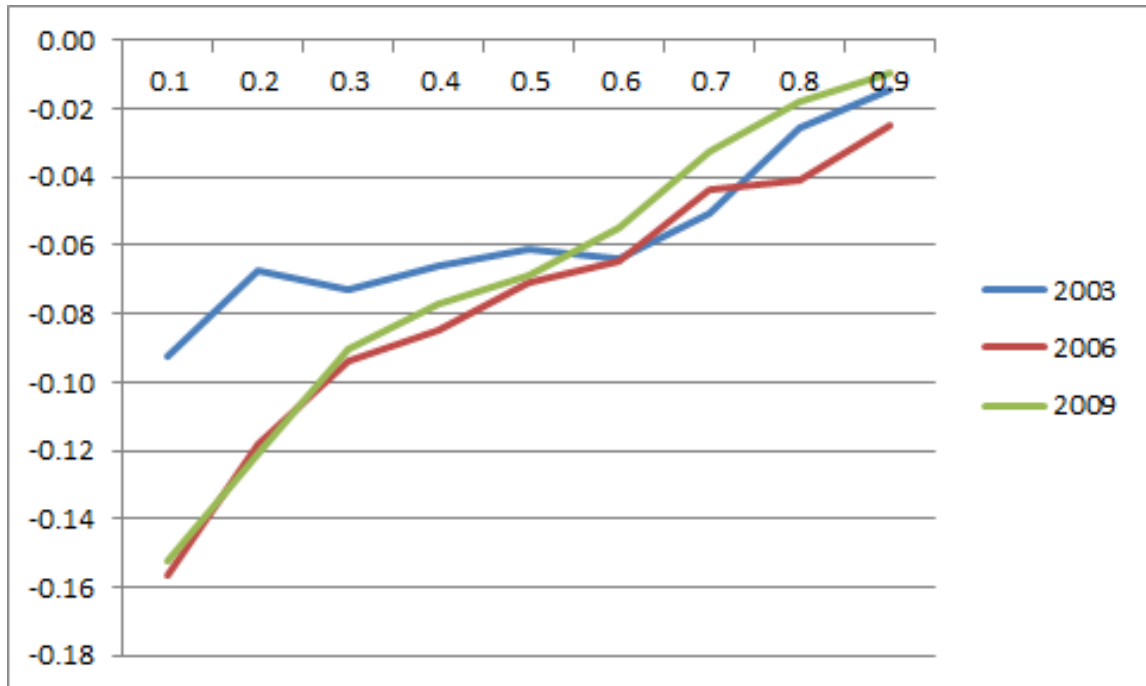
Notes: See Figure 3.5.

Figure 3.8: IV Quantile Regression Coefficients for Family Status



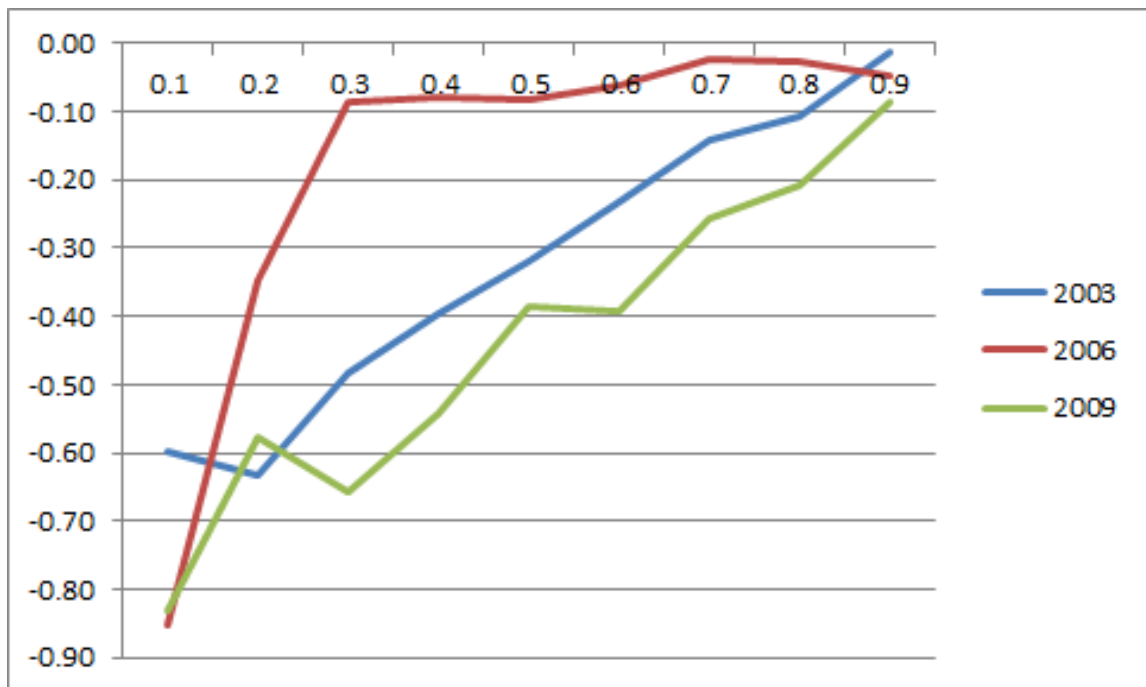
Notes: See Figure 3.5.

Figure 3.9: IV Quantile Regression Coefficients for the Number of Children



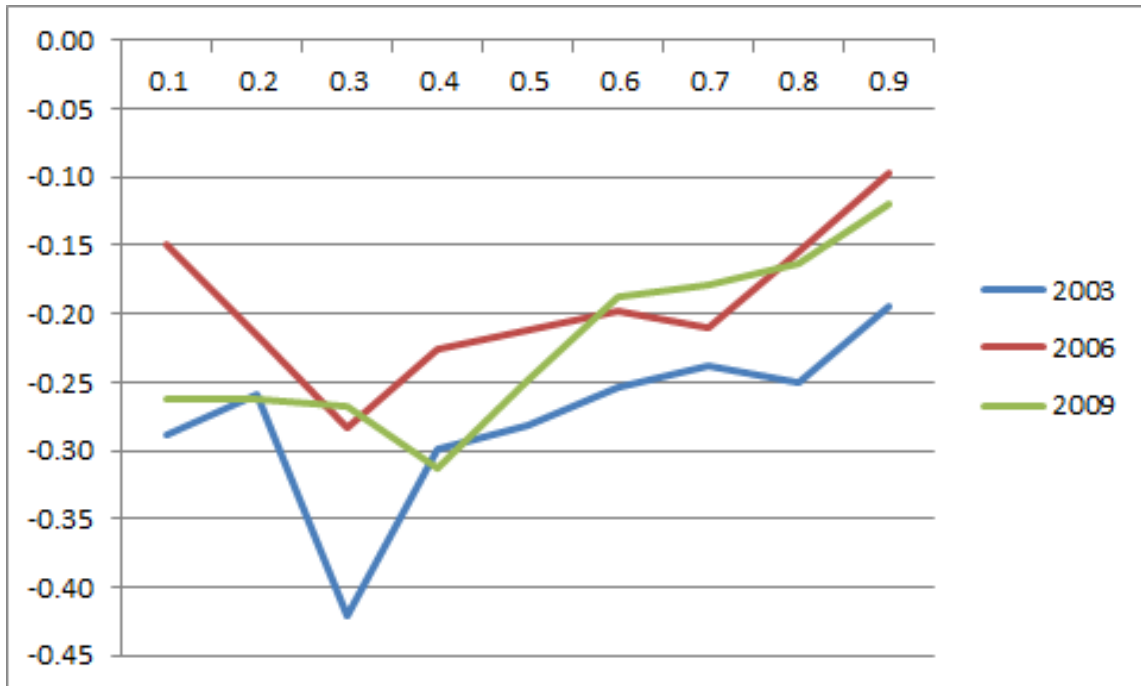
Notes: See Figure 3.5.

Figure 3.10: IV Quantile Regression Coefficients for Young Children



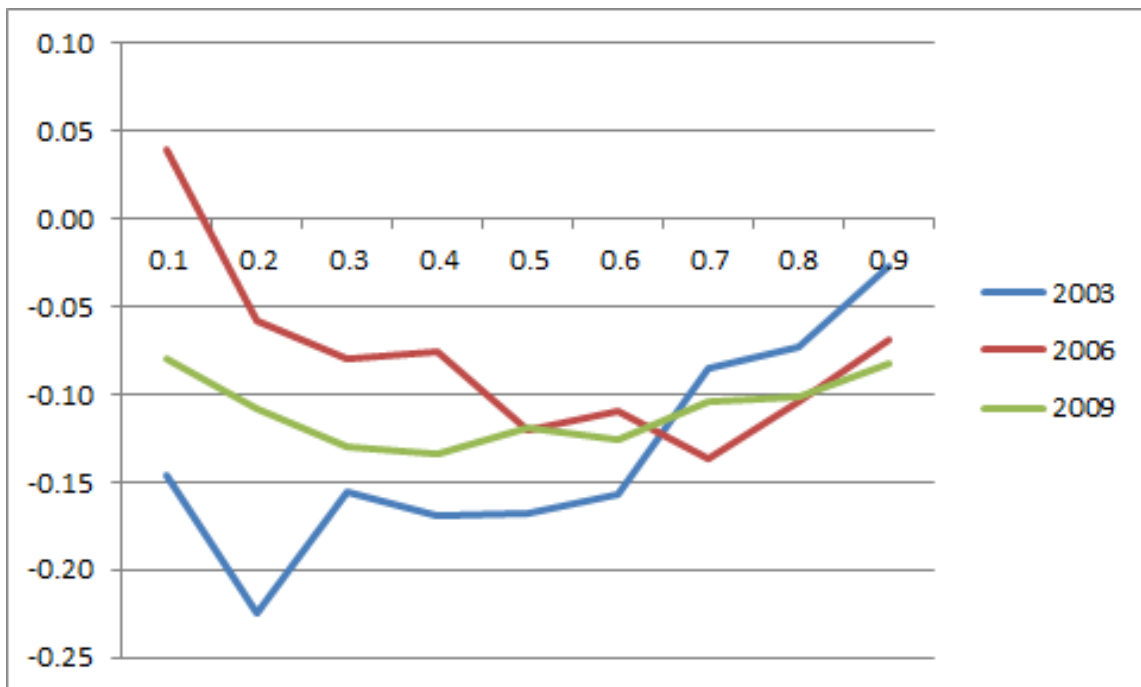
Notes: See Figure 3.5.

Figure 3.11: IV Quantile Regression Coefficients for Medium-Aged Children



Notes: See Figure 3.5.

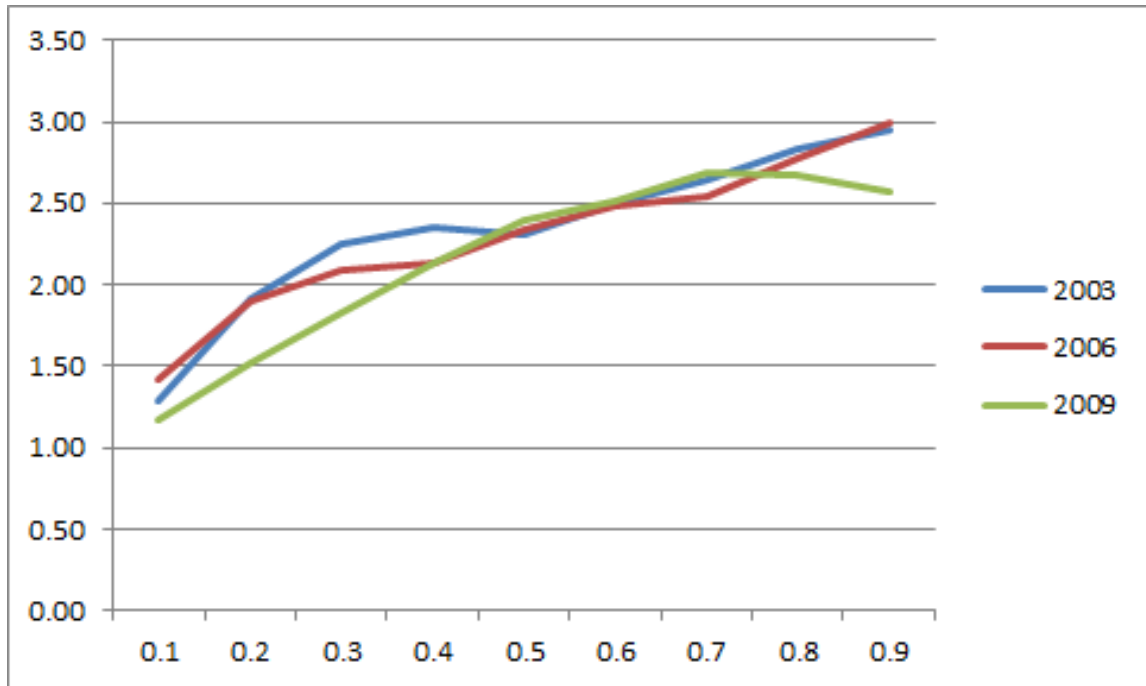
Figure 3.12: IV Quantile Regression Coefficients for Older Children



Notes: See Figure 3.5.

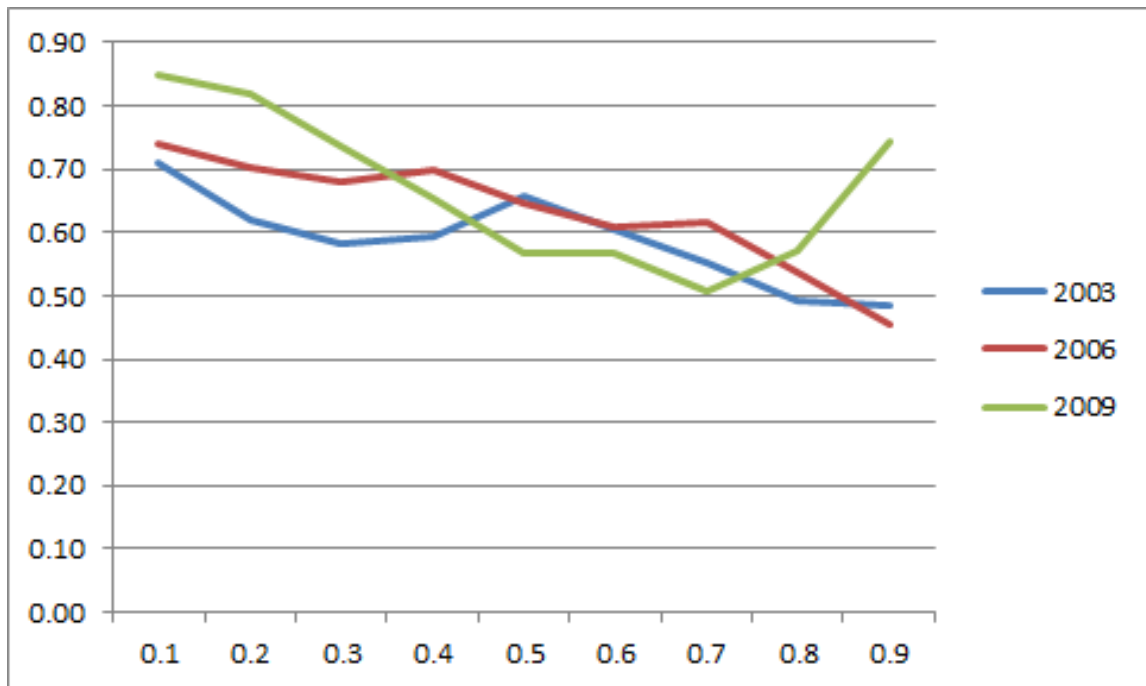


Figure 3.13: IV Quantile Regression Constants



Notes: See Figure 3.5.

Figure 3.14: Hicks Elasticity



Notes: This figures shows instrumental variable quantile regression estimates for the Hicks elasticity as described in equation (3.10) for the years 2003, 2006, and 2009 and  $q = 0.1, 0.2, \dots, 0.8, 0.9$  (horizontal axis).

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