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Game-Theoretic Analysis to Examine How Government Subsidy Policies Affect a Closed-Loop Supply Chain Decision

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Abstract: The pros and cons of government subsidy policies in a closed-loop supply chain (CLSC) setting on optimal pricing, investment decisions in improving product quality, and used product collection under social welfare (SW) optimization goal have not been examined comprehensively. This study compares the outcomes of three government policies under manufacturer-Stackelberg (MS) and retailer-Stackelberg (RS), namely (i) direct subsidy to the consumer, (ii) subsidy to the manufacturer to stimulate used product collection, and (iii) subsidy to the manufacturer to improve product quality. Results demonstrate that the greening level, used product collection, and SW are always higher under the RS game, but the rate of a subsidy granted by the government is always higher under the MS game. Profits for the CLSC members and SW are always higher if the government provides a subsidy directly to the consumer, but productivity of investment in the perspective of the manufacturer or government are less. In a second policy, the government organizations grant a subsidy to the manufacturer to stimulate used product collection, but it does not necessarily yield the desired outcome compared to others. In a third policy, the manufacturer receives a subsidy on a research and development (R&D) investment, but it yields a sub-optimal greening level. This study reveals that the outcomes of subsidy policies can bring benefit to consumers and add a degree of complication for CLSC members; government organizations need to inspect carefully among attributes, mainly product type, power of CLSC members, and investment efficiency for the manufacturer, before implementing any subsidy policies so that it can lead to an environmentally and economically viable outcome.

Keywords: production planning optimization; closed-loop green supply chain; government subsidy; stackelberg game; re-manufacturing

1. Introduction

In last two decades, a closed-loop supply chain (CLSC) is gaining increasing attention from both industry practitioners and academics due to alleged benefits for sustainable development and growing environmental awareness among consumers and environmental regulations [1–7]. Government organizations play an impressive role in the development of sustainable product manufacturing and re-manufacturing decisions; they can enforce strict legislation, as well as offer support through various subsidy policies. In Japan, the Ministry of Environment approved 5 billion yen in 2019 as a subsidy for the manufacturer to cover 33% to 50% of their equipment price to produce products with biodegradable bio-plastics (https://bioplasticsnews.com/2018/08/27/japan-
government-bioplastics/). The government of China provides a subsidy in different ways. For example, a manufacturer in Hunan province receives a one-time subsidy to improve re-manufacturing activities, whereas in Hubei province, Sevalo Construction Machinery Re-manufacturing Co. Ltd. receives 1 million RMB as a research and development (R&D) subsidy to improve re-manufacturing activities [8]. In 2016, the Chinese government introduced the “Guidance on Promoting Green Consumption” program to achieve the long-term goal of stimulating green product consumption. Through the Technology & Quality Up-gradation Support scheme for MSMEs (TEQUP), the government of India provides a subsidy up to 25% of the project cost for implementation of energy efficient technology. The support became one of the key factors influencing growth for the companies like Banyan Nation, Karma Recycling; in fact, the former company received the Dell People’s choice award for circular economy entrepreneur at the world economic forum in Davos (www.standupmitra.in/Home/SubsidySchemesForAll). To encourage the consumer to procure an energy-efficient green vehicle, the United States government provides subsidies up to $7500 for the purchase of a plug-in electric vehicle [9]. Government organizations in the USA, such as The Ohio Environmental Protection Agency (EPA), has awarded a total of $1.24 million in recycling market development grant money to upgrade and install new equipment to increase the amount of recyclables (www.recyclingtoday.com/article/ohio-epa-recycling-grants/). The European Union also put significant efforts to promote green product manufacturing and expedite product reuse, as well as introduce various financial packages to encourage the circular economy (European Commission, 2015). Recently, an innovative and flexible pan-European network of research funding organizations, supported by EU Horizon 2020, proposed a funding of 14.530 million euros (www.era-min.eu/sites/default/files/docs/call_text_2018_0.pdf). The above evidences explain that government subsidy policies are made in different ways.

Therefore, it is imperative to conduct comparative analysis for highlighting the pros and cons among those policies. Despite the necessity to explore the effect of different government subsidy policies in CLSCs under government social welfare (SW) maximization objective, comparative study is relatively sparse. This study considers omnipotence of three subsidy policies under the manufacturer-Stackelberg (MS) and retailer-Stackelberg (RS) games to pinpoint their effects and explores the answers to the following research issues:

1. How does the government social welfare optimization goal affect the optimal decision of a CLSC members?
2. Do power structures and product type affect the used product collection, pricing, and investment decision of the product?
3. Which policy stimulates the manufacturer to escalate investment in R&D and product collection activity?
4. What are the main barriers associated with each subsidy policy that overturn the government, as well as CLSC members’ sustainability target?

In an attempt to answer the above questions and provide insights, we examined outcomes of eight scenarios and compared corresponding optimal decisions. For tractability, and in line with the CLSC configuration considered by previous studies [10], we mainly focused on single period optimal decision. However, product collection and network design [11] is an important aspect, we limited this study on the manufacturer collection mode only to keep our focus on the assessment of three subsidy policies under a three-stage game framework. In Policy C, the government provides a subsidy directly to the consumer [12] to stimulate a green product purchase. In Policy RE, the government shares a fraction of manufacturer investment effort to encourage used product return. In Policy T, the manufacturer receives a fraction of R&D investment from the government as a subsidy to improve the greening level (GL) [13]. To compare outcomes of subsidy policies, we derived an optimal decision under no subsidy as the benchmark, called Policy N. To explore the influence of a powerful retailer, models were formulated under both the MS and RS game frameworks; and results are compared. This study contributes to the present literature as follows: First, comparative analysis will help practitioners to understand the behavior of pricing, investment decision for the manufacturer in used product return,
and R&D to improve GL in a CLSC. Second, the government sets the SW optimization goal and decides
the amount of the subsidy. Therefore, results can provide a guideline for them before implementing
subsidy policies. Third, according to the investment decision, green products can be categorized
as development intensive green products (DIGPs); marginal cost-intensive green products (MIGPs);
and marginal-development cost-intensive green products (MDIGPs) [14,15]. Examples belonging
to the first categories are developing LED bulbs; integrating an adaptive product business model,
energy star home appliances, technology for product-life extension, and high-speed electric cars.
All of these require a substantial amount of R&D investments. On the other hand, there is installing
lithium-ion car batteries or emission control devices, using biodegradable plastics for FMCG packaging,
and the manufacturing cost increasing with per unit product, all of which belong to second category.
To the best of the authors’ knowledge, the effect of MDIGPs on CLSCs has not been explored yet.
Therefore, this study provides a complete overview for the manufacturer on the investment decision
to produce MDIGPs. Finally, comparative study in the perspective of participating members can
assist to formulate a framework to design a subsidy policy for green product manufacturing and
re-manufacturing.

This study is organized as follows. The following subsection a brief literature review is reported to
highlight the position of our research in the literature. Assumptions and background of the models are
presented in Section 2. CLSC decisions under subsidy policies are discussed in Section 3. In Section 4,
managerial insights are drawn with numerical illustration. Finally, conclusions, limitations, and future
research are presented in Section 5.

1.1. Literature Review

It is imperative for CLSC members trading with green product to consider the evolving behavior
of consumers when making important strategic and operational decisions. Possibly, Ottman [16]
first reported the opportunities and the pitfalls in green marketing. The author noted double benefit
for the consumer, i.e., ‘personal benefit’ and ‘environmental benefit’, in green product procurement.
Since then, numerous studies related to green supply chain (GSC) management explored the properties
of optimal decision under price-GL sensitive demand in different perspectives [17–19]. We will discuss
some recent works focusing on variation of optimal decisions in different games under price-GL
sensitive demand. Ghosh and Shah [20] compared optimal decisions obtained for different games
and stated that the GL increased in the MS game, but the consumers needs to pay more. Liu and
Yi [21] established that pricing and GL changes considerably under various power structures when the
manufacturer also invest in knowing consumer preference information in the big data environment,
and the manufacturer needs to set the lowest wholesale price under the RS game. Yang and Xiao
[22] explored optimal decisions for a GSC under governmental interventions and used triangular
fuzzy numbers to describe the imprecise information. The authors found that RS game scenario is
the best for all players if governmental interventions are strong enough. Nielsen et al. [23] explored
characteristics of the three-level GSC in a two-period setting. The authors found that the manufacturer
needs to trade with the product at lower GL if the distributor dominates the GSC. Chen et al. [24]
examined pricing, along with the investment effort of both the manufacturer and retailer in a GSC.
The authors found that the total GSC profits increased if members share the R&D expenditure but not
individual of the manufacturer or retailer simultaneously. Dey et al. [14] found that the manufacturer’s
decision to produce MIGPs and DIGPs is highly sensitive to the power structure. The authors found
that a powerful retailer might want to trade with MIGPs, which leads to less amount of profit for the
manufacturer. In this direction, the recent works of Huang et al. [25] and Ranjan and Jha [26] are worthy
of mention. The findings of the above cited articles support that the game structures always made an
impact on the optimal decisions. However, the above studies explored the characteristics of a forward
SC. We extended this stream of research and studied the properties of CLSC under price-GL sensitive
demand. CLSC is one of the great interests in both business and academic researchers due to growing
consumer awareness on environmental issues and regulations. In existing literature, CLSC models
are studied to explore various perspectives. For example, Hong and Yeh [27], Ma et al. [28], and Saha et al. [29] compared optimal decisions in a CLSC under different collection mode. The authors formulated their models mainly under the MS game framework and explored the consequences where the manufacturer, retailer, or a third party, individually or jointly collects used products. On the other hand, CLSC coordination issues were comprehensively studied by Zhang and Ren [30], He et al. [31], and others. For example, Hong et al. [32], Johari and Motlagh [33], and He et al. [31] discussed effect of two-part tariff contract; Zhao et al. [34] used a commission fee contract, while a revenue-and-expense sharing contract is used by Xie et al. [35], and spanning revenue-cost sharing is used by Choi et al. [36]. On the other hand, the optimal decision under different game structures is discussed by Wang et al. [37], Gao et al. [38], Zheng et al. [39], and others. In those studies, the authors made an effort to highlight how the optimal decision changes according to game structure. We refer to recent review articles [2,40–42] on CLSC for detailed discussion. The environmental and operational measures of a CLSC network is another important aspect, where how to reduce number of vehicles to be used and the resulting carbon emissions, as well as the impact of re-manufacturing on environment, are studied extensively under an integer programming framework. We refer to the work by [43–45] for the detailed discussion in this direction. However, as mentioned earlier, it is difficult to ignore the influence of government organizations on the optimal decision in a CLSC, but literature is scanty in that direction. Zhang et al. [46] measured the supply-chain green efficiency (SCGE) of thirty-seven different industrial sectors in China and found that environmental policy management and innovation capacity of the manufacturer are important factors affecting SCGE. Researchers mostly explored the influence of a government subsidy in a forward SC. For example, Hafezalkotob [47], in addition to Sinayi and Rasti-Barzoki [12], explored the optimal decision where the consumer receives a subsidy directly from the government; Chen et al. [48] explored characteristics of optimal decisions when both the manufacturer and retailer receive a subsidy on per unit product; Safarzadeh and Rasti-Barzoki [49] discussed the optimal decision for a two-echelon SC when the manufacturer receives a subsidy on the R&D investment. To establish the position of the present study, we outline existing work on CLSCs under influence of a subsidy in Table 1.

<table>
<thead>
<tr>
<th>Study</th>
<th>Games</th>
<th>Effect of GL</th>
<th>Nature of Subsidy</th>
<th>SW Maximization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mitra and Webster [50]</td>
<td>MS</td>
<td>No</td>
<td>To the manufacturer to increase re-manufacturing activity</td>
<td>No</td>
</tr>
<tr>
<td>Ma et al. [51]</td>
<td>MS</td>
<td>No</td>
<td>To consumers to procure new products</td>
<td>No</td>
</tr>
<tr>
<td>Shu et al. [52]</td>
<td>MS</td>
<td>No</td>
<td>To the manufacturer for re-manufacturing</td>
<td>No</td>
</tr>
<tr>
<td>Xiao et al. [53]</td>
<td>MS</td>
<td>No</td>
<td>To the manufacturer and consumer jointly</td>
<td>Yes</td>
</tr>
<tr>
<td>Heydari et al. [54]</td>
<td>MS</td>
<td>No</td>
<td>Replacement subsidy to the customer and manufacturer</td>
<td>Yes</td>
</tr>
<tr>
<td>Jena et al. [55]</td>
<td>MS</td>
<td>No</td>
<td>To the manufacturer for re-manufacturing</td>
<td>No</td>
</tr>
<tr>
<td>Jena et al. [56]</td>
<td>MS</td>
<td>No</td>
<td>To the manufacturer for re-manufacturing</td>
<td>No</td>
</tr>
<tr>
<td>Guo et al. [5]</td>
<td>MS</td>
<td>No</td>
<td>To the manufacturer for re-manufacturing</td>
<td>No</td>
</tr>
<tr>
<td>Wan and Hong [57]</td>
<td>MS</td>
<td>No</td>
<td>To the manufacturer for re-manufacturing and retailer for recycling</td>
<td>No</td>
</tr>
<tr>
<td>Saha et al. [58]</td>
<td>MS</td>
<td>No</td>
<td>To the manufacturer and to consumer based on the greening level</td>
<td>No</td>
</tr>
<tr>
<td>He et al. [59]</td>
<td>MS</td>
<td>No</td>
<td>Directly to consumers</td>
<td>Yes</td>
</tr>
<tr>
<td>Present study</td>
<td>MS and RS</td>
<td>Yes</td>
<td>Directly to consumers, to the manufacturer for improving quality and to the manufacturer for re-manufacturing</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 1 demonstrates that most of the articles focused on the behavior of participants under a single subsidy policy, mostly under the MS game setting. The effect of joint influence of price-GL is also ignored. With growing awareness about green products, the influence of GL needs to be considered to obtain a pragmatic CLSC decision. Comparative study to explore preferences of the
CLSC members, consumers, and government organizations are not examined in the previous literature. In this study, the investment and pricing decisions of a CLSC member are explored by correlating the optimal decision of the government organization. This study can help practitioners to understand the pricing and investment patterns for the manufacturer under the MS and RS games in a CLSC setting. Comparative analysis conducted in this study on the efficiency of investment and consumer preference can help government organizations to cultivate a pragmatic subsidy policy.

2. Prerequisites and Assumptions

We considered eight different scenarios, namely Scenarios \(ij, i \in \{m, r\}\), which signifies MS and RS games; \(j \in \{C, RE, T, N\}\), which refers to the models where consumers receive a subsidy (C), the manufacturer receives a subsidy on the investment effort on recycling (RE), and the manufacturer receives a subsidy on the total R&D investment (T); and the benchmark decision model where the government organizations do not provide a subsidy (N), respectively. Therefore, the first index represents the game structure, and the second one represents the subsidy policy. The following notations presented in Table 2 are used to differentiate the decision and auxiliary variables in different scenarios:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w^j_i)</td>
<td>wholesale price of the new/re-manufacturing product</td>
</tr>
<tr>
<td>(p^j_i)</td>
<td>market price</td>
</tr>
<tr>
<td>(\tau^j_i)</td>
<td>collection rate of used products</td>
</tr>
<tr>
<td>(\theta^j_i)</td>
<td>greening level</td>
</tr>
<tr>
<td>(\rho^j_i)</td>
<td>per unit subsidy received by the consumer from the government, (\rho^j_i &lt; p^j_i)</td>
</tr>
<tr>
<td>(\eta^j_i)</td>
<td>subsidy received by the manufacturer on investment to improve recycling, (0 \leq \eta^j_i \leq 1)</td>
</tr>
<tr>
<td>(\mu^j_i)</td>
<td>fraction of subsidy received by the manufacturer on investment to improve GL, (0 \leq \mu^j_i \leq 1)</td>
</tr>
<tr>
<td>(\pi^j_{ki})</td>
<td>member k’s profit, (k \in {m, r})</td>
</tr>
<tr>
<td>(\pi^j_{gi})</td>
<td>SW of government</td>
</tr>
<tr>
<td>(Q^j_i)</td>
<td>sales volume</td>
</tr>
</tbody>
</table>

The following assumptions are made to establish proposed models:

1. Similar to Nielsen et al. [60]; Dey et al. [14]; Dey and Saha [61], the market demand is linearly dependent on the retail price and GL, and its functional form is \(D^j_i = a - p^j_i + \beta \theta^j_i\), where \(a\) represents potential intrinsic demand, and \(\beta > 0\) represents GL sensitivity. Therefore, higher value for \(a\) means the consumer has the better perception about the product. For analytical simplicity, the coefficient of price sensitivity is normalized with the unit [15].

2. It is assumed that re-manufacturing cost is less compared to manufacturing cost, i.e., \(c_r < c_m\) [6,57]. The manufacturer invests \(CL(\tau^j_i, \alpha) = \alpha \tau^j_i D^j_i + \kappa \tau^j_i^2\) to collect used products, where \(\alpha > 0\) represents the scaling parameter, and \(\kappa > 0\) represents the monetary benefit’s for the consumers for returning used products. If \(\alpha = 0\), this assumption is similar to Ma et al. [28], Xiao et al. [53], and Wan and Hong [57]. \(\tau^j_i\) represents the collection rate (\(0 \leq \tau^j_i \leq 1\)). While optimizing objective functions, all members have access to the same information [51,59]. A portion \(\delta (0 \leq \delta \leq 1)\) of the collected used products converts into new one [57].

3. The manufacturer bears extra cost for green technology innovation. In a recent study by Zhang et al. [62], it was found that technological innovation have significant effects on regional industrial eco-efficiency. In this study, we assume that the manufacturer produces MDIGPs, and corresponding per unit and total R&D investment costs are considered as \(\lambda_1 \theta\) and \(\lambda_2 \theta^2\), respectively. Therefore, \(\lambda_1\) and \(\lambda_2\) represents the efficiency of the manufacturer on per unit investment and investment in R&D, respectively. If \(\lambda_1 = 0\), then the manufacturer invests in producing DIGPs [18,19]. If \(\lambda_2 = 0\), then the product converts to MIGPs [63]. The fixed cost for the...
retailer and manufacturer are normalized to zero [14,15]. The government organizations provide a R&D subsidy on the total investment. As noted by Dey et al. [14], for MIGPs, the variable cost is directly proportional with the product quality, and it might not possible to recover the cost for the manufacturers. For example, installing emission reduction devices or packaging material are directly proportional to the unit product, but it is difficult for the manufacturer to recover the cost of those in the re-manufacturing process.

4. The influence of three subsidy policies was analyzed. In Policy C, the government provides a subsidy \( \rho_i^C \) on per unit product directly to consumers. Therefore, the consumers need to pay \( p_i^C - \rho_i^C \) for per unit purchase. In Policy RE, the manufacturer receives a subsidy \( \eta_i^R \), \( 0 < \eta_i^R \leq 1 \) on the investment effort on used product collection. In Policy T, the manufacturer receives a subsidy \( \mu_i^T \), \( 0 < \mu_i^T \leq 1 \) on the R&D investment [13,65] to improve GL.

5. We find optimal decisions in a three-stage game to study the influence of government decision. Under the MS and RS games, the decision sequence is defined as follows:

- Step 1: The government decides the subsidy rate \( \rho_i^C \) by maximizing social welfare;
- Step 2: In the MS game, the manufacturer decides \( w_{mi}^i, \theta_{mi}^i \), and \( \tau_{mi}^i \). In the RS game, the retailer decides the profit margin \( m_{ji}^i = p_{ji}^i - w_{ji}^i \);
- Step 3: In the MS game, the retailer decides the retail price \( p_{mi}^i \). In the RS game, the manufacturer decides \( w_{ji}^i, \theta_{ji}^i \), and \( \tau_{ji}^i \).

Therefore, the government takes the responses of the CLSC members into consideration, while deciding the subsidy rate [48,59].

3. The Models

In this section, we formulate mathematical models to examine the characteristics of optimal pricing, re-manufacturing, and investment in R&D for CLSC members and the subsidy rate of government organizations. The scenarios wherein the consumer subsidy is provided are explored in Section 3.1, and then the scenarios where the manufacturer receives a subsidy on the investment effort for improving used product collection are discussed in Section 3.2, and finally, the equilibrium results are derived in Section 3.3, where the manufacturer receives a subsidy on the R&D investment. Optimal decisions between the MS and RS games are compared to explore characteristics of GL, collection rate, SW, and the government subsidy rate.

3.1. Optimal Decisions in Policy C

The manufacturer produces MDIGPs and sells to the retailer at wholesale price \( w_{mi}^C \). The retailer sells those to the customers at a price of \( p_{mi}^C \). The demand function in Policy C is \( D_{i}^C = a - (p_{mi}^C -\rho_{mi}^C) + \beta \theta_{mi}^C \). The manufacturer collects used product directly from the consumers for re-manufacturing. The government organization decides the subsidy rate by maximizing SW. A consumer subsidy on electronic vehicles is common in countries like China, Canada, Germany, Japan, etc. [66]. The profit functions for the retailer and manufacturer, and the SW for the government in Scenarios MC and RC are obtained as follows:

\[
\pi_{ri}^C(p_{mi}^C) = (p_{mi}^C - w_{mi}^C)D_{i}^C, \tag{1}
\]

\[
\pi_{mi}^C(w_{mi}^C, \theta_{mi}^C, \tau_{mi}^C) = (w_{mi}^C - \lambda_1 \theta_{mi}^C)D_{i}^C + (c_m \delta - \alpha - c_r)\tau_{mi}^C D_{i}^C - \lambda_2 \theta_{mi}^c \theta_{mi}^C, \tag{2}
\]

\[
\pi_{gi}^C(\rho_{mi}^C) = \pi_{mi}^C + \frac{D_{i}^C \rho_{mi}^C}{2} - \rho_{mi}^C D_{i}^C. \tag{3}
\]

The government’s objective function includes the influence of profits for each member, the social aspect, i.e., consumer surplus (CS), and the total amount of the subsidy provided by the government organization [67,68]. The CS is the area of the demand curve below a given price, which can be
expressed as $\pi_i^{RE}$. Optimal decisions in Policy C under the MS and RS games are presented in Lemma 1,2, respectively. The detail derivations of optimal decisions are presented in Appendix A, B, respectively. Additional notations used throughout the study are presented at the end of Appendix A.

Lemma 1. Optimal decision in Scenario MC are obtained as follows:

\[
\begin{align*}
\rho_m^C &= \frac{(a-c_m)X_{12}}{\Delta_1}; \quad w_m^C = \frac{(aN_1-2a_m)X_{12}-YZ_k}{\Delta_1}; \quad p_m^C = \frac{(a-c_m)X_{12}}{\Delta_1}; \\
\pi_{mm}^C &= \frac{(a-c_m)^2X_{12}^2}{\Delta_1^2}; \quad \pi_{rr}^C = \frac{(a-c_m)^2X_{12}^2}{\Delta_1^2}; \quad \pi_{rr}^C = \frac{(a-c_m)^2X_{12}^2}{\Delta_1^2}; \quad Q_m^C = \frac{2(a-c_m)X_{12}}{\Delta_1}, \quad \text{where} \quad \Delta_1 = M_{1X} - X^2_{12}.
\end{align*}
\]

Lemma 2. Optimal decision in Scenario RC are obtained as follows:

\[
\begin{align*}
\rho_m^C &= \frac{(a-c_m)X_{12}}{\Delta_1}; \quad w_m^C = \frac{(aN_1-2a_mX_{12})}{\Delta_1}; \quad p_m^C = \frac{(a-c_m)X_{12}}{\Delta_1}; \quad \theta_m^C = \frac{(a-c_m)X_{12}}{\Delta_1}; \quad r_{rr}^C = \frac{(a-c_m)X_{12}}{\Delta_1}; \\
\pi_{mm}^C &= \frac{(a-c_m)^2X_{12}^2}{\Delta_1^2}; \quad \pi_{rr}^C = \frac{(a-c_m)^2X_{12}^2}{\Delta_1^2}; \quad \pi_{rr}^C = \frac{(a-c_m)^2X_{12}^2}{\Delta_1^2}; \quad Q_m^C = \frac{2(a-c_m)X_{12}}{\Delta_1}.
\end{align*}
\]

The concavity of profit functions for CLSC members and SW for the government in Scenarios MC and RC is ensured by condition $\Delta_1 > 0$ and $4X > X^2$, respectively. It is found that feasible values of GLs ($\theta_m^C$ and $\theta_r^C$) of the product exists if $\beta > 1$. Therefore, if per unit investment efficiency for the manufacturer is too low, but consumer sensitivity with green product is less, then the manufacturer cannot produce MDIGPs. The unit cost of the product increased with $\lambda_1$; in this circumstance, the manufacturer cannot compensate increasing cost. Therefore, results make sense. By comparing optimal decisions between the MS and RS games, the following theorem is proposed.

Theorem 1. In Policy C

1. The greening levels, collection rates, and social welfare are identical under both games; and the amount of the subsidy on per unit product is higher under the MS game.
2. The greening levels, collection rates, subsidy rates, and social welfare decrease with respect to $\lambda_1$ and $\lambda_2$.

We refer to Appendix C for the details of Theorem 1. Lemma 1,2 indicate that if the manufacturer is not efficient enough, then it is difficult to produce greener product. Per unit subsidy and the collection rate decreases with respect to $\lambda_1$ and $\lambda_2$, and as a result, SW decreased. Additionally, the manufacturer needs to reduce the total investment effort in improving used product collection if the manufacturing cost is high. Therefore, results are sensible. Except profits of CLSC members, game structures do no have any effect on the optimal decision. The manufacturer charges a higher wholesale price, and the retailer sets a higher retail price in the MS game because $w_m^C - w_r^C = \frac{2(a-c_m)X_{12}}{\Delta_1} > 0$ and $p_m^C - p_r^C = \frac{(a-c_m)(k(\beta-1)^2+X^2_{12})}{\Delta_1} > 0$, respectively. Due to more bargaining power, the subsidy rate is always higher in the MS game, but the consumer needs to pay the same price under the MS and RS games because $(p_m^C - p_m^C) - (p_r^C - p_r^C) = 0$. Overall, consumers remain unaffected, and the government needs to provide a higher per unit subsidy under the MS game, which does not ensure higher quality.

3.2. Optimal Decisions in Policy RE

The profit structure of CLSC members and the SW of government remain similar to the previous subsection, but the demand function converts to $D_{RE}^{RE} = a - p_i^{RE} + \beta q_i^{RE}$. For example, in China, manufacturers and government organizations, such as China’s National Development and Reform Commission in 2010, collaborated to promote the program “Comments on Boosting the Re-manufacturing Development” to encourage product reuse [52]. As discussed in Section 3.1, the profit functions for the retailer and manufacturer, and SW in Scenarios MRE and RRE are obtained as follows:

\[
\pi_{rr}^{RE}(p_r^{RE}) = (p_r^{RE} - w_r^{RE})D_r^{RE},
\]

\[
\pi_{mr}^{RE}(p_m^{RE}) = (p_m^{RE} - w_m^{RE})D_m^{RE},
\]

\[
\pi_{rr}^{RE}(p_r^{RE}) = (p_r^{RE} - w_r^{RE})D_r^{RE},
\]

\[
\pi_{mr}^{RE}(p_m^{RE}) = (p_m^{RE} - w_m^{RE})D_m^{RE},
\]

\[
\pi_{rr}^{RE}(p_r^{RE}) = (p_r^{RE} - w_r^{RE})D_r^{RE},
\]

\[
\pi_{mr}^{RE}(p_m^{RE}) = (p_m^{RE} - w_m^{RE})D_m^{RE},
\]
A similar type of subsidy policy is discussed in the forward SC setting by several researchers [59].

\[ 
\pi_{mi}^{RE}(w_i^{RE}, \theta_i^{RE}, \tau_i^{RE}) = (w_i^{RE} - \lambda_1 \theta_i^{RE} + X \tau_i^{RE})D_i^{RE} - (1 - \eta_i^{RE}) \kappa \tau_i^{RE} - \lambda \theta_i^{RE}, \]

\[ 
\pi_{gi}^{RE}(\eta_i^{RE}) = \pi_{mi}^{RE} + \frac{D^{RE}2}{2} - \kappa \eta_i^{RE} \tau_i^{RE}. \]

Derivations of optimal decisions are similar to Policy C, hence omitted. The concavity of profit functions for CLSC members and SW for the government in Scenarios MRE and RRE is ensured by condition \( \Delta_2m > 0 \) and \( \Delta_2r > 0 \), respectively. Optimal decisions under the MS and RS games are presented in Lemma 3, 4, respectively.

**Lemma 3.** Optimal decision in Scenario MRE are obtained as follows:

\[ 
\eta_{m}^{RE} = \frac{6\lambda_2}{\mathcal{M}_2}, \quad w_{m}^{RE} = \frac{(a(4M_2 - M_2X^2) + 4c_mM_2X)}{\Delta_2m}; \quad \beta_{m}^{RE} = \frac{(a-c_m)M_2X}{\Delta_2m}; \quad \pi_{mm}^{RE} = \frac{(a-c_m)^2M_2X^2\lambda_2}{\Delta_2m}; \quad \eta_{mr}^{RE} = \frac{(a-c_m)^2M_2^2X^2\lambda_2}{\Delta_2m}, \]

\[ 
\tau_{m}^{RE} = \frac{(a-c_m)X\lambda_2}{\Delta_2m}; \quad \eta_{rm}^{RE} = \frac{(a-c_m)^2X^2\lambda_2}{\Delta_2m}; \quad \pi_{mr}^{RE} = \frac{(a-c_m)^2M_2^2X^2\lambda_2^2}{\Delta_2m}; \quad \pi_{gm}^{RE} = \frac{(a-c_m)ZM_2M_2X^2\lambda_2}{\Delta_2m}, \]

\[ 
\pi_{mr}^{RE} = \frac{(a-c_m)ZM_2M_2X^2\lambda_2}{\Delta_2m}, \quad \pi_{mr}^{RE} = \frac{(a-c_m)ZM_2X^2\lambda_2}{\Delta_2m}, \]

\[ 
Q_{m}^{RE} = \frac{(a-c_m)M_2X\lambda_2}{\Delta_2m}, \quad \text{where } \Delta_2m = M_2^2\lambda_2 - M_2X^2\lambda_2. \]

**Lemma 4.** Optimal decision in Scenario RRE are obtained as follows:

\[ 
\eta_{r}^{RE} = \frac{2\lambda_2 - Z^2\hat{k}}{2(\lambda_1 + \lambda_2)}; \quad w_{r}^{RE} = \frac{M_2(2(\lambda_1 + \lambda_2) - X^2\lambda_1)}{\Delta_2r}; \quad \beta_{r}^{RE} = \frac{(a-c_m)Z(M_2X + X^2\lambda_2)}{\Delta_2r}; \quad \tau_{r}^{RE} = \frac{(a-c_m)X\lambda_2}{\Delta_2r}; \quad \eta_{mr}^{RE} = \frac{(a-c_m)^2X^2\lambda_2}{\Delta_2r}; \quad \pi_{mr}^{RE} = \frac{(a-c_m)^2M_2^2X^2\lambda_2^2}{\Delta_2r}, \]

\[ 
Q_{r}^{RE} = \frac{(a-c_m)M_2X\lambda_2}{\Delta_2r}, \quad \text{where } \Delta_2r = M_2^2\lambda_2 - X^2(M_2 + 2\lambda_2)\lambda_2. \]

Recall that optimal subsidy rates in Policy C are directly proportional with market potential, and different results are obtained in Policy RE. Although the demand increases with market potential, it does not directly affect the subsidy rate. However, the government may have to spend more because the collection rates \( \tau_{m}^{RE} \) and \( \tau_{r}^{RE} \), or overall demand \( Q_{m}^{C} \) and \( Q_{r}^{C} \), increase with market potential. The following theorem highlights the characteristics of the optimal decision.

**Theorem 2.** In Policy RE,

1. The greening levels, collection rate, and social welfare are always higher under the RS game; however, the subsidy rate is higher under the MS game.
2. The greening levels, collection rates, subsidy rates, and social welfare decrease with respect to \( \lambda_1 \) and \( \lambda_2 \).

We refer to Appendix D for the details of Theorem 2. The outcome of Theorem 2 differs from the previous one. A powerful retailer can enforce that the manufacturer produce and trade with greener product. The product collection rate is also higher under the RS game. Similar to Policy C, the subsidy rate is higher under the MS game. Therefore, one can find an indication that the sustainability goal can be achieved under the RS game in the presence of a subsidy.

### 3.3. Optimal Decisions in Policy T

In this subsection, models are formulated and the optimal decision is derived in Policy T. A similar type of subsidy policy is discussed in the forward SC setting by several researchers [59]. For example, to strengthen sustainable innovation, $400 million was allotted to fund the R&D of energy technologies as a part of the American Recovery and Reinvestment Act of 2009 [48].

The demand function in this policy becomes \( D_i^T = a - \mu_i^T + \beta \theta_i^T \), and the corresponding profit functions of the manufacturer and retailer; and SW remains similar with previous subsections, and they are obtained as follows:

\[ 
\pi_{mi}^{T}(\mu_i^T) = (\mu_i^T - w_i^T)D_i^T, \quad \pi_{mi}^{T}(\mu_i^T) = (\mu_i^T - \lambda_1 \theta_i^T)D_i^T + X \tau_i^TD_i^T - \kappa \tau_i^{T^2} - (1 - \mu_i^T) \lambda_2 \theta_i^{T^2}, \]

\[ 
\pi_{gi}^{T}(\eta_i^T) = \pi_{mi}^{T} + \frac{D^{T^2}}{2} - \kappa \eta_i^{T} \tau_i^{T^2}. \]
\[
\pi_{T}(\mu_{T}) = \pi_{ri}^{T} + \pi_{mi}^{T} + \frac{D_l^2}{2} - \mu_{T}^{T} \lambda_{2}^{T} \beta^{T}. \tag{9}
\]

Note that the manufacturer receives a subsidy on the total R&D investment, not on per unit product \(\lambda_{1}^{T}, i = m, r\). If \(\lambda_{1} = 0\) and \(\mu_{T}^{T} = 0\), the profit functions become similar to Ghosh and Saha [18], as well as Song and Gao [19], where the authors examined the optimal decision for a forward SC setting where the government does not provide any subsidy. Similar to previous subsections, we derive the optimal decision for Policy T and omit the detailed derivation. Lemma 5,6 characterize the optimal decisions under the MS and RS games, respectively.

**Lemma 5.** Optimal decision in Scenario MT are obtained as follows:

\[
\mu_{m}^{T} = \frac{\Delta_{3m}}{N_{3}^{2} \lambda_{2}}; \quad \mu_{r}^{T} = \frac{N_{2}^{3} \lambda_{2} - Z^{2} \kappa}{N_{2}^{3} \lambda_{2} + Z^{2} \kappa}; \quad \pi_{m}^{T} = \frac{N_{2} Y Z x + N_{4}(a N_{2} + 4 \kappa \lambda_{2}) \lambda_{2}}{\Delta_{3m}}; \quad \theta_{m}^{T} = \frac{(a - c_{m}) N_{5} Z x}{\Delta_{3m}}; \quad \tau_{m}^{T} = \frac{(a - c_{m}) N_{4} \lambda_{2}}{\Delta_{3m}},
\]

where \(\Delta_{3m} = N_{4}^{2} \lambda_{2} - N_{5} Z x\).

**Lemma 6.** Optimal decision in Scenario RT are obtained as follows:

\[
\mu_{r}^{T} = \frac{N_{2}^{3} \lambda_{2} - Z^{2} \kappa}{2(N_{2}^{3} \lambda_{2} + Z^{2} \kappa)}; \quad \pi_{m}^{T} = \frac{N_{2} Y Z x + N_{4}(a N_{2} + 4 \kappa \lambda_{2}) \lambda_{2}}{2 \Delta_{3m}}; \quad \theta_{m}^{T} = \frac{(a - c_{m}) N_{5} Z x}{2 \Delta_{3m}}; \quad \tau_{m}^{T} = \frac{(a - c_{m}) N_{4} \lambda_{2}}{2 \Delta_{3m}}; \quad Q_{r}^{T} = \frac{2(a - c_{m}) N_{4} \lambda_{2}}{2 \Delta_{3m}},
\]

where \(\Delta_{3m} = N_{2}^{2} \lambda_{2} - 2 N_{5} Z x\).

The concavity of profit functions for CLSC members and SW in Scenarios MT and RT is ensured by condition \(\Delta_{3m} > 0\) and \(\Delta_{3r} > 0\), respectively. Similar to Policy RE, optimal decisions differ according to the power of CLSC members, and increasing market potential does not affect the subsidy rates. By comparing optimal decisions, the following theorem is proposed.

**Theorem 3.** In Policy T

1. The greening level, collection rate, and social welfare are always higher under the RS game; however, the subsidy rates are lower under the MS game.
2. The greening levels, collection rates, subsidy rates, and social welfare decreases with respect to \(\lambda_{1}\) and \(\lambda_{2}\).

We refer to Appendix E for the details of Theorem 3. Results of Theorem 2,3 are similar. Overall, SW and GL are higher under the RS game under Policy RE and T. In all three policies, the subsidy rates are higher under the MS game, but it does not guarantee the higher SW and GL. Therefore, a shift of power from the manufacturer to the retailer in a CLSC always encourages in the perspective of achieving the sustainability goal. By combining concavity conditions, one can conclude that the profits of CLSC members and SW are concave in three subsidy policies under the MS and RS games if \(M_{1} > 0\), \(M_{2} > 0\) and \(\Delta_{1} > 0\). To obtain an overview of increment or decrements in GL, used product collection rate, profits of CLSC members, and SW, we derive the optimal decision in the absence of a subsidy.

3.4. Optimal Decisions in Absence of Subsidy

By substituting \(\rho_{i}^{T} = 0, \eta_{r}^{T} = 0, \mu_{r}^{T} = 0\) in Equation (1),(2); (4),(5); (7),(8), respectively, one can obtain the profit functions of CLSC members in the absence of a subsidy. The corresponding optimal decisions under the MS and RS games are presented in Lemma 7,8, respectively.

**Lemma 7.** Optimal decision in the absence of a subsidy under the MS game is as follows:

\[
\pi_{m}^{N} = \frac{N_{2} Y Z x + N_{4}(a N_{2} + 4 \kappa \lambda_{2}) \lambda_{2}}{\Delta_{4m}}; \quad \theta_{m}^{N} = \frac{(a - c_{m}) N_{5} Z x}{\Delta_{4m}}; \quad \tau_{m}^{N} = \frac{(a - c_{m}) N_{4} \lambda_{2}}{\Delta_{4m}}; \quad Q_{m}^{N} = \frac{2(a - c_{m}) N_{4} \lambda_{2}}{\Delta_{4m}}, \quad \text{where} \quad \Delta_{4m} = N_{4} \lambda_{2} - \kappa Z^{2}.
\]
Lemma 8. Optimal decision in the absence of a subsidy under the RS game is as follows:

\[ w_r^N = \frac{(a+c_m)N_1\lambda_2 + (YZ+c_m)M_2\kappa}{2\lambda_2}; \]
\[ p_r^N = \frac{2aN_1\lambda_2 + aM_1 + YZ + 2c_m\lambda_2\kappa}{2\lambda_2}; \]
\[ \theta_r^N = \frac{(a-c_m)\kappa}{2\lambda_2}; \]
\[ \tau_r^N = \frac{(a-c_m)X\lambda_2}{2\lambda_2}; \]
\[ \pi_{mr}^N = \frac{(a-c_m)\kappa}{4\lambda_2}; \]
\[ \pi_{mr}^N = \frac{(a-c_m)\kappa\lambda_2^2}{2\lambda_2}; \]
\[ Q_r^N = \frac{(a-c_m)\kappa\lambda_2}{\Delta_4}; \]

where \( \Delta_4 = N_2\lambda_2^2 - \kappa Z^2 \).

Unlike optimal decisions in Policy C, the outcomes differ between the MS and RS game in the absence of a subsidy. In this regard, one can conclude that government can weaken the effect of power of CLSC members by implementing Policy C. We use results of above two lemmas as the benchmark to compare outcomes.

4. Analysis and Discussions

In the previous section, results were compared to highlight the behavior of optimal decisions between the two games. In the following subsections, we evaluate gains and losses in the perspective of consumer, CLSC members, and the government organization.

4.1. Consumer’s Perspective

The following theorem highlights consumers preference among three subsidy policies.

Theorem 4. Irrespective of game structures, the greening level and sales volume are higher in Policy C.

We refer to Appendix F for the details of Theorem 4. Theorem 4 demonstrates that consumers always receive product at a higher GL in Policy C, where the government can penetrate consumers directly. Therefore, direct monetary gains stimulate consumers to buy more product. If sales volumes increase, the manufacturer can compensate investment cost, and GL is also consequently increased. Recall that GL, collection rate, and retail price are identical under both games in Policy C. Therefore, Policy C outperforms others in the perspective of consumer benefit and green product consumption. Graphical representation of GLs, sales volumes, effective prices consumer needs to pay, and ratios of GLs with effective retail price in six scenarios is presented in Figure 1a–d. Parameter values are used for numerical examples as follows: \( a = 500, \beta = 0.6, c_m = 50(\$/unit), c_r = 20(\$/unit), a = 10(\$/unit), \delta = 0.7, \kappa = 1500, \lambda_1 = 0.3, \) and \( \lambda_2 = 1. \) Note that technical restrictions on parameters values are considered to ensure optimal conditions.

Figure 1a,b justify the statement of Theorem 4. The effective retail price is less in Policy C compared to others, and GL is always higher in the RS game in all three policies. If the consumer perceives the retail price in their mind, then Policy C outperforms others because the ratio of GLs with retail price is maximum under Policy C. By comparing the ratios, consumers can figure out how much they need to pay to procure the product. Figure 1c. demonstrates that the consumer needs to pay a lesser price under Policy C. One can observe that the GLs are lower in Policy RE compared to others, and the consumer needs to pay a higher price in Policy T.
Figure 1. Graphical representation of (a) greening levels, (b) sales volumes, (c) effective retail prices, and (d) ratio of greening levels with effective retail prices in Scenarios MC, RC, MRE, RRE, MT, and RT.

4.2. Retailer and Manufacturer Perspectives

The following theorem is proposed to highlight the pros and cons for three policies in the perspective of CLSC members.

**Theorem 5.** Under both games, the collection rate of used product and profits for each member are always higher in Policy C.

We refer to Appendix G for the details of Theorem 5. The outcomes of Theorem 5 are consistent with Theorem 4. GL and sales volume are both higher in Policy C; consequently, CLSC members receive a higher profit in a green-sensitive market. Flexibility of investment for the manufacturer in improving GL and used product return is increased with market demand. In such a scenario, the retailer can also get benefited. The results demonstrate that fact. Graphical representation of the profits for the retailer and manufacturer, collection rate, total investment for the manufacturer to produce product (MI) \( MI^C_i = \lambda_1 \theta^C_i D^C_i + \lambda_2 \theta^C_i^2 \), \( MI^RE_i = \lambda_1 \theta^RE_i E^RE_i + \lambda_2 \theta^RE_i^2 \); and \( MI^T_i = \lambda_1 \theta^T_i D^T_i + (1 - \mu^T_i)\lambda_2 \theta^T_i^2 \), investment for the manufacturer in encouraging used product return (RI) \( R^C_i = \alpha \tau^C_i D^C_i + \kappa \tau^C_i^2 \), \( R^RE_i = \alpha \tau^RE_i D^RE_i + (1 - \eta^RE_i)\kappa \tau^RE_i^2 \); and \( R^T_i = \alpha \tau^T_i D^T_i + \kappa \tau^T_i^2 \), ratios of relative change of GL with investment to produce products \( \Delta \theta M^C_i = \frac{\theta^C_i - \theta^C_i^N}{M^C_i - M^C_i^N} \), and ratios of relative change of the collection rate with the investment effort to stimulate used product return \( \Delta \tau R^C_i = \frac{\tau^C_i - \tau^C_i^N}{R^C_i - R^C_i^N} \) in six scenarios is presented in Figure 2a–g.

Figure 2a–g support the statement of Theorem 5. In Policy RE, the manufacturer receives a subsidy to encourage the product collection, but it does not yield a higher return compared to Policy C. Note that the used product collection rates are less in Policy T compared to other two. Figure 2e demonstrates that the investment effort to stimulate used product return for the manufacturer is less in Policy RE; however, due to the government support, collection rate improved. In Policy T, the manufacturer has more flexibility in the R&D investment until the manufacturer can invest more in
Policy C. However, Figure 2f exhibits some doubts about the efficiency of Policy C. In Policy C, the total amount of investment for the manufacturer to improve GL is maximum, but the ratio of relative change in GL improvement is less. Therefore, higher investment does not ensure higher GL, especially in Policy C. Figure 2g demonstrates a noteworthy outcome in the perspective of designing subsidy policy. It demonstrates that the power of CLSC members should be considered before implementation of the subsidy policy. Interestingly, the manufacturer reduces the investment effort considerably under the RS game, but a reverse trend is observed under the MS game. Overall, investment efficiency in producing greener product and used product return reduced in Policy C.

Figure 2. Graphical representation of (a) profit of manufacturer, (b) profit of retailer, (c) used product return, (d) manufacturer’s R&D investment to produce product, (e) investment effort in encouraging used product collection, (f) ratios of relative change of greening levels with investment to produce greener product, and (g) ratios of relative change of return rate with investment effort to stimulate product return for the manufacturer in Scenarios MC, RC, MRE, RRE, MT, and RT.
4.3. Government Perspective

In this subsection, we compare SWs and the amount of government subsidy to explore consequence in the perspective of government organizations.

**Theorem 6.** The social welfare and the amount of government subsidy is higher in Policy C in both the games.

We refer Appendix H for the proof of Theorem 6. Theorem 6 demonstrates that the government expenditure and SW are always higher in Policy C. Therefore, the outcomes of Theorem 4–6 are very much alike. Higher subsidy cause higher profits, as well as GL, and SW consequently increased. Graphical representation of SW, the amount of government subsidy (GI) \( GI^C_i = \rho_i^C D_i^C \); \( GI^R_i = \eta_i^R G^R_i \); and \( GI^T_i = \mu_i^T \lambda_i^T i^2 \), and the ratios of relative change of GL with total amount of government subsidy \( \Delta \theta G_i = \frac{\theta_i G_i N - \theta_i G_i I}{\theta_i G_i I} \) in six scenarios is depicted in Figure 3a–c.

![Figure 3](image)

**Figure 3.** Graphical representation of (a) social welfare, (b) total amount of government subsidy in each scenario, and (c) ratios of relative change of greening levels with total amount of government subsidy in Scenarios MC, RC, MRE, RRE, MT, and RT.

The above figures support the statement of Theorem 6. However, if we investigate at the macro level, then one cannot draw a straightforward conclusion in favor of Policy C. By correlating Figure 2e with Figure 3c, the ratio of investment efficiency reflects the different consequence. In the perspective of the manufacturer and government organization, Policy C may lead to an inadequate investment decision. However, one cannot ignore the influence of consumers until they enjoy the higher privilege in Policy C because government support directly passes to the consumer. The total amount of expenditure for the government is too high in Policy C, which does not yield a higher relative improvement in GL.

4.4. When Manufacturer Produces Only DIGPs

In this study, it is assumed that the manufacturer produces MDIGPs but does not recover the cost for used product. Therefore, we conduct numerical experiment where the manufacturer produces DIGPs(\( \lambda_1 = 0 \)), which is more predominant in existing literature. The following figures represent
the relative change of profits of the SC members \( \Delta \pi^j_{ki} = \frac{\pi^j_{ki}|_{\lambda_1=0} - \pi^j_{ki}}{\pi^j_{ki}} \), collection rates \( \Delta \tau^j_{ji} = \frac{\tau^j_{ji}|_{\lambda_1=0} - \tau^j_{ji}}{\tau^j_{ji}} \); GLs \( \Delta \theta^j_{ji} = \frac{\theta^j_{ji}|_{\lambda_1=0} - \theta^j_{ji}}{\theta^j_{ji}} \); total amount of government subsidies \( \Delta GI^j = \frac{GI^j|_{\lambda_1=0} - GI^j}{GI^j} \); and SWs \( \Delta \pi^j_{gi} = \frac{\pi^j_{gi}|_{\lambda_1=0} - \pi^j_{gi}}{\pi^j_{gi}} \).

It is expected that the CLSC members receive higher profits if unit production cost decreased. Figure 4a,b reflect that nature changes profits for CLSC members, SW, and GL, which also supports the expectation. CLSC members always receive a higher profit in Policy C, and GL is always maximum. However, the nature of used product collection and the amount of government subsidy changes significantly. Increment in the used the product collection rate and the amount of government subsidy are maximum in the RS game and in Policy T. Therefore, the government needs to examine the product type to frame an effective policy.

Figure 4. Graphical representation of (a) change in profits for the manufacturer, (b) change in profits for the retailer, (c) change in product collection rates, (d) change in greening levels \( (\lambda_1 = 0) \), (e) change in amount of government subsidies, and (f) change in social welfare in Scenarios MC, RC, MRE, RRE, MT, and RT \( (\lambda_1 = 0) \).

4.5. Overall Implications

The preceding discussion offers a rich amount of contextual detail based on the analytical and numerical evaluation. Subsidies make sense to encourage R&D activities in areas that would benefit society, stimulate greener product consumption with a society’s environmental objectives, such as less contamination, cleaner air, etc., provide much-needed help to innovative startups, or support a
manufacturer in surviving financial losses due to high R&D investment. However, there has been little discussion on comparative analysis among outcomes under the government SW optimization goal.

The present study discloses some eye-opening issues. It has always been a topic of interest to consider which subsidy policy can lead to a pragmatic CLSC business model, or which is both environmentally and economically worthwhile for participating member and government organization. Based on the discussion, one can articulate that the optimal decision, preference, and implications of subsidy policies significantly change between MS and RS games. To maintain goodwill and dominate a green-sensitive market, it is always imperative for the retailer to sell greener product. However, the manufacturer receives a higher subsidy in the MS game; yet, GL and SW is higher in the RS game. Therefore, the power of a CLSC member adds a degree of conflict, and government organizations needs figure out the dynamics of power before implementing subsidy policies. Overall, Policy C under the RS game can drive toward encouraging outcomes in the perspective of consumers, retailer, and government organization.

It is commonly believed that a subsidy assists manufactures to produce greener products and trade them at low price to the consumers. To some extent, the results of the present study support the convention, but in the presence of government subsidy policy, CLSC members need to be prepared for sudden operational changes. In practice, government organizations sometimes commit to environmental policies for several years but afterward renege on their commitments. For example, the government of China recently recommended to withdraw a subsidy from the EV battery industry (https://chinaeconomicreview.com/subsidy-withdrawal-to-decimate-chinas-ev-industry/), and the government of India recently revised the amount of subsidy for the scheme “Faster Adoption and Manufacturing of Hybrid and Electric Vehicles” (https://energy.economictimes.indiatimes.com/news/power/govt-withdraws-sops-to-conventional-battery-vehicles-under-fame/65990495). If the government suddenly revises a subsidy amount due to a sudden fall of market demand, the manufacturer needs to adjust its production rate and to be prepared for adjustment of the entire operations and marketing activities. For example, when the Indian government revised the scheme and reduced the direct subsidy to consumers, car manufacturers faced market fall. A similar situation also reported in the UK is that “Subsidy cuts blamed for fall in UK sales of electrified vehicles” (www.theguardian.com/business/2019/jul/04/subsidy-cuts-blamed-for-fall-in-uk-sales-of-electrified-vehicles). Examples are similar with Policy C because, in the car industry, consumers directly receive a subsidy from government. Results indicate that Policy C clearly becomes a financial liability for government. Due to direct cash-transfer in Policy C, all the consumers enjoy a subsidy irrespective of income groups. Therefore, there is a possibility that government resources might become a drain, especially if high-income consumers take the subsidy. As observed earlier, GL and GIs are maximum in Policy C; therefore, this is where to find answer of the question how much additional amount needs to be paid for improving GL? Figure 5a, b, representing the ratio of GLs and GIs under Policy C, are drawn to obtain an overview in this direction.
By observing the vertical axis of Figure 5a–c, one can recognize the additional financial burden associated with Policy C, especially under the MS game. It is also found that in Policy RE, the government needs to spend less and consequently also lessen GL. In practice, there is a possibility that the manufacturer can strategically reduce investment effort in presence of a subsidy. Our study also supports this fact because the manufacturer clearly reduce its investment effort, as depicted in Figure 2g. Figure 2g also demonstrates that if the intention of the government is to improve used product collection, then Policy T or RE under the MS game can drive to the desirable outcome. Finally, in Policy T, the government provides a subsidy and anticipates that the manufacturer can produce greener product, while consumers benefit from low prices. However, Policy T also becomes pricier in the perspective of consumers, but the improvement of GL is maximum under this policy.

5. Conclusions

The formation of a sustainable CLSC in the presence of a government subsidy is one of the key issues because it does not make sense to pollute the world for higher profits. One the other hand, it is infeasible in the perspective of a government organization to spend large amounts that fail to create value. Therefore, it is always challenging to design a subsidy policy that can lead to pragmatic outcomes. In literature, comparative studies on optimal outcomes in the presence of government SW optimization goal are scanty.

Motivated by emerging practice, we formulated eight CLSC models to compare outcomes of three subsidy policies. The central result emerging from the analysis reveals that in Policy C, CLSC members receive higher profits, SW of the government organization higher, and the consumers receive products at a higher GL. Characteristics of the optimal decision under the MS and RS games are not concurrent; GL and used product collection are always higher in the RS game, and the government subsidy rate is always higher in the MS game. Whatever the nature of game structures, the consumer always receives product at lower price in Policy C. However, Policy C still has shortcomings. GL does improve as the R&D investment or amount of the government subsidy increases, but the rate of change is lowest. It is found that Policy C can be a substantial financial burden without too much improvement in GL and used product collection. Because the amount of the subsidy is maximum in Policy C, our study contradicts conventional beliefs that a higher subsidy level always improves the performance of the CLSC members. The present study discloses that any straightforward conclusion on the optimal preferences in the perspective of CLSC members, consumers, and government organization is challenging to be made. If the government wants to improve green product consumption among its community, then the government can implement Policy C. However, expenditure as a subsidy will increase considerably. If the government aims to utilize a subsidy expenditure in an effective way, then Policy T can lead to a decent outcome. However, SW will be less and the consumer needs to pay more. In Policy RE, a strategic manufacturer can reduce the investment effort in the presence of a
subsidy. If the intention of government is to improve used product collection, then Policy T or RE under
the MS game can drive to a desirable outcome. It is also observed that the manufacturer’s decision
to produce DIGPs or MDGIPs can also affect the outcomes of a subsidy policy. A retailer-dominated
CLSC is always advantageous for the government; in that scenario, the government can reduce the
amount of a subsidy, maximize SW, and the consumer receives greener products.

Therefore, this study can be extended in several directions. In practice, a retailer or third party is
also involved in used product collection. Sometimes manufacturer and retailer can both be involved
in collections. Therefore, one can examine optimal decisions in the different modes of collections
i.e., manufacturer, retailer, third party, or their combined collection mode. We assume that the
consumer cannot distinguish the difference between the new and re-manufactured products. However,
consumers often value the re-manufactured product less than the new product [69]. Therefore, one can
analyze the influence of subsidy policies where the CLSC members need to set different prices for new
and re-manufactured product. We restricted our analysis under single period formulation; therefore,
one can extend this analysis under two-period setting. Furthermore, it will be interesting to examine
the behavior of a CLSC decision if the members agree to cooperate with each other through coordination
contract mechanisms [70,71] under the influence of the government subsidy.

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Appendix A. Optimal Decision in Scenario MC

To obtain optimal response for the retailer in Equation (1), one needs to solve $\frac{d\pi_m^C}{dp_m^C} = 0$.

On simplification, $p_m^C(w_m^C, \theta_m^C, r_m^C, \rho_m^C) = \frac{a + \theta_m^C + \beta_m^C + \rho_m^C}{2}$. Because $\frac{d^2\pi_m^C}{dp_m^C} = -2 < 0$, the profit function
for the retailer is concave.

Therefore, substituting $p_m^C$ in Equation (2), the profit function for the manufacturer is obtained as:

$$\pi_m^C(w_m^C, \theta_m^C, r_m^C, \rho_m^C) = \frac{(w_m^C - \theta_m^C + X\theta_m^C - c_m)(a - w_m^C + \beta_m^C + \rho_m^C) - 2\gamma c^2 - 2\lambda_2 \theta_m^2}{2}$$

To obtain optimal response for the manufacturer on wholesale price and investment efforts, we
need to solve $\frac{d\pi_m^C}{dw_m^C} = 0$, $\frac{d\pi_m^C}{dr_m^C} = 0$, and $\frac{d\pi_m^C}{d\rho_m^C} = 0$, simultaneously. After simplification,
the following response is obtained:

$$w_m^C = \frac{\kappa(a + c_m + \rho_m^C)(4\lambda_2 - \beta - \lambda_1)}{\kappa(8\lambda_2 - Z^2) - X^2\lambda_2}$$

$$r_m^C = \frac{a - c_m + \rho_m^C}{\kappa(8\lambda_2 - Z^2) - X^2\lambda_2}$$

and $\theta_m^C = \frac{\kappa Z(a - c_m + \rho_m^C)}{\kappa(8\lambda_2 - Z^2) - X^2\lambda_2}$

Because, the manufacturer’s profit function is a function of three variables, we compute the
Hessian matrix ($H_m^C$) to verify concavity as follows:

$$H_m^C = \begin{bmatrix}
\frac{\partial^2 \pi_m^C}{\partial w_m^C \partial w_m^C} & \frac{\partial^2 \pi_m^C}{\partial w_m^C \partial \theta_m^C} & \frac{\partial^2 \pi_m^C}{\partial w_m^C \partial \rho_m^C} \\
\frac{\partial^2 \pi_m^C}{\partial \theta_m^C \partial \theta_m^C} & \frac{\partial^2 \pi_m^C}{\partial \theta_m^C \partial \rho_m^C} & \frac{\partial^2 \pi_m^C}{\partial \theta_m^C \partial \rho_m^C} \\
\frac{\partial^2 \pi_m^C}{\partial \rho_m^C \partial \rho_m^C} & \frac{\partial^2 \pi_m^C}{\partial \rho_m^C \partial \theta_m^C} & \frac{\partial^2 \pi_m^C}{\partial \rho_m^C \partial \rho_m^C}
\end{bmatrix}
= \begin{bmatrix}
-1 & -\frac{X}{\beta} & -\frac{\beta + \lambda_1}{\beta}
\\
-\frac{X}{\beta} & -2\kappa & \frac{\beta - \lambda_1}{\beta}
\\
-\frac{\beta + \lambda_1}{\beta} & \frac{\beta - \lambda_1}{\beta} & -2\lambda_2 - \lambda_1\beta
\end{bmatrix}$$
The values of first, second, and third principal minors are obtained as $H_{m1}^C = -1 < 0$; $H_{m2}^C = \frac{8\kappa - X^2}{4}$; and $H_{m3}^C = -(8\kappa - X^2)\lambda_2 - \kappa(\beta - \lambda_1)^2$, respectively. Therefore, profit function for the manufacturer is concave if $8\kappa > X^2$ and $(8\kappa - X^2)\lambda_2 - \kappa(\beta - \lambda_1)^2 > 0$.

Substituting optimal responses in Equation (3), the SW function for the government organization is obtained as $\pi_{tg}^C (\rho_t^C) = \frac{\kappa \lambda_2 (a - c_m + \rho_t^C)(a - c_m)(14\kappa - X^2)\lambda_2 - \kappa Z^2) + \kappa Z^2 \rho_t^C - (2X - X^2)\lambda_2\rho_t^C}{(8\kappa - X^2)\lambda_2 - \kappa(\beta - \lambda_1)^2}$. Therefore, one can obtain optimal subsidy rate by solving $\frac{d\pi_{tg}^C}{d\rho_t} = 0$. On Simplification, optimal subsidy rate is obtained as $\rho_t^C = \frac{6(a - c_m)\lambda_2}{\kappa (2\lambda_2 - Z^2) - X^2\lambda_2}$. Note that $\frac{d^2\pi_{tg}^C}{d\rho_t^2} = -\frac{2\kappa \lambda_2 \lambda_1}{\kappa (2\lambda_2 - Z^2) - X^2\lambda_2} < 0$, i.e., $\pi_{tg}^C$ is concave with respect to subsidy rate if $\Delta_1 = \kappa(2\lambda_2 - (\beta - \lambda_1)^2 - X^2\lambda_2$. By using back substitution, we obtain optimal decision as presented in Lemma 1.

The following additional notations are used throughout the article for simplicity:

$M_1 = 2\lambda_2 - Z^2$; $M_2 = 4\lambda_2 - Z^2$; $M_3 = 8\lambda_2 - Z^2$; $M_4 = 14\lambda_2 - Z^2$; $N_1 = 2\kappa - X^2$; $N_2 = 4\kappa - X^2$; $N_3 = 6\kappa - X^2$; $N_4 = 8\kappa - X^2$; $X = c_m\delta - c_r - \alpha$; $Y = a\lambda_1 - c_m\beta$, $Z = \beta - \lambda_1$.

**Appendix B. Optimal Decision in Scenario RC**

First, we substitute $m_r^C = p_r^C - w_r^C$ in Equation (1)–(3) to obtain optimal decision in the RS game. To obtain the manufacturer response first, one needs to solve $\frac{d\rho_m^C}{da} = 0$, $\frac{d\rho_m^C}{d\theta} = 0$, and $\frac{d\rho_m^C}{d\tau} = 0$ simultaneously. Therefore, the manufacturer’s response is obtained as follows:

$$w_r^C = \frac{(\kappa(2\lambda_2 + Z\lambda_1) - X^2\lambda_2)(a - m_r^C + \rho_r^C) + c_m\kappa(2\lambda_2 - (\beta - \lambda_1))}{\kappa(4\lambda_2 - Z^2) - X^2\lambda_2}$$

$$\tau = \frac{(a - c_m - m_r^C + \rho_r^C)X\lambda_2}{\kappa(4\lambda_2 - Z^2) - X^2\lambda_2}$$

and $\theta = \frac{(a - c_m - m_r^C + \rho_r^C)KZ}{\kappa(4\lambda_2 - (\beta - \lambda_1)^2 - X^2\lambda_2)}$

Because, the profit function for the manufacturer is a function of three variables, therefore, we compute corresponding Hessian matrix($H_r^C$) for the manufacturer profit function is as follows:

$$H_r^C = \begin{bmatrix}
\frac{\partial^2 \pi_{mg}}{\partial a^2} & \frac{\partial^2 \pi_{mg}}{\partial a \partial \theta} & \frac{\partial^2 \pi_{mg}}{\partial a \partial \tau} \\
\frac{\partial^2 \pi_{mg}}{\partial \theta \partial a} & \frac{\partial^2 \pi_{mg}}{\partial \theta^2} & \frac{\partial^2 \pi_{mg}}{\partial \theta \partial \tau} \\
\frac{\partial^2 \pi_{mg}}{\partial \tau \partial a} & \frac{\partial^2 \pi_{mg}}{\partial \tau \partial \theta} & \frac{\partial^2 \pi_{mg}}{\partial \tau^2}
\end{bmatrix} = \begin{bmatrix}
-2 & -X & \beta + \lambda_1 \\
-X & -2\kappa & \beta X \\
\beta + \lambda_1 & \beta X & -2(\lambda_2 + \beta + \lambda_1)
\end{bmatrix}$$

The principal minors of above Hessian matrix are $H_{r1}^C = -2 < 0$; $H_{r2}^C = 4\kappa - X^2 > 0$; and $H_{r3}^C = -2(\kappa(4\lambda_2 + Z^2) - X^2\lambda_2)$, respectively. Consequently, the profit function will be concave if $4\kappa > X^2$ and $\kappa(4\lambda_2 + Z^2) > X^2\lambda_2$.

Substituting optimal response for the manufacturer in Equation (1), profit function for the retailer is obtained as $\pi_{gr}^C (m_r^C) = \frac{2m_r^C \kappa \lambda_2 (a - c_m - m_r^C + \rho_r^C)}{\kappa(4\lambda_2 - Z^2) - X^2\lambda_2}$. Therefore, the optimal response for the retailer is obtained by solving $\frac{d\pi_{gr}^C}{d\rho_r^C} = 0$. After simplification, $m_r^C = \frac{a - c_m + \rho_r^C}{2}$. The profit function of the retailer is also concave because $\frac{d^2 \pi_{gr}^C}{d\rho_r^2} = \frac{-4\kappa \lambda_2}{\kappa(4\lambda_2 - Z^2) - X^2\lambda_2}$. Substituting optimal responses in Equation (3), the simplified value of the SW in Scenario RC is obtained as follows:

$$\pi_{gr}^C (\rho_r^C) = \frac{\kappa \lambda_2 (a - c_m + \rho_r^C)(a - c_m)(14\kappa - X^2)\lambda_2 - 3\kappa Z^2 - 3X^2\lambda_2) + \kappa Z^2 \rho_r^C - (2X - X^2)\lambda_2\rho_r^C}{4\kappa(4\lambda_2 - Z^2) - X^2\lambda_2}\lambda_2^2$$

Therefore, the optimal subsidy rate will be obtained by solving $\frac{d\pi_{gr}^C}{d\rho_r^C} = 0$. On simplification, we obtain the value of $\rho_r^C$ as presented in Proposition (2). The SW under the RS game is concave because, $\frac{d^2 \pi_{gr}^C}{d\rho_r^2} = \frac{-4\kappa \lambda_2 \lambda_1}{\kappa(4\lambda_2 - (\beta - \lambda_1)^2 - X^2\lambda_2) < 0}$ where $\Delta_1 = \kappa(2\lambda_2 - (\beta - \lambda_1)^2 - X^2\lambda_2$. By using back substitution, we obtain optimal decision as presented in Lemma 2.
Appendix C. Proof of Theorem 1

The following inequalities ensure the proof of first part of Theorem 1:
\[ \theta_m^C - \theta_m^G = 0; \tau_m^C - \tau_m^G = 0; \pi_{mGr}^C - \pi_{mGr}^G = 0; \phi_m^C - \phi_m^G = (a-c_m)(xZ^2 + X^2\lambda) > 0; \]
Differentiating optimal decisions in Lemma 1, with respect to \( \lambda_1 \) and \( \lambda_2 \), the following relations are obtained:
\[ \frac{d\theta_m^C}{d\lambda_1} - \frac{d\theta_m^G}{d\lambda_1} = \frac{(a-c_m)(xZ^2 + X^2\lambda)}{\Delta_1^2} < 0; \frac{d\theta_m^C}{d\lambda_2} - \frac{d\theta_m^G}{d\lambda_2} = \frac{(a-c_m)(xZ^2\lambda)}{\Delta_2^2} < 0; \frac{d\phi_m^C}{d\lambda_1} - \frac{d\phi_m^G}{d\lambda_1} = \frac{(a-c_m)(x(2Z^2\lambda + \Delta_1))}{\Delta_1^2} < 0; \]
\[ \frac{d\phi_m^C}{d\lambda_2} - \frac{d\phi_m^G}{d\lambda_2} = \frac{(a-c_m)^2(2xZ^2\lambda^2)}{\Delta_2^2} < 0; \frac{d\phi_m^C}{d\lambda_1} - \frac{d\phi_m^G}{d\lambda_1} = \frac{(a-c_m)^2Z^2x^2\lambda^2}{\Delta_1^2} < 0; \frac{d\phi_m^C}{d\lambda_2} - \frac{d\phi_m^G}{d\lambda_2} = \frac{(a-c_m)^2(2xZ^2\lambda^2)}{\Delta_2^2} < 0; \]

The above inequalities supports the claim in Theorem 1.

Appendix D. Proof of Theorem 2

The following inequalities ensure the proof of first part of Theorem 2:
\[ \theta_r^R - \theta_m^R = \frac{(a-c_m)(xZ^2(4X^2 + 16N_1x)\lambda_1^2 + Z^2(4X^2\lambda + 2X^2\lambda_2(5M_1 + 7N_1\lambda_2)))}{\Delta_2\Delta_2} > 0; \]
\[ \tau_r^R - \tau_m^R = \frac{(a-c_m)(xZ^2(24M_1\lambda_1\lambda_2 - M_1\lambda_2^2 + M_1X^2\lambda_2)}{\Delta_2\Delta_2} > 0; \]
\[ \pi_{rGr}^R - \pi_{mGr}^R = \frac{(a-c_m)^2(2X^2(10x - 7X)^2\lambda_1^2 + (96x^2 - 40X^2\lambda + 8X^2\lambda_2)^2 - 23(13 - X^2)\lambda_1\lambda_2 + 2x^2X^2)}{\Delta_1^2} < 0. \]

Differentiating optimal decisions in Lemma 3,4, with respect to \( \lambda_1 \) and \( \lambda_2 \), the following relations are obtained:
\[ \frac{d\theta_r^R}{d\lambda_1} - \frac{d\theta_m^R}{d\lambda_1} = \frac{(a-c_m)(M_1\lambda_1 + \lambda_2)M_2XZ\lambda_2}{\Delta_2\Delta_2} < 0; \frac{d\theta_r^R}{d\lambda_2} - \frac{d\theta_m^R}{d\lambda_2} = \frac{(a-c_m)(xXZ^2(48x^2 + 28M_2\lambda_2 + Z^2))}{\Delta_2\Delta_2} < 0; \]
\[ \frac{d\phi_r^R}{d\lambda_1} - \frac{d\phi_m^R}{d\lambda_1} = \frac{-2(a-c_m)(M_1\lambda_1 + \lambda_2)M_2XZ\lambda_2}{\Delta_2\Delta_2} < 0; \frac{d\phi_r^R}{d\lambda_2} - \frac{d\phi_m^R}{d\lambda_2} = \frac{(a-c_m)(2XZ^2\lambda_2 + 28M_2\lambda_2 + Z^2)}{\Delta_2\Delta_2} < 0; \]
\[ \frac{d\pi_{rGr}^R}{d\lambda_1} - \frac{d\pi_{mGr}^R}{d\lambda_1} = \frac{-2(a-c_m)(M_1\lambda_1 + \lambda_2)M_2XZ\lambda_2}{\Delta_2\Delta_2} < 0; \frac{d\pi_{rGr}^R}{d\lambda_2} - \frac{d\pi_{mGr}^R}{d\lambda_2} = \frac{(a-c_m)(xXZ^2(48x^2 + 28M_2\lambda_2 + Z^2))}{\Delta_2\Delta_2} < 0; \]
\[ \frac{d\pi_{rGr}^R}{d\lambda_1} - \frac{d\pi_{mGr}^R}{d\lambda_1} = \frac{-(a-c_m)^2Z^2(3M_1\lambda_1 + \lambda_2)(M_1\lambda_1 + X^2\lambda_2)}{\Delta_2\Delta_2} < 0; \frac{d\pi_{rGr}^R}{d\lambda_2} - \frac{d\pi_{mGr}^R}{d\lambda_2} = \frac{-(a-c_m)^2Z^2(3M_1\lambda_1 + \lambda_2)(M_1\lambda_1 + X^2\lambda_2)}{\Delta_2\Delta_2} < 0. \]

The above inequalities supports the claim in Theorem 2.

Appendix E. Proof of Theorem 3

The following inequalities ensure the proof of first part of Theorem 3:
\[ \theta_m^T - \theta_m^G = \frac{(a-c_m)2xZ^2(4X^2 + 4xN_2x) + N_2^2\lambda_2^2)}{\Delta_3\Delta_3} > 0; \mu_m^T - \mu_m^G = \frac{N_2^2\lambda_2^2 + (86c_m - 2Z^2N_2)}{2(N_2 + \lambda_2)} > 0; \]
\[ \tau_m^T - \tau_m^G = \frac{(a-c_m)(xN_2Z^2\lambda_2 + 2xN_2(2xM_1 - X^2\lambda_2 + 2xN_2(7N_2 - X^2\lambda_2) + 3x^2\lambda_2))}{2\Delta_3\Delta_3} > 0; \]
\[ \pi_{mGr}^T - \pi_{mGr}^G = \frac{(a-c_m)2xZ^2(4X^2 + 44\lambda_2^2 + 2Z^2x(N_2 + 4\lambda) + 3x_2 - 2Z^2\lambda_2)}{3x_2} > 0. \]

Differentiating optimal decisions in Propositions 5 and 6, with respect to \( \lambda_1 \) and \( \lambda_2 \), the following relations are obtained:
\[ \frac{d\theta_m^T}{d\lambda_1} - \frac{d\theta_m^G}{d\lambda_1} = \frac{-2(a-c_m)N_2N_2XZ\lambda_2}{\Delta_3\Delta_3} < 0; \frac{d\theta_m^T}{d\lambda_2} - \frac{d\theta_m^G}{d\lambda_2} = \frac{(a-c_m)N_2N_2XZ\lambda_2}{\Delta_3\Delta_3} < 0; \]
\[ \frac{d\phi_m^T}{d\lambda_1} - \frac{d\phi_m^G}{d\lambda_1} = \frac{-2(a-c_m)^2N_2^2\lambda_2^2}{\Delta_3\Delta_3} < 0; \frac{d\phi_m^T}{d\lambda_2} - \frac{d\phi_m^G}{d\lambda_2} = \frac{(a-c_m)^2N_2^2\lambda_2^2}{\Delta_3\Delta_3} < 0; \]
\[ \frac{d\phi_m^T}{d\lambda_1} - \frac{d\phi_m^G}{d\lambda_1} = \frac{-2(a-c_m)(xN_2Z^2\lambda_2 + 2xN_2(2xM_1 - X^2\lambda_2 + 2xN_2(7N_2 - X^2\lambda_2) + 3x^2\lambda_2))}{2\Delta_3\Delta_3} < 0; \]
\[ \frac{d\phi_m^T}{d\lambda_2} - \frac{d\phi_m^G}{d\lambda_2} = \frac{(a-c_m)(xN_2Z^2\lambda_2 + 2xN_2(2xM_1 - X^2\lambda_2 + 2xN_2(7N_2 - X^2\lambda_2) + 3x^2\lambda_2))}{2\Delta_3\Delta_3} < 0. \]

The above inequalities supports the claim in Theorem 3.
Appendix H. Proof of Theorem 6

Differences among GLs under the MS game are found as follows:

\[ \theta^C_m - \theta^RE_m = \frac{6(a-cm)^2 x_l x_\lambda (x_\lambda + \Delta_1)}{2\Delta_1^2 \Delta_2^2} > 0; \quad \theta^C_m - \theta^T_m = \frac{36(a-cm)^2 x_l^2 x_\lambda^2}{2\Delta_1^2 \Delta_2^2} > 0. \]

Similarly, differences among GLs under the RS game are found as follows:

\[ \theta^C_r - \theta^RE_r = \frac{(a-cm)z(4x_\lambda^2 + \Delta_1)(x_\lambda + \Delta_1)}{2\Delta_1^2 \Delta_2^2} > 0; \quad \theta^C_r - \theta^T_r = \frac{(a-cm)^2 x_\lambda^2 (4x_\lambda^2 + \Delta_1)}{\Delta_1^2 \Delta_2^2} > 0. \]

Differences among sales volumes under the MS game are found as follows:

\[ Q^C_m - Q^RE_m = \frac{12(a-cm)^2 x_\lambda^2 (6x_\lambda + \Delta_1)}{2\Delta_1^2 \Delta_2^2} > 0; \quad Q^C_m - Q^T_m = \frac{12(a-cm)^2 x_\lambda^2 (6x_\lambda + \Delta_1)}{2\Delta_1^2 \Delta_2^2} > 0. \]

Differences among sales volumes under the RS game are found as follows:

\[ Q^C_r - Q^RE_r = \frac{(a-cm)^2 x_\lambda^2 (4x_\lambda^2 + \Delta_1)(2x_\lambda - \Delta_2)}{\Delta_1^2 \Delta_2} > 0; \quad Q^C_r - Q^T_r = \frac{(a-cm)^2 x_\lambda^2 (4x_\lambda^2 + \Delta_1)(2x_\lambda - \Delta_2)}{\Delta_1^2 \Delta_2} > 0. \]

The above relations ensure the claim in Theorem 4.

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