a review of Variational analysis and generalized differentiation. I: Basic theory. II: Applications by Mordukhovich, Boris S.

Mordukhovich, Boris S.

Variational analysis and generalized differentiation. I: Basic theory. II: Applications. (English) Zbl 1100.49002

Grundlehren der Mathematischen Wissenschaften 330/331. Berlin: Springer (ISBN 3-540-25437-4/hbk; 3-540-25438-2/hbk). xxii, 579 p./v.1, xxii, 610 p./v.2 (2006).

Optimization has been a major motivation and driving force for developing differential and integral calculus. Modern variational analysis can be viewed as an outgrowth of the calculus of variations and mathematical programming, where the focus is on optimization of functions relative to various constraints and on sensitivity/stability of optimization-related problems with respect to perturbations. One of the most characteristic features of modern variational analysis is the intrinsic presence of nonsmoothness, i.e., we must deal with nondifferentiable functions, sets with nonsmooth boundaries, and set-valued mappings. Indeed, many fundamental objects frequently encountered in the framework of variational analysis are inevitably of nonsmooth and/or set-valued structures requiring the development of new forms of analysis that involve generalized differentiation.

The principal objective in these two volumes is to present basic concepts and principles of variational analysis unified in finite-dimensional and infinite-dimensional settings, to develop a comprehensive generalized differential theory at the same level of perfection in both finite and infinite dimensions, and to provide valuable applications of variational theory to broad classes of problems in constrained optimization and equilibrium, sensitivity and stability analysis, control theory for ordinary, functional-differential and partial differential equations, and also to selected problems in mechanics and economics. The author develops generalized differentiation by using a geometric dual-space approach, which revolves around the extremal principle, namely, a local variational counterpart of the classical convex separation in nonconvex settings.

The book consists of eight chapters, being divided into two volumes.

Chapter 1 concerns the generalized differential theory in arbitrary Banach spaces in terms of geometric dual spaces. The chapter starts with normals to sets, then proceeds to coderivatives of set-valued mappings, and then to subdifferentials to extended-real-valued functions. Chapter 2 concerns a detailed study of the extremal principle in variational analysis, which is to be put down as a variational counterpart of the convex separation principle in nonconvex settings. The princple is presented in three forms, i.e., in terms of ε-normals, Fréchet normals and basic normals. Chapter 3 is the cornerstone of the book, concerning comprehensive calculus rules for basic normals, subgradients and coderivatives in the framework of Asplund spaces. Chapter 4 gives complete characterizations and efficient applications of fundamental properties in nonlinear studies related to Lipschitzian stability, metric regularity, and covering/openness at a linear rate.

The principal objective in chapter 5 is to derive necessary optimality and suboptimality conditions for various problems of constrained optimizations and equilibria in infinite-dimensional spaces. Chapter 6 derives necessary optimality conditions in a range of optimal control problems for evolution systems by using methods of variational analysis and generalized differentiation. It is concerned with dynamical systems governed by ordinary differential equations and inclusions in Banach spaces. Chapter 7 focuses on control systems with distributed parameters governed by functional-differential and partial differential relations. In particular, it studies optimal control problems for delayed differential-algebraic inclusions that cover several important classes of control systems essentially different from ordinary ones, and for partial differential equations of parabolic and hyperbolic types that involve boundary controls of both Dirichlet and Neumann types as well as pointwise state constraints.

The book is accompanies by 1379 references, a comprehensive list of statements and a glossary of notation. Key issues of variational analysis in finite-dimensional spaces have been addressed by *R. T. Rockafellar* and *R. J.-B. Wets* [Variational analysis. Grundlehren der Mathematischen Wissenschaften. 317. Berlin: Springer (1998; Zbl 0888.49001)].

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

49-02 Research exposition (monographs, survey articles) pertaining to calculus of variations and optimal control
49J52 Nonsmooth analysis
49J53 Set-valued and variational analysis
49K27 Optimality conditions for problems in abstract spaces
49K40 Sensitivity, stability, well-posedness
90C31 Sensitivity, stability, parametric optimization

Keywords:

generalized differentiation optimization; control problem; nonsmoothness; nondifferentiable functions; sets with nonsmooth boundaries; set-valued mappings

Software:

GradSamp

Full Text: DOI