4-10-2018

System Reliability in Special Steel and Concrete Bridge Systems: Identification of Redundancy Factor Modifiers

Dan M. Frangopol
Lehigh University

Benjin Zhu

Samantha Sabatino

System Reliability in Special Steel and Concrete Bridge Systems: Identification of Redundancy Factor Modifiers

By

Dr. Dan M. Frangopol
The Fazlur R. Khan Endowed Chair of Structural Engineering and Architecture,
Professor of Structural Engineering

Dr. Benjin Zhu
Former Graduate Student

Dr. Samantha Sabatino
Former Graduate Student

ATLSS Report No. 18-07

April 10, 2018

ATLSS is a National Center for Engineering Research
on Advanced Technology for Large Structural Systems

117 ATLSS Drive
Bethlehem, PA 18015-4729

Phone: (610)758-3525
Fax: (610)758-5902

www.atlss.lehigh.edu
Email: inatl@lehigh.edu

Page Intentionally Left Blank
Outside Cover

System Reliability in Special Steel and Concrete Bridge Systems: Identification of Redundancy Factor Modifiers

By

Dr. Dan M. Frangopol
The Fazlur R. Khan Endowed Chair of Structural Engineering and Architecture,
Professor of Structural Engineering

Dr. Benjin Zhu
Former Graduate Student

Dr. Samantha Sabatino
Former Graduate Student

ATLSS Report No. 18-07

April 10, 2018

ATLSS is a National Center for Engineering Research
on Advanced Technology for Large Structural Systems

117 ATLSS Drive
Bethlehem, PA 18015-4729

Phone: (610)758-3525
Fax: (610)758-5902

www.atlss.lehigh.edu
Email: inatl@lehigh.edu

Page Intentionally Left Blank
Inside Cover

ACKNOWLEDGEMENT OF SUPPORT AND DISCLAIMER

This material is based upon work supported by the Federal Highway Administration under Cooperative Agreement No. DTFH61-11-H- 00027. Advancing Steel and Concrete Bridge Technology to Improve Infrastructure Performance; Task 8: System Reliability in Special Steel and Concrete Bridge Systems: Identification of Redundancy Factor Modifiers

SI* (MODERN METRIC) CONVERSION FACTORS				
APPROXIMATE CONVERSIONS TO SI UNITS				
Symbol	**When You Know**	**Multiply By**	**To Find**	**Symbol**
		LENGTH		
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
		AREA		
in^2	square inches	645.2	square millimeters	mm^2
ft^2	square feet	0.093	square meters	m^2
yd^2	square yard	0.836	square meters	m^2
ac	acres	0.405	hectares	ha
mi^2	square miles	2.59	square kilometers	km^2
		VOLUME		
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft^3	cubic feet	0.028	cubic meters	m^3
yd^3	cubic yards	0.765	cubic meters	m^3
		NOTE: volumes greater than 1000 L shall be shown in m^3		
		MASS		
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")
		TEMPERATURE (exact degrees)		
°F	Fahrenheit	5 (F-32)/9 or (F-32)/1.8	Celsius	°C
		ILLUMINATION		
fc	foot-candles	10.76	lux	lx
fl	foot-Lamberts	3.426	candela/m^2	cd/m^2
		FORCE and PRESSURE or STRESS		
lbf	poundforce	4.45	newtons	N
lbf/in^2	poundforce per square inch	6.89	kilopascals	kPa
APPROXIMATE CONVERSIONS FROM SI UNITS				
Symbol	**When You Know**	**Multiply By**	**To Find**	**Symbol**
		LENGTH		
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
		AREA		
mm^2	square millimeters	0.0016	square inches	in^2
m^2	square meters	10.764	square feet	ft^2
m^2	square meters	1.195	square yards	yd^2
ha	hectares	2.47	acres	ac
km^2	square kilometers	0.386	square miles	mi^2
		VOLUME		
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m^3	cubic meters	35.314	cubic feet	ft^3
m^3	cubic meters	1.307	cubic yards	yd^3
		MASS		
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
		TEMPERATURE (exact degrees)		
°C	Celsius	1.8C+32	Fahrenheit	°F
		ILLUMINATION		
lx	lux	0.0929	foot-candles	fc
cd/m^2	candela/m^2	0.2919	foot-Lamberts	fl
		FORCE and PRESSURE or STRESS		
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	lbf/in^2

*SI is the symbol for the International System of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380.
(Revised March 2003)

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

The load and resistance factors in the AASHTO LRFD bridge design specifications were calibrated, in some specific instances, to achieve uniform levels of reliability. The resulting system reliability index is affected not only by the reliability of its components, but also by other parameters, such as correlation among resistances of components and system arrangement, among others. Therefore, it is necessary to study the reliability of systems whose components are designed to have a uniform reliability level.

This report investigates this problem by using idealized systems consisting of up to 500 equally reliable components (e.g., all components have the same reliability index β_c). In this case, idealized systems are schematic system reliability diagrams that consider a specific system arrangement, which dictates the position of components (i.e., series, parallel, or mixed configuration). The system arrangement describes the definition of system failure events. The effects of system arrangement, correlations among the resistances of components, number of components in a system, coefficients of variation of load and resistances, and the mean value of the load on the system reliability index are studied. Within this report, various correlation conditions are imposed on the component resistances in order to simulate realistic dependence among components and study the correlation effect on resulting system reliability and redundancy factors. Standard tables of system reliability index associated with different system arrangements and correlation cases are generated with respect to representative cases in which the coefficients of variation of resistance and load are set to commonly used values.

The results obtained from this report indicate that the design of components based on the same reliability level might cause low system reliability in series systems and high system reliability in

parallel systems. Perfect correlation among failure modes of components occurs when both their resistances and load effects are perfectly correlated; if either the resistances or load effects associated with the components are not perfectly correlated, the failure modes of components will not be perfectly correlated. Therefore, in order to achieve a safe and economic design, modifier factors which take into account the effects of system arrangement, correlation among the resistances of components, coefficients of variation, and probability distribution type of resistance and load on the system reliability need to be considered during component design. The modifier factor proposed in this report and the factor relating to redundancy in AASHTO LRFD bridge design specifications are actually of the same nature because both factors reflect the effect of system redundancy on the component design. Nevertheless, the redundancy-related load modifier in the current AASHTO LRFD specifications is determined based on a very general classification of redundancy levels, without considering the effects of the aforementioned parameters.

For this reason, this report proposes a new definition of redundancy factor to provide an improved quantification of system redundancy levels in component design. Examples are presented to illustrate this definition. By virtue of the proposed approach for system reliability analysis, the redundancy factors of N-component systems with up to N = 500 components are evaluated considering various system arrangements, correlations among the resistances of components, coefficients of variation of load and resistances, and mean values of the load. Strength limit states in which system redundancy is taken into account from both the load side and the resistance side are also investigated.

Since structural components used in practice have their own behavior, redundancy factors of ductile, brittle, and mixed (i.e., ductile-brittle) systems are investigated. In general, material behavior can be broadly classified into two categories: brittle and ductile. Ductile materials exhibit large strains and yielding before they lose all load-carry capacities, while brittle materials reach such a state without warning. The effect of post-failure behavior on the redundancy factor of components of parallel systems is captured via an illustrative example.

In order to provide guidance to bridge designers, an Appendix containing notations, definitions of relevant terms, and examples is provided. A commentary containing the procedures for applying redundancy factors in bridge design and examples of system modeling of bridges is also enclosed at the end of the report.

In summary, a new approach based on system engineering is proposed in this report to properly consider structural redundancy of bridges. The proposed approach adopts a "pure" reliability analysis and does not require extensive studies of structural responses and load-carrying capacities. The analyses are not limited to specific bridge types (girder, box, truss, cable stayed, etc.) but, rather, use idealized series or parallel systems with different numbers of components, various arrangements, and different post-failure behaviors (ductility or brittle failure) to represent failure modes of bridges. Due to the flexibility of system modeling, the proposed approach can use different system failure modes to accommodate different consequence-acceptance attitudes of stakeholders. It provides the missing link between member-level reliability and system level reliability. Using the proposed approach, this report provides appropriate redundancy factors in tabular forms for different bridge characteristics (series/parallel, brittle/ductile, and number of elements). The work in this report focuses on strength limit states. Future work is needed to identify the system failure modes for other limit states and different bridge types as well as ad hoc system arrangements for tailored client needs and risk attitudes.

CHAPTER 1. INTRODUCTION

The most important task in structural design is to maximize the safety of structures within economic constraints. This is achieved by making the difference between the designed resistance and the load effect as large as economically practical. A large safety margin needs to be provided to allow for abnormal situations, unexpected loads, misuse, degradation, and ineffective maintenance, among others.[6] Along with the increased understanding of structural behavior and loading processes, the field of structural design has evolved significantly in the past decades. Since approximately 1931, the sole design philosophy associated with the bridge design standards prescribed by the American Association of State Highway and Transportation Officials (AASHTO) was allowable stress design (ASD). In this conventional design, the structural safety is considered by using a single safety factor subjectively determined to account for uncertainties.

Beginning in early 1970s, a new design philosophy referred to as load factor design (LFD) was introduced. LFD recognized that the live load was more variable than the dead load and uncertainties in load prediction are considered through the load factors.[15] However, no probabilistic concept was involved in the calibration of the factors for loads and resistances in LFD. Around 1990s, with the development and application of probability-based reliability theory in civil engineering, the bridge design philosophy moved from LFD to the load and resistance factor design (LRFD). [2,3,30,31]

LRFD represents a more rational approach by which the uncertainties associated with resistance and load are incorporated quantitatively into the design process.[16,21,27,4] In the LRFD bridge design specifications,[1] load and resistance factors in the strength limit state are developed from the theory of reliability based on current statistical knowledge of the variability of loads and structural performance. In the calibration process, a target reliability index is first selected to provide a minimum acceptable safety margin and then the load and resistances factors are determined to achieve a uniform reliability level of components.(19) The target reliability index is currently provided only for the design of individual components of the bridge instead of the bridge system. Generally, the target reliability index of bridge components (e.g., bridge girders) is taken as 3.5.(19,20) Component reliability currently considered in AASHTO LRFD Bridge Design Specifications is associated with component failure, usually resulting from load effects that exceed the resistance, e.g. the failure of a bridge girder or a connection. The reliability of a bridge system is greater than the reliability of its components, except when the system is non-redundant (i.e. weakest-link, also called series system).

System reliability is an important performance indicator for structures because it reflects the overall level of structural safety.(5,9,10,18,24,25,28) However, with all the components having the same reliability index (also called "equally reliable"), the system reliability is affected not only by its components reliability, but also by several other parameters, such as correlation among resistances of components and system arrangement, among others. The correlation coefficient among resistances of components is a statistical measure of the degree to which changes to the resistance value of one component predict change to the resistance value of another. System arrangement (i.e., series, parallel, or series-parallel system) describes the definition of system failure. For example, a system can be modeled as a series system if the system failure is defined as failure of any component. Alternatively, a system can be modeled as a parallel system if the system failure is defined as the failure of all components. Therefore, system reliability is affected by the system arrangement, which is mainly controlled by the number of components and the alternative load paths.

Therefore, it is necessary to investigate the reliability of systems with equally reliable components. For some structures that have dozens or hundreds of members (i.e., truss bridge), as is often the case, it is time-consuming to calculate the system reliability to check if it satisfies the predefined reliability threshold in the preliminary design stage. In this context, generating standard tables of reliability index for various types of systems consisting of different numbers of component in different correlation cases when all components have the same reliability level will be helpful in facilitating the design process.

The fundamental concepts involved in the technical approach presented are introduced in Chapter 2; the reliability of systems with equally reliable components is evaluated and standard reliability index tables associated with different types of idealized systems are generated. The domain Ω, representing system failure is expressed in terms of component failure events as:

(a) for series system $\Omega \equiv \bigcup_{k=1}^{n} \left\{ g_k \left(\boldsymbol{X} \right) < 0 \right\}$

(b) for parallel system $\Omega \equiv \bigcap_{k=1}^{n} \left\{ g_k \left(\boldsymbol{X} \right) < 0 \right\}$

(c) for series-parallel system $\Omega \equiv \bigcup_{k=1}^{n} \bigcap_{j=1}^{c_n} \left\{ g_{k,j} \left(\boldsymbol{X} \right) < 0 \right\}$

where g is the performance function for a component, $\boldsymbol{X}$ is the set of random variables associated with g, and c_n is the number of components in the n-th cut set.(3) Several idealized system models are illustrated in Figure 1. The reliability results of systems with up to 100 components associated with different system arrangements, three correlation cases, and two distribution types of resistance and load are presented in the format shown in Table 1.

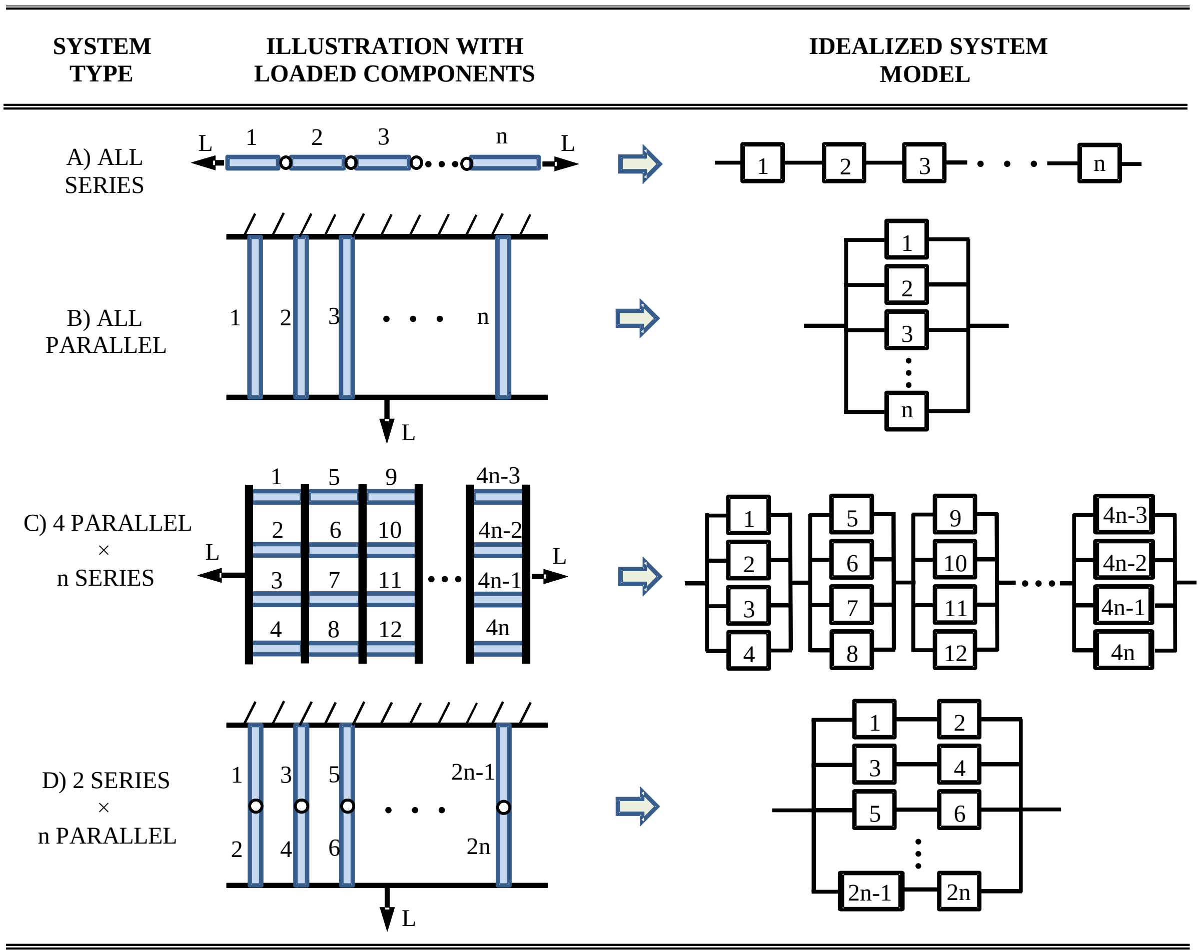


Figure 1. Graph. Idealized system models.

Research on bridge system reliability and redundancy has been extensively performed in the past decades.(11,12, 14,17,22,26,29,32) Redundancy is often separated into three different types: internal redundancy, structural redundancy, and load-path redundancy, described as follows:

(a) internal (member) redundancy describes multiple parallel elements within a member, such as a built-up member made from many different plates and other structural shapes that are bolted or riveted together. If one element were to fracture, the crack is expected to arrest and not propagate to the adjoining elements within the member;

(b) structural redundancy is based on static indeterminacy of the structure as a whole; often, continuous-span structures would have structural redundancy. Further, the system performance of the bridge also provides structural redundancy, such as the participation of secondary members, the deck, parapets, etc. Though similar to load-path redundancy in some ways, structural redundancy is not always as obvious as load-path redundancy;

(c) load-path redundancy is the simplest to identify because it is based on the number of primary load-carrying members in a span. Generally, to satisfy load-path redundancy constraints, there must be more than two primary load-carrying members.

Based on these studies, the AASHTO LRFD bridge design specifications include a factor relating to redundancy η_R on the load side in the strength limit state. Its value is determined as follows:

(a) $\eta_R \geq 1.05$ for nonredundant members;

(b) $\eta_R = 1.00$ for conventional level of redundancy; and

(c) $\eta_R \geq 0.95$ for exceptional levels of redundancy.

However, the AASHTO classification of redundancy levels is very general and subjective. In fact, the value of this factor relating to redundancy is affected by several parameters, such as system type, number of components in a structure, and correlations among the resistances of components, among others.(13,17) Past research efforts have been devoted to improve structural redundancy by calibrating η_R values based on different bridge configurations. For instance, NCHRP 406 provided a series of η_R values for girder bridges with different spans. Though such approaches provided somehow improved estimation of the safety of specific bridges, the application scope is severely limited within the bridges that were analyzed. Even for these bridges, assumptions in the analysis (e.g. live load models) can invalidate the adoption of certain η_R values in practice. For more general cases, nonlinear finite element modeling was usually used as the last resort for quantifying structural redundancy, which hinders the application of these approaches in bridge design.

Different from the previous studies (including NCHRP 406), the approach used in this report adopts a “pure” reliability analysis on the basis of component reliability without restricting the definition of such a component. As a result, this approach has the capability to represent different types of bridges including both bridge super- and substructures. Redundancy is analyzed by assembling series, parallel and series-parallel systems using such components. Since structures are directly represented by these system models, the redundancy factors proposed herein are supported by rigorous theoretical backbones.

The bridge system reliability modeling approach used in the report is the classical, also called pure, approach used in system analysis in the field of reliability engineering. The bridge components and their connectivity (series, parallel, and mixed, i.e. series-parallel) provide a representation of the real bridge system for the purposes of quantifying its system reliability and redundancy. Similar approaches have been extensively used for reliability analysis of other civil structures such as hydraulic structures, pipelines, and nuclear power plants. The work in this report aims to use this classical system reliability modeling approach to propose redundancy factors that can fit into the current practice of bridge design.

Table 1. Example standard table.

Number of components	System type	Normal distrib. N	Normal distrib. P	Normal distrib. F	Lognormal distrib. N	Lognormal distrib. P	Lognormal distrib. F
2-10	All Parallel						
10-50	All Series						
10-50	All Parallel						
10-50	5 Parallel ×*n* Series						
10-50	10 Parallel ×*n* Series						
10-50	5 Series ×*n* Parallel						
10-50	10 Series ×*n* Parallel						
50-100	All Series						
50-100	All Parallel						
50-100	5 Parallel ×*n* Series						
50-100	10 Parallel ×*n* Series						
50-100	20 Parallel ×*n* Series						
50-100	5 Series ×*n* Parallel						
50-100	10 Series ×*n* Parallel						
50-100	20 Series ×*n* Parallel						

Note: N = no correlation; P = partially correlated; F = fully correlated.

As mentioned in section 1.3.2.1 in AASHTO[(1)]: "improved quantification of ductility, redundancy, and operational classification may be attained with time, and possibly leading to a rearranging of Eq. 1.3.2.1-1, in which these effects may appear on either side of the equation or on both sides". Taking this into account, Chapter 3 proposes a redundancy factor η_R to account for the redundancy from the resistance side and considers a more detailed redundancy classification to provide an improved and rational basis for reliability-based design of structural components. This redundancy factor is defined as the ratio of the mean resistance of a component in a system when the system reliability index is prescribed (e.g., $\beta_{sys} = 3.5$), $E_{cs}(R)$, to the mean resistance of the same component when its reliability index is the same as that of the system (e.g., $\beta_c = 3.5$), $E_c(R)$. In addition to investigating the effects of the aforementioned parameters on the redundancy factor, the redundancy factors of N-component systems ($N \leq 100$) associated with different correlation cases and system arrangements are evaluated with respect to the representative case.

The materials used for structural components in practical cases have their own behavior. The materials behavior can be broadly classified into two categories: brittle and ductile. Ductile materials exhibit large strains and yielding before they fail. On the contrary, brittle materials fail suddenly and without warning. Due to the significant difference in the material's performance under load, it is necessary to take into account the material behavior of components in the evaluation of redundancy factor. Therefore, Chapter 3 investigates the redundancy factors of ductile, brittle, and mixed systems with no more than four components and studies the effects of post-failure behavior factor on the redundancy factor of parallel systems. Finally, Chapter 3 provides two types of limit states in which system redundancy is taken into account from the load and resistance side, respectively.

In summary, this report proposes a new definition of redundancy factor to provide an improved quantification of system redundancy levels in component design. Using idealized systems consisting of equally reliable components, the effects of system arrangement, correlations among the resistances of components, number of components in a system, coefficients of variation of load and resistances, and the mean value of the load on the system reliability index are studied. Standard tables of system reliability index associated with system arrangements, material behavior, and correlation cases are generated. Moreover, various correlation conditions are imposed on the component resistances in order to simulate realistic dependence among components and study the effect on resulting system reliability and redundancy factors. Overall, the analyses in this report are not directly related to bridge type. In practice, the design engineers would have to apply their judgment to decide what type of system their unique bridge should be considered.

CHAPTER 2. RELIABILITY OF SYSTEMS WITH EQUALLY RELIABLE COMPONENTS

In this chapter, firstly, the procedure for calculating the reliability of a system with equally reliable components is presented; then, a brief example is provided to illustrate the procedure; next, the effects of several parameters on the system reliability when the reliability indices of all components are designed to be 3.5 are studied by using idealized systems; finally, the system reliability indices of *N*-component systems associated with a representative case are evaluated with respect to different correlation cases. In general, it is necessary to investigate the reliability of systems with equally reliable components. In this context, generating standard tables of reliability index for various types of systems consisting of different numbers of component in different correlation cases when all components have the same reliability level will be helpful in facilitating the design process.

2.1 Procedure for Calculating the System Reliability with Equally Reliable Components

Consider a single component with random resistance R and under a random load P with given probability density functions. For the given mean value of load, $E(P)$, and the coefficients of variation of resistance and load, denoted as $V(R)$ and $V(P)$, respectively, the mean value of the component resistance, denoted as $E_c(R)$, can be determined (e.g., by using Monte Carlo Simulation (MCS) in MATLAB) to meet the predefined component reliability index level $\beta_c = 3.5$. If both R and P are normally or lognormally distributed, $E_c(R)$ can also be calculated by solving the equations in Figure 2 and Figure 3, respectively.

$$\beta_c = \frac{E_c(R) - E(P)}{\sqrt{\left(E_c(R) \cdot V(R)\right)^2 + \left(E(P) \cdot V(P)\right)^2}}$$

Figure 2. Equation. Component reliability index for normal distribution.

$$\beta_c = \frac{\ln\left[\dfrac{E_c(R)}{E(P)}\sqrt{\dfrac{1+V^2(P)}{1+V^2(R)}}\right]}{\sqrt{\ln\left[\left(1+V^2(R)\right)\left(1+V^2(P)\right)\right]}}$$

Figure 3. Equation. Component reliability index for lognormal distribution.

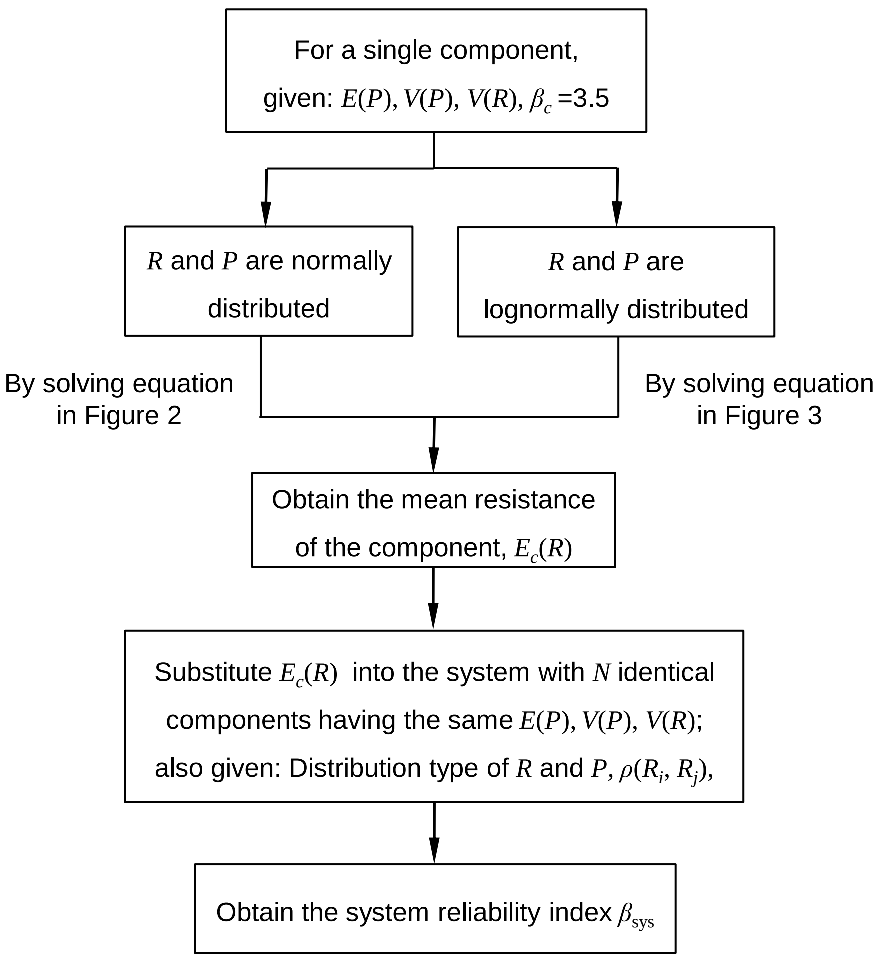


Figure 4. Graph. Procedure for determining the system reliability index.

For a system consisting of N equally reliable components whose geometries and material properties are the same as the single component just described, it is assumed that the load effect acting on each component is also P. Therefore, the component reliability in the system β_{cs} will be 3.5 if the mean resistances of components in the system $E_{cs}(R)$ are set to be $E_c(R)$. Therefore, all the components investigated in this study are equally reliable. Given the distribution type of R and P, the values of $E_{cs}(R) = E_c(R)$, $E(P)$, $V(R)$, $V(P)$, and the correlation coefficient between the resistances of components i and j, denoted as $\rho(R_i,R_j)$, the system reliability index β_{sys} can be calculated by using MATLAB.[23] The procedure for determining β_{sys} is described in the flowchart shown in Figure 4.

2.2 Example: A Three-Component Structure

An example is provided in this subsection to illustrate the procedure presented in in Figure 4. Assuming a three-component structure, three different systems can be formed: series, parallel, and series-parallel systems. The values of $E(P)$, $V(R)$, and $V(P)$ associated with the three components are arbitrarily assumed 10, 0.1, and 0.1, respectively. Three correlation cases between the resistances are considered:

(a) $\rho(R_i,R_j) = 0$, no correlation;

(b) $\rho(R_i,R_j) = 0.5$, partial correlation; and

(c) $\rho(R_i,R_j) = 1$, perfect correlation.

The aim of this section is to illustrate the procedure for determining the system reliability index; therefore, the assumed parameters are used. However, system reliability will change if different parameters are adopted. Two types of distributions are assumed for the resistances and loads of the components: normal and lognormal distribution. Based on the equations in Figure 2 and Figure 3 and the above parameters, the mean values of resistance associated with a single component for the normal and lognormal distribution when the component reliability index is 3.5 are found to be $E_{c,N}(R) = 16.861$ and $E_{c,LN}(R) = 16.384$, respectively.

By using the obtained $E_c(R)$ as the mean resistances of components in the system, the system reliability index β_{sys} associated with normal and lognormal distribution is calculated based on the program RELSYS, [8] as shown in Table 2 and Table 3, respectively. RELSYS (Reliability of Systems) is a FORTRAN 77 program which calculates the reliability of any structure that can be modeled as a combination of series and parallel systems. The program reduces the entire system to an equivalent single component and computes the system reliability. Details about this program can be found in Estes (1997) [8].

It is seen from the tables that in the no correlation and partial correlation cases, when the components reliability indices are 3.5:

(a) β_{sys} in the series and parallel system is less and larger than 3.5, respectively;

(b) β_{sys} in the series-parallel system is between those in the series and parallel systems; and

(c) β_{sys} associated with the lognormal distribution case is in general lower than that associated with the normal distribution.

For the perfect correlation case, β_{sys} has the same value as that of the component reliability index in all the systems.

Table 2. System reliability index of three-component systems when *R* and *P* follow normal distribution.

Correlation	Series system	Parallel system	SP system
$\rho(R_i,R_j) = 0$	3.205	5.478	3.507
$\rho(R_i,R_j) = 0.5$	3.222	4.460	3.494
$\rho(R_i,R_j) = 1$	3.500	3.500	3.500

Note: $E(P) = 10$; $V(P) = 0.1$; $V(R) = 0.1$; $\beta_c = 3.5$; $E_{c,N}(R) = 16.861$

Table 3. System reliability index of three-component systems when *R* and *P* follow lognormal distribution.

Correlation	Series system	Parallel system	Series-parallel system
$\rho(R_i,R_j) = 0$	3.201	4.761	3.491
$\rho(R_i,R_j) = 0.5$	3.234	4.187	3.474
$\rho(R_i,R_j) = 1$	3.500	3.500	3.500

Note: $E(P) = 10$; $V(P) = 0.1$; $V(R) = 0.1$; $\beta_c = 3.5$; $E_{c,LN}(R) = 16.384$

2.3 Effects of $V(R)$, $V(P)$, $E(P)$, $\rho(R_i,R_j)$ and N on the System Reliability

Different types of systems consisting of two, three, and four components when the reliability indices of all components are 3.5 are investigated to study the effects of $V(R)$, $V(P)$, $E(P)$, $\rho(R_i,R_j)$, and N on the system reliability. The distribution type of R and P of the components in these systems is assumed to be normal. The system reliability index as function of $V(R)$, $V(P)$, and $E(P)$ for two-component systems associated with two extreme correlation cases is plotted in Figure 5. It is found that:

(a) as $V(R)$ increases, the system reliability β_{sys} associated with the no correlation case ($\rho(R_i,R_j) = 0$) increases significantly in the parallel system while it decreases slightly in the series system;

(b) as $V(P)$ increases, β_{sys} associated with the no correlation case remains almost the same in the series system while it decreases significantly in the parallel system;

(c) β_{sys} is not affected by change in the mean values of the load in both systems associated with the no correlation case;

(d) in the perfect correlation case, β_{sys} in both systems is equal to 3.5 and is not affected by change in $V(R)$, $V(P)$, and $E(P)$.

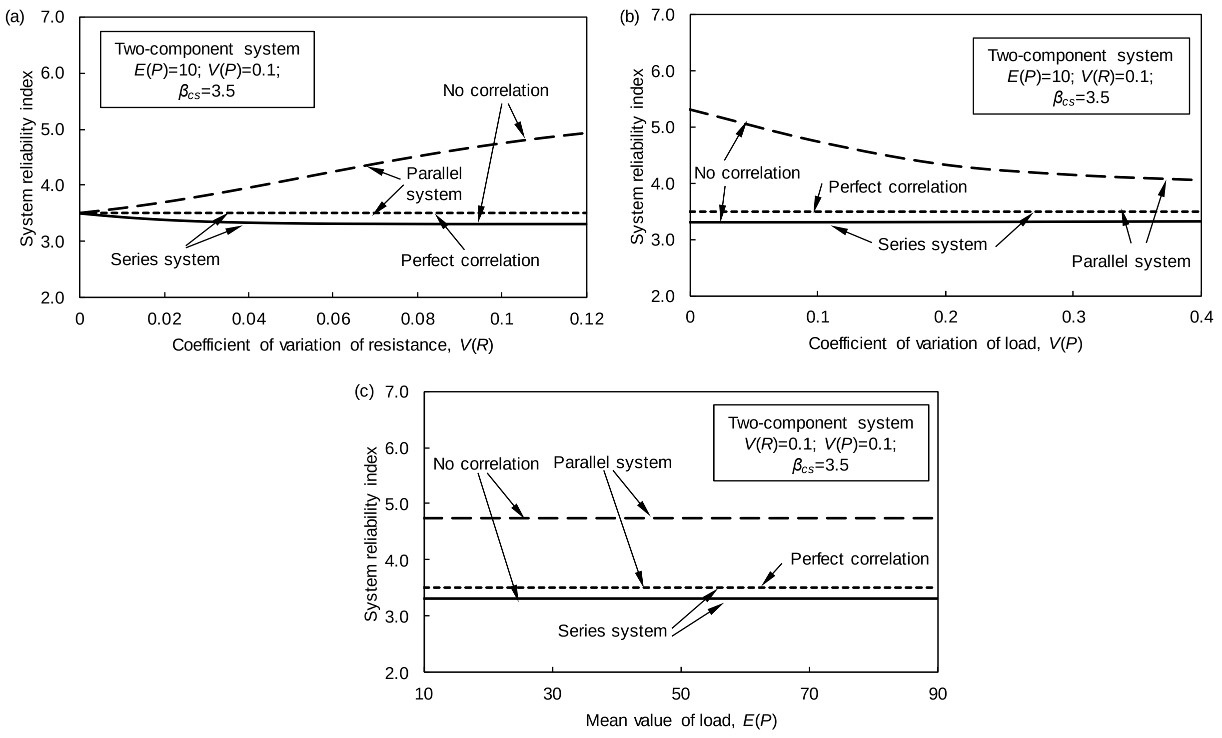


Figure 5. Graph. Effects of (a) *V*(*R*); (b) *V*(*P*); and (c) *E*(*P*) on system reliability for two-component systems associated with no correlation and perfect correlation among their resistances.

Note that the coefficient of variation of component resistance *V*(*R*) is usually less than the coefficient of variation of load effect *V*(*P*); therefore, the selected range of *V*(*R*) is smaller than *V*(*P*). Figure 5 is used to illustrate the effect of *V*(*R*), *V*(*P*) and *E*(*P*) on the system reliability. The increase / decrease of the range of these parameters will not affect the conclusions.

The effects of the considered parameters on β_{sys} in the three-component systems are shown in Figure 6. It is noted that:

(a) as *V*(*R*) increases in the no correlation case, β_{sys} shows an increasing and decreasing tendency in parallel and series systems, respectively; however, the rate of change in the parallel system is more significant than that in the series system;

(b) β_{sys} associated with the parallel system decreases significantly as *V*(*P*) increases; however, β_{sys} associated with the series system is almost not affected by *V*(*P*);

(c) for the series-parallel system, the effects of $V(R)$ and $V(P)$ on β_{sys} can almost be neglected in the no correlation case;

(d) β_{sys} does not change as the mean value of the load $E(P)$ varies in the three systems associated with the two correlation cases;

(e) $V(R)$ and $V(P)$ have no effect on β_{sys} for all systems in the perfect correlation case, as concluded from the two-component systems case;

(f) the effect of $V(R)$ and $V(P)$ on β_{sys} for all the three systems considered is more significant in the no correlation case than in the perfect correlation case.

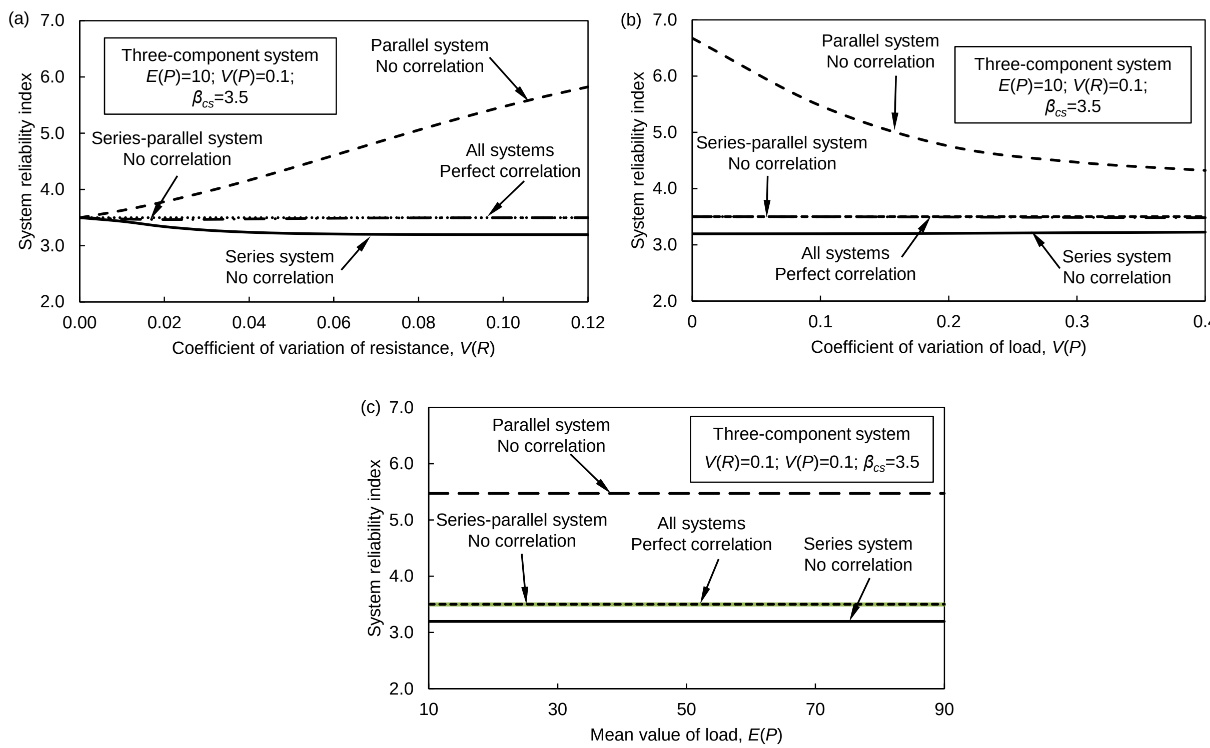


Figure 6. Graph. Effects of (a) *V*(*R*); (b) *V*(*P*); and (c) *E*(*P*) on system reliability for three-component systems associated with no correlation and perfect correlation among their resistances.

The effects of the aforementioned parameters on β_{sys} are also investigated for the four-component systems in which four different systems can be composed: series system (Figure 7(a)), parallel system (Figure 7(b)), series-parallel system A (Figure 7(c)) and series-parallel system B

(Figure 7(d)). An additional correlation case in which the correlation coefficients among the components resistances are 0.5 is studied.

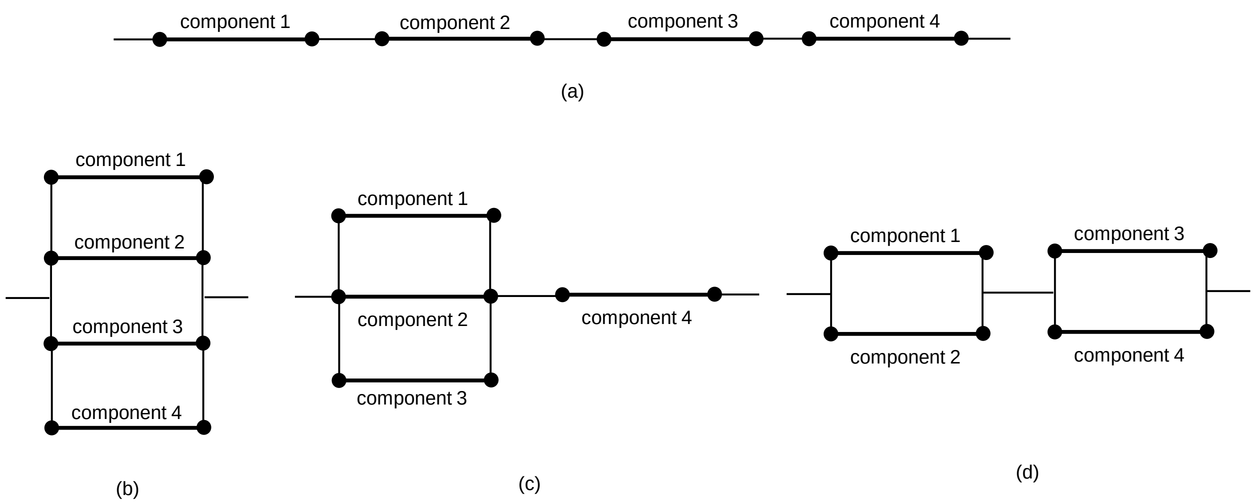


Figure 7. Graph. Four-component systems: (a) series system; (b) parallel system; (c) series-parallel system A; and (d) series-parallel system B.

The results associated with the effects of $V(R)$, $V(P)$, and $E(P)$ are presented in Figure 8, Figure 9, and Figure 10, respectively. In the no correlation and partial correlation cases, Figure 8 shows that:

(a) β_{sys} associated with both the parallel system and the series-parallel system B shows an increasing tendency as $V(R)$ increases; however, the rate of change in the parallel system is more significant than that in the series-parallel system B;

(b) β_{sys} associated with the series system decreases slowly with the increase of $V(R)$;

(c) the effect of $V(R)$ on β_{sys} in the series-parallel system A can almost be neglected. It is also noted that as the correlation among the resistances becomes stronger, the rate of change of β_{sys} due to the variation of $V(R)$ become less significant.

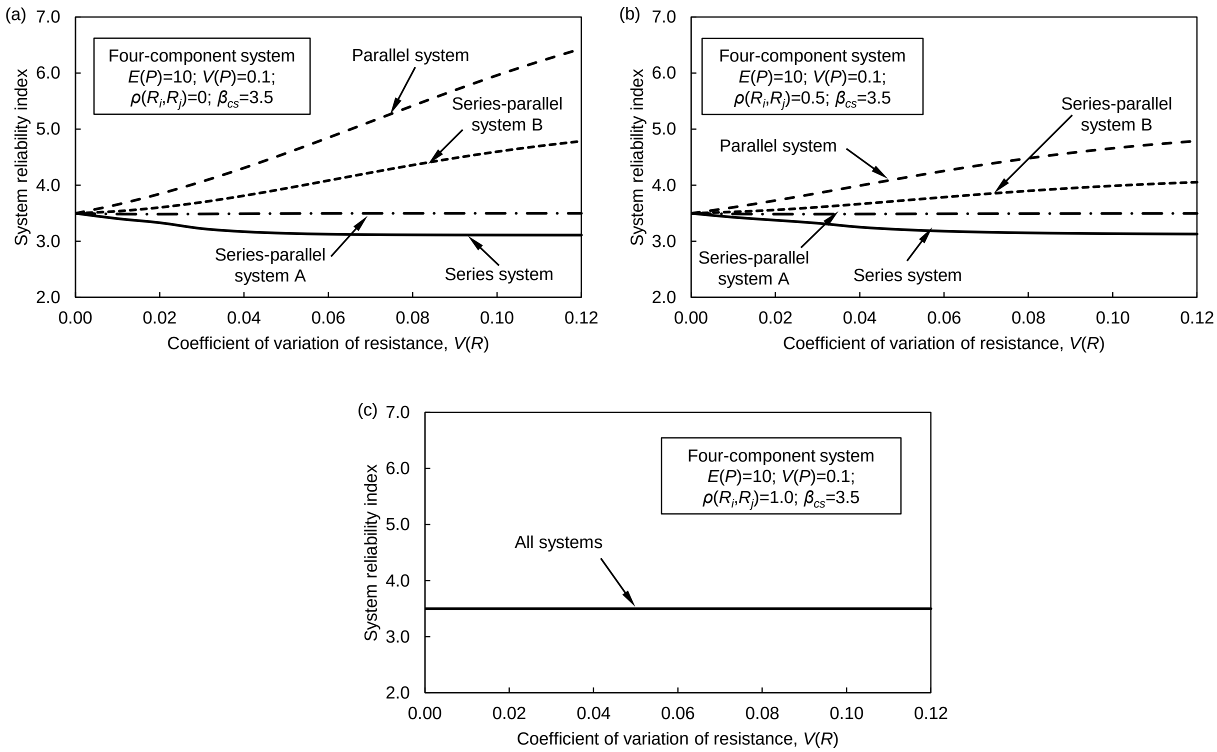


Figure 8. Graph. Effects of $V(R)$ on system reliability of four-component systems associated with the case of (a) no correlation; (b) partial correlation; and (c) perfect correlation among their resistances.

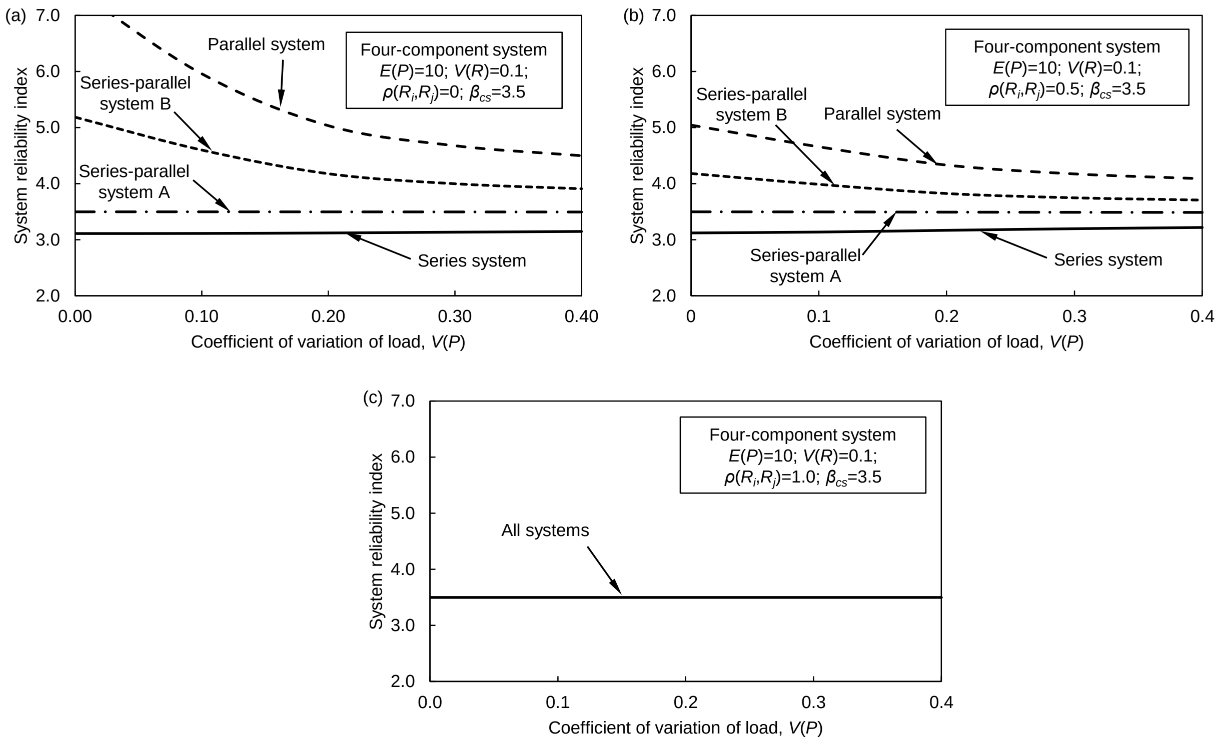


Figure 9. Graph. Effects of $V(P)$ on system reliability of four-component systems associated with the case of (a) no correlation; (b) partial correlation; and (c) perfect correlation among their resistances.

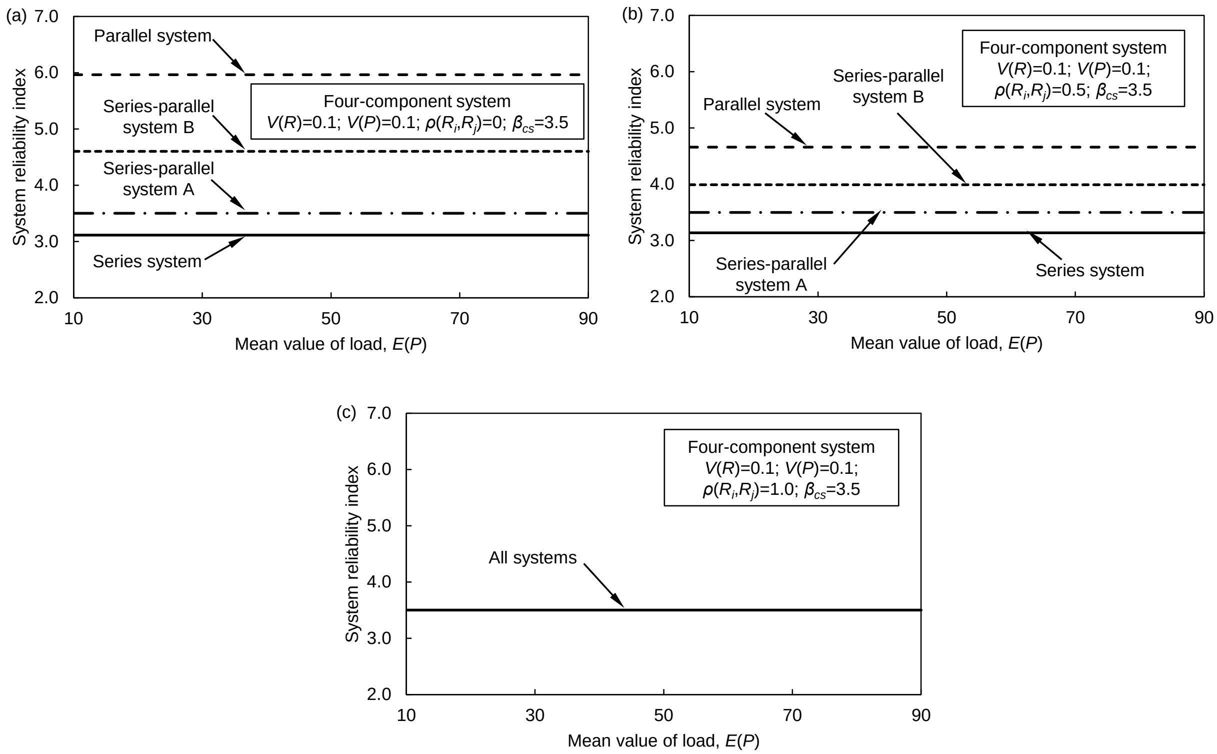


Figure 10. Graph. Effects of *E*(*P*) on system reliability of four-component systems associated with the case of (a) no correlation; (b) partial correlation; and (c) perfect correlation among their resistances.

In the no correlation and partial correlation cases, it is observed from Figure 9 that:

(a) increase of $V(P)$ reduces β_{sys} in the parallel system and series-parallel system B; and

(b) β_{sys} of the series system and series-parallel system A is almost not affected by change in $V(P)$.

In the perfect correlation case, β_{sys} of all the systems remain 3.5 independent of $V(P)$. The conclusion obtained from Figure 10 which shows the effect of $E(P)$ on the system reliability of the four-component system for different correlation cases is the same as that drawn from Figure 5 and Figure 6.

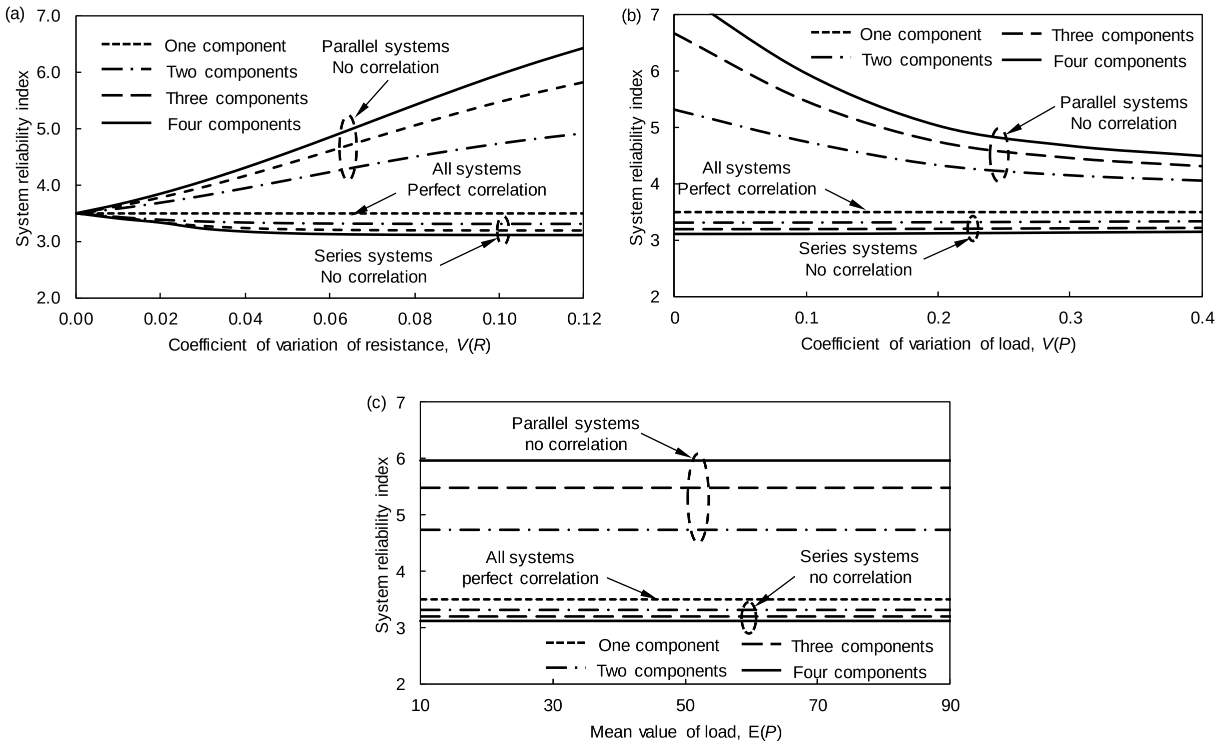


Figure 11. Graph. Effects of number of components on system reliability with the variations of (a) $V(R)$; (b) $V(P)$; and (c) $E(P)$ in the no correlation and perfect correlation cases.

Figure 11 shows the effects of number of components N on β_{sys} in different systems with the changes of $V(R)$, $V(P)$, and $E(P)$. As the number of components increases in the no correlation case, it is observed that β_{sys} in parallel system increases while its counterpart in series system decreases; while in the perfect correlation case, β_{sys} is not affected by N and remains 3.5 for all systems. It should be noted that $\rho(R_i,R_j)$ in this report refers to the correlation between the resistances of components i and j instead of the correlation between the failure modes of the components. Since the external load acting on the structural system is usually distributed to its components, the load effects associated with the components are correlated. Therefore, the failure modes of the components are usually correlated even in the case when the resistances of components are assumed to independent. In the cases in which the failure modes of components are perfectly correlated, β_{sys} is not affected by $V(R)$, $V(P)$, or $E(P)$.

2.4 Reliability of Systems with Many Equally Reliable Components

As mentioned previously, the AASHTO LRFD bridge design specifications were calibrated to achieve uniform reliability level in the component design. In practical cases, most structures have dozens or hundreds of members and it will be computationally expensive to calculate the system reliability index during the iterative design process. Therefore, it is necessary to investigate the reliability of these systems when their components reliability indices are all 3.5 so that standard system reliability tables can be generated to facilitate the design process.

Idealized systems with up to 100 components (N = 2, 3, 5, 10, 15, 20, 25, 50, 100) are analyzed in this subsection. As the number of components increases, the required computational time also increases dramatically. Therefore, a representative case in which $V(R)$ and $V(P)$ are assumed as commonly used values ($V(R) = 0.05$ and $V(P) = 0.3$) is investigated instead of studying various combinations of $V(R)$ and $V(P)$ [33,34,35,36].

Different series-parallel (SP) systems can be formed for an N-component structure, thus the following rule is used to denote these SP systems:

(a) if the subsystem of the series-parallel (SP) system is a parallel system consisting of m components and it is repeated n times in the system model, as shown in Figure 12, the series-parallel system is denoted as $mp \times ns$ SP system;

(b) if the subsystem of the series-parallel system is a series system consisting of m components and it is repeated n times in the system model, as shown in Figure 13, the series-parallel system is denoted as $ms \times np$ SP system. In this report, SP systems in which m equals to 5, 10 and 20 are investigated.

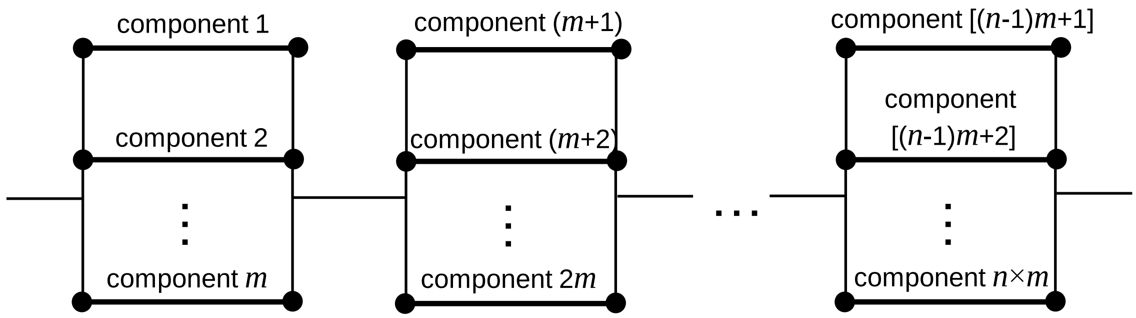


Figure 12. Graph. Schematic figure of a *mp* × *ns* series-parallel system (*n* series of *m* components in parallel).

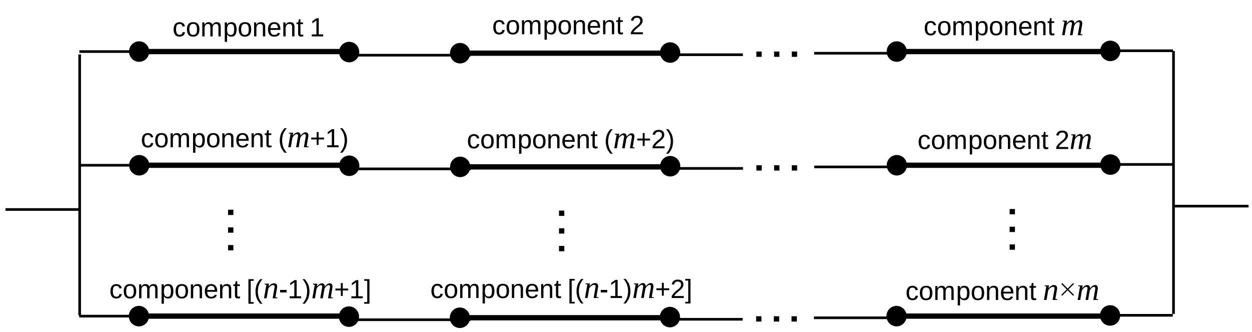


Figure 13. Graph. Schematic figure of a *ms* × *np* series-parallel system (*n* parallel of *m* components in series).

With all components reliability indices being 3.5, the system reliability indices are calculated with respect to the N-component systems (N = 2, 3, 5, 10, 15, 20, 25, 50, 100) associated with:

(a) different system types modeling (i.e., series, parallel, and series-parallel systems);

(b) three correlation cases among components resistances (i.e., $\rho(R_i,R_j)$ = 0, 0.5, and 1.0); and

(c) two distribution cases of resistance and load effects (i.e., normal and lognormal distribution).

For the representative case in which $V(R)$ = 0.05, $V(P)$ = 0.3, and $E(P)$ = 10, the mean resistance of a single component for normal and lognormal distribution when its reliability is 3.5 is found to be $E_{c,N}(R)$ = 21.132 and $E_{c,LN}(R)$ = 27.194, respectively.

Based on the given $V(R)$, $V(P)$, $E(P)$, $\rho(R_i,R_j)$, distribution type, and the obtained $E_c(R)$, the system reliability can be calculated using the following algorithm:

1. Give the mean value of the load effect $E(P)$, coefficients of variation of resistance and load effect $V(R)$ and $V(P)$, correlation between the resistances of components $\rho(R_i, R_j)$, distribution types of resistance and load, number of components N, number of simulation samples w; set the mean value of component resistance $E_{cs}(R)$ to be equal to $E_c(R)$;

2. Generate the random samples of resistance R_i and load effect P based on the above parameters; the dimensions of the R_i and P vectors are $w \times 1$;

3. Obtain the performance function for each component $g_i = R_i - P$ ($i = 1, 2, \ldots, N$); the dimensions of g_i is also $w \times 1$;

4. For series system, define an $w \times 1$ zero vector L, and the ratio of the number of $[L \mid (g_1 < 0) \mid \ldots \mid (g_N < 0)]$ to the total sample size w represents the failure probability of series system ("|" is logical OR in MATLAB; it refers to union); for the parallel system, define an $w \times 1$ unit vector Q, and the ratio of the number of $[Q \,\&\, (g_1 < 0) \,\&\, \ldots \,\&\, (g_N < 0)]$ to w is the P_f of parallel system ("&" is logical AND in MATLAB; it refers to intersection); for the $mp \times ns$ SP system, define an $w \times 1$ zero vector L and an $w \times 1$ unit vector Q, and the ratio of the number of $\{L \mid [Q \,\&\, (g_1 < 0) \,\&\, \ldots \,\&\, (g_m < 0)] \mid \ldots \mid [Q \,\&\, (g_{m\cdot(n-1)+1} < 0) \,\&\, \ldots \,\&\, (g_{m\cdot n} < 0)]\}$ to w is the P_f of the SP system; for the $ms \times np$ SP system, define an $w \times 1$ zero vector L and an $w \times 1$ unit vector Q, and the ratio of the number of $\{Q \,\&\, [L \mid (g_1 < 0) \mid \ldots \mid (g_m < 0)] \,\&\, \ldots \,\&\, [L \mid (g_{m\cdot(n-1)+1} < 0) \mid \ldots \mid (g_{m\cdot n} < 0)]\}$ to w is the P_f of the SP system; it should be noted that in the series-parallel systems, $n \times m$ is equal to the number of components N.

5. Repeat steps 1 to 4 for t times (e.g., $t = 50$) to obtain the average probability of failure of the system, then, convert it to the reliability index.

The results associated with normal and lognormal distribution are shown in Table 4 to Table 7. Note that a sufficient number of samples w is employed in order to capture relatively small failure probabilities (e.g., 10^{-6}). Figure 14 shows the effect of the number of components on the system reliability in series and parallel systems associated with different correlation and distribution cases. It is observed from this figure and Table 4 to Table 7 that:

(a) in the no correlation and partial correlation cases, the system reliability of the series and $mp \times ns$ SP systems that have the same number of parallel components (i.e., m is the same in these SP systems) shows a decreasing tendency as the number of components increases; however, the contrary is observed in the parallel and $ms \times np$ SP systems which have the same number of series components (i.e., m is the same);

(b) in the perfect correlation case, the system reliability is equal to 3.5 for different types of systems with different number of components; this was expected since for systems whose components are identical and their failure modes are perfect correlated, the system can be reduced to a single component; therefore, the system reliability in this correlation case does not change as the system type and number of components vary;

(c) for the series and parallel systems, the system reliability associated with the lognormal distribution case is larger and less than that associated with the normal distribution case, respectively;

(d) as the correlation among components resistances increases, the system reliability decreases in the parallel system while it increases in the series system.

Table 4. System reliability index of different systems associated with different correlation cases when *R* and *P* follow normal distribution; $1 \leq N \leq 20$.

System	$\rho(R_i,R_j) = 0$	$\rho(R_i,R_j) = 0.5$	$\rho(R_i,R_j) = 1.0$
1-component system	3.50	3.50	3.50
2-component system - Series system	3.368	3.393	3.50
2-component system - Parallel system	3.757	3.687	3.50
3-component system - Series system	3.293	3.338	3.50
3-component system - Parallel system	3.883	3.701	3.50
3-component system - Series-parallel system	3.478	3.466	3.50
4-component system - Series system	3.245	3.305	3.50
4-component system - Parallel system	3.968	3.802	3.50
5-component system - Series system	3.207	3.302	3.50
5-component system - Parallel system	4.019	3.815	3.50
6-component system - 2*p*×3*s* SP system	3.590	3.532	3.50
10-component system - Series system	3.097	3.196	3.50
10-component system - Parallel system	4.156	3.904	3.50
10-component system - 5*p*×2*s* SP system	3.928	3.765	3.50
10-component system - 5*s*×2*p* SP system	3.376	3.385	3.50
15-component system - Series system	3.036	3.152	3.50
15-component system - Parallel system	4.248	4.028	3.50
15-component system - 5*p*×3*s* SP system	3.867	3.716	3.50
15-component system - 5*s*×3*p* SP system	3.455	3.432	3.50
20-component syst - Series system	2.996	3.122	3.50
20-component syst - Parallel system	4.298	4.043	3.50
20-component syst - 5*p*×4*s* SP system	3.845	3.702	3.50
20-component syst - 10*p*×2*s* SP system	4.100	3.871	3.50
20-component syst - 5*s*×4*p* SP system	3.502	3.463	3.50
20-component syst - 10*s*×2*p* SP system	3.244	3.286	3.50

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$

Table 5. System reliability index of different systems associated with different correlation cases when *R* and *P* follow normal distribution; $25 \le N \le 100$.

System	$\rho(R_i,R_j) = 0$	$\rho(R_i,R_j) = 0.5$	$\rho(R_i,R_j) = 1.0$
25-component system - Series system	2.967	3.102	3.50
25-component system - Parallel system	4.339	4.050	3.50
25-component system - 5*p*×5*s* SP system	3.811	3.679	3.50
25-component system - 5*s*×5*p* SP system	3.529	3.488	3.50
50-component system - Series system	2.877	3.035	3.50
50-component system - Parallel system	4.456	4.121	3.50
50-component system -5*p*×10*s* SP system	3.755	3.632	3.50
50-component system -10*p*×5*s* SP system	3.987	3.809	3.50
50-component system -5*s*×10*p* SP system	3.620	3.549	3.50
50-component system -10*s*×5*p* SP system	3.372	3.375	3.50
100-component system - Series system	2.793	2.977	3.50
100-component system - Parallel system	4.553	4.184	3.50
100-component system -5*p*×10*s* SP system	3.691	3.590	3.50
100-component system -10*p*×10*s* SP system	3.933	3.763	3.50
100-component system -20*p*×5*s* SP system	4.143	3.903	3.50
100-component system -5*s*×20*p* SP system	3.689	3.592	3.50
100-component system -10*s*×10*p* SP system	3.448	3.422	3.50
100-component system -20*s*×5*p* SP system	3.239	3.279	3.50

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$

Table 6. System reliability index of different systems associated with different correlation cases when *R* and *P* follow lognormal distribution; $1 \leq N \leq 20$.

System	$\rho(R_i,R_j) = 0$	$\rho(R_i,R_j) = 0.5$	$\rho(R_i,R_j) = 1.0$
1-component system	3.50	3.50	3.50
2-component system - Series system	3.419	3.444	3.50
2-component system - Parallel system	3.607	3.572	3.50
3-component system - Series system	3.382	3.410	3.50
3-component system - Parallel system	3.668	3.613	3.50
3-component system - Series-parallel system	3.494	3.482	3.50
5-component system - Series system	3.328	3.378	3.50
5-component system - Parallel system	3.728	3.655	3.50
10-component system - Series system	3.273	3.331	3.50
10-component system - Parallel system	3.800	3.696	3.50
10-component system - 5*p*×2*s* SP system	3.673	3.609	3.50
10-component system - 5*s*×2*p* SP system	3.405	3.424	3.50
15-component system - Series system	3.241	3.312	3.50
15-component system - Parallel system	3.823	3.729	3.50
15-component system - 5*p*×3*s* SP system	3.643	3.594	3.50
15-component system - 5*s*×3*p* SP system	3.436	3.444	3.50
20-component system - Series system	3.216	3.295	3.50
20-component system - Parallel system	3.854	3.739	3.50
20-component system - 5*p*×4*s* SP system	3.627	3.590	3.50
20-component system - 10*p*×2*s* SP system	3.743	3.666	3.50
20-component system - 5*s*×4*p* SP system	3.459	3.464	3.50
20-component system - 10*s*×2*p* SP system	3.335	3.376	3.50

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$

Table 7. System reliability index of different systems associated with different correlation cases when *R* and *P* follow lognormal distribution; 25 ≤ *N* ≤ 100.

System	$\rho(R_i,R_j) = 0$	$\rho(R_i,R_j) = 0.5$	$\rho(R_i,R_j) = 1.0$
25-component system - Series system	3.204	3.281	3.50
25-component system - Parallel system	3.871	3.755	3.50
25-component system - 5*p*×5*s* SP system	3.611	3.570	3.50
25-component system - 5*s*×5*p* SP system	3.471	3.469	3.50
50-component system - Series system	3.158	3.251	3.50
50-component system - Parallel system	3.927	3.788	3.50
50-component system - 5*p*×10*s* SP system	3.576	3.548	3.50
50-component system - 10*p*×5*s* SP system	3.695	3.624	3.50
50-component system - 5*s*×10*p* SP system	3.513	3.500	3.50
50-component system - 10*s*×5*p* SP system	3.393	3.420	3.50
100-component system - Series system	3.116	3.219	3.50
100-component system - Parallel system	3.971	3.819	3.50
100-component system - 5*p*×10*s* SP system	3.547	3.525	3.50
100-component system - 10*p*×10*s* SP system	3.668	3.610	3.50
100-component system - 20*p*×5*s* SP system	3.761	3.676	3.50
100-component system - 5*s*×20*p* SP system	3.553	3.523	3.50
100-component system - 10*s*×10*p* SP system	3.425	3.440	3.50
100-component system - 20*s*×5*p* SP system	3.326	3.369	3.50

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$

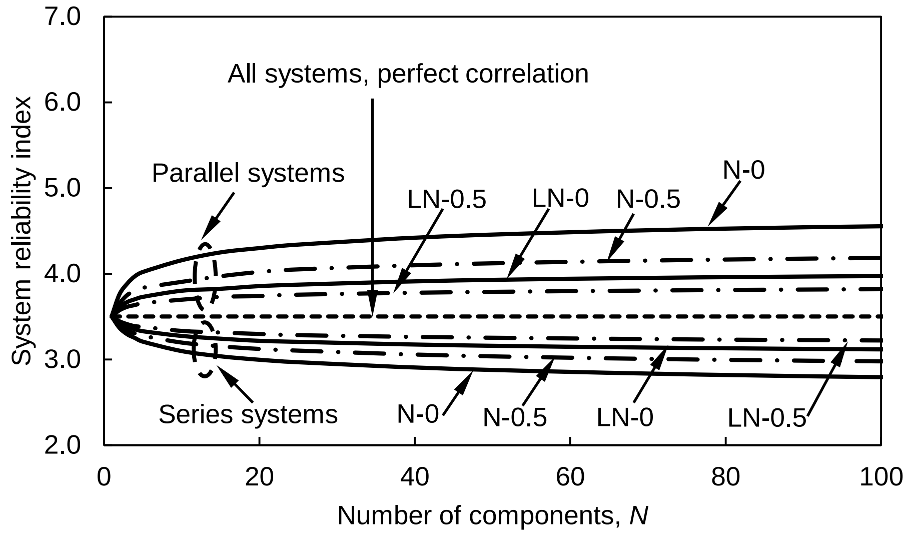


Figure 14. Graph. Effect of number of components on system reliability associated with normal and lognormal distributions (Note: "N" denotes normal distribution; "LN" denotes lognormal distribution; "0" denotes $\rho(R_i,R_j) = 0$; and "0.5" denotes $\rho(R_i,R_j) = 0.5$).

CHAPTER 3. SYSTEM RELIABILITY-BASED REDUNDANCY FACTOR FOR COMPONENT DESIGN

In this chapter, Section 3.1 introduces the definition of the proposed redundancy factor; Section 3.2 provides two examples to illustrate this definition; Sections 3.3 and 3.4 investigate the effects of several parameters on the redundancy factor using idealized systems; Section 3.5 evaluates the redundancy factors of *N*-component systems associated with a representative case; Section 3.6 studies the redundancy factors of ductile, brittle, and mixed systems with no more than four components and also investigates the effect of post-failure behavior factor on the redundancy factor of parallel systems; finally, Section 3.7 provides two types of limit states in which system redundancy is taken into account from the load and resistance side, respectively.

3.1 Definition of the Redundancy Factor

Consider a single component with resistance *R* and load *P* which are random variables. Given the mean value of load $E(P)$, the coefficients of variation of resistance and load $V(R)$ and $V(P)$, and the predefined component reliability index $\beta_c = 3.5$, the mean value of the component resistance $E_c(R)$ can be determined (e.g., by using Monte Carlo Simulation in MATLAB). For two particular cases in which both *R* and *P* of the component are normally or lognormally distributed, $E_c(R)$ can be directly calculated by solving the equations in Figure 2 and Figure 3, respectively. The $E_c(R)$ obtained will be used as the reference value to be compared with the mean value of component resistance in a system to yield the redundancy factor.

For a system consisting of *N* equally reliable components whose geometries and material properties are the same as the single component just described, different types of systems can be formed: series, parallel and series-parallel systems.[7,17] Given the distribution type of *R* and *P*, the values of $E(P)$, $V(R)$, $V(P)$, $\rho(R_i,R_j)$, and the target system reliability index β_{sys} that is assumed to be 3.5, the mean value of component resistance in the system $E_{cs}(R)$ can be calculated by using MATLAB.[23] After obtaining the mean resistance of a component in a system when the system reliability index is 3.5, $E_{cs}(R)$, and the mean resistance of the same component when the component reliability is 3.5, $E_c(R)$, the redundancy factor, denoted as η_R, which is defined as the ratio of $E_{cs}(R)$ to $E_c(R)$, can be determined. The procedure for determining η_R is described in the flowchart shown in Figure 15.

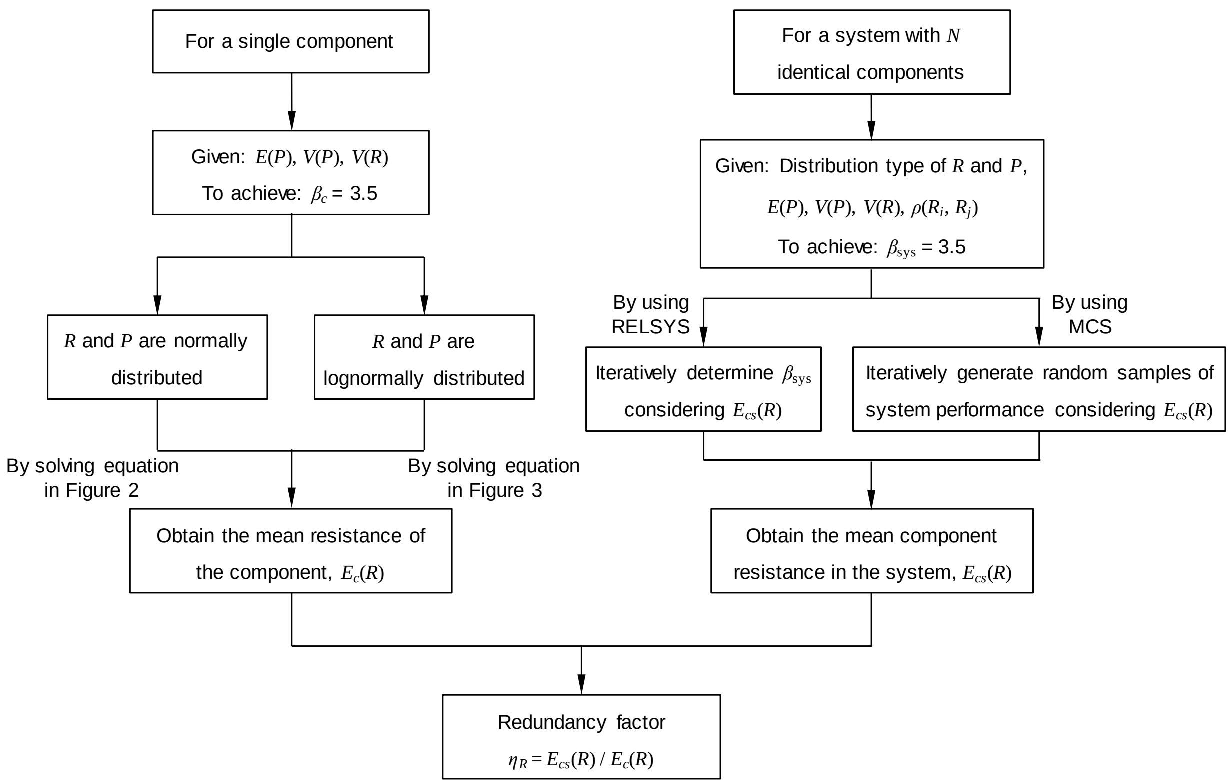


Figure 15. Graph. Flowchart of the procedure for determining the redundancy factor.

It should be noted that $\rho(R_i,R_j)$ refers to the correlation between the resistances of components i and j instead of the correlation between the failure modes of the two components. Since the external load acting on the system leads to load effects in all the components, the load effects are correlated. Therefore, the failure modes of the components are correlated even when $\rho(R_i,R_j) = 0$.

3.2 Examples

Two examples are provided in this subsection to illustrate the above concepts. In the first example, the number of the investigated equally reliable components is three; two different systems can be formed: series and parallel, as shown in Figure 16. Normal distribution is assumed for the resistances and loads of the components. The values of $E(P)$, $V(R)$, and $V(P)$ associated with the three components are assumed as 10, 0.1, and 0.1, respectively. Three correlation cases among the resistances of components are considered:

(a) $\rho(R_i,R_j) = 0$, no correlation;

(b) $\rho(R_i,R_j) = 0.5$, partial correlation; and

(c) $\rho(R_i,R_j) = 1$, perfect correlation.

Based on the equations in Figure 2 and Figure 3 and the parameters mentioned previously, the mean values of resistance associated with a single component for the normal and lognormal distribution are found to be $E_{c,N}(R) = 16.861$ and $E_{c,LN}(R) = 16.384$, respectively.

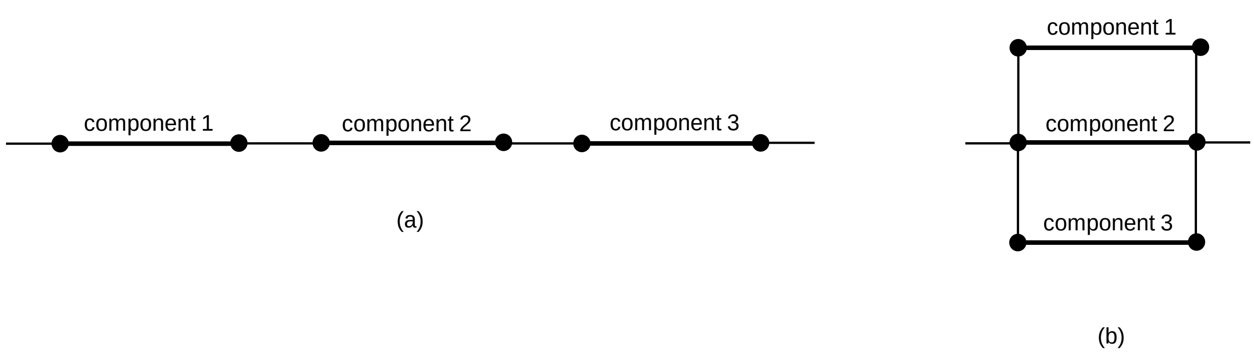


Figure 16. Graph. Three-component systems: (a) series system; and (b) parallel system.

Assuming the target system reliability index $\beta_{sys} = 3.5$, the mean values of component resistance $E_{cs}(R)$ corresponding to the two systems associated with the normal distribution case are calculated by combining RELSYS[8] with MATLAB[23]. The redundancy factors η_R and the corresponding components reliability indices β_{cs} are also obtained, as presented in Table 8.

Table 8. $E_{cs}(R)$, η_R and β_{cs} of three-component systems when *R* and *P* follow normal distribution.

Correlation	Series system $E_{cs}(R)$; η_R; β_{cs}	Parallel system $E_{cs}(R)$; η_R; β_{cs}
$\rho(R_i,R_j) = 0$	17.685; 1.049; 3.78	13.684; 0.812; 2.17
$\rho(R_i,R_j) = 0.5$	17.651; 1.047; 3.77	14.817; 0.879; 2.69
$\rho(R_i,R_j) = 1$	16.861; 1.000; 3.50	16.861; 1.000; 3.50

Note: $E(P) = 10$; $V(P) = 0.1$; $V(R) = 0.1$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 16.861$

Another type of distribution usually followed by resistance and load effect is lognormal distribution. Therefore, an additional case in which the resistance and load effect of components have lognormal distributions while $E(P)$, $V(R)$, and $V(P)$ remain 10, 0.1, and 0.1, respectively, is studied. By performing the same procedure, the mean values of component resistance $E_{cs}(R)$, the redundancy factors η_R8o, and the components reliability indices β_{cs} associated with the lognormal case are shown in Table 9.

Table 9. $E_{cs}(R)$, η_R and β_{cs} of three-component systems when *R* and *P* follow lognormal distribution.

Correlation	Series system $E_{cs}(R)$; η_R; β_{cs}	Parallel system $E_{cs}(R)$; η_R; β_{cs}
$\rho(R_i,R_j) = 0$	17.045; 1.040; 3.78	14.092; 0.860; 2.43
$\rho(R_i,R_j) = 0.5$	16.985; 1.037; 3.76	14.969; 0.914; 2.86
$\rho(R_i,R_j) = 1$	16.384; 1.000; 3.50	16.384; 1.000; 3.50

Note: $E(P) = 10$; $V(P) = 0.1$; $V(R) = 0.1$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,LN}(R) = 16.384$

It is noted from Table 8 and Table 9 that in the no correlation and partial correlation cases:

(a) the redundancy factors η_R associated with series system are greater than 1.0; this indicates that the mean resistance required for each component in a series system is larger than that needed for a single component; therefore, the component reliability indices β_{cs} in the two correlation cases are larger than 3.5;

(b) in the parallel system, the obtained conclusion is contrary to that of the series system.

By comparing the results in Table 8 and Table 9, it is noticed that in the no correlation and partial correlation cases, the difference in η_R between normal and lognormal distributions is less than 6%.

In the second example, the number of the investigated components is extended to four and three different systems are studied: series system, parallel system, series-parallel system, as shown in Figure 17. The mean value of the resistance associated with a single component $E_c(R)$ when its reliability index is 3.5 is still 16.861 and 16.384 for the normal and lognormal distribution case, respectively, as obtained in the previous three-component example. The value of $E_c(R)$ is independent of the system type, correlation among the resistances of components, and number of components in a system because it is only related to the parameters associated with a single component.

Based on the same assumptions for the values of $E(P)$, $V(R)$ and $V(P)$ used in the previous three-component example, the mean values of component resistance, the redundancy factors, and the component reliability indices of the four-component systems associated with normal and lognormal distribution are provided in Table 10 and Table 11. It is observed that the values of η_R associated with the partial correlation case are between the redundancy factors associated with the two extreme correlation cases.

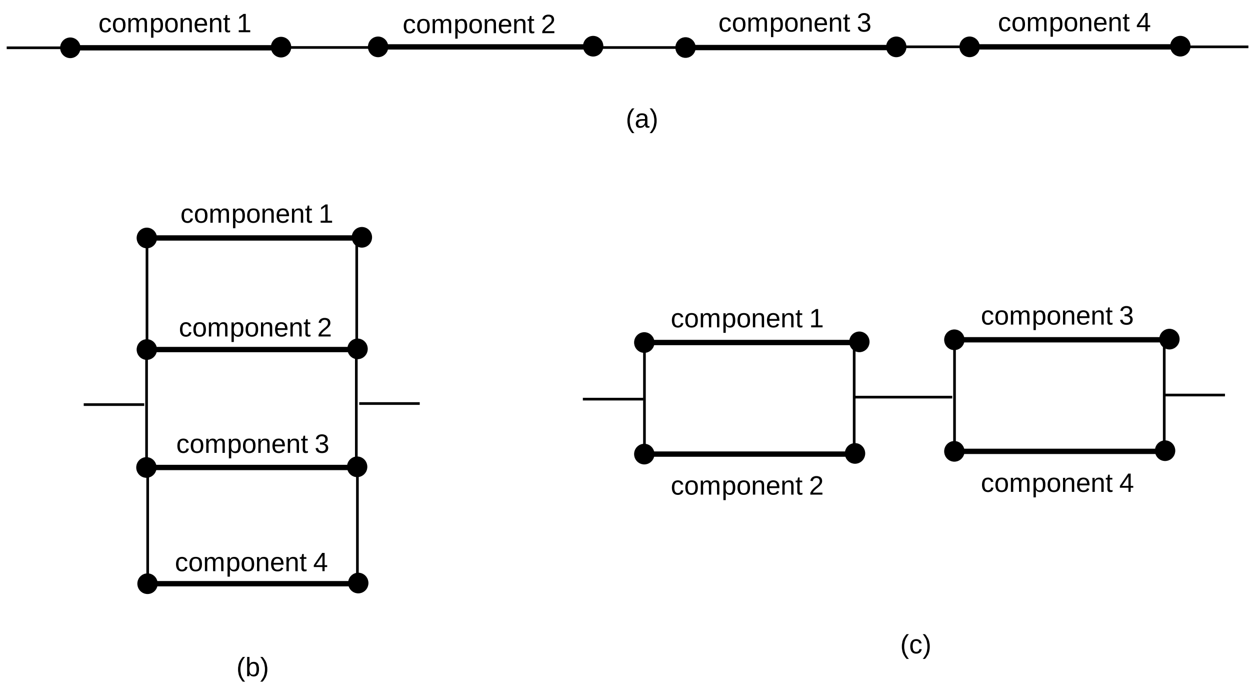


Figure 17. Graph. Four-component systems: (a) series system; (b) parallel system; and (c) series-parallel system.

Table 10. $E_{cs}(R)$, η_R and β_{cs} of four-component systems when *R* and *P* follow normal distribution.

Correlation	Series system $E_{cs}(R)$; η_R; β_{cs}	Parallel system $E_{cs}(R)$; η_R; β_{cs}	Series-parallel system $E_{cs}(R)$; η_R; β_{cs}
$\rho(R_i,R_j) = 0$	17.902; 1.062; 3.85	13.239; 0.785; 1.95	14.811; 0.878; 2.69
$\rho(R_i,R_j) = 0.5$	17.860; 1.059; 3.84	14.478; 0.859; 2.54	15.732; 0.933; 3.07
$\rho(R_i,R_j) = 1.0$	16.861; 1.000; 3.50	16. 861; 1.000; 3.50	16. 861; 1.000; 3.50

Note: $E(P) = 10$; $V(P) = 0.1$; $V(R) = 0.1$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 16.861$

When comparing the results of Table 8 to Table 11, it is found that as the number of components increases from three to four, η_R and the component reliability index β_{cs} associated with the series system increase while their counterparts associated with the parallel system decrease. Details about the effects of the number of components and other parameters on the redundancy factors are discussed in the next section.

Table 11. $E_{cs}(R)$, η_R **and** β_{cs} **of four-component systems when** R **and** P **follow lognormal distribution.**

Correlation	Series system $E_{cs}(R)$; η_R; β_{cs}	Parallel system $E_{cs}(R)$; η_R; β_{cs}	Series-parallel system $E_{cs}(R)$; η_R; β_{cs}
$\rho(R_i,R_j) = 0$	17.215; 1.051; 3.85	13.688; 0.835; 2.23	15.083; 0.921; 2.91
$\rho(R_i,R_j) = 0.5$	17.136; 1.046; 3.82	14.695; 0.897; 2.73	15.694; 0.958; 3.19
$\rho(R_i,R_j) = 1.0$	16.384; 1.000; 3.50	16.384; 1.000; 3.50	16.384; 1.000; 3.50

Note: $E(P) = 10$; $V(P) = 0.1$; $V(R) = 0.1$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,LN}(R) = 16.384$

3.3 Effects of $V(R)$, $V(P)$, $E(P)$ and N on the Redundancy Factor η_R

In the two examples presented previously, it was shown that the mean value of the component resistance $E_{cs}(R)$ is affected by the coefficient of variation of resistance $V(R)$, the coefficient of variation of load $V(P)$, the mean value of load $E(P)$, and correlation among the resistances of components $\rho(R_i,R_j)$. In addition to these parameters, the number of components N in a system has an impact on $E_{cs}(R)$. Therefore, different types of systems consisting of two, three, and four components are investigated to study the effects of $V(R)$, $V(P)$, $E(P)$, $\rho(R_i,R_j)$, and N on the redundancy factor η_R of components in these systems. The distribution type of R and P of the components in these systems is assumed to be normal, and $\beta_c = \beta_{sys} = 3.5$.

The effects of $V(R)$, $V(P)$, and $E(P)$ on the redundancy factor η_R in two-component systems associated with two extreme correlation cases are plotted in Figure 18. It is noted from this figure that:

(a) as $V(R)$ increases, η_R associated with the no correlation case increases in the series system while it decreases significantly in the parallel system;

(b) as $V(P)$ increases, η_R associated with the no correlation increase in both systems but more significantly in the parallel system;

(c) η_R is not affected by changes in the mean values of the load in both systems associated with the no correlation case;

(d) in the perfect correlation case, η_R in both systems are equal to 1.0 and it is not affected by changes of $V(R)$, $V(P)$, and/or $E(P)$.

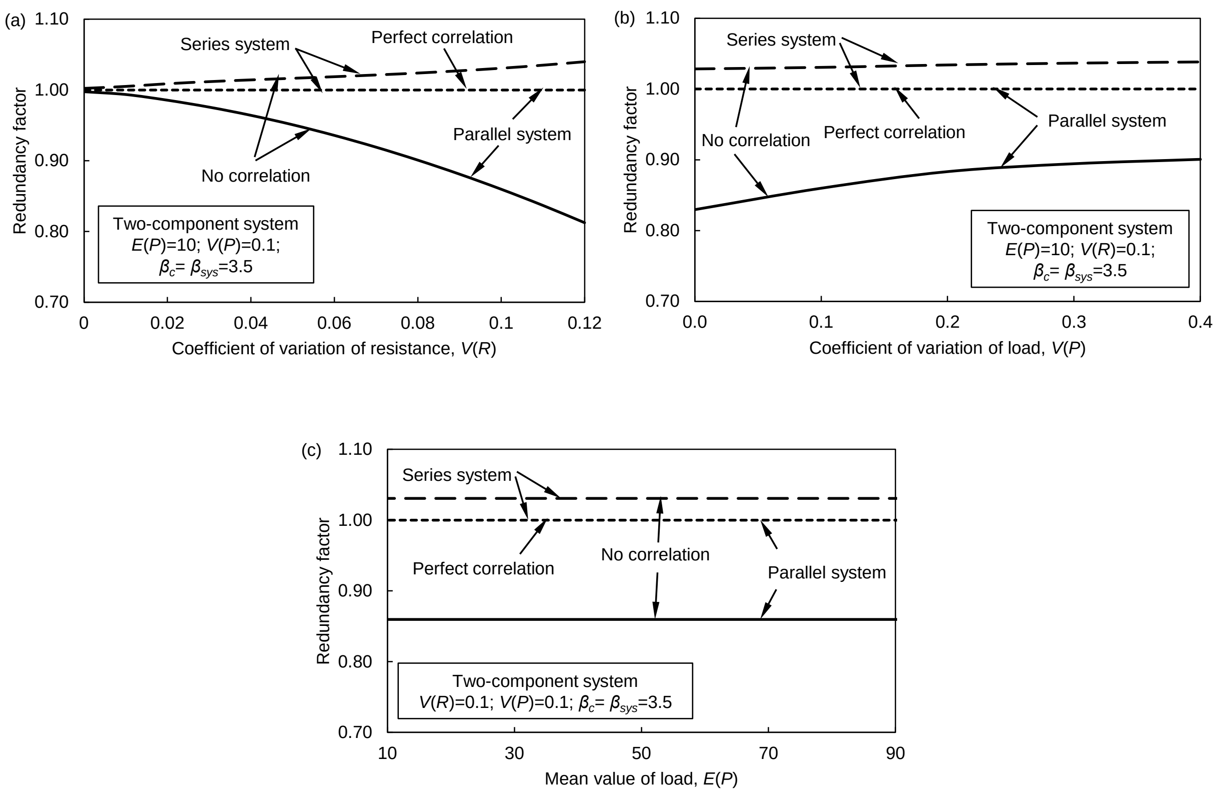


Figure 18. Graph. Effects of (a) $V(R)$; (b) $V(P)$; and (c) $E(P)$ on η_R in two-component systems.

These observations can be explained by the results presented in Figure 19 which shows the effects of $V(R)$ and $V(P)$ on the mean resistance of the single component $E_c(R)$ and the mean component resistance in the two systems $E_{cs}(R)$ associated with two correlation cases. It is found that:

(a) as $V(R)$ or $V(P)$ increases, $E_c(R)$ and $E_{cs}(R)$ in two systems corresponding to both correlation cases increase;

(b) in the no correlation case, the variation of $E_{cs}(R)$ in the series system due to the change of $V(R)$ or $V(P)$ is more significant than that of $E_c(R)$; therefore, $\eta_R = E_{cs}(R) / E_c(R)$ will increase as $V(R)$ or $V(P)$ increases;

(c) however, in the parallel system, the increase of $E_{cs}(R)$ due to the increase of $V(R)$ in the no correlation case is less significant than the increase of $E_c(R)$; therefore, η_R associated with the no correlation case in parallel system decreases (see Figure 18(a));

(d) as $V(P)$ increases in the no correlation case, the distance between the curves associated with $E_c(R)$ and $E_{cs}(R)$ of the parallel system decreases; thus, η_R increases along with the increase of $V(P)$ (see Figure 18(b));

(e) for the perfect correlation case, $E_{cs}(R) = E_c(R)$; hence, η_R = 1.0and $V(R)$ and $V(P)$ have no effect on the redundancy factor.

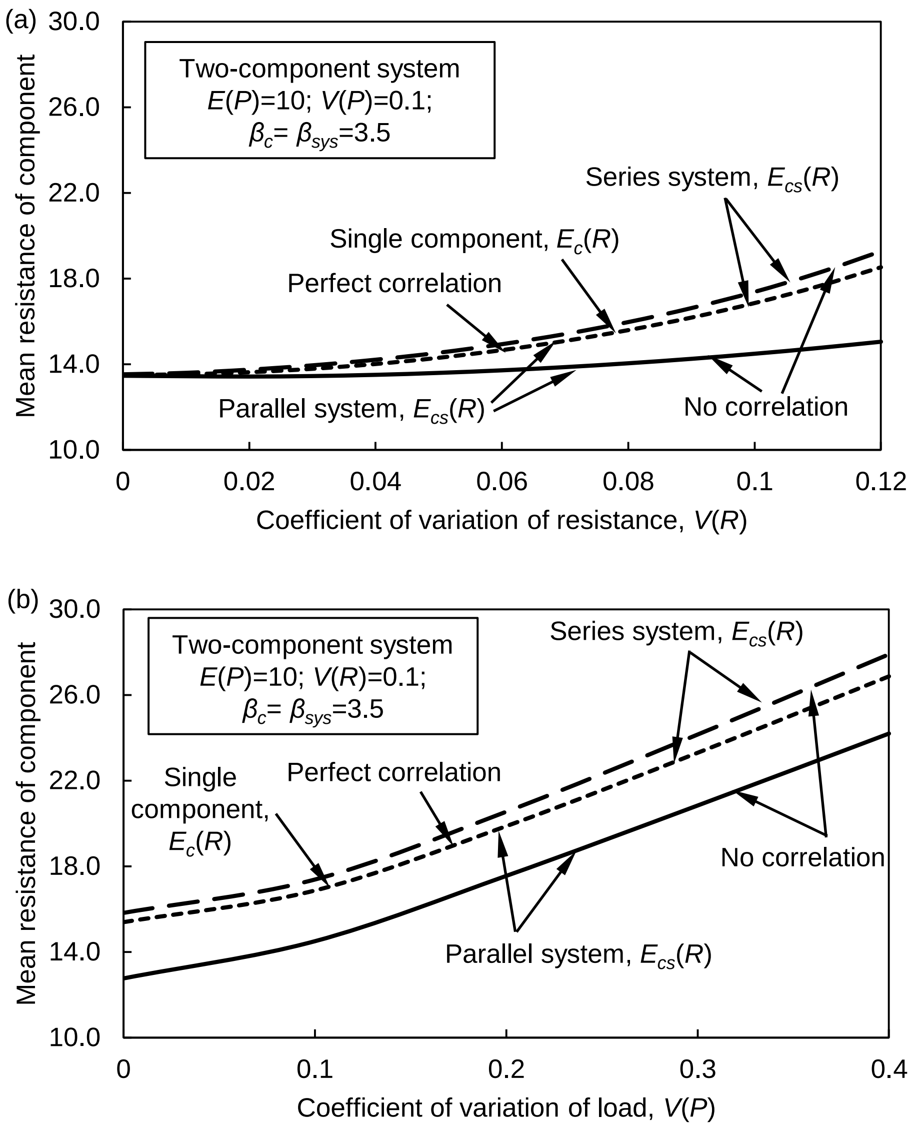


Figure 19. Graph. Effects of (a) $V(R)$; and (b) $V(P)$ on $E_c(R)$ and $E_{cs}(R)$ in two-component systems.

It should be noted that when $V(R)$ increases, both the mean $E_c(R)$ and standard deviation $\sigma_c(R)$ of the component resistance increase, as shown in Figure 20; the rate of increase of $\sigma_c(R)$ is more significant than that of $E_c(R)$ so that $V(R) = \sigma_c(R) / E_c(R)$ shows an increasing tendency. The redundancy factor as function of $V(R)$, $V(P)$, and $E(P)$ in three-component systems is plotted in Figure 21. From this figure it is observed that:

(a) as $V(R)$ increases in the no correlation case, η_R shows an increasing and decreasing tendency in series and parallel systems, respectively; the reason for this has been explained in the case of the two-component systems;

(b) as $V(P)$ increases, η_R in series and parallel system associated with the no correlation case increases and the changes are more significant in the parallel system than those in the series system;

(c) $E(P)$ has no effect on η_R of the two systems associated with both correlation cases;

(d) in the perfect correlation case, η_R of both systems is not affected by the variation of $V(R)$ and $V(P)$;

(e) the effects of $V(R)$ and $V(P)$ on η_R of both systems decrease with increasing correlation among resistances.

The effects of the aforementioned parameters on η_R are also investigated for the four-component systems in which three different systems can be considered, as shown in Figure 17. In addition to the no correlation and perfect correlation cases, a partial correlation case in which the correlation coefficients among the resistances are 0.5 is studied. The results are presented in Figure 22 to Figure 24. It is noted from Figure 22 that in the no correlation and partial correlation cases, as $V(R)$ increases, η_R associated with the series system increases while η_R associated with both the parallel and series-parallel systems show a decreasing tendency. It is also seen that as the correlation among the resistances becomes stronger, the sensitivity of η_R to the changes in $V(R)$ decreases.

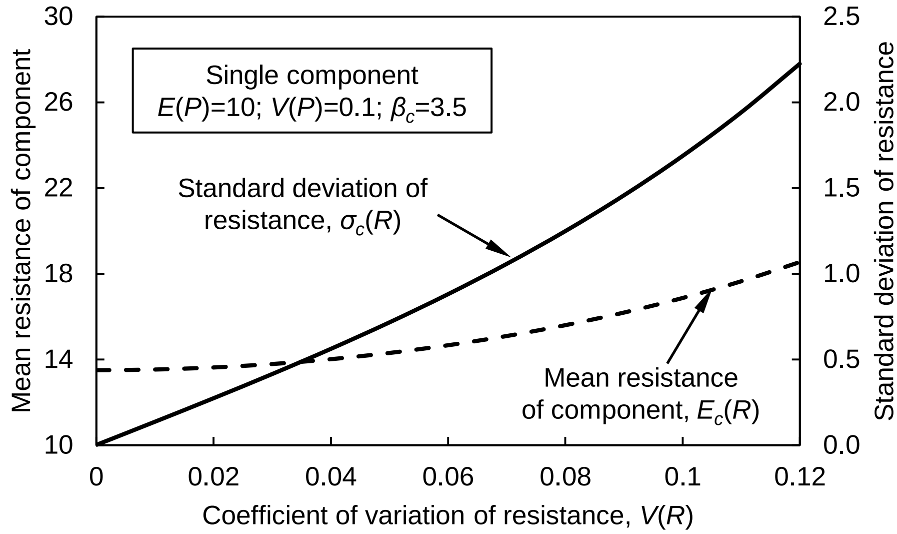


Figure 20. Graph. Effects of $V(R)$ on the mean and standard deviation of component resistance.

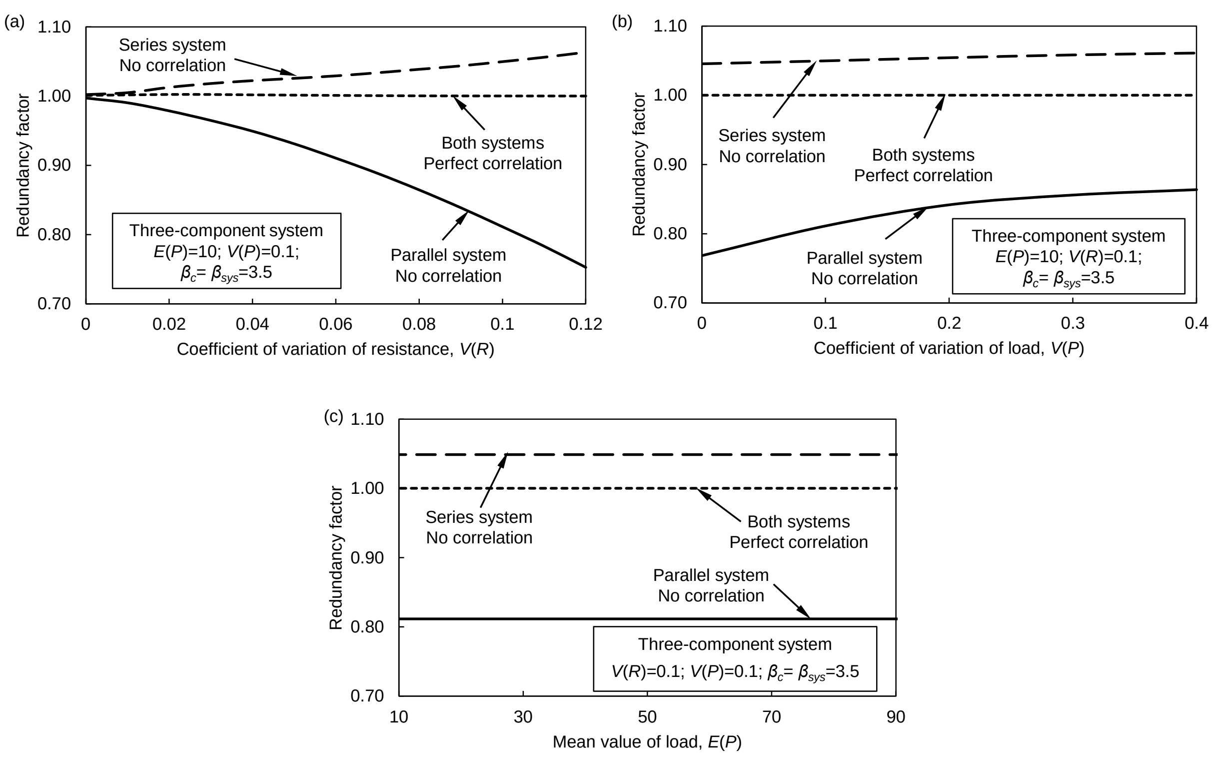


Figure 21. Graph. Effects of (a) $V(R)$; (b) $V(P)$; and (c) $E(P)$ on η_R in three-component systems.

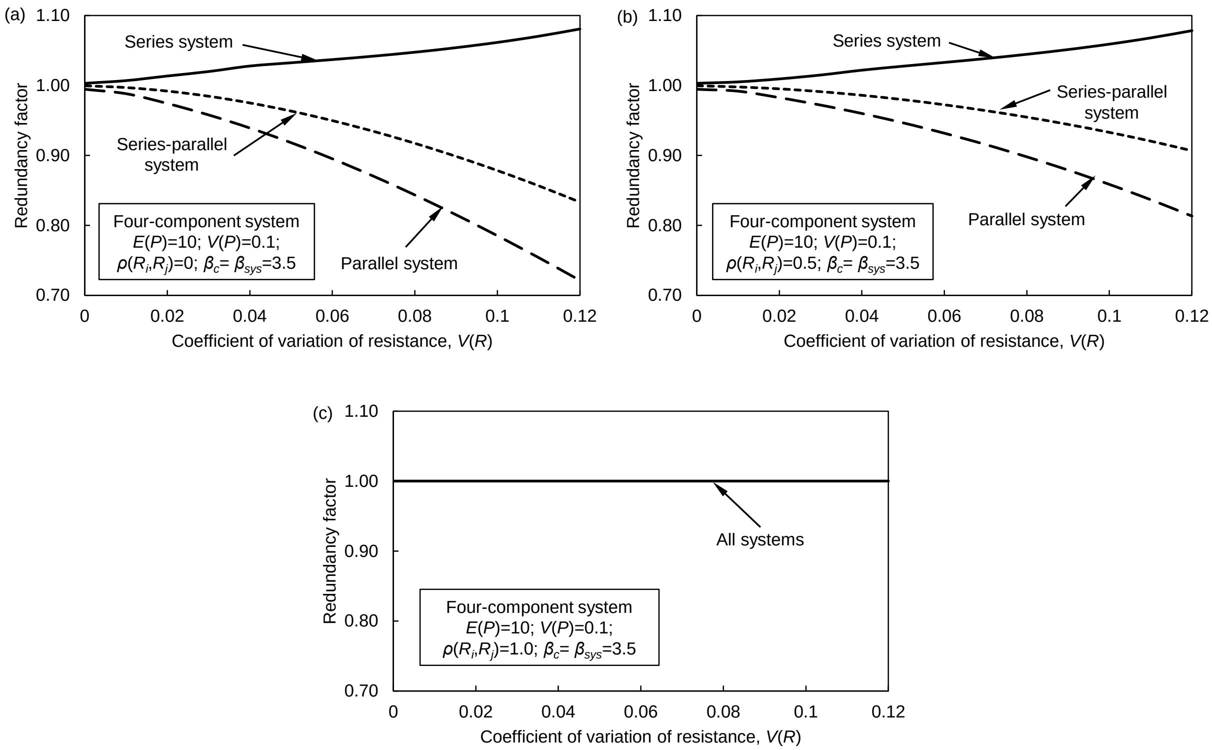


Figure 22. Graph. Effects of $V(R)$ on η_R in four-component systems associated with the case of (a) no correlation; (b) partial correlation; and (c) perfect correlation.

In the no correlation and partial correlation cases, Figure 23 shows that increasing $V(P)$ leads to a larger redundancy factor in series, parallel and series-parallel systems. In the perfect correlation case, η_R of all systems is 1.0 for any value of $V(P)$. The conclusion obtained from Figure 24 which shows the effect of $E(P)$ on η_R in four-component systems is the same as that drawn from Figure 18 and Figure 21. The effects of number of components N on the redundancy factor η_R in different systems with variations of $V(R)$, $V(P)$, and $E(P)$ are plotted in Figure 25. As N increases in the no correlation case, it is observed that:

(a) η_R in series systems increases while its counterpart in parallel systems decreases; and

(b) the change of η_R due to the variation of $V(R)$ or $V(P)$ is more significant than that due to the variation of $E(P)$.

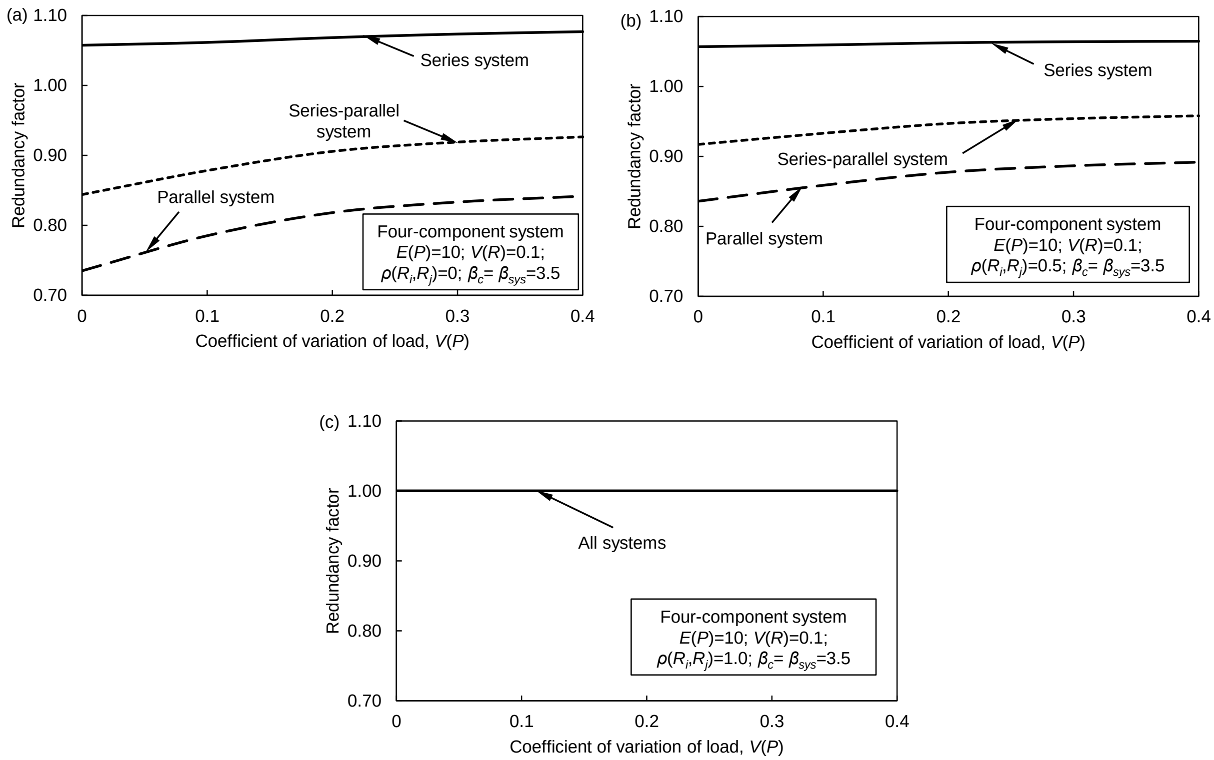


Figure 23. Graph. Effects of $V(P)$ on η_R in four-component systems associated with the case of (a) no correlation; (b) partial correlation; and (c) perfect correlation.

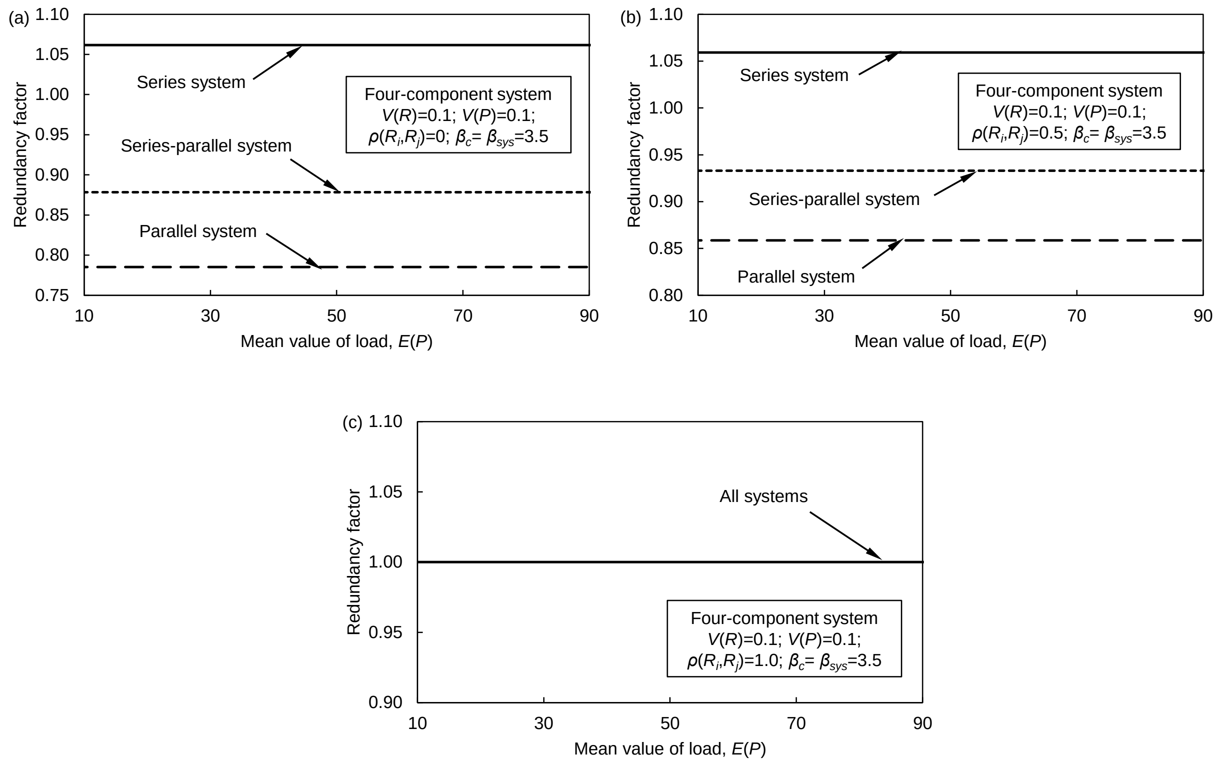


Figure 24. Graph. Effects of $E(P)$ on η_R in four-component systems associated with the case of (a) no correlation; (b) partial correlation; and (c) perfect correlation.

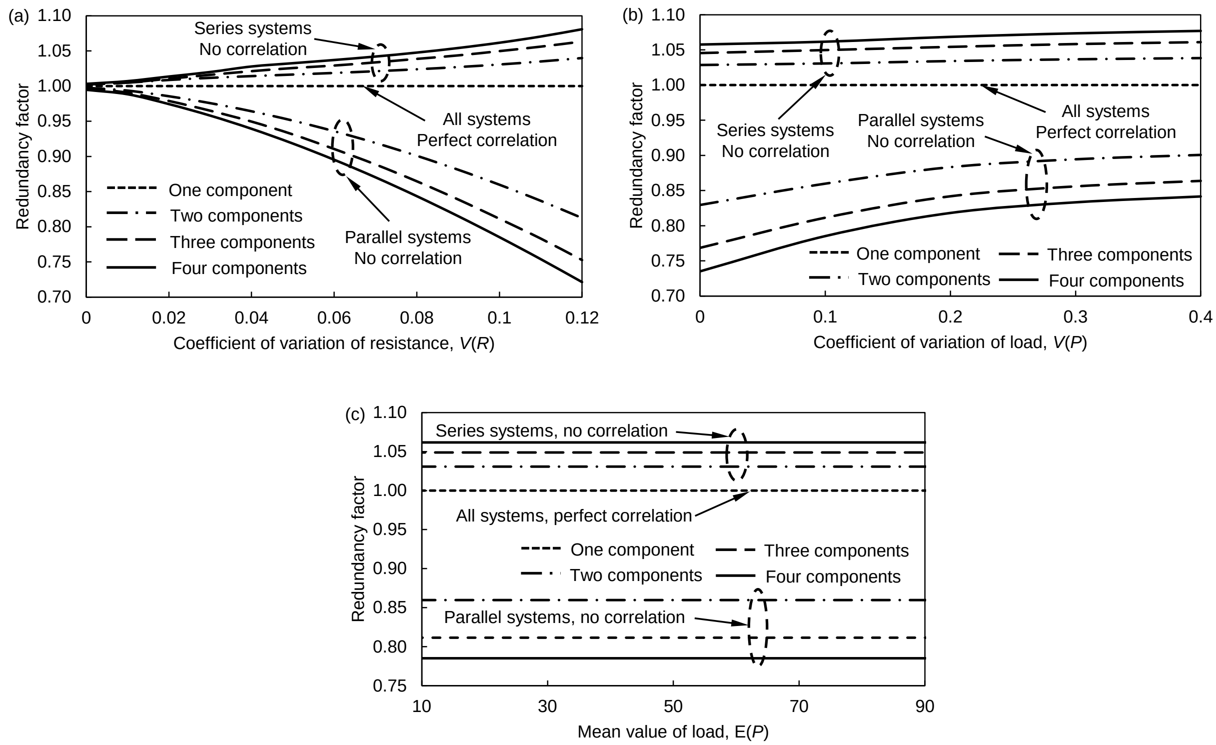


Figure 25. Graph. Effects of number of components on η_R with the variations of (a) $V(R)$; (b) $V(P)$; and (c) $E(P)$ in two extreme correlation cases.

3.4 Effect of System Modeling on the Redundancy Factor η_R

Structural systems can be modeled as series, parallel, or series-parallel combination of potential failure modes, depending on the definition of system failure. Different models lead to different relationships between component and system reliability, and thus the redundancy factor will be different. In this section, the effect of system modeling on the redundancy factor is studied using steel girder bridges consisting of 4, 6, 8, 10, and 12 girders. The system being investigated within this example is the flexural support provided by the steel girders. Bridge girders are thus modeled as components. Herein, the effects of bridge deck and girders can be approximately considered by the correlation among load effects in girders. If more information regarding the bridge deck and secondary members become available, a more complex analysis could be carried out by considering these member as separate subsystems.

For a steel girder bridge with four girders numbered from 1 to 4 (girders 1 and 4 are exterior girders and girders 2 and 3 refers to interior girders), four different system models can be considered according to different definitions of the girders system failure:

(a) series model: the system fails if any girder fails;

(b) parallel model: the system fails only if all girders fail;

(c) series-parallel model I: the system fails if either of the exterior girders and either of the interior girders fail simultaneously, denoted as $2s \times 2p$ series-parallel model; and

(d) series-parallel model II: the system fails if any two adjacent girders fail simultaneously, denoted as $2p \times 3s$ series-parallel model, as shown in Figure 26.

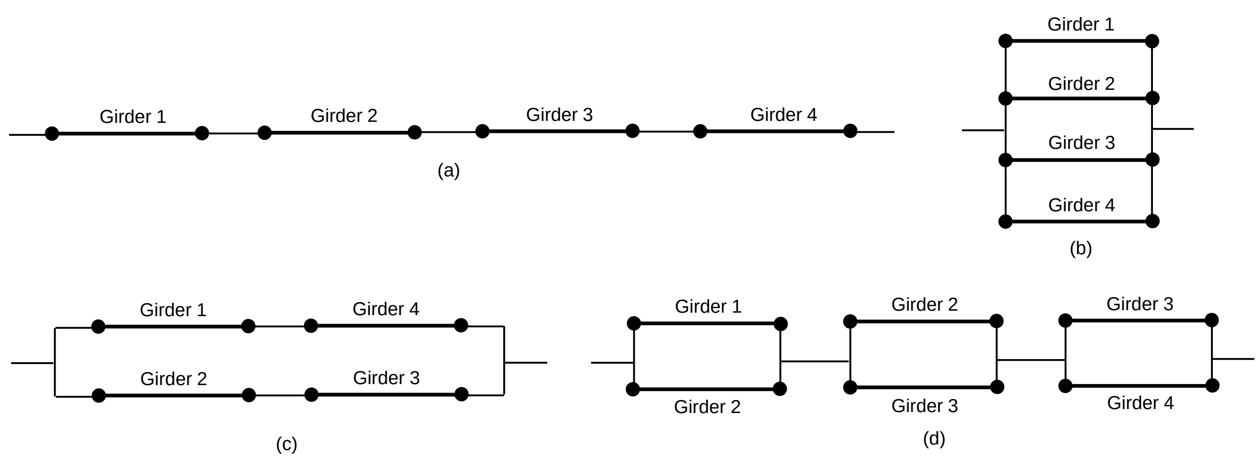


Figure 26. Graph. Four-girder bridge systems: (a) series, (b) parallel, (c) 2s×2p series-parallel, and (d) 2p×3s series-parallel.

The resistance R and load effect P are assumed to be the same for each girder and to follow normal distribution. Three correlation cases among the resistances of girders are considered:

(a) $\rho(R_i,R_j) = 0$, no correlation;

(b) $\rho(R_i,R_j) = 0.5$, partial correlation; and

(c) $\rho(R_i,R_j) = 1.0$, perfect correlation.

Although the girders may fail in different modes, only the flexural failure mode is analyzed. Since the mean value of the load effect does not affect the redundancy factor when $V(R)$ and $V(P)$ are fixed, the mean value of the bending moment due to vertical loads is assumed to be $E(P)$ = 7500 kN·m. Two cases associated with the coefficients of variation of R and P are studied:

(a) Case A: $V(R)$ = 0.05, $V(P)$ = 0.3; and

(b) Case B: $V(R)$ = 0.1, $V(P)$ = 0.4.

Based on this information, the required mean resistance of each girder $E_c(R)$ when the component reliability index β_c is 3.5 is found to be 1.58×10^4 kN·m for Case A and 2.01×10^4 kN·m for Case B. Next, assuming the system reliability index β_{sys} = 3.5, the mean resistances of each girders $E_{cs}(R)$ in the four systems associated with Cases A and B are calculated using a Monte Carlo Simulation-based program (MathWorks 2009). Finally, the associated redundancy factors and the associated reliability indices β_{cs} of girders are obtained by the ratio $E_{cs}(R)$ / $E_c(R)$. Note that it is implicitly assumed that the ratio of mean to nominal resistance and the ratio of dead to live load is constant for the bridge systems analyzed in this report. The results are presented in Table 12 and Table 13 as matrices in function of system modeling and correlation cases.

Table 12. Redundancy factors and reliability indices of girders in the 4-girder bridge systems associated with Case A ($V(R)$ = 0.05, $V(P)$ = 0.3).

Models	$\rho = 0$ η_R; β_{cs}	$\rho = 0.5$ η_R; β_{cs}	$\rho = 1.0$ η_R; β_{cs}
Series	1.041; 3.76	1.032; 3.70	1.0; 3.50
Parallel	0.934; 3.08	0.956; 3.22	1.0; 3.50
2*s*×2*p* SP	0.988; 3.42	0.995; 3.48	1.0; 3.50
2*p*×3*s* SP	0.983; 3.40	0.992; 3.45	1.0; 3.50

Note: ρ denotes $\rho(R_i,R_j)$.

Table 13. Redundancy factors and reliability indices of girders in the 4-girder bridge systems associated with Case B ($V(R)$ = 0.1, $V(P)$ = 0.4).

Models	$\rho = 0$ η_R; β_{cs}	$\rho = 0.5$ η_R; β_{cs}	$\rho = 1.0$ η_R; β_{cs}
Series	1.076; 3.83	1.066; 3.79	1.0; 3.50
Parallel	0.842; 2.75	0.892; 3.00	1.0; 3.50
2*s*×2*p* SP	0.946; 3.25	0.975; 3.39	1.0; 3.50
2*p*×3*s* SP	0.938; 3.21	0.968; 3.36	1.0; 3.50

Note: ρ denotes $\rho(R_i,R_j)$.

It is observed that:

(a) the redundancy factor and the girder reliability index associated with series system are the highest while their counterparts associated with parallel system are the lowest;

(b) for the two series-parallel systems, 2*s*×2*p* SP system (system fails if either of the exterior girders and either of the interior girders fail) provides higher redundancy factor and girder reliability index than 2*p*×3*s* SP system (system fails if any two adjacent girders fail);

(c) as the correlation among the resistances of girders increases, the redundancy factor and the girder reliability index decrease in the series system but increase in the parallel and series-parallel systems;

(d) the results of series system in Case B are higher than those in Case A; however, contrary findings are observed for the parallel and series-parallel systems;

(e) in the perfect correlation case, the redundancy factors of all the systems are 1.0 and thus the associated reliability indices of girders are 3.5.

For a 6-girder bridge, three different system models are formed based on different system failure definitions:

(a) series system;

(b) parallel system; and

(c) 2*p*×5*s* series-parallel system (the system fails if any two adjacent girders fail).

Assuming the distribution type of *R* and *P* and the associated distribution parameters of the 6-girder bridge systems are same as those used in the 4-girder systems, the redundancy factors and the reliability indices of girders of the 6-girder bridge systems associated with Cases A and B are calculated and the results are presented in Table 14 and Table 15, respectively. It is seen that compared with Case A, Case B provides lower results for the parallel and series-parallel systems but higher results for the series system.

In the evaluation of the redundancy factors of the 8-, 10-, and 12-girder bridge systems, an additional system modeling in which the system fails if any three adjacent girders fail is investigated. Therefore, the 8-girder bridge is modeled as a 2*p*×7*s* and 3*p*×6*s* series-parallel system if the failure of the system is defined as the failure of any two and three adjacent girders, respectively. Similarly, two different series-parallel systems are also formed for 10- and 12-girder bridge, respectively, based on the above failure definitions.

Table 14. Redundancy factors and reliability indices of girders in the 6-girder bridge systems associated with Case A ($V(R)$ = 0.05, $V(P)$ = 0.3).

Models	$\rho = 0$ η_R; β_{cs}	$\rho = 0.5$ η_R; β_{cs}	$\rho = 1.0$ η_R; β_{cs}
Series	1.052; 3.83	1.040; 3.75	1.0; 3.50
Parallel	0.922; 3.01	0.946; 3.16	1.0; 3.50
2*p*×5*s* SP	0.994; 3.46	1.000; 3.50	1.0; 3.50

Note: ρ denotes $\rho(R_i,R_j)$.

Table 15. Redundancy factors and reliability indices of girders in the 6-girder bridge systems associated with Case B ($V(R)$ = 0.1, $V(P)$ = 0.4).

Models	$\rho = 0$ η_R; β_{cs}	$\rho = 0.5$ η_R; β_{cs}	$\rho = 1.0$ η_R; β_{cs}
Series	1.098; 3.93	1.083; 3.86	1.0; 3.50
Parallel	0.818; 2.62	0.874; 2.90	1.0; 3.50
2*p*×5*s* SP	0.956; 3.30	0.985; 3.43	1.0; 3.50

Note: ρ denotes $\rho(R_i,R_j)$.

Table 16. Redundancy factors and reliability indices of girders in the 8-girder bridge systems associated with Case A ($V(R)$ = 0.05, $V(P)$ = 0.3).

Models	$\rho = 0$ η_R; β_{cs}	$\rho = 0.5$ η_R; β_{cs}	$\rho = 1.0$ η_R; β_{cs}
Series	1.059; 3.88	1.046; 3.78	1.0; 3.50
Parallel	0.914; 2.95	0.941; 3.13	1.0; 3.50
2*p*×7*s* SP	1.001; 3.51	1.005; 3.53	1.0; 3.50
3*p*×6*s* SP	0.973; 3.33	0.986; 3.41	1.0; 3.50

Note: ρ denotes $\rho(R_i,R_j)$.

Table 17. Redundancy factors and reliability indices of girders in the 8-girder bridge systems associated with Case B ($V(R)$ = 0.1, $V(P)$ = 0.4).

Models	$\rho = 0$ η_R; β_{cs}	$\rho = 0.5$ η_R; β_{cs}	$\rho = 1.0$ η_R; β_{cs}
Series	1.113; 4.00	1.096; 3.92	1.0; 3.50
Parallel	0.803; 2.55	0.860; 2.83	1.0; 3.50
2*p*×7*s* SP	0.969; 3.36	0.995; 3.48	1.0; 3.50
3*p*×6*s* SP	0.911; 3.08	0.951; 3.28	1.0; 3.50

Note: ρ denotes $\rho(R_i,R_j)$.

With the aforementioned parameters related to the resistances and load effects of girders, the redundancy factors and the reliability indices of girders associated with 8-, 10-, and 12-girder bridge systems are obtained, as shown in Table 16 to Table 21. It is noticed that:

(a) the results of the series-parallel systems associated with failure of any two adjacent girders are higher than those of the series-parallel systems associated with failure of any three adjacent girders; and

(b) the results of the series-parallel systems in Case B are lower than those in Case A.

Table 18. Redundancy factors and reliability indices of girders in the 10-girder bridge systems associated with Case A ($V(\boldsymbol{R})$ = 0.05, $V(\boldsymbol{P})$ = 0.3).

Models	ρ = 0 η_R; β_{cs}	ρ = 0.5 η_R; β_{cs}	ρ = 1.0 η_R; β_{cs}
Series	1.064; 3.90	1.050; 3.81	1.0; 3.50
Parallel	0.908; 2.92	0.937; 3.10	1.0; 3.50
2*p*×9*s* SP	1.006; 3.54	1.009; 3.56	1.0; 3.50
3*p*×8*s* SP	0.978; 3.37	0.990; 3.43	1.0; 3.50

Note: ρ denotes $\rho(R_i,R_j)$.

Table 19. Redundancy factors and reliability indices of girders in the 10-girder bridge systems associated with Case B ($V(\boldsymbol{R})$ = 0.1, $V(\boldsymbol{P})$ = 0.4).

Models	ρ = 0 η_R; β_{cs}	ρ = 0.5 η_R; β_{cs}	ρ = 1.0 η_R; β_{cs}
Series	1.126; 4.05	1.106; 3.96	1.0; 3.50
Parallel	0.794; 2.50	0.854; 2.81	1.0; 3.50
2*p*×9*s* SP	0.977; 3.40	1.002; 3.51	1.0; 3.50
3*p*×8*s* SP	0.921; 3.13	0.958; 3.31	1.0; 3.50

Note: ρ denotes $\rho(R_i,R_j)$.

Table 20. Redundancy factors and reliability indices of girders in the 12-girder bridge systems associated with Case A ($V(\boldsymbol{R})$ = 0.05, $V(\boldsymbol{P})$ = 0.3).

Models	ρ = 0 η_R; β_{cs}	ρ = 0.5 η_R; β_{cs}	ρ = 1.0 η_R; β_{cs}
Series	1.070; 3.94	1.053; 3.83	1.0; 3.50
Parallel	0.904; 2.89	0.934; 3.08	1.0; 3.50
2*p*×11*s* SP	1.010; 3.56	1.012; 3.57	1.0; 3.50
3*p*×10*s* SP	0.982; 3.39	0.992; 3.45	1.0; 3.50

Note: ρ denotes $\rho(R_i,R_j)$.

Table 21. Redundancy factors and reliability indices of girders in the 12-girder bridge systems associated with Case B ($V(R)$ = 0.1, $V(P)$ = 0.4).

Models	$\rho = 0$ η_R; β_{cs}	$\rho = 0.5$ η_R; β_{cs}	$\rho = 1.0$ η_R; β_{cs}
Series	1.136; 4.09	1.113; 3.99	1.0; 3.50
Parallel	0.786; 2.46	0.848; 2.78	1.0; 3.50
$2p\times11s$ SP	0.985; 3.43	1.009; 3.54	1.0; 3.50
$3p\times10s$ SP	0.927; 3.16	0.964; 3.34	1.0; 3.50

Note: ρ denotes $\rho(R_i,R_j)$.

It is observed from Table 12 through Table 21 that:

(a) series and parallel systems provide the highest and lowest redundancy factors, respectively;

(b) the results of the series-parallel systems associated with any two adjacent girders failure ($2p\times ns$) are higher than those associated with any three adjacent girders failure ($3p\times ns$); the maximum difference is approximately 0.058 in the no correlation case in Table 21, and the minimum difference is 0.003 in the partial correlation case in Table 12;

(c) compared with Case A, Case B provides lower results for the parallel and series-parallel systems but higher results for the series system;

(d) as the number of girders increases, the redundancy factor of the series and series-parallel systems associated with failure of any m (m = 2, 3) adjacent girders increases; however, contrary findings are associated with the parallel systems.

Therefore, the difference in the redundancy factors between the series and parallel systems reaches the maximum in the 12-girder systems associated with Case B (see Table 21). The corresponding mean resistances of girders $E_{cs}(R)$ in the 12-girder series and parallel systems associated with Case B are 2.28×10^4 kN·m and 1.58×10^4 kN·m, respectively. It is seen that the designed mean resistances of girders in the series system are about 50% larger than those in the parallel system.

3.5 Redundancy Factors of Systems with Many Components

In the previous section, η_R is evaluated with respect to the systems consisting of no more than 12 components. However, in most practical cases, a structure usually consists of dozens or hundreds of members; therefore, it is necessary to investigate the redundancy factors of systems with many components. In this section, a self-developed program based on Monte Carlo Simulation (MCS) is used to determine these redundancy factors. The algorithm of this program using MATLAB[23] is described as follows:

1. Given the mean value of the load effect $E(P)$, coefficients of variation of resistance and load effect $V(R)$ and $V(P)$, correlation between the resistances of components $\rho(R_i, R_j)$, distribution types of resistance and load, number of components N, number of simulation samples w, and the initial guess for the mean value of component resistance $E_{cs}(R)$;

2. Generate the random samples of resistance R_i and load effect P based on the above parameters, and the dimensions of the R_i and P vectors are $w \times 1$;

3. Obtain the performance function for each component $g_i = R_i - P$ ($i = 1, 2, \ldots, N$); the dimensions of g_i is also $w \times 1$;

4. For series system, define an $w \times 1$ zero vector L, and the ratio of the number of $[L \mid (g_1 < 0) \mid \ldots \mid (g_N < 0)]$ to the total sample size w represents the failure probability of series system ("|" is logical OR in MATLAB; it refers to union); for the parallel system, define an $w \times 1$ unit vector Q, and the ratio of the number of $[Q \,\&\, (g_1 < 0) \,\&\, \ldots \,\&\, (g_N < 0)]$ to w is the P_f of parallel system ("&" is logical AND in MATLAB; it refers to intersection); for the $mp \times ns$ SP system, define an $w \times 1$ zero vector L and an $w \times 1$ unit vector Q, and the ratio of the number of $\{L \mid [Q \,\&\, (g_1 < 0) \,\&\, \ldots \,\&\, (g_m < 0)] \mid \ldots \mid [Q \,\&\, (g_{m\cdot(n-1)+1} < 0) \,\&\, \ldots \,\&\, (g_{m\cdot n} < 0)]\}$ to w is the P_f of the SP system; for the $ms \times np$ SP system, define an $w \times 1$ zero vector L and an $w \times 1$ unit vector Q, and the ratio of the

number of $\{Q \& [L \mid (g_1 < 0) \mid \ldots \mid (g_m < 0)] \& \ldots \& [L \mid (g_{m\cdot(n-1)+1} < 0) \mid \ldots \mid (g_{m\cdot n} < 0)]\}$ to w is the P_f of the SP system; it should be noted that in the series-parallel systems, $n \times m$ is equal to the number of components N.

5. Repeat steps 1 to 4 for t times (e.g., t = 50) to obtain the average probability of failure of the system; then, convert it to the reliability index.

These programs are run on a computational server located in the Computational Laboratory for Life-cycle Structural Engineering, in the ATLSS Engineering Research Center at Lehigh University.

When using the MCS-based program for finding the reliability index of systems, it is found that as N increases, the computational time required increases dramatically. Therefore, the aforementioned search algorithm that requires a group of initial values is not efficient when combined with the MCS-based program. In order to reduce the computing time, a simple algorithm based on the effects of number of the components on the redundancy factor is used in combination with the MCS-based program to find $E_{cs}(R)$ and η_R. The procedure of this algorithm consists of the following steps:

1. Determine an initial guess value of $E_{cs}(R)$ based on the effects of number of components N on the redundancy factors in different systems. For example, it was found previously that $E_{cs}(R)$ associated with series (or series-parallel) system increases as N increases; however, this increase is less significant as N becomes larger. Therefore, the initial guess of $E_{cs}(R)$ for the 100-component series system can be obtained by increasing the $E_{cs}(R)$ of 50-component series system by Δ percent ($0.5 \leq \Delta \leq 1$). On the contrary, increasing N leads to lower $E_{cs}(R)$ in parallel systems. Hence, the initial guess of $E_{cs}(R)$ for the 100-component parallel system can be determined by reducing the $E_{cs}(R)$ of 50-component parallel system by Δ percent ($0.5 \leq \Delta \leq 1$).

2. Substitute the initial value from step 1 to the MCS-based algorithm described previously to obtain the system reliability index β_{sys}.

3. Checkpoint: if $|\beta_{sys} - 3.5| \leq Tol$ (Tol refers to tolerance and it is set to be 10^{-4} in this study), then return this initial value; otherwise go to the next step.

4. Checkpoint: if the $\beta_{sys} < 3.5$, increase the initial value by δ percent ($0.1 \leq \delta \leq 0.3$); if $\beta_{sys} > 3.5$, reduce the initial value by δ percent ($0.1 \leq \delta \leq 0.3$);

5. Repeat steps 2-4 until $E_{cs}(R)$ is found.

$E_{cs}(R)$ can usually be found within four loops. A flowchart for this algorithm combined with the MCS-based program is presented in Figure 27. Three correlation cases ($\rho(R_i,R_j)$ = 0; 0.5; and 1.0) among the resistances of components and two distribution types (normal and lognormal) of the loads and resistances are investigated. The coefficients of variation of resistance and load are 0.05 and 0.3, respectively, as stated previously. The mean value of load acting on each component $E(P)$ is also assumed to be 10. Based on the equations in Figure 2 and Figure 3 and the given parameters, the mean values of resistance associated with a single component for the normal and lognormal distribution are found to be $E_c(R)$ = 21.132 and $E_c(R)$ = 27.194, respectively. By combining the MCS-based program with the simple algorithm, the redundancy factors of different types of N-component systems (N = 2, 3, 5, 10, 15, 20, 25, 50, 100) are evaluated. The mean resistances of components and redundancy factors associated with the normal distribution case are presented in Table 22 to Table 27, and the results associated with the lognormal distribution case listed in Table 28 to Table 33.

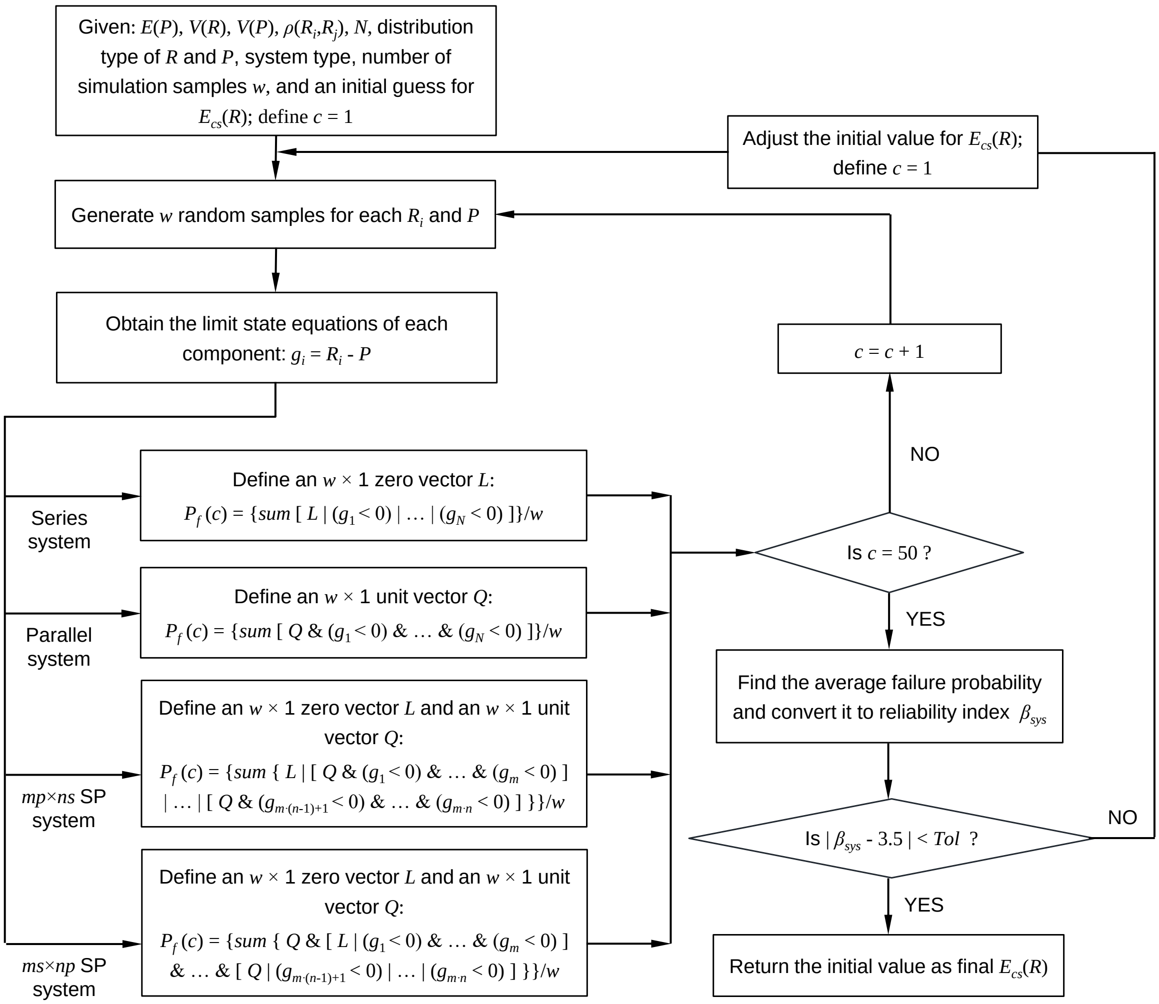


Figure 27. Graph. Flowchart for the algorithm combined with MCS-based method.

Table 22. $E_{cs}(R)$ and η_R of different systems associated with the case of $\rho(R_i,R_j) = 0$ using the MCS-based method when R and P follow normal distribution; $1 \leq N \leq 20$.

System	$E_{cs}(R)$	η_R
1-component system	21.132	1.000
2-component system - Series system	21.582	1.021
2-component system - Parallel system	20.310	0.961
3-component system - Series system	21.835	1.033
3-component system - Parallel system	19.960	0.945
4-component system - Series system	21.998	1.041
4-component system - Parallel system	19.737	0.934
5-component system - Series system	22.123	1.047
5-component system - Parallel system	19.591	0.927
6-component system - Series system	22.231	1.052
6-component system - Parallel system	19.477	0.922
6-component system - 2*p*×3*s* SP system	20.850	0.987
10-component system - Series system	22.495	1.064
10-component system - Parallel system	19.196	0.908
10-component system - 5*p*×2*s* SP system	19.870	0.940
10-component system - 5*s*×2*p* SP system	21.530	1.019
15-component system - Series system	22.730	1.076
15-component system - Parallel system	18.994	0.899
15-component system - 5*p*×3*s* SP system	20.015	0.947
15-component system - 5*s*×3*p* SP system	21.300	1.008
20-component system - Series system	22.855	1.082
20-component system - Parallel system	18.867	0.893
20-component system - 5*p*×4*s* SP system	20.108	0.952
20-component system - 10*p*×2*s* SP system	19.425	0.919
20-component system - 5*s*×4*p* SP system	21.130	1.000
20-component system - 10*s*×2*p* SP system	21.955	1.039

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

Table 23. $E_{cs}(R)$ and η_R of different systems associated with the case of $\rho(R_i,R_j) = 0$ using the MCS-based method when R and P follow normal distribution; $25 \leq N \leq 100$.

System	$E_{cs}(R)$	η_R
25-component system - Series system	22.987	1.088
25-component system - Parallel system	18.773	0.888
25-component system - 5*p*×5*s* SP system	20.193	0.955
25-component system - 5*s*×5*p* SP system	21.030	0.995
50-component system - Series system	23.321	1.104
50-component system - Parallel system	18.510	0.876
50-component system - 5*p*×10*s* SP system	20.370	0.964
50-component system - 10*p*×5*s* SP system	19.682	0.931
50-component system - 5*s*×10*p* SP system	20.770	0.983
50-component system - 10*s*×5*p* SP system	21.540	1.019
100-component system - Series system	23.631	1.118
100-component system - Parallel system	18.306	0.866
100-component system - 5*p*×10*s* SP system	20.551	0.972
100-component system - 10*p*×10*s* SP system	19.846	0.939
100-component system - 20*p*×5*s* SP system	19.293	0.913
100-component system - 5*s*×20*p* SP system	20.550	0.972
100-component system - 10*s*×10*p* SP system	21.300	1.008
100-component system - 20*s*×5*p* SP system	21.980	1.040

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

Table 24. $E_{cs}(R)$ **and** η_R **of different systems associated with the case of** $\rho(R_i,R_j) = 0.5$ **using the MCS-based method when** R **and** P **follow normal distribution;** $1 \le N \le 20$**.**

System	$E_{cs}(R)$	η_R
1-component system	21.132	1.000
2-component system - Series system	21.480	1.016
2-component system - Parallel system	20.590	0.974
3-component system - Series system	21.680	1.026
3-component system - Parallel system	20.355	0.963
4-component system - Series system	21.808	1.032
4-component system - Parallel system	20.202	0.956
5-component system - Series system	21.910	1.037
5-component system - Parallel system	20.080	0.950
6-component system - Series system	21.981	1.040
6-component system - Parallel system	20.000	0.946
6-component system - 2*p*×3*s* SP system	21.025	0.995
10-component system - Series system	22.190	1.050
10-component system - Parallel system	19.795	0.937
10-component system - 5*p*×2*s* SP system	20.309	0.961
10-component system - 5*s*×2*p* SP system	21.512	1.018
15-component system - Series system	22.360	1.058
15-component system - Parallel system	19.654	0.930
15-component system - 5*p*×3*s* SP system	20.425	0.967
15-component system - 5*s*×3*p* SP system	21.350	1.010
20-component system - Series system	22.453	1.063
20-component system - Parallel system	19.549	0.925
20-component system - 5*p*×4*s* SP system	20.490	0.970
20-component system - 10*p*×2*s* SP system	19.990	0.946
20-component system - 5*s*×4*p* SP system	21.245	1.005
20-component system - 10*s*×2*p* SP system	21.835	1.033

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

Table 25. $E_{cs}(R)$ and η_R of different systems associated with the case of $\rho(R_i,R_j)$ = 0.5 using the MCS-based method when R and P follow normal distribution; $25 \leq N \leq 100$.

System	$E_{cs}(R)$	η_R
25-component system - Series system	22.530	1.066
25-component system - Parallel system	19.481	0.922
25-component system - 5*p*×5*s* SP system	20.540	0.972
25-component system - 5*s*×5*p* SP system	21.175	1.002
50-component system - Series system	22.768	1.077
50-component system - Parallel system	19.277	0.912
50-component system - 5*p*×10*s* SP system	20.703	0.980
50-component system - 10*p*×5*s* SP system	20.190	0.955
50-component system - 5*s*×10*p* SP system	20.980	0.993
50-component system - 10*s*×5*p* SP system	21.545	1.020
100-component system - Series system	23.005	1.089
100-component system - Parallel system	19.124	0.905
100-component system - 5*p*×10*s* SP system	20.840	0.986
100-component system - 10*p*×10*s* SP system	20.305	0.961
100-component system - 20*p*×5*s* SP system	19.890	0.941
100-component system - 5*s*×20*p* SP system	20.840	0.986
100-component system - 10*s*×10*p* SP system	21.385	1.012
100-component system - 20*s*×5*p* SP system	21.880	1.035

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

Table 26. $E_{cs}(R)$ **and** η_R **of different systems associated with the case of** $\rho(R_i,R_j) = 1.0$ **using the MCS-based method when** R **and** P **follow normal distribution;** $1 \leq N \leq 20$**.**

System	$E_{cs}(R)$	η_R
1-component system	21.132	1.000
2-component system - Series system	21.125	1.000
2-component system - Parallel system	21.124	1.000
3-component system - Series system	21.124	1.000
3-component system - Parallel system	21.124	1.000
4-component system - Series system	21.124	1.000
4-component system - Parallel system	21.124	1.000
5-component system - Series system	21.124	1.000
5-component system - Parallel system	21.124	1.000
6-component system - Series system	21.127	1.000
6-component system - Parallel system	21.127	1.000
6-component system - 2p×3s SP system	21.127	1.000
10-component system - Series system	21.131	1.000
10-component system - Parallel system	21.130	1.000
10-component system - 5p×2s SP system	21.130	1.000
10-component system - 5s×2p SP system	21.130	1.000
15-component system - Series system	21.131	1.000
15-component system - Parallel system	21.131	1.000
15-component system - 5p×3s SP system	21.131	1.000
15-component system - 5s×3p SP system	21.131	1.000
20-component system - Series system	21.132	1.000
20-component system - Parallel system	21.132	1.000
20-component system - 5p×4s SP system	21.132	1.000
20-component system - 10p×2s SP system	21.132	1.000
20-component system - 5s×4p SP system	21.132	1.000
20-component system - 10s×2p SP system	21.132	1.000

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

Table 27. $E_{cs}(R)$ **and** η_R **of different systems associated with the case of** $\rho(R_i,R_j) = 1.0$ **using the MCS-based method when** R **and** P **follow normal distribution;** $25 \leq N \leq 100$**.**

System	$E_{cs}(R)$	η_R
25-component system - Series system	21.132	1.000
25-component system - Parallel system	21.132	1.000
25-component system - 5*p*×5*s* SP system	21.132	1.000
25-component system - 5*s*×5*p* SP system	21.132	1.000
50-component system - Series system	21.132	1.000
50-component system - Parallel system	21.132	1.000
50-component system - 5*p*×10*s* SP system	21.132	1.000
50-component system - 10*p*×5*s* SP system	21.132	1.000
50-component system - 5*s*×10*p* SP system	21.132	1.000
50-component system - 10*s*×5*p* SP system	21.132	1.000
100-component system - Series system	21.133	1.000
100-component system - Parallel system	21.133	1.000
100-component system - 5*p*×10*s* SP system	21.133	1.000
100-component system - 10*p*×10*s* SP system	21.133	1.000
100-component system - 20*p*×5*s* SP system	21.133	1.000
100-component system - 5*s*×20*p* SP system	21.133	1.000
100-component system - 10*s*×10*p* SP system	21.133	1.000
100-component system - 20*s*×5*p* SP system	21.133	1.000

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

Table 28. $E_{cs}(R)$ and η_R of different systems associated with the case of $\rho(R_i,R_j) = 0$ using the MCS-based method when R and P follow lognormal distribution; $1 \leq N \leq 25$.

System	$E_{cs}(R)$	η_R
1-component system	27.194	1.000
2-component system - Series system	27.839	1.024
2-component system - Parallel system	26.292	0.967
3-component system - Series system	28.209	1.037
3-component system - Parallel system	25.874	0.951
5-component system - Series system	28.596	1.051
5-component system - Parallel system	25.441	0.935
10-component system - Series system	29.115	1.070
10-component system - Parallel system	24.922	0.916
10-component system - 5*p*×2*s* SP system	25.864	0.951
10-component system - 5*s*×2*p* SP system	27.960	1.028
15-component system - Series system	29.349	1.079
15-component system - Parallel system	24.674	0.907
15-component system - 5*p*×3*s* SP system	26.082	0.959
15-component system - 5*s*×3*p* SP system	27.710	1.019
20-component system - Series system	29.561	1.087
20-component system - Parallel system	24.501	0.901
20-component system - 5*p*×4*s* SP system	26.208	0.964
20-component system - 10*p*×2*s* SP system	25.286	0.930
20-component system - 5*s*×4*p* SP system	27.550	1.013
20-component system - 10*s*×2*p* SP system	28.600	1.052
25-component system - Series system	29.650	1.090
25-component system - Parallel system	24.368	0.896
25-component system - 5*p*×5*s* SP system	26.328	0.968
25-component system - 5*s*×5*p* SP system	27.390	1.007

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,LN}(R) = 27.194$

Table 29. $E_{cs}(R)$ and η_R of different systems associated with the case of $\rho(R_i,R_j) = 0$ using the MCS-based method when R and P follow lognormal distribution; $50 \leq N \leq 100$.

System	$E_{cs}(R)$	η_R
50-component system - Series system	30.098	1.107
50-component system - Parallel system	24.014	0.883
50-component system - 5p×10s SP system	26.569	0.977
50-component system - 10p×5s SP system	25.668	0.944
50-component system - 5s×10p SP system	27.100	0.997
50-component system - 10s×5p SP system	28.040	1.031
100-component system - Series system	30.470	1.120
100-component system - Parallel system	23.695	0.871
100-component system - 5p×10s SP system	26.831	0.986
100-component system - 10p×10s SP system	25.874	0.951
100-component system - 20p×5s SP system	25.147	0.925
100-component system - 5s×20p SP system	26.825	0.986
100-component system - 10s×10p SP system	27.790	1.022
100-component system - 20s×5p SP system	28.643	1.053

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,LN}(R) = 27.194$

Table 30. $E_{cs}(R)$ **and** η_R **of different systems associated with the case of** $\rho(R_i,R_j) = 0.5$ **using the MCS-based method when** R **and** P **follow lognormal distribution;** $1 \leq N \leq 25$**.**

System	$E_{cs}(R)$	η_R
1-component system	27.194	1.000
2-component system - Series system	27.678	1.018
2-component system - Parallel system	26.596	0.978
3-component system - Series system	27.931	1.027
3-component system - Parallel system	26.292	0.967
5-component system - Series system	28.198	1.037
5-component system - Parallel system	26.009	0.956
10-component system - Series system	28.610	1.052
10-component system - Parallel system	25.637	0.943
10-component system - 5*p*×2*s* SP system	26.318	0.968
10-component system - 5*s*×2*p* SP system	27.806	1.023
15-component system - Series system	28.768	1.058
15-component system - Parallel system	25.451	0.936
15-component system - 5*p*×3*s* SP system	26.463	0.973
15-component system - 5*s*×3*p* SP system	27.625	1.016
20-component system - Series system	28.889	1.062
20-component system - Parallel system	25.311	0.931
20-component system - 5*p*×4*s* SP system	26.556	0.976
20-component system - 10*p*×2*s* SP system	25.890	0.952
20-component system - 5*s*×4*p* SP system	27.500	1.011
20-component system - 10*s*×2*p* SP system	28.300	1.041
25-component system - Series system	28.975	1.065
25-component system - Parallel system	25.235	0.928
25-component system - 5*p*×5*s* SP system	26.649	0.980
25-component system - 5*s*×5*p* SP system	27.429	1.009

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,LN}(R) = 27.194$

Table 31. $E_{cs}(R)$ **and** η_R **of different systems associated with the case of** $\rho(R_i,R_j) = 0.5$ **using the MCS-based method when** R **and** P **follow lognormal distribution;** $50 \leq N \leq 100$**.**

System	$E_{cs}(R)$	η_R
50-component system - Series system	29.290	1.077
50-component system - Parallel system	24.969	0.918
50-component system - 5*p*×10*s* SP system	26.796	0.985
50-component system - 10*p*×5*s* SP system	26.187	0.963
50-component system - 5*s*×10*p* SP system	27.190	1.000
50-component system - 10*s*×5*p* SP system	27.865	1.025
100-component system - Series system	29.537	1.086
100-component system - Parallel system	24.748	0.910
100-component system - 5*p*×10*s* SP system	27.038	0.994
100-component system - 10*p*×10*s* SP system	26.344	0.969
100-component system - 20*p*×5*s* SP system	25.784	0.948
100-component system - 5*s*×20*p* SP system	27.000	0.993
100-component system - 10*s*×10*p* SP system	27.690	1.018
100-component system - 20*s*×5*p* SP system	28.247	1.039

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,LN}(R) = 27.194$

Table 32. $E_{cs}(R)$ and η_R of different systems associated with the case of $\rho(R_i,R_j)$ = 1.0 using the MCS-based method when R and P follow lognormal distribution; $1 \leq N \leq 25$.

System	$E_{cs}(R)$	η_R
1-component system	27.194	1.000
2-component system - Series system	27.190	1.000
2-component system - Parallel system	27.190	1.000
3-component system - Series system	27.190	1.000
3-component system - Parallel system	27.190	1.000
5-component system - Series system	27.190	1.000
5-component system - Parallel system	27.190	1.000
10-component system - Series system	27.198	1.000
10-component system - Parallel system	27.198	1.000
10-component system - 5*p*×2*s* SP system	27.198	1.000
10-component system - 5*s*×2*p* SP system	27.198	1.000
15-component system - Series system	27.198	1.000
15-component system - Parallel system	27.198	1.000
15-component system - 5*p*×3*s* SP system	27.198	1.000
15-component system - 5*s*×3*p* SP system	27.198	1.000
20-component system - Series system	27.198	1.000
20-component system - Parallel system	27.198	1.000
20-component system - 5*p*×4*s* SP system	27.198	1.000
20-component system - 10*p*×2*s* SP system	27.198	1.000
20-component system - 5*s*×4*p* SP system	27.198	1.000
20-component system - 10*s*×2*p* SP system	27.198	1.000
25-component system - Series system	27.201	1.000
25-component system - Parallel system	27.201	1.000
25-component system - 5*p*×5*s* SP system	27.201	1.000
25-component system - 5*s*×5*p* SP system	27.201	1.000

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,LN}(R) = 27.194$

Table 33. $E_{cs}(R)$ **and** η_R **of different systems associated with the case of** $\rho(R_i,R_j) = 1.0$ **using the MCS-based method when** R **and** P **follow lognormal distribution;** $50 \leq N \leq 100$**.**

System	$E_{cs}(R)$	η_R
50-component system - Series system	27.201	1.000
50-component system - Parallel system	27.201	1.000
50-component system - 5*p*×10*s* SP system	27.201	1.000
50-component system - 10*p*×5*s* SP system	27.201	1.000
50-component system - 5*s*×10*p* SP system	27.201	1.000
50-component system - 10*s*×5*p* SP system	27.201	1.000
100-component system - Series system	27.203	1.000
100-component system - Parallel system	27.203	1.000
100-component system - 5*p*×10*s* SP system	27.203	1.000
100-component system - 10*p*×10*s* SP system	27.203	1.000
100-component system - 20*p*×5*s* SP system	27.203	1.000
100-component system - 5*s*×20*p* SP system	27.203	1.000
100-component system - 10*s*×10*p* SP system	27.203	1.000
100-component system -20*s*×5*p* SP system	27.203	1.000

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,LN}(R) = 27.194$

It is observed from these tables that in the no correlation and partial correlation cases:

(a) η_R of the series and *mp*×*ns* SP systems that have the same number of parallel components (i.e., *m* is the same in these SP systems) become larger as the number of components increases; however, the contrary is observed in the parallel and *ms*×*np* SP systems which have the same number of series components (i.e., *m* is the same); and

(b) the redundancy factors associated with normal and lognormal distributions are close; this indicates that the effect of distribution type on the redundancy factor is not significant. In the perfect correlation case ($\rho(R_i,R_j) = 1.0$), the redundancy factors are always equal to 1.0 regardless of system type, number of components in the system, and distribution type.

For the investigated systems associated with different correlation cases, the component reliability indices β_{cs} can be found after the $E_{cs}(R)$ is obtained. Table 34 and Table 35 present the component reliability indices β_{cs} of series and parallel systems consisting of N components (N = 1, 2, 3, 5, 10, 15, 20, 25, 50, 100) associated with three correlation cases when R and P follow normal and lognormal distribution, respectively.

Figure 28 shows the variations of the component reliability and redundancy factor in series and parallel systems due to the increase in the number of components. It is noticed that:

(a) as the number of components increases, the component reliability index increases in series systems, while it decreases in parallel systems;

(b) for series systems, the component reliability indices associated with the normal distribution are higher than those associated with the lognormal distribution in the no correlation and partial correlation cases; however, for parallel systems, contrary conclusions are found;

(c) the effect of the distribution type of R and P on the redundancy factor is not significant, especially in the series systems;

(d) in the perfect correlation case, the component reliability index is equal to 3.5 and the redundancy factor equals 1.0, which indicates that they are not affected by any of the aforementioned parameters.

Table 34. Component reliability index of different systems associated with different correlation cases when *R* and *P* follow normal distribution; $1 \leq N \leq 100$.

Component / System	$\rho(R_i,R_j) = 0$	$\rho(R_i,R_j) = 0.5$	$\rho(R_i,R_j) = 1.0$
1-component	3.50	3.50	3.50
2-component system - Series system	3.63	3.60	3.50
2-component system - Parallel system	3.26	3.34	3.50
3-component system - Series system	3.71	3.66	3.50
3-component system - Parallel system	3.15	3.27	3.50
5-component system - Series system	3.79	3.73	3.50
5-component system - Parallel system	3.04	3.19	3.50
10-component system - Series system	3.90	3.81	3.50
10-component system - Parallel system	2.92	3.10	3.50
15-component system - Series system	3.97	3.86	3.50
15-component system - Parallel system	2.86	3.06	3.50
20-component system - Series system	4.00	3.89	3.50
20-component system - Parallel system	2.82	3.03	3.50
25-component system - Series system	4.04	3.91	3.50
25-component system - Parallel system	2.79	3.01	3.50
50-component system - Series system	4.14	3.98	3.50
50-component system - Parallel system	2.71	2.94	3.50
100-component system - Series system	4.23	4.05	3.50
100-component system - Parallel system	2.65	2.90	3.50

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

Table 35. Component reliability index of different systems associated with different correlation cases when *R* and *P* follow lognormal distribution; $1 \leq N \leq 100$.

Component / System	$\rho(R_i,R_j) = 0$	$\rho(R_i,R_j) = 0.5$	$\rho(R_i,R_j) = 1.0$
1-component	3.50	3.50	3.50
2-component system - Series system	3.57	3.55	3.50
2-component system - Parallel system	3.38	3.42	3.50
3-component system - Series system	3.62	3.59	3.50
3-component system - Parallel system	3.33	3.38	3.50
5-component system - Series system	3.66	3.62	3.50
5-component system - Parallel system	3.27	3.35	3.50
10-component system - Series system	3.72	3.67	3.50
10-component system - Parallel system	3.20	3.30	3.50
15-component system - Series system	3.75	3.68	3.50
15-component system - Parallel system	3.17	3.27	3.50
20-component system - Series system	3.78	3.70	3.50
20-component system - Parallel system	3.15	3.25	3.50
25-component system - Series system	3.79	3.71	3.50
25-component system - Parallel system	3.13	3.24	3.50
50-component system - Series system	3.84	3.74	3.50
50-component system - Parallel system	3.08	3.21	3.50
100-component system - Series system	3.88	3.77	3.50
100-component system - Parallel system	3.03	3.18	3.50

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,LN}(R) = 27.194$

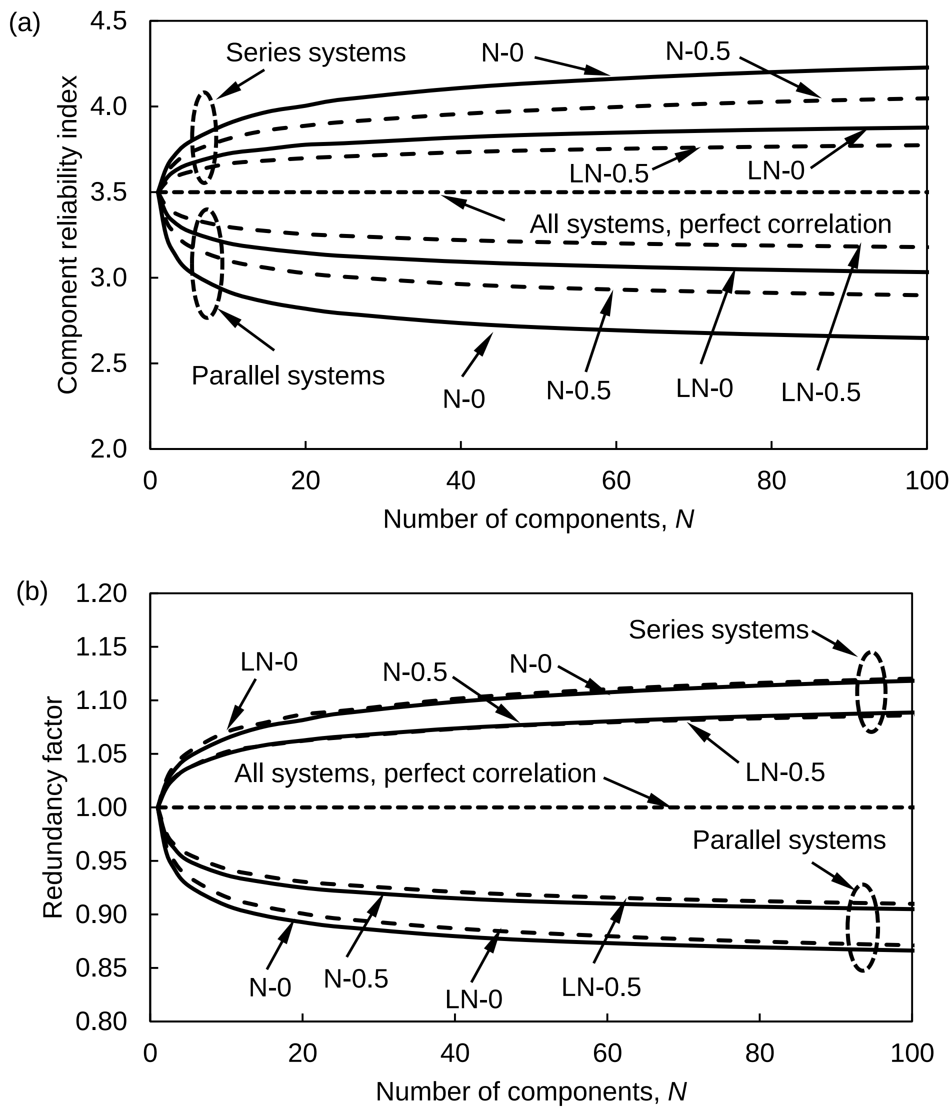


Figure 28. Graph. The effects of number of components on (a) component reliability index β_{cs}; and (b) redundancy factor η_R (Note: "N" denotes normal distribution; "LN" denotes lognormal distribution; "0" denotes $\rho(R_i,R_j) = 0$; "0.5" denotes $\rho(R_i,R_j) = 0.5$; $V(R)$=0.05; $V(P)$=0.3; $E(P)$=10; and β_c =3.5).

3.6 Redundancy Factors of Ductile and Brittle Systems

This section firstly investigates the redundancy factors of ductile and brittle systems with up to four components. Then, the redundancy factor of systems consisting of both ductile and brittle components (denoted as "mixed system") is evaluated. Finally, the effects of post-failure material behavior factor on the redundancy factor are studied. Ductile system in this study refers to the system whose components are all ductile, that is, the components resistances are not reduced after failure. If a component resistance is decreased to zero after failure, it is called brittle component and the system consisting of brittle component is named brittle system.

3.6.1 Redundancy Factor of Ductile Systems

For a single ductile component whose R and P are treated as normally distributed random variables with $V(R)$, $V(P)$, and $E(P)$ equal to 0.05, 0.3, and 10, respectively, the mean resistance of component $E_c(R)$ is found to be 21.132 to make the component reliability index equal to 3.5. Then, consider a system consisting of two ductile components which are identical with the single component just mentioned. Two systems can be formed: series and parallel. Since failure of any component in the series system leads to system failure, the reliability index of the series system is not affected by the material behavior of the components. Consequently, the redundancy factor of series system is also independent of the components material behavior. Therefore, the evaluation of redundancy factor in Section 3.6 is mainly focused on the parallel and series-parallel systems.

For the two-component ductile parallel system, the resistances of the two components are denoted as R_1 and R_2, respectively. The load acting on the system is $2P$ so that the load effect distributed to each component is P. The ductile behavior in this report refers to elastic-perfectly-plastic. For a ductile component, the loads it takes before and after its failure are the same, which is equal to its resistance. Therefore, the applied load will not redistribute inside the system if either component fails. The statistical parameters associated with R and P in this system are the same as those associated with the single component mentioned previously. Since the failure modes of ductile systems are independent of the failure sequence of components, the limit state equation of the parallel system is

$$g = R_1 + R_2 - 2P = 0$$

Figure 29. Equation. Limit state equation associated with failure of a two-component ductile parallel system.

Three correlation cases among the resistances of components are considered:

(a) $\rho(R_1,R_2) = 0$, no correlation;

(b) $\rho(R_1,R_2) = 0.5$, partial correlation; and

(c) $\rho(R_1,R_2) = 1.0$, perfect correlation.

Based on the statistical parameters and limit state equation, the mean resistances of components in the two-component ductile parallel system associated with the three correlation cases ($\rho(R_1,R_2)$ = 0, 0.5, 1.0) when the system reliability is 3.5 are calculated using the MCS-based method mentioned in Section 3.5. The results are found to be $E_{cs}(R)$ = 20.810, 20.950, and 21.132, respectively. Dividing $E_{cs}(R)$ by $E_c(R)$ yields the redundancy factors: 0.985 if $\rho(R_1,R_2) = 0$, 0.991 if $\rho(R_1,R_2) = 0.5$, and 1.0 if $\rho(R_1,R_2) = 1.0$. The associated reliability indices of components are 3.40, 3.45, and 3.50, respectively.

In the following example, the number of components in the parallel system is extended from two to three and four, and two types of probability distribution for both resistances and loads (i.e., normal and lognormal) are considered. For the lognormal distribution case, the mean resistance associated with a single component when its $V(R)$, $V(P)$, and $E(P)$ are 0.05, 0.3, and 10, respectively, is $E_c(R) = 27.194$. Assuming the load acting on the three- and four-component system is 3*P* and 4*P*, respectively, the limit state equations associated with the three- and four-component parallel system are

$$g = R_1 + R_2 + R_3 - 3P = 0$$

Figure 30. Equation. Limit state equation associated with failure of a three-component ductile parallel system.

$$g = R_1 + R_2 + R_3 + R_4 - 4P = 0$$

Figure 31. Equation. Limit state equation associated with failure of a four-component ductile parallel system.

where R_i (i = 1,2,3,4) is the resistance of the i_{th} component. By performing the same procedure, the mean resistances, redundancy factors, and component reliability indices associated with normal and lognormal distributions are presented in Table 36. It is noticed that:

(a) increasing the correlation among the resistances of components leads to a higher redundancy factor and component reliability in both distribution cases; and

(b) the redundancy factor and component reliability index associated with lognormal distribution are slightly higher than those associated with normal distribution.

Table 36. Redundancy factor of three-component ductile parallel system associated with normal and lognormal distribution.

Correlation	Normal distribution: Mean resistance	Normal distribution: Redundancy factor	Normal distribution: Component reliability	Lognormal distribution: Mean resistance	Lognormal distribution: Redundancy factor	Lognormal distribution: Component reliability
$\rho(R_i,R_j) = 0$	20.699	0.980	3.37	26.925	0.990	3.46
$\rho(R_i,R_j) = 0.5$	20.910	0.989	3.44	27.065	0.995	3.49
$\rho(R_i,R_j) = 1$	21.132	1.000	3.50	27.194	1.000	3.50

Note: $V(R) = 0.05$; $V(P) = 0.3$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$; $E_{c,LN}(R) = 27.194$

In addition to the four-component ductile parallel system whose limit state equation is presented in Figure 31, a four-component $2p\times 2s$ ductile series-parallel system shown in Figure 17(c) is studied. There are two failure modes associated with its system failure and the corresponding limit state equations are listed in Figure 32. By using the MCS-based method, the mean resistance, redundancy factor, and component reliability index of the four-component ductile parallel and series-parallel systems associated with the normal distribution case are presented in Table 37. It is found that in the no correlation and partial correlation cases, the redundancy factor and component reliability associated with the series-parallel system are higher than those associated with the parallel system.

$$g_1 = R_1 + R_2 - 2P = 0$$

$$g_2 = R_3 + R_4 - 2P = 0$$

Figure 32. Equation. Limit state equations associated with failure of the four-component ductile series-parallel system.

Table 37. Redundancy factor of four-component ductile systems.

Correlation	Parallel system/Mean resistance	Parallel system/Redundancy factor	Parallel system/Component reliability	Series-parallel system/Mean resistance	Series-parallel system/Redundancy factor	Series-parallel system/Component reliability
$\rho(R_i,R_j) = 0$	20.660	0.978	3.36	21.160	1.001	3.51
$\rho(R_i,R_j) = 0.5$	20.893	0.989	3.43	21.231	1.005	3.53
$\rho(R_i,R_j) = 1$	21.132	1.000	3.50	21.132	1.000	3.50

Note: $V(R) = 0.05$; $V(P) = 0.3$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

3.6.2 Redundancy Factor of Brittle Systems

Since a brittle component will not take any load after its fracture failure, the applied load will distribute to other remaining components in brittle systems. Therefore, for a brittle system, different failure sequences lead to different load distributions and thus different failure modes. In order to illustrate the procedure for calculating the redundancy factor of brittle systems, the following two-, three-, and four-component systems are analyzed with respect to the three correlation cases mentioned previously.

Consider the two-component parallel system described in the previous section. Assuming both components are brittle, two different failure modes are anticipated:

(a) mode I caused by failure of component 1 followed by component 2; and

(b) mode II caused by failure of component 2 followed by component 1.

Therefore, the system failure can be evaluated by using the series-parallel system model shown in Figure 33. Based on the assumption that the load applied on the two-component parallel system is $2P$, the limit state equations associated with the two failure modes are given in Figure 34. In Mode I, the load component 1 takes before its failure is R_1; therefore, its limit state equation is g_1 in Figure 34. Since a brittle component loses its capacity after failure, the applied load $2P$ is taken only by component 2 by component 1 fails. Therefore, the limit state equation of component 2 given the failure of component 1 is g_3 in Figure 34. Assuming the resistances and load of components follow normal distribution with the coefficients of variation being 0.05 and 0.3, respectively, the mean resistance that makes the reliability index of each component be 3.5 is 21.132, as presented previously. With the limit state equations and other statistical parameters associated with the components (e.g., $V(R) = 0.05$, $V(P) = 0.3$, $E(P) = 10$), the mean resistance of components associated with three correlation cases when both R and P follow normal distribution are found to be: 21.585 if $\rho(R_i,R_j) = 0$, 21.481 if $\rho(R_i,R_j) = 0.5$, and 21.132 if $\rho(R_i,R_j) = 1.0$. Dividing these resistances by $E_c(R)$ = 21.132 yields the redundancy factors: 1.021 if $\rho(R_i,R_j)$ = 0, 1.017 if $\rho(R_i,R_j)$ = 0.5, and 1.0 if $\rho(R_i,R_j)$ = 1.0. The associated reliability indices of components are: 3.63 if $\rho(R_i,R_j)$ = 0, 3.60 if $\rho(R_i,R_j)$ = 0.5, and 3.50 if $\rho(R_i,R_j)$ = 1.0. It is noticed that the redundancy factor of the brittle parallel system decreases as the correlation among the resistances of components becomes stronger.

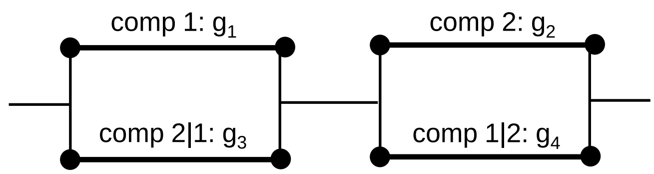


Figure 33. Graph. Failure modes of two-component brittle parallel system.

$$g_1 = R_1 - P = 0 \qquad g_2 = R_2 - P = 0$$

$$g_3 = R_2 - 2P = 0 \qquad g_4 = R_1 - 2P = 0$$

Figure 34. Equations. Limit state equations associated with failure of the two-component brittle parallel system.

As the number of brittle components in the parallel system increases to three, the possible failure modes increase to six, as shown in Figure 35. Assuming the load acting on the three-component system is 3*P*, the limit state equations associated with all the failure modes are listed in Figure 36. Assuming the same statistical parameters of components as those used in the two-component system (e.g., $V(R)$ = 0.05, $V(P)$ = 0.3, $E(P)$ = 10), the mean resistances, redundancy factors, and the associated reliability indices of components considering three correlation cases are obtained, as presented in Table 38. It is observed that:

(a) the redundancy factor associated with lognormal distribution is slightly higher than that associated with normal distribution;

(b) the redundancy factors and reliability indices of components associated with both distributions decrease as the correlation among resistances of components increases; and

(c) the components reliability related to normal distribution is higher than that related to lognormal distribution.

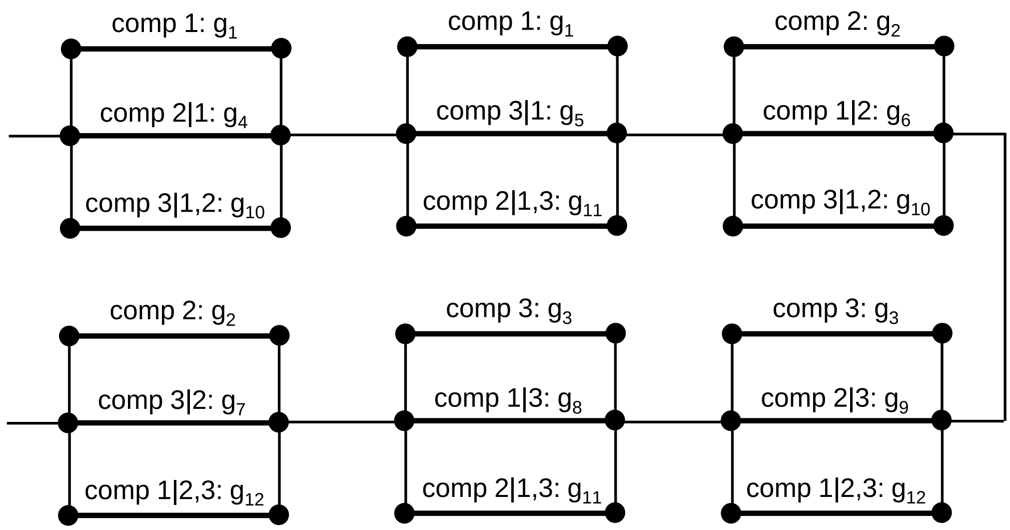


Figure 35. Graph. Failure modes of three-component brittle parallel system.

$$g_1 = R_1 - P = 0 \qquad g_2 = R_2 - P = 0 \qquad g_3 = R_3 - P = 0$$

$$g_4 = R_2 - 1.5P = 0 \qquad g_5 = R_3 - 1.5P = 0 \qquad g_6 = R_1 - 1.5P = 0$$

$$g_7 = R_3 - 1.5P = 0 \qquad g_8 = R_1 - 1.5P = 0 \qquad g_9 = R_2 - 1.5P = 0$$

$$g_{10} = R_3 - 3P = 0 \qquad g_{11} = R_2 - 3P = 0 \qquad g_{12} = R_1 - 3P = 0$$

Figure 36. Equations. Limit state equations associated with failure of the three-component brittle parallel system.

Table 38. Redundancy factor of three-component brittle parallel system associated with normal and lognormal distribution.

Correlation	Normal distribution: Mean resistance	Normal distribution: Redundancy factor	Normal distribution: Component reliability	Lognormal distribution: Mean resistance	Lognormal distribution: Redundancy factor	Lognormal distribution: Component reliability
$\rho(R_i,R_j) = 0$	21.827	1.033	3.71	28.190	1.037	3.62
$\rho(R_i,R_j) = 0.5$	21.672	1.026	3.66	27.940	1.027	3.59

$\rho(R_i,R_j) = 1$	21.132	1.000	3.50	27.194	1.000	3.50

Note: $V(R) = 0.05$; $V(P) = 0.3$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$; $E_{c,LN}(R) = 27.194$

For the four-component brittle parallel system, the number of possible failure modes is 24 (i.e., 4!). The system can be modeled as the $4p \times 24s$ series-parallel system shown in Figure 37 and the failure sequence for each failure mode is presented in the sub-parallel systems. The associated limit state equations are listed in Figure 38. A four-component brittle system can also be formed as a $2p\times2s$ series-parallel system (Figure 17(c)) other than the parallel system just analyzed. The load acting on this series-parallel system is $2P$. The failure modes of each sub-parallel system are similar to those of the two-component brittle system shown in Figure 33. Therefore, the system has totally four failure modes, as presented in Figure 39, and the associated limit state equations are given in Figure 40.

With the limit state equations and the statistical parameters of the resistances and load (i.e., $V(R) = 0.05$, $V(P) = 0.3$), the mean resistances, redundancy factors, and reliability indices of components associated with the four-component brittle parallel and $2p\times2s$ series-parallel systems when R and P are normally distributed are calculated. The results are displayed in Table 39. It is noted that the redundancy factors and components reliability indices associated with both systems are almost the same and they decrease as the correlation among the resistances of components increases.

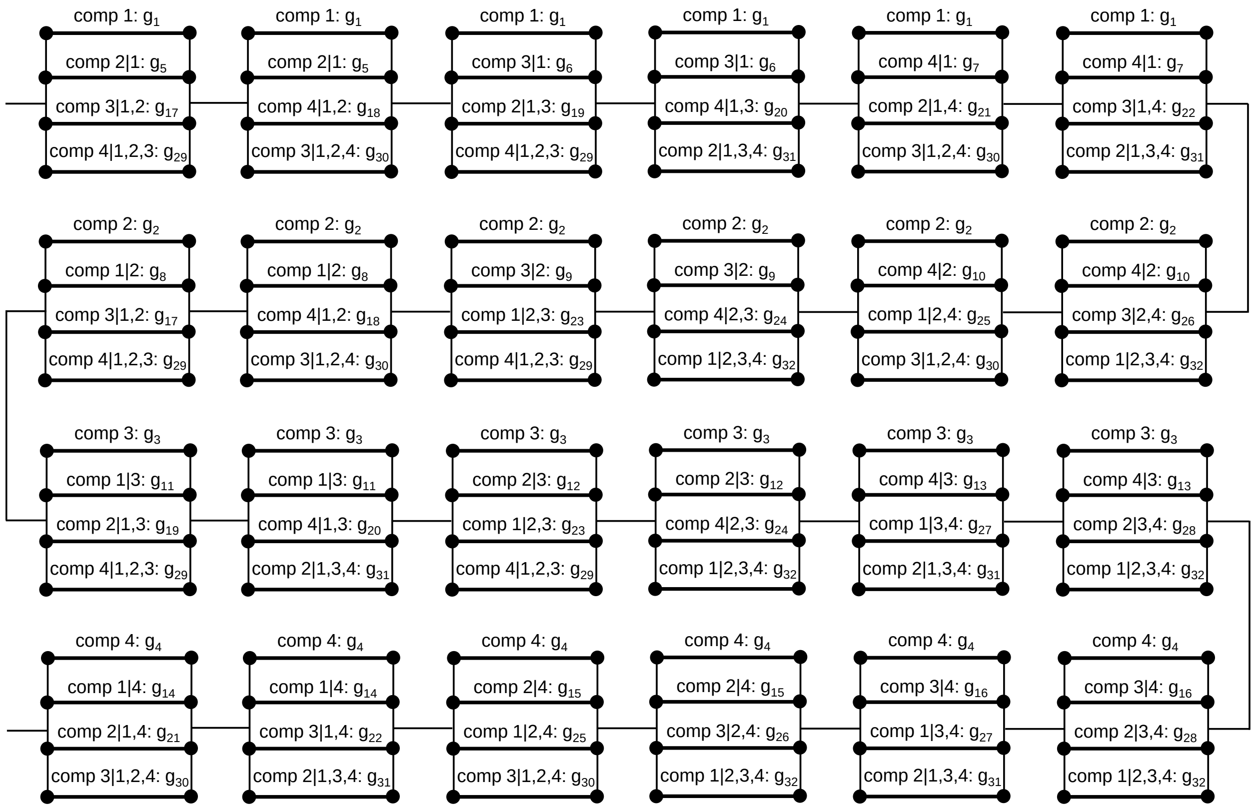


Figure 37. Graph. Failure modes of four-component brittle parallel system.

$$g_1 = R_1 - P = 0 \qquad g_2 = R_2 - P = 0 \qquad g_3 = R_3 - P = 0 \qquad g_4 = R_4 - P = 0$$

$$g_5 = R_2 - 1.33P = 0 \qquad g_6 = R_3 - 1.33P = 0 \qquad g_7 = R_4 - 1.33P = 0 \qquad g_8 = R_1 - 1.33P = 0$$

$$g_9 = R_3 - 1.33P = 0 \qquad g_{10} = R_4 - 1.33P = 0 \qquad g_{11} = R_1 - 1.33P = 0 \qquad g_{12} = R_2 - 1.33P = 0$$

$$g_{13} = R_4 - 1.33P = 0 \qquad g_{14} = R_1 - 1.33P = 0 \qquad g_{15} = R_2 - 1.33P = 0 \qquad g_{16} = R_3 - 1.33P = 0$$

$$g_{17} = R_3 - 2P = 0 \qquad g_{18} = R_4 - 2P = 0 \qquad g_{19} = R_2 - 2P = 0 \qquad g_{20} = R_4 - 2P = 0$$

$$g_{21} = R_2 - 2P = 0 \qquad g_{22} = R_3 - 2P = 0 \qquad g_{23} = R_1 - 2P = 0 \qquad g_{24} = R_4 - 2P = 0$$

$$g_{25} = R_1 - 2P = 0 \qquad g_{26} = R_3 - 2P = 0 \qquad g_{27} = R_1 - 2P = 0 \qquad g_{28} = R_2 - 2P = 0$$

$$g_{29} = R_4 - 4P = 0 \qquad g_{30} = R_3 - 4P = 0 \qquad g_{31} = R_2 - 4P = 0 \qquad g_{32} = R_1 - 4P = 0$$

Figure 38. Equations. Limit state equations associated with failure of the four-component brittle parallel system.

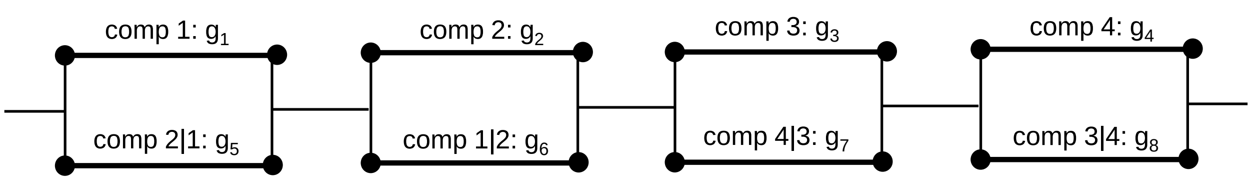


Figure 39. Graph. Failure modes of four-component brittle series-parallel system.

$$g_1 = R_1 - P = 0 \qquad g_2 = R_2 - P = 0 \qquad g_3 = R_3 - P = 0 \qquad g_4 = R_4 - P = 0$$

$$g_5 = R_2 - 2P = 0 \qquad g_6 = R_1 - 2P = 0 \qquad g_7 = R_4 - 2P = 0 \qquad g_8 = R_3 - 2P = 0$$

Figure 40. Equations. Limit state equations associated with failure of the four-component brittle series-parallel system.

Table 39. Redundancy factor of four-component brittle systems.

Correlation	Parallel system - Mean resistance	Parallel system - Redundancy factor	Parallel system - Component reliability	$2p\times2s$ Series-parallel system - Mean resistance	$2p\times2s$ Series-parallel system - Redundancy factor	$2p\times2s$ Series-parallel system - Component reliability
$\rho(R_i,R_j) = 0$	21.999	1.041	3.75	22.009	1.042	3.76
$\rho(R_i,R_j) = 0.5$	21.805	1.032	3.70	21.805	1.032	3.70
$\rho(R_i,R_j) = 1$	21.132	1.000	3.50	21.132	1.000	3.50

Note: $V(R) = 0.05$; $V(P) = 0.3$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

By comparing the results associated with the ductile and brittle systems consisting of up to four components (Table 36 to Table 39), the following conclusions are drawn:

1. In both ductile and brittle parallel systems, the redundancy factor associated with lognormal distribution is slightly higher than that associated with normal distribution.
2. The redundancy factor of ductile parallel system is at most 1.0 while its counterpart of brittle parallel systems is at least 1.0. Increasing the correlation among the resistances of components leads to a higher and lower redundancy factor in ductile and brittle system, respectively. In the ductile case, the redundancy factor associated with the $2p\times2s$ series-parallel system is higher than that associated with the four-component parallel system; while in the brittle case, the redundancy factors associated with both systems are almost the same.

3.6.3 Redundancy Factor of Ductile-Brittle Systems

In the above two subsections, the redundancy factor of systems consisting of only ductile or brittle components is investigated. However, there might be some cases where both types of material behaviors are involved in the system. Therefore, it is necessary to study the redundancy factor of systems having both ductile and brittle components (called "mixed systems"). In order to be consistent with the previous sections, the systems consisting of two, three, and four components are studied in this subsection.

For the two-component parallel system, there is only one combination for the mixed system: one component is ductile and the other one is brittle (denoted as "1 ductile & 1 brittle"). However, as the number of components increases, the number of combinations for mixed systems also increases. For three-component parallel system, two mixed systems are considered: 1 ductile & 2 brittle, and 2 ductile & 1 brittle. Similarly, three mixed systems can be formed for four-component parallel system: 1 ductile & 3 brittle, 2 ductile & 2 brittle, and 3 ductile & 1 brittle. For the four-component 2*p*×2*s* series-parallel system, there are two combinations associated with the 2 ductile & 2 brittle case:

(a) 2 ductile & 2 brittle Case A, where 2 ductile components are located in the same sub-parallel system; and

(b) 2 ductile & 2 brittle Case B, where 2 ductile components are located in two sub-parallel systems, as shown in Figure 41.

Therefore, four mixed systems are investigated with respect to the 2*p*×2*s* series-parallel system.

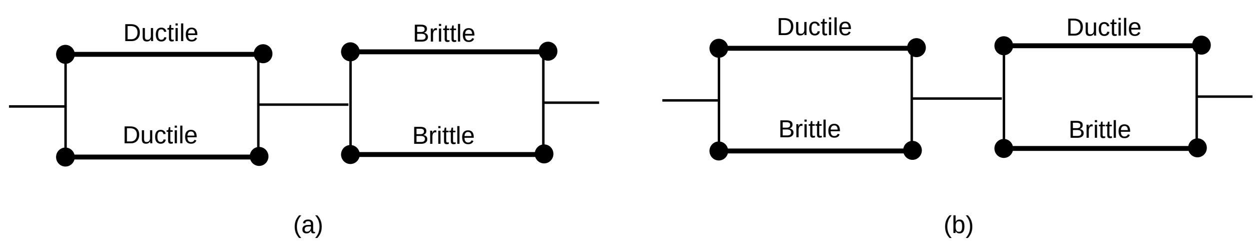


Figure 41. Graph. Four-component mixed series-parallel systems: (a) 2 ductile & 2 brittle Case A; and (b) 2 ductile & 2 brittle Case B.

Since a mixed system has brittle component(s), the failure mode of the system is determined by the failure sequence of components. Therefore, all the possible failure modes and the associated limit state equations need to be identified as those presented in the brittle case. Consider the two-component mixed parallel system as an example: its failure modes are the same as those in the brittle case, as shown in Figure 33; however, the associated limit state equations are different from those in the brittle case due to the existence of a ductile component in this mixed system. Assuming component 1 is ductile, component 2 is brittle, and the applied load is 2*P*, the limit state equations are:

$$g_1 = R_1 - P = 0 \qquad g_2 = R_2 - P = 0$$

$$g_3 = R_1 + R_2 - 2P = 0 \qquad g_4 = R_1 - 2P = 0$$

Figure 42. Equations. Limit state equations associated with failure of the two-component mixed parallel system.

g_1 and g_3 are associated with the failure mode I where component 2 fails after component 1. Since component 1 is ductile, the load it takes after failure is still R_1; therefore, the applied load is taken by both components 1 and 2 in the limit state equation g_3. However, for the other failure mode where the brittle component (component 2) fails first, the load acting on the survived component (component 1) is 2*P* because the brittle component cannot take any load once it fails. The limit state equations associated with this failure mode are g_2 and g_4.

Similarly, the limit state equations associated with three- and four-component mixed systems can be identified based on the failure modes shown in Figure 35, Figure 37, and Figure 39. Assuming *R* and *P* are normally distributed with $V(R) = 0.05$ and $V(R) = 0.3$, the mean resistance, redundancy factors, and reliability indices of components of the mixed systems considering three correlation cases are presented in Table 40, Table 41, and Table 42.

It is observed that:

(a) the redundancy factors of the investigated mixed parallel systems are all at least 1.0 due to the existence of brittle component(s) in the systems;

(b) compared with the redundancy factors in the ductile (Table 36 and Table 37) and brittle (Table 38 and Table 39) cases, the results associated with the mixed systems are in between;

(c) for the $2p\times2s$ series-parallel system, the redundancy factors associated with 2 ductile & 2 brittle cases A and B are the same; and

(d) the redundancy factor in mixed system decreases as the correlation among the resistances of components increases; this is similar to the finding in brittle system.

Table 40. Redundancy factor of mixed systems associated with the case $\rho(R_i,R_j) = 0$ when R and P follow normal distribution.

System	$E_{cs}(R)$	η_R	β_{cs}
2-component parallel system - 1 ductile & 1 brittle	21.280	1.007	3.55
3-component parallel system - 1 ductile & 2 brittle	21.630	1.024	3.65
3-component parallel system - 2 ductile & 1 brittle	21.300	1.008	3.55
4-component parallel system - 1 ductile & 3 brittle	21.850	1.034	3.71
4-component parallel system - 2 ductile & 2 brittle	21.640	1.024	3.65
4-component parallel system - 3 ductile & 1 brittle	21.319	1.009	3.56
4-component series-parallel system (2×2 SP system) 1 ductile & 3 brittle	21.850	1.034	3.71
4-component series-parallel system (2×2 SP system) 2 ductile & 2 brittle Case A	21.680	1.026	3.66
4-component series-parallel system (2×2 SP system) 2 ductile & 2 brittle Case B	21.680	1.026	3.66
4-component series-parallel system (2×2 SP system) 3 ductile & 1 brittle	21.440	1.015	3.59

Note: $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_c(R) = 21.132$

Figure 43 shows the effects of the number of brittle components in the mixed parallel system on the redundancy factor. It is seen that:

(a) in the no correlation and partial correlation cases, the redundancy factor in mixed parallel system becomes larger as the number of brittle components increases; and

(b) increasing the correlation among the resistances of components leads to a higher redundancy factor in ductile system while a lower redundancy factor in mixed and brittle system.

Table 41. Redundancy factor of mixed systems associated with the case $\rho(R_i,R_j)$ = 0.5 when *R* and *P* follow normal distribution.

System	$E_{cs}(R)$	η_R	β_{cs}
2-component parallel system - 1 ductile & 1 brittle	21.260	1.006	3.53
3-component parallel system - 1 ductile & 2 brittle	21.530	1.019	3.62
3-component parallel system - 2 ductile & 1 brittle	21.290	1.007	3.55
4-component parallel system - 1 ductile & 3 brittle	21.700	1.027	3.67
4-component parallel system - 2 ductile & 2 brittle	21.550	1.020	3.62
4-component parallel system - 3 ductile & 1 brittle	21.318	1.009	3.55
4-component series-parallel system (2×2 SP system) 1 ductile & 3 brittle	21.700	1.027	3.67
4-component series-parallel system (2×2 SP system) 2 ductile & 2 brittle Case A	21.585	1.021	3.63
4-component series-parallel system (2×2 SP system) 2 ductile & 2 brittle Case B	21.585	1.021	3.63
4-component series-parallel system (2×2 SP system) 3 ductile & 1 brittle	21.420	1.014	3.59

Note: $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_c(R) = 21.132$

Table 42. Redundancy factor of mixed systems associated with the case $\rho(R_i,R_j)$ = 1.0 when *R* and *P* follow normal distribution.

System	$E_{cs}(R)$	η_R	β_{cs}
2-component parallel system - 1 ductile & 1 brittle	21.132	1.000	3.50
3-component parallel system - 1 ductile & 2 brittle	21.132	1.000	3.50
3-component parallel system - 2 ductile & 1 brittle	21.132	1.000	3.50
4-component parallel system - 1 ductile & 3 brittle	21.132	1.000	3.50
4-component parallel system - 2 ductile & 2 brittle	21.132	1.000	3.50
4-component parallel system - 3 ductile & 1 brittle	21.132	1.000	3.50
4-component series-parallel system- (2×2 SP system) 1 ductile & 3 brittle	21.132	1.000	3.50
4-component series-parallel system (2×2 SP system) 2 ductile & 2 brittle Case A	21.132	1.000	3.50
4-component series-parallel system (2×2 SP system) 2 ductile & 2 brittle Case B	21.132	1.000	3.50
4-component series-parallel system (2×2 SP system) 3 ductile & 1 brittle	21.132	1.000	3.50

Note: $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_c(R) = 21.132$

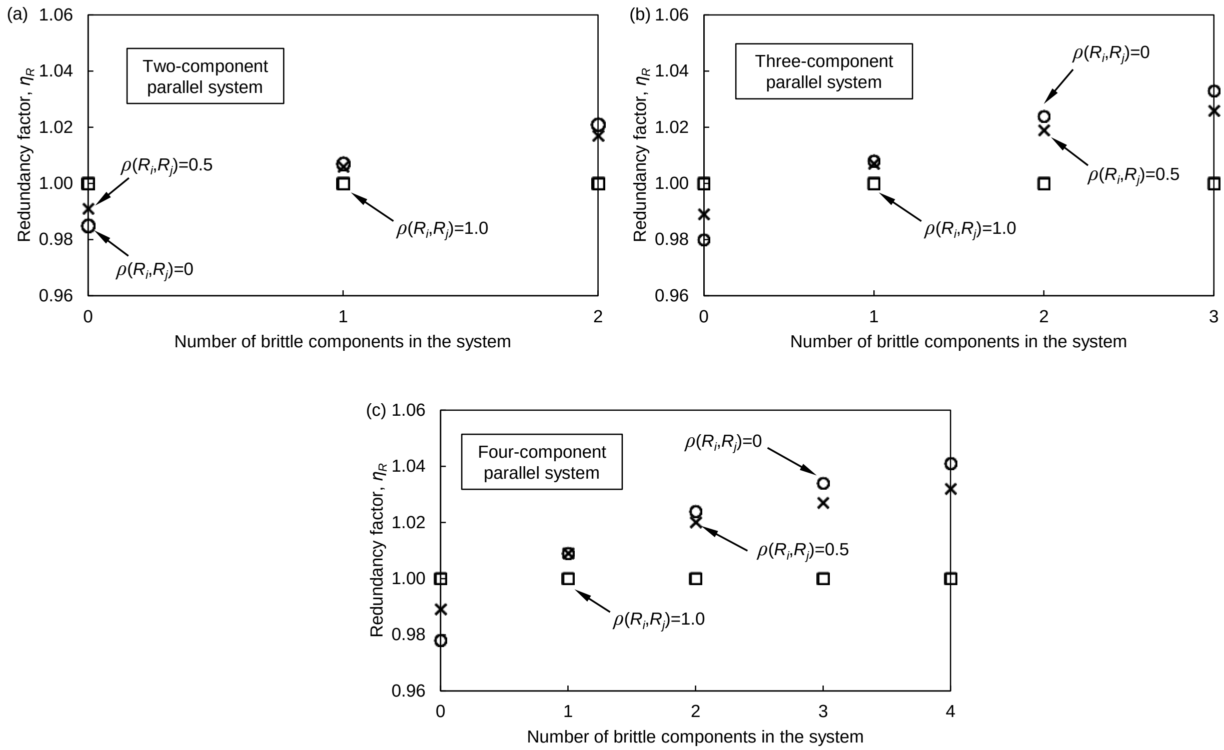


Figure 43. Graph. Effects of the number of brittle components on the redundancy factor in the parallel systems consisting of (a) two components; (b) three components; and (c) four components.

3.6.4 Effects of Post-failure Material Behavior Factor on Redundancy Factor

The post-failure behavior factor (denoted as "δ") represents the percentage of strength remaining after failure. In the previous subsections, the redundancy factor of the ductile and brittle cases in which the post-failure behavior factor is 1 and 0, respectively, was investigated. It was found that the results associated with the two extreme post-failure behavior cases are significantly different. However, it is not clear how the redundancy factor varies when δ increases from 0 to 1. Therefore, in this subsection, the redundancy factor associated with intermediate post-failure behavior ($0 < \delta < 1$) is studied with respect to parallel systems consisting of two, three, and four components. The failure modes of these systems are similar to those of brittle systems. The only difference is that the remaining capacity of component *i* after its failure is $\delta \cdot R_i$ instead of zero.

Assuming the post-failure behavior factors of all the components are the same and the resistances and load of components follow normal distribution with the aforementioned parameters (e.g., $V(R) = 0.05$, $V(P) = 0.3$), the effects of the post-failure behavior factor on the redundancy factor in two-, three-, and four-component parallel systems associated with three correlation cases are shown in Figure 44 and Figure 45.

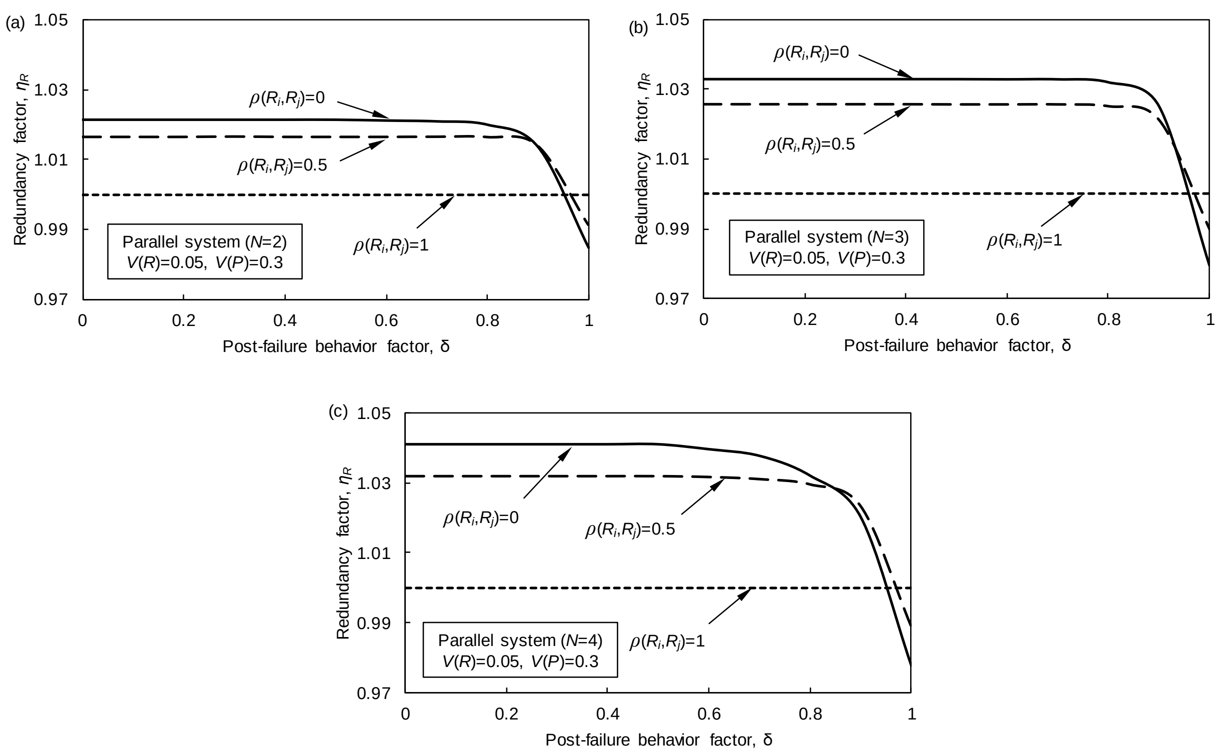


Figure 44. Graph. Effects of post-failure behavior factor δ on the redundancy factor η_R in the parallel systems consisting of (a) two components; (b) three components; and (c) four components.

It is noticed that:

(a) as δ increases in the no correlation and partial correlation cases, the redundancy factors associated with the three parallel systems initially remain the same and then decrease significantly;

(b) the values of δ corresponding to the turning points of the curves are greater than 0.5 in all systems; this indicates that for most intermediate post-failure behavior cases, the redundancy factors are almost the same as those in the brittle behavior case;

(c) in the no correlation and partial correlation cases, the region of δ in which the redundancy factor remains the same increases as the correlation among the resistances of components becomes stronger, while in the perfect correlation case, the redundancy factor is not affected by δ;

(d) as δ increases from 0 to 1, the sequence of the redundancy factors associated with the three correlations becomes reverse in all systems (e.g., the redundancy factor associated with the $\rho(R_i,R_j) = 0$ case is the highest in the brittle case but the lowest in the ductile case);

(e) the differences in the redundancy factors associated with the three parallel systems becomes less significant along with the increase of δ; when δ = 1 (i.e., ductile), the redundancy factors associated with the three parallel systems are almost the same.

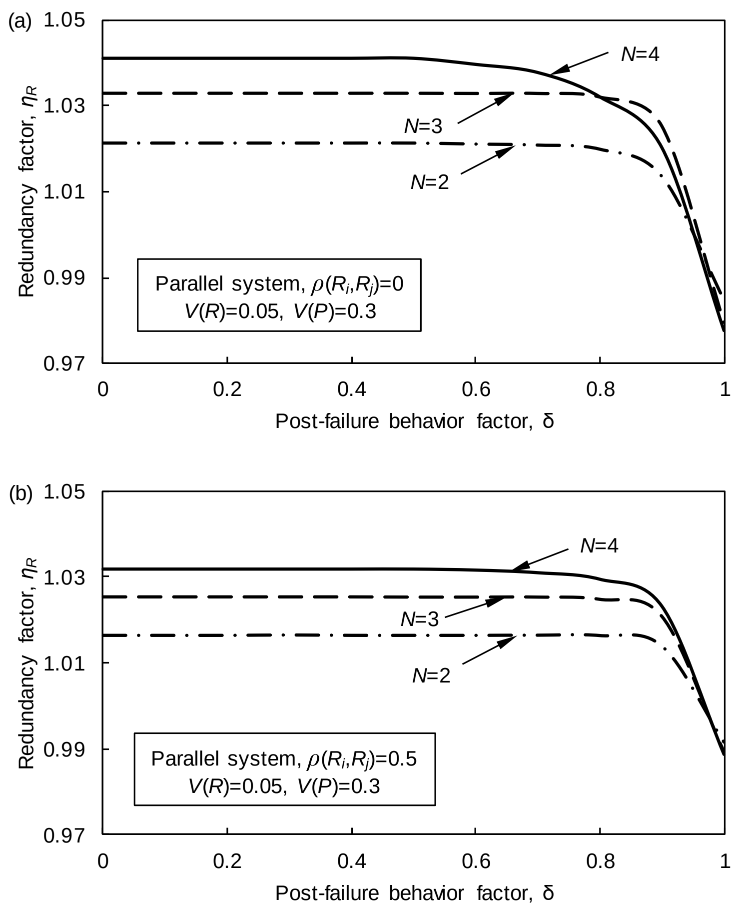


Figure 45. Graph. Effects of post-failure behavior factor δ on redundancy factor η_R in the (a) no correlation case; and (b) partial correlation case.

In the calculation of redundancy factor, the mean resistance of components ($E_{cs}(R)$) when the system reliability index is 3.5 is obtained. Substituting $E_{cs}(R)$ into the component reliability analysis yields the reliability indices of components. Figure 46 and Figure 47 plot the effects of the post-failure behavior factor on the component reliability index associated with the three parallel systems considering three correlation cases. The conclusions from these two figures are similar to those regarding redundancy factors drawn from Figure 44 and Figure 45. In addition, it is seen that the reliability index of components when δ = 0 (brittle) is greater than 3.5, while its value when δ = 1 (ductile) is less than 3.5. This is because the brittle systems are much less redundant than ductile systems and, therefore, larger redundancy factor (η_R > 1.0) needs to be applied to penalize the brittle components and the components are designed more conservatively ($\beta_{cs} > 3.5$); while in the ductile case, smaller redundancy factor ($\eta_R < 1.0$) can be used to achieve a more economical component design ($\beta_{cs} < 3.5$).

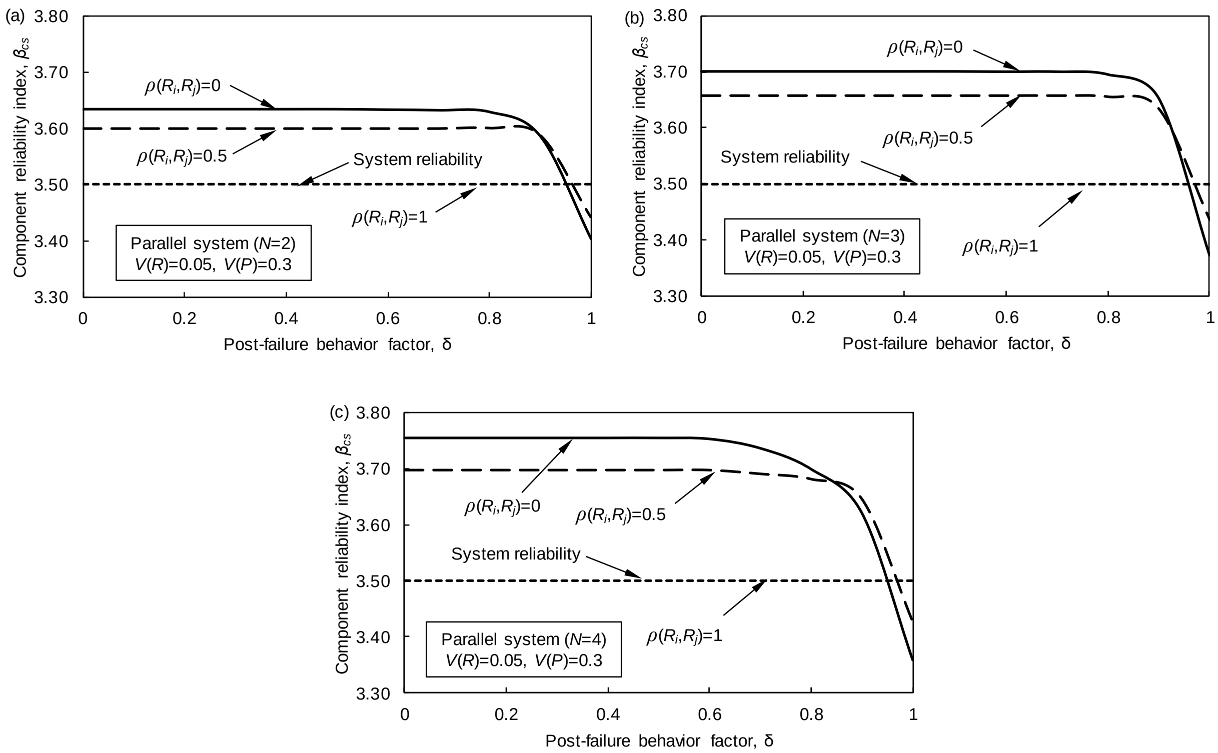


Figure 46. Graph. Effects of post-failure behavior factor δ on component reliability index in the parallel systems consisting of (a) two components; (b) three components; and (c) four components.

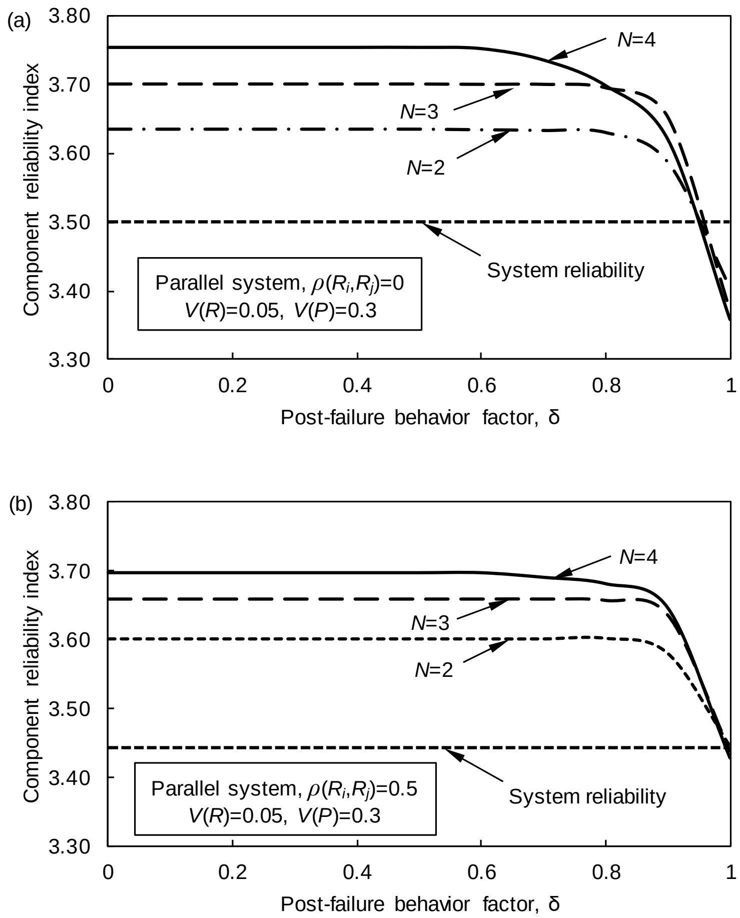


Figure 47. Graph. Effects of post-failure behavior factor δ on component reliability index in the (a) no correlation case; and (b) partial correlation case.

3.7 Limit States for Component Design

In the current AASHTO LRFD bridge design specifications,[(1)] each component and connection shall satisfy the following equation for each limit state during the design:

$$\sum \eta_i \gamma_i Q_i \leq \phi R_n = R_r$$

Figure 48. Equation. Strength limit state in AASHTO specification.

in which, γ_i is a load factor; Q_i is force effect; ϕ is a resistance factor; R_n is nominal resistance; R_r is factored resistance; and η_i is a load modifier relating to ductility, redundancy, and operational classification, given as

$$\eta_i = \eta_D \eta_R \eta_l$$

Figure 49. Equation. Load modifier.

where η_D is a factor relating to ductility; η_R is a factor relating to redundancy; and η_l is a factor relating to operational classification. Therefore, the equation in Figure 48 can be rewritten as follows

$$\sum \eta_D \eta_R \eta_l \gamma_i Q_i \leq \phi R_n = R_r$$

Figure 50. Equation. Strength limit state in AASHTO specification.

It is noticed that, as stated previously, η_R is considered on the load effect side in the above limit state equation and its value is determined based on a very general classification of redundancy levels:

(a) $\eta_R \geq 1.05$ for nonredundant members;

(b) $\eta_R = 1.00$ for conventional level of redundancy; and

(c) $\eta_R \geq 0.95$ for exceptional levels of redundancy.[1]

However, in this report, the proposed redundancy factor, which is also denoted as η_R, is more specifically investigated for different system arrangements, different correlation cases among the resistances of components, and different number of components in the system. This factor takes into account the system redundancy from the resistance side.

The procedure for applying this redundancy factor in structural design consists of two steps: (a) calculating the resistance R'_r

$$\phi R_n^{'} = R_r^{'} \geq \sum \eta_D \eta_l \gamma_i Q_i$$

Figure 51. Equation. Resistance without including redundancy factor.

Equation in Figure 51 does not consider the factor relating to redundancy on the load effect side; therefore, the effect of redundancy is not reflected in the resistance R'_r;

(b) applying the redundancy factor η_R to the resistance R'_r to obtain the final factored resistance R_r, as:

$$R_r = \eta_R R_r^{'}$$

Figure 52. Equation. Factored resistance.

By substituting the equation in Figure 52 into equation in Figure 51, the equation in Figure 51 can be rewritten as follows:

$$\sum \eta_D \eta_l \gamma_i Q_i \leq \phi R_n^{'} = R_r^{'} = \frac{R_r}{\eta_R}$$

Figure 53. Equation. Strength limit state.

Multiplying both sides of the equation in Figure 53 by η_R yields

$$\sum \eta_D \eta_R \eta_l \gamma_i Q_i \leq \phi \eta_R R_n^{'} = \eta_R R_r^{'} = R_r$$

Figure 54. Equation. Strength limit state.

where $\eta_R R_n^{'} = R_n$, and $\eta_R R_r^{'} = R_r$. It is seen that the equation in Figure 54 is actually the same as that in Figure 50 which is the limit state equation used in the current AASHTO specifications. The only difference is that the value of η_R in the equation in Figure 54 is based on a more detailed classification (i.e., considering the effects of system type, correlation among components resistances, and number of components, among others) than that used in the equation in Figure 50. Therefore, if the redundancy factor η_R is considered from the load side, equation in Figure 54 is used as the limit state equation for component design; however, if the redundancy factor η_R is taken into account from the resistance side, the limit state equation becomes

$$\sum \eta_D \eta_l \gamma_i Q_i \leq \phi \phi_R R_n = \phi_R R_r$$

Figure 55. Equation. Strength limit state.

where ϕ_R is the redundancy modifier, given by

$$\phi_R = \frac{1}{\eta_R}$$

Figure 56. Equation. Redundancy modifier.

CHAPTER 4. CONCLUSIONS

This report firstly investigates in Chapter 2 the reliability of systems with equally reliable components. In particular, the case with codified reliability index components (i.e., β_c = 3.5) used for the calibration of the AASHTO LRFD bridge design specifications is studied. By using idealized systems consisting of components having the same reliability, the effects of system arrangement, correlations among the resistances of components, number of components in a system, coefficients of variation of load and resistances, and the mean value of the load on the system reliability index are studied. For the representative case in which the coefficient of variation of resistance $V(R)$ = 0.05 and the coefficient of load $V(P)$ = 0.3, the reliability indices of N-component systems ($N \leq 100$) associated with three correlation cases, two distribution types, and different system arrangements are evaluated using the MCS method.

The obtained results show that the design based on equally reliable components leads to low reliability in series systems but high reliability in parallel systems in the no correlation and partial correlation cases. Therefore, in order to improve the reliability of series systems to avoid under capacity design and to reduce the reliability of parallel systems to avoid over capacity design, modifier factors that relate to redundancy by taking into account the effects of system arrangement and correlations among components on the system reliability should be considered during the component design.

To achieve this objective, the second part of this report (i.e., Chapter 3) proposes a redundancy factor to provide a rational system reliability-based design of structural members. This factor is defined as the ratio of the mean resistance of a component in a system when the system reliability index is prescribed (e.g., β_{sys} = 3.5) to the mean resistance of the same component when its reliability index is the same as that of the system (e.g., β_c = 3.5). By using idealized systems described in Chapter 2, the effects of the system arrangement, correlations among the resistances of components, number of components in a system, coefficients of variation of load and resistances, and mean value of the load on the redundancy factor are investigated. For the representative case mentioned previously, the redundancy factors of N-component systems associated with different correlation cases and system models are evaluated using the MCS-based program ($N \leq 100$). Since the structural components in practical cases have their own material behavior (e.g., ductile, brittle), redundancy factors of ductile, brittle, and mixed systems with no more than four components are studied and the effect of post-failure behavior factor on the redundancy factor of parallel systems is investigated. Finally, two types of limit states in which system redundancy is taken into account from the load and resistance side, respectively, are provided.

The results show the effects of the aforementioned parameters on the system reliability and redundancy factor and the convenience of using the standard tables in estimating the system reliability and designing components with redundancy factor. Specifically, the following conclusions are drawn:

1. For the no correlation and partial correlation cases, the increase of the coefficient of variation of resistance leads to:

(a) increase of system reliability in parallel systems and

(b) decrease of system reliability in series systems.

Contrary to this effect, as the coefficient of variation of load increases, the reliability of the parallel system decreases .

2. In the no correlation and partial correlation cases, the system reliability of the series and $mp \times ns$ SP systems that have the same number of parallel components (i.e., m is the same

in these SP systems) shows a decreasing tendency as the number of components increases; however, the contrary is observed in the parallel and $ms \times np$ SP systems which have the same number of series components (i.e., m is the same).

3. As the correlation among resistances of components increases, the effects of $V(R)$, $V(P)$, N, and distribution type on the system reliability are less significant. In the case of perfect correlation, these parameters have no effect on the system reliability and the system reliability index is always equal to its components reliability index.
4. If the target system reliability index is 3.5, the design based on components with the same reliability index of 3.5 is safe for all the systems in the perfect correlation case and also for the parallel system in the no correlation case. However, for the series and some series-parallel systems in the no correlation case, the system reliability will be less than the predefined threshold. Therefore, system modifier factors are needed to be applied to some critical components so that the system reliability can be improved to meet the threshold. On the contrary, for the parallel and some series-parallel systems whose reliability are higher than the threshold, system factor modifiers are necessary to be applied to the components in order to avoid this over-conservatism.
5. The redundancy factor η_R proposed in this report and the factor relating to redundancy in the AASHTO LRFD bridge design specifications are of the same nature. The major difference is that the factor relating to redundancy in the AASHTO LRFD specifications is determined based on a general classification of redundancy levels while the proposed redundancy factor η_R in this report is much more rational since it is based on a comprehensive system reliability-based approach considering several parameters including

the system type, correlation among the resistances of components, and number of components in the system.

6. During the design process, the system redundancy can be considered from the load side by using the equation in Figure 54 or from the resistance side by applying the equation in Figure 55.

   In the no correlation and partial correlation cases:

   (a) increasing the coefficient of variation of resistance leads to higher redundancy factor in series system but lower redundancy factor in parallel system; and

   (b) as the coefficient of variation of load increases, the redundancy factors associated with both series and parallel systems increase.

7. For the $mp \times ns$ SP systems having the same number of parallel components (i.e., m is same in these systems), the effect of the number of components on the redundancy factors is similar to that in the series system; whereas, for the $ms \times np$ SP systems that have the same number of series components (i.e., m is same), the variation of the redundancy factor as function of the number of components is similar to that in the parallel system.

8. The redundancy of parallel system is significantly affected by the material behavior of components: the redundancy factors of the components of these systems associated with ductile and brittle case are less and greater than 1.0, respectively.

9. Based on the results associated with the mixed parallel systems, the redundancy factors of their components are at least 1.0 due to the existence of brittle components. Increasing the correlation among the resistances of components leads to lower redundancy factor in the mixed parallel systems.

10. Future work is needed to identify the system failure modes for other limit states and different bridge types as well as ad hoc system arrangements for tailored client needs and risk attitudes. In addition, components with different reliability levels should be considered under the premise that LRFD is calibrated for all structural members including girders, columns, piers, pile shafts, among others.

REFERENCES

1. American Association of State Highway and Transportation Officials (AASHTO). (2010). LRFD bridge design specifications, 5th Ed., Washington, DC.
2. Ang, A.H.-S., and Tang, W.H. (1975). Probability concepts in engineering planning and design, vol. 1, John Wiley & Sons, New York.
3. Ang, A.H.-S., and Tang, W.H. (1984). Probability concepts in engineering planning and design, vol. 2, John Wiley & Sons, New York.
4. Babu, S. G. L., and Singh, V. P. (2011). Reliability-based load and resistance factors for soil-nail walls. *Canadian Geotechnical Journal*, 48 (6), 915-930.
5. Bruneau, M. (1992). Evaluation of system-reliability methods for cable-stayed bridge design. *Journal of Structural Engineering*, 118(4), 1106-1120.
6. Burdekin, F.M. (2007). General principles of the use of safety factors in design and assessment. *Engineering Failure Analysis*, 14(3), 420-433.
7. Ditlevsen, O., and Bjerager, P. (1986), Methods of structural systems reliability. *Structural Safety*, 3 (3-4), 195-229.
8. Estes, A. C., and Frangopol, D. M. (1998). RELSYS: a computer program for structural system reliability. *Structural Engineering & Mechanics*, 6(8), 901–99.
9. Estes, A.C., and Frangopol, D.M. (1999). Repair optimization of highway bridges using system reliability approach. *Journal of Structural Engineering*, 125(7), 766-775.
10. Estes, A. C., and Frangopol, D. M. (2001). Bridge lifetime system reliability under multiple limit states. *Journal of Bridge Engineering*, 6(6), 523-528.

11. Frangopol, D. M., and Curley, J. P. (1987). Effects of damage and redundancy on structural reliability. *Journal of Structural Engineering*, 113(7), 1533-1549.

12. Frangopol, D. M., Iizuka, M., and Yoshida, K. (1992). Redundancy measures for design and evaluation of structural systems. *Journal of Offshore Mechanics and Arctic Engineering*, 114(4), 285-29.

13. Ghosn, M., and Moses, F. (1998). Redundancy in highway bridge superstructures, NCHRP Report 406. *Transportation Research Board*, Washington, DC.

14. Ghosn, M., Moses, F., and Frangopol, D.M. (2010). Redundancy and robustness of highway bridge superstructures and substructures. *Structure and Infrastructure Engineering*, 6(1-2), 257-278.

15. Hansell, W. C., and Viest, I. M. (1971). Load factor design for steel highway bridges, AISC Engineering Journal, American Inst. *Steel Construction*, 8(4), 113-123.

16. Hsiao, L., Yu, W., and Galambos, T. (1990). AISI LRFD Method for Cold-Formed Steel Structural Members. *Journal of Structural Engineering*, 116(2), 500–517.

17. Hendawi. S., and Frangopol, D. M. (1994), System reliability and redundancy in structural design and evaluation. *Structural Safety*, 16 (1-2), 47-71.

18. Imai, K., and Frangopol, D.M. (2002). System reliability of suspension bridges. *Structural Safety*, 24(2-4), 219-259.

19. Kulicki, J.M., Mertz, D.R. and Wassef, W.G. (1994). LRFD design of highway bridges. NHI Course 13061, *Federal Highway Administration*, Washington, D. C.

20. Kulicki, J. M., Prucz, A., Clancy, C. M., Mertz, D. R., and Nowak, A. S. (2007). Updating the calibration report for AASHTO LRFD code. Final Report, Project No. NCHRP 20-7/186, *Transportation Research Board of the National Academies*, Washington, D.C.

21. Lin, S., Yu, W., and Galambos, T. (1992). ASCE LRFD Method for Stainless Steel Structures. *Journal of Structural Engineering*, 118(4), 1056–1070.

22. Liu, W. D., Neuenhoffer, A., Ghosn, M. and Moses, F. (2001). Redundancy in highway bridge substructures, NCHRP Report 458. *Transportation Research Board*, Washington, DC.

23. MathWorks. (2009). Statistical toolbox, version 7.9, MathWork, Inc., Cambridge, MA.

24. Moses, F. (1974). Reliability of structural systems. *Journal of the Structural Division*, Proceedings of the American Society of Civil Engineers, vol. 100, No. ST 9, September 1974.

25. Moses, F. (1982) System reliability development in structural engineering. *Structural Safety*, 1(1), 3–13.

26. Okasha N. M., and Frangopol D. M. (2009). Time-dependent redundancy of structural systems. *Structure and Infrastructure Engineering*, 6(1–2), 279–301.

27. Paikowsky, S. G. (2004). Load and resistance factor design (LRFD) for deep foundations, NCHRP Report 507. *Transportation Research Board*, Washington, DC.

28. Rakoczy, A. M. (2012). Development of system reliability models for railway bridges, Ph.D. Dissertation, University of Nebraska.

29. Saydam D., and Frangopol D. M. (2011). Time-dependent performance indicators of damaged bridge superstructures. *Engineering Structures*, 33(9), 2458-2471.

30. Thoft-Christensen, P., and Baker, M.J. (1982). Structural reliability theory and its applications. Springer-Verlag, Berlin, Heidelberg, New York.

31. Thoft-Christensen, P., and Murotsu, Y. (1986). Application of structural systems reliability theory. Springer-Verlag, Berlin, Heidelberg, Tokyo.

32. Zhu, B., and Frangopol, D. M. (2012). Reliability, redundancy and risk as performance indicators of structural systems during their life-cycle. *Engineering Structures*, vol. 41, 34-49.

33. Reid, S. G. (2013). Fiducial confidence in risk-based engineering decision-making. 11th International Conference on Structural Safety & Reliability (ICOSSAR) 2013, Leiden, The Netherlands: CRC Press.

34. Sykora, M. and Holicky, M. (2014). Reliability verification of industrial heritage buildings. Engineering Mechanics, 21(6), 381-392.

35. Perez-Rocha, L.E., Fernandez, M.A., Manjarrez, L.E., et al. (2013). A simplified methodology to obtain reliability indexes for calibration of design code for buildings. 4th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, 1303-1320.

36. Holicky, M. (2007). Safety design of lightweight roofs exposed to snow loads. Engineering Sciences, vol 58: 51–57.

GLOSSARY OF NOTATIONS

E	=	mean value
$E_c(R)$	=	mean resistance of a single component
$E_{cs}(R)$	=	mean resistance of a component in a system
g	=	performance function
w	=	number of simulation samples
N	=	number of components in a system
P	=	load
P_f	=	probability of failure
Q	=	force effect
R	=	resistance of a component
V	=	coefficient of variation of a random variable
β_c	=	reliability index of a single component
β_{cs}	=	reliability index of a component in a system
β_{sys}	=	reliability index of a system
ϕ	=	resistance factor
η_D	=	a factor relating to ductility
η_I	=	a factor relating to operational classification
η_R	=	redundancy factor
ρ	=	correlation coefficient
δ	=	post-failure behavior factor

APPENDIX: NOTATIONS, RELEVANT TERMS, DEFINITIONS, ADDITIONAL TABLES, AND EXAMPLES

Notations

CDF = cumulative distribution function

$E(X)$ = mean value of the random variable X

$E_c(R)$ = mean resistance of a single component

$E_{cs}(R)$ = mean resistance of a component in a system

F_j = failure of component j

g = performance function

M = safety margin

P = load

PDF = probability density function

P_f = probability of failure

P_s = probability of survival

Q = load effect

R = resistance of a component

$V(X)$ = coefficient of variation of the random variable X

X = random variable

β_c = reliability index of a single component

β_{cs} = reliability index of a component in a system

β_{sys} = reliability index of a system

δ = post-failure behavior factor

η_R = component redundancy factor

μ_M = mean of the safety margin

μ_Q = mean of the load effect

μ_R = mean of the component resistance

$\rho(X_1, X_2)$ = correlation coefficient between random variables X_1 and X_2

σ_M = standard deviation of the safety margin

σ_Q = standard deviation of the load effect

σ_R = standard deviation of the component resistance

$\Phi^{-1}(\cdot)$ = inverse of the standard normal cumulative distribution

$\cap$ = union of events

$\cup$ = intersection of events

Relevant terms

Bridge component

Bridge component reliability

Bridge component redundancy

Bridge failure modes

Bridge modeling

Bridge sub-system

Bridge sub-system redundancy

Bridge sub-system reliability

Bridge system

Bridge system reliability

Bridge system redundancy

Brittle component

Correlation among bridge component resistances

Correlation among bridge failure modes

Correlation among bridge loads

Correlation between two random variables

Ductile component

Post-failure behavior

Semi-brittle component

Definitions

Bridge component

According to AASHTO, a bridge component may be regarded as a physical piece of material that comprises a bridge. For example, structural bridge components include girders, decks, and piers. In this report, the word component is encountered when analyzing system reliability block diagrams; the components are arranged in specific configurations within the system reliability block diagram. A girder can fail in many ways (e.g., bending, shear) and in various locations. A component in the reliability model used in this study is a place where a limit state could occur. Therefore, the component failure refers to the event that the limit state represented by the component is reached or violated. For example, a bridge girder can fail due to bending moment and shear force, each of these failure events can be represented by a component failure.

Component *j*, representing a specific bridge part (e.g., girder) is idealized in Figure A-1.

Figure A-1. Graph. Component *j*.

Bridge system

The bridge system is the combination of all the components comprising the bridge structure. An example of an eight-component bridge system is shown in Figure A-2.

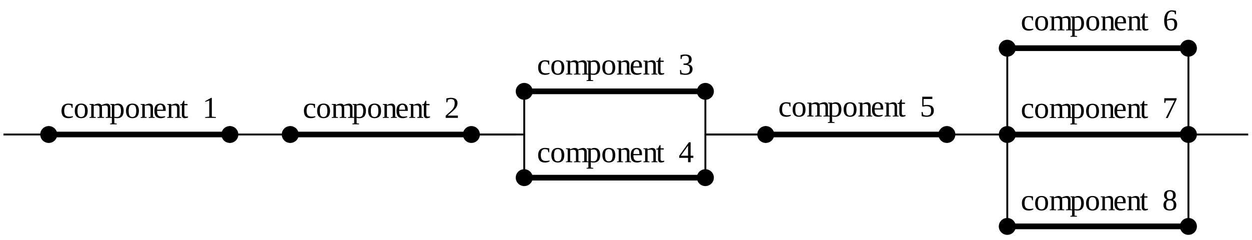


Figure A-2. Graph. Eight-component bridge system.

Bridge sub-system

A bridge sub-system is considered a group of components within the bridge system. As an example, three sub-systems (i.e., A, B, and C) of the bridge system in Figure A-2 are highlighted in Figure A-3. If F_j represents the failure of component *j*, the failure of sub-system A requires the failure of component 1 *or* component 2, *or both*, expressed mathematically as the event $F_1 \cup F_2$. Similarly, sub-system B fails when both component 3 *and* 4 fail (i.e., $F_3 \cap F_4$). The failure of sub-system C occurs when component 5 fails *or* components 6, 7, *and* 8 fail simultaneously (i.e., $F_5 \cup (F_6 \cap F_7 \cap F_8)$).

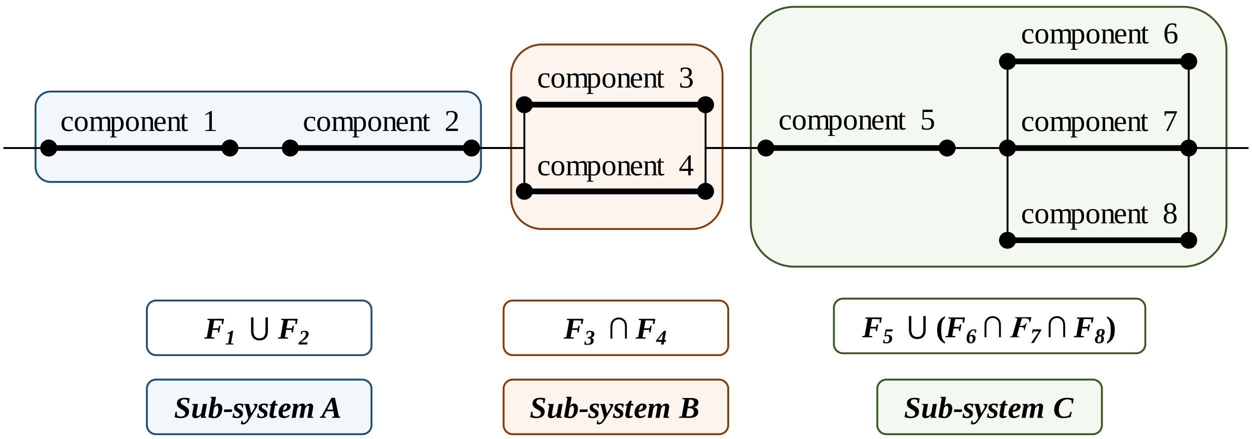


Figure A-3. Graph. Sub-systems of a bridge system.

Bridge failure modes

A failure occurs when a component or system stops performing its required function. A failure mode is defined as the mechanism that is responsible for the non-operation of a component, sub-system, or system. For structural components within bridge systems, failure modes investigated include shear, bending, fatigue, yielding, rupture, and cracking, among others.

Bridge modeling

Bridge modeling refers to the configuration of the system reliability block diagram, including series, parallel, and series-parallel. An example of each type of these systems is shown in Figure A-4. Systems A, B, C, D, and E contain the same four components arranged in different configurations. Series, parallel, and series-parallel systems are depicted in Figure A-4a, Figure A-4b, and Figure A-4c-e, respectively.

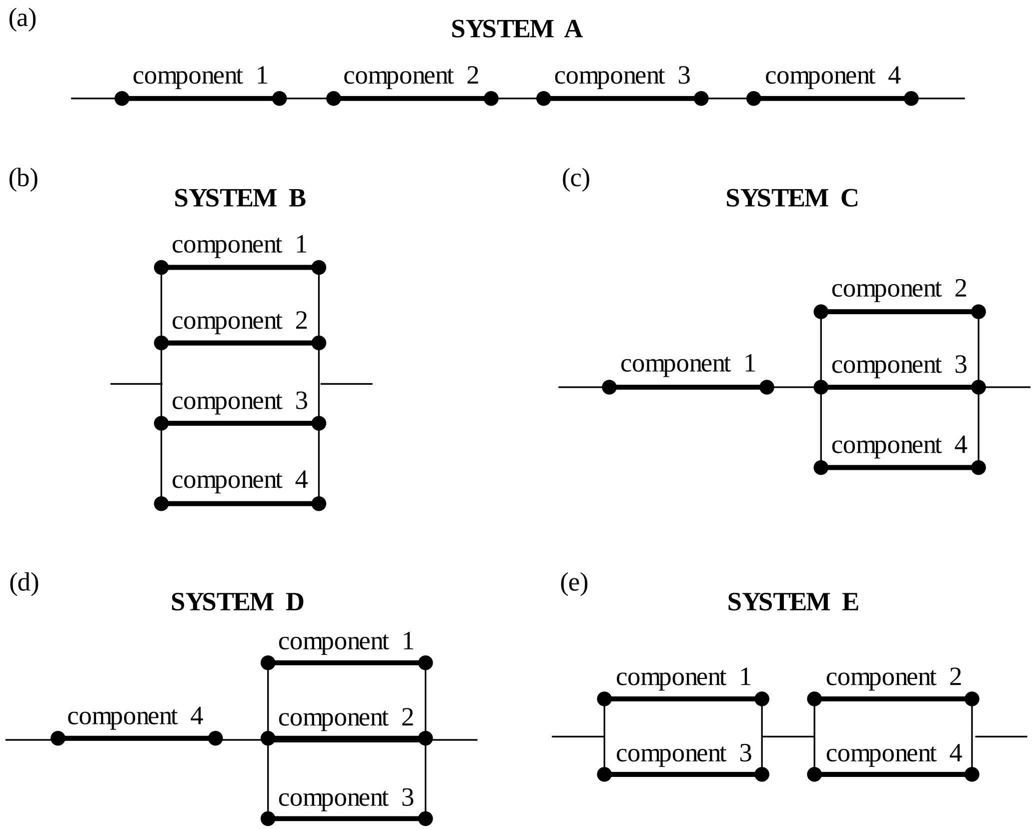


Figure A-4. Graph. Five configurations of a four-component system.

In order to provide guidance for the discretization of superstructure continuums, the process of system modeling associated with different types of bridges is discussed herein. For a box girder bridge, it can be assumed that the critical sections are arranged in either series, parallel, or a series-parallel. Additionally, for a bridge that consists of steel girders that support a reinforced concrete deck, the girders' and deck's failure modes may be arranged in a series-parallel configuration; this type of bridge is modeled in Figure A-15.

Bridge component redundancy

Component redundancy is a measure of how much its failure contributes the failure of the bridge system. AASHTO defines component redundancy as "the quality of a bridge component that enables it to perform its design function in a damaged state." As an example, rupture or yielding of an individual component may not cause collapse or failure of the whole bridge system (AASHTO 2010).

Bridge system redundancy

If a system is redundant, there exists more than one way of fulfilling the requirements of system operation (i.e., non-failure). Similarly, bridge redundancy is defined as "the capability of a bridge structural system to carry loads after damage or the failure of one or more of its members" in The Manual for Bridge Evaluation (AASHTO 2008).

Bridge sub-system redundancy

Similar to the definition of bridge system redundancy, if a sub-system is redundant, there exists more than one way of fulfilling the requirements of sub-system operation (i.e., non-failure).

Bridge component reliability

In general, bridge component reliability can be defined as the probability that this component will adequately perform its purpose for a period of time under specified environmental conditions. The reliability assessment of bridge components can be expressed as a problem of supply and demand, which is modeled by means of random variables. For instance, if R and Q are the resistance and the load effect corresponding to a specific bridge component respectively, the probability that Q will not exceed R represents the reliability of the structural component investigated. The resistance of a bridge component greatly depends upon the material it is composed of, the environmental conditions, in addition to its dimensions. In contrast, the load effect depends upon hazards that the bridge component is subject to and also on the bridge characteristics.

The probability of failure of a component is defined as the probability of violating any of the limit state functions that define its failure modes. Limit states associated with bridge components are expressed with equations relating the resistance of the structural component to the load effects acting on this component. The safety margin is expressed as follows:

$$M = R - Q$$

Figure A-5. Equation. Safety margin.

where M is the safety margin, R represents the resistance, and Q denotes the load effect. Another way of expressing the safety margin is called the performance function g. For example, the structural behavior of a bridge component may be described by the following performance function:

$$g = R - Q$$

Figure A-6. Equation. Performance function.

The safety margin M is a random variable with probability density function (PDF) $f_M(m)$. As shown in Figure A-8, the area under the PDF upper bounded by $m = 0$ represents the probability of failure. The reliability index of a component is defined as (see Figure A-8):

$$\beta_c = \frac{\mu_M}{\sigma_M}$$

Figure A-7. Equation. Reliability index of a component.

where μ_M and σ_M are the mean and standard deviation of the safety margin, respectively.

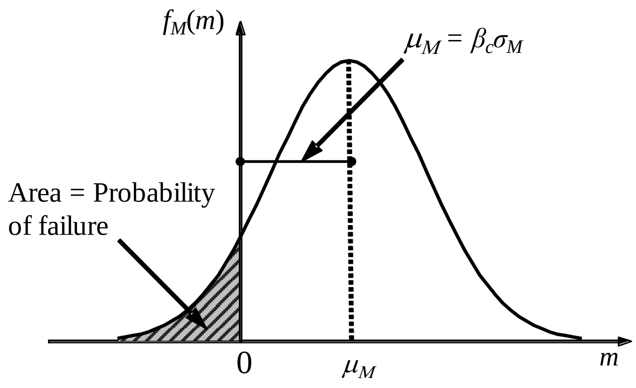


Figure A-8. Graph. PDF of the safety margin f_M (m).

If R and Q are independent, the reliability index of a component becomes

$$\beta_c = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}$$

Figure A-9. Equation. Reliability index.

where μ_R, μ_Q and σ_R, σ_Q are the means and standard deviations, respectively. Furthermore, on the assumption that the safety margin M is normally distributed, the reliability index can be expressed as:

$$\beta_c = \Phi^{-1}\left(P_s\right) = \Phi^{-1}\left(1 - P_f\right)$$

Figure A-10. Equation. Relation between probability of failure, probability of survival, and reliability index.

where $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution function (CDF), P_f is the probability of failure, and $P_s = 1 - P_f$ is the probability of safety.

Bridge system reliability

Bridge system reliability is calculated considering the system reliability model (also called the system reliability block diagram). Bridge systems that are composed of multiple components can be classified as series, parallel, or combined series-parallel. In general, the failure events comprising the system reliability model may be represented as events in series (defined as union, ⋃) or in parallel (denoted as intersection, ⋂).

Systems whose components are connected in series are such that the failure of *any* of these components constitutes the failure of system. These types of systems (i.e., series systems) have no redundancy and are also known as "weakest link" systems; the reliability of this type of system requires that none of the components fail. An idealized series system is shown in Figure A-14a. For series systems, the domain Ω, representing system failure, is expressed in terms of component failure events as:

$$\Omega \equiv \bigcup_{k=1}^{n} \{ g_k(\boldsymbol{X}) < 0 \}$$

Figure A-11. Equation. Failure domain for series systems.

Conversely, if system failure requires the failure of *all* its components, then the system may be idealized as a parallel system. If any of the components survive in a parallel system, the system will not fail. For instance, a bridge system may constitute a parallel system if there is adequate ductility and sufficient reserve capacity in all components. Clearly, a parallel system is a redundant system and may be represented as shown in Figure A-14b. For parallel systems, the system failure domain Ω is expressed as:

$$\Omega \equiv \bigcap_{k=1}^{n} \{ g_k(\boldsymbol{X}) < 0 \}$$

Figure A-12. Equation. Failure domain for parallel systems.

Figure A-14c depicts an idealized series-parallel system with n sub-systems of 4 parallel components in series. In general, the failure domain of a series-parallel system may be expressed in terms of component failure events as

$$\Omega \equiv \bigcup_{k=1}^{n} \bigcap_{j=1}^{c_n} \left\{ g_{k,j}\left(\boldsymbol{X}\right) < 0 \right\}$$

Figure A-13. Equation. Failure domain for series-parallel systems.

where c_n is the number of components in the nth cut set.

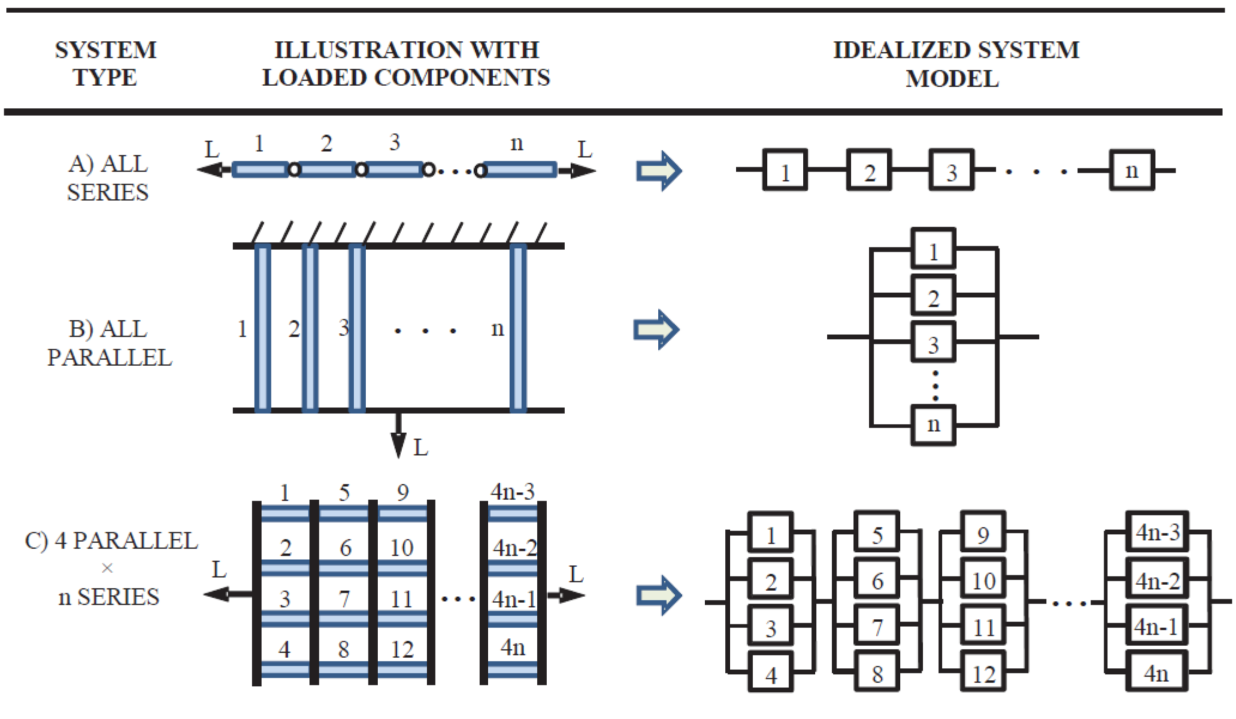


Figure A-14. Graph. Idealized series, parallel, and series-parallel systems.

An example of a system reliability model for a bridge with a reinforced concrete deck and steel girders is shown in Figure A-15. Figure A-15a presents the transverse cross section of the investigated bridge superstructure while Figure A-15b shows the idealized system reliability model. In this system reliability model, it is assumed that failure of the entire bridge superstructure (i.e., the system) is modeled as a series-parallel system consisting of a failure of the deck or the failure of any two adjacent girders.

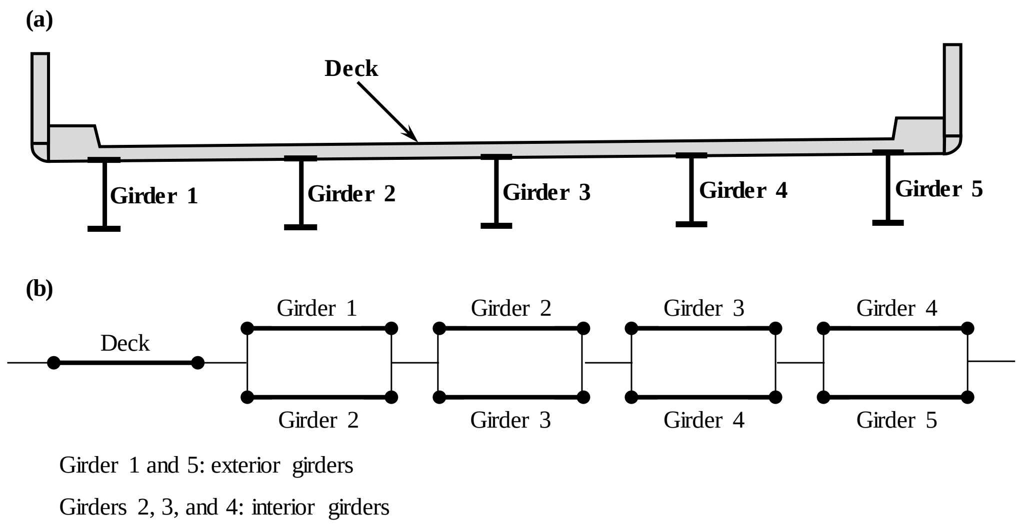


Figure A-15. Graph. (a) Transverse cross-section and (b) system reliability model of the superstructure of a bridge.

Considering that F_j represents the failure of component j, the event set describing the failure of the entire bridge system is:

$$F_{system} = F_{deck} \bigcup \left(F_{girder1} \bigcap F_{girder2}\right) \bigcup \left(F_{girder2} \bigcap F_{girder3}\right) \bigcup \left(F_{girder3} \bigcap F_{girder4}\right) \bigcup \left(F_{girder4} \bigcap F_{girder5}\right)$$

Figure A-16. Equation. System failure event.

Bridge sub-system reliability

Similar to the definition of bridge system reliability, bridge sub-system reliability is calculated considering the sub-system’s reliability block diagram. The sub-system’s reliability model is representative of a sub-system’s reliability in an event diagram format. As an example, the reliability associated with sub-systems A, B, and C within Figure A-3 are calculated considering the event sets $F_1 \cup F_2$, $F_3 \cap F_4$, and $F_5 \cup (F_6 \cap F_7 \cap F_8)$, respectively.

Post-failure behavior

The structural response of bridge components beyond the elastic limit can be characterized by brittle, ductile, or mixed (ductile-brittle) behavior. The parameter utilized to express the post-failure behavior is δ, with $\delta = 1$ and $\delta = 0$ representing a ductile and brittle component, respectively. δ is called the post-failure behavior factor. Hardening and softening behavior of components also affect the system reliability. For hardening behavior, perfect plastic post-failure can provide a conservative estimation of the system reliability and redundancy. Similar, the softening behavior after initial failure can be approximated by brittle failure. To accurately consider hardening and softening behavior requires the knowledge of the rate of capacity increase/decrease after initial failure. This rate is usually material- and/or structure-specific, which indicates that it cannot be generalized.

For component *j*, the representative force-deformation relationships, considering the post-failure behavior of the component, is shown in Figure A-17.

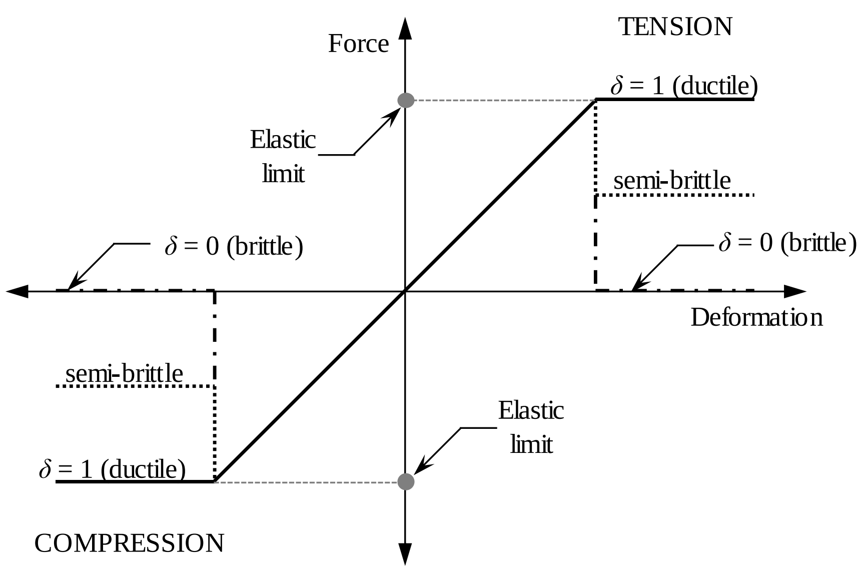


Figure A-17. Graph. Force-deformation relationship considering the effect of the post-failure behavior δ.

Ductile component

As defined by AASHTO, ductility refers to a “property of a component or connection that allows inelastic response.” Additionally, if, by means of confinement or other measures, a bridge component or connection can sustain inelastic deformations without significant loss of load-carrying capacity, this component can be considered ductile (AASHTO 2010).

Brittle component

Brittle behavior, or “the sudden loss of load-carrying capacity immediately when the elastic limit is exceeded,” refers to the post-failure behavior of a bridge component. According to AASHTO, “brittle behavior is undesirable because it implies the sudden loss of load-carrying capacity immediately when the elastic limit is exceeded” (AASHTO 2010).

Semi-brittle component

A component which exhibits post-failure behavior in between the brittle and ductile response extremes. A semi-brittle component is represented by $0 < \delta < 1$.

Correlation between two random variables

Correlation indicates the amount of relative dependency among random variables. The most common measure of dependence between two quantities is the Pearson's correlation coefficient, commonly called "the correlation coefficient." It is obtained by dividing the covariance of the two investigated variables by the product of their standard deviations. The correlation coefficient $\rho(X_1, X_2)$ between two random variables X_1 and X_2 with expected values μ_{X1} and μ_{X2} and standard deviations σ_{X1} and σ_{X2} is defined as:

$$\rho(X_1, X_2) = \frac{E[(X_1 - \mu_{X1})(X_2 - \mu_{X2})]}{\sigma_{X1} \cdot \sigma_{X2}}$$

Figure A-18. Equation. Correlation coefficient between two random variables.

Statistical independence between the two random variables X_1 and X_2 implies $\rho(X_1, X_2) = 0$ (i.e, no correlation). In contrast, perfect correlation between the two random variables X_1 and X_2 implies $\rho(X_1, X_2) = 1$. Figure A-19 depicts different correlations between two random variables X_1 and X_2.

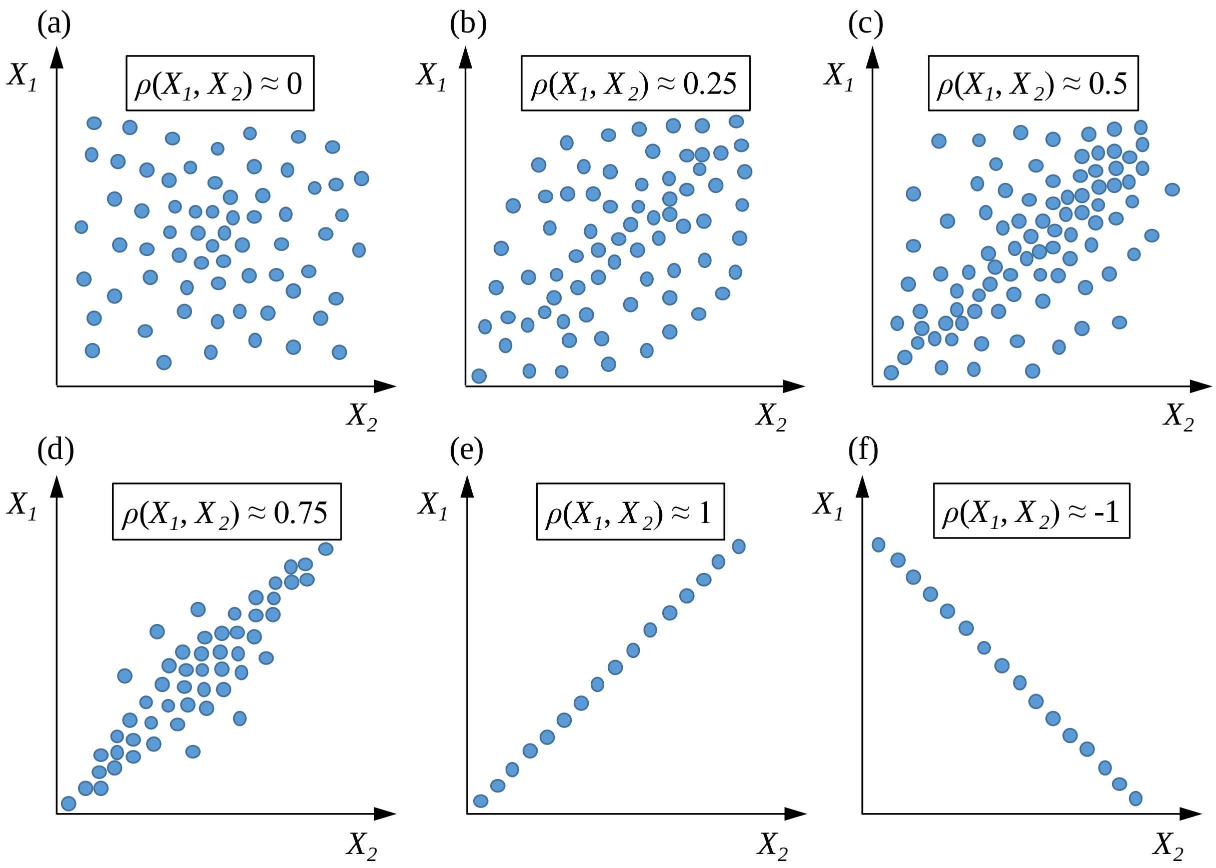


Figure A-19. Graph. Correlation coefficient $\rho(X_1, X_2)$ between two random variables X_1 and X_2.

Correlation among bridge loads

Correlation among bridge loads occurs when the loads applied to a structural system are related. Typically, separate loads are independent when the occurrence of one load has no bearing on the occurrence of the other load. As an example, the dead load and live load on a bridge are independent; in other words, these two loads have no correlation.

Since statistical independence between two random variables X_1 and X_2 (i.e., $\rho(X_1, X_2) = 0$) implies that the two variables are not related, the value of one variable has no influence on the value of the other variable. An example of statistically independent variables with application to bridges is the loading applied to the structure. For example, traffic loading and dead loads are independent and traffic loads and seismic loads are also independent; the amount of traffic has no bearing on the occurrence and magnitude of dead or seismic loads.

Correlation among bridge component resistances

Correlation among bridge resistances occurs when the structural resistances associated with multiple components (e.g., $R_1, R_2, \ldots, R_N$) are related. In practice, the resistances of bridge components could be correlated, like in the case of multiple interior bridge girders. However, the resistances of an interior girder and the deck of a bridge (see Figure A-15a for an example) are typically not correlated (e.g., independent).

Correlation among bridge failure modes

Correlation among bridge failure modes occurs when some or all of the components of limit state functions of the two failure modes are related (e.g., same load present in the two failure modes, same resistances present in the two failure modes). Typically, only the correlation among the random variables involved in the reliability assessment are known and/or quantified. However, the correlation among different performance functions (e.g., $g_1, g_2, \ldots, g_N$) may also be calculated considering the relationships among all the investigated random variables.

Since perfect correlation between two random variables (i.e., $\rho = 1$) implies that the two variables are fully interrelated, the value of one variable has complete influence on the value of the other variable. An example with application to bridges is found when analyzing the bending and shear failure of a simply supported span of a two lane bridge with two independent truck loads (e.g., two trucks following each other in the same lane). The simply supported span with the two applied truck loads P_1 and P_2 is shown in Figure A-20. The free body diagram of the span is also shown in Figure A-20 where vertical reaction forces at the left and right support are denoted as Y_1 and Y_2, respectively. Because the expressions for the vertical reactions at the supports of the span are functions of similar terms (i.e., P_1 and P_2), there exists some inherent correlation between the reaction forces (i.e., $\rho(Y_1, Y_2) > 0$) . For this particular span, the two modes of failure considered are bending and shear.

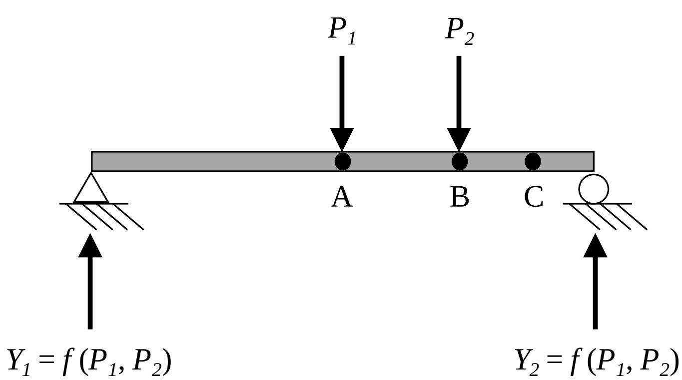


Figure A-20. Graph. Correlation coefficient $\rho(X_1, X_2)$ between two random variables X_1 and X_2.

The bending moment at point A is denoted as M_A and is a function of both truck loads P_1 and P_2. Similarly, the shear force at point C, V_C is also a function of both truck loads P_1 and P_2. In general, the expressions for moment and shear in the span contain terms representing the magnitude of the two applied loads. Because both equations used to calculate M_A and V_C contain P_1 and P_2, the bending moment at point A and shear force at point C are correlated. In this case, the magnitude of the applied loads are independent (i.e., no correlation) but their load effects are correlated. The bending moment at points A and B, M_A and M_B, respectively, will be correlated, as well.

Furthermore, the failure modes associated with bending and shear of the bridge may be analyzed. The performance function associated with bending is

$$g_M = R_M - Q_M$$

Figure A-21. Equation. Performance function associated with bending failure.

where R_M is the bending resistance and Q_M is the bending load effect. Similarly, the performance function corresponding to shear failure is expressed as:

$$g_V = R_V - Q_V$$

Figure A-22. Equation. Performance function associated with shear failure.

where R_V is the shear resistance and Q_V is the shear load effect. Since the two load effects Q_M and Q_V are correlated, it is evident that the performance functions (i.e., failure modes) presented in Figure A-21 and Figure A-22 are also correlated. Therefore, even if the resistances in bending and shear are independent, their respective failure modes will always be correlated.

Additional tables with reliability and redundancy factors for systems up to 500 components

In the main report, the system reliability index is investigated with respect to the ductile and brittle systems having a small number of components and it is found that the system reliability index is affected by the number of components in the system. However, most real structures are composed of dozens or hundreds of components. Therefore, it is necessary to evaluate the system reliability index of ductile and brittle systems that consist of many iso-reliability components so that standard tables of system reliability index can be generated to facilitate the component design process.

For a structure consisting of *N* ductile components, different types of systems can be formed: series, parallel, and series-parallel. The reliability analysis of a series system is independent of the material behavior of its components. Therefore, only parallel and series-parallel systems are analyzed. The series-parallel systems where the number of the series or parallel components in the sub-series or sub-parallel systems equal to 5, 10 and 20 are investigated.

The system reliability index is not affected by the mean value of the load acting on the system when all the components in the system are identical and have the same component reliability. Therefore, the following assumption is made for the loads applied to the parallel and series-parallel systems: (a) for an *N*-component parallel system, the load it is subject to is $N \cdot P$, where *P* is the load acting on a single component which is used to calculate $E_c(R)$; (b) for a $mp \times ns$ series-parallel system which has *m* components in each sub-parallel system, the load acting on system is $m \cdot P$; and (c) for a $ms \times np$ series-parallel system which has *n* sub-series systems, the load acting on system is $n \cdot P$. In this manner, the load effect of each component in the intact parallel and series-parallel systems are also *P*.

Since the failure modes of ductile systems are independent of the failure sequence, the limit state equation of an *N*-component parallel system is

$$g = \sum_{i=1}^{N} R_i - N \cdot P = 0$$

Figure A-23. Equation. Limit state equation associated with failure of the *N*-component parallel system in the ductile case.

For a $mp \times ns$ series-parallel system which has *n* possible failure modes, the limit state equation associated with one of the failure modes is

$$g = \sum_{i=m(k-1)+1}^{m \cdot k} R_i - m \cdot P = 0$$

Figure A-24. Equation. Limit state equation associated with failure of the $mp \times ns$ series-parallel system in the ductile case.

where $m(k\text{-}1)+1$ and $m \cdot k$ denote the first and last component in the k_{th} sub-parallel system, respectively. For a $ms \times np$ series-parallel system, the number of its possible failure modes is m^n. The limit state equation associated with one of the failure modes can be written as

$$g = \sum_{i=k}^{k+n-1} R_i - n \cdot P = 0$$

Figure A-25. Equation. Limit state equation associated with failure of the $ms \times np$ series-parallel system in ductile case.

where components *k*, *k*+1,…, *k*+*n*-1 are located in different sub-series systems.

After identifying the limit state equations of the *N*-component ($N \leq 500$) parallel and series-parallel systems and assuming that the coefficients of variation of resistance and load to be 0.05 and 0.3, respectively, the system reliability indices associated with two distribution types (normal and lognormal) and two correlation cases ($\rho(R_i,R_j) = 0$ and 0.5) when the reliability indices of all components are 3.5 are calculated. Table A-1 and Table A-2 present the obtained results along with the system reliability indices of series systems. Figure A-26 shows the effects of the number of components on the system reliability index in the series and parallel systems.

It is observed from the figure and tables that (a) the effect of N on the system reliability index in the parallel system depends on the value of N: when N is small, increasing N leads to higher system reliability index in the parallel system, and the change is less significant as the correlation among the resistances of component becomes stronger; however, as N continues increasing, the its effect on the system reliability index becomes insignificant; (b) for the $mp \times ns$ series-parallel systems that have the same number of parallel components (i.e., m is the same in these systems), the system reliability index decreases along with the increase of N; this is similar to the finding observed in the series systems; (c) as the correlation among the resistances of components increases, the system reliability index exhibits a decreasing trend in the series system while an increasing tendency in the parallel system; and (d) in the series system, the system reliability index associated with normal distribution is lower than that associated with the lognormal distribution; while in the parallel system, contrary conclusion is observed.

Table A-1. System reliability index of ductile systems associated with different correlation cases when *R* and *P* follow normal distribution; $100 \le N \le 500$.

Component / System	$\rho(R_i,R_j) = 0$	$\rho(R_i,R_j) = 0.5$	$\rho(R_i,R_j) = 1.0$
100-component system - Series system	2.793	2.977	3.50
100-component system - Parallel system	3.709	3.604	3.50
100-component system - 5*p*×20*s* SP system	3.409	3.390	3.50
100-component system - 10*p*×10*s* SP system	3.531	3.478	3.50
100-component system - 20*p*×5*s* SP system	3.615	3.532	3.50
200-component system - Series system	2.716	2.923	3.50
200-component system - Parallel system	3.710	3.605	3.50
200-component system - 5*p*×40*s* SP system	3.359	3.363	3.50
200-component system - 10*p*×20*s* SP system	3.501	3.457	3.50
200-component system - 20*p*×10*s* SP system	3.588	3.517	3.50
300-component system - Series system	2.669	2.892	3.50
300-component system - Parallel system	3.711	3.607	3.50
300-component system - 5*p*×60*s* SP system	3.339	3.344	3.50
300-component system - 10*p*×30*s* SP system	3.475	3.439	3.50
300-component system - 20*p*×15*s* SP system	3.571	3.510	3.50
400-component system - Series system	2.640	2.871	3.50
400-component system - Parallel system	3.711	3.608	3.50
400-component system - 5*p*×80*s* SP system	3.325	3.338	3.50
400-component system - 10*p*×40*s* SP system	3.465	3.432	3.50
400-component system - 20*p*×20*s* SP system	3.558	3.498	3.50
500-component system - Series system	2.617	2.855	3.50
500-component system - Parallel system	3.712	3.610	3.50
500-component system - 5*p*×100*s* SP system	3.306	3.328	3.50
500-component system - 10*p*×50*s* SP system	3.456	3.426	3.50
500-component system - 20*p*×25*s* SP system	3.550	3.494	3.50

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_{cs} = 3.5$

Table A-2. System reliability index of ductile systems associated with different correlation cases when *R* and *P* follow lognormal distribution; $100 \le N \le 500$.

Component / System	$\rho(R_i,R_j) = 0$	$\rho(R_i,R_j) = 0.5$	$\rho(R_i,R_j) = 1.0$
100-component system - Series system	3.116	3.219	3.50
100-component system - Parallel system	3.556	3.526	3.50
100-component system - 5*p*×20*s* SP system	3.409	3.426	3.50
100-component system - 10*p*×10*s* SP system	3.469	3.469	3.50
100-component system - 20*p*×5*s* SP system	3.509	3.500	3.50
200-component system - Series system	3.074	3.194	3.50
200-component system - Parallel system	3.556	3.527	3.50
200-component system - 5*p*×40*s* SP system	3.386	3.413	3.50
200-component system - 10*p*×20*s* SP system	3.452	3.456	3.50
200-component system - 20*p*×10*s* SP system	3.495	3.486	3.50
300-component system - Series system	3.055	3.180	3.50
300-component system - Parallel system	3.557	3.527	3.50
300-component system - 5*p*×60*s* SP system	3.375	3.401	3.50
300-component system - 10*p*×30*s* SP system	3.439	3.451	3.50
300-component system - 20*p*×15*s* SP system	3.490	3.476	3.50
400-component system - Series system	3.037	3.165	3.50
400-component system - Parallel system	3.557	3.528	3.50
400-component system - 5*p*×80*s* SP system	3.368	3.397	3.50
400-component system - 10*p*×40*s* SP system	3.436	3.445	3.50
400-component system - 20*p*×20*s* SP system	3.484	3.479	3.50
500-component system - Series system	3.026	3.159	3.50
500-component system - Parallel system	3.558	3.529	3.50
500-component system - 5*p*×100*s* SP system	3.361	3.392	3.50
500-component system - 10*p*×50*s* SP system	3.432	3.442	3.50
500-component system - 20*p*×25*s* SP system	3.481	3.474	3.50

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_{cs} = 3.5$

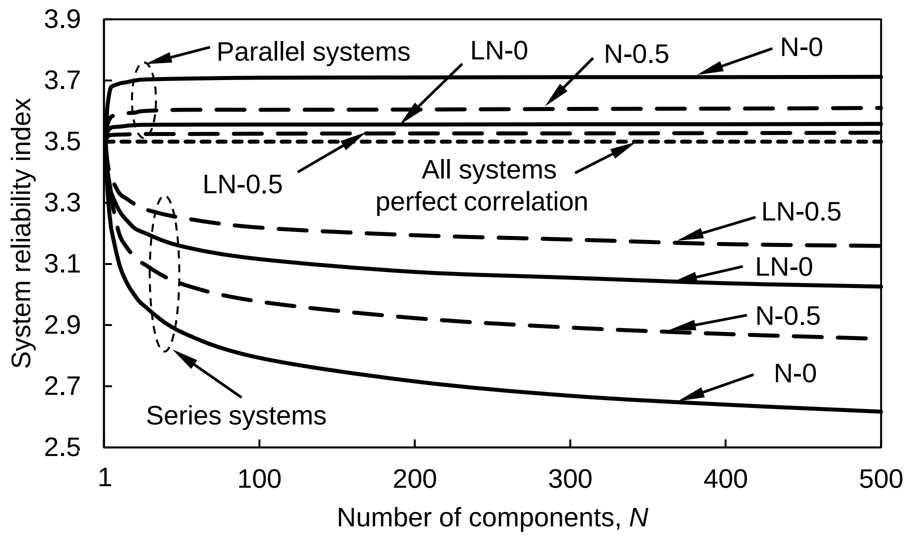


Figure A-26. Graph. Effects of the number of components on the reliability index of ductile systems (Note: "N" denotes normal distribution; "LN" denotes lognormal distribution; "0" denotes $\rho(R_i,R_j) = 0$; and "0.5" denotes $\rho(R_i,R_j) = 0.5$).

Assuming the coefficients of variation of resistance and load to be 0.05 and 0.3, respectively, the redundancy factors associated with two probability distributions (normal and lognormal) and two correlation cases ($\rho(R_i,R_j) = 0$ and 0.5) when the system reliability is 3.5 are obtained using the MCS-based method, as listed in Table A-3 to Table A-6. These results are also plotted in Figure A-27 which shows the effects of the number of components on the redundancy factor of series and parallel systems.

It is observed that (a) the redundancy factors of the ductile series systems are the same as those of the regular series systems (without considering the material behavior) presented in the main portion of this report; this is due to the fact that the failure modes of the series system are independent of the material behavior of components; (b) the effect of *N* on the redundancy factor in the parallel system also depends on the value of *N*: when *N* is small, the redundancy factor decreases as *N* increases and the change is less significant as the correlation among the resistances of component becomes stronger; however, as N continues increasing, its effect on the redundancy factor becomes insignificant; (c) for the $mp \times ns$ series-parallel systems that have the same number of components in the sub-parallel system (i.e., m is the same in these systems), the redundancy factor becomes larger as N increases; this is similar to the finding observed in the series systems; (d) as the correlation among the resistances of components becomes stronger, the redundancy factor increases in the series system but decreases in the parallel system; and (e) in the series system, the redundancy factors associated with both distributions (i.e., normal and lognormal) are very close; however, in the parallel system, the differences in the redundancy factors associated with the two distributions are more significant.

Table A-3. Redundancy factor of ductile systems associated with the case $\rho(R_i,R_j) = 0$ when R and P follow normal distribution; $100 \leq N \leq 500$.

System	$E_{cs}(R)$	η_R
100-component system - Series system	23.626	1.118
100-component system - Parallel system	20.519	0.971
100-component system - 5*p*×20*s* SP system	21.428	1.014
100-component system - 10*p*×10*s* SP system	21.026	0.995
100-component system - 20*p*×5*s* SP system	20.794	0.984
200-component system - Series system	23.921	1.132
200-component system - Parallel system	20.519	0.971
200-component system - 5*p*×40*s* SP system	21.576	1.021
200-component system - 10*p*×20*s* SP system	21.132	1.000
200-component system - 20*p*×10*s* SP system	20.878	0.988
300-component system - Series system	24.112	1.141
300-component system - Parallel system	20.498	0.970
300-component system - 5*p*×60*s* SP system	21.639	1.024
300-component system - 10*p*×30*s* SP system	21.195	1.003
300-component system - 20*p*×15*s* SP system	20.921	0.990
400-component system - Series system	24.217	1.146
400-component system - Parallel system	20.498	0.970
400-component system - 5*p*×80*s* SP system	21.703	1.027
400-component system - 10*p*×40*s* SP system	21.238	1.005
400-component system - 20*p*×20*s* SP system	20.942	0.991
500-component system - Series system	24.323	1.151
500-component system - Parallel system	20.498	0.970
500-component system - 5*p*×100*s* SP system	21.745	1.029
500-component system - 10*p*×50*s* SP system	21.280	1.007
500-component system - 20*p*×25*s* SP system	20.963	0.992

Note: $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

Table A-4. Redundancy factor of ductile systems associated with the case $\rho(R_i,R_j)$ = 0.5 when *R* and *P* follow normal distribution; $100 \leq N \leq 500$.

System	$E_{cs}(R)$	η_R
100-component system - Series system	23.013	1.089
100-component system - Parallel system	20.815	0.985
100-component system - 5*p*×20*s* SP system	21.449	1.015
100-component system - 10*p*×10*s* SP system	21.195	1.003
100-component system - 20*p*×5*s* SP system	21.026	0.995
200-component system - Series system	23.203	1.098
200-component system - Parallel system	20.815	0.985
200-component system - 5*p*×40*s* SP system	21.555	1.020
200-component system - 10*p*×20*s* SP system	21.259	1.006
200-component system - 20*p*×10*s* SP system	21.090	0.998
300-component system - Series system	23.309	1.103
300-component system - Parallel system	20.815	0.985
300-component system - 5*p*×60*s* SP system	21.660	1.025
300-component system - 10*p*×30*s* SP system	21.322	1.009
300-component system - 20*p*×15*s* SP system	21.132	1.000
400-component system - Series system	23.414	1.108
400-component system - Parallel system	20.815	0.985
400-component system - 5*p*×80*s* SP system	21.703	1.027
400-component system - 10*p*×40*s* SP system	21.343	1.010
400-component system - 20*p*×20*s* SP system	21.132	1.000
500-component system - Series system	23.457	1.110
500-component system - Parallel system	20.815	0.985
500-component system - 5*p*×100*s* SP system	21.703	1.027
500-component system - 10*p*×50*s* SP system	21.364	1.011
500-component system - 20*p*×25*s* SP system	21.132	1.000

Note: $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

Table A-5. Redundancy factor of ductile systems associated with the case $\rho(R_i,R_j)$ = 0 when R and P follow lognormal distribution; 100 ≤ N ≤ 500.

System	$E_{cs}(R)$	η_R
100-component system - Series system	30.457	1.120
100-component system - Parallel system	26.759	0.984
100-component system - 5*p*×20*s* SP system	27.928	1.027
100-component system - 10*p*×10*s* SP system	27.439	1.009
100-component system - 20*p*×5*s* SP system	27.085	0.996
200-component system - Series system	30.784	1.132
200-component system - Parallel system	26.759	0.984
200-component system - 5*p*×40*s* SP system	28.119	1.034
200-component system - 10*p*×20*s* SP system	27.575	1.014
200-component system - 20*p*×10*s* SP system	27.248	1.002
300-component system - Series system	31.028	1.141
300-component system - Parallel system	26.759	0.984
300-component system - 5*p*×60*s* SP system	28.227	1.038
300-component system - 10*p*×30*s* SP system	27.656	1.017
300-component system - 20*p*×15*s* SP system	27.303	1.004
400-component system - Series system	31.137	1.145
400-component system - Parallel system	26.759	0.984
400-component system - 5*p*×80*s* SP system	28.255	1.039
400-component system - 10*p*×40*s* SP system	27.711	1.019
400-component system - 20*p*×20*s* SP system	27.303	1.004
500-component system - Series system	31.246	1.149
500-component system - Parallel system	26.759	0.984
500-component system - 5*p*×100*s* SP system	28.309	1.041
500-component system - 10*p*×50*s* SP system	27.738	1.020
500-component system - 20*p*×25*s* SP system	27.357	1.006

Note: $V(P)$ = 0.3; $V(R)$ = 0.05; β_c = 3.5; β_{sys} = 3.5; $E_{c,LN}(R)$ = 27.194

Table A-6. Redundancy factor of ductile systems associated with the case $\rho(R_i,R_j)$ = 0.5 when *R* and *P* follow lognormal distribution; 100 ≤ *N* ≤ 500.

System	$E_{cs}(R)$	η_R
100-component system - Series system	29.533	1.086
100-component system - Parallel system	26.976	0.992
100-component system - 5*p*×20*s* SP system	27.847	1.024
100-component system - 10*p*×10*s* SP system	27.439	1.009
100-component system - 20*p*×5*s* SP system	27.221	1.001
200-component system - Series system	29.777	1.095
200-component system - Parallel system	26.976	0.992
200-component system - 5*p*×40*s* SP system	27.928	1.027
200-component system - 10*p*×20*s* SP system	27.575	1.014
200-component system - 20*p*×10*s* SP system	27.248	1.002
300-component system - Series system	29.913	1.100
300-component system - Parallel system	26.949	0.991
300-component system - 5*p*×60*s* SP system	28.010	1.030
300-component system - 10*p*×30*s* SP system	27.602	1.015
300-component system - 20*p*×15*s* SP system	27.330	1.005
400-component system - Series system	29.995	1.103
400-component system - Parallel system	26.949	0.991
400-component system - 5*p*×80*s* SP system	28.037	1.031
400-component system - 10*p*×40*s* SP system	27.629	1.016
400-component system - 20*p*×20*s* SP system	27.357	1.006
500-component system - Series system	30.077	1.106
500-component system - Parallel system	26.949	0.991
500-component system - 5*p*×100*s* SP system	28.064	1.032
500-component system - 10*p*×50*s* SP system	27.656	1.017
500-component system - 20*p*×25*s* SP system	27.357	1.006

Note: $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,LN}(R) = 27.194$

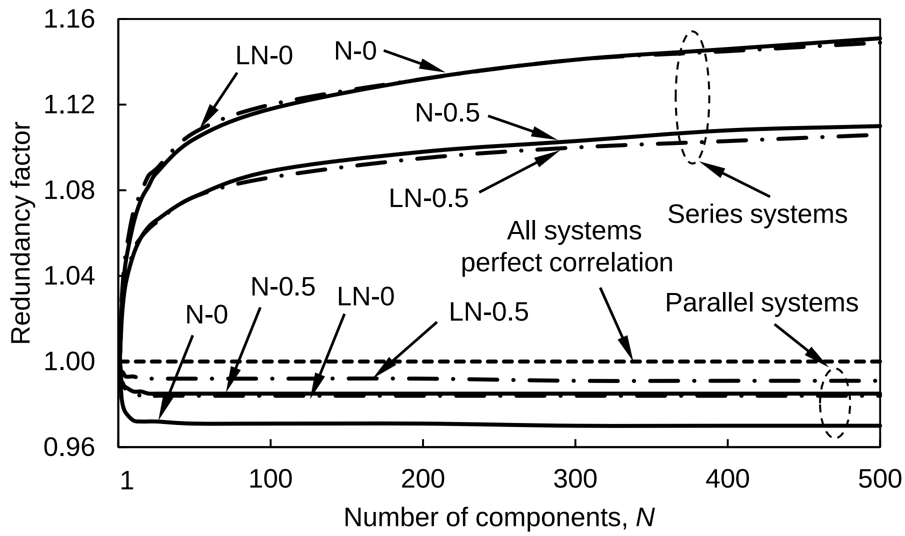


Figure A-27. Graph. Effects of the number of components on the redundancy factor in ductile systems (Note: "N" denotes normal distribution; "LN" denotes lognormal distribution; "0" denotes $\rho(R_i,R_j) = 0$; and "0.5" denotes $\rho(R_i,R_j) = 0.5$).

The associated component reliability indices of the *N*-component systems are presented in Table A-7 and Table A-8. The results are also plotted in Figure A-28. It is noted that (a) the effects of *N* and $\rho(R_i,R_j)$ on the reliability index of components are similar to those on the redundancy factor; (b) in the series system, the component reliability index associated with normal distribution is higher than that associated with lognormal distribution; however, contrary finding is observed in the parallel system; and (c) the differences in the component reliability index due to different distributions and different correlation cases associated with the ductile parallel systems are less significant than those associated with the regular parallel systems (presented in main portion of this report); this indicates that including the ductile behavior impairs the effects of distribution type and correlation on the reliability index of components.

Table A-7. Component reliability index of different systems associated with different correlation cases when *R* and *P* follow normal distribution; $1 \leq N \leq 500$.

Component / System	$\rho(R_i,R_j) = 0$	$\rho(R_i,R_j) = 0.5$	$\rho(R_i,R_j) = 1.0$
1-component	3.50	3.50	3.50
2-component system - Series system	3.63	3.60	3.50
2-component system - Parallel system	3.40	3.45	3.50
3-component system - Series system	3.71	3.66	3.50
3-component system - Parallel system	3.37	3.44	3.50
5-component system - Series system	3.79	3.73	3.50
5-component system - Parallel system	3.35	3.43	3.50
10-component system - Series system	3.90	3.81	3.50
10-component system - Parallel system	3.33	3.41	3.50
15-component system - Series system	3.97	3.86	3.50
15-component system - Parallel system	3.33	3.41	3.50
20-component system - Series system	4.00	3.89	3.50
20-component system - Parallel system	3.33	3.40	3.50
25-component system - Series system	4.04	3.91	3.50
25-component system - Parallel system	3.33	3.40	3.50
50-component system - Series system	4.14	3.98	3.50
50-component system - Parallel system	3.32	3.40	3.50
100-component system - Series system	4.23	4.05	3.50
100-component system - Parallel system	3.32	3.40	3.50
200-component system - Series system	4.31	4.10	3.50
200-component system - Parallel system	3.32	3.40	3.50
300-component system - Series system	4.36	4.14	3.50
300-component system - Parallel system	3.31	3.40	3.50
400-component system - Series system	4.40	4.16	3.50
400-component system - Parallel system	3.31	3.40	3.50
500-component system - Series system	4.42	4.18	3.50
500-component system - Parallel system	3.31	3.40	3.50

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,N}(R) = 21.132$

Table A-8. Component reliability index of different systems associated with different correlation cases when *R* and *P* follow lognormal distribution; $1 \leq N \leq 500$.

Component / System	$\rho(R_i,R_j) = 0$	$\rho(R_i,R_j) = 0.5$	$\rho(R_i,R_j) = 1.0$
1-component	3.50	3.50	3.50
2-component system - Series system	3.57	3.55	3.50
2-componentsystem - Parallel system	3.47	3.49	3.50
3-component Series system	3.62	3.59	3.50
3-component Parallel system	3.46	3.49	3.50
5-component Series system	3.66	3.62	3.50
5-component Parallel system	3.46	3.48	3.50
10-component Series system	3.72	3.67	3.50
10-component Parallel system	3.45	3.48	3.50
15-component Series system	3.75	3.68	3.50
15-component Parallel system	3.45	3.48	3.50
20-component Series system	3.78	3.70	3.50
20-component Parallel system	3.45	3.48	3.50
25-component Series system	3.79	3.71	3.50
25-component Parallel system	3.45	3.48	3.50
50-component Series system	3.84	3.74	3.50
50-component Parallel system	3.44	3.48	3.50
100-component Series system	3.88	3.77	3.50
100-component Parallel system	3.44	3.48	3.50
200-component Series system	3.91	3.80	3.50
200-component Parallel system	3.44	3.48	3.50
300-component Series system	3.94	3.81	3.50
300-component Parallel system	3.44	3.47	3.50
400-component Series system	3.95	3.82	3.50
400-component Parallel system	3.44	3.47	3.50
500-component Series system	3.96	3.83	3.50
500-component Parallel system	3.44	3.47	3.50

Note: $E(P) = 10$; $V(P) = 0.3$; $V(R) = 0.05$; $\beta_c = 3.5$; $\beta_{sys} = 3.5$; $E_{c,LN}(R) = 27.194$

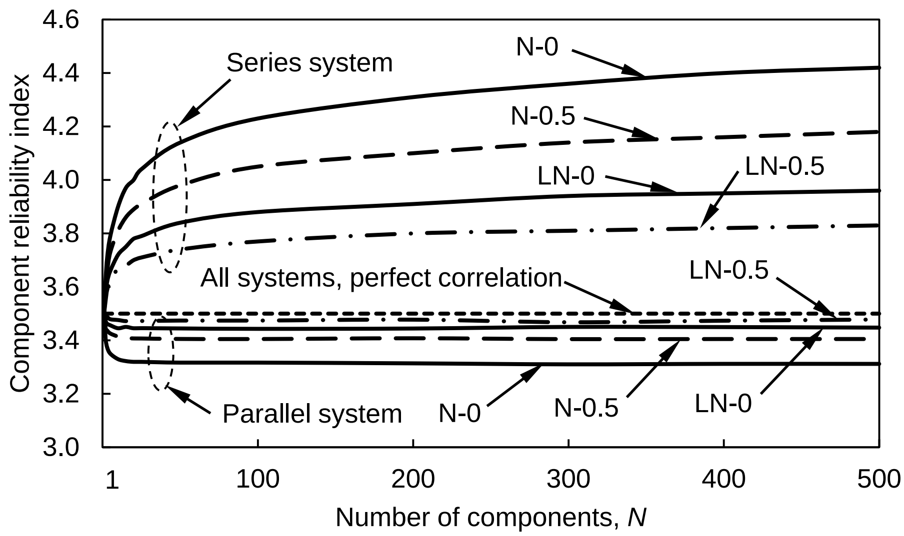


Figure A-28. Graph. Effects of the number of components on the reliability index of components in ductile systems (Note: "N" denotes normal distribution; "LN" denotes lognormal distribution; "0" denotes $\rho(R_i,R_j)$ = 0; and "0.5" denotes $\rho(R_i,R_j)$ = 0.5).

Example 1

A steel girder bridge with 4 girders is investigated within this illustrative example. In order to describe different series-parallel systems, the following rule is used: if the sub-system of a series-parallel (SP) system consists of *m* parallel components and it is repeated *n* times in the system model, as shown in Figure A-29, this series-parallel system is denoted as a $mp \times ns$ SP system.

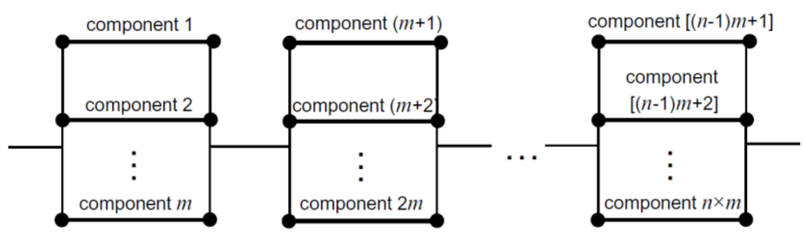


Figure A-29. Graph. Schematic figure of the *mp*×*ns* series-parallel system.

For a steel girder bridge with four girders numbered from 1 to 4 (girders 1 and 4 are exterior girders and girders 2 and 3 refer to interior girders), four different system models can be considered according to different definitions of system failure: (a) series model: the system fails if any girder fails; (b) parallel model: the system fails only if all girders fail; (c) series-parallel model I: the system fails if either of the exterior girders and either of the interior girders fail, denoted as $2s \times 2p$ series-parallel model; and (d) series-parallel model II: the system fails if any two adjacent girders fail, denoted as $2p \times 3s$ series-parallel model, as shown in Figure A-30.

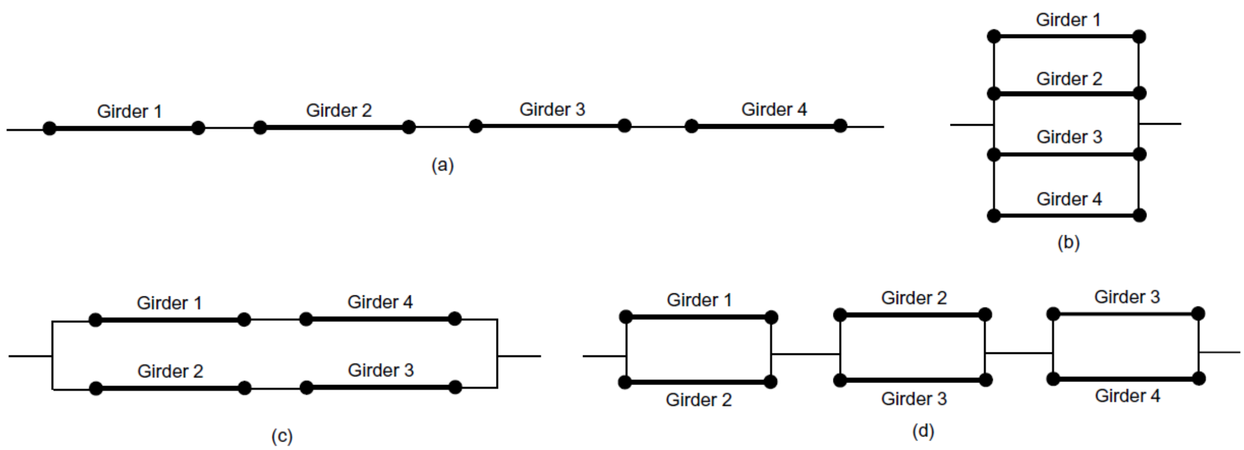


Figure A-30. Graph. Four-girder bridge systems: (a) series, (b) parallel, (c) 2*p*×2*s* series-parallel, and (d) 2*p*×3*s* series-parallel.

For each girder, the resistance R and load effect Q are assumed to follow normal distributions. The mean values and standard deviations of R are the same for all girders; so are the mean values and standard deviations of Q. Three correlation cases among the resistances of girders are considered herein: (a) $\rho(R_i,R_j) = 0$, no correlation; (b) $\rho(R_i,R_j) = 0.5$, partial correlation; and (c) $\rho(R_i,R_j) = 1.0$, perfect correlation.

Although the girders may fail in different modes, only the flexural failure mode is analyzed. The mean value of the load effect, which is vertical bending moment, is assumed to be $E(Q) = 7500$ kN·m. Two cases associated with the coefficients of variation of R and Q are studied: (a) Case A: $V(R) = 0.05$, $V(Q) = 0.3$; and (b) Case B: $V(R) = 0.1$, $V(Q) = 0.4$. Based on this information, the required mean resistance of each girder $E_c(R)$ when the component reliability index β_c is 3.5 is found to be 1.58×10^4 kN·m for Case A and 2.01×10^4 kN·m for Case B. Next, assuming the system reliability index $\beta_{sys} = 3.5$, the mean resistances of each girders $E_{cs}(R)$ in the four systems associated with Cases A and B can be calculated. Finally, the redundancy factors η_R and reliability indices β_{cs} associated with the girders are obtained using the ratio $E_{cs}(R)$ / $Ec(R)$. The results are presented in Table A-9 and Table A-10 as matrices in function of system modeling and correlation cases for the four girder bridge system. Note that the redundancy factors within Table A-9 and Table A-10 can also be found in the following standardized tables provided in the main report: Table 12, Table 13, Table 22, Table 24, and Table 26.

Table A-9. Redundancy factors and reliability indices of girders in the 4-girder bridge systems associated with Case A ($V(R)$ = 0.05, $V(Q)$ = 0.3).

Models	ρ = 0.0 η_R	ρ = 0.0 β_{cs}	ρ = 0.5 η_R	ρ = 0.5 β_{cs}	ρ = 1.0 η_R	ρ = 1.0 β_{cs}
Series	1.041	3.76	1.032	3.70	1.0	3.50
Parallel	0.934	3.08	0.956	3.22	1.0	3.50
2*p*×2*s* SP	0.988	3.42	0.995	3.48	1.0	3.50
2*p*×3*s* SP	0.983	3.40	0.992	3.45	1.0	3.50

Note: ρ denotes $\rho(R_i, R_j)$.

Table A-10. Redundancy factors and reliability indices of girders in the 4-girder bridge systems associated with Case B ($V(R)$ = 0.1, $V(Q)$ = 0.4).

Models	ρ = 0.0 η_R	ρ = 0.0 β_{cs}	ρ = 0.5 η_R	ρ = 0.5 β_{cs}	ρ = 1.0 η_R	ρ = 1.0 β_{cs}
Series	1.076	3.83	1.066	3.79	1.0	3.50
Parallel	0.842	2.75	0.892	3.00	1.0	3.50
2*p*×2*s* SP	0.946	3.25	0.975	3.39	1.0	3.50
2*p*×3*s* SP	0.938	3.21	0.968	3.36	1.0	3.50

Note: ρ denotes $\rho(R_i, R_j)$.

It is observed that: (a) the redundancy factor and the girder reliability index associated with series system are the highest while their counterparts associated with parallel system are the lowest; (b) for the two series-parallel systems, 2*s*×2*p* SP system (system fails if either of the exterior girders and either of the interior girders fail) provides higher redundancy factor and girder reliability index than 2p×3s SP system (system fails if any two adjacent girders fail); (c) as the correlation among the resistances of girders increases, the redundancy factor and the girder reliability index decrease in the series system but increase in the parallel and series-parallel systems; (d) the results of series system in Case B are higher than those in Case A; however, contrary findings are observed for the parallel and series-parallel systems; and (e) in the perfect correlation case, the redundancy factors of all the systems are 1.0 and thus the associated reliability indices of girders are 3.5. Additionally, note that the redundancy factors derived are often outside of the bounds of 0.95 to 1.05 as defined in AASHTO LRFD. This means that some designs *may contain unnecessary conservatism* (i.e., $\eta_R < 1$) and some are *not conservative enough* (i.e., $\eta_R > 1$).

Example 2

A highway bridge example is presented herein to demonstrate the application of the proposed redundancy factor. The span length of the simply supported bridge is 20 m. The deck consists of 18 cm of reinforced concrete and 8 cm surface layer of asphalt. The roadway width is 8.2 m with 0.2 m wide railing on each side. The space between two adjacent railing columns is 3 m; therefore, there are 7 railing columns on each side of the bridge. The slab is supported by four I-beam steel girders as shown Figure A-31. Assuming the same dimensions of the steel girders, the goal of the design is to determine the bending resistance of the girders using the proposed redundancy factors.

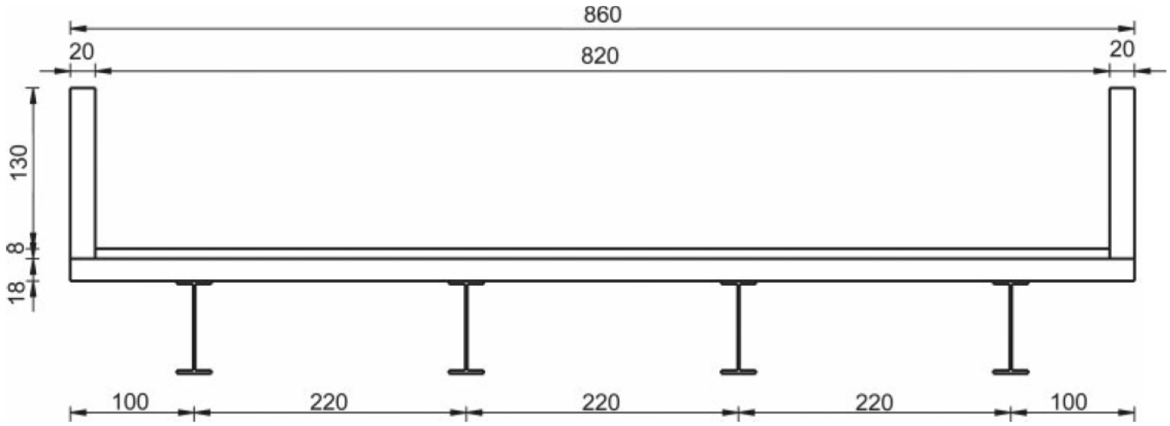


Figure A-31. Graph. The cross-section of the bridge (dimensions are in cm).

The total bending moment acting on each girder consists of the moments due to both dead and live loads. The maximum bending moment occurs at the mid-span cross-section of the girder. Therefore, the moment capacity at mid-span cross-section governs during the design and the limit state equation for flexure failure of the girder *i* at the mid-span cross-section is:

$$g_i = M_{U,i} - M_{L,i}$$

Figure A-32. Equation. Performance function associated with bending failure.

where $M_{U,i}$ and $M_{L,i}$ are the ultimate moment capacity and bending moments acting on girder i, respectively. The next step is to estimate the load effects on each girder due to dead and live loads.

According to AASHTO (2010), vehicular live loading on the roadways of bridges, designated HL-93, shall consist of the design truck or design tandem and the design lane load. In this example, a combination of the design truck and lane load is used. Based on the influence line for the bending moment at the mid-span cross-section, the most unfavorable longitudinal loading position associated with the design truck was determined. In addition, the bridge is subject to the lane load of 9.34 kN/m that is uniformly distributed along the bridge. The maximum bending moment at the mid-span cross-section when both lanes are loaded is M'_{LL} = 3379 kN m.

Since only one lane is loaded for exterior girders, the multiple presence factor m is 1.2 and, thus, the associated lateral load distribution factors are found to be m_{ext} = 0.81. However, for interior girders, the multiple presence factor m is 1.0 because both lanes are loaded; therefore, the lateral load distribution factor is m_{int} = 0.81. With the maximum bending moment at mid-span cross-section and the lateral load distribution factors of each girder, the maximum bending moments due to live load acting on exterior and interior girders are: $M_{LL,ext} = M_{LL,int}$ = 2736 kN m. Since the lateral load distribution factors of exterior and interior girders are identical, the obtained maximum live load bending moments of exterior and interior girders are the same.

The dead load herein refers to the self-weight of the superstructure. For exterior girders, the dead load consists of the weights of the slab, asphalt pavement, railings, and steel girder; however, for interior girders, the self-weight of the railings is not included since it is generally taken by the exterior girders. Therefore, only the weights of the slab, asphalt pavement, and steel girders are considered. Assuming the weights of the slab and asphalt pavement between the exterior and interior girders are uniformly distributed, the weights of slab and asphalt pavement distributed on exterior and interior girder are $w_{s,ext}$ = 7.99 kN/m (slab, exterior girder), $w_{s,int}$ = 9.5 kN/m (slab, interior girder), $w_{a,ext}$ = 3.0 kN/m (asphalt pavement, exterior girder), and $w_{a,int}$ = 4.0 kN/m (asphalt pavement, interior girder), respectively. The uniform railing weight on the exterior girder is $w_{r,ext}$ = 0.44 kN/m. The self-weight of each girder is assumed to be $w_{g,i}$ = 1.96 kN/m. With all the uniform loads obtained previously, the total distributed dead loads for the exterior and interior girder are w_{ext} = 13.41 kN/m and w_{int} = 15.46 kN/m, respectively. Therefore, the dead load bending moments acting on the exterior and interior girders at the mid-span cross-section are: $M_{DL,ext}$ = 671 kN·m and $M_{DL,int}$ = 773 kN· m.

Based on the live load and dead load bending moments obtained previously, the total bending moment is found to be $M_{L,ext}$ = 3407kN·m (for exterior girder) and $M_{L,int}$ = 3509kN·m (for interior girder). Assuming that the resistance and load effect in Figure A-32 are normally-distributed random variables, the total bending moments associated with exterior and interior girders just mentioned are used herein as the mean value of the load effects acting on girders. The coefficients of variation of girder resistance and load effect are assumed to be 0.05 and 0.3, respectively. Therefore, the mean resistances for exterior and interior girders when the reliability index of each girder is 3.5 are found to be $E_c(M_{U,ext})$ = 7200kN·m (for exterior girder) and $E_c(M_{U,int})$ = 7415kN·m (for interior girder), respectively.

For the analyzed bridge, three types of systems are studied herein based on three different definitions of system failure: (a) the system fails if any girder fails (series system); (b) the system fails only if all girders fail (parallel system); and (c) the system fails if any two adjacent girders fail (series-parallel system), as shown in Figure A-33. In addition, three correlation cases among the resistances of girders are investigated herein: (a) $\rho(R_i,R_j) = 0$; (b) $\rho(R_i,R_j) = 0.5$; and (c) $\rho(R_i,R_j) = 1.0$.

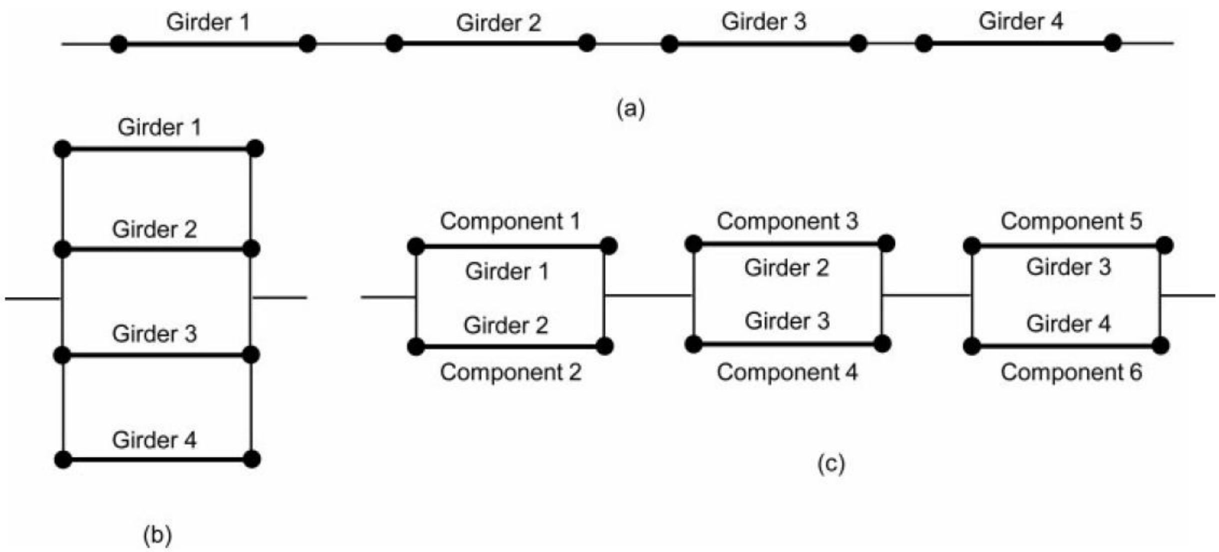


Figure A-33. Graph. Three system models of the analyzed bridge: (a) series system; (b) parallel system; and (c) series-parallel system.

By considering the idealized systems consisting of equally reliable components, the redundancy factors of the three systems associated with the three correlation cases are found in Table 22, Table 24, and Table 26 of the main report and are succinctly tabulated in Table A-11. Multiplying the mean resistances of girders obtained previously by the redundancy factors yields the designed mean resistances of girders in series, parallel, and series-parallel systems, as listed in Table A-12. Since the dimensions of the girders are assumed to be the same, as previously mentioned, the larger bending moment between the exterior and interior girders is selected as the final mean resistance of girder $E_{cs}(M_U)$, as shown in the last column of Table A-12. It is seen that the final design resistance of girder is the same as that of the interior girder; this is because the total load effect acting on interior girder is larger than that on exterior girder.

Table A-11. The redundancy factors of the three systems.

Correlation case	Series system	Parallel system	Series-parallel system
$\rho(R_i,R_j) = 0$	1.041	0.934	0.987
$\rho(R_i,R_j) = 0.5$	1.032	0.956	0.995
$\rho(R_i,R_j) = 1.0$	1.000	1.000	1.000

Note: $V(R) = 0.05$; $V(P) = 0.3$

Table A-12. The designed mean resistance of exterior $E_{cs}(M_{U,ext})$ and interior girders $E_{cs}(M_{U,int})$ in the four-component systems.

System type	Correlation case	$E_{cs}(M_{U,ext})$ (kN·m)	$E_{cs}(M_{U,int})$ (kN·m)	$E_{cs}(M_U)$ (kN·m)
Series system	$\rho(R_i,R_j) = 0$	7495	7719	7719
Series system	$\rho(R_i,R_j) = 0.5$	7430	7652	7652
Series system	$\rho(R_i,R_j) = 1.0$	7200	7415	7415
Parallel system	$\rho(R_i,R_j) = 0$	6725	6926	6926
Parallel system	$\rho(R_i,R_j) = 0.5$	6883	7089	7089
Parallel system	$\rho(R_i,R_j) = 1.0$	7200	7415	7415
Series-parallel system	$\rho(R_i,R_j) = 0$	7106	7319	7319
Series-parallel system	$\rho(R_i,R_j) = 0.5$	7164	7378	7378
Series-parallel system	$\rho(R_i,R_j) = 1.0$	7200	7415	7415

Note: $E(M_{L,ext})$ = 3407 kN · m; $E(M_{L,int})$ = 3509 kN · m; $V(R) = 0.05$; $V(P) = 0.3$; $E_{c,N}(M_{U,ext})$ =7200 kN · m; $E_{c,N}(M_{U,int})$ =7415 kN ·m

The corresponding component reliability indices of exterior (β_{ext}) and interior (β_{int}) girders and the associated system reliability indices (β_{sys}) of the three systems are presented in Table A-13. It is seen that the system reliability indices in all correlation cases are no less than 3.5. Therefore, they satisfy the predefined reliability level $\beta_{target} = 3.5$. For the no correlation and partial correlation cases, the component reliability indices (β_{ext} and β_{int}) associated with series system are much higher than those associated with other systems while their counterparts associated with parallel system are much lower. This reflects the effect of system modeling on the design of structural components.

When computing the redundancy factors presented in Table A-11 associated with series-parallel system, different correlations among the resistances of six components are considered: $\rho(R_i,R_j)$ = 0, 0.5, and 1.0 (i, j = 1, 2, 3, . . ., 6). However, it should be noted that in Figure A-33(c), components 2 and 3 refer to the same girder (Girder 2) and Girder 3 also represents both components 4 and 5, which indicates that components 2, 3 and components 4, 5 are perfectly correlated. Hence, the series-parallel system actually consists of four components instead of six components. In order to distinguish these two cases, the system considering the perfect correlation between components 2, 3 and components 4, 5 is named "4-component series-parallel system'' while the system that doesn't take perfect correlation into account is called "6-component series-parallel system''. Therefore, for the no correlation and partial correlation cases, the redundancy factors in Table A-11 associated with the 6-component series-parallel system are slightly higher than the redundancy factors associated with the 4-component series-parallel system.

Table A-13. The reliability indices of exterior and interior girders and the system reliability index.

System type	Correlation case	β_{ext}	β_{int}	β_{sys}
Series system	$\rho(R_i,R_j) = 0$	3.95	3.75	3.58
Series system	$\rho(R_i,R_j) = 0.5$	3.89	3.70	3.60
Series system	$\rho(R_i,R_j) = 1.0$	3.69	3.50	3.50
Parallel system	$\rho(R_i,R_j) = 0$	3.26	3.08	3.61
Parallel system	$\rho(R_i,R_j) = 0.5$	3.40	3.22	3.63
Parallel system	$\rho(R_i,R_j) = 1.0$	3.69	3.50	3.69
Series-parallel system	$\rho(R_i,R_j) = 0$	3.60	3.42	3.62

Series-parallel system	$\rho(R_i,R_j) = 0.5$	3.65	3.47	3.61
Series-parallel system	$\rho(R_i,R_j) = 1.0$	3.69	3.50	3.50

Note: $E(M_{L,ext})$ = 3407 kN · m; $E(M_{L,int})$ = 3509 kN · m; $V(R)$ = 0.05; $V(P)$ = 0.3; $E_{c,N}(M_{U,ext})$ =7200 kN · m; $E_{c,N}(M_{U,int})$ =7415 kN ·m

By taking the perfect correlation between components 2, 3 and components 4, 5 into account, the redundancy factors associated with the 4-component series-parallel system are found to be 0.983 (no correlation case) and 0.991 (partial correlation case), and 1.0 (perfect correlation case). The designed mean resistances of girders and the associated reliability indices of girders and system based on these redundancy factors are listed in Table A-14 and Table A-15, respectively. It is observed that the final mean resistance $E_{cs}(M_U)$ and the system reliability index β_{sys} without considering perfect correlation (Table A-12 and Table A-13) is slightly higher than those considering perfect correlation (Table A-14 and Table A-15); this indicates that the design based on the 6-component series-parallel system is safer than that based on the 4-component series-parallel system. Therefore, the redundancy factors from the regular idealized system that doesn't consider the perfect correlation among some components can be used as a good approximation of the true redundancy factors associated with the series-parallel system to determine the designed mean resistance of girders. This finding shows the necessity of generating standard tables using the regular idealized systems for different number of components, different system models, and different correlations. After these standard tables are generated, the redundancy factor corresponding to a specific system can be found from these tables and then directly used in the design.

Table A-14. The designed mean resistance associated with the 4-component series-parallel system.

System type	Correlation case	$E_{cs}(M_{U,ext})$ (kN·m)	$E_{cs}(M_{U,int})$ (kN·m)	$E_{cs}(M_U)$ (kN·m)

Series-parallel system	$\rho(R_i,R_j) = 0$	7078	7289	7289
Series-parallel system	$\rho(R_i,R_j) = 0.5$	7139	7348	7348
Series-parallel system	$\rho(R_i,R_j) = 1.0$	7200	7415	7415

Note: $E(M_{L,ext})$ = 3407 kN · m; $E(M_{L,int})$ = 3509 kN · m; $V(R)$ = 0.05; $V(P)$ = 0.3; $E_{c,N}(M_{U,ext})$ =7200 kN · m; $E_{c,N}(M_{U,int})$ =7415 kN ·m

Table A-15. The reliability indices of exterior and interior girders and the system reliability index associated with the 4-component series-parallel system.

System type	Correlation case	β_{ext}	β_{int}	β_{sys}
Series-parallel system	$\rho(R_i,R_j) = 0$	3.58	3.39	3.59
Series-parallel system	$\rho(R_i,R_j) = 0.5$	3.63	3.44	3.58
Series-parallel system	$\rho(R_i,R_j) = 1.0$	3.69	3.50	3.50

Note: $E(M_{L,ext})$ = 3407 kN · m; $E(M_{L,int})$ = 3509 kN · m; $V(R)$ = 0.05; $V(P)$ = 0.3; $E_{c,N}(M_{U,ext})$ =7200 kN · m; $E_{c,N}(M_{U,int})$ =7415 kN ·m

References

AASHTO (2010). LRFD bridge design specifications, 5th Ed., Washington, DC.

AASHTO (2008). Manual for Bridge Evaluation. American Association of State Highway and Transportation Officials, Washington, DC.

Zhu, B., Frangopol, D. M., and Kozy, B. M. (2014). System reliability and the redundancy factor by simplified modeling. *Life-cycle of structural systems: Design, assessment, maintenance and management*, H. Furuta, D. M. Frangopol and M. Akiyama (Eds.). Taylor & Francis Group, London, 614-618.

COMMENTARY: APPLICATION OF REDUNDANCY FACTORS IN THE DESIGN OF BRIDGES AND EXAMPLES OF SYSTEM RELIABILITY MODELING OF EXISTING BRIDGES

Generalized procedure for applying redundancy factors in bridge design

This commentary summarizes methodologies for the practical application of the proposed redundancy factors. More specifically, the approach to determine the redundancy factor η_R contained in the strength limit state outlined in the AASHTO LRFD bridge design specifications (please refer to Figure 50 within the main report) is discussed in detail. One key part of the adopted methodology is the bridge system reliability modeling. In particular, when building a system reliability model, several fundamental engineering principles are integrated into the approach, including identification of relevant failure modes, uncertainty quantification, and probabilistic considerations. Overall, this section of the commentary describes the general procedure for determining the bridge redundancy factors.

Bridge modeling, system reliability, and redundancy evaluation framework

The generalized framework for determining the bridge redundancy factors is listed in Figure C-1. The procedure begins with identifying relevant failure modes, then developing a system reliability model considering different combinations (e.g., series, parallel, series-parallel) of individual failure modes, finding the redundancy factors based on the provided tables, and finally applying these redundancy factors to the strength limit state outlined in the AASHTO LRFD bridge design specifications.

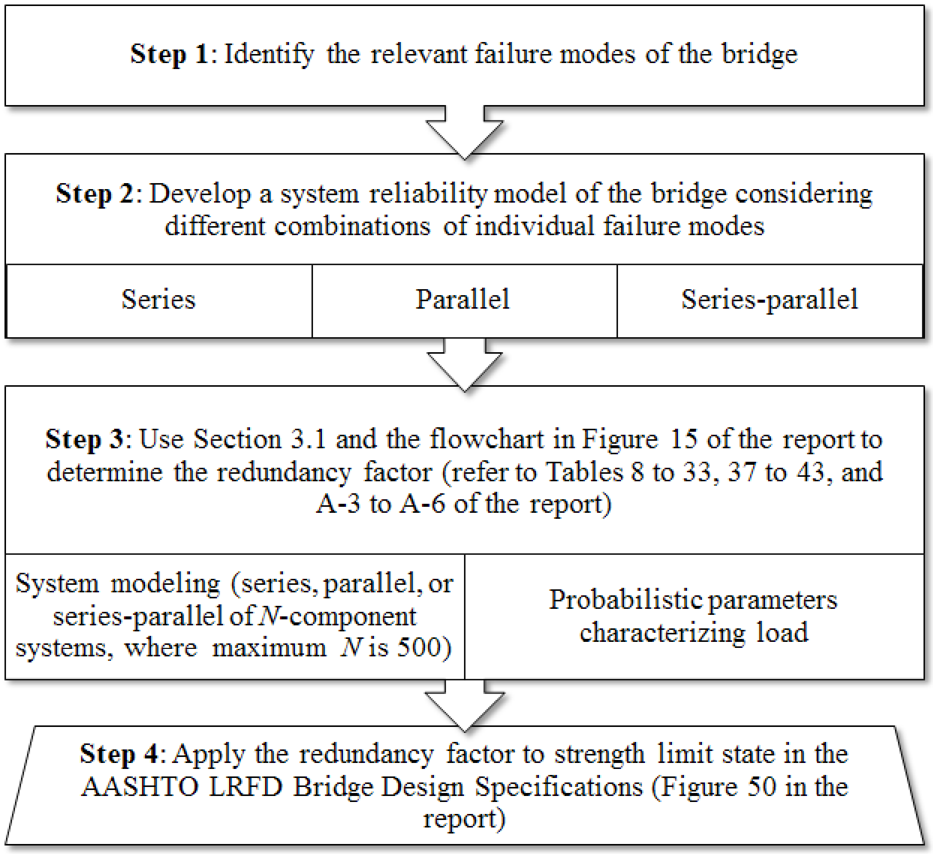


Figure C-1. Graph. Flowchart describing bridge modeling, system reliability, and redundancy evaluation.

The first step of the framework involves identifying the relevant failure modes of the analyzed bridge system. For instance, Example 2 in the appendix of the main report considers flexural failure of a bridge superstructure system consisting of a reinforced concrete deck supported by four I-beam steel girders. After calculating the total bending moments acting on the exterior and interior girders at the mid-span cross-section, in addition to the mean resistances corresponding to exterior and interior girder when the reliability index of each girder is 3.5, the system reliability model corresponding to bridge system is established.

For the system analyzed in Example 2, three types of system models, based on three different definitions of system failure are developed, as shown in Figure A-33 of the appendix in main report. The first model defines system failure as the failure of any girder (series system) and the second model dictates that the system fails only if all girders fail (parallel system). The third reliability model denotes that the system fails if any two adjacent girders fail (series-parallel system). Next, considering these reliability models and the correlation among resistance of girders, the redundancy factors of the three systems are found in Table 22, Table 24, and Table 26 of the report. Finally, the redundancy factors found in this example can be directly used within the strength limit state equation within AASHTO LRFD bridge design specifications.

System reliability modeling

This section outlines the methodology for developing system reliability models corresponding to bridge systems. First, fundamental nomenclature and definitions are clarified. Additionally, several examples of modeling simple structures (e.g., beams and trusses) and bridges, in general, are presented.

Definitions

Element

For the purposes of this report, an element is defined as any physical piece of material that comprises a bridge system. Examples of bridge elements include structural members such as girders, decks, and piers. Each element has particular material, geometrical, and physical properties that contribute to its overall internal capacity. Hazards and loading events may affect elements in various ways (e.g., uniformly, selectively), depending upon the location and intensity of loadings.

Component

A component is defined as a "place" where a limit state could occur. The reliability of each element of a bridge may be evaluated with respect to various limit states and at different locations. For instance, a bridge girder can fail in many ways (e.g., bending, shear) and in various locations (e.g., mid-span for bending, at the support for shear). Components characterizing the main failure modes of a system are arranged in specific configurations (i.e., series, parallel, series-parallel) to form an idealized representation of the reliability performance of the system denoted as the system reliability model.

Applications

This section of the commentary presents a collection of examples of system reliability modeling regarding various applications including beams, trusses, and simple bridges.

Beams

Consider a simply supported beam structure, as shown in Figure C-2. The beam is subjected to a distributed load *w* and two points of interest are established: one at midspan where the bending moment is largest, point A, and one near the left support where the shear force is maximum, point B. It is assumed that the resistance in shear is weaker near the left support than that near the right support.

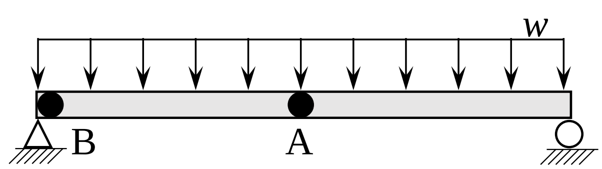


Figure C-2. Graph. Simply supported beam with uniform load.

The bending moment at point A is denoted as M_A and the shear force at point B as S_B. The performance function associated with bending at point A is

$$g_M = R_M - Q_M$$

Figure C-3. Equation. Performance function associated with bending at point A.

where R_M is the bending resistance and Q_M is the bending load effect. Similarly, the performance function corresponding to shear failure at point B is expressed as

$$g_S = R_S - Q_S$$

Figure C-4. Equation. Performance function corresponding to shear failure at point B.

where R_S is the shear resistance and Q_S is the shear load effect. The load effects Q_M and Q_S are correlated because they both are dependent on the same uniform load w; thus, the performance functions (i.e., failure modes) presented in Figure C-3 and Figure C-4 are also correlated. Considering shear and bending failure modes, the beam system in Figure C-2 may be idealized as two components arranged in series, as shown in the reliability model in Figure C-5.

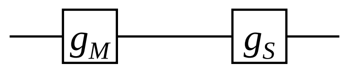


Figure C-5. Graph. System reliability model of the beam in Figure C-2 considering bending and shear failure.

The event set describing the failure of the beam system is

$$F_{beam} = (g_M < 0) \cup (g_S < 0)$$

Figure C-6. Equation .Event set describing the failure of the beam.

where the union ∪ represents the occurrence of event $g_M < 0$, event $g_S < 0$, or both events.

Trusses

Consider the ten-bar symmetric truss system shown in Figure C-7. The truss is subjected to two concentrated loads W.

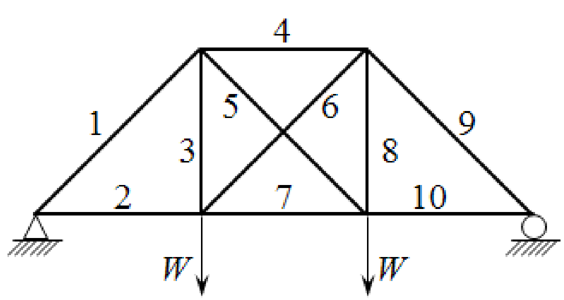


Figure C-7. Graph. 10 bar symmetric truss (adopted from Frangopol and Curley 1987)

In order to determine a system reliability model, several collapse mechanisms are considered. If bar 1 or bar 9 fails, then the entire truss fails. Additionally, if failure of two members, including only failure of member 3 or member 4 or member 5 is considered, failure of both bars 3 and 1, or 3 and 4, or 3 and 5, or 3 and 6, or 3 and 8, or 3 and 9, or 4 and 1, or 4 and 5, or 4 and 6, or 4 and 8, or 4 and 9, or 5 and 1, or 5 and 6, or 5 and 8, or 5 and 9, will cause the truss system to collapse. This system failure model considering failure of one member, or failure of two members including members 3 or 4 or 5, is depicted in the system reliability model shown in Figure C-8.

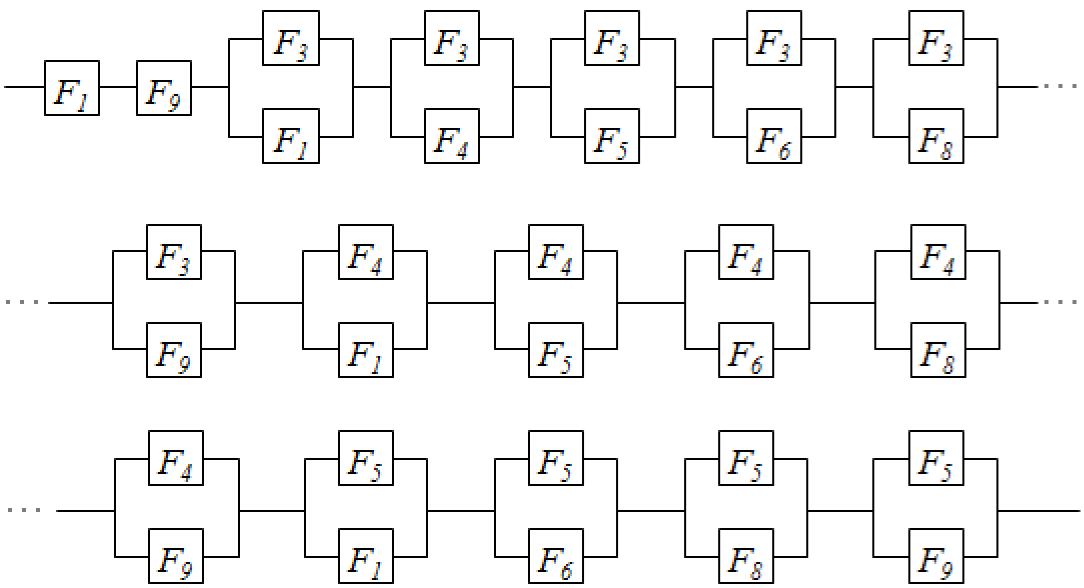


Figure C-8. Graph. System reliability model for the 10 bar truss in Figure C-7, considering failure of one member and failure of two members including failure of members 3, 4, or 5.

Considering that F_j represents the failure of bar *j*, the event set describing the failure of the entire truss structure under the assumption that one member fails or two member fail, including members 3, 4 or 5, is

$$\begin{aligned} F_{truss} = F_1 \cup F_9 \cup (F_3 \cap F_1) \cup (F_3 \cap F_4) \cup (F_3 \cap F_5) \cup (F_3 \cap F_6) \cup (F_3 \cap F_8) \\ \cup (F_3 \cap F_9) \cup (F_4 \cap F_1) \cup (F_4 \cap F_5) \cup (F_4 \cap F_6) \cup (F_4 \cap F_8) \\ \cup (F_4 \cap F_9) \cup (F_5 \cap F_1) \cup (F_5 \cap F_6) \cup (F_5 \cap F_8) \cup (F_5 \cap F_9) \end{aligned}$$

Figure C-9. Equation. Event set describing the failure of the truss structure.

where the intersection ∩ represents the simultaneous occurrence of all events investigated.

Bridge modeling – general

Bridges are designed/constructed using a wide array of methods/materials and, at the same time, subjected to a variety loads. These factors affect the resistance and load effects parameters which are embedded in component performance functions. This section contains an example that illustrates the process of developing a system reliability model considering a variety of different limit states, such as strength (e.g., bending, shear), serviceability (e.g., maximum deflection), and fatigue-and-fracture (e.g., fatigue cracking).

In general, three components, or performance functions may be considered to form a system reliability model of this bridge. Figure C-10 depicts, at the most basic level, the system reliability model of the bridge; three basic failure modes are defined for this bridge: (1) strength g_{st}, (2) serviceability g_{ser}, and (3) fatigue g_f.

Figure C-10. Graph. System reliability model for investigated steel bridge.

This reliability model may be further refined when more information about each of the failure mode is included; for example, the single reliability block representing strength limit states in Figure C-10 may be considered a sub-system consisting of three components, shear g_s, bending g_b, and torsion g_t failure modes arranged in series. Additionally, multiple serviceability limit states may be included within the model by breaking down the component related to serviceability within Figure C-10 into two components representing deflection g_δ and vibration comfort g_v. Similarly, the component corresponding to fatigue limit states in Figure C-10 may be further broken down into a series sub-system composed of multiple fatigue critical details (e.g., bolted g_{bc} and welded g_{wc} connections). The refined system reliability model for the investigated bridge is shown in Figure C-11.

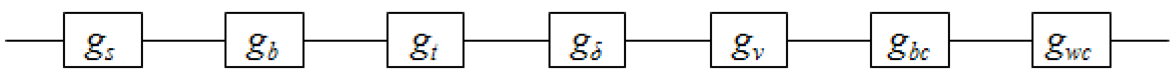


Figure C-11. Graph. Refined system reliability model for the investigated bridge.

A similar approach is applied by Estes and Frangopol (2001), where a hypothetical series system for a girder consisting of components relating to failure by shear, moment, and excessive deflection is developed (see Figure C-12).

Figure C-12. Graph. Hypothetical Series System model of typical Girder (Estes and Frangopol 2001).

Using the general approach outlined herein, one can idealize any type of bridge systems including girder, cable-stayed, and suspension bridges.

Examples of system modeling

This section of the commentary presents several examples of modeling bridge systems, as found in recently published literature. More specifically, the system reliability modeling of girder and suspension bridges are discussed herein. Additional examples of system reliability modeling of existing bridges can be found for several types of bridges including prestressed girder (Akgül and Frangopol 2004b) and riveted railway (Imam *et al.* 2012) bridges.

Example 1: Girder bridge (Colorado Bridge E-17-AH)

Bridge description

Component and system reliability of the Colorado Bridge E-17-AH are analyzed in this example with respect to several limit states (Estes and Frangopol 1999). Colorado Bridge E-17-AH is made up of three equal length (13.3 m) simple spans, as shown in Figure C-13. The deck consists of 22.9 cm of reinforced concrete and a 7.6-cm surface layer of asphalt. There are two lanes of traffic in each direction with an average daily traffic of 8,500 vehicles. The roadway width is 12.18 m with 1.51 m of pedestrian sidewalks and hand railings on each side. The bridge provides 6.8 m of clearance for the railroad spur that runs underneath and there is no skew or curvature. The slab is supported by nine standard-rolled, compact, non-composite steel girders as shown in Figure C-13(b). End and intermediate diaphragms at the third points are used to stiffen the girders. Each girder is supported at one end by a fixed bearing and an expansion bearing at the other end.

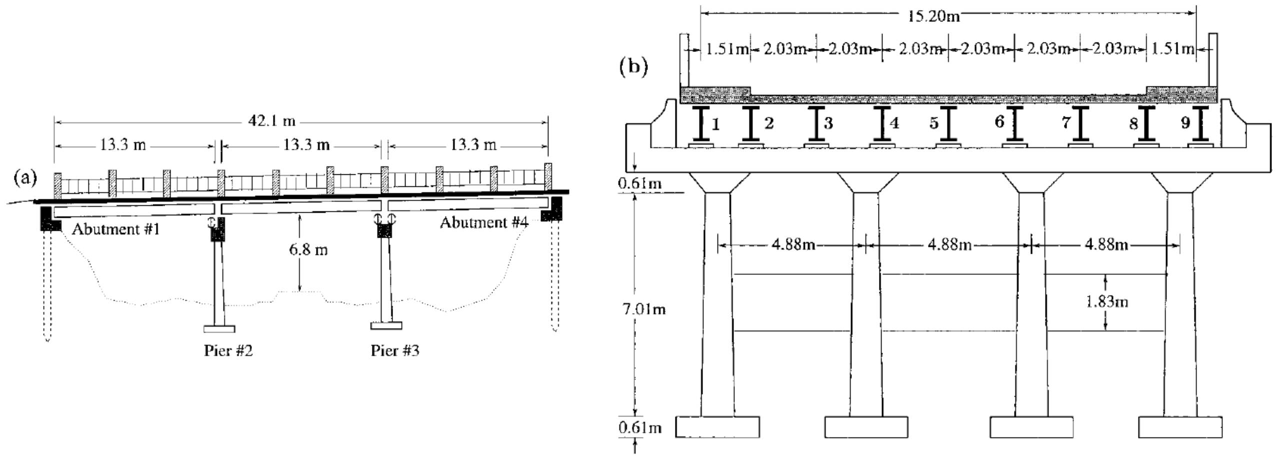


Figure C-13. Graph. Colorado State Highway Bridge E-17-AH: (a) elevation and (b) cross section views (Estes and Frangopol 1999).

Component reliability

Sixteen different failure modes, as listed in Table C-1, are analyzed with respect to the E-17-AH Bridge. Each failure mode i is characterized by a limit state $g(i) = 0$, such that $g(i) \leq 0$ and $g(i) > 0$ define the failure and safe states, respectively. The failure modes analyzed include moment failure of the slab, moment and shear failure of the girders, and multiple failure modes of the pier cap, columns, and footings. The girders are classified as exterior (i.e., girders 1 and 9 in Figure C-13(b)) that carry emergency vehicle and pedestrian traffic, interior-exterior (i.e., girders 2 and 8 in Figure C-13(b)) that act as exterior girders for normal traffic, and interior (i.e., girders 3 to 7 in Figure C-13(b)). Limit state equations were developed separately for each type of girder.

Table C-1. Limit state equation, failure mode, and reliability index (adopted from Estes and Frangopol 1999).

Limit state equation	Failure mode	Reliability index β
$g(1) = 0$	Concrete deck, flexure	5.51
$g(2) = 0$	Interior girder, shear	6.22
$g(3) = 0$	Interior girder, flexure	2.44
$g(4) = 0$	Exterior girder, flexure	4.02
$g(5) = 0$	Exterior girder, shear	7.13
$g(6) = 0$	Interior-exterior girder, flexure	2.79
$g(7) = 0$	Interior-exterior girder, shear	6.43
$g(8) = 0$	Pier cap, shear	3.83
$g(9) = 0$	Pier cap, positive moment	8.82
$g(10) = 0$	Pier cap, negative moment	8.75
$g(11) = 0$	Top column, crushing	5.80
$g(12) = 0$	Bottom column, crushing	5.72
$g(13) = 0$	Footing, one-way shear	7.69
$g(14) = 0$	Footing, two-way shear	5.28
$g(15) = 0$	Footing, flexure	2.60
$g(16) = 0$	Expansion bearing, crushing	7.84

Prior to considering any deterioration, 24 random variables were identified that included material strength, model uncertainty, girder distribution factors, and material dimensions that could not be directly measured. The parameters that define these random variables were adopted from existing literature. The notations used to define these random variables and their mean values and standard deviations are shown in Table C-2. Limit-state equations that are defined by capacity minus demand for each of the sixteen failure modes in Table C-1 were developed in terms of the twenty-four random variables in Table C-2 (Estes and Frangopol 1999).

Table C-2. Random variables used in the reliability analysis of the E-17-AH Bridge (adopted from Estes and Frangopol 1999).

Random variables	Units	Notation	Mean value	Standard deviation
Uncertainty factor: reinforcing steel area in concrete	--*	λ_{rebar}	1.0	0.015
Yield stress of steel reinforcing in deck	MPa	f_y	386.1	42.5
Uncertainty factor: effective depth of rebar in concrete	--	λ_{deff}	1.0	0.02
Model uncertainty: flexure in concrete	--	γ_{mfc}	1.02	0.061
Uncertainty factor: weight of truck on bridge	--	γ_{trk}	1.38	0.1656
Uncertainty factor: live load shear on interior girders	--	V_{trk-i}	1.38	0.1656
Yield strength of steel in girders	MPa	F_y	252.5	29.0
Uncertainty in live load girder distribution: interior girders	--	DF_i	1.309	0.163
Uncertainty in live load girder distribution: interior-exterior girders	--	DF_{i-e}	1.14	0.142
Uncertainty in live load girder distribution: exterior girders	--	DF_e	0.982	0.122
Uncertainty factor: impact on girders	--	I_{beam}	1.14	0.114
Live load moment on interior girders (kNm)	--	M_{trk-i}	579.4	69.6
28-day compressive strength of concrete	MPa	$f'c$	19.0	3.42
Uncertainty factor: weight of asphalt	--	λ_{asph}	1.0	0.25
Uncertainty factor: weight of concrete	--	λ_{conc}	1.05	0.105
Uncertainty factor: weight of steel	--	λ_{steel}	1.03	0.082
Model uncertainty: shear in steel	--	γ_{msg}	1.14	0.137
Model uncertainty: flexure in steel	--	γ_{mfg}	1.11	0.128
Uncertainty factor: live load shear on exterior girders	--	V_{trk-e}	1.13	0.1356
Live load moment on exterior girders	kNm	M_{trk-e}	474.1	56.9
Model uncertainty: shear in concrete	--	γ_{msc}	1.075	0.108
Area of shear reinforcement/bar spacing	mm	A_v/s	4.52	0.18
Model uncertainty: eccentricity in short columns	--	γ_{mcc}	0.85	0.085
Modulus of elasticity: steel	GPa	E_s	199.9	12.0

*Random variables without units listed are dimensionless.

System reliability

Considering all possible failure modes, a series-parallel model for Colorado Bridge E-17-AH is developed and shown in Figure C-14. In this figure, the performance functions $g(i)$ associated with the individual failure modes correspond to the limit-state equations $g(i) = 0$, indicated in Table C-1. For example, $g(1)$ refers to failure of the concrete bridge deck that is shown in series in Figure C-14. Because of the large end and center diaphragms in the superstructure, which will transfer load, it is assumed that the failure of any three adjacent girders is required for the superstructure to fail and that the concrete deck is identical throughout an individual span. Assuming no deterioration of the structure over time, and considering no correlation between the resistances of the girders, the system reliability for the bridge is $\beta_{sys} = 2.51$.

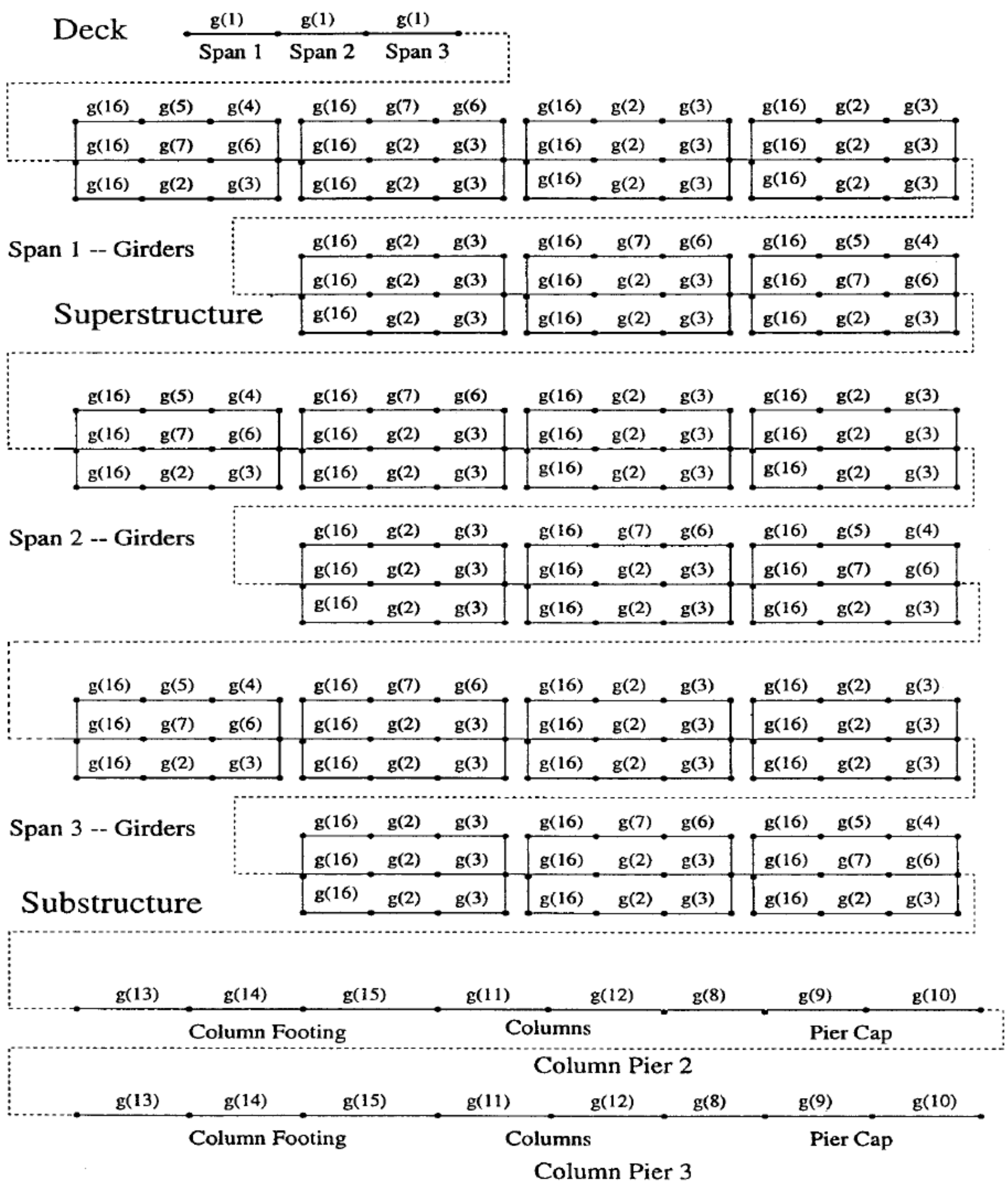


Figure C-14. Graph. Series-Parallel Model for Bridge E-17-AH: Deck, Superstructure, and Substructure (Estes and Frangopol 1999).

Although the model provided in Figure C-14 is thorough and complete, it is possible to simplify it further by making some reasonable assumptions. The failure modes associated with very high reliabilities (e.g., $\beta_i > 6$), which contribute little to the reliability of the system, are eliminated. Further, if the spans are assumed to be perfectly correlated, and the symmetry within a span is considered, the system model can be reduced to the model depicted in Figure C-15(a), where any three adjacent girders must still fail for the system to fail. The girders are numbered 1–5 as shown in Figure C-13(b). Using the simplified model in Figure C-15(a) without deterioration, the system reliability is equal to $\beta_{sys} = 2.54$, which is very close to the reliability index $\beta_{sys} = 2.51$, which is associated with the more complex model shown in Figure C-14.

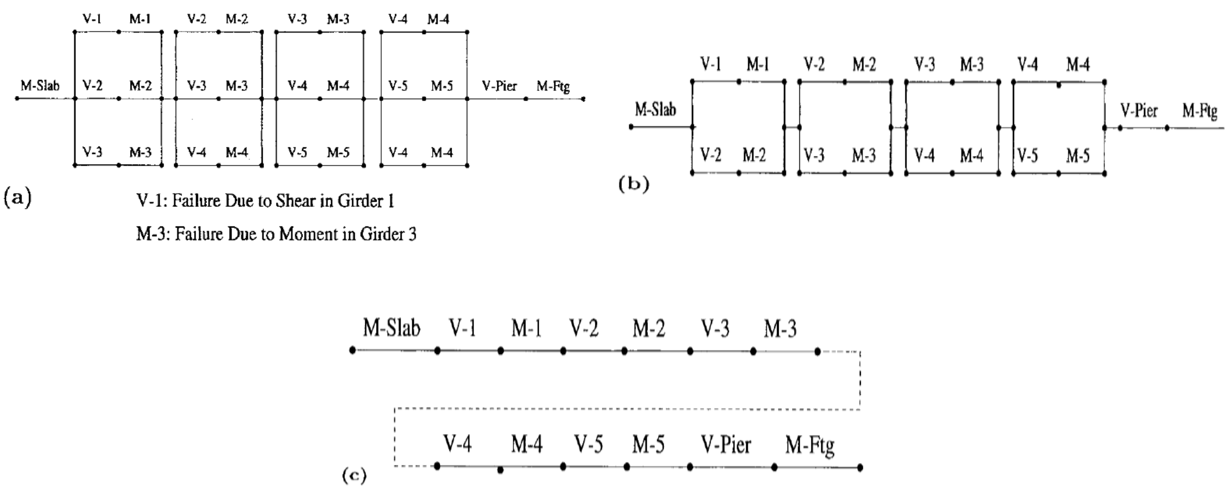


Figure C-15. Graph. Simplified series-parallel model for the E-17-AH Bridge considering that failure of any (a) three adjacent girders, (b) two adjacent girders, (c) girder is required for system failure (Estes and Frangopol 1999).

The system model and correlation between random variables affects the overall bridge reliability. In the previous computation where $\beta_{sys} = 2.54$, it was assumed that the girder resistances were uncorrelated, (i.e., $\rho(R_i,R_j) = 0.0$, where $\rho(R_i,R_j)$ is the correlation coefficient between the resistance of girders i and j). Using the model shown in Figure C-15(a), the system reliability is computed β_{sys} = 2.49 when $\rho(R_i,R_j)$ = 0.5 and β_{sys} = 2.31 when $\rho(R_i,R_j)$ = 1.0. The system failure event is altered as shown in Figure C-15(b), where only any two adjacent girders need to fail for the system to fail, and in Figure C-15(c), where only one girder must fail. The system reliability results for all three models shown in Figure C-15 are indicated in Table C-3.

Table C-3. Bridge system reliability results using different system failure models (please refer to Figure C-15) and different correlation between girder resistances (Estes and Frangopol 1999).

System failure event	**Correlation between girder resistances**		
	$\rho(R_i,R_j)$ = 0.0	$\rho(R_i,R_j)$ = 0.5	$\rho(R_i,R_j)$ = 1.0
Failure of any girder	1.97	2.06	2.23
Failure of any two adjacent girders	2.50	2.41	2.26
Failure of any three adjacent girders	2.54	2.49	2.31

For the model in Figure C-15(c), which is entirely a series system, the increased correlation between the resistances improves the system reliability. For the series-parallel system models in Figure C-15(a) and (b), the increased correlation between the resistances decreases the system reliability. When there is perfect correlation between the resistances, the three models produce very close results. The effects of correlation between other random variables on bridge system reliability could also be investigated along with other variations in the system model. Such analyses emphasize the importance of accurate input data for reliability computations. The results obtained are only as good as the parameters of the random variables, the correlation structure among variables, and the system model that produces them.

Example 2: Girder bridge (Colorado Bridge E-17-LE)

Using a similar methodology as the one outlined in the previous example, the system reliability model for Colorado Bridge E-17-LE is presented in Akgül and Frangopol (2004a). For steel I-beam and plate girder bridges, Akgül and Frangopol (2003) derived limit state equations for the ultimate capacity of steel girders with respect to flexure, shear, and serviceability with respect to permanent deformation under overload. Once the values of random variables, deterministic parameters, and constant coefficients, for Colorado Bridge E-17-LE, are obtained, component reliability indices for the slab and the girders are calculated for each bridge based on the computer program RELSYS (Estes and Frangopol 1999). The cross section of the superstructure and the system failure model are shown in Figure C-16.

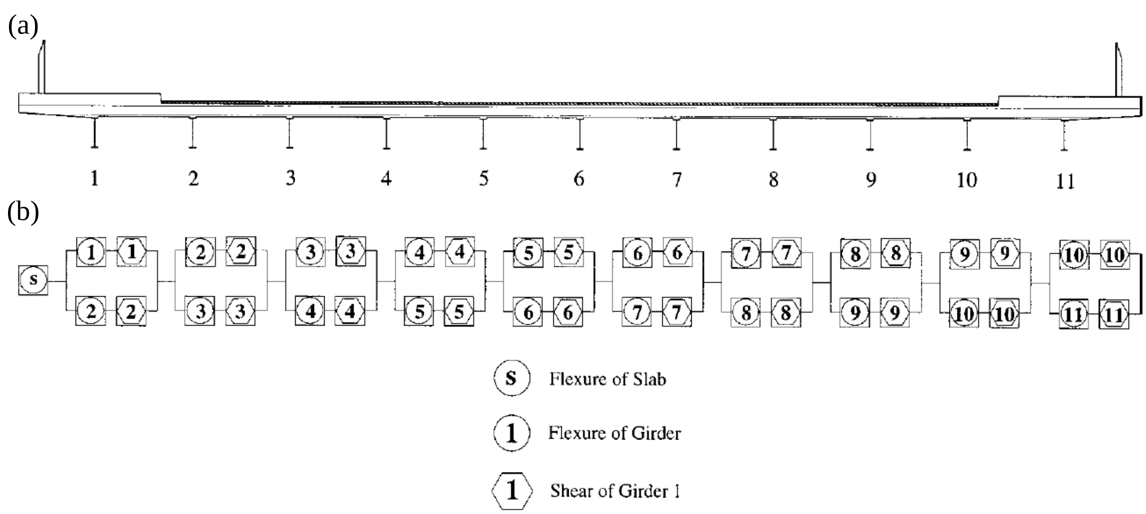


Figure C-16. Graph. (a) Cross section and (b) system reliability model associated with Colorado Bridge E-17-LE (adopted from Akgül and Frangopol 2004a)

The system reliability index, as shown in Table C-4, is calculated based on the system failure model shown in Figure C-16(b). System failure is defined as failure of a series-parallel system: failure of slab in flexure, or failure of any two adjacent girders (i.e., flexural failure of girder at maximum moment location, or shear failure of girder at maximum shear location, or both), or both.

Table C-4. Reliability indices for Colorado Bridge E-17-LE

Limit state function	Reliability index
Slab, flexure	3.89
Steel girder, flexure	4.13
Steel girder, shear	5.80
Steel girder, serviceability	5.00
Series-parallel subsystem	.4.28
System with slab	3.82
System without slab	4.10

Colorado Bridge E-17-LE consists of eleven girders, reflected by the large system failure model shown in Figure C-16(b). Calculated component and system reliability indices for the bridge E-17-LE are listed in Table C-4. Component reliability indices are calculated for flexure of the slab, flexure of the steel girder at the critical moment section, shear of the steel girder, and serviceability of steel girder at the critical moment section for permanent deformation under overload. Three system reliability indices are listed in Table C-4; reliability indices for a series-parallel subsystem based on failure of any two adjacent girders in flexure or shear mode, an overall system reliability index including slab in the failure model, and an overall system reliability index based on girder failures only.

Example 3: Suspension bridge (Innoshima Bridge)

Bridge description

The Innoshima bridge is analyzed in detail by Imai and Frangopol (2001, 2002) in terms of structural analysis, reliability analysis, and quantification of uncertainty in loading and geometric properties. This suspension bridge links the islands of Honshu and Shikoku in the Hiroshima prefecture of Japan. The center span measures 770 m with two side spans of 250 m. The roadway is 20 m from safety fence to safety fence and accommodates four lanes of traffic. The suspended structure consists of two stiffening trusses spaced 26 m apart. Lateral trusses, spaced 10 m apart, connect the two stiffening trusses. The lateral trusses are braced by upper and lower diagonal members. Plate girders are supported on the upper chords of the lateral trusses. A pedestrian way is also supported on the lower chords of the lateral trusses. The height of towers is 135.85 m and each tower consists of two shafts connected by two horizontal struts and cross bracing. Bridge schematics are shown in Figure C-17. Imai and Frangopol (2001) presented reliability-based design assumptions used for probabilistic assessment of this bridge. Both two- and three-dimensional models were used for system reliability computations.

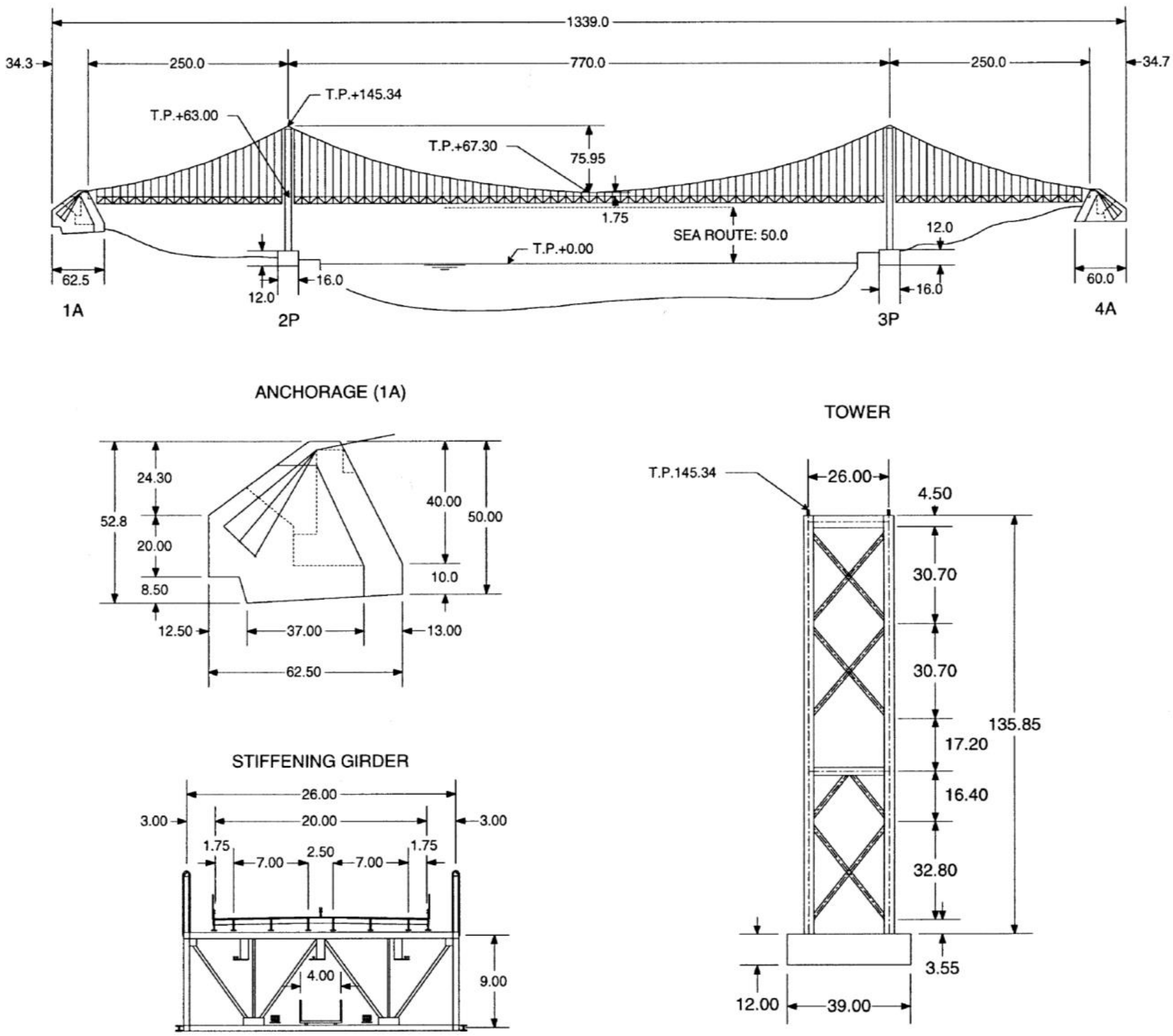


Figure C-17. Graph. General view of the Innoshima Bridge, dimensions are in m (Imai and Frangopol 2002).

Component reliability

The design of the Innoshima Bridge considers dead load D, live load L, wind load W, temperature *T*, and support displacement SD. This study considers the dead, live, and wind loads as random and temperature and support displacements as deterministic (Imai and Frangopol 2001). For complete details regarding the structural models used in this reliability evaluation, please refer to Imai and Frangopol (2002). Considering a two-dimensional model, limit state functions may be established for main bridge components. The limit state corresponding to the main cable is

$$g = A_c\sigma_c - T_c(\mathrm{D, L, T, SD}) = 0$$

Figure C-18. Equation. Limit state corresponding to the main cable.

where A_c=cross-section area of the main cable, σ_c = rupture strength of main cable, and T_c = tension in the main cable. The design of the hanger rope considers the second-order effect due to bending in the main cable, B_M, and errors associated with manufacturing, E_M, and erection, E_E, processes. The limit state function for a hanger rope is

$$g = 4R_h - T_h(\mathrm{D, L, T, SD}) - E_M - E_E - B_M = 0$$

Figure C-19. Equation. Limit state equation for a hanger rope.

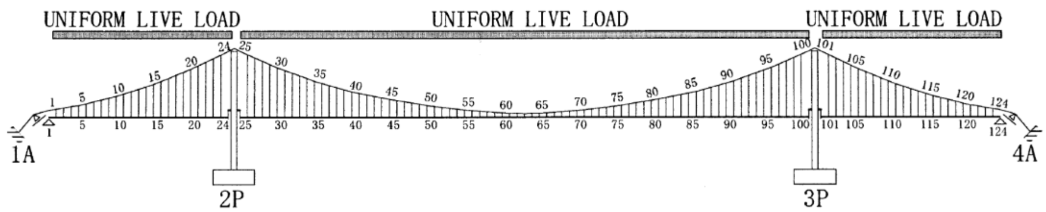
where R_h = rupture strength of one hanger rope and T_h = tension in the hanger rope. The stiffening girders (upper chord) possess the following limit state

$$g = A_G\sigma_G - \frac{M_z(\mathrm{D, L, T, SD})}{h} = 0$$

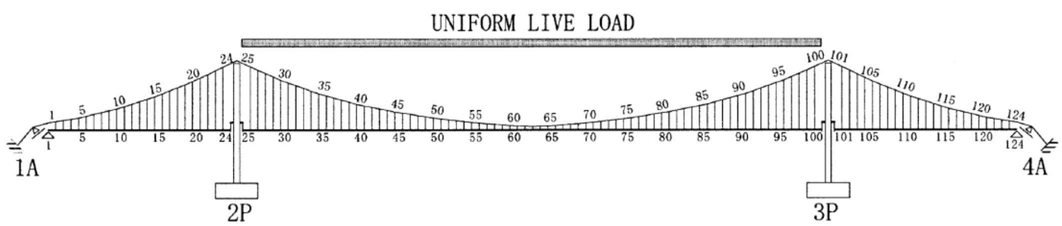
Figure C-20. Equation. Limit-state equation for stiffening girders (upper chord).

where A_G = cross-section area of the upper chord, σ_G = yield stress of the upper chord, $M_z(\mathrm{D, L, T, SD})$ = bending moment in the vertical direction, and h = height of the stiffening truss.

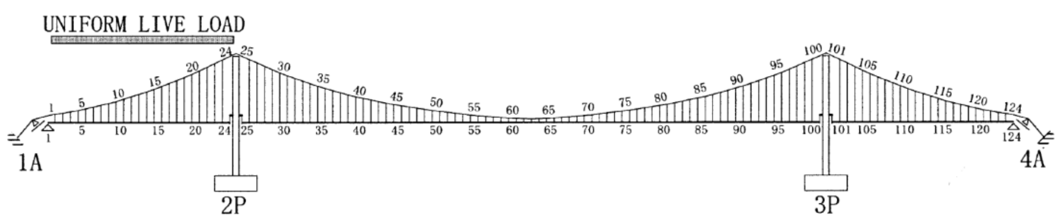
Three cases of live load conditions are considered as indicated in Figure C-21. In the first case, it is considered that the live loads exist on the whole bridge (Case 1), which is the critical case for the main cables. In the second case, it is considered that the live loads are on the central span (Case 2), and the third case assumes that live loads exist on the side span only (Case 3) which is the critical case for the stiffening girders. The component reliability indices are computed for every cross section. They are plotted in Figure C-22 and Figure C-23 for live loads Case 1 and Case 3, respectively.



(a) Live Load from 1A to 4A (Case 1)



(b) Live Load from 2P to 3P (Case 2)



(c) Live Load from 1A to 2P (Case 3)

Figure C-21. Graph. Uniform live load cases (Imai and Frangopol 2002).

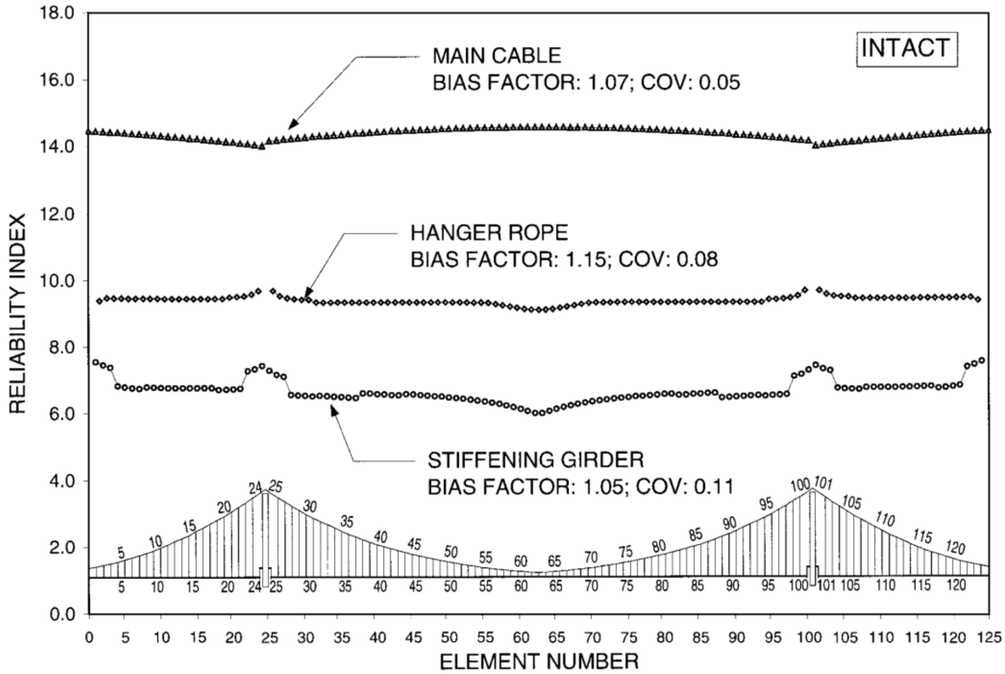


Figure C-22. Graph. Reliability indices of main cable, hanger rope, and stiffening girder: load Case 1 (D, L, T, SD), live load over all three spans (Imai and Frangopol 2001)

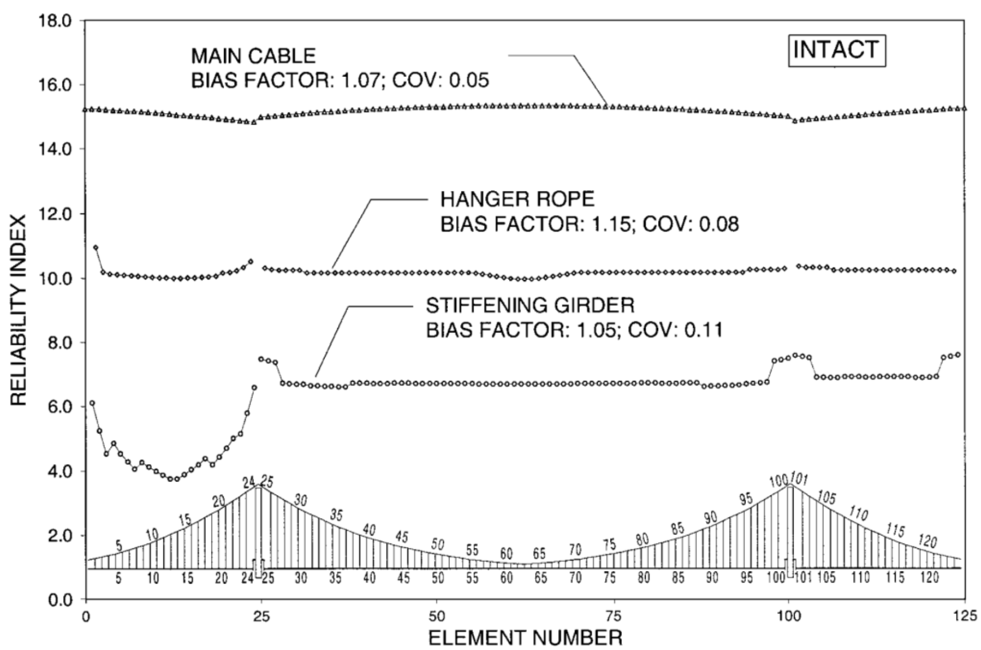


Figure C-23. Graph. Reliability indices of main cable, hanger rope, and stiffening girder: load Case 3 (D, L, T, SD), live load over side span (Imai and Frangopol 2001).

From Figure C-22 and C-23 it can be noted that the reliability indices of stiffening girder are smaller than those of the hanger ropes and main cables. The reliability indices of the stiffening girder associated with Case 3 are smaller than those associated with Case 1. Since the girder element 12 (G-12) has the smallest reliability index under live load on the side span (Case 3) it is assumed that this element will fail first. The reliability indices of cables, hanger ropes, and girders after failure of girder element 12 are shown in Figure C-24.

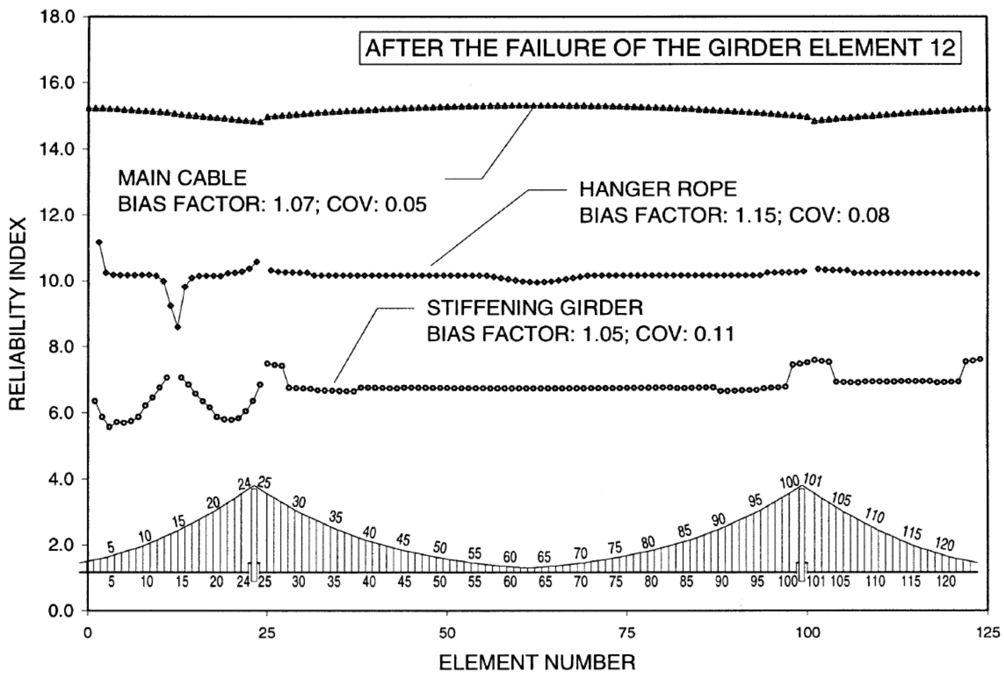


Figure C-24. Graph. Reliability indices of main cable, hanger rope, and stiffening girder after the failure of girder element 12; load combination: D (pre- and post-dead loads), L (line and uniform; case 3 in Figure C-21), T and SD (Imai and Frangopol 2002).

System reliability

As evidenced by Figure C-24, the reliability indices of the stiffening girder elements are the smallest and those of the cable elements are the largest; overall, the reliabilities of cables, hanger ropes, and girders are very different. By comparing the results in Figure C-24 with those associated with the intact state, it can be concluded that due to load redistribution occurring after failure of girder element 12, the reliability indices of girder and hanger rope elements adjacent to the failed girder element are substantially affected. However, the reliabilities of main cable elements are not affected. Based on these findings, the bridge is modeled as a series of parallel subsystems within the girder and hanger rope elements in the side span. Each parallel subsystem consists of two failure modes including failure of two girder elements, or failures of one girder and one hanger rope element within the side span 1A-2P. It is assumed that the stiffening girder element fails first in each parallel subsystem. Based on these assumptions, the series of parallel subsystems is shown in Figure C-25.

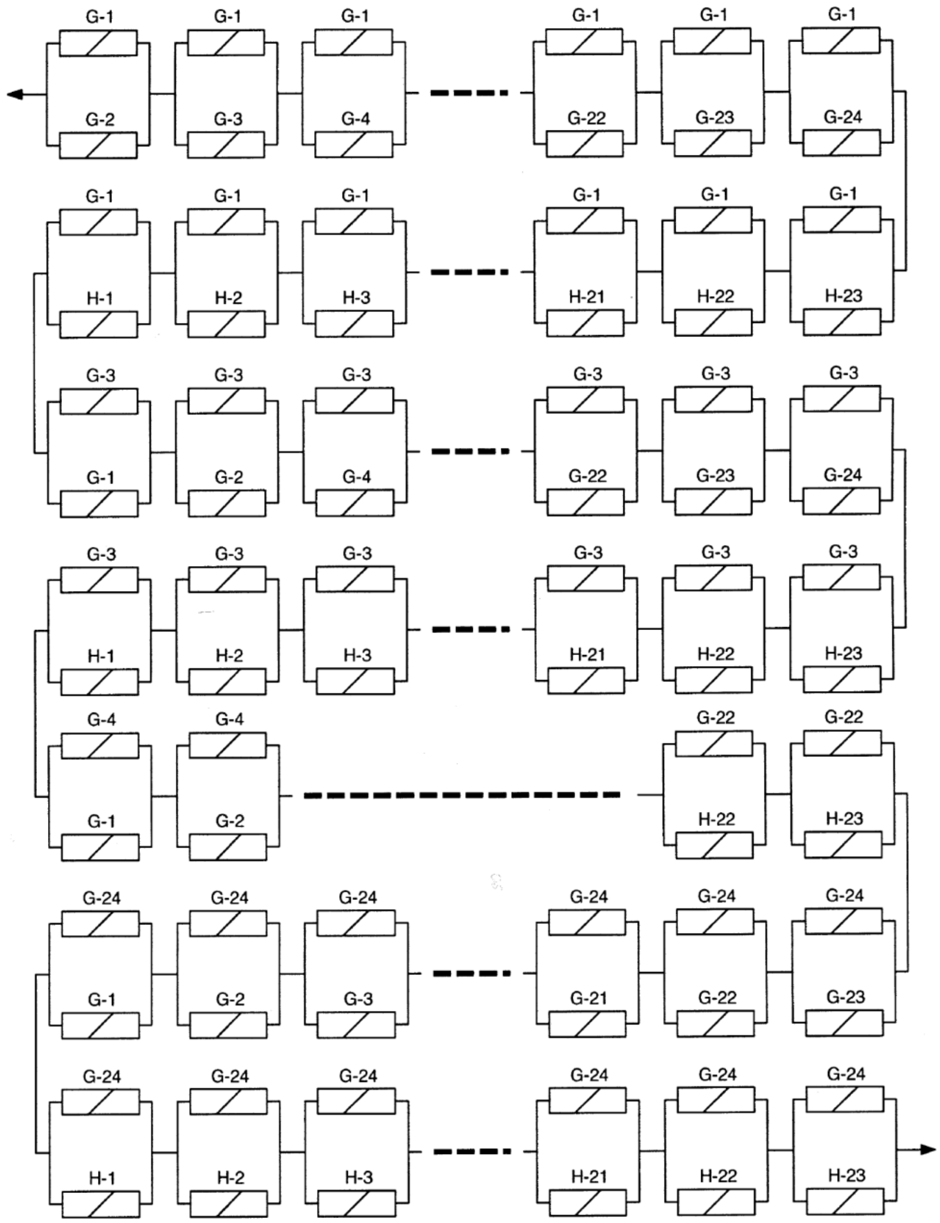


Figure C-25. Graph. Series of parallel subsystems of the Innoshima Bridge (Imai and Frangopol 2002).

The reliability indices of parallel subsystems and component reliabilities are depicted in Figure C-26 to C-30 following the failure of girder elements 1, 3, 12, and 21, respectively. The reliability index of the series of parallel subsystems in Figure C-25, calculated using RELSYS-

FEAP (Imai and Frangopol 2001), is bounded by 4.609 and 4.625 as lower and upper bounds, respectively.

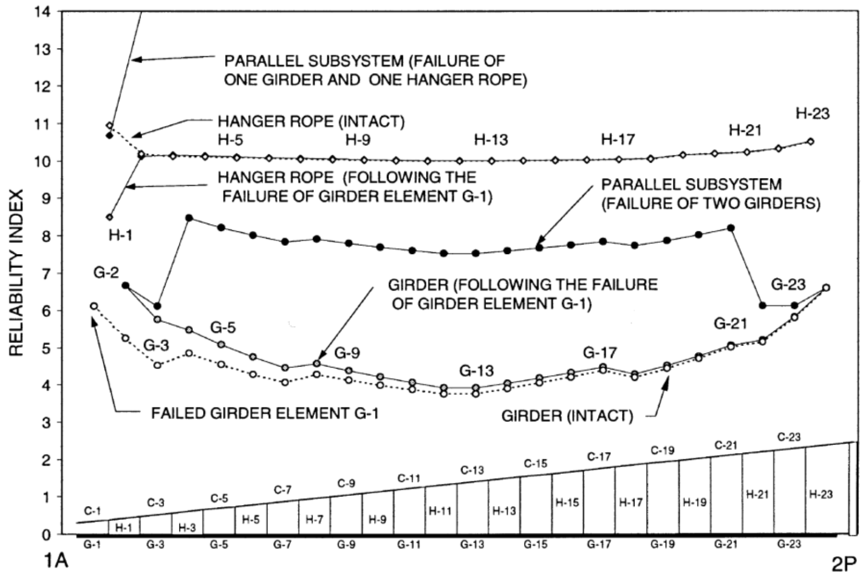


Figure C-26. Graph. Components and parallel subsystems after the failure of girder element 1; load combination: D (pre- and post-dead loads), L (line and uniform; case 3 in Figure C-21), T and SD (Imai and Frangopol 2002).

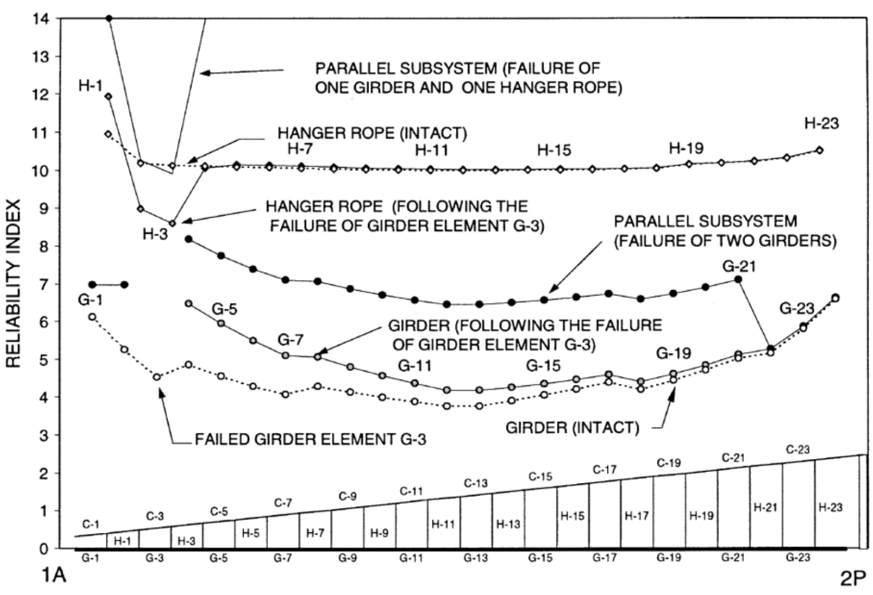


Figure C-27. Graph. Components and parallel subsystems after the failure of girder element 3; load combination: D (pre- and post-dead loads), L (line and uniform; case 3 in Figure C-21), T and SD (Imai and Frangopol 2002).

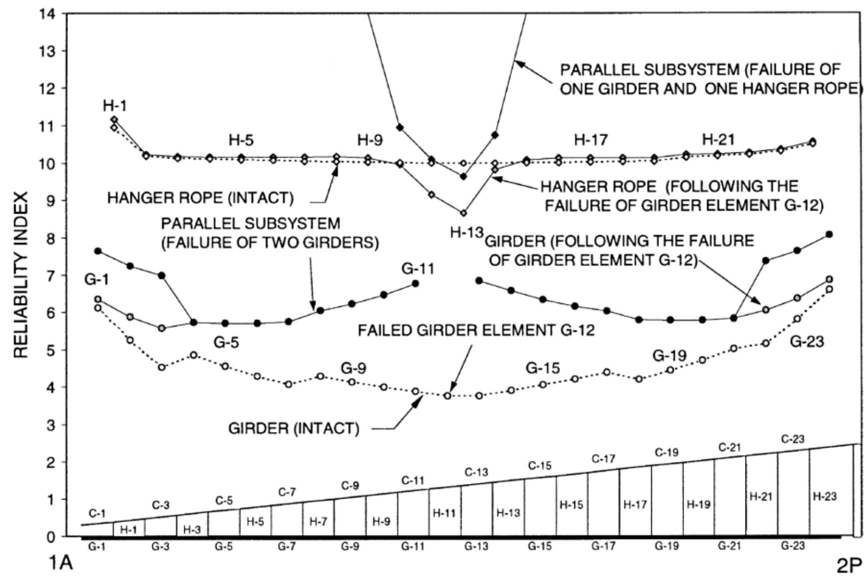


Figure C-28. Graph. Components and parallel subsystems after the failure of girder element 12; load combination: D (pre- and post-dead loads), L (line and uniform; case 3 in Figure C-21), T and SD (Imai and Frangopol 2002).

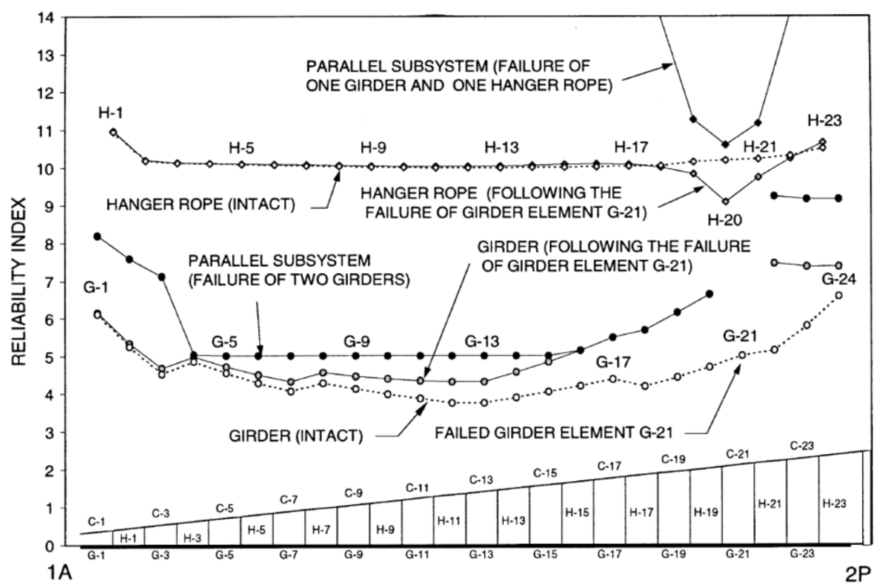


Figure C-29. Graph. Components and parallel subsystems after the failure of girder element 21; load combination: D (pre- and post-dead loads), L (line and uniform; case 3 in Figure C-21), T and SD (Imai and Frangopol 2002).

A general approach to evaluate the reliability of structures exhibiting geometrically nonlinear elastic behavior was proposed in Imai and Frangopol (2002). Based on the results of robust reliability calculations of an existing suspension bridge, several conclusions are made. For both live and wind loads, the reliability indices of main cables, hanger ropes, and stiffening girders are very different. The reliability indices of stiffening girders are the smallest, and those of main cable are the largest. This difference in reliability indices is partially due to the different safety factors of main cables, hanger ropes, and stiffening girders. This is also explained by the fact that the main cables and hanger ropes have a brittle behavior and the stiffening girders are made of ductile steel.

References

Akgül, F., and Frangopol, D.M. (2003). Rating and reliability of existing bridges in a network. *Journal of Bridge Engineering*, ASCE, 8(6), 383–393.

Akgül, F., and Frangopol, D.M. (2004a). Lifetime performance analysis of existing steel girder bridge superstructures. *Journal of Structural Engineering*, 130(12), 1875-1888.

Akgül, F., and Frangopol, D.M. (2004b). Computational platform for predicting lifetime system reliability profiles for different structure types in a network. *Journal of Computing in Civil Engineering*, ASCE, 18(2), 92-104.

Estes, A.C., and Frangopol, D.M. (1999). Repair optimization of highway bridges using system reliability approach, *Journal of Structural Engineering*, ASCE, 125(7), 766-775.

Estes, A.C., and Frangopol, D.M. (2001). Bridge lifetime system reliability under multiple limit states. *Journal of Bridge Engineering*, ASCE, 6(6), 523-528.

Frangopol, D.M., and Curley, J.P. (1987). Effects of damage and redundancy on structural reliability. Journal of Structural Engineering, ASCE, 113(7), 1533-1549.

Imai, K., and Frangopol, D.M. (2001). Reliability-based assessment of suspension bridges: application to the Innoshima bridge. *Journal of Bridge Engineering*, ASCE, 6(6), 398-411.

Imai, K., and Frangopol, D.M. (2002). System reliability of suspension bridges. *Structural Safety*, Elsevier, 24(2-4), 219-259.

Imam, B. M., Chryssanthopoulos, M.K., and Frangopol, D.M. (2012). Fatigue system reliability analysis of riveted railway bridge connections. *Structure and Infrastructure Engineering*, Taylor & Francis, 8(10), 928-938.