Numerical analysis of ultrasonic guided waves propagation in highly attenuative viscoelastic material

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Abstract
The propagation of ultrasonic guided waves in viscoelastic isotropic material has been investigated. Based on the plane theory, a numerical model of the guided waves propagating is developed in the frequency domain by employing the SFEM (spectral finite element method). To verify the proposed method, thin bitumen on the steel substrate is examined and compared with the single plate in terms of the dispersion and attenuation. From the dispersion and attenuation of the displacement curves, the propagating properties can be obtained, which depends not only on the viscous parameter, but also on those of the substrate. The guided wave attenuates rapidly at the location near the source, and with the receiver distance increasing, it becomes slowly, compared with single bitumen, the attenuation of amplitude for the guided waves propagating in the viscoelastic is tend to gently. The phenomenon shows propagation distance will increase in bilayer material cause of the substrate influence.

Introduction
Viscoelastic materials are employed in a number of technical areas. Especially in the oil, gas, and petro-chemical industries highly attenuative materials, such as bitumen, are often used as coatings in order to protect pipe networks from corrosion. As a consequence, the study of wave propagation in highly attenuative materials has been a subject of extensive investigation in the literature. It is of great importance in a variety of applications ranging from nondestructive testing of composite structures to properties evaluating of thin film used in aircraft, spacecraft, or other engineering industries.

When guided waves propagating in highly attenuative viscoelastic materials, the multiple propagating modes, the frequency dependent dispersion, and the frequency dependent attenuation will appear due to the interaction of the boundaries and viscosity dissipation. While an accurate knowledge of their dispersive and attenuation properties is indispensable for evaluating such highly attenuative materials.

There are many methods of wave propagating in isotropic and anisotropic viscoelastic materials, including analysis method and numerical method. Analysis method is accurate but is limited to the complex material structure. Finite element method is flexible as numerical simulation which can be solved in time domain and frequency domain. A common feature of the matrix method used in layered linear elastic materials is that a root searching routine is employed to find the real roots in the frequency domain. In viscoelastic material, the search scheme needs to be performed in the complex-valued domain. The search scheme can be time consuming and possibly missing roots.
Semi-analysis finite element (SAFE) method is developed as an alternative way to tackle wave propagation problem. The general SAFE approach for extracting dispersive solutions uses a finite discretization of the cross-section of the waveguide alone, the method solutions are obtained in a stable manner from an eigenvalue problem, and thus do not require the root searching algorithms, missing solutions can be avoid. To enhance the accuracy of the solution, a more refined discretization may be necessary.

The spectral finite element method (SFEM) is a numerical method evolved from the Fourier Transform. In this method first the wave equation is transformed into the frequency domain using appropriate forward transforms. The governing equation in the transformed domain is then solved exactly or almost exactly and the results are post-processed to get all the relevant parameters in the frequency domain. SFEM employs this exact solution as an interpolating function for element formulation. The constants of integration are made to satisfy the boundary conditions in the frequency domain and thus all the requirements are satisfied at each discrete frequency. Using the inverse-FFT (IFFT), The time domain data are recovered.

In this paper, the propagating properties for ultrasonic guided waves in the isotropic viscoelastic material are studied. A single plate made of bitumen of high damping, which is commonly used for protection against corrosion in the pipeline industry, is chosen to demonstrate the proposed method. Then the wave modes of a layer structure with thin bitumen on the steel substrate are examined and compared with the single-phase system in terms of the dispersion and attenuation. Knowledge of these properties is important for evaluating and detecting material structure damage.

**Basic theory and simulation method**

Considering an infinite plate of finite coating thickness $h$ of a coating-substrate system. Highly attenuative isotropic material, bitumen, is chosen to be a coating, and substrate is anisotropic material, steel. The geometry of the problem is shown in Figure 1.

![Fig.1 Geometry of the coating-substrate system.](image)

**1. Wave propagations in the viscoelastic material**

This section reviews the linear viscoelastic models that will be used in the SFEM formulation proposed in the present work. As well know, the general tensorial stress-strain (constitutive) relations

$$\sigma_{ij} = c_{ijkl} e_{kl}$$

considering the viscous damping in the constitutive relation, the ideal stress-strain relations is modified to include damping by adding terms containing time derivatives of the strains; that is

$$\sigma_{ij} = c_{ijkl} e_{kl} + \eta_{ijkl} \frac{\partial e_{kl}}{\partial t} \quad i, j, k, l = 1, 2, 3. $$

it can be expressed in frequency domain

$$\sigma_{ij} = (c_{ijkl} + i\omega \eta_{ijkl}) e_{kl}$$
Here $\sigma_{ij}$ and $e_{kl}$ are the stress and strain tensors, respectively; $\omega$ is the angular frequency.

Elastic stiffness matrix $c_{ijkl}$ contains the storage moduli and viscous matrix $\eta_{ijkl}$ contains the loss moduli.

2. SFEM formulation for ultrasonic guide waves propagation

The amplitude of wave propagation in lossy medium steadily decreases as its energy is dissipated, or absorbed by the medium. And the corresponding strain field is

$$e_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

(4)

For fields varying only with the y coordinate, ignoring mechanic force, the equation of motion reduces to

$$\frac{\partial \sigma_{ij}}{\partial x_j} = -\rho \frac{\partial^2 u_i}{\partial t^2}$$

(5)

The paper presents two materials of bitumen and steel, which can be described as isotropic linearly viscoelastic and anisotropic linearly elastic, respectively. For an anisotropic linearly elastic material of density $\rho$, the plane strain displacement equation is written as follows in the frequency domain:

$$C_{11}^* \frac{\partial^2 \tilde{u}_1}{\partial x_1^2} + C_{66}^* \frac{\partial^2 \tilde{u}_1}{\partial x_2^2} + (C_{12}^* + C_{66}^*) \frac{\partial^2 \tilde{u}_2}{\partial x_1 \partial x_2} = -\rho \omega^2 \tilde{u}_1$$

(6)

$$C_{22}^* \frac{\partial^2 \tilde{u}_2}{\partial x_2^2} + C_{66}^* \frac{\partial^2 \tilde{u}_2}{\partial x_1^2} + (C_{12}^* + C_{66}^*) \frac{\partial^2 \tilde{u}_1}{\partial x_1 \partial x_2} = -\rho \omega^2 \tilde{u}_2$$

(7)

Here $\tilde{u}_1$ and $\tilde{u}_2$ are transformed displacement variables depending on frequency,

$$\tilde{u}_1 = \int_0^{\infty} u_1 e^{-i\omega t} dt, \quad \tilde{u}_2 = \int_0^{\infty} u_2 e^{-i\omega t} dt, \quad \text{and} \quad \omega = 2\pi f.$$  

In practice, the matrix $C_{ij}^*$ is composed of the elastic stiffness tensor, $C_{ij}$, and the viscosity tensor, $\eta_{ij}$. For elastic material, viscosity coefficient $\eta_{ij}$ can be ignored, $\eta_{ij} = 0$.

Finite element formulation

The partial differential equations can be solved by a finite element software (COSMOL). The equations can be solved in the frequency domain with complex coefficients when the material is viscoelastic. In this case, the corresponding PDE problem can be written in the specific following form:

$$\nabla \cdot (-c \nabla \tilde{u}) + a \tilde{u} = 0$$

(8)

The symbol $\nabla$ is the vector differential operator, $\tilde{u}$ is the Fourier transform of the displacement vector, $c$ is a 2×2 matrix composed of four submatrices, and $a$ is a 2×2 matrix as a absorption coefficient.
Considering continuity of stress and displacement at the interface between the two layers, laser-generated boundary condition can be set Neumann, that is

\[ n \cdot (c \nabla \vec{u}) = g \quad (9) \]

Here \( g \) is the laser source located at a single point located at the origin, other boundaries can be considered the traction-free conditions; that is, \( g = 0 \).

**Simulated results and discussion**

1. **Model of simulation**

In order to obtain the properties of ultrasonic guide waves propagating in viscoelastic materials with high damping, the paper presents a highly attenuative isotropic material, bitumen, and analysis the propagation characterize about a viscoelastic coating with an elastic anisotropic substrate, steel, which properties are given in Table 1.

<table>
<thead>
<tr>
<th>properties used in the calculation</th>
<th>thickness (mm)</th>
<th>Density ( \rho ) (g/cm(^3))</th>
<th>( C_{11} )</th>
<th>( C_{22} )</th>
<th>( C_{12} )</th>
<th>( C_{44} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>bitumen</td>
<td>0.2</td>
<td>1.5</td>
<td>0.52+0.023i</td>
<td>0.08+0.24i</td>
<td>10.5</td>
<td>8.5</td>
</tr>
<tr>
<td>steel</td>
<td>20</td>
<td>7.86</td>
<td>26.9</td>
<td>26.9</td>
<td>10.5</td>
<td>8.5</td>
</tr>
</tbody>
</table>

2. **Results and discussion**

Bitumen is a highly attenuative material, from Table 1 we can see that shear waves is much more attenuative than longitudinal waves. When the ultrasonic wave propagating along the surface of single bitumen with thickness of 0.2mm, attenuation curves of displacement can be describe in fig 2. It plots displacement curves of three points with 4mm, 8mm, 12mm distance to excited source, respectively. Fig 2 shows ultrasonic wave appear monopole Rayleigh, which vertical displacement reduce by exponential form of complex number with receive distance increasing. It is different wave mode from anisotropic thin plate materials. For bitumen, the guided wave attenuates rapidly at the location near the source, and with the receiver distance increasing, it tends to gently.

When the bitumen is put on the steel substrate with 20 mm thickness, the wave propagation along the surface of bitumen coating has presented great changes. There are three main features in the waveform. One is the surface skimming longitudinal wave denoted by \( sP \), which is an outward displacing unipolar wave. The other two are the surface shear wave front denoted by \( sS \) and the main initially negative-going dipolar Rayleigh wave denoted by \( R \), the \( sP \) and \( sS \) waves mark the intersection with the surface of that longitudinal wave and shear wave fronts respectively, which originate from the center of the extended acoustic source. The displacement component normal to the surface is calculated, and the numerical results by inverse FFT at different source-receiver displaces are shown in Fig 3.

Fig 3 (a), (b), (c) plot the vertical displacement in distance of 4, 8, and 12mm, \( sP \) wave appears first, then \( sS \) and Rayleigh waves arrived, it is because that \( sP \) is faster than \( sS \) and Rayleigh, and \( sS \) velocity is close to Rayleigh waves. From fig 3 we can also see, the displacement transformation of \( sP \) and \( sS \) waves is not more than Rayleigh. The viscous of the materials can be described by imagining displacement. The comparison between real and imagining displacement curves suggests that the attenuation of Rayleigh waves reflect the viscoelastic characterization, it shows in fig 4.

The wave modes transformed in fig 4 illustrate that the combination of coating and substrate properties determines the character of the ultrasonic wave dispersion. Compared the ultrasonic guide waves in two-layer system with those in single plate, as fig 5 shows, we can see that time-
displacement curves attenuated with receiver-distance increase. But attenuation of amplitude is slower than single plate. The phenomenon shows propagation distance will increase in bilayer material cause of the substrate influence.

From the above discussion, it can be seen that wave mode attenuation in a layered materials is highly related to its energy concentration in the viscoelastic layer. The more the energy is concentrated in the viscoelastic layer, the greater the attenuation for the wave mode.

From the results of the numerical simulated, the average dissipated power will diminish faraway the laser source. It explained that attenuation shows faster in near the source than in the far field. With receive distance increasing, attenuation turns to slow and the dispersion intensifies obviously in the waveforms. More specific information on the elastic and viscoelastic properties of coating and substrate materials can be obtained from these dispersion curves.

**Conclusion**

The propagation of ultrasonic guided waves in viscoelastic isotropic materials has been studied in the frequency domain by employing the SFEM. A general formulation relating the guided waves propagation in viscoelastic isotropic materials has been derived. It shows the viscoelastic characterization of materials can be determined by complex stiffness parameter. The attenuation of viscoelastic materials is related to receiving point and substrate material, some quantitatively investigations are in progress.

![Dispersion Curves](image)

**Fig.2** Ultrasonic wave propagating along the surface of single bitumen with thickness of 0.2mm, the distance of source and receiver is 4mm, 8mm, 12mm, respectively.

![Displacement Curves](image)

**Fig.3** Displacement curves of the guide waves propagating along the bitumen coating.
Fig. 4 Comparison between real displacement (a) and imagining displacement (b) at receiving point 8mm.

Fig. 5 The peak strain energy attenuative curves with increasing of receive distance.

References


