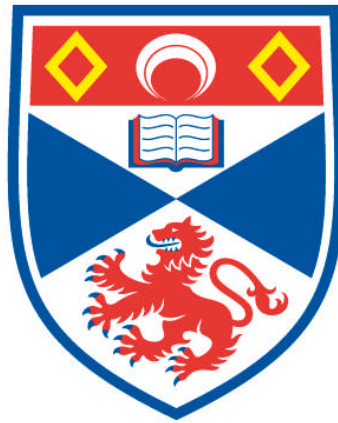


**CARL FRIEDRICH GEISER AND FERDINAND RUDIO:
THE MEN BEHIND THE FIRST INTERNATIONAL
CONGRESS OF MATHEMATICIANS**

Stefanie Ursula Eminger

**A Thesis Submitted for the Degree of PhD
at the
University of St Andrews**



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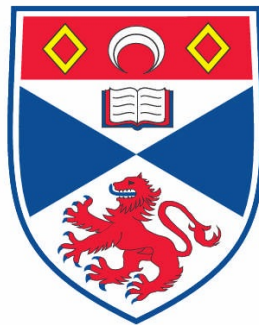
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Carl Friedrich Geiser and Ferdinand Rudio:
The Men Behind the First International Congress of
Mathematicians

Stefanie Ursula Eminger



This thesis is submitted in partial fulfilment for the degree of
PhD

at the
University of St Andrews

2014

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Abstract

The first International Congress of Mathematicians (ICM) was held in Zurich in 1897, setting the standards for all future ICMs. Whilst giving an overview of the congress itself, this thesis focuses on the Swiss organisers, who were predominantly university professors and secondary school teachers. As this thesis aims to offer some insight into their lives, it includes their biographies, highlighting their individual contributions to the congress. Furthermore, it explains why Zurich was chosen as the first host city and how the committee proceeded with the congress organisation.

Two of the main organisers were the Swiss geometers Carl Friedrich Geiser (1843-1934) and Ferdinand Rudio (1856-1929). In addition to the congress, they also made valuable contributions to mathematical education, and in Rudio's case, the history of mathematics. Therefore, this thesis focuses primarily on these two mathematicians.

As for Geiser, the relationship to his great-uncle Jakob Steiner is explained in more detail. Furthermore, his contributions to the administration of the Swiss Federal Institute of Technology are summarised. Due to the overarching theme of mathematical education and collaborations in this thesis, Geiser's schoolbook *Einleitung in die synthetische Geometrie* is considered in more detail and Geiser's methods are highlighted.

A selection of Rudio's contributions to the history of mathematics is studied as well. His book *Archimedes, Huygens, Lambert, Legendre* is analysed and compared to E W Hobson's treatise *Squaring the Circle*. Furthermore, Rudio's papers relating to the commentary of Simplicius on quadratures by Antiphon and Hippocrates are considered, focusing on Rudio's translation of the commentary and on *Die Mönchen des Hippokrates*. The thesis concludes with an analysis of Rudio's popular lectures *Leonhard Euler* and *Über den Antheil der mathematischen Wissenschaften an der Kultur der Renaissance*, which are prime examples of his approach to the history of mathematics.

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1. Introduction

The first International Congress of Mathematicians (ICM) took place in Zurich in 1897. It was hosted by the Swiss Federal Polytechnic, which had been founded only four decades previously, but had already established an excellent reputation. At the time considered to be a trial congress, it laid the foundations for future congresses. Indeed, the emphasis both on scientific quality and on the social side that is evident in the 1897 ICM is reflected in all ICMs that have been held since. Undoubtedly this will be the case with the upcoming 2014 ICM in Seoul.

Scholars have investigated the history of the ICMs and the (political) circumstances that led to Zurich being the first host city, but to my knowledge nobody has as yet studied the backgrounds of the Swiss organisers. Some became famous mathematicians, but most of them have since been forgotten. In this thesis I hope to shed some light on their lives and achievements, highlighting their contributions to the congress. In particular, I have looked at the two main organisers, the geometers Carl Friedrich Geiser and Ferdinand Rudio. Neither achieved lasting mathematical fame, but both made valuable contributions in related fields, such as education and the history of mathematics.

Geiser was Jakob Steiner's grandnephew and an accomplished geometer, however, his contributions to the Polytechnic and to Swiss education are even more important. His textbook *Einleitung in die synthetische Geometrie* combines his interests, and so I chose to study this book instead of his research papers.

Rudio, on the other hand, is best known as initiator of the Euler project *Leonhardi Euleri Opera omnia*. He was also a keen historian of mathematics with an interest in education. Some of his historical publications are still read today, but to my knowledge they have not yet been analysed.

Chapter 2 contains an extensive biography of Geiser, including information on his family and his teaching. In section 2.2, his connection with Steiner is

explained in more detail. Section 2.3 attempts to illustrate Geiser's contributions to the administration of the Polytechnic.

Chapter 3 contains a biography of Rudio, with section 3.2 summarising Rudio's involvement with the Euler project. Both Rudio's and Geiser's publications are listed in appendix C.

Chapter 4 begins with an account of the events leading up to the first ICM, the work of the organising committee, and the congress itself. Geiser and Rudio's contributions are highlighted in sections 4.1.4 and 4.1.5, respectively. Section 4.2 contains the biographies of the members of the Swiss organising committee. In the case of Fritz Bützberger, I include a sub-section (4.2.5.1) dedicated to his work on Steiner, as this nicely ties in with Geiser's Steiner connection and Rudio's interest in the history of mathematics.

Chapter 5 contains firstly an analysis of Geiser's aforementioned textbook, which was written in order to help students prepare for their studies at the Polytechnic. Secondly, in section 5.2 some letters that he sent to his friend Julius Gysel, a secondary school teacher, are considered. This is accompanied by a biography of Gysel.

Chapter 6 contains analyses of some of Rudio's most important works on the history of mathematics: the book *Archimedes, Huygens, Lambert, Legendre*, his works concerning the commentary of Simplicius on quadratures by Antiphon and, more importantly, Hippocrates, and the two popular lectures that he gave as part of the Zurich "Town Hall Lecture Series". In sections 6.1 and 6.2 I also summarise how the papers were received and where they are referred to by other scholars. Furthermore, I compare *Archimedes, Huygens, Lambert, Legendre* to Ernest William Hobson's *Squaring the Circle*.

A note on referencing: due to the large number of works used, I chose to create individual reference lists for each chapter, as I believe that this will

make it easier to find the works referred to in each chapter. The bibliography lists all the works that I consulted for this thesis.

Most of my sources are in German (and some in French), but a number of specialist terms, particularly within education and politics, cannot simply be translated. I therefore use some words in the original, but they are explained in the glossary in appendix A. Words included in this glossary are marked with an asterisk the first time that they appear in the text.

Unless stated otherwise, all the translations in the text are my own work. Furthermore, I include my translations of some speeches at the ICM, some of Geiser and Rudio's papers, as well as some letters, in appendix E. I believe that this will make it easier for the reader to look up passages that are referred to in the text, but the documents are also interesting to read in their own right.

With regard to German names, I use the English spellings "Weierstrass" and "Gauss" when referring to Karl Weierstrass and Carl Friedrich Gauss, but do not replace "ß" with "ss" otherwise and keep the original German spelling.

The appendix also contains a short overview of the first decades of the Polytechnic and its influence on Swiss education (appendix B). This is by no means a comprehensive account, but it is merely intended to provide some background information.

2. Carl Friedrich Geiser (1843 – 1934)

2.1 Life

Carl¹ Friedrich Geiser was born on 26 February 1843 in Langenthal, at the time of his birth a town of about 3,000 inhabitants², in canton* Bern. According to Kuert, there are records of Geisers living in Ob- und Nid- u. Oberaargau dating back to the 15th century; in particular in connection with the abbey St. Urban, whose dominion included the region [7].

Geiser's grandfather Friedrich (*1768) was a butcher; his wife was Barbara Hofer (*1774). The couple had five children: Katharina (*1798), Anna Barbara (*1810), Jakob (*1812), Friedrich, and Maria Elisabeth (*1818). According to Bützberger's³ family tree of the Geisers [4a], Katharina had a daughter named after her, but I have not been able to find any further information about Geiser's aunts and uncles.

Geiser's father, Friedrich (1816-1857)⁴, married Elisabeth Begert (1815-1892), of Ersigen, canton Bern, on 28 October 1836. He was a butcher as well, and worked in Kirchberg, canton Bern, at the time of his wedding [7]. The cause of his premature death is explained in the parish register: he fell off a horse [ibid.]. In several biographies of Geiser, his father's profession is erroneously given as 'butcher, innkeeper, and member of the cantonal parliament' [cf. 32, p. 372; 10, p. 286]. According to Kuert, this describes Friedrich Geiser-Rüegger⁵, grandfather of the historian Karl Geiser, but not Carl Friedrich Geiser's father [7].

¹ In some biographies and references Geiser's first name is spelt "Karl". However, as he always signed "C. F. Geiser" on his letters, I used "Carl" throughout.

² http://www.langenthal.ch/dl.php/de/52f1ed8adc8c6/2014.02.03_Einwohnerstatistik_ab_1764.pdf, accessed 28/03/2014. The population has since quintupled.

³ See section 4.2.5.

⁴ Some of the dates of birth given in the Geiser family tree by Bützberger [4a] do not match the dates in the Langenthal parish registers, as given by Kuert [7]. I chose to include the latter as they are consistently earlier in the month. Bützberger's dates might have been the dates of christening rather than the dates of birth.

⁵ A short biography by Kuert can be found on the website of the town Langenthal: http://www.langenthal.ch/de/portrait/geschichte/?action=showinfo&info_id=4821, accessed 28/03/2014. F Geiser-Rüegger, K Geiser, and C F Geiser are all included in the website's section on 'notable people'.

Friedrich and Elisabeth had four sons: Johann Heinrich⁶ (1839-1873), Theodor (*1840), Jakob (1841-1887), and Carl Friedrich. Both Johann and Jakob became butchers like their father and grandfather. There is no record of Theodor's occupation in the family tree, or of his date of death. It is conceivable that he was still alive at the time that Bützberger undertook his research. The family tree in figure I (p. 30) is based on Bützberger's work [4a].

Geiser himself married Emma Gessner (1842-1899) on 19 September 1872 in Zurich. Emma was from Zurich; her parents were Eduard Gessner and Susanna née Brunner. Eduard (1799-1862) was a printer and treasurer in a crown land administration. He was a grandson of the writer and painter Salomon Gessner, and attended Pestalozzi's school in Yverdon [15, p. 169]. There is a drawing of "family Gessner-Brunner" in the Graphic Collection of the Zentralbibliothek in Zurich, dating to 1838. It shows 'Susanna Gessner-Brunner (1806-1881), wife of Eduard Gessner (1799-1862), a grandson of Salomon Gessner' with her three children Susanna, Charlotte, and Arnold. It is likely that these were Emma's mother and older siblings [6].

Emma and Carl Friedrich had three daughters: Emma Elisabeth Charlotte (1873-1909/10), Susanne Charlotte (*1874), and Ida Hedwig (*1879). There is no further information about the two younger daughters in Bützberger's family tree; in all likelihood they outlived him. It seems that the eldest, Emma, became a primary school teacher and studied Romance philology at the University of Zurich [48]. She married an engineer from Küssnacht, one I F H Iltis⁷, on 05 May 1902. An annotation on the family tree reveals that she 'died from surgery in America 1909/10' and that she had 'one baby girl' [4a]. Indeed, a search on the Ellis Island Foundation website confirms that a 32 year-old Emma Iltis and a 2 year-old Charlotte Iltis from "Kusanacht" arrived at Ellis Island in 1905. Age and town match, so it is likely that this was Geiser's daughter. Furthermore, one Jean Iltis from Hannover, also 32 years of age, arrived in 1904; he might have been Emma's husband [46]. The godfather of Geiser's youngest daughter Hedwig was none other than the Swiss writer

⁶ According to Bützberger, Johann Heinrich; according to Kuert, Johann Friedrich.

⁷ Or J F H Iltis

Gottfried Keller (1819-1890). Keller was a friend of Geiser (see below). He wrote two poems for his goddaughter: *Du bist nun in die Welt getreten* ("Now you stepped into this world") for her christening in 1879, and *Ich han doch einen faulen Götti* ("Don't I have a lazy godfather") in 1885, possibly written for her birthday⁸. According to the ETH School Board*, Geiser lived with his unmarried daughter when he retired, but it is not apparent whether this was Susanne or Hedwig [50, p. 84]. A Miss Geiser accompanied him at the 1932 ICM, but after Geiser's death, the School Board arranged for Geiser's life insurance to be paid out to 'Mrs H Keller-Geiser', presumably a married Hedwig [5p].

Hardly any information on Geiser's private life is given in obituaries, and descriptions of Geiser by friends and former students imply that he was a very private man, revealing little about his family.

Let us return to Geiser's childhood, which he spent in his native town. He first attended secondary school there, followed by the Kantonsschule* in Bern. Already at school he showed a particular aptitude for mathematics, which is illustrated by an anecdote in [30, p. 522]:

New coins were introduced during his time at secondary school in Langenthal. Being good at calculating, he had to assist with exchanging the old batzen⁹ for francs and rappen.

Geiser then studied mathematics at the Swiss Federal Polytechnic* in Zurich from 1859-1861, and for two further years in Berlin. At the time, most talented mathematics students went to Berlin for a few years, in order to be taught by Karl Weierstrass, Ernst Eduard Kummer, Leopold Kronecker, and Jakob Steiner. For Geiser, Steiner was the particular attraction; after all, he was his granduncle. Steiner's work significantly influenced Geiser's own

⁸ These poems are published in W Morgenthaler et al (eds.), *Historisch-Kritische Gottfried Keller-Ausgabe* 14, Stroemfeld, Basel, 2011.

⁹ A "batzen" was a coin used in canton Bern and elsewhere in Switzerland, which allegedly got its name from the Bernese bear embossed on it. Due to a monetary act in 1850 Swiss francs and rappen were introduced in all cantons. Cf. article by A-M Dubler in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D13677.php>, accessed 28/03/2014.

mathematical taste. However, being a good student, Geiser also caught the eyes of Weierstrass and Kronecker. They procured private lessons for him so that he could support himself, as ‘he found himself on his own soon [after] a disagreement between Steiner and his relatives’ [32, p. 372].

In 1863 Geiser returned to Zurich and habilitated* as a Privatdozent* at the Polytechnic. In addition, together with Theodor Reye he acted as replacement for Johann Wolfgang von Deschwanden, whose untimely death in 1866 left the chair of descriptive geometry vacant. Wilhelm Fiedler filled it a year later.

Geiser obtained his doctorate from the University of Bern on 28 July 1866 [13, p. 10] for his thesis *Beiträge zur synthetischen Geometrie*,¹⁰ which he dedicated to Siegfried Heinrich Aronhold and Bruno Elwin Christoffel. Geiser has the distinction of having been Ludwig Schläfli’s first doctoral student. According to Kollros [30, p. 523], Geiser solved a number of problems given by Steiner in *Aufgaben und Lehrsätze* (1852)¹¹ in his thesis, using Plücker’s formulae and a geometric relation that he discovered¹².

Geiser became a Titularprofessor* in 1869 and was appointed to an ordinary professorship* in higher mathematics and synthetic geometry with effect of 01 January 1873 [13, p. 11]. In 1895 the School Board filed a request to the Bundesrat, asking that Geiser be appointed for life, ‘in recognition of his service to the Polytechnic’ [5]. Geiser retired on 01 October 1913. As Robert Gnehm, at the time President of the ETH, wrote in a letter to the Federal Department of Home Affairs on 14 May 1913:

As per his letter of 30 April 1913 Prof Dr C F Geiser submits a request for retiring at the end of the current academic year [...] Albeit Mr Geiser is still fit, both physically and mentally, we will have to comply with his request in view of his age – he turned *seventy* on 26 February 1913 – and his exceptionally long service. We will also have to consider

¹⁰ “Contributions to Synthetic Geometry”

¹¹ *Crelle’s Journal* **49**, 273-278; also [44, p. 613-620]

¹² Geiser published his discovery in *Über eine geometrische Verwandtschaft zweiten Grades* (“On a Geometric Relation of Second Degree”) in 1866.

that the loss of one of his eyes, which he suffered a few years ago, makes carrying out certain tasks very arduous from time to time [...]

(As quoted in [13, p. 11])

Hermann Weyl became his successor, recommended to Gnehm by Geiser's friend Georg Frobenius [9, p. 443]. As Frei and Stambach write, 'due to Geiser's authority his chair at the ETH was very prestigious' [13, p. 10].

In addition to his teaching duties, Geiser was Head of the Department VI A, for Mathematics Teachers, from 1905-1909. Before that he was Deputy Head to Jérôme Franel for a couple of years. Furthermore, Geiser regularly acted as examiner for the Polytechnic's entrance and final examinations.

Geiser was awarded his *venia docendi*, his teaching entitlement, on 20 October 1863 [5d, p. 160]:

In consequence of a petition by Mr C F Geiser of Langenthal as of 27 April 1863, pertaining to granting the *venia docendi* for mathematical subjects; after reviewing expert opinion as of 19 Oct. by Professors Clausius, Christoffel, Durège & Zeuner on the written scientific work submitted by the petitioner, whereby they comment favourably on the scientific qualification of Mr Geiser

We decree that:

- 1) Mr Geiser shall provisionally be given permission to give lectures on pure and applied mathematics at the Polytechnic as a Privatdozent, however the definite habilitation shall be subject to the decision of the School Board.
- 2) Notification to Mr Geiser, returning the documents, to the Director and to the Treasurer.

Geiser lectured on algebraic, analytic, differential, infinitesimal, and synthetic geometry, and on invariant theory. He started his career in the Department for Elective Courses, teaching one lecture course. However, he was soon allocated more teaching hours. In the academic year 1865/66 he also

started teaching in the Engineering Department, giving tutorials for courses taught by Deschwanden. He then assisted Christoffel with the tutorials for Christoffel's courses in the Engineering and Teacher Departments until he became Titularprofessor. In his first few years as Privatdozent Geiser also led free mathematics examples classes with Reye. In the summer semester of 1866 Geiser took over Deschwanden's lectures on "Elements of Descriptive Geometry", which were part of the mathematical Preparatory Course (see appendix B). Moreover, he taught three weekly tutorials accompanying Orelli's mathematics lectures in the Preparatory Course in 1866/67 [5a]. With the introduction of the Department for Mathematics and Physics Teachers in 1866, Geiser taught a lecture course, "Introduction to Synthetic Geometry", in this department. He repeated it in almost every semester for a number of years (see section 2.2). Inspired by his teaching experience in this course, Geiser published his lecture material in his widely recognised book *Einleitung in die synthetische Geometrie* (1869). The book is discussed in section 5.1.

After being promoted to Titularprofessor and then to Professor he primarily taught in the Department for Mathematics and Physics Teachers. Some of his lecture courses also appear in the course catalogue of the Electives Department. Furthermore, he lectured on various topics in geometry in the Engineering Department. In particular, he took over Heinrich Weber's lectures on analytic geometry in the Engineering Department when Weber left in 1875 [14, p. 42]. However, as several of his biographers note, he was a very demanding teacher and preferred the mathematicians to the engineers. This feeling was mutual: '[...] the engineers were not particularly interested in his lectures. They were a bit too removed from the rest of mathematics and seemed too theoretical to [the engineers]' [32, p. 372]. Pólya also recounts an anecdote illustrating Geiser's standing as a lecturer, although he does not specify whether this lecture was for mathematics or engineering students¹³:

In German universities there was a kind of anonymous vote about the performance of the professor. If the students liked the class, they

¹³ In any case, it was not a lecture that Pólya could have witnessed himself, as he came to the ETH in 1914, after Geiser had retired. However, Geiser still took an interest in his alma mater: 'He lived somewhere a few miles from Zürich and walked to the ETH' [40, p. 41]. Pólya also remembers 'that he told good stories' [ibid.], which both Meissner and Kollros confirm in their obituaries [32; 30].

stomped. If they did not, they shuffled their feet. He was not popular so at the end of one of his classes there was a great shuffling. When it died down, he said very calmly: “May I ask the concerned gentlemen to shuffle with only two feet?” ‘

[40, p. 41]

However, Albert Einstein, arguably Geiser’s most famous student, enjoyed his lectures¹⁴. He wrote to Walter Leich in 1930 that ‘Geiser was dry only in the large lectures, otherwise I owe him the most of all’ (quoted in [43, p. 103]). Einstein was a student at the Polytechnic from 1896-1900. During his first year he attended Geiser’s lectures on analytic geometry and on determinants, in his second year he took Geiser’s courses on invariant theory and on infinitesimal geometry [41, p. 161]. He had a particular interest in the latter course, and described the lectures as ‘true masterpieces of pedagogical art, which later helped me very much in wrestling with general relativity. But apart from this I did not care much about higher mathematics during my university years’ (quoted in [43, p. 103])¹⁵. In particular, Geiser introduced his students to the foundations of infinitesimal geometry: Gaussian theory of surfaces and Riemann analysis. Einstein later drew an analogy between Gaussian surfaces and his own static gravitational fields [43, p. 104]. Reich describes the content of Geiser’s lectures that Einstein attended in more detail in [41, p. 164-166], highlighting that Geiser explored concepts from both geometry and invariant theory. In her opinion, the friendships with Steiner and Christoffel are of particular importance with regard to Geiser [41, p. 163].

In addition to his courses on geometry, Geiser taught in the “Mathematical Seminar” for a number of years. This was founded by Christoffel and modelled on the mathematical seminars at German universities. Starting in

¹⁴ Geiser refers to Einstein and some of his results in his talk on Reye [23, p. 169-171]; a translation is included in appendix E.3.2.

¹⁵ Reiser even writes that ‘Einstein was less interested in mathematical speculation than in the visible process of physics [at the time]. Nor did the Mathematician Gayser [sic!], whose teaching was subsequently influential in the development of the theory of relativity, greatly interest the young student’ [42, p. 49]. It seems that Einstein did not enjoy his mathematics lectures much, but maybe Geiser’s lectures were the lesser evil?

1878, he first taught with Fiedler and Frobenius, then only with Frobenius (from 1882), with Adolf Hurwitz from 1893 onwards, and with Hermann Minkowski in 1901. The seminars ran every semester, but Geiser did not teach all of them [5, appendices 1866-1913].

It seems that Geiser supervised only one doctoral student during his career. In 1914, one Karl Merz submitted his thesis on *Parallelfächen und Centralflächen eines besonderen Ellipsoides und die Steinersche Fläche*¹⁶. His first supervisor was Geiser; his second one was Marcel Grossmann [5b, p. 54]. The ETH could confer doctorates only from 1908 onwards and as Geiser retired in 1913 he would not have had an opportunity to supervise more doctoral students.

In addition to teaching mathematics, Geiser lectured on ballistics at the Polytechnic's Military Department. As Scherrer writes, he also taught seminars on the theory of firearms and supervised the associated shooting practices, which took place 'early in the morning on [summer] Sundays' [32, p. 374].

Apart from being a lecturer, Geiser served as the Polytechnic's Director twice, from 1881-1887 and from 1891-1895. His influence in this capacity is explained in more detail in section 2.3. Furthermore, he was one of the main organisers and president of the first International Congress of Mathematicians, which took place in Zurich in August 1897. See chapter 4, in particular section 4.1.4.

As indicated above, he was a good storyteller, but very private and modest. Nevertheless, he built up an extensive network of contacts all across Switzerland and neighbouring countries: 'Geiser was friends with many famous men; he knew almost all of the important mathematicians of his time' [30, p. 525]. As Meissner puts it, 'this prototype Swiss wanted to live in a European world intellectually; his mastery of languages enabling him to do

¹⁶ "Parallel Surfaces and Central Surfaces of a Special Ellipsoid and the Steiner Surface"

so' [32, p. 372]. These contacts and friendships proved to be very useful for his work as Director of the Polytechnic and organiser of the 1897 ICM. In particular, he established good relations with his colleagues at the Polytechnic and its governing staff. Most notable among them are Karl Kappeler and Gnehm, two of the three School Board Presidents during Geiser's time [29, p. 157]. Unfortunately, there are hardly any letters or notes preserved in Geiser's scientific estate at the ETH, or in other archives, which could give an indication of the extent of his contacts network. A number of letters that he wrote to his friend and former student Julius Gysel are analysed in section 5.2.2, translations of the letters can be found in appendix E.2.1. They indicate that Geiser tried to use his contacts to further Gysel's career, and that he corresponded with Schläfli and Luigi Cremona, amongst others. For example, Geiser asked Gysel to pass on his best wishes to Schläfli while Gysel stayed in Bern, and mentioned meeting Cremona (see section 5.2.2). On the occasion of Schläfli's 70th birthday on 15 January 1884 a letter of congratulations signed by 18 lecturers in Zurich was published in [16]. Geiser was behind this. Schläfli mentions in a letter to Gysel [2]:

As regards the letter of congratulations arranged by Geiser [...] I was surprised that Geiser came up with the idea. I suspect that Regierungsrat Affolter initiated this: I visited him shortly before Christmas at his behest [...] and told him that I would be 70 the following month. Not only did Geiser post the address by the 18 Zurichers in the *Bauzeitung*, but, as I surmise, he also sent copies of the same to Beltrami and Cremona. I was showered with congratulations.
(Italics by the author)

One of Geiser's friends who was not a mathematician was Gottfried Keller¹⁷, who became the godfather of Geiser's youngest daughter Hedwig

¹⁷ Gottfried Keller (1819-1890) was a Swiss poet and writer. From 1861-1876 he worked as Zurich's First Official Secretary (Erster Staatsschreiber). Among his most famous works are the novel *Der grüne Heinrich* (*Green Henry*, 1855, final version 1879) and the novellas *Romeo und Julia auf dem Dorfe* (*A Village Romeo and Juliet*, 1855/6, final version 1875) and *Kleider machen Leute* (*Clothes Make the Man*, 1874). Cf. biography by

(see above). Geiser was among the speakers when Keller received his honorary doctorate from the University of Zurich in 1869, and became an ‘understanding younger friend’ to Keller [11, p. 429]. Ermatinger also recounts a little anecdote about them [11, p. 445]:

On 01 February 1872 Keller attends an organ concert by Theodor Kirchner in [the church] St. Peter in the company of Professor Geiser. As they leave the church they see a beautiful aurora on the sky. All of a sudden Keller remarks that it occurred to him during the concert that he would have to change something in his legends manuscript.¹⁸

Geiser was among the first generation of Polytechnic graduates, and subsequently among the first members of its alumni association, the *Gesellschaft Ehemaliger Polytechniker (GEP)*, which was founded in 1869. Some of the GEP’s main objectives were to help recent graduates find adequate jobs and to provide a network of contacts, but the society helped shape the university throughout the decades (see appendix B). It was renamed *ETH Alumni Vereinigung* in 2000; today it has more than 20,000 members [33, p. 23] and over 40 regional and professional groups [8]. Geiser served as the GEP’s second president from 1870-1875 and was made an honorary member upon retiring from the post. The reason for his stepping down was that he was ‘piled with work’ [35]. His experience as president probably came in useful when he was Director, as the GEP’s executive committee maintained close relations with the School Board [31, p. 9].

Like many Swiss scientists in the 19th century, Geiser was a member of the *Schweizerische Naturforschende Gesellschaft*, the Swiss Society for Natural Scientists, which he joined in 1865. He chaired the society’s central committee

U Amrein in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D12024.php>, accessed 04/04/2014.

¹⁸The “legends manuscript” refers to Keller’s novella cycle *Sieben Legenden (Seven Legends, 1872)*. Apparently Keller re-wrote the ending of the seventh legend, *Das Tanzlegendchen (A Legend of the Dance)*, after attending the concert that Ermatinger mentions.

from 1898-1904¹⁹. According to Neuenschwander, Geiser and Hermann Amandus Schwarz tried to found an independent section for mathematics within the society in 1871. However, due to the low number of mathematical contributions at society meetings this attempt remained unsuccessful [9, p. 29-30]. Plancherel reports that a mathematical section chaired by Schwarz featured in three of the society's annual meetings in the early 1870s. Apart from Schwarz, Geiser also gave talks in these sectional meetings: *Über die Fresnelsche Wellenfläche* (1871)²⁰ and *Zur Erinnerung an Jakob Steiner* (1873)²¹; but no talk in 1874 [39, p. 206]. However, it seems that there was not enough interest to sustain the sectional meetings after Schwarz left for Göttingen in 1875. Neuenschwander suggests that most mathematicians in Switzerland preferred to publish in the journals of the various regional societies [9, p. 29-30]. In fact, Geiser was also a member of the *Naturforschende Gesellschaft in Zürich* from 1883 onwards.

Furthermore, Geiser was a member of the Euler-Kommission for two years, but had to step down due to health reasons. The commission was chaired by his colleague Rudio (see section 3.2). Geiser was also among the mathematicians who constituted the Steiner-Schläfli Committee in the 1890s. The committee organised Steiner's exhumation and raised money for a tombstone on Schläfli's grave (see section 4.2.5.1). At the Polytechnic, he was a member of the Library Committee, which was again chaired by Rudio.

Geiser became one of the first three honorary members of the *Schweizerische Mathematische Gesellschaft (SMG)*²² in 1911, shortly after the SMG's foundation. The other two were Hermann Kinkelin and Heinrich Weber. Furthermore, he was made a foreign member of the *Leopoldina*²³ on 09 December 1888 [27, p.

¹⁹ The central committee relocated to a different town in Switzerland every six years, meaning that society members in these towns made up the respective committees. Geiser became president when it was Zurich's turn to provide the committee.

²⁰ "On Fresnel's Wave Surface"

²¹ "In memoriam Jakob Steiner". A translation is included in appendix E.3.1. See also section 2.2.

²² "Swiss Mathematical Society"

²³ Founded in 1652 as the "Academia Naturae Curiosorum", the Leopoldina is the oldest continuously existing learned society in the world. Geiser knew it as "Deutsche Akademie der Naturforscher Leopoldina"; it was renamed and

214], and he received the award of “Officier de l’académie” from the French government in 1889 [5g]. Geiser was awarded two honorary doctorates, one from the University of Bern, on 10 September 1917, on the occasion of the 50th anniversary of the conferral of his PhD; and one from the ETH, on the occasion of his 75th birthday in 1918. In both cases, both his contributions to mathematics and to school education were recognised [cf. 36; 38].

Geiser undoubtedly was a talented mathematician, but did not make as much of an impact on mathematics as many of his colleagues at the Polytechnic. A contributing factor to this may have been his chosen field – as mentioned above, his mathematical interests were in various areas of geometry, particularly algebraic, synthetic, and also analytic geometry. At the time, geometry was one of the main areas of mathematical research, but it lost its importance during the 20th century. If one were to compare Geiser’s mathematical impact to that of his colleagues, one would have to bear in mind that many mathematicians at the Polytechnic, such as Hurwitz or Minkowski, ventured into new areas of mathematics, which Geiser did not. Furthermore, Geiser did not publish many papers from 1881 onwards; in fact, he probably did not have much time to conduct research due to his teaching and administrative duties. Frobenius commented on Geiser’s mathematical work in a letter to Gnehm on 27 June 1913:

The papers of my old friend Geiser [...] show no pretension whatsoever. But there is an original thought in each of his papers, which characterises that piece of work and which no one has yet expressed in this form.

(Quoted in [13, p. 14])

Geiser’s first publication was not his PhD thesis, but *Einige geometrische Betrachtungen* (1866)²⁴. Here he investigates a number of relations between points and planes, using poles and polar planes. Most of the proofs are

reconstituted as the German Academy of Sciences in 2007. Cf. <http://www.leopoldina.org/de/home/>, accessed 01/04/2014.

²⁴ “Some Geometrical Observations”

synthetic, but he also gives analytic proofs of two theorems²⁵. In the same year he published *Über die Normalen der Kegelschnitte*²⁶, in which he verifies a result given by Steiner in *Crelle's Journal* 55. In particular, he uses recursion to find the number of conics that 'go through α points, touch β straight lines, and have γ other straight lines as their normals; where α, β, γ are positive integers (including zero) and $\alpha+\beta+\gamma = 5$ ' [20, p. 381].

Among his more important papers are *Über zwei geometrische Probleme* (1867)²⁷ and *Über die Doppeltangenten einer ebenen Kurve vierten Grades* (1869)²⁸. In the first paper, he derives, by means of synthetic geometry, what is now called a "Geiser involution":

If seven of the nine points of intersection of two cubic curves remain fixed, while the eighth point describes a straight line G , then the ninth point traverses a curve C_8 of degree eight, on which the seven fixed points are triple points.²⁹

[21, p. 80]

Nowadays Geiser involutions are considered a special class of Cremona transformations, but Geiser does not connect his results to Cremona's work. However, he discusses Cremona transformations in his review of Cremona's *Opere matematiche* [19, p. 455], again without any reference to his own research.

In the second paper, he investigates the relationship between the 28 double tangents of a quartic plane curve C_4 and the 27 straight lines of a cubic surface F_3 . He followed this result up in *Über Flächen vierten Grades, welche eine*

²⁵ In particular: 'If a point moves on a plane, then the intersection of its polar planes with three fixed quadratic surfaces describes a cubic surface', first stated without proof by Steiner in 1859 [17, p. 220-221]; and 'The locus of the centres of all quadratic conic surfaces that pass through six independent points in space is a quartic surface [...]' [17, p. 227].

²⁶ "On the Normals of Conics"

²⁷ "On Two Geometrical Problems"

²⁸ "On the Double Tangents of a Quadratic Plane Curve"

²⁹ A more involved and "modern" definition can be found in [26, p. 177-178]. Geiser also investigates the analogous case for the eight points of intersection of three quadratic curves [cf. 21, p. 83-88].

Doppelkurve zweiten Grades haben (1869)³⁰, in which he verifies a result by Jordan (whose inspiration in turn was Geiser's investigation) [30, p. 524].

Furthermore, a minimal surface bears his name. In *Notiz über die algebraischen Minimumsflächen* (1870)³¹, he shows that the intersection of an algebraic minimal surface and the plane at infinity takes the form of straight lines and the absolute imaginary spherical circle [30, p. 524].

In the correspondence between Felix Klein and David Hilbert we find a reference to Geiser's paper *Zur Theorie der Flächen zweiten und dritten Grades* (1868)³². Klein asks Hilbert in 1887 whether he knows of any papers on pencils of quadratic surfaces. Hilbert points him to Geiser's paper (p. 215) and a paper by Sturm (*Crelle's Journal* 70) [12, p. 27; 31].

In addition to his mathematical papers, Geiser wrote biographies of Steiner, Christoffel, and Reye. His paper *Zur Erinnerung an Theodor Reye* [23] (see appendix E.3.2) is arguably the most interesting of the three. Whilst he focuses on the lives and achievements of Steiner and Christoffel in their respective biographies the talk on Reye is much more comprehensive. In fact, we learn relatively little about Reye: Geiser touches upon the main events in Reye's life and summarises his major mathematical contributions, but refrains from characterising Reye's personality, as is the case in the other biographies. Seeming more interested in placing Reye's life into its historic context, he illustrates the circumstances in which Reye lived and taught – and, by extension, also Geiser himself. As he mentions in the introduction, the *Schweizerische Bauzeitung* asked him for an obituary of Reye, but [23, p. 158]:

The need for giving an idea of the scientific importance of this researcher that covers at least the main points; the wish to discuss the questions concerning the “theoretical” instruction at technical colleges as well, questions that tie in with the activities of the teacher and that continued to raise an interest in him when they were discussed extensively later on, even when he was not personally affected by them anymore; and above all the want for depicting people's fates in

³⁰ “On Quartic Surfaces With a Quadratic Double Curve”

³¹ “Note on Algebraic Minimal Surfaces”

³² “On the Theory of Quadratic and Cubic Surfaces”

connection with the events of the day, have of course led me far beyond the scope of the *Bauzeitung*.

Reading the biography one gets the feeling that Geiser saw this as an opportunity to comment on a number of topics, such as the place of mathematics in university education and German imperialism, without being perceived as speaking his mind.

The biography is split into seven parts, with the addition of the aforementioned introduction, in which Geiser also summarises Reye's major achievements. The different sections cover:

- I. Reye's life until his habilitation
- II. Culmann and his connection with Reye
- III. Reye's work on geometry whilst at the Polytechnic
- IV. Fiedler at the Polytechnic
- V. The debate on the place of mathematics in an engineer's training
- VI. Reye's mathematical work during his years in Strasbourg
- VII. Universities and political attitudes in pre-WWI Germany

The last section concludes with a short account of Reye's years as a pensioner and his death.

As mentioned by Gnehm in his letter [13] (see above), Geiser was a spry pensioner. However, he suffered from an eye disease³³, which caused him discomfort for several decades. As he explains himself, he had already had a 'difficult operation' in 1899, which was followed by a second operation and eventually the loss of his right eye [3b]³⁴. In a note of congratulations for his 80th birthday, fellow GEP members wish him 'all the best for future years, in

³³ In [3b] he mentions that he required an operation to treat "Staar". Unfortunately, this could refer to "grauer Star", which means cataract, or "grüner Star", meaning glaucoma.

³⁴ Being a mathematician, he gives a mathematical analogy to describe his feelings: 'None of the operations caused me any pain – but I often contemplated the unchallengeable arithmetic example: that if I lost the second eye as well, there would not be a third one to replace it.' [3b]

particular that the current treatment of his eye complaint will have a favourable outcome' [37]. According to Geiser, he spent this birthday 'completely blind' [3b], but he regained his eyesight to the extent that he could still read and write.

His former assistant Meissner portrays the retired Geiser as follows:

His mind was agile until the last year of his life. At the age of 89 he still wanted to be taken to Zurich to hear Chiesa's lectures on Dante, for which he prepared himself meticulously. Bernard Shaw's Saint Joan occupied his mind and judgement. At the age of 90, he still read Shakespeare; he even did mathematics. Later on, the visitor would find his venerable figure sitting in an armchair, very similar to Pope Julius II, and full of energy and dignity. Nothing about this old man was callow or vague. He prefers causing an affront to getting involved in anything ambiguous. He always knows how to keep silent and never lets himself be coaxed, but what he says is credible. One can never hoax him; when one wants to twist his arm he becomes suspicious. His fondness for country life is connected to his straightforward nature. His opinion on right and wrong is simple. He is a good Swiss, who still wants to be led to the ballot box at the age of 90, and who takes his duties seriously, because he respects both himself and his country. At the bottom of his heart this European remained a man from Oberaargau.

[32, p. 374]

Meissner further reports that Geiser's daughters took care of him in his old age, for which he was 'very grateful' [ibid.] and 'for once showed his [thanks] to other people' [ibid.].

Carl Friedrich Geiser died on 07 March 1934³⁵.

³⁵ According to the School Board, Geiser planned to bequeath the scientific part of his private library to the ETH's mathematics reading room. Kollros and Plancherel, who inspected the library at the School Board's request, report that it contained 'a large number of valuable books, in particular complete editions of known mathematicians and very rare and historically important books' [5v]. Note that Geiser designated Kiefer (see section 4.2.14) and Scherrer (see appendix D) to be his executors.

Figure I: Geiser Family Tree (Based on [4a]).

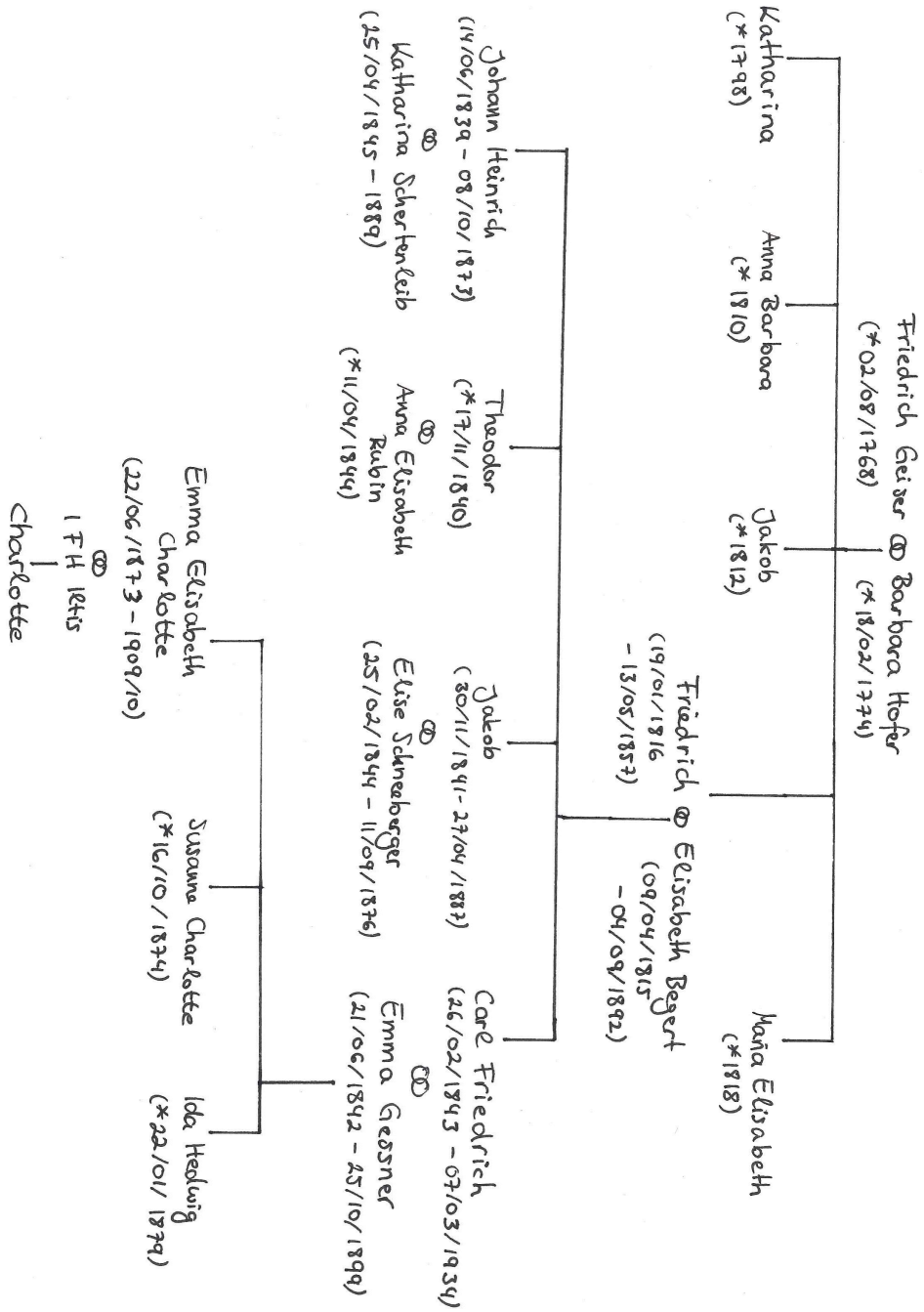
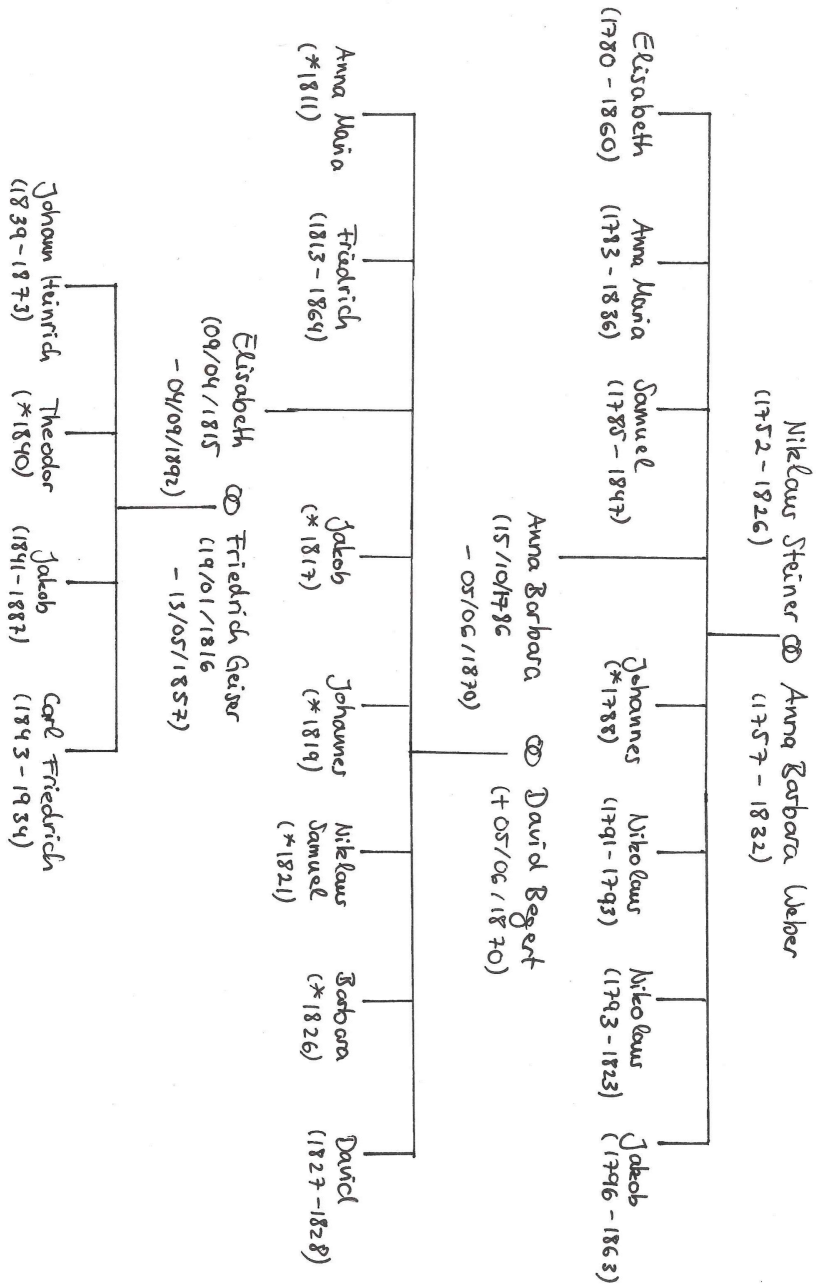


Figure II: Steiner – Geiser Family Tree (Based on [4] and <http://www-history.mcs.st-andrews.ac.uk/Biographies/Steiner.html>).



2.2 Connection with Steiner

As mentioned above, Geiser was one of Steiner's grandnephews. Jakob Steiner (1796-1863) was the youngest child of Niklaus Steiner (1752-1826) and Anna Barbara née Weber (1757-1832). Their fourth child, Anna Barbara (1786-1870), married David Begert (†1870)³⁶ of Ersigen, canton Bern. Like Anna Barbara's parents, the couple had eight children. Their third, Elisabeth (1815-1892), married Friedrich Geiser (1816-1857) of Langenthal, canton Bern, in 1836. These were Geiser's parents (see section 2.1). A more extensive family tree is given in figure II (p. 31).

Steiner was one of Geiser's lecturers in Berlin and his inspiration to study mathematics. Unfortunately, there are no records that could indicate the nature of their relationship. We do know that Geiser held Steiner in high esteem throughout his life and that he was partial to synthetic geometry, probably due to his uncle's influence. However, whilst Steiner encouraged Geiser to study [cf. 32, p. 372], it seems that he was not too fond of him. As Sidler writes to Bützberger [4c]:

A flaw of Steiner's character is his conduct towards Geiser. Steiner knew the outstanding talent & the enthusiasm of his grandnephew. But Steiner ignored him in his will completely, be it due to jealousy, be it due to an aversion to Geiser's mother [...].

Indeed, apart from the money that he donated to the Berlin Academy for the Steiner Prize, Steiner bequeathed most of his fortune to the children and grandchildren of his eldest sister Elisabeth, with the majority of the money going to her eldest son, Jakob Mathys³⁷ [4c; 24, p. 48-49]. It seems that Geiser never displayed any resentment towards Steiner, otherwise Bützberger would not have replied to Sidler [4d], 'it is a nice trait that [people] are not cross with [Steiner] because [he ignored them in his will], but hold his memory in high

³⁶ Bützberger gives different dates for David Begert: in **Hs 194: 4** and **Hs 194: 26-30** he notes that Begert died on 05/06/1870, but a note in **Hs 194: 154** suggests that Begert lived from 1786-1854).

³⁷ Jakob Mathys was a lower-rank bailiff in Koppigen, canton Bern. Both Sidler and Bützberger express their contempt for him in their letters [4c; 4d].

regard'. However, Geiser was probably more embittered about being disregarded in the will than he would have admitted. In an undated³⁸ manuscript preserved in Steiner's estate in the ETH Library Archive [3b] Geiser explains that:

The material content of the will & particularly the motives that Steiner pleaded at the time [...] caused me to avoid making any verbal or written comments on this matter for many years. But eventually I found the reason for the almost lunatic deficits in the ever increasing illness & loneliness [...]

Here Geiser still tries to explain his uncle's behaviour and takes pity on him, but his bitterness is implied in his comment. It echoes the sentiment that he expressed, albeit in much more general terms and without reference to any personal experience, in his 1873 paper *Zur Erinnerung an Jakob Steiner* [22, p. 246; p. 250] – see appendix E.3.1.

Steiner did not even pass his mathematical manuscripts on to Geiser; instead, he bequeathed them to his old friends Georg Sidler³⁹ and Schläfli. Schläfli received all the papers relating to his correspondence with Steiner from 1848-1856, which was later published by J H Graf⁴⁰ [3a; 3b]. The other manuscripts were given to Sidler, who 'deposited them in the Town Library [in Bern], for the attention of the *Schweizerische Naturforschende Gesellschaft*', as Graf writes to Bützberger in 1917 [4b; italics by the author]. However, he continues, 'it seems that the managers of the library at that time hardly

³⁸ In this manuscript Geiser refers to a book by H Fischer on the work of the Federal Matura Committee, which seems to have been published in 1927. Thus, he must have written the draft in 1927 or in subsequent years.

³⁹ Georg Sidler (1831-1907) was among the first teaching staff at the Polytechnic in 1855. He spent most of his career as mathematics teacher at the Kantonsschule Bern and as professor for mathematics and astronomy at the University of Bern. He also served on the canton's Matura Examination Committee. See S Eminger, <http://www-history.mcs.st-andrews.ac.uk/history/Biographies/Sidler.html>.

⁴⁰ Johann Heinrich Graf (1852-1918), professor of mathematics at the University of Bern who was appointed as Schläfli's successor in 1892. He edited a large part of Schläfli's correspondence, chaired the Steiner-Schläfli Committee, and wrote numerous papers on the history of mathematics in Switzerland. See biography by S Eminger: <http://www-history.mcs.st-andrews.ac.uk/Biographies/Graf.html>.

understood how to preserve these precious manuscripts and placed the boxes in a corner of the Town Library's attic, where, with open roof hatches, the valuable papers were at the mercy of wind and weather' [ibid.]⁴¹.

Before that, Sidler gave some of the manuscripts to Geiser. Among them were the manuscripts that were published as *Die Theorie der Kegelschnitte in elementarer Darstellung*⁴², volume I of *Jacob Steiners Vorlesungen über synthetische Geometrie*⁴³ [18]. The two volumes of these *Vorlesungen* cover two of the lecture courses that Steiner taught most frequently. Geiser explains the origin of this book in a bit more detail in [3b]:

Some time after my habilitation as Privatdozent for mathematics, particularly synthet[ic] geom[etry], at the Fed[eral] Polyt[echnic] [Sidler], who had been my teacher at the Bernese Kantonsschule & knew of my relation to St[einer], offered to give me some of the manuscripts for an in-depth examination and possible editing. Since he approved of the educational tendency of my lectures (for trainee teachers) & was very interested in my first scientific efforts, the elementary sections were considered initially. I compiled a modest volume: "Theory of Conics in Elementary Treatment" based on a comprehensive collection of material [footnote: which Steiner referred to as "Popular Conics"], which was entirely within the scope of higher years at the Realschule, and partly also at the Gymnasium. At the same time, "The Theory of Conics Based on Proj[ective] Properties" by Prof Schröter was published. [Schröter] had attended St[einer]'s lectures at the time & had also been given St[einer]'s manuscripts. The two vol[ume]s share the title: "J. St[einer]'s Lect[ures] on Synth[etic] Geometry" 1867.

⁴¹ Graf found these papers in 1888 and gave some of them to Bützberger for editing. See section 4.2.5.1 for Bützberger's work on Steiner.

⁴² "The Theory of Conics in Elementary Representation"

⁴³ "Jakob Steiner's Lectures on Synthetic Geometry", Leipzig, 1867. Volume I was edited by Geiser, Volume II was edited by H Schröter and covers *Die Theorie der Kegelschnitte gestützt auf projectivische Eigenschaften* ("The Theory of Conics Based on Projective Properties").

In the preface to the first volume Geiser explains that he also used a number of manuscripts where Steiner had jotted down ideas, and Sidler's notes from Steiner's lectures [cf. 18, p. v-vi]. In addition to the lectures Geiser edited two further papers by Steiner: *Geometrische Betrachtungen und Lehrsätze* (1866/67)⁴⁴ and *Konstruktion der Fläche zweiten Grades durch 9 Punkte* (1868)⁴⁵.

In [3b] Geiser indicates that he intended to edit more of Steiner's manuscripts, but for a number of reasons this never happened. Firstly, it seems that Mathys contested Steiner's will: 'Referring to a remark by Steiner "that his manuscripts are worth more than the rest of his fortune", the main beneficiary made corresponding claims to Sidler, which were rejected.' Geiser reports that he had to put any further publications on hold until the matter was settled: 'dejected, I put the manuscripts aside'. Then, the Berlin Academy tasked Weierstrass with the publication of Steiner's complete works⁴⁶. As a result, Geiser explains, he could not proceed with any publications independently. He further writes that Weierstrass asked him for papers to be included in the second volume. Due to the conditions set out by the Academy, only papers that were already prepared for publication could be published posthumously, which was not the case with most of the manuscripts in Geiser's possession. However, he sent Weierstrass his edition of Steiner's two famous papers on maxima and minima⁴⁷. Steiner had published these in a French translation only, and Geiser thought that the German originals deserved to be published, as they were more accurate than the translations. This idea was first rejected by a publisher, but Weierstrass included the papers and a few annotations by Geiser in *Gesammelte Werke II* [cf. 3b; 44, p. v-vii].

⁴⁴ "Geometric Observations and Theorems"

⁴⁵ "Construction of the Quadratic Surface Through 9 Points"

⁴⁶ K Weierstrass (ed.), *Jacob Steiner's Gesammelte Werke*, 2 volumes, G. Reimer, Berlin, 1881-1882.

⁴⁷ *Über Maxima und Minima bei den Figuren in der Ebene, auf der Kugelfläche und im Raume überhaupt* ("On Maxima and Minima of Shapes in the Plane, on the Sphere and in Space in General"), 1. Abhandlung in [44, p. 177-240]; 2. Abhandlung in [44, p. 241-308]. Steiner submitted these papers to the Parisian Academy in 1841, but only published French translations, as Liouville asked for the first paper to be included in his *Journal*. Both papers were published in *Crelle's Journal* 24 in French, as *Sur le maximum et le minimum des figures dans le plan, sur la sphère et dans l'espace en général*. As Geiser and Weierstrass note, the translation is inaccurate in places.

As Geiser reports, the publication of *Gesammelte Werke* coincided with him becoming Director of the Polytechnic. This position and later his presidency of the Federal Matura* Committee (see section 2.3) led to an increased workload, which left him little time for research and editing⁴⁸. Furthermore, organising the first ICM would have kept him busy, although he does not mention it. And finally, his eyes caused him trouble from the late 1890s onwards (cf. section 2.1; [3b]), which surely contributed to him spending less time on the Steiner manuscripts. He mentions that the correspondence between Schläfli and Steiner (published by Graf after Schläfli's death) was of great interest to him, and helped him when studying Steiner's manuscripts. Apparently he sent some remarks relating to the correspondence to the publisher, rather hastily written due to a short editing process [3b]. If Geiser prepared any manuscripts for publication later on they were never published. It seems that he passed the manuscripts on to the *Schweizerische Naturforschende Gesellschaft* or the Berlin Academy. He also destroyed most of his notes [cf. 3a].

Steiner influenced Geiser's mathematical taste; without him, Geiser may not have focused on synthetic geometry. As a lecturer Geiser introduced his students to this branch of geometry for a number of years. He taught lecture courses on synthetic geometry almost every semester from 1866-1877, typically "Introduction to Synthetic Geometry" in the first, and "Synthetic Geometry" in the second semester. During the subsequent ten years he offered one course on "Synthetic Geometry" per year. In his first few years as Privatdozent he taught a course entitled "Synthetic Geometry based on Steiner" twice (in 1865 and 1867).

As mentioned in section 2.1, Geiser solved some problems given by Steiner in *Aufgaben und Lehrsätze* (1852) in his PhD thesis. A number of his later

⁴⁸ In a separate note [3a] he responds to the preface of *Gesammelte Werke*, in which Weierstrass recounts the difficulties that he encountered in the editing process:

The difficulties for the manuscripts in my possession were significantly greater: in a number of cases my scientific understanding was not sufficient, then my teaching obligations changed from synth[etic] geom[etry] to analyt[ic] & algebra[aic] [geometry], for a long period administr[ative] duties impeded my scientific research (more accurate details are not important here).'

mathematical papers were inspired by theorems and problems that Steiner had stated without proof (see section 2.1).

Moreover, Geiser wrote a biography of Steiner, *Zur Erinnerung an Jakob Steiner* (see appendix E.3.1), which he presented at the annual meeting of the *Schweizerische Naturforschende Gesellschaft* in Schaffhausen in 1873. As Geiser says himself, no proper biography of Steiner had been written until then: ‘Will I be reprimanded, because I now attempt to draw a delicate outline of his figure based on family ties, personal and scientific relationships and the memories they evoke?’ [22, p. 217]. Note that this is one of the few personal remarks that Geiser adds to the text. Interestingly, he dedicates a comparatively long section to the historic developments in geometry despite not showing much interest in the history of mathematics, save his interest in Steiner. However, this interest was undoubtedly motivated by their personal connection, regardless of his bitterness over his uncle’s will. Geiser’s biography testifies to his admiration for Steiner; in fact, it almost seems as if he chose to talk about his favourite aspects of Steiner. Almost half of the talk is dedicated to Steiner’s mathematical achievements (including the historical survey). Whilst he puts Steiner’s childhood into a wider context, he omits a number of events that occurred in Steiner’s later life. Furthermore, he singles out a few of Steiner’s friends, but ignores others. When talking about Steiner’s lectures, Geiser focuses on methodology rather than content – perhaps not surprising given Geiser’s own interest in education.

Bützberger quotes from Geiser’s talk in his 1896 paper *Jakob Steiner bei Pestalozzi in Yverdon* and used it for his unpublished Steiner biography (see section 4.2.5.1)⁴⁹; Graf also used it for his more extensive Steiner biography [24].

⁴⁹ Bützberger’s estate in the ETH Library Archive contains Graf’s Steiner biography (**Hs 194: 10**). On p. 1 there is a handwritten reference to an obituary of Steiner written by Geiser and published in *Die Schweiz: Illustrierte Zeitschrift für Literatur und Kunst* in 1863, calling it ‘the most valuable biogr[aphical] source’. Unfortunately I have not been able to obtain a copy of this obituary.

2.3 Impact at the Polytechnic and on Education

As mentioned in section 2.1, Geiser served as Director of the Polytechnic twice, first from 1881-1887 and then again from 1891-1895. In 1881 the lecturing staff were given more privileges (cf. appendix B); one of these was that the Lecturers' General Assembly ("Professorenkonferenz") now elected the Polytechnic's Director, with the School Board only making the official appointment. Before that the Director was chosen by the School Board, the staff had no say in the matter. Geiser was thus the first Director to be elected by his colleagues [cf. 3b]. He also served as Deputy Director from 1887-1891 and from 1895-1899. Note that the Director and his Deputy were always appointed for two years at a time.

Whilst the School Board President was responsible for all the policy decisions and staff appointments at the Polytechnic, the Director seems to have had a more administrative role – possibly remotely comparable to the roles of Proctor and Academic Registrar at the University of St Andrews. Unfortunately, I have not been able to find any descriptions of the Director's duties; the information in this paragraph is based on comments found in various books and biographies.

The Polytechnic's regulations, set out in the course catalogue for each semester, state that students had to submit any course choices and other paperwork, such as proof of address, to the Director's Office. Thus, Geiser would have had to deal with administrative matters such as matriculation, student records, and possibly graduation. Furthermore, it seems that he was in charge of student discipline. An incident in 1885 suggests this: During the 1880s, many students protested against the restrictive regulations and strict discipline inflicted on them by the Polytechnic's management (see appendix B). As Guggenbühl reports in [25, p. 105-106]:

[...] in 1885, when the young Polytechnicians thought that Director Geiser continuously treated them unkindly, and eventually felt provoked to lodge open protest, a student demonstration [took place], which members of the University joined, too, and subsequently [there was] once again an academic revolt. The direct cause for this was an

arguably untenable rumour. They said that Geiser refused to suspend teaching on the occasion of the funeral of two young Polytechnicians [...]. As a result, [students] performed rough music in the evening hours of 09 February in front of his house in nearby Küsnacht, using a variety of cacophonous instruments, including a bass drum.

Frei and Stambach, who describe the incident in [14, p. 44], note that 'it was probably only because of Kappeler's skilful comportsment that the situation calmed down again'. According to the School Board meeting at which the incident was discussed, a tactful approach was indeed advisable, as some students got into trouble with the police during the demonstration [5e]. The Board also stresses that the allegations against Geiser were 'altogether untrue and fictitious' [5e, p. 22].

As Director, Geiser represented the Polytechnic at various events in Switzerland and abroad. For example, he told his friend Gysel in 1882 that he was invited to attend the opening ceremony of the Gotthard [1a] (see also section 5.2.2). In 1895 he represented the Polytechnic at the centenary celebrations of the Ecole Normale in Paris [5i]. It seems that he retained some of these representative duties after his retirement as Director. As an example, he attended anniversary celebrations of various secondary schools in Switzerland, such as the Kantonsschule in Grisons [5m]. In a similar vein, he substituted for the ETH Director at a meeting in Bern in 1912, where the contributions of higher education institutions at the Swiss National Exhibition in 1914 were discussed [5u].

As Geiser writes, 'in addition to the usual duties' as Director he faced 'extensive negotiations with the authorities of those schools which asked that their graduates be admitted without any entrance examinations' [3b] after taking up his post. In particular, Geiser became "Consultant to the School Board on Matura Affairs" ("Referent des Schulrates für Maturitätsangelegenheiten"). There is no record in the Board minutes as to when exactly he was appointed, but in minutes from 1921 concerning a

pensions increase it is noted that Geiser took up that post at the beginning of the 1880s [5c]. When he stepped down from his post of Director in 1895, the Board asked him to continue working as Consultant, which he did until his retirement, despite doubts [3b]:

Due to this extensive adm[inistrative] function I got thoroughly acquainted with the organisation and administration of the Polyt[echnic], gained a comprehensive overview of Swiss secondary schools, & had the opportunity to meet many teachers, headmasters, and education ministers, which proved to be very stimulating. But despite the interest with which I addressed myself to these matters and despite the ample recognition for my efforts from superior authorities, I am increasingly plagued by the question whether I am not far too distracted from my actual job as academic teacher which is based on research.

In this role Geiser conducted negotiations on the subject of entrance examinations with various schools in Switzerland. Due to a change in legislation in 1881 (see appendix B) the Polytechnic discontinued its Preparatory Course and existing treaties with secondary schools [45]. Thus, new treaties had to be negotiated, a project that Geiser became heavily involved in. To this end he visited schools across the entire country, either by himself or with a School Board representative. For example, he told Gysel in 1883 that he would travel to Frauenfeld with Kappeler to conduct treaty negotiations [1b] (see also section 5.2.2); in 1891 he visited the secondary school in St. Gallen [5h]. He continued these school visits during the 20th century, as is attested in the minutes of several Board meetings [e.g. 5r, p. 83-85; 5k; 5l]. As Consultant Geiser gave evaluations of schools in question at Board meetings, and indicated whether or not a treaty would be worthwhile [e.g. 5s]. Furthermore, he corresponded with schools and educational ministers on behalf of the School Board [e.g. 5r, p. 85; 5t], and sat on the Polytechnic's Entrance Examinations Committee.

In essence, pupils who obtained their Matura from a “treaty school” were exempt from the Polytechnic’s entrance examinations. Before signing any treaties the School Board and its delegates, Geiser for example, inspected the quality of the curriculum and the teaching staff in order to ensure that certain standards were met. If applicable, the Board set out certain conditions that the schools had to meet before entering into an agreement. For example, based on Geiser’s observations, the Board decided that the secondary school in canton Schwyz had to reduce teaching hours for sciences in favour of languages [5s, p. 124] – which is quite interesting as the Polytechnic was generally more concerned with raising the standards of science teaching. Furthermore, Polytechnic lecturers acted as examiners at the Matura examinations at the “treaty schools”. From about 1900 onwards, the School Board regularly appointed representatives to attend examinations⁵⁰, among them Geiser and many of his colleagues.

Geiser soon became an expert on secondary school education in Switzerland. It is likely that this reputation prompted the Swiss government to appoint him President of the first Federal Matura Committee in 1891. Geiser remained in this post until 1909. As Geiser reports in [3b], he first chaired a small committee tasked with reviewing the existing Matura procedures. The full Matura Committee then conducted negotiations with secondary schools regarding the contents of their Matura programmes. Moreover, he writes, due to the specific regulations for medical students he had to familiarise himself with the requirements for medical exams in other countries. He also comments that changes to the Polytechnic’s admissions policy in the 1880s, such as raising the entrance age and level of academic prerequisites, led to disputes between the Polytechnic and individual schools. In particular, these changes touched upon the debate of federal vs. cantonal sovereignty (see also appendix B).

⁵⁰ Possibly this happened already during the 19th century, but there are no references that I could find in the minutes.

By all accounts Geiser was very skilled in administrative matters; his extensive network of contacts (see section 2.1) also helped him in his task. His biographers also attribute his influence to his close friendships with the Polytechnic's Presidents and Directors. As Meissner puts it [32, p. 372]:

He was the paladin⁵¹ and confidant of [...] Kappeler, who generously delegated many tasks to him, occasionally even representation in Bern⁵².

Furthermore, Plancherel notes that Geiser 'played a discreet but important role in appointments of mathematics professors' [39, p. 213]; Meissner echoes this observation [32, p. 372-373]. Unfortunately, there is no proof of this in the School Board minutes, but we find Geiser in a few appointment committees, e.g. for Edouard Meissner and for Ernst Fiedler (see section 4.2.7). Note, however, that this was after Geiser's time as Director. During the 19th century in particular, the School Board Presidents were in charge of staffing. Whilst both Kappeler and Bleuler were known to trust only their judgement, it is conceivable that they asked selected professors for their opinion. In the case of mathematics, the Polytechnic recruited a large number of first-rate mathematicians, indicating that someone was very good at recognising talent. However, unfortunately I am not able to give more details on Geiser's involvement.

In addition to the roles discussed above, Geiser was a member of the first Swiss Delegation of the International Commission on Mathematical Instruction (ICMI), which was established at the 1908 ICM in Rome at the initiative of David Eugene Smith. Geiser's Swiss colleagues were the ICMI's first Secretary General, Henri Fehr from the University of Geneva, and the Bernese professor Johann Heinrich Graf. The delegates were appointed at a meeting in Karlsruhe, Germany, in 1909. Supported by a sub-committee the

⁵¹ Meissner does not refer to the Knights of Charlemagne here, but rather uses 'paladin' in its figurative sense to describe a loyal friend or follower. This reflects its original meaning: in Antiquity 'palatinus' described a high-ranking official in the court of the Roman Emperor. In German 'paladin' can be used derisively, but I do not think that this is the case here.

⁵² i.e. at the Swiss Federal Council

delegates 'would address themselves to a meticulous investigation on mathematical instruction in Switzerland at all levels' [34; see also 47]. Due to his experience Geiser was in an excellent position to conduct such a study.

His interest in education is also reflected by some of the conferences that he attended during his career: In 1889 for example, he attended the *Congrès international pour l'enseignement supérieur et l'enseignement secondaire*, which was part of the World Exhibition in Paris [5f].

An example of his administrative achievements at the Polytechnic is the establishment of a Civil Fund for Widows and Orphans. Kollros notes that Franel, Geiser, Gnehm, and Albin Herzog were involved in creating the fund [28, p. 169], but it is possible that other professors were involved in this project as well. At their meeting on 04 July 1899 the School Board enacted a petition to the Swiss Federal Council on the subject of such a fund [5q]. At this time Geiser had just stepped down from his post as Deputy Director; his successor was Herzog, who had been Director, and whose successor in turn was Gnehm.

Moreover, Geiser was one of the School Board delegates on the committee that organised the 50th anniversary celebrations of the Polytechnic, together with the Deputy President Gustave Naville, Director Gnehm and soon-to-be Director Franel [cf. 5n].

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3. Ferdinand Karl Rudio (1856 – 1929)

A shorter but similar version of this biography was published as part of S Eminger, Carl Friedrich Geiser, Ferdinand Rudio and Jérôme Franel: three organisers of the first International Congress of Mathematicians in Zurich in 1897, *conference volume of the History of Mathematics & Teaching of Mathematics conference, Sárospatak, Hungary, 23-27 May 2012*.

3.1 Life

Ferdinand Karl Rudio was born on 02 August 1856 in Wiesbaden. His father Heinrich was an official in the Duchy of Nassau; his mother Louise née Klein (+1879) was the daughter of well-known Forester J J Klein. Rudio had three sisters. One of them, Mathilde, later married Rudio's university friend and mountaineering companion Constantin von Monakow¹ [2].

Rudio first attended the Gymnasium* for four years and transferred to the Realgymnasium*, also in his hometown, in 1870. He had a particular talent for languages and for mathematics, which was taught by Wilhelm Unverzagt and Eduard Fürstenau. However, Rudio was an excellent student in general:

At Easter 1874 the gifted young man passed his final examinations with “excellent” as the overall grade. The same grade was achieved in almost all of the individual examination subjects, too. According to the presiding provincial school inspector it was the best Matura examination in the history of the Hessen-Nassau Province².

[7, p. 115]

¹ Russian-Swiss neurologist (1853-1930) who pioneered in experimental brain research. He studied medicine at the University of Zurich from 1872-1877 and later became director of the brain anatomy institute there. Cf. biography by U Boschung in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D14556.php>, accessed 19/03/2014.

² The Prussian Hessen-Nassau Province was created in 1868 as a consequence of the Austro-Prussian war.

Rudio moved to Zurich in 1874. He studied at the Polytechnic's Engineering Department for three semesters before he transferred to the Department for Mathematics and Physics Teachers. Among his lecturers in Zurich were Karl Culmann, Wilhelm Fiedler, Geiser, Hermann Amandus Schwarz and Heinrich Friedrich Weber. Rudio had to suspend his studies when he was taken ill with typhoid in spring 1876. He spent spring 1877 in Italy before returning to Zurich to finish his studies, and went to Berlin in the autumn of that year. There he attended lectures by Kummer and Weierstrass, in particular their mathematical seminar. The Minister of Education awarded him a prize for his achievements in the seminar. After a year in Berlin he studied in Paris for a couple of months.

In 1880 Rudio received a doctorate from the University of Berlin for his thesis *Über diejenigen Flächen, deren Krümmungsmittelpunktsflächen konfokale Flächen zweiten Grades sind*³. His supervisors were Kummer and Weierstrass. Fueter explains that Rudio used hyperelliptic integrals in order to solve the differential equations involved, thus demonstrating his mastery of analysis [7, p. 124]. Some of his later publications⁴ also concerned these results, and, according to Fueter, prompted Arthur Cayley to write a paper⁵ [ibid.].

As was custom at German universities, Rudio had to defend a thesis in a public disputation as part of the doctoral examination process. Rudio's thesis was that "the value of a mathematical discipline cannot be measured according to its applicability to empirical sciences". His opponent was Carl Runge, who claimed that: "the value of a mathematical discipline is to be assessed according to its applicability to empirical sciences". Schröter remarks in [7, p. 117] that 'the two of them remained good friends nonetheless'.

Rudio's former teacher Geiser, whom he met on a journey to Switzerland, suggested that he should habilitate at the Zurich Polytechnic. Schwarz

³ "On Those Surfaces That Have Quadratic Confocal Surfaces Through Their Centres of Curvature"

⁴ E.g. *Zur Theorie der Flächen, deren Krümmungsmittelpunkte konfokale Flächen 2. Grades sind* (1883); *Über eine spezielle Fläche vierter Ordnung mit Doppelkegelschnitt* (1889)

⁵ *On Rudio's Inverse Centro-Surface* (*Quarterly Journal of Mathematics* **22**, 1887, 156-158)

however, a former teacher as well, offered him a post in Göttingen. Encouraged by his friend Carl Schröter, Rudio moved back to Zurich in 1881. He stayed there for the rest of his life, becoming a citizen of Zurich in 1888. In the same year he married Maria Emma Müller (*1867) from Rheinfelden (canton Aargau). The couple had three daughters: Emmy (*1889), Elisabetha (*1890), and Alice Anna Hedwig (*1892).

At the Polytechnic Rudio first taught as a Privatdozent, in 1885 he became a Titularprofessor and in 1889 he was appointed to a full professorship in mathematics, which he held until his retirement in 1927⁶. He regularly gave introductory lectures in higher mathematics for students in architecture, chemistry and forestry, which were followed up by lectures in mathematical physics and analytic mechanics. The aim of his lectures was to impart the basics of mathematics and mathematical thinking. When teaching mathematics students he alternated between lecturing on mathematics and on the history of mathematics. He had a reputation for being ‘an excellent lecturer of mathematics and an adept in its history’ [1].

Rudio was interested in a variety of mathematical areas, particularly in historical topics, which is reflected in his publications. His purely mathematical papers mainly cover problems in geometry, such as geodesic lines on various surfaces, but he also worked on calculus of variations, mechanics, and group theory. An example of the latter is *Über primitive Gruppen* (1888), in which Rudio proves a theorem by Jordan on transitivity of simple groups. In addition, he wrote two textbooks on analytic geometry, *Die Elemente der analytischen Geometrie der Ebene*⁷ (1888, with Heinrich Ganter) and *Die Elemente der analytischen Geometrie des Raumes*⁸ (1891), both of which were inspired by his lectures. A full list of publications is in appendix C.2.

⁶ Depending on the publication the year of his retirement was 1926, 1927 or 1928. I have chosen the year 1927 as it is the one given in the ETH publications such as: http://www.ethistory.ethz.ch/materialien/professoren/listen/alle_profs/, accessed 22/03/2012.

⁷ “Elements of Analytic Geometry in the Plane”

⁸ “Elements of Analytic Geometry in Space”

In 1892 his book *Archimedes, Huygens, Lambert, Legendre* was published, which was one of several works on the topic of squaring the circle, Simplicius's comments on quadratures and Hippocrates's lunes (see chapter 6). Although he had received good training in languages during his school years, Rudio 'started studying Greek with enthusiasm' [7, p. 122] in order to be able to read the original sources. As Glaus puts it: Rudio's 'philological aptitude and meticulousness gave him access to the Greek sources and to experts such as Hermann Diels or Heron-editor Wilhelm Schmidt' [2]. His biographical work includes papers on Gotthold Eisenstein, Friedrich Hultsch and Georg Sidler. Together with Adolf Hurwitz he also edited Eisenstein's letters to Moritz Abraham Stern.

In 1881 Rudio became a member of the *Naturforschende Gesellschaft Zürich*. He served on the society's committee for several decades (as president from 1898-1900) and edited its quarterly *Vierteljahrsschrift* from 1894-1912. Together with Schröter he wrote *Notizen zur schweizerischen Kulturgeschichte*⁹, a regular section in the *Vierteljahrsschrift* covering developments in science and the Zurich academic community. To mark the society's 150th anniversary, Rudio wrote *Geschichte der Naturforschenden Gesellschaft 1746-1896*. The society honoured his commitment by making him an honorary member in 1912.

Seven years later he received an honorary doctorate from the University of Zurich, in recognition of his services to the libraries in Zurich and to the Euler project. When Rudolf Wolf, astronomy professor and director of the Polytechnic Library, died in 1893, Rudio was appointed his successor. Incidentally, Wolf was also Rudio's predecessor as editor of the *Vierteljahrsschrift*. Rudio created a catalogue of the Library's complete collection (in fact, the last one to be printed, in 1896) and was in charge of reconstructing the Library, which reopened in 1900. In doing so, 'he contributed to the reputation of the Library [which] was praised as exemplary after the reconstruction' [8]. He stepped down from his post in 1919. Moreover, he greatly contributed to founding the Central Library in Zurich, for example by fundraising.

⁹ "Notes on Swiss Cultural History"

Ferdinand Rudio died shortly after his retirement, on 21 June 1929.

3.2 Contribution to Euler's *Opera Omnia*

Today Rudio is mostly remembered for his contributions to the Euler project [cf. 4, p. 102-103; p. 510]. However, several excellent papers on the Euler project and its history have already been written. Therefore, I will give a short and by no means exhaustive summary of Rudio's involvement, but I chose to concentrate on his other works on the history of mathematics instead (see chapter 6).

As discussed in section 4.1.5, Rudio suggested at the 1897 ICM that Euler's complete works should be published. This suggestion is evidence of his interest in projects of a bibliographical nature, and in Euler in particular. In fact, Rudio's first recorded work on Euler was a talk he gave in 1883 as part of the Town Hall Lecture Series. The talk was later reprinted and sold in order to raise funds for the Euler project (see section 6.3.3).

The idea of publishing Euler's complete works was not a new one. Several attempts had already been made in Germany, Russia and Belgium, but no more than a few volumes were printed in each case [5, p. 26]. On the occasion of the bicentenary of Euler's birth in Basel in 1907, Rudio called on the representatives of the Swiss government and the Academies in Berlin and St Petersburg to publish Euler's works. His proposal was supported by Geiser, Alfred Kleiner from Zurich (Einstein's doctoral supervisor, incidentally), and Christian Moser from Bern, who had also been involved with the Steiner-Schläfli committee (see section 4.2.5.1). Three months later, at the annual meeting of the *Schweizerische Naturforschende Gesellschaft*, he proposed that a committee should investigate the feasibility of such a project:

There is a duty of honour that remains to be fulfilled and which should not be held off any longer: the complete edition of Euler's works! Admittedly this mammoth task can only be accomplished by the collaboration of many [individuals], but this year the entire mathematical world is primarily looking at Switzerland since an

energetic initiative is expected of Euler's home country. And the Swiss Society for Natural Scientists may not avoid this task!

[6, p. 217]

The society appointed a committee of eleven, the so-called "Euler-Kommission", with Rudio as president. Geiser was one of the members but he had to step down two years later due to health reasons [3, p. 12]. The committee established publication guidelines, decided on the contents of each volume and raised the necessary funds. Fueter notes that Rudio was one of the 'driving forces' behind the fundraising in particular [7, p. 129-130]. The project was authorised in 1909 and an editing committee was appointed, consisting of Rudio and the German mathematicians Paul Stäckel and Adolf Krazer. Rudio became chief editor; in this capacity he supervised the publication of over 30 volumes. In addition to determining the editing procedures and proofreading each volume, he was also responsible for corresponding with the publishers and all the editors across the world. Fueter describes Rudio's meticulous preparations:

Every editor receives the manuscript of his volume already completely compiled, so that his work merely involves inspecting and possibly annotating [it]. To this end Rudio was able to buy all works containing Euler's papers and all of his independent books second-hand.

[7, p. 130-131]

Furthermore, Rudio edited the first and second parts of *Leonhard Euleri Opera Omnia: Series Prima: Commentationes Arithmeticae* (1915 and 1917, respectively) and contributed to three more volumes. He also wrote regular reports on the progress of the Euler edition in Swiss journals, and gave a talk on the project at the 1912 ICM in Cambridge (see section 4.1.5).

Rudio was very well equipped for the project as he had a 'broad mathematical knowledge and a good education in languages and history' [2]. As Fueter puts it [7, p. 129]:

To describe Rudio's merits with regard to initiating and realising this grand work means to write an account of the entire history of the same.

Because it was he who was the driving force right from the beginning, who worked for it with unflinching energy and managed to inspire many others.

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4. The First International Congress of Mathematicians, Zurich 1897

In this chapter, the events leading to the choice of Zurich as the host town are outlined, as well as the organisation of the congress itself. Biographies of all members of the Swiss organising committee are given (with the exception of Geiser and Rudio, see chapters 2 and 3, respectively), and the contributions that the individual members made to the organisation are highlighted (for Geiser and Rudio's contributions, see sections 4.1.4 and 4.1.5, respectively).

4.1 Background and Organisation

Sections 4.1.1-4.1.3 are an edited version of S Eminger, *Viribus unitis! shall be our watchword: the first International Congress of Mathematicians, held 9-11 August 1897 in Zurich*, *BSHM Bulletin* 27, 2012, 1-14.

Parts of sections 4.1.4-4.1.5 have been published in S Eminger, Carl Friedrich Geiser, Ferdinand Rudio and Jérôme Franel: three organisers of the first International Congress of Mathematicians in Zurich in 1897, *conference volume of the History of Mathematics & Teaching of Mathematics conference, Sárospatak, Hungary, 23-27 May 2012*.

4.1.1 Historical Developments

During the 19th century, science became increasingly important and popular. The industrial revolution, and towards the end of the century inventions such as steam power and telegraphs, raised society's awareness of science and technology; and, as economic prosperity increased, higher education became more important. However, teaching was no longer considered the main activity of a university professor; research started to play an equally crucial part in the job description. This resulted in an ever-growing number of scientists slowly starting to collaborate internationally [55, p. 1]. In mathematics, this happened rather late in comparison to sciences such as astronomy, geology, or cartography.

One of the results of this new significance of science was the foundation of scientific societies; the first mathematical society was founded in Moscow in 1864. Other societies followed in most European countries and in North America. As more and more research was done, the number of mathematical journals and books published each year increased ‘at a rapid pace’ [55, p. 2]. One of these journals was the *Jahrbuch über die Fortschritte der Mathematik*¹, first published in 1871. The *Jahrbuch* was the first of many bibliographical catalogues that provided mathematicians across the world with an overview of current research and developments in their respective fields. Soon it ‘became indispensable for mathematical research’ [ibid.].

However, all the editors and the reviewers were German. The editors appealed for international cooperation and indeed, from the second volume on, some of the reviewers were non-German. The number of different countries represented increased with each new volume. Interestingly, none of the reviewers were French. The French first published *Répertoire bibliographique des sciences mathématiques* in 1885. It was a catalogue of all mathematical publications, divided into various sections and subsections. The French editors were soon joined by international colleagues, too.

For quite a while, French and German mathematicians did not have any means of communicating mathematical ideas apart from exchanging letters directly. The Swedish mathematician Gösta Mittag-Leffler published papers by both French and German mathematicians in his journal *Acta Mathematica* (founded in 1882), thus providing ‘a privileged place for communication between German and French researchers where the patriotic sensibilities of the various protagonists would not be offended’ [28].

Comparatively late, in 1894, a joint international bibliographical project was launched, the *Enzyklopädie der mathematischen Wissenschaften*², with the first issue being published in November 1898. Though of German origin, the emphasis of the project was on German-French cooperation. In fact, German and French mathematicians worked together to produce a French edition of

¹ “Yearbook on the Progress of Mathematics”

² “Encyclopaedia of Mathematical Sciences”

the encyclopaedia, which was ‘not merely a translation, but an adaptation’ as Dyck puts it in the preface to the first volume [60, p. XVIII]. The initiators of the project were Felix Klein in Göttingen, Heinrich Weber in Strasbourg and Franz Meyer, at the time professor at the Mining Academy in Clausthal. In contrast to the earlier bibliographical publications, the encyclopaedia included papers in a range of mathematical fields and also catered for physics, mechanics, and astronomy.

Unsurprisingly, the relations between France and Germany had greatly suffered due to the Franco-Prussian War (1870-1871). The French saw the reason for Prussia’s victory in its scientific superiority and therefore wanted to catch up with the scientific developments in Germany. Germany on the other hand was a young, strong empire with all the German states united, and it had imperialistic aspirations, wanting “a place in the sun”. A lot of effort was put into expanding and improving the Empire’s naval fleet, which was paid for with French reparations. Science was considered to be key to military development and as a result, the German government supported scientific research.

Whilst scientific progress was furthered in the name of patriotism in France and Germany, the respective governments did little to support international cooperation. In fact, political tension grew across Europe, yet the end of the nineteenth century also saw a drastic increase in scientific exchange and cooperation. A lot of this change was brought about by individuals or scientific societies rather than by governmental bodies.

In mathematics, most of the early international collaborations concerned bibliographical projects. Georg Cantor in Halle was one of the first to express the necessity of international collaboration beyond the bibliographical level. He was a fervent advocate of the idea of a mathematical society in Germany and proposed in 1888 that ‘German and French mathematicians should meet at a neutral site’ [55, p. 3], e.g. in Belgium, Switzerland or the Netherlands.

Leaving international cooperation aside for a moment and looking at the state of mathematical cooperation in Germany itself, it is clear that Cantor’s

ideas were in accordance with the spirit of the time. Until the 1890s, there were hardly any opportunities for German-speaking mathematicians to cultivate friendships amongst each other. A few meetings of societies such as the *Gesellschaft deutscher Naturforscher und Ärzte*³ included mathematical sections where they could present their work. However, in Jena in 1890, a number of mathematics and science teachers founded the *Verein zur Förderung des Unterrichts in der Mathematik und in den Naturwissenschaften*⁴. This had become necessary because of attempts to reform higher education at the time. Furthermore, mathematics and science teachers wanted to stand their ground against the interests of the arts teachers [10, p. 257].

In the same year the German Mathematical Society was founded and Cantor became its first president. At the time he already had the idea of an international congress of mathematicians. At first, he was not taken very seriously by some of his colleagues, as is shown by a letter that Walther von Dyck wrote to Felix Klein in August 1890:

Recently G. Cantor wrote me about very high-flying plans regarding international congresses of mathematicians. I really do not know whether that is a real need.

[55, p. 3]

From 1894-1896, Cantor was in correspondence on the subject of international congresses with a number of mathematicians, including Aleksander Vasilyev, Charles Hermite, Camille Jordan, Henri Poincaré, Charles-Ange Laisant, Émile Lemoine, Klein and von Dyck. Cantor argued that a congress would serve as a much-needed international forum where the ever-growing mathematical community could present and discuss their work without prejudice. He himself needed such a forum to present his work, as not all his German colleagues approved of his new and radical ideas in set theory. The fact that he began to stress his non-German origin – his father came from

³ “Society for German Natural Scientists and Physicians”

⁴ “Society for the Promotion of Mathematics and Science Education”

Denmark, and Cantor himself was born in St Petersburg – made him fall out of favour with the German mathematicians even more.

One of the German mathematicians who did not see eye to eye with Cantor was Klein. However, he recognised the need for international cooperation when he attended the Congress of Mathematicians and Astronomers in Chicago in August 1893. It was one of the satellite conferences held on the occasion of the Chicago World's Columbian Exposition, organised in order to celebrate the 400th anniversary of Columbus's discovery of America. The mathematical congress had 45 participants, four of whom came from countries other than the United States. These four international mathematicians were all Europeans. In fact, the centres of mathematics were all European at this time, 'yet a mathematical conference as early as 1893 with participants from two continents was a historical event' [55, p. 5].

Klein went to Chicago in his capacity as Imperial Commissioner of the German Emperor Wilhelm II. He took with him papers of several of his colleagues and also gave an opening address, *The Present State of Mathematics*. In his speech he pointed out the threat to mathematics of being split into different branches, the necessity of international collaboration and the benefits that mathematical societies brought to mathematics. He said that mathematicians 'must form international unions, and I trust that this present World Congress [...] will be a step in that direction' [49, p. 135].

Klein and Heinrich Weber became the leading figures in organising an international congress on the German side. They got much more support from their peers than Cantor had received a few years previously, as German mathematicians expressed the wish for an international congress of mathematicians to be organised, particularly 'in view of the successes achieved by international communication in other areas of science' [10, p. 258]. However, nothing was done about organising such a congress: In 1895, the German Mathematical Society claimed to support the idea of an international congress in principle after French mathematicians had presented the idea to their German colleagues at the society's annual meeting the year before, but

they refused to organise it [55, p. 7]. As for Cantor, he eventually abandoned the project, probably due to the fact that his ideas had met with so much resistance. He did attend the first congress though.

Two of Cantor's most supportive correspondents on the subject of international congresses were the French mathematicians Charles-Ange Laisant and Émile Lemoine. They presented the idea of an international congress in the first volume of their journal *L'Intermédiaire des mathématiciens* and explained that it came from both French and foreign mathematicians. Besides Laisant and Lemoine, Cantor could also claim Poincaré's support [28].

The idea that an international congress should be organised began to spread across Europe and beyond from 1894 onwards. The French and the American mathematical societies backed the idea of an international congress, but neither offered to organise it. It was agreed, however, that the congress should be permanent, be held at regular intervals of three to five years and follow a number of rules that were to be established. The French Mathematical Society at least declared that it would support a trial congress.

4.1.2 Organising the Congress

Cantor had proposed that such a trial should be held in a neutral country, Switzerland or Belgium, in 1897, and that the first actual congress should be held in Paris in 1900. The choice of the host country remained open for quite a while, but in December 1895 it became clear that Switzerland was preferred over Belgium, especially 'in view of the Swiss tradition of promoting international interests' [57, p. 7]. Moreover, Klein and Weber suggested that the congress should be held in Zurich. The presidents of the German and French mathematical societies approved of this suggestion and contacted Carl Friedrich Geiser at the Federal Polytechnic in Zurich, as Geiser himself explained in a letter to all his colleagues in Zurich, inviting them to a preliminary meeting on 21 July 1896 [8k]. The presidents made a very good choice in approaching Geiser, as 'in addition to many leaders in politics and economics [he] knew almost all important mathematicians of this time in

Germany, France and Italy in person; he was even friends with many of them' [59, p. 372]. Furthermore, he 'had proven himself to be very skilled in administrative matters, in particular in his capacity as Director of the Federal Polytechnic' [33, p. 1] (see sections 2.1. and 2.3). The fact that Geiser was in Zurich might not have been the primary reason for asking the Swiss to organise the congress, however it seems to have been a contributing factor rather than just an added bonus.

The mathematicians who attended the preliminary meeting in July 1896 unanimously decided that they would host an international congress. An organising committee was elected, comprising the Polytechnic professors Geiser, Ferdinand Rudio, Adolf Hurwitz, Jérôme Franel, and the assistants Gustave Dumas and Johann Jakob Rebstein as secretaries. Geiser was elected president. Out of the committee members, Rudio in particular made a name for himself in helping to organise the congress. He edited the congress minutes and was involved in drafting the congress regulations with Geiser. Moreover, he became one of the driving forces behind the publication of Euler's complete works (see sections 4.1.5 and 3.2, respectively).

At the first meeting in November 1896, Hermann Minkowski and the Polytechnic's Director Albin Herzog joined the committee. It is worth noting that the organisation of the congress was completely in the hands of mathematicians at the Polytechnic to begin with. Frei and Stambach claim that the reason for this was that the University of Zurich was 'not very well-staffed' in mathematics at the time [33, p. 1]. Eduard Gubler and Heinrich Burkhardt from the University of Zurich joined the organising committee in December 1896 and January 1897, respectively. The organising committee grew throughout the months leading up to the congress; new members were lecturers at the Polytechnic (e.g. Arthur Hirsch and Marius Lacombe) and mathematics teachers at secondary schools in Zurich (most importantly Walter Gröbli and Fritz Bützberger).

Geiser had contacted various mathematicians after the preliminary meeting in July 1896 and asked them for opinions and suggestions. At the first meeting

of the organising committee (at that time still consisting of seven members) on 12 November 1896, Geiser could report that 'the idea was received favourably everywhere where the calling of a congress had been announced' [8a]. Then Rudio gave an account of the meeting of the German Mathematical Society's executive committee in Frankfurt in July to which he had been invited. He brought back a number of valuable suggestions concerning the date and duration of the congress and its financing, as well as publications associated to the congress and the invitations. The Germans also requested that the congress should cover only developments in more general areas of mathematics such as bibliography rather than being 'a collection point for talks and communications' [ibid.].

In accordance with the wishes expressed by the majority of mathematicians the committee decided that the congress should be held from 09-11 August 1897. The French *Société pour l'avancement des sciences* had a meeting at roughly the same time, but the Zurich committee decided that the three days they had designated would be the most suitable [8a; 8d].

The committee then decided that there should be three general meetings and a number of individual sections. The general meetings were to provide an opportunity for discussing business matters and for four talks 'of a more universal significance for which specific invitations would have to be issued, in particular with regard to the international nature of the congress' [73, p. 4]. Furthermore there were to be a number of sections for subject-specific talks. The format of the congress was very similar to the format of scientific meetings at the time; in the proceedings Rudio points out that the meetings of the *Schweizerische Naturforschende Gesellschaft* served as a particular inspiration [ibid.]. The committee also decided to publish the congress proceedings after the congress, but refrained from releasing a celebratory publication before the congress due to financial reasons.

Following a proposal by the German Mathematical Society, the committee chose to send out the invitations to the congress to individual mathematicians rather than to mathematical societies. Moreover, the Germans recommended that the organising committee should be enlarged by a number of foreign mathematicians and nominated Klein as their representative. The Zurich

committee then decided that Geiser should invite the following mathematicians to join the enlarged committee: Gösta Mittag-Leffler (Sweden) for Scandinavia, Henri Poincaré for France, Luigi Cremona for Italy, Franz Mertens for Austria and Andrej Markov for Russia. Klein was commissioned to designate representatives for the UK and the USA. His choices, announced at the committee's next meeting in December 1896, were Alfred George Greenhill and George William Hill.⁵

Thus, the invitations to the congress that were sent out to 2000 mathematicians and mathematical physicists all over the world in January 1897 were signed by an impressive list of mathematicians, 'comprising [...] 3 categories, the "Zurichers", the "Swiss" (beyond Zurich) and the "foreigners", in total 21 members' [8c]. Most of the international members on the inviting committee seem to have been chosen due to their reputations in the mathematical community. Their names were well known, therefore adding weight to the invitations as well as emphasising the international nature of the congress. Moreover, they had excellent contacts with most of the mathematicians in their respective countries; Hill for example was the president of the American Mathematical Society at the time. Most of the members of the inviting or international committee helped to distribute the invitations in their respective countries. They were joined by mathematicians in countries that were not represented on the committee, including Paul Mansion in Belgium, Pieter Hendrik Schoute in the Netherlands and Cyparissos Stéphanos in Greece. These mathematicians also attended the congress, whereas most of the foreign members on the inviting committee did not. Poincaré, who had been invited to give one of the plenary lectures, had to cancel a few days before the congress due to a family bereavement [8i; 10, p. 260]. As for the others, there are no records in the committee minutes as to why they did not attend. In fact, the committee decided in late July that 'an autograph letter should be addressed to Mr Greenhill, in which he is asked to

⁵ As for the choice of Greenhill, the German and French versions of the minutes differ: According to the French version [81], Klein suggested that Greenhill should be the British representative on the committee, but according to the German version [8b] Klein only nominated Hill as the American representative and recommended that the organising committee should contact the Mathematical Society in London.

attend the congress in his capacity as signatory of the invitation' [8g]. Apart from distributing the invitations, the members of the international committee were invited to give their opinions on the future format of such congresses.

Rudio had proposed in December 1896 that the organising committee should elect four sub-committees with three members each. These sub-committees dealt with the following areas: finances (president: Gröbli), board and lodging (president: Rudio), amusement (president: Herzog) and reception (president: Hurwitz) [8b]. The amusement and reception committees immediately set to work and drafted a congress programme. A preliminary programme was attached to a second invitation that was sent out to all mathematicians in May 1897, reminding them of the congress and asking them to return their applications by 01 August.

The attendance fees⁶ paid by the participants and the accompanying ladies covered about half of the total cost of the congress, which amounted to 11,243.35 Franks [8j]. The rest was paid for with subsidies from the Swiss government and the municipalities of both the canton and the town Zurich, as well as with donations from individuals (mostly industrialists and merchants) and the Polytechnic's alumni society *Gesellschaft ehemaliger Polytechniker* [8j]. Here Geiser's and also Herzog's excellent contacts proved very useful. In addition to sending official petitions, they met their friends in the relevant authorities, which Rudio did not approve of – he would have preferred to use the official channels only [8d].

Most of the congress took place in the buildings of the Polytechnic. Curiously, there are no references to the congress whatsoever in the School Board meetings [93]. Admittedly, its President, Bleuler, was a member of the organising committee, but one would expect that the committee had to apply to use the rooms. There are also very few references to venues in the committee minutes. It is possible that these things were organised outside of

⁶ Attendance fees were 25 Franks for participants and 15 Franks for accompanying ladies [8m]. The fees covered admittance to all general and all section meetings as well as all the banquets and outings in the official congress programme.

the official meetings. The cashbook lists compensations to a number of staff members, probably the Polytechnic's janitors and secretaries [8j; cf. also 8f; 8h].

4.1.3 The Congress Itself

On the evening before the congress, on Sunday 08 August, the international inviting committee met in order to discuss several administrative matters. However, only eleven members (out of 21) attended this meeting: Geiser, Bleuler, Dumas, Franel, Hirsch, Klein, Mertens, Minkowski, Mittag-Leffler, Rudio and Von der Mühl. Furthermore, Brioschi, Laisant, Vasilyev and Weber attended the meeting upon special invitations [73, p. 13].

The committee discussed and eventually approved the congress programme and the agenda items that had to be presented to the congress participants. This included the regulations on which the congress was to be based and a number of resolutions, which had to be adopted by the participants at one of the general meetings. Geiser had drafted both the regulations and the resolutions. Laisant had devised a very detailed organisation plan and it seems that the committee used some of his suggestions when drafting the regulations. According to the regulations, a congress executive committee was to be elected at the first general meeting, consisting of a president, two general secretaries (one native German speaker and one native French speaker) who were also the official translators, four secretaries (one each for German, French, Italian and English) and eight ordinary members⁷. Suitable candidates were nominated at the meeting of the international committee on the Sunday, the choices were confirmed by the congress participants the day after. Not surprisingly, Geiser was elected president and Rudio and Franel became the general secretaries⁸.

⁷ Art. 3 of the congress regulations [73, p. 14]

⁸ The secretaries were Eduard von Weber (German), Émile Borel (French), Vito Volterra (Italian) and James Pierpont (English). The ordinary members were Nikolai Bugaev, Francesco Brioschi, Felix Klein, John Sturgeon Mackay, Gösta Mittag-Leffler, Émile Picard, Henri Poincaré and Heinrich Weber. As Poincaré could not attend the congress, Franz Mertens was elected as a ninth member. Cf. [8n] and [73, p. 30].

The reception committee spent the entire Sunday at the train station, welcoming the mathematicians, 'many of whom were fortunately also accompanied by their ladies' [73, p. 22] and issuing the congress cards and vouchers for the banquets and the outings. In addition, each participant also received either French or German copies of the programme, the regulations and the resolutions, as well as an illustrated guidebook of Zurich, published by the official transport committee of the town Zurich.

This was the congress programme [73, p. 17-18]:

Sunday 08 August

- Arrival
- Reception and refreshments in the Tonhalle

Monday 09 August

- First general meeting
- Banquet
- Steamboat outing to Rapperswil⁹

Tuesday 10 August

- Section meetings

Wednesday 11 August

- Second general meeting
- Banquet on the Uetliberg¹⁰

⁹ Rapperswil in the canton St Gallen is a municipality situated on the northern shore of Lake Zurich. According to the proceedings it took the congress participants a little more than an hour to get there by steamboat. The boat was to be met by an illuminated gondola parade when approaching Zurich in the evening, but the parade had to be cancelled due to bad weather. However, many official and private buildings on the lakefront were illuminated [73, p. 44].

¹⁰ Zurich's local mountain, accessible by train and a popular destination for a day out.

Geiser officially opened the congress at the first general meeting the next morning. The two plenary speakers were Poincaré (his paper was read out by Franel) and Hurwitz. In addition, Rudio spoke *Über die Aufgaben und die Organisation der internationalen mathematischen Kongresse*¹¹. He presented the resolutions prepared by the organising committee and gave examples of areas where collaborations between mathematicians of various countries were not only possible but in fact necessary, such as a mathematical bibliography and publishing the complete works of Euler (see section 3.2).

The plenary speakers had been chosen by the organising committee, or, more precisely, by a sub-committee that was formed in February 1897 and comprised Geiser, Hurwitz and Minkowski. Amberg and Franel were assigned to it later on. The task of this sub-committee was choosing the speakers for the general meetings and for the sessions of the individual sections. For the general meetings, they were looking for ‘general talks by men whose names would have a certain ring to them’ [8e]. After some debate and some re-scheduling, the four plenary speakers were Henri Poincaré and Adolf Hurwitz at the first general meeting, as mentioned above, and Giuseppe Peano and Felix Klein at the second meeting. As for the individual sections, Geiser approached a number of mathematicians directly (including all members of the international committee), asking them whether they were interested in giving a talk or whether they could recommend any colleagues. The five sections were:

- Algebra and Number Theory
- Analysis and Theory of Functions
- Geometry
- Mechanics and Physics
- History and Bibliography

¹¹ “On the Duties and the Organisation of the International Mathematical Congresses”; cf. appendix E.1.5.

A total of 24 talks were given in the sessions of these sections. Comparing this to the 21 plenary lectures and the approximately 180 invited talks in 19 different sections scheduled for the next ICM in Seoul in 2014 [86], one can see that the ICMs have come a long way since the very first one in Zurich. Incidentally, the original intention was not to count the Zurich congress at all, but to regard it as a trial congress and then count the Paris congress in 1900 as the first proper one. However, partly due to the great success of the Zurich congress it is regarded as the first ICM¹².

Admittedly, the organisers of the ICM in Seoul can expect several thousand participants. The Zurich congress had 242 participants in total, of which 38 were ladies. Most of these ladies were the wives of the participating mathematicians, or their daughters. Geiser for example was accompanied by his wife Emma and two of his daughters, Charlotte and Hedwig [73, p. 68]. Rudio's wife Maria attended, too, as did a 'Miss Elisabeth Rudio' from Wiesbaden – maybe an unmarried sister or aunt who came to visit [73, p. 74]. Only four female mathematicians attended the congress¹³, which is not surprising given that the congress was held at the end of the 19th century. However, the congress organisers advanced a more modern view on female students than many of their international colleagues. Charlotte Angas Scott wrote to Wilhelm Fiedler in 1897, asking him 'whether ladies will be welcome – as mathematicians, of course?' [17]. Boesch Trüb notes that Fiedler, who neither organised nor attended the congress, could give her a positive answer, and suggests that this was due to the fact that women had been allowed to study in Zurich for several decades already¹⁴ [ibid.].

¹² However, with the exception of the first few, ICMs have not been numbered due to the controversy surrounding exclusion policy at the 1920 and 1924 ICMs, which barred mathematicians from the former Central Powers in WWI from attending these ICMs [27, p. 19-21].

¹³ They were Iginia Massarini (Rome), Vera von Schiff (St Petersburg), Charlotte Angas Scott (Bryn Mawr), and Charlotte Wedell (Göttingen). However, none of them gave a talk – in fact, the first woman to give a talk at an ICM was H P Hudson in Cambridge in 1912 [26, p. 52].

¹⁴ The University of Zurich first admitted female students in the 1860s; it was the first state-accredited university in the world to award a degree to a woman (in 1867 – in contrast, women were given the right to vote on a national level only in 1971!). Most higher education institutions in Switzerland followed suit, but numbers remained

The time of the congress also explains the fact that the vast majority of the attendants were European. At the time, the major centres of mathematical research were at European universities, two prominent examples being Paris and Göttingen. Sixteen countries were represented at the ICM, with Switzerland, Germany, France and Italy accounting for roughly two thirds of the male participants. Whilst the organising committee coordinated the date of the congress with meetings of German and French scientific societies, it could not consider every country. In the same year there was a congress in Kiev that most Russian mathematicians would have attended [81]. Furthermore, the British Association of Mathematicians had a meeting in Canada [ibid.], which might explain the surprisingly low number of British participants¹⁵.

Most of the male attendants were mathematicians and mathematical physicists who held lectureships or professorships at university, but the list of participants also includes a relatively large number of secondary school teachers, as well as a few publishers and representatives of various Swiss authorities.

Although the number of participants at ICMs (and hence the number of talks), and the number of countries represented by said participants have increased considerably since 1897, the regulations on which the congresses are based have not changed all that much since then. Of course, they have been edited and amended over the decades, in particular as the congresses are now organised under the auspices of the International Mathematical Union (IMU). The IMU's *Guidelines* are more detailed than the regulations on which the Zurich congress was based; the committees now have to consider gender balance and an appropriate distribution of countries, in particular developing countries, when inviting speakers, etc. But the essence of those guidelines is

low for several decades, particularly at such a technology-oriented institution as the Polytechnic (where the most popular subject for women was pharmaceuticals). Furthermore, most female students were foreigners – but then girls at state schools in Zurich were only allowed to take their Matura exams from about 1900 onwards [39, p. 114-118].

¹⁵ The three British attendants were the Cambridge lecturers Ernest William Hobson and Joseph Larmor, and the schoolteacher John Sturgeon Mackay from Edinburgh.

still the same as that of the 1897 regulations: that the congresses should provide mathematicians from all over the world with an opportunity to get to know each other and to discuss mathematical questions, regardless of their nationalities [cf. 47]. So, with the exception of part d), one could say that the first article of the regulations of 1897 is still valid today:

Art. 1

The congress has the purpose of:

- a) Furthering the personal relations between the mathematicians of various countries;
- b) Providing [...] an overview over the current state of the various fields of mathematical sciences and their applications, as well as the treatment of individual problems of particular importance;
- c) [Discussing] the tasks and the organisation of future international congresses;
- d) Preparing a solution for problems on bibliography, terminology etc., which require international cooperation.

[73, p. 14]

Another part of the regulations which is of particular interest is the following, as it highlights both the international nature of the congress and the fact that the host country was Switzerland [ibid.]:

Art. 4

The official publications of the congress are to be in German and French. In the main meetings and the sessions of the individual sections, votes and talks in Italian or English are permitted as well.

Despite the second part of the article, German and French were predominant. Two talks were given in Italian, and one talk was scheduled in

English¹⁶. However, due to popular demand and the fact that hardly any native English speakers attended the congress that talk was given in German [73, p. 45]. Given the tense political situation between Germany and France the all-round bilingualism of the congress was probably a very wise choice. The organising committee saw to a fair distribution of languages with regard to the talks and ensured that German and French versions of every printed matter relating to the congress were available. Of course, this came very naturally to the Swiss: their country was multilingual, as was the committee itself.

As mentioned above, the organising committee was keen to plan the social programme of the congress. On 31 January 1897, a slightly disgruntled Minkowski wrote in a letter to David Hilbert [33, p. 3]:

The schedules for the outings etc. at the congress have already been drafted; here too the scientific part comes last again.

Of course, the committee had to start work somewhere, and without a doubt they felt very honoured that Switzerland had been chosen as the first host country. Offering the participants a range of opportunities to explore Zurich and its surroundings probably also helped to get funding from cantonal and federal governments, as it was a chance to promote the 'tourist region Switzerland' [80]. But the minutes of the committee meetings and the speeches given at the congress itself suggest that the social aspect was prioritised, and that the mathematical part was almost thought of as taking care of itself. This is nicely illustrated by Hurwitz's welcoming speech at the reception on 08 August: Rather than talking about mathematical collaborations and mathematics in general, he emphasised the social side of the congress, stressing the 'hermitic' [73, p. 23] mathematician's need to talk to colleagues. Apart from having the opportunity to discuss scientific problems, he hoped that the congress participants would 'enjoy the cheerful and informal company of [their] peers, enhanced by the knowledge that representatives of

¹⁶ *On Pasigraphy, its Present State and the Pasigraphic Movement in Italy* by Ernst Schröder from Karlsruhe

various nations feel connected in peace and friendship by the most ideal interests' [ibid.; cf. appendix E.1.2]. Hurwitz considered these "most ideal interests" to be the search for knowledge and scientific truth, rather than political or economic interests. Westermann suggests that the conference attendees 'presented and affirmed a certain image of a mathematician and scholar to one another' [80].

Similarly, Rudio claimed in his talk on 09 August that 'international congresses of mathematicians would have a right to exist even if their only purpose was to bring mathematicians of all countries of the Earth closer together' [73, p. 32-33]. The importance of personal relations was stressed in practically every speech given at the congress. Mathematicians were distinguished from one another only by their mathematical preferences but not by their nationalities. Both the congress proceedings (including the speeches) and the organising committee's minutes imply subtly that one of the objectives of the congress was to overcome, or to attempt to overcome, animosities between French and German mathematicians. Although mathematical collaborations were discussed, e.g. publishing Euler's works, the predominant opinion at the time seemed to be that real mathematical progress could only be achieved by individuals. Geiser nicely explains this in his opening speech: 'Surely none of us will believe that in future the solution of great problems in science will be the result of such meetings' [73, p. 27]. But despite (or possibly because of) the solitary nature of mathematical research, great value was attached to exchanging ideas and establishing friendships with other mathematicians. As the European countries became more and more imperialistic and also nationalistic towards the end of the 19th century, this was all the more important. Klein summed up his feelings in his plenary lecture [73, p. 300]:

The mathematical congress is drawing to a close. Although it is too early to discuss its results, we may express a sensation that dominates each and every one of us. I am talking about the overwhelming impression of the variety of mathematical views and interests, which greatly hinders communication [between mathematicians]. The

diversity of languages almost pales in comparison to the diversity of mathematical mindsets.

And yet we all feel the desire for communication equally strongly. There is no better proof of this than the number of peers who have gathered for this first international congress. We will try to consider our science as a great entity, as a harmony; not just for the sake of philosophical knowledge, but also from a practical point of view: we have to defend and often regain the importance of our science.

However, despite this appeal to unity, Klein was also very quick to secure the 1904 ICM for Germany after France had been approved to host the 1900 congress. It was not so easy to distinguish mathematics from politics after all: 'In the cause of the universality of scientific knowledge, the mathematicians worked on an international standardisation of their terminology, and stressed the national research contributions at the same time' [80].

In a nutshell, it can be said that the first international congress of mathematicians was a success. It paved the way for future congresses, and the fact that the ICMs are not only still held today, but have increased significantly in size, importance and popularity since 1897 is a tribute to the work of the organising committee. Geiser could not have foreseen such a development, he could only have hoped for it when he bid farewell to the congress participants at the second general meeting on 11 August 1897 [73, p. 60]:

And if, at the end of this lovely day, I call out a cordial farewell to you all on behalf of my colleagues in Zurich, then I may also assume to speak in accordance with the kind invitation of our peers from France when I add:

Auf Wiedersehen in Paris – See you in Paris!

4.1.4 Geiser's Contribution

As discussed above, Geiser's reputation contributed to the choice of Zurich as the first ICM host. In July 1896 he invited¹⁷ mathematicians in Zurich to the preliminary meeting in the Polytechnic, where it was decided that the congress should be held in Zurich and where the organising committee was elected. It was almost self-evident that Geiser chaired the committee.

As president, Geiser chaired the meetings of the organising committee and coordinated the work of the four sub-committees. He regularly updated the committee on any developments and funding that they had received. At the first meeting he proposed the date of the congress, and that the proceedings should be published [8a]. He also requested the meal on 11 August [8c], devised the seating plan for the banquet [8i], and was involved in confirming the congress programme. Furthermore, he was the committee's main contact person and corresponded with a number of mathematicians and officials on the subject of the congress. As an example, he invited the members of the international committee [8a; 8b]. Moreover, it seems that he used his contacts to secure a grant from the government [8d].

Together with Hurwitz and Minkowski, and also Amberg and Franel, Geiser formed the sub-committee that chose the speakers and determined the different sections¹⁸. In this capacity, Geiser invited mathematicians to give talks. He also suggested that the sub-committee 'ensures that talks are also given in languages other than German' [8e].

Geiser and Rudio became the two main organisers. The minutes suggest that they disagreed on several occasions: establishing the sub-committees [8b], choosing a publisher [8c] and applying for funding (see section 4.1.2), for example. However, it seems that they worked well together otherwise. Amongst other things, they were both involved in drafting the congress regulations.

The congress committee was elected in the first general meeting, and Geiser was elected president 'by acclamation' [73, p. 30]. He delivered the opening

¹⁷ Unfortunately, the letter does not specify which mathematicians received it. See appendix E.1.1.

¹⁸ This sub-committee was formed upon Rudio's suggestion.

and closing speeches (see appendices E.1.3 and E.1.4), but he did not give a mathematical talk. In his addresses he praised the beauty of his home country and its famous mathematicians, and emphasised the importance of personal relations between mathematicians of various countries. Like Rudio and Hurwitz, he viewed the congress as an opportunity for celebrations and a 'social event, memorable in its own right' [80].

Moreover, he chaired the meeting of an international organising committee on 08 August, in which the congress regulations and resolutions were approved, and the congress committee was suggested [8n] (see section 4.1.3). According to the minutes, he also chaired a meeting of all individual section chairs [8g], but did not chair a section himself.

Geiser was one of the vice-presidents of the 1900 ICM in Paris, and attended the subsequent 1904 ICM in Heidelberg together with one of his daughters. He was a member of the international committee of the 1912 ICM in Cambridge, but did not attend the congress. He only signed the invitation and would have been involved in distributing them. In his late eighties he still attended the 1932 ICM in Zurich, again with one of his daughters. He was a member of the congress's honorary committee. To this day, Geiser's name remains connected to the first ICM in Zurich.

4.1.5 Rudio's Contribution

Rudio made a name for himself in the international mathematical community through his involvement with the first International Congress of Mathematicians. Having joined the organising committee at the preliminary meeting in July 1896 he quickly became one of the main organisers beside his colleagues Geiser and Hurwitz. Rudio had the chance to talk to German mathematicians at the annual meeting of the German Mathematical Society and brought back a number of valuable suggestions concerning the date and duration of the congress, invitations to the congress and fundraising. He proposed in December 1896 that the organising committee should elect four sub-committees, dealing with the following areas: finances, board and lodging

(of which he was president), amusement, and reception [8b] (see section 4.1.2). Furthermore, he suggested that a special sub-committee should choose the congress speakers; and that the alumni society should be asked for financial support. When the committee discussed the social programme of the congress, he was often the spokesperson for the amusement and board & lodging committees, and he was heavily involved in organising the outings and the collation on the Sunday evening. Moreover, he set the price of the ladies' tickets and suggested that mathematicians across the world should be asked for their ideas on the congress, for future reference. He was also responsible for organising the section on History and Bibliography; at the congress he chaired this section. He was also scheduled to give a talk in this section, on the Second International Bibliographic Congress, which took place in Brussels from 02-04 August 1897 and which he had intended to attend. However, 'the preparations for the Zurich congress, which demanded his attention completely even in the last few days, contrary to his expectations, prevented him from going to Brussels and thus made the talk obsolete' [73, p. 54].

Rudio drafted most of the committee's communications, such as the official invitations and circulars to the international committee. As general secretary of the congress he edited the congress proceedings. Pierpont writes in his review of the proceedings [69, p. 486] that:

Mathematicians will be grateful to Professor Rudio for this very complete and attractive report. The book contains so much of general interest that it will be welcome to all.

At the first general meeting on 09 August 1897 Rudio spoke *Über die Aufgaben und die Organisation der internationalen mathematischen Kongresse* (see appendix E.1.5). He presented the congress resolutions and gave examples of areas where collaborations between mathematicians of various countries were necessary, such as a mathematical bibliography. Another example was the publication of Euler's complete works, a project that occupied him throughout the rest of his life (see section 3.2).

Rudio was a vice president of the 1912 ICM in Cambridge. Rather surprisingly, this is the only other congress that he attended – perhaps his numerous duties and his work on the Euler edition did not allow him to attend any of the other congresses. However, in Cambridge he reported on the history and progress of the Euler edition in his talk *Mitteilungen über die Eulerausgabe*¹⁹ (in section IV (b) – Didactics), thus returning to his 1897 ICM talk.

4.2 The Swiss Organising Committee

At the initial meeting of all mathematicians in Zurich on 21 July 1896, when it was decided that they would host the first international congress of mathematicians, an organising committee was elected, consisting of ‘Messrs Prof. Geiser, Franel, Hurwitz, Rudio, Prof. Weber, assistant Dumas and Dr. Rebstein’ [8a]. During the subsequent months, the committee grew continuously; most of the committee members lectured at the Polytechnic, but some were lecturers at the University of Zurich or secondary school teachers. Whilst some of the committee members became well-known mathematicians (or already were at the time), most of them remained relatively unknown even during their lifetimes, and are certainly unknown today. As well as giving a short biography of each committee member, I will also highlight the role they played in organising the congress as is evident in the minutes of the organising committee.

4.2.1 Ernst Julius Amberg (1871 – 1952)

Ernst Julius Amberg was born in Zurich on 06 September 1871. His parents both descended from old farming families, but moved to Zurich after their wedding. In one funeral speech [78, p. 6] a brother, Heinrich, is mentioned, but we do not know of any other siblings. Amberg attended the Gymnasium, being top of the class in his Matura examinations. He studied mathematics at

¹⁹ “Notes on the Euler Edition”

the Polytechnic and obtained a diploma as a mathematics teacher in 1894. Three years later, in 1897, he received his doctorate from the University of Zurich for his thesis *Über einen Körper, dessen Zahlen sich rational aus zwei Quadratwurzeln zusammensetzen*²⁰. His supervisors were the astronomer Alfred Wolfer (1854-1931) and Adolf Hurwitz. In 1947, his doctorate was renewed to celebrate its 50th anniversary²¹.

At the time of the ICM Amberg was an assistant at the Polytechnic, before becoming a teacher at the Kantonsschule in Frauenfeld (canton Thurgau). In 1903 he became Gröbli's successor at the Gymnasium in Zurich, teaching mathematics. He held this post until 1938; he also served as the school's director from 1916-1938. During the Second World War, so a few years after his retirement, he worked as a substitute teacher in various Gymnasiums.

Amberg, the 'small, stocky mathematics teacher with firm footsteps' [46, p. 12] seems to have loved teaching, and wanted to make mathematics accessible to weaker pupils, too. While his classes are praised, 'the pupils, on their part, probably were a bit frightened of the strictness of their rector' [46, p. 11]. Among his pupils was Albert Einstein's second son Eduard. In a letter to his father from September 1921, concerning their holiday, Eduard comments that 'the rector [Amberg] is not amused about this kind of skiving' [52, p. 270]. In a letter to his wife from May 1901, Albert Einstein himself comments that Amberg recommended him as a teacher to Jakob Rebstein²², who seems to have asked Einstein to come to teach at the Polytechnic [13, p. 170].

Before starting his teaching post in Zurich, Amberg also took on a job as an actuary at a life insurance company. He worked for insurance and reinsurance companies alongside his school duties throughout most of his life, and continued to write expert opinions even after his retirement.

²⁰ "On a Solid Whose Numbers Are Rationally Composed of Two Square Roots"

²¹ At some universities in Germany, Switzerland and Austria, faculties can nominate certain PhD certificates to be renewed after 50 years. This entails an award ceremony, so can be seen as honouring the recipient's work since completing their PhD.

²² Not the Jakob Rebstein on the organising committee, though this Rebstein also worked at the Polytechnic. See also section 4.2.18.

In 1912 he also became assistant professor for mathematics and analytic geometry at the ETH, and was promoted to Titularprofessor in 1918. Furthermore, he also lectured on teaching skills for mathematics both at the ETH and at the University of Zurich. Amberg lectured at the ETH until his retirement in 1938.

It seems that he did not publish much apart from his thesis. The ETH-Library holds an expertise that he wrote in 1906: *Finanzielle Tragweite einer Alters- und Invaliden-Versorgung der Beamten und Angestellten der Stadt Zürich*²³. He seems to have been a member of a committee investigating general education (the committee's report was published in the *Schweizerische Bauzeitung* in 1951). In 1933 he published an article on how future mathematics teachers were educated at the Polytechnic and the University of Zurich.

Amberg was a keen mountaineer and a member of the Swiss Alpine Club (SAC) for 61 years. For six years he served as president of the Uto section, i.e. the Zurich section of the SAC. Together with Anton Züblin he was the first to climb Piz Gannaretsch (3040m) and Piz Vatgira (2983m), both in canton Grisons. Furthermore, he also served as an officer in the Swiss army.

He was married, but did not have any children. Ernst Amberg died on 15 March 1952.

Amberg joined the organising committee in November 1896: Rebstein, the German-speaking secretary, could not attend the meeting on 12 November and Amberg covered for him. He joined the committee then. In May 1897 he joined the sub-committee that chose the plenary speakers and ensured a fair distribution of languages. This sub-committee already consisted of Geiser, Hurwitz and Minkowski, and Franel joined together with Amberg. At the meeting on 27 July 1897 Amberg was elected secretary, thus replacing Rebstein who had to step down from his post due to military service. Amberg's deputy was Hirsch. Furthermore, Amberg was in charge of setting

²³ "Financial Consequences of an Old-Age and Disability Pension Scheme of the Civil Servants and Employees of the Town Zurich"

the fees²⁴. He did not give a talk at the congress, but was elected secretary of section I: Arithmetic and Algebra.

As headmaster of the Gymnasium Zurich Amberg was an official Swiss delegate at the 1932 ICM in Zurich. Furthermore, he was one of the Swiss delegates at the ICMI meeting in the same year [87].

4.2.2 Christian Beyel (1854 – 1941)

Christian Beyel, from Zurich, was born on 29 November 1854. His father, Christian Melchior Beyel, was a bookseller. Beyel junior studied at the Engineering Department of the Polytechnic from 1872-1876. He worked as an engineer for the Swiss North-Eastern Railways (Schweizerische Nordostbahn) for a year, but moved to Göttingen in 1877 in order to do further studies in mathematics. A year later he returned to the Polytechnic as the assistant of W Fiedler and Wilhelm Ritter²⁵. He also matriculated at the University of Zurich in 1878, but seems to have left it soon after, according to the matriculation register [91].

In January 1882 Beyel received his doctorate from the University of Zurich for his thesis *Centrische Kollineation nter [sic!] Ordnung in der Ebene vermittelt durch Ähnlichkeitspunkte von Kreisen*²⁶; it was re-conferred to him on the occasion of its 50th anniversary. In 1883 he habilitated at the Polytechnic, as Privatdozent. He held his post until 1834, lecturing mainly on geometry, in particular projective geometry. Beyel was a member of the German Mathematical Society.

In 1889 he married Lydia Magdalena Schalch (1860-1946). Their son Franz did a PhD in German literature and worked as a teacher in Basel.

²⁴ The committee minutes do not explain which fees were meant, but presumably they refer to the prices of the individual tickets for the social events.

²⁵ Karl Wilhelm Ritter (1847-1906): Professor of graphic statics, and bridge and railway construction. He served as Director of the Polytechnic from 1887-1891 [84].

²⁶ "Centric Collineation of n^{th} Order in the Plane by Means of Similarity Points of Circles"

Beyel was a prolific writer. His most important book is *Der mathematische Gedanke in der Welt. Plaudereien und Betrachtungen eines alten Mathematikers*²⁷ (1922), in which he writes about mathematical topics and mathematics in everyday life for lay people. Reprints of this book are still sold today; a review [94] describes it as ‘a declaration of love to mathematics’. Beyel also wrote a number of geometry books, such as *Geometrische Studien*²⁸ (1886) and *Darstellende Geometrie: Mit einer Sammlung von 1800 Dispositionen zu Aufgaben aus der darstellenden Geometrie*²⁹ (1901). The latter is a concise collection of the basic principles of orthogonal parallel projection intended for students; it is based on Beyel’s lectures [56]. Moreover, Beyel published numerous papers on geometry: Two examples are *LVII Sätze über das orthogonale Dreieck*³⁰ (1889) and *Der kubische Kreis mit Doppelpunkt*³¹ (1897).

In addition to his mathematical publications he wrote several articles on the state of the cinema, on political issues, and against immoral literature. He was one of the founders of the *Schweizerische Monatshefte für Politik und Kultur*³², for which he regularly reviewed books.

Christian Beyel died on 16 January 1941.

Beyel joined the organising committee in December 1896, but he did not hold a particular position.

4.2.3 Hermann Bleuler (1837 – 1912)

Hermann Bleuler was born on 22 November 1837 in Hottingen (nowadays part of the town Zurich). His father Johann Caspar (1801-1882) was a

²⁷ “The Mathematical Thought in the World. Causeries and Reflections of an Old Mathematician”. The term “causeries” describes a type of short humorous essays characterised by a personal approach and linguistic jokes.

²⁸ “Geometric Studies”

²⁹ “Descriptive Geometry: With a Collection of 1800 Exercises on Descriptive Geometry”

³⁰ “LVII Theorems on the Orthogonal Triangle”

³¹ “The Cubic Circle With Double Points”

³² “Swiss Monthly for Politics and Culture”, founded in 1921. Its current name is “Swiss Month” (*Schweizer Monat*).

merchant and silk manufacturer; he was also a member of Zurich's cantonal parliament for some years. Hermann's mother Sophie Regula née Arter (1811-1880) came from a family of merchants. Hermann, the second oldest child of the family, had two sisters and three brothers, one of whom became a doctor; the other two became planters in Guatemala.

At the age of eight he entered the boarding school for boys "zum Felsenhof" in Männedorf (canton Zurich). This international school 'replaced the then missing secondary school' [14, p. 85]. Afterwards Bleuler attended the Gymnasium and then the Industrieschule* in Zurich. He was one of the first students at the Polytechnic: He matriculated at the Engineering Department in 1855, the school's first year. After his graduation in 1858, as one of the Polytechnic's first eight civil engineers, he worked as an engineer for two years, at the machine factory Bell & Co in Kriens (canton Luzern).

Bleuler had a stellar career in the Swiss army, in which he enrolled in 1861. Already a year later he became head of the Federal Artillery Bureau in Aarau and secretary of the Federal Artillery Committee. He was promoted to captain in 1864, to major in 1868, to lieutenant colonel in 1869 and to colonel in 1871. In 1870 he was appointed to chief instructor of the artillery, a position that he held for eighteen years. As such he helped to improve the standard and training of the artillery. He also invented a 'ground-breaking' [20] field howitzer. In 1883 he became commanding officer of the 6th division, and in 1891 he took command of the 3rd corps. In addition, he became a member of the National Defence Committee.

Bleuler became a member of the Federal School Board in 1881, the year when the Polytechnic's regulations were revised. As a result, the age at entry for students was raised, students had a greater freedom to choose their courses in their last two years, the standard of technical education was raised, and the teaching staff were allowed to elect the school's Director – the first one being Geiser (see appendix B). Moreover, the School Board now consisted of seven members instead of five, of whom some came from industry [cf. 39, p.

54-56; p. 104]. Bleuler became Vice-President after Alfred Escher's³³ death in 1882, and in 1888 he succeeded Karl Kappeler as president of the School Board. Both the President and Vice-President used to be appointed for life, but Bleuler retired in 1905 due to health reasons. Incidentally, he was a good friend of Geiser. He helped to improve the Polytechnic's reputation by supporting the construction of engineering laboratories and appointing excellent teaching staff. Following Kappeler's method, Bleuler often attended lectures of teaching candidates incognito to assess their qualities (see appendix B). On the whole, the Polytechnic continued to boom during his presidency, particularly economically. However, when the Polytechnic professors demanded the right to award doctorates, more freedom in their research and greater freedom of choice for their students at the beginning of the 20th century, Bleuler used his position and connections to the government to thwart these reform attempts³⁴ [cf. 39, p. 136-144].

In addition to his positions in the army and at the Polytechnic, he also served as president of the GEP from 1885-1888.

In 1873, Bleuler married Emma Huber whose father had also been a silk manufacturer. The couple had a son, Walter (*1875). In 1887, the family moved into the so-called "Villa Bleuler", which was built for them. Today the building houses the Swiss Institute of Cultural Studies [1].

Hermann Bleuler died on 07 February 1912 after a long illness.

Bleuler signed the invitation circular from January 1897 in his capacity as School Board president. Later he was invited to join the organising committee, which he did in May 1897. He also attended the congress, as one of few non-mathematicians. His wife Emma provided some entertainment for the

³³ Alfred Escher (1819-1882): Swiss politician and railways pioneer whose influence on Swiss politics and economy in the 19th century remained unequalled. Escher was instrumental in founding the Polytechnic and served as Vice-President of the School Board from 1854-1882. Cf. biography by M Bürgi in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D3626.php>, accessed 19/03/2014; and the website of the Alfred Escher-Stiftung: www.alfred-escher.ch.

³⁴ His successor Robert Gnehm was much more forward-looking and supported the reforms.

accompanying ladies at the congress: 'In the afternoon [Tuesday, 10 August] the ladies were invited by Mrs School Board President Bleuler' [73, p. 55].

4.2.4 Heinrich Burkhardt (1861 – 1914)

Heinrich Friedrich Karl Ludwig Burkhardt was born in Schweinfurt (Franconia) on 15 October 1861. His father Carl Heinrich Theodor, who worked as an assessor in the district court, died when Heinrich was six years old. He and his younger sister were raised by his mother Caroline Louise née Heyde in her native town Ansbach. Heinrich attended both primary school and Gymnasium there. He was a very studious child and loved to read. Among his teachers at the Gymnasium was Siegmund Günther³⁵, who recognised his particular talent for mathematics early on and encouraged him to study mathematics at university.

He began his studies in 1879, initially in Munich, both at the university and at the technical college (Technische Hochschule = TH). Among his teachers at the university were Philipp Seidel and Alfred Pringsheim; among those at the TH were Alexander von Brill and Jacob Lüroth. Burkhardt completed his studies in 1886 with a teaching diploma. However, he did not stay in Munich for the entirety of his studies: In 1881 Burkhardt went to Berlin for a year to attend lectures by Hermann von Helmholtz, Leopold Kronecker, Ernst Kummer and Karl Weierstrass. And from 1883-1884 he studied in Göttingen, attending Hermann Amandus Schwarz's lectures in particular. As a student, Burkhardt received a scholarship from the Maximilianeum Foundation³⁶.

³⁵ Siegmund Günther (1848-1923): German geographer, mathematician and historian of mathematics. He taught at various universities and Gymnasiums across Germany. In Ansbach he taught mathematics and physics from 1876-1886. Cf. biography by J Hohmann, in *Neue Deutsche Biographie* 7, 1966, 266-267: <http://www.deutsche-biographie.de/sfz24547.html>, accessed 19/03/2014.

³⁶ The Maximilianeum Foundation was founded by King Maximilian II in 1852. It provides a small number of highly gifted students with free board and accommodation, and funding for language courses etc. In Burkhardt's time, it offered students the opportunity to improve their knowledge of 'modern languages, music, dance, and fencing' [57, p. 186]. See also <http://www.maximilianeum.de/>.

Apart from his teaching diploma, he also received his doctorate from the University of Munich in 1886. Burkhardt's thesis, supervised by Gustav Bauer, was entitled *Beziehungen zwischen Invariantentheorie und der Theorie algebraischer Integrale und ihrer Umkehrungen*³⁷. He then worked as Dyck's assistant at the TH in Munich for a year, before moving back to Göttingen in winter 1887. This time, Klein's reputation was his reason for studying there [57, p. 188]. In 1889 Burkhardt habilitated at the University of Göttingen, in 1894 he became Titularprofessor. He lectured on various aspects of geometry, function theory, Galois theory and variational calculus, amongst others. In fact,

the selection of material covered in the lectures given during that period is very comprehensive in view of the fact that in the whole of Germany only few people wanted to study mathematics at the time.

[57, p. 188].

During his time in Göttingen he also spent a few months in Paris (winter 1893/94) in order to attend lectures by Poincaré, Picard and Félix Tisserand. In 1897 Burkhardt was appointed to an ordinary professorship at the University of Zurich, succeeding Arnold Meyer³⁸. His inaugural lecture was entitled *Mathematisches und naturwissenschaftliches Denken*³⁹. In Zurich, Burkhardt mainly lectured on algebraic analysis, and differential and integral calculus, but he also held more specialised seminars. In 1908 he returned to Munich to teach at the TH. One of Burkhardt's objectives as a teacher was to make mathematics accessible to science students, whilst providing mathematics students with adequate training. He had a number of PhD students both in Zurich and in Munich. Burkhardt became an extraordinary member of the Royal Bavarian Academy of Sciences⁴⁰ in 1909; in 1921 he became an ordinary member.

³⁷ "Relations Between Invariant Theory and the Theory of Algebraic Integrals and Their Converses"

³⁸ Arnold Meyer (1844-1896): ordinary professor of mathematics at the University of Zurich; his research interest was in number theory. Cf. obituary by A Lang in *Schweizerische Pädagogische Zeitschrift* 7 (4), 1897, 200-209.

³⁹ "Mathematical and Scientific Thinking"

⁴⁰ Königliche Bayerische Akademie der Wissenschaften

Among Burkhardt's publications, his textbooks *Funktionentheoretische Vorlesungen*⁴¹ (1897-1899) and *Vorlesungen über die Elemente der Differential- und Integralrechnung und ihre Anwendung zur Beschreibung von Naturerscheinungen*⁴² (1907) are particularly noteworthy:

E Lampe once said about [them]: "The many virtues of Burkhardt's lectures caused the individual volumes to have become the most widely read books on theory of functions among students. In answer to the question which books a candidate studied before the exam, the professor is regularly given the titles of Burkhardt's writings. And they deserve to be so widely spread because they are written in a plain and clear manner yet contain a wealth of profound material in a moderately large space.

[57, p. 191-192]

Burkhardt also wrote a number of research papers, mainly on problems in theory of functions, group theory and trigonometric series. These 'stand out due to the extensive breadth of their viewpoint, a standardisation of the relevant literature and a detailed consideration of the historic background' [44]. His most important work is *Entwicklungen nach oszillierenden Funktionen und Integration der Differentialgleichungen der mathematischen Physik*⁴³ (1901-1908), a report of more than 1,800 pages [57, p. 192]. In addition, he wrote several articles for the *Enzyklopädie der mathematischen Wissenschaften*, mainly on topics in group theory. He was also one of the encyclopaedia's editors. Burkhardt was asked to edit Euler's works on the mechanics of elastic solids, but could not perform this work due to his premature death [65, p. 566].

⁴¹ "Lectures on the Theory of Functions"

⁴² "Lectures on the Elements of Differential and Integral Calculus and Their Application for the Description of Natural Phenomena"

⁴³ "Developments of Oscillating Functions and Integration of Differential Functions of Mathematical Physics"

In 1897 he married Mathilde Büdinger, daughter of the history professor Max Büdinger (1828-1902). Burkhardt was a theatre enthusiast and a good pianist; he liked Wagner's compositions in particular.

Heinrich Burkhardt died on 02 November 1914 in Munich.

At the committee meeting on 21 January 1897, Geiser could announce that Burkhardt had agreed to sign the invitation to the congress. After that meeting he was also asked to join the reception committee. He attended most of the committee meetings in 1897. Burkhardt was the only professor from the University of Zurich on the organising committee.

Burkhardt also attended the 1904 ICM in Heidelberg and the 1912 ICM in Cambridge. As a member of the international committee of the latter he represented the Royal Bavarian Academy of Sciences in Munich.

4.2.5 Fritz Bützberger (1862 – 1922)

Fritz (Friedrich) Bützberger, of Bleienbach (canton Bern), was born on 26 March 1862. After attending primary school in his native village and secondary school in Langenthal⁴⁴ he attended the Gymnasium in Burgdorf. In 1880 he began his studies at the Engineering Department of the Polytechnic, but after a year he transferred to the Department for Mathematics and Physics Teachers. He graduated from the Polytechnic in 1884 and began working as a mathematics teacher at his former secondary school in Langenthal, after having worked at a school in Solothurn for a few months.

Whilst teaching in Langenthal he also studied towards a doctorate at the University of Bern⁴⁵. He completed his thesis *Ein mit der Theorie algebraischer*

⁴⁴ Geiser also attended this school.

⁴⁵ According to the university's matriculation register, a Fritz Bützberger matriculated in Bern in the academic year 1882/83, when Bützberger was still studying at the Polytechnic. His qualification is given as 'secondary school certificate 26 Oct. 1882 Langenthal', but this is not in accordance with biographies of him. However, the date of birth and hometown match [83]. Bützberger also appears on the matriculation registers in 1884/85, in 1888, and in 1890, although there is no payment noted in the latter. Whilst both the 1888 and 1890 registers refer to the 1884 one, there is no reference to any previous matriculation numbers that Bützberger might have

*Flächen zusammenhängendes planimetrisches Problem*⁴⁶ in 1888. The doctorate was conferred “summa cum laude”, with highest honours [37, p. 134]⁴⁷. Bützberger was the 11th of Schläfli’s 12 doctoral students.

Bützberger became a mathematics teacher at the Kantonsschule in Zurich in 1896. From 1899 onwards he taught in the school’s technical track only, thus preparing future engineers for their university studies. Furthermore, he also taught mathematics at Zurich’s adult education centre. At the University of Zurich he delivered lectures on descriptive geometry to future secondary school teachers. In 1903 he took up a teaching post at the technical school in Burgdorf.

Bützberger was a ‘first-rate teacher and author of several much valued textbooks with ample exercises’ [48, p. 422]. His textbooks include *Lehrbuch der ebenen Trigonometrie mit vielen Aufgaben und Anwendungen*⁴⁸ (Zurich, 4th edition 1910), *Lehrbuch der Stereometrie*⁴⁹ (Zurich, 3rd edition 1916), and *Lehrbuch der Arithmetik und Algebra für Mittelschulen*⁵⁰ (Zurich, 2nd edition 1920). All of them were reviewed favourably and were reprinted several times. Salkowski calls *Lehrbuch der Stereometrie* an ‘established book’ [74], whilst Barneck comments on Bützberger’s style in *Lehrbuch der Arithmetik ...*: ‘The presentation is broad and comprehensible for pupils’ [12]. Zacharias commends *Lehrbuch der ebenen Trigonometrie ...* for similar reasons [81]:

The [writing] is succinct and clear; the definitions and theorems are succinct and easy to learn throughout; historic remarks make for interesting reading; numerous exercises, parts [sic!] theoretical, partly practical, are included in the individual sections [...]

been assigned in the 1884 register. It is possible that they were two different people, just with the same hometown and date of birth.

⁴⁶ “A Planimetric Problem Connected to the Theory of Algebraic Surfaces”

⁴⁷ In Germany, doctorates are conferred with Latin honours, with the most common honours being “summa cum laude” (with highest honours) and “magna cum laude” (with great honours). This is similar in Switzerland.

⁴⁸ “Textbook on Planar Trigonometry With Many Exercises and Applications”

⁴⁹ “Textbook on Stereometry”

⁵⁰ “Textbook on Arithmetic and Algebra for Middle Schools”

Moreover, Bützberger published several papers, among them *Eiförmige Drehkörper* (1917), which is aimed at secondary school pupils. In this paper he determines centroids of ovoids, using the Guldinus theorem. As he mentions in the introduction, ‘one of my pupils, a keen ornithologist, inspired me to solve [the problems treated here]’ [21, p. 218]. Bützberger also wrote a very readable biography of the Bernese mathematician Georg Sidler⁵¹ [23], his friend and former teacher. In 1912 he gave a talk at the annual meeting of the Swiss Mathematical Society.

He had a particular interest in Jakob Steiner and published a couple of biographical papers on the geometer. More importantly, he organised and edited Steiner’s papers from 1823-26 on the request of the Bernese Society for Natural Scientists. As Bützberger remarks in [7n, p. 49, footnote 1], Graf discovered Steiner’s handwritten manuscripts covering the period from 1814-1826 in the attic of the Town Library in Bern. Graf then passed the documents on to Bützberger ‘to put them in order and to good use’ [ibid.]⁵². The collection of Bützberger’s papers stored in the ETH Library Archive is briefly analysed in section 4.2.5.1.

Bützberger was married to Rosa Kohler. The couple had two children: a son, Fritz, and a daughter, Marie⁵³. Fritz Bützberger died on 01 November 1922.

⁵¹ Bützberger’s estate [7] contains letters from colleagues expressing their thanks for the Sidler biography. As an example, I quote Hurwitz’s letter [7B]:

[...] Your paper on Prof Sidler held great interest for me. I did not know how versatile and – in particular with regards to education – proficient a mathematician Prof Sidler had been until reading it. I have always admired him as a noble man [...]

⁵² In a letter to Bützberger, Graf explains that he found the manuscripts in 1888 and passed them on to Bützberger ‘ca. 1892’ [7y].

⁵³ There is little information about Bützberger’s personal life in the obituaries and his estate. In a letter to U Hoepli and his wife in 1920 [7A], Bützberger mentions his son and daughter. Presumably he would have mentioned any further children. He also refers to his brother Ernst and his brother-in-law Hardmeyer.

Ulrico Hoepli (1847-1935) was a Swiss bookseller who emigrated to Milan. He became one of Italy’s most important publishers, specialising in sciences and Italian classics, and an influential art patron. Unfortunately, I have not been able to determine how Bützberger knew him. Cf. biography by V Rothenbühler, in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D30470.php>, accessed 19/03/2014.

Bützberger joined the organising committee (or enlarged committee as it is called in the minutes) in December 1896. At his first committee meeting on 08 December he joined the reception committee chaired by Hurwitz. At the meeting on 31 July 1897, Rudio suggested that a ‘correspondence and mail room should be established and a postal service organised’ [8g]; Bützberger accepted this task.

4.2.5.1 Bützberger’s Work on Steiner

As mentioned above, Bützberger was an avid Steiner scholar. His scientific estate in the ETH Library Archive contains a host of letters and notes connected to his research. The correspondence indicates that Bützberger’s research lasted for several decades. Furthermore, it also shows that he was in touch with a number of his colleagues on the organising committee – Geiser in particular – as well as fellow mathematicians in Switzerland and abroad. Correspondence with Johann Heinrich Graf, Julius Gysel, and Sidler deserves a special mention here, but the list also includes Moritz Cantor, Arnold Emch, Wilhelm Fiedler, and Theodor Reye.

Whilst I am not able to do all the material in the estate justice within the scope of this thesis, I will give a brief overview.

As one would expect, there are sheets and slips of paper with notes⁵⁴ on the topic of his Steiner research, some of them in French or English (e.g. [7m]). There are also a number of manuscripts and drafts of papers relating to Steiner. Among these, a handwritten, 125-page biography of Steiner [7a] and a draft of a book on Steiner’s mathematical manuscripts 1823-1826 [7b; 7c] stand out. It seems that this book, *Jakob Steiners Nachlass aus den Jahren 1823–1826*, was never published. As Burckhardt notes in [18], the manuscript is held in the ETH Library (Burckhardt refers to the copy in **Hs 92** in the ETH Library

⁵⁴ Among these is an undated invoice for a student, one A Spaerry, in which Bützberger charges him 55 Franks for 10.5 hours of private mathematics lessons plus ‘2 notebooks [and] 1 sheet of plotting paper’ [7r]. Presumably the only reason why it was kept are a few notes on Steiner that Bützberger scribbled on the back, but it gives a small insight into Bützberger’s life as a teacher.

Archive, which contains papers relating to Steiner's works). However, Bützberger did publish two papers on Steiner's life: *Jakob Steiner bei Pestalozzi in Yverdon*⁵⁵ (1896), and *Zum 100. Geburtstag Jakob Steiners*⁵⁶ (1896), in which he focuses on Steiner's notebooks as a student at Pestalozzi's school and at university. Letters from Geiser suggest that it may have been the same paper: In November 1895 he arranges for Bützberger's paper to be published in *Schweizerische Pädagogische Zeitschrift* and puts him in touch with its editor, Friedrich Fritschi⁵⁷ [7u; 7v]. Furthermore, he proposes a re-publication in *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*. Indeed, in February 1896 he recommends the paper to the journal's editor Immanuel Carl Volkmar Hoffmann⁵⁸, as Geiser himself 'could not provide [Hoffmann] with the desired article [on Steiner]' [7w]. These short letters nicely illustrate Geiser's remarkable networking skills, which are also apparent in his letters to Gysel (section 5.2.2). Translations of the full letters are in appendix E.2.2.

There are several postcards in Bützberger's estate in which colleagues congratulate him on his paper – "paper" in the singular, without any specification as to which one is being referred to. Nevertheless, a card by Cantor from 1896 [7s] represents the sentiment echoed in these cards:

Dear Colleague!

Thank you very much indeed for your exceptionally intriguing paper on Steiner's background. Like all our peers, I dare say, I eagerly anticipate the continuation of your publications.

Yours respectfully
M. Cantor

⁵⁵ "Jakob Steiner at Pestalozzi's in Yverdon" [22]

⁵⁶ "On the Occasion of Jakob Steiner's 100th Birthday"

⁵⁷ Friedrich Fritschi (1851-1921), Swiss teacher, politician, and editor of *Schweizer Lehrerzeitung*. Cf. biography by M Bürgi in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D3636.php>, accessed 13/05/2014.

⁵⁸ Volkmar Hoffmann (1825-1905), a German teacher, founded and edited *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*. Cf. http://personen-wiki.slub-dresden.de/index.php/Hoffmann,_Immanuel_Carl_Volkmar, accessed 13/05/2014.

In fact, it was Geiser who suggested that Bützberger send his paper to Cantor [7u; italics by the author]:

Should you have reservations about entrusting your fine work to a non-mathem[atical] journal, do contact Prof. Cantor in Heidelberg, so that the *Zeitschrift für Mathematik & Physik* includes the article. Do not hesitate to say that I asked you to send the paper if you think that this would have an impact.

However, for some reason unknown to us the paper was not published in Cantor's journal. Perhaps Bützberger did not send it to Cantor after all, and Cantor commented on an already published version of it.

Bützberger must have spent much of his time trying to track down any remaining relatives and friends of Steiner, as well as pictures of and documents relating to him. Over the years he built up quite an extensive network of contacts that could provide him with information. Among them was his own father-in-law Johann Kohler, who lived about 20 km from Steiner's hometown⁵⁹. Letters in Bützberger's estate suggest that Kohler did a lot of research in the region [7h]. Friends of Bützberger, such as Johann Petri, and relatives of Steiner, e.g. one J Werner-Mathys⁶⁰, helped as well. Gysel tried to track down a certain Conrad Maurer for him, who seems to have been Steiner's mathematics teacher at Pestalozzi's school [7z].

Bützberger showed a particular interest in Steiner's family tree, especially how Steiner and Geiser were related. His findings are mentioned in Geiser's biography (section 2.2).

Another very profitable source of information was Sidler, who inherited some of Steiner's manuscripts [cf. 38, p. 48-51] and for whom Steiner had been

⁵⁹ Johann Kohler (1843-1908), a member of the cantonal parliament, lived in Forst, a hamlet outside of Bützberg in canton Bern. Bützberg is part of the municipality Thunstetten.

⁶⁰ There is not much information about him in any of the letters, except that he lived in Fribourg and owned three of Steiner's books [7F]. As Sidler explains, a Jakob Mathys, one of Steiner's nephews, inherited most of Steiner's money [7E]; Werner-Mathys was probably one of his descendants.

a fatherly friend. Indeed, Bützberger reports in his Sidler biography that 'Sidler visited [Steiner] almost daily during [Steiner's] bitter time of suffering. His dear widowed mother [...], who had lived with him since 1861, ministered to the terminally ill geometer as well, for which he was very grateful' [23, p. 70]. In fact, in 1906 Bützberger asks Sidler's permission to include this anecdote in his 'soon to be completed work'⁶¹ [7k]. In his reply [7E], Sidler suggests changes to the manuscript and gives background information on some points, such as Steiner's character traits and his heirs. Moreover, he recounts some anecdotes that explain Steiner's difficulties with writing and the end of his friendships with Carl Jacobi, Lejeune Dirichlet, and his doctor Johann Schneider. Sidler also comments on the rumour about Steiner's illegitimate daughter. Apparently Bützberger planned to ask Geiser for comments on the manuscript as well [7F].

Furthermore, Sidler responds to Bützberger's accusations against Graf and tries to calm him down: According to Bützberger, Graf included some of Bützberger's results regarding Steiner's years in Yverdon in his own papers but failed to reference them. In his Steiner biography, Graf remarks that the Yverdon section is based on Bützberger's paper. Comparing the two papers today, some passages are almost identical and would require better referencing [cf. 38, p. 2-6; 22, p. 20-26]. However, Bützberger seems most outraged about Graf's 1905 paper, in which he is not referred to at all. The matter of dispute is the year of Steiner's arrival in Yverdon: Bützberger, believing that he settled the issue, cannot understand why Graf revived the debate. This is explained in letters to one Dr Israel, in which Bützberger complains about Graf [7C], and to Graf himself, in which he expresses his displeasure [7f]. In his reply, Graf insists that this must have been a misunderstanding and encloses a postcard with Steiner's birthplace, 'to prove

⁶¹ Bützberger probably referred to *Über bizentrische Polygone...* here, his only Steiner-related publication in the 20th century. As he explains to Sidler why he included a 'biographic preamble', we can assume that this work was not purely a biography. Sadly, Sidler did not live to read the finished book.

As Kiefer writes in [48, p. 423], Bützberger also worked on an extensive Steiner biography, and he expresses the hope that it might be published posthumously. However, it seems that the biography remained unpublished.

[that he] is not cross with [Bützberger]' [7e] – a rather curious reaction! Whilst Bützberger had a point, his reaction was rather dramatic given the scale of this academic dispute. In his letter, Sidler urges Bützberger to remain professional:

Your work should become a classic in memory of the great geometer & it should go without saying that the preface should be nobly written. As an example, look at the second part of Poncelet's *Propriétés des figures projectives*. Surely everybody laments that Poncelet got too carried away with polemics there; this tarnishes Poncelet's memory.

[7E; italics by the author]

The other letters that Bützberger and Graf exchanged suggest that their relationship was generally professional [7g; 7x; 7y].

In fact, both were members of the Steiner-Schläfli Committee, as was Geiser, along with Sidler and five more mathematicians⁶². In a circular of October 1895 the committee explains that its main objective was to raise money for tombs for Steiner and Schläfli⁶³. As Graf writes in [36, p. 19], Bützberger and a colleague, Christian Moser, found Steiner's lost grave, and Sidler donated a small tombstone. However, as the graveyard was closed down, the committee successfully applied for permission to exhume Steiner. The committee organised Steiner's re-interment and the erection of a grand tombstone on Schläfli's grave in 1896 to celebrate Steiner's centenary and the anniversary of Schläfli's death (who had died in 1895) [cf. 36⁶⁴; 6G]. It seems that the committee disbanded afterwards.

⁶² These mathematicians were: Hermann Kinkelin from Basel, Hugo Schiff from Florence, and Gottlieb Huber, Christian Moser, and Eduard Ott, all from Bern. Graf was president.

⁶³ As Neuenschwander writes in [61, p. 34; p. 77-78; p. 83], the Swiss Mathematical Society founded a Steiner Committee and Steiner Archive in 1930, with the objective of safeguarding and publishing Steiner's manuscripts. In 1937, it was renamed Steiner-Schläfli Committee – there do not seem to be any links to the committee referred to in the text here. It disbanded in 1956, after having published Schläfli's complete works (*Gesammelte Abhandlungen*, three volumes: 1950, 1953, 1956). It seems that they only published one of Steiner's works, *Allgemeine Theorie über das Berühren und Schneiden der Kreise und der Kugeln*, edited by R Fueter, F Gonseth (1931).

⁶⁴ Graf includes the most important correspondence between the committee and authorities, the commemorative speeches by Geiser and himself, and congratulatory letters from a number of mathematicians. Among them we find Schwarz, Cremona,

Letters from Reye [7D] and Emch [7t] to Bützberger suggest that he might have planned to publish Steiner's posthumous works. As Hollcroft writes, Bützberger 'died before he had completed the work of editing the Steiner manuscripts' [45, p. 794]. However, he did publish *Über bizenrische Polygone, Steinersche Kreis- und Kugelreihen und die Erfindung der Inversion* in 1913⁶⁵, dedicating a separate section for each of the three topics. The 'carefully and clearly written' [30, p. 415] book was reviewed favourably, with particular praise for the historical background: 'The numerous historical and biographical facts add particular value to this book' [7p]. Furthermore, reviewers note that Bützberger used elementary geometric methods (instead of analytical methods), such as reciprocal radii. In the second section in particular, he treats 'Steiner series of circles and of spheres; here [he] follows Geiser's view: "Einleitung in die synthetische Geometrie", last chapter "Das Prinzip der reziproken Radien"' [54]. An analysis of Geiser's book is given in section 5.1.

All the reviews available to me also agree that the third section is the most intriguing one. Danzer for example writes [7o]:

I think that Bützberger's book is very interesting; the first and second sections, in which hardly any new material is included, less so, but certainly the third section, which provides an insight into "Master Steiner's" workshop.

Specifically, Bützberger cites a document that he found among Steiner's manuscripts, which proves that Steiner did invent the inversion, as had been suspected previously [cf. 7q; 30, p. 414]. Presumably in preparation for the book Bützberger copied down a dozen relevant papers in a scrapbook, from 1896 onwards. Among the papers are excerpts from J Plücker (*Crelle's Journal* **11**, 1831), papers on Steiner's solution of the Malfatti problem (by H Schröter,

Brioschi, Beltrami, and Rudio. Schläfli's friend Gysel and Cantor both apologise for their absence – in Cantor's case, floods made rail travel impossible [cf. 36, p. 21-22]. Curiously, Graf also describes the exhumation itself, concentrating on the shape and measurements of Steiner's skull. Photographs of the skull are included, as 'surely people would like to own a picture of it' [36, p. 13].

⁶⁵ "On Bicentric Polygons, Steiner Series of Circles and of Spheres, and the Invention of Inversion"

1873; W Goit, 1877; and J Petersen, 1879), and Geiser's 1896 paper *Das räumliche Sechseck und die Kummersche Fläche* [71].

As an aside, a couple of letters that Bützberger exchanged with Sidler have mathematical content as well. Sidler lent his friend mathematical books from his extensive library and pointed out further reading on the subject of the geometry of triangles, Sidler's particular interest [71]. In 1898 Sidler sent Bützberger a copy of a proof by Droz-Farny⁶⁶ concerning a property of triangles⁶⁷, 'as I presume that you will take as much pleasure in it as I have' [7].

4.2.6 Gustave Dumas (1872 – 1955)

Gustave Dumas was born on 25 March 1872 in L'Etivaz (canton Vaud). His father was a priest. Dumas attended secondary school in Lausanne; after having completed his baccalaureate he stayed there to study mathematics at the university. Having obtained his diploma, he gained a second one from the Sorbonne, again in mathematics. Despite his university studies he seems to have been assistant at the Polytechnic at the time of the congress.

Dumas then went to Berlin for some months, where he attended lectures by Georg Frobenius, Schwarz and Kurt Hensel. He returned to Paris, and in 1904 he was awarded a doctorate for his thesis *Sur les fonctions à caractère algébrique*

⁶⁶ Arnold Droz-Farny (1856-1912) was a Swiss mathematics and science teacher. See J-L Ayme, A Purely Synthetic Proof of the Droz-Farny Line Theorem, *Forum Geometricum* 4, 2004, 219-224 for more information. Note that he published his Line Theorem without proof in 1899 – it is not the theorem in Sidler's letter.

⁶⁷ According to Sidler the theorem was first proved by 'Brocard junior' (presumably he meant Henri Brocard) in *Mathésis* in 1896, but Droz-Farny's proof was much simpler. The theorem is the following [7]:

In the plane of any given triangle ABC there are two (always real) points P & Q, which have the property that when one extends each of the rays AP, BP, CP or AQ, BQ, CQ up to the points of intersection with the opposite sides A', B', C' or A'', B'', C'', the resulting 6 line segments are of equal length:
 $AA' = BB' = CC' = AA'' = BB'' = CC''$

These points P & Q are the foci of the ellipse with least surface area circumscribed around the triangle ABC.

*dans le voisinage d'un point donné*⁶⁸. Two years later he habilitated at the Polytechnic in Zurich, with the paper *Sur quelques cas d'irréductibilité des polynômes à coefficients rationnels*⁶⁹. In both of these papers he used new notions introduced by Hensel [71, p. 121]. Dumas taught higher mathematics as a Privatdozent at the Polytechnic; he was promoted to Titularprofessor in 1913.

In the same year he was appointed to a professorship in mathematics at the Engineering School⁷⁰ of the University of Lausanne. He became an ordinary professor in 1916. Dumas stayed at his alma mater until he retired in 1942, teaching mainly differential and integral calculus to future engineers and mathematicians. Among his students was Georges de Rham, who became Dumas's assistant in the mid-1920s. Dumas's own research interests would be classed 'as classical algebraic geometry (over the complex field)' [25, p. 201] today. He was also very interested in Poincaré's work. His mathematical papers cover various topics in algebra, analysis and geometry, and include *Note relative aux abaques à alignement*⁷¹ (1906), *Sur les singularités des surfaces*⁷² (1912), and his lectures *Notes de calcul différentiel et intégral*⁷³ (1925). In addition, Dumas wrote a couple of papers on technical education in the French-speaking part of Switzerland.

Dumas became a member of the Swiss Mathematical Society (*Schweizerische Mathematische Gesellschaft*, SMG) when it was founded in 1910. He gave a number of talks at the society's annual meetings and served as secretary-treasurer from 1920-1922. From 1922-1924 and again from 1930-1931 he was president of the SMG. In 1944 he was made an honorary member of the society.

In 1923 he co-founded the so-called *Colloque mathématique des Universités romandes*, later renamed as *Cercle mathématique de Lausanne*. This was a 'very

⁶⁸ "On Algebraic Functions in the Vicinity of a Given Point"

⁶⁹ "On Some Cases of Irreducible Polynomials With Rational Coefficients"

⁷⁰ Ecole d'Ingénieurs de l'Université de Lausanne

⁷¹ "Note on Alignment Charts"

⁷² "On Singularities of Surfaces"

⁷³ "Notes on Differential and Integral Calculus"

rigorous group' [25, p. 207] that organised lectures and meetings until the 1980s.

Furthermore, Dumas was a member of the Euler-Kommission from 1919-1943. He was awarded an honorary doctorate from the University of Lausanne when he retired in 1942.

Apart from mathematics and education, Dumas also had a strong interest in literature and philosophy.

He died on 11 July 1955.

Dumas joined the organising committee at the preliminary meeting in July 1896, as the French-speaking secretary. His committee minutes are kept in [8], part 2. He is mentioned by name only a couple of times, when specific jobs were given to him: He wrote the letter to Greenhill, inviting him again to attend the congress [8g], and finalised the congress programme with Geiser, Franel and Hirsch on 02 August. Rudio and Franel, the two general secretaries at the congress, also had their personal secretaries, Hirsch and Dumas. It can be assumed that Dumas, as the native French speaker, was Franel's secretary. Dumas did not give a talk at the congress, but he was among the signatories of the invitations.

Dumas attended more ICMs than most of his colleagues. Representing the University of Lausanne, he attended the 1920 ICM in Strasbourg, the 1928 ICM in Bologna, and the 1932 ICM in Zurich. He gave a talk in Bologna, entitled *Sur les singularités des surfaces*, in section II-B (geometry). Dumas also served on the organising committee of the 1932 congress in Zurich.

4.2.7 Ernst Fiedler (1861 – 1954)

Ernst Fiedler was the oldest son of the mathematician Wilhelm Fiedler. Wilhelm Fiedler married Elise Springer in 1860, in Chemnitz, and Ernst was born there on 22 July 1861. He had two brothers and four sisters, among them Alfred Fiedler (1863-1894), who lectured on zoology at the University of Zurich.

The family moved to Prague in 1864. Three years later Wilhelm Fiedler was appointed to fill the chair of descriptive geometry at the Polytechnic in Zurich, which had been vacant since Wolfgang von Deschwanden's death (see section 2.1). The family stayed in Zurich then, and they became Swiss citizens in 1875.

Ernst attended the Gymnasium in Zurich and, from 1879 onwards, the Polytechnic, where he studied mathematics at the Department for Mathematics and Physics Teachers. Among his lecturers were his own father, Frobenius and Geiser. In 1882 he moved to Berlin, where he attended lectures by Kummer, Kirchhoff, Helmholtz and Weierstrass in particular. Two years later he moved to Leipzig in order to study under Klein and the philosophy professor Wilhelm Wundt. They both supervised his doctoral thesis, *Über eine besondere Klasse der Modulargleichungen der elliptischen Funktionen*⁷⁴, which he completed in 1885. He then became Privatdozent for mathematics at the Polytechnic and assistant teacher at the Kantonsschule in Zurich. In 1889 he became professor for mathematics at the Industrieschule in Zurich. Fiedler was influential in the school's development, especially after having become Director of the school in 1904, when it was re-organised and renamed as Oberrealschule. He held this post until his retirement in 1926. Under his guidance, the school became 'an acknowledged institution preparing for studies at the Polytechnic' [89]. Fiedler seems to have been very good at choosing his teaching staff; he also supported extra-curricular activities, founding both the school's orchestra and rowing club [32, p. 95].

Fiedler did not produce any mathematical research papers; he was very much a schoolteacher and not a research mathematician. His publications include a couple of secondary school textbooks on descriptive geometry, a graduation speech entitled *Lebenserfahrung und Bescheidenheit*⁷⁵ that he gave at his school in 1908, and several papers on ballistics and military education.

⁷⁴ "On a Special Class of Modular Equations of Elliptic Functions"

⁷⁵ "Experience of Life and Modesty", Zurich, 01/10/1908

Early on Fiedler made a name for himself in the Swiss army, which he joined in 1881. After a swift career he became the then youngest colonel at the age of forty-three. From 1889 onwards he lectured on ballistics and shooting theory at the Polytechnic, instead of mathematics. He retired from this post in 1923.

In 1899, the Polytechnic's Board decided that the school's assistants would no longer be asked to work during the entry examinations. Instead, Bleuler asked Fiedler to assist with the exams in descriptive geometry, conducted by his father, who remarked that 'due to the amount of candidates it is absolutely necessary to have two experts in the hall' [6a]. Since similar requests were sent to Fiedler in the two following years, we can assume that he was responsive to Bleuler's pleas [6].

Fiedler was a member of the *Schweizerische Gesellschaft für Schulgesundheitspflege*, a society taking charge of all aspects of health and hygiene in Swiss schools. He was also a consultant of the *Schweizerische Rektorenkonferenz*⁷⁶ and served on the supervisory board of the Teachers' College in Küsnacht. It is due to Fiedler that shorthand was introduced as an optional subject in secondary schools. Furthermore, he was the creator and first curator of the Archive of Secondary Schools. Following a nervous breakdown, Fiedler became heavily involved in the temperance movement of the late 19th and early 20th centuries, educating the public on the dangers of alcohol abuse.

He married Lina Knoch in 1886; the marriage lasted until Lina's death in 1949. The couple had four sons and one daughter. Two of his sons also studied at the Polytechnic: Karl (1892-1965), the oldest son, became a civil engineer; the third son Max (1893-1944) was a mechanical engineer.

Ernst Fiedler died in 1954⁷⁷.

⁷⁶ "Swiss Universities Association"

⁷⁷ The obituary in [31] gives 06 October as his day of death, whereas [89] claims that he died on 16 July.

Fiedler is only mentioned once in the committee minutes: he attended the meeting on 08 December 1896. Having said that, there does not exist an attendance record for every meeting so he may have attended more meetings. He attended the congress but did not give a talk.

Fiedler attended two further congresses, the 1908 ICM in Rome and the 1912 ICM in Cambridge, but did not give a talk at either.

4.2.8 Jérôme Franel (1859 – 1939)

Jérôme Franel was born on 29 November 1859 in Travers (canton Neuchâtel) where he grew up with his twelve siblings. His place of origin, however, was Provence (canton Vaud). After attending the industrial school in Lausanne he studied at the Polytechnic in Zurich for four years, at the Department for Mathematics and Physics Teachers. He then continued his studies in Berlin, where his teachers included Weierstrass, Kronecker and Kummer, and in Paris, where he attended Charles Hermite's lectures in particular. He graduated with a degree in mathematics from the Paris Academy in 1883 and returned to Switzerland to teach at his old school in Lausanne for a couple of years. In 1886 he was appointed to a professorship in mathematics in French, succeeding Eduard Méquet. He held this post until his retirement in 1929. At the beginning, he was the only mathematician who lectured in French; later on a second chair for mathematics in French was created. Franel co-supervised (at least) four PhD students, three jointly with Hurwitz and one with Hermann Weyl.

Franel served as the Polytechnic's Director from 1905-1909. In this capacity he fought for more liberal study regulations and for the Polytechnic's right to award doctorates. He also succeeded Geiser as president of the Federal Matura Committee, from 1909-1915, and 'he had a fortunate influence on the development of Swiss middle schools' [51, p. 440] – however, this influence is not elaborated. On several occasions Franel acted as intermediary between the Polytechnic and secondary schools or the *Gesellschaft ehemaliger Polytechniker*. Furthermore, he supported the students from French-speaking Switzerland

throughout his time at the Polytechnic. Together with Geiser, Herzog and Robert Gnehm he founded the Polytechnic's Civil Fund for Widows and Orphans.

In recognition of his work the University of Zurich awarded him an honorary doctorate in 1901. Four years later he was made an honorary citizen of the town Zurich; this happened on the occasion of the Polytechnic's 50th anniversary. As Director, Franel was heavily involved in organising the celebrations.

Franel was first and foremost a teacher and not a researcher. His former student and later colleague and friend Louis Kollros speaks of him very fondly, claiming that Franel 'was one of the School's most popular teachers' [51, p. 439]. His colleague George Pólya commented that 'Old Franel [was] interesting. He always dressed in the manner of an earlier generation' [70, p. 75]. He also said about Franel [70, p. 76]:

He is not very much remembered as a mathematician, but he was an especially attractive kind of person and a very good teacher. He gave the introductory lectures on calculus in French for several decades. He had a real interest in mathematics, but he was more interested in French literature. Teaching occupied a good deal of his time but in French literature he had to read everything available. He had no time left to do mathematics. But when he retired he suddenly tackled two of the great problems: the Riemann Hypothesis and 'Fermat's last theorem'.

Franel wrote a couple of papers on problems in geometry, but then turned to analysis and number theory. He published most of his papers in the 1890s, including work on Euler sums, a fundamental formula by Kronecker and the Riemann zeta function. Franel also regularly contributed to the French journal *L'Intermédiaire des mathématiciens*. His most important paper was *Les suites de Farey et le problème des nombres premiers*⁷⁸ which was published in the *Göttinger Nachrichten* in 1924, a few years before his retirement. In this short paper he

⁷⁸ "Farey Sequences and the Problem of Prime Numbers"

proved that it is possible to link the Farey sequence to the Riemann hypothesis. The German mathematician Edmund Landau, who held a professorship at the University of Göttingen at the time, then wrote a few papers on the same topic based on and expanding Franel's ideas. Guthery says of Franel's proof [40, p. 176]:

That the relationship between a series of fractions so simple can be connected to a mathematical hypothesis so profound with such economy is the mark of a teacher of mathematics of the very highest order.

Franel seems to have been married, and he had two daughters, Jeanne (*1889) and Marie-Louise (*1893).

Jérôme Franel died in Zurich on 21 November 1939.

Franel joined the organising committee in July 1896, where he was responsible for the French translations. Furthermore, he was on the committee for board and lodging and was asked to join the sub-committee that chose the speakers. With Geiser, Dumas and Hirsch he also edited the final congress programme. At the congress itself he acted as the general secretary for French. Franel did not give a talk himself, but he read out Poincaré's talk in the first general meeting, which he is remembered for. On a more trivial note, he was also the first to propose a toast (to Switzerland) at the congress banquet on the Monday.

Despite having been general secretary, Franel never had the chance to edit the congress proceedings in French. Originally, the organising committee had decided to publish German and French editions of the proceedings. But since the talks were to be published in the language in which they were given, the committee decided that the two editions would be identical for the most part, and that only a German edition should be published, with French translations of the most important speeches.

Franel was a member of the organising committee of the 1932 ICM in Zurich again, but did not give a talk at the congress.

4.2.9 Walter Gröbli (1852 – 1903)

Walter (originally Walther) Gröbli was born on 23 September 1852 in Oberuzwil (canton St. Gallen). He spent his childhood and attended primary school in his native village. Later the family moved to Töss (now part of the town Winterthur) in canton Zürich, where Walter attended the Industrieschule. His parents were Isaak and Elisabetha Gröbli, née Grob. Walter had two older brothers, Joseph Arnold and Hermann, and a younger sister. His four younger brothers all died in their infancy.

Isaak Gröbli (1822-1917) was a jacquard weaver who invented the “Schifflickmaschine”, a shuttle embroidery machine, in 1863. Soon the machine worked 10 times faster than a hand embroidery machine and became widely used, but it brought Isaak Gröbli only modest wealth. At the age of 64 he moved to Gossau (canton St. Gallen) to set up his own small embroidery business, supported by his son Hermann. The oldest son Arnold emigrated to the United States in 1876, where he refined his father’s invention. Letters preserved in Gröbli’s scientific estate [9] show that Arnold took an interest in his younger brother’s career.

Walter, however, was not interested in weaving, but in mathematics. He completed his school education at the Kantonsschule in St. Gallen. Encouraged and supported by his father, he studied mathematics at the Polytechnic from 1871-1875, at the Department for Mathematics and Physics Teachers. His lecturers included Heinrich Weber and Schwarz, ‘both of whom had great influence on his further scientific career’ [11, p. 3]. Weber in particular got Gröbli, whom he referred to as the ‘best student he had ever had’ [58, p. 25], interested in vortex theory and hydrodynamics. Probably encouraged by Weber and Schwarz, Gröbli went to Berlin to hear Kirchhoff, Helmholtz, Kummer and Weierstrass in 1875. In Berlin he caused a stir by solving the prize problem on vortex motion posed by Kirchhoff.

In 1876 Gröbli obtained his doctorate from the University of Göttingen for his thesis *Spezielle Probleme über die Bewegung gradliniger, paralleler*

*Wirbelflächen*⁷⁹. His supervisor was Schwarz⁸⁰; his oral examiners were Schwarz for mathematics and Johann Benedikt Listing for physics. Upon his return to Zurich in 1877 he habilitated as Privatdozent at the Polytechnic. Along with the request he had to submit at least two papers and three references; his referees were W Fiedler, Frobenius, and Geiser. The whole process, from application to appointment, took a remarkable two weeks [2]. Gröbli also became Frobenius's assistant for the following six years. He held his teaching post until 1894; he mainly lectured on hydrodynamics.

In 1883 he was appointed professor of mathematics at the Gymnasium in Zurich. By all accounts he was a very demanding (school) teacher, but impressed his pupils with 'his phenomenal proficiency in mental arithmetic, which is not widely spread among higher mathematicians and of which astounding stories were told' [58, p. 28]. Despite his mathematical talent and encouragement from his former professors and his colleagues, Gröbli did not pursue a career as a research mathematician; he was content with being a schoolteacher.

Gröbli married Emma Bodmer in 1899, but there are no verified records of whether the couple had any children. In one note [9a], Thomann writes that they had a son, Walter (1900-1975), in another [9d], that they did not and that the couple got divorced before Gröbli's death.

Even after he had stepped down from his post at the Polytechnic, Gröbli continued to take great interest in the latest mathematical research. He was also very interested in languages, reading English and German literature, and learning Italian in his forties.

⁷⁹ "Special Problems on the Motion of Rectilinear Parallel Vortices"

⁸⁰ There is some confusion as to who was Gröbli's actual supervisor. In [9e] the University of Göttingen confirms that Schwarz was his supervisor, but Rott suggests in [9c] that Heinrich Weber (1842-1913), professor of mathematics at the Polytechnic from 1870-1875, afterwards in Königsberg (until 1883), was in fact Gröbli's actual supervisor. Weber is best known for his textbook on partial differential equations of theoretical physics, known as "Riemann-Weber". A former student of Kirchhoff and Helmholtz himself, he was familiar with their work and would have been a likely choice of supervisor. Cf. biography by E F Robertson, J J O'Connor: http://www-history.mcs.st-andrews.ac.uk/Biographies/Weber_Heinrich.html, accessed 19/03/2014.

His main passion (besides mathematics) though was mountaineering. He climbed most of the more major peaks in the Alps; in many cases he was the first person to do so. Examples are climbing Piz Ela (3339m) in 1880 and traversing the north route of the Tödi (3614m). In addition, he was a keen hiker. Gröbli was a very active member of the Swiss Alpine Club: he led many mountaineering trips and served on the executive committee of the Club's local branch for many years. He also wrote several reports on his excursions for the Club's journal. A letter from the Federal Topographic Bureau [9b] suggests that Gröbli combined his two passions in measuring the heights of mountains and mountain passes.

On 26 June 1903, he led a group of 16 of his pupils on a hike on Piz Blas (3019m, canton Grisons). Due to bad weather conditions, the group had to choose a different route, but they were still caught in an avalanche. Walter Gröbli and two pupils, Ernst Hofmann and Adolf Odermatt, perished on the mountain; another pupil, Richard Liebmann, died of his injuries later on.

It is hard to say whether Gröbli would have written any mathematical papers had he not died so early. But as it stands his only publication was his doctoral thesis. The basic model that he investigated had been discovered by his former professor Helmholtz (*Crelle's Journal*, 1858), but:

The subject matter [of Gröbli's thesis] was the motion of three vortices, the motion of four vortices assuming the existence of an axis of symmetry, and the motion of $2n$ vortices assuming the existence of n symmetry axes.

[11, p. 9-10]

He had already worked on a similar topic for his dissertation at the Polytechnic. The thesis was cited a number of times in the late 19th century, by G R Kirchhoff, D N Goryachev and H Lamb for example. People then forgot about it until 1949, when J L Synge published a paper on the converse of the problem that Gröbli investigated. The thesis is still cited today, in fact, 'few

will write a thesis that will be the subject of attention a century later' [11, p. 20].

At the time of the congress Gröbli had already given up his teaching post at the Polytechnic. He joined the enlarged organising committee in December 1896. At the meeting on 08 December he was elected president of the finance committee (one of the four sub-committees). As such he was responsible for creating the committee's budget and asking individuals (merchants, manufacturers etc.) for donations.

4.2.10 Salomon Eduard Gubler (1845 – 1921)

Eduard Gubler was born on 07 July 1845. His place of origin was the village Wila in canton Zurich. According to the matriculation register at the University of Bern for the academic year 1870-1871 [82], Gubler graduated from the Polytechnic in 1870. He was a student at the University of Bern, but there are no records of when he left. However, he is listed as one of Schläfli's students [37, p. 143; 19, p. 18], and as a letter from Schläfli to Gysel [5] indicates, they visited each other occasionally. According to Graf, Gubler was one of Schläfli's former students who attended his funeral, along with Geiser, Bützberger and Gysel [37, p. 144].

Gubler 'spent his entire career in secondary education' [31, p. 83]⁸¹; he taught mathematics and geometry at both the women's teachers' college and the girls' Gymnasium in Zurich. In 1894 his doctoral thesis *Verwandlung einer hypergeometrischen Reihe in Anschluss an das Integral* $\int_0^\infty J_{(x)}^a e^{-bx} x^{c-1} dx$ ⁸² was published. His supervisor was Graf, which suggests that Gubler received his

⁸¹ Fehr writes "enseignement secondaire" in [31], but in fact Gubler taught both at secondary schools and at the University of Zurich.

⁸² "Transformation of a Hypergeometric Series Connected to the Integral $\int_0^\infty J_{(x)}^a e^{-bx} x^{c-1} dx$ „

doctorate from the University of Bern. However, there are no records of when he actually wrote and submitted the thesis.

In 1896 Gubler habilitated as Privatdozent at the University of Zurich with the paper *Über ein discontinuierliches Integral*⁸³. At the university, Gubler mainly lectured on algebraic analysis, number theory, higher algebra, planar and spherical trigonometry, integral calculus, and methodology of mathematics teaching at secondary schools.

Gubler's research interest was in the theory of Bessel functions, following the school of thought established by Schläfli's student Graf. Together with Graf, Gubler published *Einleitung in die Theorie der Bessel'schen Funktionen*⁸⁴ (1898-1900). Most of his publications are schoolbooks though, such as *Mündliches Rechnen. 25 Übungsgruppen. Zum Gebrauch an Mittelschulen*⁸⁵, and *Grundlehren der Geometrie für Sekundarschulen*⁸⁶ (1907). He also published a book on Leonardo da Vinci's mathematical works⁸⁷. Furthermore, he wrote reports on mathematics teaching in Swiss schools for various journals. The most important of these was *Der mathematische Unterricht an den höhern Mädchenschulen der Schweiz*⁸⁸ (1912).

Gubler co-founded the *Swiss Society of Mathematics Teachers* in 1901. In addition he was on the *Swiss Committee for Mathematics Teaching*. He retired from his teaching posts in 1914.

Eduard Gubler died in Zurich on 06 November 1921.

Gubler joined the enlarged organising committee in December 1896. A couple of months later he joined the welcoming committee, together with Hirsch and Burkhardt, in order to provide additional support.

Gubler attended both the 1904 ICM in Heidelberg and the 1908 ICM in Rome, but did not give any talks.

⁸³ "On a Discontinuous Integral"

⁸⁴ "Introduction to the Theory of Bessel Functions"

⁸⁵ "Mental Arithmetic. 25 Exercises. For the Use at Middle Schools"

⁸⁶ "Basic Principles in Geometry for Secondary Schools"

⁸⁷ *Leonardo da Vinci's mathematische Arbeiten* (1897)

⁸⁸ "Mathematics Teaching at Higher Girls' Schools in Switzerland"

4.2.11 Albin Herzog (1852 – 1909)

Albin Herzog was born on 26 October 1852 in Homburg (canton Thurgau). His father Johann was a teacher. Albin first attended primary school in his native village, then secondary school in Steckborn, and then the Kantonsschule in Frauenfeld, Thurgau's capital. In some obituaries [66; 76] it is commented that he always walked to school, an hour each way, regardless of the weather. His mathematics teachers in Frauenfeld were Wilhelm Schoch and his future colleague Rebstein.

Herzog was a very good student and matriculated at the Polytechnic in Zurich in 1870. He studied mathematics at the Department for Mathematics and Physics Teachers until 1874. The lecturers who influenced him most were Schwarz, Karl Culmann and Weber, as well as Geiser. After his graduation Herzog received a prize for solving a problem posed at his department, and in 1875 he received a doctorate from the University of Zurich for his thesis *Bestimmung einiger specieller Minimalflächen*⁸⁹.

In the same year he became Ludwig Kargl's⁹⁰ assistant and habilitated as Privatdozent at the Polytechnic. Herzog took on Kargl's lectures during the professor's illness and after his death. In 1877 Herzog was appointed to a full professorship in applied mechanics, which was quite unusual for the time given his young age. He held this post until his death in 1909.

Herzog was head of the Mechanical-Technical Department for some years, but stepped down from this post when he succeeded his good friend Geiser as Director of the Polytechnic (1895-1899). Geiser acted as Herzog's deputy. Together they founded the Polytechnic's Civil Fund for Widows and Orphans, but Herzog seems to have been the driving force behind the project. During his term in office he also actively supported his younger colleagues in creating an engineering laboratory. As a result, the Polytechnic was one of the first technical universities that could offer its students the opportunity to do practical work.

⁸⁹ "Determining Some Special Minimal Surfaces"

⁹⁰ Ludwig Kargl (1846-1875), from Vienna, professor for applied mechanics, mechanical engineering and geostatics at the Polytechnic; died of tuberculosis [84].

The ‘exceptionally gifted’ [35] teacher Herzog was renowned for his inspiring lectures. He was one of the first to bridge the gap between the mathematical basis of mechanics and practical applications of the subject. At the time there were two schools of thought in mechanics: one that advocated classical mechanics based on Lagrange’s work, and one that tried to explain mechanics in popular terms. Deeming the first as too analytical and missing rigour in the second, Herzog based his lectures on Bernoulli’s work and a synthetic approach. He was very successful with this method; his talent for combining theory and applications ‘are already apparent in his thesis [...], according to his competent friend [...] Geiser’ [77, p. 90]. In special, highly popular seminars he introduced his students to more advanced work by Maxwell and Minkowski, amongst others. The School Board named him a “life-long teacher” [67].

Engineering companies often asked him to help them solve practical problems, and in particular to recommend young engineers to them. Herzog was committed to further talented students; he also knew the name and background of every student at the Polytechnic [77, p. 88]. When the young Einstein failed the Polytechnic’s entry exam in 1895, Herzog recommended him to the Kantonsschule in Aarau, where Einstein indeed obtained the Matura. Having been a keen singer in his student years, Herzog supported the student choir throughout his life.

Beside his teaching and administrative duties, Herzog still found time to do research. He was ‘always busy with some new problem’ [77, p. 89], but subjected his work to a very strict self-censorship. His published research papers concern problems in applied mechanics, such as truss theory (1890 and 1891)⁹¹ and the properties of gearing mechanisms (1901)⁹².

In 1877, Herzog married Elise Bucher from Regensberg (canton Zurich), the daughter of Nationalrat* Bucher. They had three sons, two of whom became jurists and one became a chemist, and one daughter. The family lived in

⁹¹ *Beitrag zur Theorie des Fachwerkes, “Contribution to Truss Theory”*

⁹² *Über den Beschleunigungszustand eines Kurbelviereckes, “On the State of Acceleration of a Four-Bar Crank”*

Hottingen, and Herzog was president of the municipality's school authority (1889-1893). He resumed his position when Hottingen was incorporated into Zurich in 1893. Furthermore, he was a member of the Great City Council of Zurich from 1895-1898.

On the occasion of the Polytechnic's 50th anniversary in 1905, Herzog was one of its professors who were awarded honorary citizenship of the town Zurich. The other mathematicians honoured in this way were Franel, Graf and Lacombe. Herzog was a member of the GEP and the *Schweizerische Naturforschende Gesellschaft*. The street "Herzogstrasse" in the District 7 in Zurich was named after him in 1910 [85].

Having suffered from diabetes and frequent headaches, Albin Herzog died on 13 June 1909 from a stroke.

The original committee invited Herzog to join the organising committee in his capacity as Director of the Polytechnic in November 1896. He was tied up with business on 12 November 1896, but attended all the other committee meetings. He chaired the amusement committee, which organised the congress outings and dinners together with the reception committee. Herzog, Rudio and Hurwitz were the most active committee members with regards to organising the social side of the congress.

Herzog's position and connections proved useful for securing the necessary subventions from the authorities. While Geiser used his contacts in the Bundesrat* and the Kantonsrat* Zurich, Herzog discussed the town Zurich's financial contribution with councillor Grob [7d].

At the congress itself, Herzog chaired section IV: Mechanics and Mathematical Physics.

4.2.12 Arthur Hirsch (1866 – 1948)

Arthur Hirsch was born on 19 July 1866 in Königsberg, at the time a Prussian city. He attended both primary and secondary school there, and completed his school education in 1882. Hirsch studied mathematics, physics and

philosophy in his hometown and in Berlin. Among his teachers in Königsberg were Hilbert and his future colleague Hurwitz, who remarked that Hirsch was ‘one of his most talented students in Königsberg’ [3, p. 9]. In 1892 he received his doctorate from the University of Königsberg for his thesis *Zur Theorie der linearen Differentialgleichungen mit rationalem Integral*⁹³.

A year later he moved to Zurich where he habilitated as Privatdozent for mathematics at the Polytechnic, upon the recommendation of Hurwitz. He also became Hurwitz’s assistant, and took over some of his lectures when Hurwitz had to reduce his workload due to illness after 1900. Hirsch became Titularprofessor in 1897, and in 1903 he was appointed to an ordinary professorship, succeeding Minkowski in his chair for higher mathematics [34, p. 4]. He taught ‘differential equations, variational calculus and hypergeometric integrals of higher order’ [88], mainly to future engineers, ‘but he did not leave too many marks’ [ibid.]. Hirsch acted as co-advisor and second examiner for a number of PhD theses between 1916 and 1926, but none of the PhD candidates became influential in mathematics. He was Deputy Head of the Department for Mathematics and Physics Teachers for more than a decade⁹⁴; Pólya recalls that he also was ‘Department Head at the ETH in the first years I was there’ [70, p. 52].

Hirsch published a few papers in *Mathematische Annalen*, primarily on differential equations and integrals. Examples of his publications are *Die Existenzbedingungen des verallgemeinerten kinetischen Potentials*⁹⁵ (1898) and *Über bilineare Relationen zwischen hypergeometrischen Integralen höherer Ordnung*⁹⁶ (1899).

⁹³ “On the Theory of Linear Differential Equations With a Rational Integral”

⁹⁴ In 1909 Hirsch was listed as Deputy Head in the School Board minutes for the first time; he was re-elected every other year until 1921. After that the minutes do not contain the list of Department Heads anymore [93]. Pólya went to Zurich in 1914, but it is not apparent when Hirsch was meant to have been Department Head.

⁹⁵ “The Necessary Conditions for the Existence of the Generalised Kinetic Potential”

⁹⁶ “On Bilinear Relations Between Hypergeometric Integrals of Higher Order”

Hirsch was a member of the Swiss Mathematical Society; he also attended some of the German Mathematical Society's annual meetings. Unlike many of his German colleagues who stayed in Zurich for the rest of their lives, it seems that he did not acquire Swiss citizenship.

Arthur Hirsch retired in 1936 and died on 18 November 1948 in Zurich.

Hirsch joined the organising committee in December 1896. At the meeting on 21 January 1897 Hurwitz and Rudio suggested that Hirsch join the reception committee (alongside Burkhardt and Gubler), partly in order to help deal with the congress publications. As assistant German-speaking secretary he was also involved in finalising the congress programme (with Geiser, Franel and Dumas). During the congress he acted as Rudio's personal secretary.

Like his colleagues Dumas and Franel, Hirsch was on the organising committee for the 1932 ICM in Zurich, but did not give a talk during the congress.

4.2.13 Adolf Hurwitz (1859 –1919)

Adolf Hurwitz was born on 26 March 1859 in Hildesheim, at the time in the Kingdom of Hanover, now in Lower Saxony. His father Salomon (1813-1885) owned the Hurwitz-Deitelzweig Company, a hand-weaving mill producing bed linen amongst other things. Salomon was the oldest of the six children of the bookkeeper Jacob Isaac Hurwitz (1787-1852). In 1851 he married his distant cousin Elise (1822-1862), daughter of Moses Heinemann Wertheimer, who owned a private bank in Hanover. Salomon and Elise had three sons, Max, Julius, and Adolf, but their daughter Jenny died at the age of one. After Elise's death, Hurwitz's aunt Rosette kept house.

Salomon Hurwitz offered his sons a good education, encouraging them to engage in music, gymnastics, Jewish traditions and smoking, 'as he could scarcely imagine a proper gentleman without a cigar or even better a pipe' [3, p. 2]. The three brothers all had a particular talent for mathematics.

Adolf Hurwitz, a good gymnast and pianist, attended the science branch of the Andreanum⁹⁷, the Gymnasium in Hildesheim. His mathematics teacher there was Hermann Schubert⁹⁸, who recognised Hurwitz's mathematical talents early on and encouraged him to do independent research. In fact, he even met Adolf and his brother Julius regularly for additional classes. At the age of seventeen, Hurwitz published a paper together with his teacher: *Über den Chaslesschen Satz $\alpha\mu + \beta\nu$* ⁹⁹.

It was Schubert who persuaded Salomon Hurwitz to let his son study mathematics and secured a scholarship for him, funded by a Mr E Edwards. Schubert also recommended his pupil to Klein, and thus, in spring 1877, Hurwitz began his studies at the technical college (TH) in Munich, where Klein lectured at the time – mainly on number theory. After one semester only Hurwitz went to Berlin (1877-1878) to study analysis under Weierstrass and Kronecker. Most students went to Berlin towards the end of their studies [72, p. 859]. There he became friends with Ferdinand Rudio, whom he met in the university's mathematics society. Rudio recalls in [72, p. 860] that Hurwitz introduced him to Klein, Brill and Arthur Cayley (who was visiting Klein) when he passed through Munich in 1879 with his friend Alfred Amsler¹⁰⁰ – this deeply impressed the two young Polytechnic students.

Soon after his return to Munich, he followed Klein, by that time a good friend, to Leipzig. A year later, in 1881, Hurwitz received his doctorate for his thesis *Grundlagen einer independenten Theorie der elliptischen Modulfunctionen*

⁹⁷ During Hurwitz's time, the Andreanum comprised both the classic Gymnasium and the Realgymnasium, which he attended. It became a separate school in 1885. Cf. <http://www.andreanum.de/die-schule/chronik>, accessed 19/03/2014.

⁹⁸ Hermann Cäsar Hannibal Schubert (1848-1911) was one of the founders of enumerative geometry. He taught mathematics at the Andreanum in Hildesheim until 1876, afterwards at the Johanneum in Hamburg. Cf. biography by E F Robertson, J J O'Connor: <http://www-history.mcs.st-andrews.ac.uk/Biographies/Schubert.html>, accessed 19/03/2014.

⁹⁹ "On Chasles's Theorem $\alpha\mu + \beta\nu$ "

¹⁰⁰ Alfred Amsler (1857-1940): Swiss mathematician, eldest son of Jakob Amsler-Laffon. Cf. biography by R Amsler in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D30356.php>, accessed 19/03/2014.

*und Theorie der Multiplikatorgleichungen erster Stufe*¹⁰¹, dedicated to his sponsor Edwards. His supervisor was, of course, Klein, who not only introduced Hurwitz to number theory and geometry, but, even more importantly, also to Riemann's ideas, 'which at the time were not yet common knowledge as they are today [1920]. In a manner of speaking, knowing them meant moving up into a higher class of mathematicians' [42, p. 371].

Hurwitz spent the year 1881/82 in Berlin, again to hear Weierstrass and Kronecker. In 1882 he habilitated at the University of Göttingen, where he got to know Moritz Stern¹⁰² and Wilhelm Weber¹⁰³. But already in 1884 Hurwitz was appointed to an extraordinary professorship* at the University of Königsberg, upon the recommendation of Ferdinand von Lindemann. There he met David Hilbert and Hermann Minkowski, both of whom became close friends [ibid.]:

Over the course of eight years we must have rummaged through every nook of mathematical knowledge on countless, at times daily walks, and Hurwitz, with his knowledge that was just as extensive and eclectic as it was well-grounded and well-ordered, always was our leader.

Hurwitz spent the summer of 1888 in Stockholm where he visited Mittag-Leffler. In 1892, he was offered professorships at both the Polytechnic in Zurich (as Frobenius's successor) and the University of Göttingen (as Schwarz's successor). He chose to go to Zurich, as Rudin explains [72, p. 856]:

Bleuler, at the time President of the Polytechnic's School Board, had gone to Königsberg himself and had already come to an agreement

¹⁰¹ "Basic Principles of an Independent Theory of Elliptic Modular Functions and Theory of Multiplier Equations of First Degree"

¹⁰² Moritz Abraham Stern (1807-1894), German mathematician. Stern was the first Jew to be appointed to an ordinary professorship at a German university; cf. biography by S Eminger:

<http://www-history.mcs.st-andrews.ac.uk/Biographies/Stern.html>.

¹⁰³ Wilhelm Eduard Weber (1804-1891), German physicist. Cf. biography by E F Robertson, J J O'Connor:

<http://www-history.mcs.st-andrews.ac.uk/Biographies/Weber.html>, accessed 19/03/2014.

with Hurwitz, when the offer from Göttingen arrived. Probably it is due to this fact only that the brilliant Hurwitz became ours and, until a few weeks ago, placed a great part of his creative powers and his whole talent into the service of our country.

At the Polytechnic, he first lectured on differential geometry, but in 1902 he succeeded Minkowski as lecturer in the mathematical seminars, teaching algebra, number theory and theory of functions. He 'was among the outstanding and most successful university lecturers' [50] of mathematics, and had a large number of PhD students, among them Amberg and his own brother Julius, who had begun studying mathematics at the age of 33. Hurwitz supported many young mathematicians, among them George Pólya¹⁰⁴, whom he recommended to the Polytechnic's School Board. They became good friends, and Pólya recalls in [70, p. 25] that:

We had a special way we worked. I would visit him and we would sit in his study and talk mathematics – seldom anything else – until he finished his cigar. Then we would go for a walk, continuing the mathematical discussion.

His approximately one hundred publications cover a wide range of problems in the theory of functions, number theory, algebra and geometry. He 'had a great mathematical breadth, as much as was possible in this time' [ibid.]. During his time in Königsberg he wrote a number of papers where he investigated algebraic problems using Riemann's methods, and a couple of papers where he applied function theory to separating the roots of transcendental equations. Moreover, he worked on the theory of arithmetic continuous fractions; an example is *Über die Entwicklung komplexer Größen in*

¹⁰⁴ Pólya was appointed to the ETH in 1914. He was one of the main editors of Hurwitz's works. Pólya describes his relationship with Hurwitz in [70, p. 25]. For a more detailed description, see G L Alexanderson, *The Random Walks of George Pólya*, The Mathematical Association of America, 2000, 35-38.

*Kettenbrüchen*¹⁰⁵. In Zurich he worked on algebraic number fields, ideal theory, and invariant theory. One example is *Über lineare Formen mit ganzzahligen Variablen*¹⁰⁶, where he proved Minkowski's theorem on linear forms. Furthermore, he wrote papers on lemniscates, Fourier series and quaternions. His only book was *Vorlesungen über die Zahlentheorie der Quaternionen*¹⁰⁷, published in 1919. In addition, he left more than thirty notebooks or diaries, in which he jotted down mathematical ideas. A few mathematical structures are named after him: The Hurwitz polynomial, a polynomial with real coefficients whose roots all have a negative real part; the Hurwitz quaternion, a quaternion whose four coefficients are either all integers or all half-integers; and the Hurwitz zeta function $\zeta(s,q) = \sum_{n=0}^{\infty} \frac{1}{(q+n)^s}$ where s and q are complex variables and $\text{Re}(s) > 1$, $\text{Re}(q) > 0$. He also solved the problem of determining the nature of roots of the characteristic equations used by his colleague Aurel Stodola to model the regulation of hydraulic turbines. The solution, which does not require the differential equations to be solved, is known as the Routh-Hurwitz stability criteria [16].

Together with Rudio, Hurwitz edited Gotthold Eisenstein's letters to Stern (1895).

Hurwitz was elected an honorary member of the mathematical societies in Kharkov, Hamburg and London. Moreover, he was a corresponding member of the mathematical society in Göttingen (from 1892 onwards; he became a non-resident member in 1914) and a non-resident member of the Academia dei Lincei in Rome.

In 1892, he married Ida Samuel, whose father had been a physician in Königsberg and a good friend. The couple had three children: Lisbeth (*1894), Eva (*1896) and Otto (*1898). Eva began studying mathematics at the ETH in 1915, but she dropped out a few years later and turned to 'extreme-

¹⁰⁵ "On the Development of Complex Quantities in Continuous Fractions"

¹⁰⁶ "On Linear Forms With Integer Variables"

¹⁰⁷ "Lectures on the Number Theory of Quaternions"

revolutionary' politics, much to her parents' dismay [3, p. 12]. Lisbeth pursued a career as a social welfare worker and Otto studied chemistry.

Apart from being a gifted mathematician, Hurwitz was also an accomplished pianist. Already in Göttingen he regularly played chamber music with various academics. In Zurich, he regularly hosted musical soirées. Einstein played with Hurwitz and Lisbeth every week during his time in Zurich (1912-1914). Furthermore, Hurwitz was interested in literature and philosophy.

Although he retained his German citizenship, he never moved back to Germany. Few German universities could offer a chair as prestigious as the one he held in Zurich, and some universities, like Leipzig, still did not appoint any Jews. Furthermore, the Polytechnic was very accommodating and life in Zurich very agreeable [3, p. 11].

Hurwitz had suffered from severe migraine since his youth; in Berlin and in Königsberg he was taken ill with typhoid. Following his recovery from pneumonia, the family moved out of the city of Zurich, but Hurwitz continued to fall ill. In 1905 one of his kidneys had to be removed, and soon the other one became diseased, too. Hurwitz continued to lecture as long as possible, and often held the seminars in his living room.

Adolf Hurwitz died in his home on 18 November 1919.

Hurwitz joined the organising committee at the initial meeting in July 1896 and became one of the main organisers of the congress. He was president of the reception committee, which was responsible for organising the social part of the congress together with the amusement committee, and for providing all congress publications (invitations, programmes, regulations, resolutions, posters, badges etc.) in both German and French. The reception committee seems to have had the highest workload out of the four sub-committees, and Hurwitz suggested that a publications committee be set up in order to relieve his committee of some of its duties. In the end, the organising committee opted for Rudio's suggestion of enlarging the existing reception committee. Together with Geiser and Minkowski, Hurwitz was responsible for choosing

the plenary speakers (on Rudio's suggestion). He also asked Klein to contribute to the preparations [8c]. Furthermore, Hurwitz wrote up the attendance list, which was given to every congress participant. He also reviewed the musical entertainment during the steamboat excursion.

On 08 August, Hurwitz spent the entire day 'at the train station, welcoming the arriving mathematicians [...], handing out the congress cards and organising accommodation for the arrivals if desired' [73, p. 22]. In the evening, at the collation in the Tonhalle, he gave a welcome speech, in which he stressed the importance of personal relations between mathematicians of different countries.

At the congress itself, he chaired section II: Theory of Functions and Analysis. More importantly, he was one of the four plenary speakers, and the only one of them representing Switzerland. He gave his talk *Entwicklung der allgemeinen Theorie der analytischen Funktionen in neuerer Zeit*¹⁰⁸, which Hilbert describes as 'exemplary due to the clear and concise style as well as the successful selection of this so extensive topic' [42, p. 375], in the first general meeting on 09 August.

4.2.14 Adolf Kiefer (1857 – 1929)

Adolf Kiefer was born in Selzach (canton Solothurn) on 22 June 1857. His father Jakob was a farmer, as well as the village's mayor and a member of the cantonal parliament. Adolf first attended primary school in Selzach, then the Bezirksschule¹⁰⁹ in Grenchen, and finally the Kantonsschule in Solothurn. In 1876 he matriculated at the Polytechnic in order to study mathematics and

¹⁰⁸ "Development of the General Theory of Analytic Functions in Recent Times"

¹⁰⁹ In canton Solothurn the Bezirksschule (literally: "district school") was a secondary school aimed at pupils who obtained good to very good grades in primary school. It prepared them both for apprenticeships and further education colleges. Contrary to Kiefer's education, most pupils did continue their studies at a Kantonsschule. Cf. newspaper article in the *Solothurner Zeitung* by E Seifert: <http://www.solothurnerzeitung.ch/solothurn/kanton-solothurn/als-die-bezirksschule-noch-die-einzige-alternative-zur-oberschule-war-126929743>, accessed 19/03/2014.

physics in the Department for Mathematics and Physics Teachers. Four years later he obtained his diploma as mathematics teacher.

His first teaching post was at the Concordia Institute, a college in Zurich, from 1881-1882. He also seems to have attended lectures at the University of Zurich, though the university's matriculation register only lists his son Adolf (1892-1951) [92]. In any case Kiefer senior obtained his doctorate from the University of Zurich in 1881 for his thesis *Der Kontakt höherer Ordnung bei algebraischen Flächen*¹¹⁰.

From 1882-1894 Kiefer taught geometry and technical drawing at the Kantonsschule in Frauenfeld (canton Thurgau), primarily at the school's vocational section, the Industrieschule. In 1886 he became deputy head and two years later headmaster. However, 'much to the regret of his superiors, colleagues and pupils' [75, p. 444] he left the school in 1894, as he became Director of the Concordia Institute. The college was closed after the First World War, but Kiefer found teaching posts at the Kantonsschule, the technical college and the teachers' college in Zurich.

Kiefer published almost forty papers, most of which are on geometry. Among his papers are *Über Kräftezerlegung*¹¹¹ (1904), *Über die Kettenlinie*¹¹² (1915), *Von der Cykloide*¹¹³ (1917), *Zum Normalenproblem bei den Flächen zweiten Grades*¹¹⁴ (1921), and *Zwei spezielle Tetraeder*¹¹⁵ (1925). He became an honorary member of the *Schweizerische Naturforschende Gesellschaft* in 1928.

Adolf Kiefer retired in 1926 after having fainted in his classroom. He died three years later, on 15 November 1929.

Kiefer joined the enlarged committee, as well as the committee in charge of board and lodging, in December 1896. His duties included organising accommodation for the Swiss secondary school teachers who attended the

¹¹⁰ "Contacts of Higher Order of Algebraic Surfaces"

¹¹¹ "On the Decomposition of Forces"

¹¹² "On the Catenary"

¹¹³ "On the Cycloid"

¹¹⁴ "On the Problem of Normals of a Quadratic Surface"

¹¹⁵ "Two Special Tetrahedrons"

congress – a certain Mr Bertsch, of the Concordia Institute in Zurich, offered Swiss teachers free accommodation there in July 1897.

4.2.15 Gustav Künzler

Gustav Künzler obtained a diploma as a mathematics teacher from the Polytechnic in 1888. He then habilitated and worked as Privatdozent there. In the list of members of the *Naturforschende Gesellschaft Zürich* from 1899 he is referred to as ‘professor at the Technical College Biel’. Künzler attended the annual meeting of the *Schweizerische Naturforschende Gesellschaft* in 1898 and gave a talk there: *Doppelcurven von abwickelbaren Flächen*¹¹⁶.

Künzler was Privatdozent at the Polytechnic at the time of the congress. He joined the organising committee in December 1896, but did not have any particular position. At the congress he acted as secretary in section III: Geometry.

4.2.16 Marius Lacombe (1862 – 1938)

Marius Lacombe was born on 07 February 1862 in Lausanne. His place of origin was Begnins (canton Vaud) and it seems that he grew up there. Lacombe studied mathematics at the Polytechnic’s Engineering Department and became a professor at the University of Lausanne in 1892. In 1894 he was appointed to a professorship at the Polytechnic in Zurich: Lacombe filled the newly created chair of descriptive geometry in French. Before that, Franel was the only mathematician at the Polytechnic who taught in French. The second professorship was established due to ‘popular demand’ [63]; and the Swiss government hoped that German-speaking students too would attend Lacombe’s lectures.

¹¹⁶ “Double Curves of Developable Surfaces”

Lacombe stayed in Zurich for 14 years. In 1908 he moved back to Lausanne, apparently because of family reasons, as the Swiss government put it in a notice in [64]:

Like us, many will indeed be sorry that this teacher, who is so popular among his colleagues and in particular also among his students, leaves our university in order to return to his native region due to family matters.

Lacombe stayed in Lausanne for the rest of his life. He taught descriptive, projective and analytic geometry at the Engineering School of the University of Lausanne¹¹⁷ until he retired in 1927. He served as the school's Director from 1911-1918. In his last year in office, he founded the university's Laboratory of Materials Testing, which still exists today.

Lacombe was more teacher than research mathematician. The only publications listed in the Swiss libraries' union catalogue concern mathematics education: *L'enseignement des mathématiques élémentaires dans le Ct. de Vaud*¹¹⁸ (1893) and *L'enseignement mathématique à l'Ecole d'Ingénieurs de Lausanne*¹¹⁹ (1911, with Graf). Lacombe believed that his students, all future engineers, should know how to use mathematics to solve technical problems, rather than be taught pure mathematics [62]:

Even though he is not an engineer, Mr Lacombe is very well informed about everything that concerns technical education. He is not one of those who consider mathematics to be simple gymnastics or an adornment of the mind. He knows that it is a means and not a goal, and he wants his students to know how to apply the tool that he gives to them in practice. The problems and exercises – particularly in

¹¹⁷ The *Ecole d'ingénieurs de l'Université de Lausanne* was the technical department of the University of Lausanne. It originated from a private technical school founded in 1853 that was based on the model of the *Ecole centrale* in Paris. In 1946 the school was renamed as *Ecole polytechnique de l'Université de Lausanne (EPUL)*; in 1969 it became an independent institution. As *Ecole polytechnique fédérale de Lausanne (EPFL)* it now is the second federal university in Switzerland. A brief history of the institution can be found at <http://information.epfl.ch/historique>, accessed 19/03/2014.

¹¹⁸ "Elementary Mathematics Education in Canton Vaud"

¹¹⁹ "Mathematics Education at the Engineering School of Lausanne"

descriptive geometry – that he poses his students are always applications of real-life cases that every engineer will face in his career. In his lectures at the Engineering School he chooses to ignore problems that are only of a purely theoretical or virtuosic interest.

He was one of the first in Switzerland to introduce his students to new techniques used in industry, such as photogrammetry.

In 1905, on the occasion of the Polytechnic's 50th anniversary, he was awarded honorary citizenship of the town Zurich, alongside his colleagues Franel and Herzog. Lacombe also received an honorary doctorate from the University of Lausanne.

Marius Lacombe died on 19 March 1938 in Chexbres (canton Vaud).

Lacombe joined the enlarged organising committee in December 1896. He was one of the members of the finance committee and also organised a special breakfast for the congress participants on the last congress day. At the congress itself he chaired section III: Geometry.

4.2.17 Hermann Minkowski (1864 – 1909)

Hermann Minkowski was born on 22 June 1864 in Aleksotas, at the time a Russian town, today part of Kaunas in Lithuania. He was the youngest son of Lewin (ca. 1825-1884), a grain merchant, and Rachel née Taubmann (ca. 1827-1904). The Minkowskis were Lithuanian Jews; in 1872 they emigrated to Königsberg, at the time a Prussian city, due to Russia's anti-Semitic politics. Hermann's oldest brother Max (1844-1930) took over the family business, but he was also an art collector and the French consul in Königsberg. The second brother Oskar (1858-1931) was a physician, best known for his work on diabetes, and father of astrophysicist Rudolph Minkowski (1895-1976). Apart from Max and Oskar, Minkowski also had an older sister, Fanny (1863-1954), and a younger brother, Toby (1873-1906).

Minkowski attended the Altstädtisches Gymnasium in Königsberg, where he obtained his leaving certificate already at the age of fifteen. In 1880 he matriculated at the University of Königsberg to study mathematics. His main teachers were Heinrich Weber and Woldemar Voigt. During three semesters in Berlin he attended Weierstrass and Kronecker's lectures in particular. In Königsberg he met Hilbert and Hurwitz; Hilbert in particular remained a very close friend throughout the rest of his life.

In 1885 Minkowski received his doctorate for his thesis *Untersuchungen über quadratische Formen, Bestimmung der Anzahl verschiedener Formen, welche ein gegebenes Genus enthält*¹²⁰. His supervisor was Lindemann. Two years later he habilitated at the University of Bonn as Privatdozent, and in 1892 he was appointed to an extraordinary professorship. In 1894 he succeeded Hilbert in Königsberg, but already two years later he moved to Zurich as he had been offered an ordinary professorship of higher mathematics at the Polytechnic. Minkowski only stayed in Zurich for a few years: in 1902 he went back to Germany to fill a new chair at the University of Göttingen, which Friedrich Althoff¹²¹ had created specifically for him. Minkowski stayed there until his death; he primarily worked with Hilbert and Klein. He had a number of doctoral students, among them Constantin Carathéodory and Kollros.

¹²⁰ "Studies on Quadratic Forms, Determining the Number of Different Forms That Are Contained in a Given Genus"

¹²¹ Friedrich Theodor Althoff (1839-1908), employee in the Prussian Ministry of Education. Although he never became minister, he had great influence on the Prussian system of higher education, and, by extension, on universities in the other German-speaking countries. Nicknamed 'Bismarck of German higher education', he played a decisive role in many university appointments in the late 19th and early 20th centuries. Furthermore, he founded and funded many research institutes and associations. Although Althoff's methods were criticised (for example, he often overrode the universities' decisions in favour of his own judgement), the excellence of the German universities at the time was mainly due to his work. In particular, his influence was crucial to Göttingen's reputation as a world-leading centre for mathematics and physics. With regard to appointing professors, it is interesting to note that Althoff's methods were similar to those of the Polytechnic's School Board: He, too, often attended the candidates' lectures incognito, and attached great importance to scientific excellence and originality. See biographies by F Schnabel, in *Neue Deutsche Biographie* **1**, 1953, 222-224: <http://www.deutsche-biographie.de/sfz726.html>; and by B Thomann: <http://www.rheinische-geschichte.lvr.de/persoenlichkeiten/A/Seiten/FriedrichAlthoff.aspx>, accessed 19/03/2014.

Minkowski was a very versatile mathematician; his publications cover problems in number theory, group theory, geometry, quadratic forms, and mathematical physics. He wrote his first more major work *Mémoire sur la théorie des formes quadratiques à coefficients entiers*¹²² at the age of seventeen. In 1882 the Académie des Sciences in Paris set a problem on the decomposition of integers into a sum of five squares, based on work by Eisenstein. Minkowski's solution was a much more general treatment of the theory of quadratic forms with integer coefficients, for which he received the Grand Prix des Sciences Mathématiques in 1883, jointly with the British mathematician Henry Smith. He elaborated on this topic in his doctoral thesis.

Based on his early works on quadratic forms and their reduction, as well as his studies of Hermite's work in number theory, Minkowski developed his "geometry of numbers". It is related to classic Euclidean geometry, but Minkowski uses the properties of convex solids in a lattice to solve problems in number theory. One of his results is known as Minkowski's theorem: a convex solid in n -dimensional space, whose centre is a point in the lattice and whose volume is 2^n , contains at least two more points in the lattice [43, p. 346]. Furthermore, he found that the number of classes of positive quadratic forms with n variables and a given determinant is finite. These and many more theorems were published in his seminal work *Geometrie der Zahlen*¹²³ (1896). Hermite, who greatly admired Minkowski's work and recognised its importance, commented on the book in a letter to Auguste Laugel: "'Je crois voir la terre promise' – I have seen the Promised Land" [43, p. 348].

In 1907 Minkowski published his second major book on the geometry of numbers, *Diophantische Approximationen*¹²⁴, which originated from his lectures in Göttingen in 1903/04. Devised as a more accessible approach to Minkowski's theorems than his first book, its focus is on applying the geometry of numbers to approximating real and complex quantities with real numbers, to algebraic number fields and to quadratic forms. Furthermore,

¹²² "Note on the Theory of Quadratic Forms With Integer Coefficients"

¹²³ "Geometry of Numbers"

¹²⁴ "Diophantine Approximations"

Minkowski published a few long papers in various journals; among them *Über die Annäherung an eine reelle Größe durch rationale Zahlen*¹²⁵ (1901), in which he developed continuous fractions for real numbers using parallelograms; and *Dichteste gitterförmige Lagerung kongruenter Körper*¹²⁶ (1904), one of his papers on the problem of stacking congruent solids. In other papers he used convex solids to prove problems in geometry and in analysis.

Despite the importance of his geometry of numbers, today Minkowski is mainly remembered for his contributions to the theory of relativity. He had a great interest in mathematical physics, particularly in electrodynamics. His first paper was on hydrodynamics though, published in 1888. Two years later he wrote to Hilbert that he was ‘completely contaminated with physics [and] would have to be put in quarantine for ten days before Hurwitz and [Hilbert] would deem him mathematically pure and admit him to [their] mathematical walks’ [43, p. 355].

In his most important work, *Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern*¹²⁷ (1908), Minkowski used Einstein and Lorentz’s work on the theory of relativity to develop the mathematical foundation for four-dimensional electrodynamics¹²⁸. He realised that all natural laws are invariant to any Lorentz transformations and are thus independent of space and time. Furthermore, he created the so-called “Minkowski space”: a four-dimensional space-time formed by the three ordinary space dimensions and time as the fourth dimension. He presented this structure in his talk *Raum und Zeit*¹²⁹ that he gave in Cologne in 1908. Essentially, Minkowski provided the mathematical setting for Einstein’s special relativity. Einstein did not immediately recognise the significance of these results, but he used them later on when formulating his general relativity. Incidentally, Einstein was one of

¹²⁵ “On Approximating Real Quantities Using Rational Numbers”

¹²⁶ “The Most Dense Stacking of Congruent Solids in a Lattice”

¹²⁷ “Fundamental Equations for the Electromagnetic Processes in Moving Solids”

¹²⁸ Geiser refers to Minkowski’s paper in *Erinnerung an Theodor Reye* (see appendix E.3.2) as he uses Einstein and Lorentz’s results to support his argument that mathematics has a valuable place in education for its own sake.

¹²⁹ “Space and Time”

Minkowski's students at the Polytechnic, 'but skived the majority of his lectures' [90].

Minkowski was very influential in both pure mathematics and mathematical physics. Dumas wrote about him [29, p. 141]:

The future will show the importance of Minkowski's unfinished work. One of his merits [...] was his independence of any tradition. Not fundamentally attaching himself to any school, he knew how to be his own master. He paved new ways and discovered unknown paths.

In 1897 Minkowski married Auguste Adler (1875-1944) in Strasbourg. They had two daughters, Lily (1898-1983), who later married the engineer Reinhold Rüdenberg, and Ruth (1902-1983), later married to Franz Buschke, a radiologist. Hermann and Auguste's three grandsons also became scientists [76]. A moon crater and an asteroid are named after Hermann Minkowski.

Hermann Minkowski died on 12 January 1909 in Göttingen, due to a ruptured appendix.

Minkowski joined the organising committee in December 1896 – he might not yet have been in Zurich for the preliminary meeting in July. He joined the amusement committee and was appointed to the sub-committee that was responsible for choosing the speakers. He suggested inviting Hilbert to give a talk in case Klein could not attend [8d], as it was, Klein did attend the congress but Hilbert did not. Minkowski also offered to give a talk himself in one of the section meetings, but for reasons that are not explained in the minutes he did not after all. At the congress, he chaired section I: Arithmetic and Algebra.

Minkowski acted as one of the secretaries at the 1900 ICM in Paris, and gave a talk in section I at the 1904 ICM in Heidelberg, entitled *Zur Geometrie der Zahlen*¹³⁰. At this point he represented the University of Göttingen, likewise at the 1908 ICM in Rome.

¹³⁰ "On the Geometry of Numbers"

4.2.18 Johann Jakob Rebstein (1840 – 1907)

As mentioned in section 4.2.1 (footnote 22) there seem to have been two Jakob Rebsteins at the Polytechnic at the time of the ICM. The ETH website only lists Johann Jakob Rebstein among its professors [84]. A small note in Rebstein's biographical dossier informs us that the other Jakob Rebstein taught mathematics from 1898-1932 (and brought Einstein to the ETH to teach). I have not been able to find any other information on this other Rebstein, apart from a paper entitled *Der Massenausgleich des Kuppelstangenantriebs bei elektrischen Lokomotiven* (SBZ 62 (8), 1913, 105-109), written by J Buchli and "Prof Dr J Rebstein in Winterthur". As we shall see, this would have been outside of Johann Jakob Rebstein's research interests. Furthermore, there is no apparent connection to Winterthur, and he passed away in 1907.

Johann Jakob Rebstein was born on 04 May 1840 in Töss (canton Zurich) as the oldest son in a family of six children. His father was a baker and innkeeper; his mother's father had been a country doctor and surgeon. Rebstein grew up in his native village and attended primary school there. He was a very intelligent boy [15, p. 73]:

Schaggi¹³¹ Rebstein was soon regarded as a sort of wonder boy in his village; in particular he was very good at mental arithmetic.

Afterwards he attended first secondary school and then the Industrieschule in Winterthur. It is reported that he solved geometry problems in his free time. In 1857 he matriculated at the Engineering Department of the Polytechnic. A year later he transferred to the Department for Mathematics and Physics Teachers, as he felt that he lacked technical drawing skills. He still kept an interest in engineering though: his favourite lectures at his new department were Johannes Wild's¹³² on topography. In his holidays he could apply his knowledge of geometry by surveying fields for the farmers in Töss; but he wanted to become a teacher, not a geometer.

¹³¹ Affectionate form of Jakob

¹³² Johannes Wild (1814-1894), professor for topography and geodesy at the Polytechnic from 1855-1889 [84].

Rebstein graduated from the Polytechnic in 1860, and went to study at the Collège de France in Paris for a year. His original plan was to complete his education in Göttingen, but Kappeler, at the time president of the Polytechnic's School Board, recommended him to the Kantonsschule in Frauenfeld (canton Thurgau). Rebstein had hoped for an academic career at the Polytechnic, but Kappeler's persuasiveness and the fact that his father had just died, leaving his mother and siblings with only a small income, made him take up the post as mathematics and physics teacher [15, p. 75]. He taught in Frauenfeld until 1877. From 1877-1898 he was professor of mathematics and physics at the Kantonsschule in Zurich. In addition he taught first mathematics, then geodesy at the Polytechnic's Elective Department as a Privatdozent; he habilitated in 1873.

In 1895 he was awarded a doctorate for his thesis *Bestimmung aller reellen Minimalflächen, die eine Schaar ebener Curven enthalten, denen auf der Gauss'schen Kugel die Meridiane entsprechen*¹³³. A year later he became Titularprofessor at the Polytechnic and in 1898 he was appointed to a full professorship for cadastral surveying¹³⁴, adjustment theory and actuarial mathematics. He seems to have held this post until a few weeks before his death.

Rebstein mainly taught at the Polytechnic's School of Cultural Engineering¹³⁵, but also at the Engineering and Elective Departments. For a number of years he was head of the School of Cultural Engineering. He was also heavily involved in creating the Polytechnic's Civil Fund for Widows and Orphans, together with Herzog and Geiser.

Today Rebstein is primarily known for his pioneering work as a geometer and surveying expert. At the time when he moved to Frauenfeld, geometers in the Grand Duchy of Baden were experimenting with traverse, a new

¹³³ "Determining All Real Minimal Surfaces That Contain a Family of Planar Curves, Which Correspond to the Meridians on the Gaussian Sphere"

¹³⁴ Cadastral surveying is used to determine and maintain the boundaries of land parcels. The data is used for entries in registers of real estate as well as a variety of maps. Cadastral surveys can only be conducted by authorised land surveying offices – in Switzerland these can be cantonal or, on a smaller scale, private. See <http://www.cadastre.ch/>, accessed 30/07/2014.

¹³⁵ Kulturingenienschule: founded in 1889 as part of the Department for Agriculture and Forestry.

surveying technique. Rebstein first heard about this method when visiting relatives in Baden and was instrumental in introducing it in Switzerland. Indeed, traverse replaced the less accurate method of triangulation in Switzerland towards the end of the 19th century.

Rebstein was an expert in using the method of least squares and the theory of errors and applying them to cadastral techniques. In 1863, at the age of 23, he was appointed surveying expert for canton Thurgau. He held this position until 1881; later he was also a surveying expert for the cities of St. Gallen (1881-1894), Zurich (1886-1892), and Luzern (1894-1907). He 'never conducted any major surveying project himself' though, but 'examined and revised those of others, among them experienced and proficient men' [15, p. 77]. In fact, leading experts in geodesy such as Gauß¹³⁶, Helmert¹³⁷ and Jordan¹³⁸ regularly asked him to review their works.

For many years Rebstein was a member of the examining board of the Swiss Concordat of Geometers. In 1868 he was elected into the board as a substitute (to replace Wild) and became a permanent member then. He served as the board's president from 1887 until his death. Moreover, he also chaired the commission of geometers who measured the perimeter, i.e. the surface area, of the Rhine in canton St. Gallen for a number of years. In 1888 he joined the Swiss Committee of Geodesy; he remained a member until his death. The committee contributed to the Austro-German project of measuring and determining the shape of the Earth's surface.

¹³⁶ Friedrich Gustav Gauß (1829-1915): German geodesist, he was one of the main developers of the cadastral land register of Prussia. He also published log tables. Cf. biography by W Großmann, in *Neue Deutsche Biographie* 6, 1968, 108: <http://www.deutsche-biographie.de/sfz20037.html>, accessed 19/03/2014.

¹³⁷ Friedrich Robert Helmert (1843-1917): German geodesist and mathematician. He developed methods to determine geoids and established the mathematical and physical foundations of modern geodesy, e.g. by introducing the method of least squares and the theory of errors. Cf. biography by R Sigl, in *Neue Deutsche Biographie* 8, 1969, 497-498: <http://www.deutsche-biographie.de/sfz29587.html>, accessed 19/03/2014.

¹³⁸ Wilhelm Jordan (1842-1899): German geodesist and mathematician. He lectured at the polytechnics in Stuttgart, then Karlsruhe, and then Hannover; he was instrumental in reorganising geodetic training in Germany in the 1870s. Cf. biography by W Großmann, in *Neue Deutsche Biographie* 10, 1974, 604-605: <http://www.deutsche-biographie.de/sfz37836.html>, accessed 19/03/2014.

Among Rebstein's publications his *Lehrbuch über praktische Geometrie mit besonderer Berücksichtigung der Theodolitmessung*¹³⁹ (1868) stands out, as most of his publications are surveying and actuarial reports. Examples of his cadastral work are *Die Kartographie der Schweiz, dargestellt in ihrer historischen Entwicklung*¹⁴⁰ (1883), a report for the Swiss National Exhibition, and *Mitteilungen über die Stadtvermessung von Zürich*¹⁴¹ (1892).

His actuarial papers include expert opinions on the civil funds of Swiss railway companies (1904 and 1906, both with G Schärtlin) and reports on the civil funds at the Polytechnic and the University of Zurich. Rebstein conducted many studies in his capacity as auditor of the Swiss Life Insurance and Pensions Service. In fact, in 1905 the University of Zurich awarded him an honorary doctorate 'in recognition of his outstanding contributions to actuarial sciences' [68, p. 153].

Rebstein was a very active member of the GEP throughout his life and served as the association's president from 1881-1885. Later on he was made an honorary member. Furthermore, he was a member of the *Schweizerische Naturforschende Gesellschaft* for most of his life (he joined in 1864) and chaired the *Thurgauische Naturforschende Gesellschaft*¹⁴² for a number of years. Like many of his colleagues at the Polytechnic (and on the organising committee), Rebstein was a keen hill walker and mountaineer. He seems to have been married and had children, but there are no records of their names.

For the last ten years of his life he suffered from a kidney disease. Jakob Rebstein died on 14 March 1907 in Zurich.

In the minutes of the organising committee we find references to "assistant Rebstein", "Dr Rebstein" and "Prof Rebstein" [8] across various meetings. It is possible that "assistant" and "Dr" refers to the same person. A "Dr Rebstein" joined the organising committee at the preliminary meeting in July 1896 as the

¹³⁹ "Textbook on Practical Geometry With Special Consideration of Theodolite Surveying"

¹⁴⁰ "Cartography in Switzerland, Presented in Its Historic Development"

¹⁴¹ "Notice on Surveying the Town Zurich"

¹⁴² "Thurgau Society for Natural Scientists"

German-speaking secretary [8a]. His substitute was Amberg, who took over in late July 1897, when Dr Rebstein stepped down from his post due to military service [8f]. Whilst “Dr” suggests a more junior colleague it is not impossible that this could refer to the Rebstein we just met, as the Swiss army regularly calls back old recruits for training exercises. The minutes further inform us that a “Prof Rebstein” joined the finance committee whilst a “Dr Rebstein” joined the amusement committee [8b]. This suggests that there were two Rebsteins. However, we know that Johann Jakob Rebstein attended the congress, not the other one, from the information given in [73, p. 74].

4.2.19 Heinrich Friedrich Weber (1843 – 1912)

Heinrich Friedrich Weber was born on 07 November 1843 in Magdala, a little town close to Weimar. He had five brothers; their father was a merchant. After completing his school education at the Gymnasium in Weimar, Weber moved to Jena in order to study physics, mathematics and philosophy at the university there. Among his lecturers the physicist Ernst Abbe and the philosopher Kuno Fischer had the biggest influence on him. In 1865 Weber received his doctorate for a thesis entitled *Neue Probleme der Diffraktionstheorie des Lichtes*¹⁴³, supervised by Abbe. Afterwards he continued to do research, under Kirchhoff, but he earned his living as a private teacher in the house of the German politician August Dennig in Pforzheim.

In 1870 Weber moved to Karlsruhe, where he worked as Gustav Wiedemann’s¹⁴⁴ assistant at the Polytechnic School. A year later he became Helmholtz’s assistant in Berlin, and in 1874 he moved to Hohenheim, where he taught physics and mathematics at the Royal Academy of Württemberg. Already a year later he was offered a professorship at the Polytechnic in Zurich [79, p. 45]:

¹⁴³ “New Problems in the Theory of Light Diffraction”

¹⁴⁴ Gustav Heinrich Wiedemann (1826-1899): German physicist; he taught at the universities or polytechnics, respectively, in Basel, Braunschweig, Karlsruhe and Leipzig. Cf. biography by R Reiger, in *Allgemeine Deutsche Biographie*, 1910, 67-70: <http://www.deutsche-biographie.de/sfz85405.html>, accessed 19/03/2014.

When he noticed a short elderly gentleman among his students [at the Academy], he did not pay much attention to him; however he was quite surprised when, directly after the lecture, that gentleman asked him whether he would like to accept a professorship in Zurich. It was Kappeler, the former president of the School Board, who had attended the lecture incognito. When appointing new members of staff, Kappeler wanted to consider only his direct personal opinion.

At the Polytechnic Weber mainly lectured on technical physics. He also supervised more than forty-three doctoral students; quite a few of them became professors at universities across Europe [79, p. 51]. His most famous student, however, was Einstein, who often worked in Weber's laboratories. But despite being a 'lecturer beyond comparison' [79, p. 50], he also had his shortcomings: As a 'typical representative of classical physics' [4]¹⁴⁵ he did not teach Maxwell's theories, nor 'the foundations of physics, as he did not teach theoretical or mathematical physics' [24, p. 67]. Therefore, Einstein often skived his lectures, on which Weber commented: "You are a clever boy, Einstein, a very clever boy indeed. But you have a great shortcoming: you don't listen to anyone!" [4]. However, 'he and his institute at least helped further sensitise Einstein to the importance of measurement for testing theory and for finding the best fit between theory and empirical reality' [24, p. 68].

In fact, as 'a pioneer in electrical engineering in Switzerland and Germany' [53, p. 788] he helped to establish a system of units of measurement, together with physicists such as Lord Rayleigh, Silvanus Thompson, Friedrich Kohlrausch, Elettore Mascart, and Lord Kelvin. The latter was a good friend of Weber's. Among other things he experimented with alternating and direct current, with heat conduction, with blackbody radiation and with specific heat.

¹⁴⁵ Here [4] refers to a particular note in Weber's biographical dossier, which contains a number of quotes and the reference "Seelig, p. 47". Presumably someone copied these quotes from a paper or book on Einstein by (Carl) Seelig.

Among Weber's publications, *Die spezifische Wärme der Elemente Kohlenstoff, Bor und Silizium*¹⁴⁶ (1874), *Der absolute Wert der Siemensschen Quecksilbereinheit*¹⁴⁷ (1884), and *Die Entwicklung der Lichtemission glühender fester Körper*¹⁴⁸ (1887) are of particular importance. His 1874 paper inspired Einstein to develop the Einstein solid in 1907¹⁴⁹.

Furthermore, he was also interested in meteorology. He joined the Federal Meteorological Commission in 1881; in 1902 he became vice-president and eight years later president.

However, Weber's most important achievement was the physics institute at the Polytechnic, which opened in 1890. The previous physics laboratories were too small and the equipment was out of date. For several years Weber tried to get the Swiss government's permission to build at least an extension of the existing institute, though he really wanted laboratories that could cater for any future developments in (electrical) engineering. It was only when Werner Siemens declared his support for 'Weber's vision [which] proved decisive in winning the backing of Kappeler and Geiser' [24, p. 54] that the government gave in and provided the necessary funds. Weber's ideas turned out to be highly successful: for a number of years the institute was the finest of its kind in the world and thus added to the Polytechnic's growing reputation. It was 'especially designed to train electrical engineers or applied physicists [which] was what the Swiss wanted their tax money spent on' [24, p. 55].

Weber married Anna Hochstetter in 1875. The couple had three daughters and five sons, all of which became academics: Oskar: chemist; Friedrich: geologist; Ernst: civil engineer and astronomer; Helmut and Richard: physicians [79, p. 46].

Heinrich Weber died on 24 May 1912.

Weber joined the organising committee at the preliminary meeting in July 1896. He did not attend all the committee meetings though and did not get

¹⁴⁶ "The Specific Heat of the Elements Carbon, Boron and Silicium"

¹⁴⁷ "The Absolute Value of Siemens's Mercury Unit"

¹⁴⁸ "The Development of Light Emission of Glowing Solids"

¹⁴⁹ *Die Plancksche Theorie der Strahlung und die Theorie der spezifischen Wärme*

assigned any particular jobs (based on the minutes). He was the only physicist and only one of two non-mathematicians on the committee.

4.2.20 Adolf Weiler (1851 – 1916)

Adolf Weiler was born on 27 December 1851 in Winterthur (canton Zurich). After having completed his primary and secondary school education in his native town he matriculated at the Department for Mathematics Teachers at the Polytechnic in 1868, graduating in 1871. He worked as a private tutor for a year before studying in Germany, first at the University of Göttingen and then at the Friedrich-Alexander-University in Erlangen, where he obtained a doctorate in 1873. His supervisor was Klein, and his thesis was entitled *Über die verschiedenen Gattungen der Komplexe 2. Grades*¹⁵⁰.

Upon his return to Switzerland, Weiler first worked as a mathematics lecturer at the Ryffel Institute in Stäfa (canton Zurich). In 1878 he was appointed to a teaching post for mathematics at the women's teachers' college in Zurich. Weiler also habilitated as Privatdozent at both the Polytechnic and the University of Zurich. He stayed at the Polytechnic, where he was also W Fiedler's assistant, from 1875-1901 and started his job at the University in 1891. There he taught analytic and descriptive geometry, later on he also lectured on map projection. In 1899 he became a Titularprofessor for geometry at the university. At the beginning of the 20th century Weiler supervised five doctoral students in total, but none of them became influential mathematicians.

Weiler's research interests lay in Steiner geometry; in particular, he was interested in complexes and congruencies of rays. He published a range of papers on problems in descriptive geometry, axonometry and map projections. Two examples of his papers are *Neue Behandlung der*

¹⁵⁰ "On the Different Classes of Complexes of Second Order"

*Parallelprojektionen und der Axonometrie*¹⁵¹ (1889) and *Geometrisches über einige Abbildungen der Kugel in der Kartenprojektion*¹⁵² (1903).

Adolf Weiler died on 01 May 1916.

Weiler joined the organising committee in December 1896, but he did not have a specific position and is not mentioned in the committee minutes (except for the attendance records).

¹⁵¹ "New Treatment of Parallel and Axonometric Projections"

¹⁵² "Geometric Observations on Some Mappings of the Sphere in Map Projection"

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5. Geiser's Schoolbook and Letters to a Schoolteacher

This chapter illustrates Geiser's interest in teaching. First, his schoolbook, one of his major works, is analysed. An account of the life of Julius Gysel, a schoolteacher, headmaster, and Geiser's friend, follows. Gysel can be seen as an example of a late 19th/early 20th century schoolmaster with whom Geiser worked in order to improve school education (see section 2.3).

5.1 *Einleitung in die synthetische Geometrie*

5.1.1 Background and Motivation

In 1863, Geiser habilitated at the Polytechnic as a Privatdozent. As part of his teaching duties he offered an introductory course on synthetic geometry for a number of years (see chapter 2). As a result of his lectures and in the hope of improving mathematics education in general, Geiser wrote his textbook *Einleitung in die synthetische Geometrie. Ein Leitfaden beim Unterrichte an höheren Realschulen und Gymnasien*¹, published in 1869 [15]. He explains his motives for writing the book in the very interesting preface, given in full here:

When a new publication emerges from the mighty stream of geometry textbooks, which did not emanate from the circle of well-versed educationalists, but traces its origin back to a junior lecturer, then a justification of the same may only be found in itself. But may the author at least be permitted to explain and to account for its purposes and objectives in a preface.

For several years now, the author of this book has been entrusted with the obligatory instruction in synthetic geometry, which is supposed to initiate the students at the Department for Mathematics Teachers at the Swiss Polytechnic in the afore-mentioned science. In his lectures he has continually experienced a series of gaps in the preparatory training of his audience. These gaps had to

¹ "Introduction to Synthetic Geometry. A Guide for Instruction at Higher Realschulen and Gymnasien". Henceforth referred to as *Synthetische Geometrie*.

be filled in the first instance, before any attention could be given to the actual subject matter.

Out of this necessity a course of lectures emerged: "Introduction to Synthetic Geometry". Several repetitions and diverse revisions of this course supplied the content for this little book at hand.

To begin with, the need to train the visual-spatial ability of the audience with as little prerequisites as possible prevailed. Therefore, upon completion of the section on plane geometry, the derived theorems in the first main section on the "Theory of Transversals" were already transferred into space as far as possible. The theorems on the planar triangle are followed by the corresponding theorems on the solid triangle and the tetrahedron. In addition to harmonic points and rays, harmonic planes are examined as well. Furthermore, a separate chapter is dedicated to linear transformations in the plane and in space. The relationship between planar and three-dimensional shapes turns out to be even more intimate in the second main section on "Circle and Sphere". In this section, every chapter contains theorems from both plane geometry and solid geometry, which illustrate one another. When deriving the fundamental theorems on radical axes, points of similarity and the harmonic properties of circles, it is demonstrated that in some cases the three-dimensional observations can even lead to the desired result more easily than the calculations required by plane geometry for proving these theorems.

The author admits that the chosen approach, though in fact only presupposing the basic elements of plane geometry and solid geometry (apart from a few simple trigonometric formulae), will not be an easy one for pupils to follow, as it demands full attention and a sufficient knowledge of the preceding course material at every moment. In his lectures, he has experienced again and again that just the first steps in synthetic geometry are the hardest ones. However, he believes that through a thorough treatment of the material presented a sufficient understanding can be achieved, also on the level for which this "introduction" has been written.

Sure enough, this will necessitate geometry becoming more important in school than is currently the case. Realschulen, which prepare their pupils for polytechnic schools, will be more inclined to this expansion of instruction in geometry, since technical education is primarily constructive and therefore requires a developed visual-spatial ability. This will increasingly have to be the case, as descriptive geometry in its natural development draws more and

more on the purely synthetic direction. Therefore, this book may also be regarded as a resource for descriptive geometry; in particular if the theories, theorems and constructions contained in it are always accompanied by a practical implementation in the form of figures, which require meticulous drawing.

Admittedly, at Gymnasien this last consideration will be omitted. However, one can state reasons no less substantial to support the view that instruction in geometry should be expanded at these institutions as well; even be treated as the centre of mathematical instruction in the higher years. Only when this happens will it be possible to achieve and retain the proper position of mathematics as a discipline that stimulates and trains the mind amongst the classical-philosophical sciences.

May the author be allowed yet a personal remark: In strictly scientific circles, achievements such as the one at hand are often regarded with great contempt and disdain. This has not kept him from daring to publish the same. He is aware of the fact that he has not taken on this task as a result of the, nowadays admittedly fairly widespread, addiction to prolific writing. In fact, he believes to be serving science by attempting to smooth and to alleviate the paths leading to science to the best of his humble abilities. Incidentally, surely he will be allowed to point out that even the greatest mathematicians of his home country, Switzerland, did not disdain to see to spreading science in the wider population. But surely even the most rigorously minded will not want to reproach men such as Leonhard Euler and Jakob Steiner for this endeavour of theirs.

May this attempt to make synthetic geometry accessible to school be recommended to teachers and pupils of this science, for consideration free of prejudice; and if the Swiss educational establishments in particular receive it favourably, then a gladly held wish of the author will come true.

C. F. G.

[15, p. iii-vi]

Interestingly, Geiser does not go into much detail about the actual book, but offers his personal opinions on the place of mathematics in school education and on academic arrogance. References to the latter frame the preface, in a manner of speaking: First, Geiser suggests that educationalists might not approve of his work due to his young age and lack of relevant (at

least in the eyes of the experts) qualifications and experience. In the penultimate paragraph, he hints at the contemptuousness of his colleagues, who might have regarded such a work as beneath them. One could argue that neither of these remarks has lost their relevance more than 150 years later. A number of universities in the UK now run outreach projects, but a look at the University of St Andrews's access webpage [40] reveals that many of these projects focus on encouraging pupils to apply to university, as well as offering guidance for the application process. Other outreach projects involve explaining (science) research to the public, e.g. at science fairs. The ETH, Geiser's alma mater, organises events, such as public lectures and departmental visits, which introduce the wider public to scientific concepts and research methods [38]. None of these aim at improving a pupil's scientific training before embarking on their studies (at least not explicitly), as opposed to Geiser's book. With regard to potential attacks from educationalists, which Geiser hints at, education experts still argue about how best to teach and what to teach, developing new frameworks in the process, such as the Curriculum for Excellence in Scotland.

It is not surprising that Geiser felt that he had to defend his book against possible accusations, or indeed that he feared such attacks. He was only 20 years old when he started teaching, and 26 years old when the book was published. It is not unreasonable to assume that some veteran teachers would have questioned his expertise, or indeed that his university colleagues would have wondered why he did not invest his time into doing research and publishing papers. However, Geiser *did* do research in the 1860s; in addition to his thesis he published nine papers in the period 1866-1869, which together account for 40% of his research publications [cf. 24, p. 526-528]. Furthermore, he also edited a few of Steiner's papers, including volume I of *Jacob Steiners Vorlesungen über synthetische Geometrie*, in the years 1866-1868 (see section 2.2). It is possible that Steiner's lecture notes inspired him to write his own book, but if that was the case, he certainly does not say so in *Synthetische Geometrie*. Unsurprisingly, *Steiners Vorlesungen* covers more advanced topics than *Synthetische Geometrie*, but one could regard Geiser's book as a prequel, which

introduces the student to the fundamental concepts of synthetic geometry that are essential for understanding Steiner's lectures.

Whatever influence the editing process might have had on Geiser's own writings, he gives an indication of his heavy workload in the preface to *Steiners Vorlesungen*: apologising for a lack of coherence, he remarks that 'in the last two years in particular, the author [Geiser] was deprived of his best working hours due to his own research and an often burdensome teaching load, so that he could only attend to editing his first draft every now and then' [16, p. vi]. Writing an entire textbook on top of this indicates his interest in education and his commitment to raising its standards, which remained apparent throughout the rest of his life.

Looking at Geiser's remarks about the place of geometry in mathematics education in schools, he touches upon a heated debate that occupied mathematics teachers and mathematics professors as well as engineers and educationalists in Switzerland, but predominantly in Germany, during the late 19th and early 20th centuries. Geiser gives an account of the so-called "Engineers Movement" ("Ingenieurbewegung") at German universities and polytechnics, which started in the late 1890s, in his biography of Theodor Reye [17, p. 166-171] (see appendix E.3.2). As he wrote the biography about half a century after *Synthetic Geometry*, it is interesting to note that his opinion did not change significantly during the years. Admittedly, he does not explicitly state his views on the Engineers Movement in the Reye biography, but it seems that he thought that mathematics deserved a prominent position in education. See also appendix B for a short account of the corresponding debate at the Polytechnic².

Whilst mathematics and engineering professors at higher education institutions argued about the place of mathematics in relation to the applied sciences, the debate took a different shape at secondary schools. There, the question was where mathematics stood in relation to the humanities. Looking specifically at geometry, Geiser acknowledges that whilst it has real-life applications, it is also an art and trains the mind (similarly to classics, for

² Cf. also [19, p. 79-99; p. 125-128]

example). Even today mathematics is generally considered a science, although (pure) mathematics can in fact be seen as an art, much closer to philosophy than to, say, chemistry.

5.1.2 Structure and Content

Returning to *Synthetic Geometry*, let us now look at the content of the book. Geiser splits it into two parts: I) “Theory of Transversals”, and II) “Circle and Sphere”. Each part contains four chapters, numbered consecutively:

- Chapter 1: “Transversals in a Triangle”
- Chapter 2: “Triangle and Tetrahedron. Complete Planar Figures”
- Chapter 3: “Harmonic and Involutionary Structures”
- Chapter 4: “Linear Dependencies in the Plane and in Space”
- Chapter 5: “Powers. Similarity Points”
- Chapter 6: “Harmonic Properties of Circles and Spheres”
- Chapter 7: “Applications”
- Chapter 8: “The Principle of Conjugate Radii”

Each chapter will be summarised in the subsequent paragraphs. Then, some examples from the book will be given, with comments, before Geiser’s style and approach will be discussed.

Chapter 1 – Theory of Transversals: Transversals in a Triangle

Section §1: Geiser begins with the basic properties of transversals through a triangle, in particular the properties of three points on a straight line. First of all he claims that when drawing a transversal through an arbitrary triangle with infinite sides³, either of the following cases hold:

1. The transversal intersects the triangle in two bounded and one extended side, or
2. The transversal intersects the triangle in three extended sides.

³ Geiser calls the sides AB , AC and BC “bounded sides”, and the infinite extensions of these sides “extended sides” of the triangle.

This yields six line segments. Geiser then proves that ‘the product of three non-adjointing line segments [...] is equal to the product of the other three’ [15, p. 1]. This relation is used to prove that the three points of intersection lie on a straight line. Geiser hardly ever names the theorems that he includes in the book, but it seems that he is proving Menelaus’ Theorem here. He uses his result again to prove a fundamental property of perspective triangles [15, p. 3]; here Geiser also explains how to make use of this property in practical constructions. He then proves a few further theorems; for example that the three base points of perpendicular lines drawn from an arbitrary point on a triangle’s circumscribed circle to the three sides lie on a straight line [15, p. 4].

Section §2: Next, Geiser looks at theorems involving three straight lines through a point. He shows that if the same relation of products as found in §1 holds for the corners A, B, C of a triangle and the points A', B', C' located on the sides of the triangle (such that either all three points are on bounded sides or that one is on a bounded side and the other two lie on extended sides), then the segments AA', BB', CC' intersect in one point [15, p. 6-7]. This is Ceva’s Theorem, but again Geiser does not mention this.

He uses this result to prove some well-known properties of triangles, namely that:

- The medians intersect in the centroid;
- The angle bisectors intersect in the centre of the incircle⁴;
- The altitudes intersect in the orthocentre.

Lastly, Geiser derives what is essentially Thales’s theorem, but without actually referring to the name. Instead, he claims that:

If the base of a triangle remains fixed, while the apex A changes in such a way that the ratio of the two sides remains constant, then the

⁴ Note that Geiser does not refer to the lines as angle bisectors in the first instance, but defines them as lines drawn ‘from the corners [of the triangle] to the points where the incircle touches the opposite sides’ [15, p. 7-8]. In a subsequent paragraph he proves that the three angle bisectors intersect in one point, and remarks that this point is indeed the incentre. He may have chosen to prove the same property twice due to the complication added by the fact that he had to work with angles, or because this led into showing how to construct the angle bisector of a triangle’s ‘inaccessible point’ [15, p. 9-10], or for reasons unknown.

bisectors of the angles at A intersect the base in two fixed points A' and a' (where the sides of the triangles are unbounded).

[15, p. 10]

Stating that A moves along a fixed curve with the angle $A'Aa'$ being a right one, Geiser arrives at Thales's theorem, expressed in terms of ratios of a triangle's sides.

Section §3: In the next section, Geiser uses the fact that perpendiculars on the sides of a triangle meet in one point if a certain relationship between the segments created by the triangle's corners and the base points of the perpendiculars is satisfied to prove a few more well-known properties, specifically that:

- The perpendicular bisectors intersect in the circumcentre;
- The altitudes intersect in the orthocentre⁵; and
- Thales's theorem (again without mentioning Thales at all).

Lastly, he defines medians and derives a result that connects medians and sides of two triangles [cf. 15, p. 14].

In this section, the proofs contain considerably more algebra than the proofs in the previous and directly succeeding sections.

Chapter 2 – Theory of Transversals: Triangle and Tetrahedron. Complete Planar Figures

Section §4: In this section, entitled “the curious points of the triangle”, Geiser proves a few of the theorems from Chapter 1⁶ without making use of transversals, but rather by means of points and sides of a triangle. He also defines the incircle and excircles of a triangle. Furthermore, he points out relationships between various points of the triangle, e.g. that the centroid S

⁵ Here, Geiser gives a different proof than in §2.

⁶ Namely, the theorems concerning the nature of the centroid, the circumcentre, the incentre, and the orthocentre. In order to prove that the angle bisectors meet at the incentre, Geiser introduces the notion of the distance of a point from a given straight line.

lies on the same line as the orthocentre H and the circumcentre M , with the relation $HS = 2HM$ [15, p. 16].

Geiser then makes use of the theorems that he introduced so far to find 'solutions of some simple problems' [15, p. 19]. By solving, he really means proving by means of geometric construction; he explains how to derive certain results by drawing straight lines and circles, bisecting angles and employing the properties of particular points in a triangle. Thus, these proofs also serve as instructions. Examples of these problems are proving that for four triangles constructed by means of four straight lines the respective circumcircles intersect in a single point, and that the respective orthocentres of such triangles are collinear.

Section §5: In this section, Geiser introduces what he calls "körperliches Dreieck" or "Dreikant". This translates to "solid triangle", which I will use for want of a better English expression⁷. Geiser remarks that 'as is commonly known, spherical triangles do not differ significantly from solid triangles' [15, p. 23]. He continues that:

We will assume the most fundamental terms with regard to solid and spherical triangles, from which we will derive a number of properties corresponding to various theorems regarding planar triangles, which were derived in the previous sections.

[15, p. 23]

Geiser proves these theorems for solid triangles, giving the analogous versions, or consequences, for spherical triangles after each proof. He remarks that he could have used spherical trigonometry instead, but does not go into

⁷ Geiser defines solid triangles as follows:

Three arbitrary planes in space, i.e. of which no two are parallel and all three of which do not intersect in a single line in particular, divide space into eight portions, each of which is a solid triangle [...]. The angles defined by the planes are generally known as the angles of the solid triangle. Meanwhile, the angles created by the lines of intersection of the planes are called the sides of the triangles. [15, p. 23]

any detail [cf. 15, p. 29]. An example is the theorem that the great circle arcs through the corners and midpoints of the respective facing sides intersect in the centroid of the spherical triangle. In order to derive this theorem, Geiser proves that the planes through the edges and median lines of the respective facing sides in a solid triangle intersect in its centroidal axis [15, p. 23-24]. Using the definition of a cone of revolution containing the edges of a solid triangle⁸, he describes the nature of the in- and circumcentre of a spherical triangle; he also shows that the analogous statement of the relationship proved in §4 [15, p. 16] holds true for solid triangles.

Lastly, Geiser shows that when a plane intersects the sides of a skew quadrilateral, resulting in eight sections, then the product of four non-adjacent sections equals the product of the other four sections [15, p. 30]. The proof of this theorem, and its converse, is again more algebraic than the previous proofs in this chapter.

Section §6: In this section, Geiser moves into three dimensions and considers analogous results of the theorems proved so far for the tetrahedron:

Elementary geometry consists of planar, spherical and spatial geometry, depending on the region in which its constructions are performed. The theory of the planar triangle in planimetrics corresponds to the theory of the spherical triangle in [...] spherical geometry, and in spatial geometry we can consider it to correspond to the theory of the trilateral pyramid or tetrahedron. Holding on to this analogy, one can easily conjecture how certain planimetric theorems can be applied to space when looking at them more closely. However, it is not guaranteed that the conjecture will hold in every case.

[15, p. 31]

⁸ According to Geiser, the cone's axis is the axis of the solid triangle's edges, which in turn is the line of intersection of the median planes of a solid triangle.

Among the theorems that do hold in three dimensions, he proves that the bimedians (the line segments between the midpoints of two opposing edges) intersect in the tetrahedron's centroid, and that the planes through the midpoints of the edges and perpendicular to them intersect in the circumcentre. As an example where the analogy does not hold in space, Geiser shows that in general, the altitudes of a tetrahedron do not intersect. Furthermore, he investigates what happens when one inscribes eight spheres in a tetrahedron and constructs their respective centres [15, p. 34-35], and derives results from the fact that the tangent plane of a sphere through the four vertices intersects the opposing triangular face in a straight line [15, p. 35-37].

Section §7: Here, Geiser defines the n -gon, first in two and then in three dimensions, as well as the complete n -gon (where $n-1$ edges join in each vertex). Referring to figures, he gives some examples of different kinds of n -gons, e.g. concave and convex. Moreover, he derives the number of edges and diagonal vertices in complete polygons, polylaterals and polyhedra⁹.

Chapter 3 – Theory of Transversals: Harmonic and Involutory Structures

Section §8: Using a theorem proved in §2, namely that an angle bisector in a triangle divides the opposite side, with the two segments having the same ratio as the adjacent sides [15, p. 9-11], Geiser introduces the notion of a harmonic range¹⁰. He explains how to construct the harmonic conjugate point if three points are given, both by means of a triangle and of a semicircle above the real projective line. The latter construction is used in turn to illustrate how

⁹ Geiser distinguishes between different types of polygons [15, p. 38-42]:

- n -Eck (n -gon): consists of n points, no three of them collinear, joined by straight lines; both in two and in three dimensions
- n -Seit (n -lateral): consists of n straight lines, no three of them meeting in the same point; both in two and in three dimensions
- n -Flach (n -hedron): consists of n planes, no four of which meet in the same point; in three dimensions only

¹⁰ Geiser uses the expression "harmonische Punkte" (harmonic range) rather than the (nowadays) more common "harmonische Teilung" (harmonic division).

the distance AA' changes while the distance BB' remains fixed¹¹, including what happens when A lies on the midpoint of BB' . Moreover, he proves that if a straight line crosses two circles intersecting at a right angle in the points A, A' and B, B' , respectively, such that the line segment AA' describes the diameter of one circle, then the four points constitute a harmonic range. Geiser then applies this result to a few constructions, identifying the limitations in each case.

Section §9: First, Geiser defines harmonic rays; later on he also defines harmonic planes. Expressing the relation between the angles formed by the rays (and later on the planes) first in terms of sine and then in terms of tangent, he shows that any transversal crosses four harmonic rays in four harmonic points [15, p. 49-50], and a similar result for harmonic planes [15, p. 54-55]. He also shows that some properties of a harmonic range hold true for harmonic rays and planes as well, e.g. that a harmonic ray or plane is uniquely determined by the other three rays or planes, respectively. Lastly, he makes use of the harmonic properties of a quadrilateral to demonstrate how to find this fourth point, or ray, or plane, without the use of a compass [15, p. 56-58].

Section §10: In this section Geiser investigates involutions of points¹². He begins by defining hyperbolic and elliptic involutions, and points out the differences between the two structures. In particular, he defines fixed points¹³, and concludes that two pairs of points uniquely determine the nature of an involution. Tying in with the proof, where he uses a property of circles, he explains how to construct an involution when two pairs of points are given,

¹¹ Geiser generally uses A, A', B, B' to denote the four points in a harmonic range, where B divides the segment AA' internally and B' divides AA' externally (or vice-versa).

¹² Instead of the term “involution” he uses “system of points” here, and “system of rays” in §11. Towards the end of the section, he introduces the term “involution”, but claims that it is an older term, used to denote six points on a straight line that satisfy certain conditions. His choice of words in this paragraph suggests that a “system of points” (or rays) denotes a more advanced structure. [15, p. 68]

¹³ He refers to fixed points as “double points” (Doppelpunkte) or “asymptotic points” (Asymptotenpunkte).

and how to find one of the points of a third pair. Finally, he gives a relation between such six points in both a hyperbolic and an elliptic involution.

Section §11: This last relation also holds for the sines of the angles between three pairs of rays in an involution of rays, as Geiser shows in this section [15, p. 63-65]. After defining an involution of rays, including a hyperbolic involution, he proves that the properties listed in §10 have analogues for rays in involution. Furthermore, he defines the axes of an involution of rays, a circular involution¹⁴, and the power of involutions of rays. Returning to involutions of points, Geiser defines special cases: the parabolic involution (where the fixed points coincide) and the equilateral hyperbolic involution (where one of the fixed points lies in infinity) [15, p. 67]. He concludes the section, and thus the third chapter, by explaining how to find the sixth point in an involution by means of a ruler only.

Chapter 4 – Theory of Transversals: Linear Dependencies in the Plane and in Space

Section §12: Geiser observes that a point M correlates two planes E_1 and E_2 uniquely by means of central projection, which, according to Geiser, preserves shapes and involutions. He then uses central projection to prove a theorem regarding the perspective alignment of two triangles, which was introduced in §1. Furthermore, he demonstrates the harmonic properties of a complete quadrilateral, which allow for the construction of involutions without the use of metric proportions.

Section §13: In this section, Geiser explains how to construct harmonic ranges by means of linear transformations, both in the plane and in three dimensions. Linear transformations are used in their geometric sense here, not in terms of functions. For example, Geiser proves that every two non-parallel straight lines can be crossed by one unique line through a given point p ; by finding the fourth point p' in the harmonic range which includes the two

¹⁴ According to Geiser, a circular involution of rays is obtained by rotating a right angle around its vertex, and consists of the sides of the angle at different positions [15, p. 66].

points of intersection and p one can establish a linear correspondence of the space through p and p' [15, p. 78]. As an example of an application he gives the following example:

If a plane crosses the six edges of a tetrahedron, and if on each edge one determines the fourth point in the harmonic range containing the two vertices and the point of intersection with the plane, the fourth point being assigned to the latter, then one obtains six points, which have the property that the three line segments connecting the pairs [of points] on opposite edges of the tetrahedron intersect in one and the same point.
[15, p. 77]

Chapter 5 – Circle and Sphere: Powers. Similarity Points

Section §14: At the beginning of the second part of the book, Geiser refers back to figure 41 in §10 to define the “power of [a] point [P] with respect to [a] circle; that is, inner power when P lies inside the circle, and outer power when P lies outside of the circle” [15, p. 80]¹⁵. He then summarises some special cases, e.g. when the radius of the circle is infinitely large or infinitely small. Next, Geiser investigates the locus of the points that have the same (inner or outer) power with respect to two arbitrary circles. Using a construction in three dimensions, i.e. involving intersecting spheres, Geiser defines the radical axis of two circles. Finally, he explains how to draw the radical axis of two non-intersecting circles; for this, he defines the “orthogonal circle”, whose centre is the radical centre, i.e. the point of intersection of the three radical axes of three circles (one axis for each pair of circles).

Section §15: In this section Geiser defines what is commonly known as Apollonian circles, i.e. two families of circles where every circle of one family intersects all the circles in the other family orthogonally. However, Geiser

¹⁵ In particular [15, p. 80]:

- Inner power: the constant product of the two segments on any chord in a circle, through a given point P ;
- Outer power: the constant product of the two segments on any secant through a circle and a given point P .

does not use this name at all, but calls such families of circles “conjugate pencils of circles”. He defines an elliptic pencil of circles, which he calls “pencil of the first kind”, and a hyperbolic pencil of circles, which he calls “pencil of the second kind”. Then, Geiser proves that if a transversal crosses an elliptic pencil of circles, it creates an elliptic involution (of points); if it crosses a hyperbolic pencil of circles, then it creates a hyperbolic involution. He also explains how to construct the fixed points of the involutions, and covers the special cases of parabolic pencils and pencils where all the circles are concentric.

He further illustrates how to construct pencils of circles by means of a family of spheres through a given circle k , and defines the radical plane of two spheres (giving two methods of construction). Geiser then proves the three-dimensional versions of certain properties of circles and radical axes proved in §14, but in terms of spheres and radical planes. These include that the three radical planes generated by three spheres intersect in the same line, and that four spheres generate six radical planes. Similarly, he defines pencils of spheres, which have analogous properties to pencils of circles, e.g. that a plane intersects a pencil of circles in a pencil of spheres. Finally, he remarks that there are infinitely many conjugates of pencils for every pencil of spheres, indicating that this is a more advanced topic [15, p. 97].

Section §16: Geiser now studies similarity points of two circles and of two spheres. In particular, he defines the inner and outer similarity points, which form a harmonic range with the centres of the circles, or spheres, respectively. He then looks at how the similarity points change for special cases, e.g. when the two circles do not intersect, when they intersect, when they have the same size, or when they are concentric. With regard to spheres he explains that two non-intersecting, non-touching spheres lie in a cone of rotation, where their outer similarity point is the cone’s apex [15, p. 100].

Section §17: Having investigated the similarity points of two circles, or spheres, respectively, Geiser now lists the properties of similarity points of

three circles, or spheres, respectively. Beginning in two dimensions, he proves the fact that the resulting six points lie, in groups of three each, on four straight lines (the radical axes) using a fundamental theorem proved in §1. He then treats a few special cases, e.g. when the three circles touch. Geiser remarks that the analogous statements hold for three dimensions, and proceeds to show that the 12 similarity points of four spheres lie on 16 straight lines, with three points being collinear.

Chapter 6 – Harmonic Properties of Circle and Sphere

Section §18: In this section, Geiser introduces the notion of poles and polars with respect to a circle. Note that he does not define them with respect to conic sections. In fact, he proves that all the poles assigned to a point with respect to a particular circle are collinear (and lie on the polar) by means of harmonic points and rays. He explains the reciprocal relationship between the two structures, and lists special cases; where the pole lies when the polar is a tangent to the circle, or when the polar lies at infinity, for example. Geiser explains how to construct polars with respect to a given circle using a ruler only. Using first the properties of a complete tetragon and then employing stereometry, he shows that the polars of all the points on a straight line intersect in the pole of the line, and vice-versa [15, p. 110-111].

Section §19: Geiser first defines triads of harmonic points or rays with respect to a circle¹⁶, of which there are infinitely many. He gives specific properties of triads of points, e.g. the locus of the points, as well as special cases such as when a point lies on the circumference of the circle, or at infinity. Furthermore, he states that pairs of points that form a triad with a given point p lie on the polar of p , and, in addition, form an involution. The type of involution depends on the location of the polar with respect to the circle; analogous statements hold for pairs and triads of rays. Geiser then explains

¹⁶ In particular, he defines a diagonal triangle determined by a tetragon, whose vertices lie on the circumference of the circle. For each side of the triangle, the pole is the opposite vertex or the intersection of the other two sides; and for each vertex, the polar is the opposite side (or extension thereof). He calls such a triangle a triad of the circle, specifically, a triad of harmonic points when considering the vertices, and a triad of harmonic rays when considering the sides of the triangle [15, p. 112].

how the “polar figure” of any given figure can be obtained by means of a process that he calls “polarisation”¹⁷. He illustrates this principle with a few examples: that the reciprocals of harmonic points are harmonic rays, and that the reciprocal of a complete quadrilateral is a complete tetragon, preserving harmonic properties.

Section §20: Having explained the properties of conjugate harmonic poles with respect to a sphere, Geiser shows that all the points that are harmonic to a given point p , with respect to a sphere, lie on the polar plane of p , with respect to the sphere. He gives alternative ways to find the polar plane, and considers properties of the plane, for example: ‘If a point p' moves across an arbitrary plane P , then its polar plane rotates around the pole p of P .’ [15, p. 122]. Using the converse, Geiser defines reciprocal polars with respect to a sphere, and lists their properties.

Section §21: In this section, Geiser defines what he calls a “polar web” of a sphere K with respect to a plane E : all the points and lines in E , where each point is associated with the line of intersection of E and the point’s polar plane with respect to K . He distinguishes elliptic polar webs (where E and K do not intersect) and hyperbolic polar webs (where E and K intersect; the web is then defined with respect to the circle of intersection). Having given various properties of these two types of web, he defines their powers and triads of harmonic points in a polar web. Then he proceeds to investigating quadruples of harmonic points with respect to a sphere; among other properties he points out that the points of a quadruple and the associated polar planes form a tetrahedron [15, p. 130]. Finally, he transfers reciprocation into three dimensions, listing the reciprocals of various geometric elements.

¹⁷ I presume that the correct English term is “reciprocation”, as Geiser defines polarisation as the process of replacing each point by its polar and each line by its pole with respect to a given circle, to obtain the polar figure, or better, “reciprocal”. I will use the terms “reciprocal” and “reciprocation” from here onwards.

Chapter 7 - Applications

Section §22: Now Geiser investigates the ‘harmonic properties of the orthogonal circle and the orthogonal sphere’ [15, p. 133]. To begin, he expresses a theorem proved in §8 in terms of poles and polars, and deduces that the polars of a point p with respect to all the circles in a pencil meet in the antipodal point p' of p , and vice-versa. As in the previous chapter, Geiser then looks at specific properties and special cases. For instance, he proves that for a circle K , which intersects three circles orthogonally, the polars of a point a on K with regard to these circles meet in the antipode of a [15, p. 135-136]. Next, he defines a web of circles¹⁸ and lists some of its properties. With regard to three dimensions he remarks:

The analogous observations in space shall not be considered in more detail than is necessary to determine the locus of all points whose polar planes with respect to four arbitrary spheres in space meet in one point.
[15, p. 137]

Section §23: This section is dedicated to Pascal’s Theorem. However, Geiser states it with respect to circles only, and includes it as an application of the material previously covered in the book:

In a hexagon, inscribed in a circle, we have that the points of intersection of the opposing sides are collinear.

This theorem is called Pascal’s Theorem. Although it has no relevance to the problems covered in this book, but instead finds its most important consequences in a different field entirely, it shall be considered in more detail here: as an example that the result can be achieved by different methods.

[15, p. 139]

Geiser gives three proofs, involving: extension of three of the sides of the hexagon to obtain a triangle and applying theorems from §1 and §7; the fact

¹⁸ According to Geiser, a web of circles comprises all the circles that intersect their orthogonal circle at right angles. These circles can form pencils of circles.

that for the six similarity points of three circles, every three are collinear, yielding four lines, as discussed in §17; pairs of points in involution on a transversal crossing a circle with a) one inscribed tetragon, b) two inscribed tetragons. He then explains how Brianchon's Theorem can be obtained from Pascal's Theorem by applying the principle of reciprocation¹⁹, and gives a special case that he proved already in §1 (Pascal) and §2 (Brianchon).

Section §24: Now Geiser looks at a more practical problem:

When one has to perform practical constructions, it is often the case that a circle is not given by the centre and the radius or by three points, but that one only knows a necessary and sufficient number of conditions that [the circle] has to comply with. Thus, it becomes a question of actually constructing such a circle, or, if the problem allows for several solutions, the limited number of such circles. I will examine one [example] out of the array of such problems: Finding a circle that is tangent to three given circles. – Considering that both a straight line and a point are special cases of a circle, which occur when the radius is taken to be infinitely big or infinitely small, respectively, then it follows that this problem contains a number of more specialised problems.

[15, p. 147]

Geiser first looks at the case when the three given circles are not mutually tangential, and finds eight circles that touch the initial three in some way. In particular, he is concerned with identifying the one circle that excludes the three circles. Using radical axes, similarity points, polars, and tangent lines, he then investigates the locus of three given circles that are tangent to two circles μ and μ' , where μ excludes them all, and μ' includes them. This is the Problem of Apollonius, but Geiser neither attributes it to Apollonius nor does he comment on the problem's long history.

¹⁹ Most textbooks refer to duality rather than reciprocation, also with regard to Pascal's and Brianchon's theorems. Geiser does not mention duality at all, which can be explained by the fact that he focused on circles rather than using the more general conics.

Section §25: Geiser proceeds to the analogous problem in three dimensions: finding a sphere that is tangent to four given spheres. He identifies 16 possible solutions, but decides to investigate only the cases where the sphere a) excludes, and b) includes the given spheres. Moreover, he uses the second method from §24, as the first one ‘gradually reduces the problem to simpler and simpler problems’ [15, p. 156]. Similarly to the previous section, the solution involves constructing powers, similarity points, poles, and planes.

Chapter 8 – The Principle of Conjugate Radii

Section §26: Geiser begins the last chapter by defining (unique) conjugate points with respect to a given circle, called transformation circle, with its centre being called transformation centre and the square of its radius representing the transformation power. He then defines the principle of conjugate radii, i.e. the fact that the conjugate points corresponding to all the points in a figure form a new figure, ‘which is related to the first one in a variety of ways, and often provides tools to discover geometric truths that would have been difficult to spot otherwise’ [15, p. 160]. Next, Geiser explores applications of this principle, showing for example that a straight line is transformed into a circle and vice-versa and giving further results, e.g. that transformation preserves angles of intersection. However, he also shows that the conjugate points of the points of a circle form a circle again, which leads to a number of properties regarding two conjugate circles, such as the fact that they belong to the same pencil as the transformation circle.

He then gives the analogous statements for three dimensions, i.e. transformation with respect to a transformation sphere. Examples of such analogues are: the conjugate points of points of a sphere lie on a sphere, too, and the transformation figure of a circle in space is a circle on the same sphere.

Section §27: Here, Geiser finally gives a real-life application of the material covered in his book: He defines ‘stereographic projection of the sphere onto the plane, [which] is used to produce maps’ [15, p. 169]. He gives some properties of meridians and circles of latitude, e.g. that they are orthogonal,

and explains that the scale of the maps differs according to latitude [15, p. 170-171]. Moving away from maps, he concludes that some of the theorems proved in the previous chapters can be derived by means of stereographic projection, albeit under different conditions. He leaves the formulation of such theorems as an exercise to the reader [15, p. 171-172].

Section §28: In this last section, Geiser investigates a number of problems:

- Can two circles with arbitrary radii be transformed into circles with equal radii?
- Can two circles be transformed into two concentric circles?
- Can three circles be transformed into three circles whose centres are collinear?

He then studies the space between two circles, where one circle lies completely within the other circle (first for the general case, then for two concentric circles), and explains the notion of commensurable and incommensurable series of circles. He then looks at analogous theorems in space, using spheres and series of spheres. He defines two series of spheres and claims that they are either both commensurable, or both incommensurable, which he proves by means of algebraic expressions and geometric constructions.

5.1.3 Geiser's Style and Method

In a nutshell, *Synthetische Geometrie* is well written and in fact quite enjoyable to read. Never having studied synthetic geometry myself, I found the material that Geiser covers very accessible; his explanations are easy to follow and to understand. Furthermore, a number of figures illustrate some of the concepts and constructions in the book. It is thus conceivable that budding Polytechnic students would have found *Synthetische Geometrie* useful in giving them the necessary basics of synthetic geometry.

However, it is very theoretical and requires the reader to engage in independent study. Whilst the geometric constructions are explained well and the steps are easy to follow, one does have to sit down with pen and paper and work the problems out for oneself. This can be seen as an example of Geiser's teaching talents: he does not hand his readers everything on a silver plate, but encourages them to discover geometry themselves. Curiously, however, Geiser includes no exercises at all, as is common practice in the majority of introductory textbooks; he does not even refer the reader to a collection of problems, for example. There are a few worked examples, but they are rather abstract and take the form of definitions and proofs rather than exercises [15; e.g. p. 12-14; p. 19-20; p. 49-51; p. 62-63; p. 72-73; p. 139-146; § 25; § 28]. A student of pure mathematics should not have any problems with this, but for school leavers and engineering students the material may have seemed rather academic and not particularly tangible – similarly to his lectures, which were not very popular with the engineers (see section 2.1). Moreover, Geiser hardly includes any references to practical applications of the material covered (an exception is §27, where he explains how stereographic projection is used to create maps).

Unfortunately, I have not been able to find any books that I could compare *Synthetische Geometrie* to. A large number of geometry textbooks were published during the second half of the 19th century, but most of them are either more advanced than Geiser's book, or treat different branches of geometry, or both. Holzmüller places *Synthetische Geometrie* into a historical context in [23], but covers several areas of geometry (see section 5.1.4).

Books that serve a similar purpose in their respective fields would include T Reye's *Synthetische Geometrie der Kugeln ...* [31] and *Geometrie der Lage* [30] as well as W Fiedler's *Darstellende Geometrie ...* [14]. A much more recent example would be C Durell, *Projective Geometry* [13]. However, note that all of the above books are aimed at university students and require a more profound mathematical knowledge than *Synthetische Geometrie*.

An example that predates Geiser's book, but caters to a similar audience, would be M Ohm's *Die reine Elementar-Mathematik*, in particular volume III

[29]. It is much more application-oriented than *Synthetische Geometrie* and contains a wealth of exercises and examples.

From a modern perspective, Geiser's style is rather wordy, and he keeps algebraic expressions to a minimum. However, this is due in part to the subject matter as well as to the era. Contemporaries of Geiser should not have found his style unusual.

To summarise, *Synthetische Geometrie* is a coherent, comprehensive introduction to synthetic geometry. It contains the basic tools needed to engage in further study of the subject, as well as some more curious and complicated problems, which provide the necessary inspiration for this. A greater emphasis on practical applications and a few historical notes would have been desirable, though. In general, the book is well structured, the explanations are concise, and the constructions are easy to reproduce. However, a few additional figures, a clearer division into paragraphs and a collection of practice problems would have made the book even better.

5.1.3.1 §18: Pole and Polar with Respect to a Circle

As an example of Geiser's approach, let us look at Chapter 6, Section 18, on poles and polars [15, p. 106-111]. Here Geiser combines concepts that he derived previously, such as harmonic properties of points and rays. At six pages it is of middle length, compared to the other sections. It contains two figures, which Geiser uses in his proofs, but it is rather theoretical, like most of the book.

In the first paragraph Geiser defines two harmonic conjugate points p and p' on a line with respect to (w.r.t.) the points of intersection of the line with a given circle M . He explains why there are infinitely many harmonic "poles" assigned to a point p [cf. 15, p. 106]. He then wants to prove that 'the harmonic conjugate poles of a point p are always distributed on a certain straight line' [15, p. 106], and adds that if this is true, then the line is perpendicular to the

line through p and the centre of the circle M 'because of symmetry' [ibid.]. The proof of this is purely geometrical; Geiser uses harmonic ranges, harmonic rays, and angles between them. Adding the observation that it does not matter whether the point p is inside or outside of the circle M , and using a theorem (without proof) to explain why the harmonic conjugate poles of p lie on a tangent of M if p lies on the circle, Geiser arrives at the main theorem of the section: 'The harmonic poles that are conjugate to a point p w.r.t. a circle M lie on a straight line P , which is called the *polar* of the point p ' [15, p. 107]. He then summarises how to find the polar geometrically, and the three cases depending on the location of p . He does similar summaries for his next theorem, which he gives without proof, that every polar has one and only one pole [cf. 15, p. 108]. Furthermore, he repeats an earlier observation [e.g. cf. 15, p. 52-53; p. 70-71] by stating that the pole of a straight line at infinity is the centre of the circle.

Next, Geiser explains how to construct the polar of a point w.r.t. a given circle using a ruler only, and, using a complete quadrilateral, proves that the line one obtains is indeed a polar. From this he deduces another theorem: 'If a point moves along a straight line P , then its polar [w.r.t.] a circle M rotates about a fixed point p , which is the pole of P ' [15, p. 110]. Remarking that the proof has already been given, he uses a lemma, without proof, to show that the proof is easier if P does not cross the circle [cf. 15, p. 110]. He then gives a stereometric proof of the theorem, which does not rely on harmonic properties.

Geiser states the converse of the theorem that the polars of all the points on a straight line go through its pole, noting that it 'does not require any specific proof' [15, p. 111]. However, he gives special cases of the two theorems, again without proof. Finally, Geiser briefly explains how to construct the pole of a given straight line w.r.t. a circle using a ruler only.

As mentioned above, the section is rather abstract, but this is in accordance with the rest of the book and could well be due to the nature of synthetic geometry. The proofs are purely geometrical, which is typical of Geiser here; there are only a few proofs that require equations. Whilst Geiser states and

uses a number of results without proof, he gives alternative proofs for a couple of results. Other sections display a similar approach. This is quite useful for students as it offers a different angle to the problem, thus broadening their mathematical horizon, but it also provides a perhaps more intuitive proof. Some readers might feel that he omits information in using theorems without proof, but then the book was intended as an introduction. As it stands, there are quite a lot of proofs for students to learn already.

Bearing in mind that synthetic geometers sought to purge all metric aspects and use an axiomatic method, Geiser's book gives a good introduction into the synthetic approach, which is illustrated by this chapter. Whether it was because of the nature of the subject or because of his intended audience, or both, Geiser explains how to construct certain results using pen and paper. This makes the material a bit more accessible and helps the reader to understand the subject material. It would also have been useful for future engineers who had to learn technical drawing. However, for engineering students in particular, some explanations as to the real-life applications of poles and polars, in this example, would have been desirable. Geiser leaves his readers with a lot of information, but they would have to turn to other, possibly more advanced, books in order to understand the significance of the material treated in *Synthetische Geometrie*.

5.1.4 Reception

Unfortunately, there are not many surviving reviews of Geiser's textbook. In particular, there is no indication of whether or not any of Geiser's colleagues belittled his work, as he feared in the introduction (see above). However, the reviews that I have been able to access, as well as a number of obituaries of Geiser, generally commend the book.

Today, reprints of *Synthetische Geometrie* can still be bought from Amazon and other online retailers. This indicates that there is at least some interest in

the book²⁰. A quick search on worldcat.org reveals that copies of *Synthetische Geometrie* are held in a number of (university) libraries across the world: primarily in Germany, Switzerland and the USA, but also in France, the UK, Slovenia, Poland, and Canada [41].

One Dr Kretschmer of Frankfurt a. O.²¹ writes in a review that:

Mr Geiser's intention is to prepare beginners with modest previous knowledge for synthetic geometry. But such a preparation can only be achieved by firmly encouraging independent study, i.e. by means of a well-structured and comprehensive collection of problems, on the basis of which the main theorems of synthetic geometry are examined. [...] Mr Geiser only developed a number of simple and stimulating problems of elementary geometry clearly and elaborately [...], so that even those with a very mediocre training will understand him.

[26]

G Holzmüller²² echoes the sentiment of this review in volume I of his *Elemente der Stereometrie*, an attempt to unite all the important results in solid geometry known at the time [23]. In the introduction, he calls Geiser's book 'excellent', in tandem with textbooks by H R Baltzer and T Reye [23, p. III]. He continues:

The intention of Geiser's textbook, which treats [aspects of] both plane and solid geometry, is merely to introduce [the reader] to certain methods of more recent geometry. [23, p. IV]

²⁰ Of course, modern technology makes it easier to produce facsimiles and digital copies of old books. However, as the production must still be worthwhile, it is likely that the book is still read and appreciated today (even if by a very small audience).

²¹ Probably Eduard Ernst Kretschmer; he seems to have been a teacher at the Gymnasium in Frankfurt a. O. During the 1860s and 1870s he published several papers on geometric problems, as well as on the place of mathematics and sciences in Gymnasium education. (Information based on author's profiles on www.worldcat.org and on zbmath.org).

²² Gustav Holzmüller (1844-1914) was a German mathematics teacher. He published several papers on geometry and analysis, as well as several mathematics textbooks, of which the four-volume *Elemente der Stereometrie* ("Elements of Solid Geometry") (Leipzig, 1900-1902) is the most important. In addition, he advocated establishing secondary schools that focused on the natural sciences as opposed to Latin. Cf. biography by G Kirschmer in *Neue Deutsche Biographie* **9**, 1972, 578: <http://www.deutsche-biographie.de/sfz33631.html>, accessed 05/11/2013.

But, as he remarks in the second chapter, it is a 'very good elementary introduction' [23, p. 140]. Furthermore, he gives an indication of topics that were not covered in schools: he comments that he could list a range of books 'that are [all] excellent in their own ways, but only treat problems within the limitations of solid geometry as taught in schools, i.e. within Euclidean geometry' [23, p. IV]. According to Holzmüller, they do not treat topics such as poles and polars, inversions, Apollonian circles, radical axes, or stereographic projection [cf. 23, p. IV]. These are precisely topics that Geiser included, and as he wrote the book with the intention of equipping Polytechnic students with the necessary knowledge of synthetic geometry, we can hazard a guess that they might not have featured on the standard secondary school syllabus. In all likelihood, Holzmüller would have referred to German schools here, but taking Geiser's introduction into account, it seems unlikely that there was a big difference between German and Swiss schools in this respect. Of course, there are limitations to Geiser's book as well; he does not cover three-dimensional drawing or map projections in the tradition of Ptolemy or Mercator, which Holzmüller identifies as missing from textbooks as well [ibid.]. However, descriptive geometry seems to have fared better at the Polytechnic than at most (German) universities, where, according to Holzmüller, it was not taught 'until recently' [23, p. IV-V].

As mentioned above, Geiser does not give any sources or inspiration for his book; we just know that it originated from his lecture course. However, it seems unlikely that he prepared the lectures without referring to any previous works. Steiner's *Vorlesungen* come to mind immediately, especially given that Geiser edited them at the time he wrote *Synthetische Geometrie*, as discussed above. Holzmüller gives a comprehensive historic overview of relevant geometry books and papers available until 1900, 'which provided scientific or educational progress' [23, p. 141]. Geiser's book is included, and, moreover, it is put into a historic context. Whilst I will briefly list some books that Geiser may have consulted, more information on the books can be found in the original [23, p. 133-141]. Of course, several of Steiner's books feature in Holzmüller's survey, including *Vorlesungen*; in fact, he even claims that

‘Steiner has to be considered the greatest geometer of all times’ [23, p. 137]. It is likely that Geiser studied relevant treatises by mathematicians such as Pascal, Desargues, Euler, Lambert, Monge, Carnot, Brianchon, Poncelet, Möbius, and Staudt²³. It is important to bear in mind that this is pure speculation, but as all of these men made significant contributions to the development of geometry, it is conceivable that Geiser was familiar with their work. Holzmüller himself bases his survey on Pohlke’s²⁴ *Darstellende Geometrie* (Berlin 1860/76), ‘virtually a classic, which intends to unite the whole of [geometry] in the smallest of spaces’ [23, p. 139]. Geiser may have had access to the first volume while writing his own book.

Among books published after *Synthetische Geometrie*, Holzmüller highlights works by W Fiedler [14] (‘probably the most outstanding of the more recent textbooks’, [23, p. 140]) and Reye [30], as well as Thomae, Rulf and Milinowski; he also lists some authors who wrote good schoolbooks [cf. 23, p. 140-141].

A reference to Geiser’s ‘exquisite little book’ [22] can be found in a one-page paper by the Hungarian engineer Josef Herzog²⁵, *Ueber die zeichnerische Parallelschaltung von Wechselstromwiderständen*. In this paper Herzog shows how to find the impedance z that corresponds to two impedances z_1 and z_2 in a parallel circuit. To this end he defines conjugate points with regard to a base circle; but he refers his readers to *Synthetische Geometrie* for more information.

A further reference to Geiser’s book, more specifically to section §25, where he treats the three-dimensional generalisation of the Problem of Apollonius, appears in a paper entitled *Ein artilleristisches Problem* by Fritz Bützberger [12],

²³ I have kept Holzmüller’s chronology here.

²⁴ Karl Wilhelm Pohlke (1810-1876), a German painter and professor of projective geometry and perspective at the Royal Bauakademie in Berlin. Among mathematicians, he is known for Pohlke’s Theorem, ‘one of the most remarkable contributions to mathematics by an artist.’ Quote taken from http://stubber.math-inf.uni-greifswald.de/mathematik+kunst/kuenstler_pohlke.html, which provides a comprehensive biography of Pohlke; accessed 05/11/2013.

²⁵ Josef Herzog (+1915) studied at the Technical University in Vienna and worked as an electrical engineer for Ganz Works in Budapest. He developed methods to calculate electrical power output and published a number of papers on alternating current, some of them with the Dutch engineer Clarence Feldmann. Cf. obituary of Herzog in: *Schweizerische Bauzeitung* **66** (1), 1915, 10.

Geiser's colleague on the ICM organising committee. Bützberger first summarises the proof of a problem solved by Haentzschel in *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht* 47: using the Problem of Apollonius to determine the location of an enemy gun, given the times when three listening posts in a plain hear gunfire. Bützberger now assumes that the battle takes place on hilly ground, and introduces a fourth listening post in order to solve the analogous problem. For his proof he utilises the three-dimensional version of Apollonius's problem, hence the reference to *Synthetische Geometrie*, as well as Fiedler's *Cyklographie* [cf. 12].

One of Bützberger's research interests was Steiner, and his scientific estate contains a manuscript regarding 'Jakob Steiner's handwritten estate from 1823-26'²⁶ [5] (see section 4.2.5.1). In this manuscript Bützberger discusses Steiner's work on commensurable series of circles and spheres, respectively. Explaining that Clausen's proof of the theorems in question is too advanced for students, Bützberger remarks:

It goes without saying that Steiner did not obtain these [theorems] by means of such difficult and involved calculations, but by using the principle of conjugate radii, that is, by means of the simple and natural method that Professor Geiser employed in order to derive the most curious of these formulae. I believe that by taking the liberty of giving short proofs of all of Steiner's formulae, which follow this example, I will be able to reconstruct Steiner's train of thought.

[5, p. 89 (§12)]

²⁶ There exist two copies of the manuscript, one is handwritten and includes a biography (Hs 194: 6), the other one is typed and includes annotations but no biography (Hs 194: 7). However, none of Bützberger's publications bears such a title. In 1913 he published *Über bizenrische Polygone, Steinersche Kreis- und Kugelreihen und die Erfindung der Inversion* (Leipzig). Emch writes in a review that 'Professor Bützberger presents the results of a critical and historical investigation on bicentric polygons, Steinerian series of circles and spheres, and the discovery of inversion' (*AMS Bulletin*, May 1914, 412). It seems that this might be the published manuscript.

Here, Bützberger refers to §28 of *Synthetische Geometrie*, the section on series of circles and series of spheres²⁷.

L Kollros includes a reference to the same section in *Quelques théorèmes de géométrie* [25]. He proves a number of theorems on families of conics that Steiner stated without proof. Some of these (discussed in §3 of the paper) concern the circles tangent to two given circles and families of circles formed by them, in particular those circles that allow for commensurable series of circles or spheres²⁸ [25, p. 37]. In § 3.III, Kollros considers a family of spheres obtained by rotating a certain family of circles around their axes. Steiner then stated that there exists a second family of spheres, where every sphere touches every sphere in the first family. Moreover, if there is a commensurable series of spheres in the first family, then there exists a commensurable series of spheres in the second family, and the following relation holds:

Let u be the number of cycles of the first family and n the number of its elements (spheres), and let U and N be the cycles and elements of the second family, respectively. Then $\frac{u}{n} + \frac{U}{N} = \frac{1}{2}$.

This is one of the most curious theorems in geometry.

[35, p. 136]²⁹

As mentioned above, Steiner does not give a proof of this, but he mentions a special case, where $u=1$, $n=3$, $U=1$ and $N=6$. Geiser proves the general case at the very end of *Synthetische Geometrie* [15, p. 179-183]. This is one of the most algebraic passages in the book, and he derives several equations in order to obtain Steiner's relation above. It must be noted, however, that he does not

²⁷ Indeed, in his review of Bützberger's *Über bizenrische Polygone...* Lampe writes that the chapter on Steiner's series follows Geiser's approach in the last chapter of *Synthetische Geometrie* (zbmath.org/?q=an:02622004, accessed 28/11/2013). This also suggests that it might be the published manuscript.

²⁸ Geiser defines a commensurable series of spheres as follows: He takes two arbitrary spheres M_1 and M_2 , with the smaller of the two lying completely inside the larger one. Moreover, he assumes that there is a third sphere M_3 in the space between M_1 and M_2 and tangent to both. Then he considers a series of spheres, of which the first one touches M_1 , M_2 and M_3 , the second one touches M_1 , M_2 , M_3 , and the sphere preceding it in the series, etc. According to Geiser, the series is commensurable if it closes, i.e. if the last sphere in the series touches M_1 , M_2 and M_3 and the first sphere in the series, which may happen after one or several cycles around M_3 . The series is incommensurable if it does not close [cf. 15, p. 177].

²⁹ Steiner originally published the paper in *Crelle's Journal* 2 (2), 1927, 190-193.

claim to be proving this particular relation, but rather he wants to determine the number of elements of two series of circles, where one series is located between two concentric circles [15, p. 179-180]. He presents Steiner's relation as a result of his proof, and only after having obtained it he explains that it relates to series of spheres as in Steiner's paper. Interestingly, Bützberger considers Geiser's proof to use a 'simple and natural method' [5, p. 89], whereas Kollros finds Geiser's calculations 'rather long' [25, p. 43], and indeed his proof is much shorter. – Of course, these two comments are not mutually exclusive.

Geiser then gives the special case (see above) as a 'simple example' [15, p. 183] and shows that the relation becomes: $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$. With this equation he concludes his book – given the style of the book, one would have expected an explanation of a concept or construction rather than an equation, but on the other hand it is a simple example at the end of a rather difficult passage. As in the rest of *Synthetische Geometrie*, Geiser makes no reference to Steiner when discussing this relation, thus keeping in line with his habit of not mentioning any historical background of the definitions and theorems that he covers in the book.

5.2 Letters to Julius Gysel

As discussed in section 2.1, Geiser had a large network of contacts across Europe, which comprised colleagues, politicians, writers, and also former students. One such student was Julius Gysel, a mathematics teacher and headmaster from Schaffhausen. Several of the letters that Geiser wrote to Gysel over the course of their long friendship survived. They give an insight not only into their friendship, but also into Geiser's work and habits, and will be discussed in this chapter. However, Gysel is an interesting person in his own right, and a good example of a late 19th century secondary school teacher. Like many of his colleagues, he took an academic approach to his teaching, and published research papers. Passionate about the sciences, he significantly improved the quality of his school, the Kantonsschule Schaffhausen.

An account of Gysel's life is given before Geiser's letters are analysed.

5.2.1. Julius Gysel (1851 – 1935)

Julius Gysel was born on 11 August 1851 in Wilchingen, a small town in canton Schaffhausen, where the family had been living since at least the 16th century [32]. His father was Johannes Gysel (1815-1875), a council clerk; he also served as president of the town council and later on as Kantonsrat. His mother was Maria Verena Waldvogel (1821-1903) [7]. Gysel grew up in his hometown with his four siblings and attended elementary school there. He then attended the Realschule in Neunkirch (canton Schaffhausen), followed by four years at the Gymnasium in Schaffhausen. There he discovered his passion for mathematics, but he excelled in all other subjects as well.

After being awarded his Matura in 1869, Gysel studied mathematics at the Polytechnic until 1872. Moreover, he attended lectures on history and art history by Johannes Scherr and Gottfried Kinkel, respectively. Among his mathematics teachers was Geiser, in fact, 'his favourite teacher was the influential Konrad [sic!] Friedrich Geiser' [36, p. 11]. The two men formed a life-long friendship. Geiser also suggested the topic for Gysel's doctoral thesis³⁰, which he submitted at the University of Zurich in 1874 and dedicated to Geiser [4]. Subsequently, Gysel attended mathematics lectures at the University of Bern for a year, notably those by Georg Sidler and Ludwig Schläfli. He remained good friends with the latter until Schläfli's death.

Whilst writing his thesis, Gysel worked as a supply teacher for mathematics at the Kantonsschule in Schaffhausen from 1872-1874. He must have made a very good impression, as he was offered a full position upon his return from Bern the following year despite his young age. As was common at the time, the school had two tracks, a classical one and science one. Gysel taught mathematics in the classical track until his retirement in 1926. In

³⁰ *Synthetische Untersuchung eines Orthogonalflächensystems*, 1874 ("A Synthetic Analysis of a System of Orthogonal Surfaces").

addition, he took on responsibility for the physics classes in both tracks until 1919.

According to Schnyder [33, p. 4] and Scherrer [32], Gysel's pupils were always well prepared for university studies, notably at the Polytechnic. Indeed, former pupils attest to the quality of Gysel's teaching:

Gysel taught as if at university. In physics, we were presented with experimental lectures, broken up by a few revision classes. At this level of education, this is a risk. The success [of this method], with exceptions, was due to the physics experiments, which fascinated the pupils, on the one hand; and to the well-structured, educational lectures on the other hand, paired with excellent experimental skills.

[36, p. 12-13]

However, as Scherrer notes, 'according to [Gysel's] own confession, neither particular aptitude, nor specific training or manual skill seemed to predestine him for teaching physics' [32]. Gysel preferred mathematics, and he was well aware of the challenges that faced any teacher of mathematics:

According to Gysel himself, mathematics is the least popular and least enjoyable subject for many pupils, because it demands strict, implacable logic and a selfless devotion to the subject matter. However, he was capable of teaching with such clarity that less talented pupils also profited [from his lessons]; this was because [their] teacher was never scathing about their shortcoming, but tried to support them as much as possible, with great wisdom and patience.

[32]

Despite all the educational reforms since Gysel's death, his observation about the popularity of the school subject mathematics still holds true today – surely many of today's mathematics teachers would agree with him.

Gysel greatly influenced the school not only as a teacher, but also in his capacity as headmaster in particular. In 1881 he was appointed deputy headmaster, 'which could be seen as an exceptional mark of confidence' [27, p.

8]. Not only had his predecessor been held in high regard, but also, more importantly, 'the powers of the deputy headmaster were vested in a representative of the sciences' [ibid.]. Three years later, Gysel was appointed headmaster, which was the first time that a science teacher was appointed to such a senior position [10, p. 70]. He only stepped down in 1909. His appointment as headmaster coincided with a general change in education across Switzerland and Germany: during the second half of the 19th century, and particularly towards its end, the sciences became more and more important in the curricula of secondary schools; polytechnics were not only founded, but were also awarded university status [19, p. 79-80; p. 127-128]. Gysel's passion for physics experiments and technological progress (see paragraph below) is representative of his time. Similarly, his appointment can be seen as an indication of the gradual revaluation of scientists.

During his time in office the school prospered: teaching of the sciences was enhanced, the laboratories were expanded, a third track for future teachers was established in 1897 [9, p. 46], the number of pupils doubled, and, as his crowning achievement, the school moved into a new building in 1902.

Gysel aka "Tschuli" was quite popular among his pupils, not only as a teacher, but also as headmaster. As K Bächtold writes, 'it has been attested a hundredfold that the headmaster was an enthusiastic and inspiring teacher, respected and loved by the pupils' [9, p. 48]. The school's pupil corporation, Scaphusia³¹, awarded him honorary membership in recognition of his benevolence towards the society³². Furthermore, the dialect writer Albert

³¹ The Scaphusia, founded in 1858, is one of the oldest pupil corporations (Schülerverbindung) in Switzerland [cf. 39]. In general, these corporations follow the organisational structure and traditions of the better known student corporations (Studentenverbindung), whose nearest equivalent in the English-speaking world are the US-American fraternities.

³² In [10] K Bächtold dedicates an entire chapter to the development of the corporation during Gysel's term in office. In a nutshell, the Scaphusia abandoned some of its pretentiousness during that time, though it is not apparent whether this was due to the influence of 'plain and wise' [10, p. 70] Gysel with his 'matter-of-fact attitude' [ibid.] or a general movement among its members. In any case, he was successful in convincing his pupils to make their morning pints a less frequent occurrence than before [10, p. 70-71].

Bächtold³³ honoured Gysel's influence on his own life in his novel *De Studänt Rübme* (1947)³⁴: The headmaster in the novel, called "Vatter" ("father"), is based on Gysel. According to his biographer K Bächtold, A Bächtold saw a father figure in Gysel [9, p. 48]. The writer described his teacher as follows:

Tschuli: something powerful, clear and straight; you know at first sight that there are no backdoors here, nor any intrigues. [...] You will find such a man only once in a hundred years.

[9, p. 48]

Throughout his life, Gysel educated himself further, both in his subjects and in educational matters. He advocated extended and better teaching of physics, and set up the school's physics laboratories. Indeed, 'he witnessed and participated in the astounding development of technology' [32] during his professional life. An example of this interest is the X-ray chamber that he set up in the new school building, and which was used by the local hospital as well. He continued working in the chamber even after his retirement. The new school was also equipped with eight laboratories and two libraries [21, p. 142] as well as current transformers and a dynamo, which generated the energy needed for physics experiments [21, p. 143].

Gysel was also a keen member of the *Naturforschende Gesellschaft Schaffhausen*, his local Society for Natural Scientists, from 1876 onwards. From 1905-1920 he served as the Society's vice president, and was made an honorary member in 1922. Over the course of the years, he gave a number of talks at the society's meetings as well as elsewhere, predominantly on topics

³³ Albert Bächtold (1891-1981) from Wilchingen (canton Schaffhausen) wrote all of his novels in the dialect of his home region. They are essentially a literary autobiography; their hero Peter Rebmann experiences similar stages in his life as the writer A Bächtold [cf. 9].

³⁴ The title translates as *The Student Rübme*, "Rübme" being the diminutive of the surname "Rebmann". The novel is based on A Bächtold's school days and pays homage to the teachers at the Kantonsschule Schaffhausen. A Bächtold attended the school from 1907-1911, first in the science track, then in the teacher training one. Gysel became A Bächtold's patron [cf. 9]. As his biographer K Bächtold notes, A Bächtold's writing was inspired by Gysel's advice: 'Do not just look at something – you have to look beyond it' [9, p. 65], which was characteristic of Gysel's attitude.

in physics. In addition, he taught experimental physics courses for teachers, and a popular lecture course on radiology in 1920/21 [36, p. 15]. Gysel became a member of the *Deutsche Mathematiker-Vereinigung* in 1897. He was also among a large number of Swiss secondary schoolteachers who attended the First International Congress of Mathematicians in Zurich.

His teaching and administrative duties left him little time for research. The list of his publications comprises only seven items: apart from his doctoral thesis, Gysel published three papers on problems in geometry as part of his school's Osterprogramme³⁵. As Uehlinger recounts [36, p. 14], the 1894 paper was, according to Gysel, the most widely recognised one, and included in L Henneberg's *Die graphische Statik der starren Systeme* (Leipzig, 1911). The other publications are a report on the Kantonsschule's new building (1903), an obituary of his friend Jakob Amsler-Laffon (1912), and the chapter on *Mathematics, Astronomy, Technology and Physics* in the centennial publication of the *Naturforschende Gesellschaft Schaffhausen* (1923).

On top of his school duties, Gysel took on a number of positions in various committees, which again reflect his interest in education and in technological progress. From 1889-1920 he served on the Erziehungsrat; he also acted as examiner at Matura examinations and the Polytechnic's entry exams for many years. Moreover, he was member of the town's library committee from 1888-1929, responsible for mathematics, astronomy, physics, technology and alpinism. For a while, he was on a committee studying the implementation of electric lighting, and for some years he was a member of the supervisory board of the Light- and Waterworks Schaffhausen.

In recognition of his contributions, he was awarded the Freedom of the City³⁶ Schaffhausen in 1922.

³⁵ *Beiträge zur analytischen Geometrie der Kurven und Flächen 2. Grades* (1877); *Über die sich rechtwinklig schneidenden Normalen einer Fläche 2. Grades* (1885); *Zur Konstruktion des Schwerpunktes einer ebenen Vielecksfläche* (1894).

³⁶ In Switzerland, worthy individuals are made an "Ehrenbürger", literally an honorary citizen. However, this honour is more comparable to the British Freedom of the City than to an honorary citizenship, but in general it does not grant any particular privileges.

On 03 August 1876, Gysel married Barbara Carolina Bollinger³⁷ from Schaffhausen. In various obituaries their 59-year long marriage is described as a happy one. The couple had two sons and two daughters, as well as nine grandchildren. One of their sons, also called Julius (1881-1972), graduated as a mechanical engineer from the ETH in 1906 and worked for an electric power company. In his obituary, his parents' home is described as a place with 'simple living, hard work, and a lot of music' [28, p. 264].

Throughout his life, Gysel was in excellent health – as Schnyder writes, 'during the 53½ years of teaching, [he] was never off sick, even for a single period' [33, p. 5]. A keen mountaineer, he climbed 33 mountains over 3000 m, and kept up his hobby until his late 70s. He co-founded his local branch of the Swiss Alpine Club³⁸, and led numerous youth hikes [34].

Julius Gysel died on 23 August 1935.

5.2.1.1 Letters from Ludwig Schläfli

A number of letters and postcards that Schläfli sent to Gysel between 1874 and 1888 are kept in Gysel's estate in Schaffhausen's Town Archive³⁹. On the whole, they tell us more about Schläfli than about Gysel, but they do offer a few snippets of information about the relationship between the two mathematicians. Schläfli mainly writes about his travels to various Swiss towns, often accepting or declining Gysel's invitations to visit him. Furthermore, he reports on his time at Felice Casorati's house in Rezzonico by Lake Como [2c, 2d], and writes about his feelings about various honours [2e, 2f]. See appendix E.2.3 for a translation of the letters.

³⁷ There are various spellings of her name; this version is taken from the parish register (register 1-37/4 Wilchingen, p. 204-205 [7]).

³⁸ SAC Sektion Randen, in 1886

³⁹ These are included in [2]; all translations by the author. I have only had access to the letters kept in this archive. Note that the timespan of their correspondence is given as 1874-1894 in Graf's obituary of Schläfli [18, p. 151].

Gysel matriculated at the University of Bern in April 1874 [37; note that his name is spelt “Gisel”], and the first letters from Schläfli date to August of the same year. The two men must have become friends quite quickly; after all, Gysel was still a student, albeit with a doctorate. In fact, Schläfli writes in August 1874 that he mentioned Gysel’s thesis to Alfred Enneper⁴⁰, who ‘expressed the wish to get a copy’ [2a]. Occasionally, Schläfli makes remarks about mathematical papers, such as when he enquires whether Gysel received a manuscript on spherical harmonics [2b]. Gysel also seems to have sent Schläfli papers, although it is not clear whether Gysel wrote them or not [2g].

Apart from exchanging letters, the two men visited each other every once in a while. One of the main topics of Schläfli’s letters is arranging these visits, and he seems to have been on cordial terms with Gysel’s family as well. In the early letters he sends Gysel’s parents his regards, later on Gysel’s family, such as in [2f], when he writes: ‘I send your wife and children my warmest regards, but I will not promise that I will teach any of them’. In a couple of letters from 1875 Schläfli sends regards to J H Graf, who later became Schläfli’s successor; he also mentions him in a few other letters. It appears that the three men met up occasionally.

Schläfli’s health deteriorated in the 1890s, but it seems that Gysel kept up his visits to Bern. On the last letter [2h], there is a handwritten remark: ‘saw Schläfli for the last time in Bern, in May 1894’, which was probably added by Gysel himself.

5.2.2. Letters from Geiser

The Stadtarchiv Schaffhausen houses 19 letters that Geiser sent to Gysel between 1874 and 1890 [3]. As with Schläfli’s letters, it is likely that more letters were exchanged. Moreover, without Gysel’s replies to Geiser, their

⁴⁰ Alfred Enneper (1830-1885), professor of mathematics at the University of Göttingen. His specialty was differential geometry. Cf. biography in S Gottwald (ed.), *Lexikon bedeutender Mathematiker*, 1990: <http://www.ub.uni-heidelberg.de/helios/fachinfo/www/math/homoeid/Gottwald/Enneper.htm>, accessed 30/04/2014.

correspondence remains incomplete, but nevertheless, the letters give an indication of their work, and their friendship. Translations of the letters are in appendix E.2.1.

As mentioned above, Geiser was Gysel's teacher at the Polytechnic, and it seems that he continued to act as a mentor for Gysel. The subject of the first few letters is Gysel's doctoral thesis, which Geiser deems 'very neat, apart from a few minor editorial details' [3a]. He also makes suggestions with regard to its publication. Although Schläfli was not Gysel's supervisor, he must have read Gysel's thesis as well, since Geiser writes that 'Schläfli will probably pour the lye of his criticism over it' [ibid.].

It is possible that Geiser advised Gysel to attend Schläfli's lectures in Bern, as he writes in [3b]: 'I am extremely happy that you get along so well with Schläfli. And if you increasingly feel how infinitely much you can learn from him, then this will do your self-awareness good'. However, Geiser still regarded Gysel as his student:

Now, hurry up with your doctorate, so that people can see your competence and, if I may say so myself, that you have learnt something from me in particular. I am very pleased indeed that you have been my student & have now stepped forward with such a neat achievement [i.e. Gysel's thesis].

[3a]

Apart from feedback on his thesis, Geiser also gave Gysel career advice. In [3c], Geiser suggests that Gysel should apply to teach in Schaffhausen again. Alternatively, he offers to have a word with the Cantonal Minister of Education. It seems that Schläfli proposed that Gysel could go into academia – as Geiser writes, 'the path indicated by Schläfli is such a honourable one that you definitely have to follow it' [3c]. However, we know that Gysel went back to Schaffhausen.

Throughout their correspondence, Geiser advises Gysel of vacancies for mathematics professors. In 1875 [3d] he recommends that Gysel apply for a post at the Academy in Lausanne, if only to call attention to him. Three years

later, Geiser put in a good word for Gysel at the Kantonsschule in Porrentruy (canton Jura), as he writes: ‘in such a way that I have reason to hope (although it is not absolutely certain) that you will be approached’ [3f]. The reason for this were Gysel’s complaints about his salary in Schaffhausen, and Geiser expresses the hope that Gysel would either be offered a pay rise or find a better-paid position. As Gysel stayed in Schaffhausen, we can assume that he did not get, or want, the post in Porrentruy, but unfortunately there are no records of whether his salary was increased. Geiser advises his friend of a vacancy again in 1886, this time at the Kantonsschule in Zurich [3m]. The letter is marked as ‘highly confidential!!! [sic!]’: Geiser writes that the School Board asked him to sound Gysel out on his interest in the post and asks Gysel to discuss the matter in person, but makes it clear that it is ‘just a mutually non-committal enquiry’ [3m]. Once again, Gysel ended up staying in Schaffhausen, despite Geiser’s threat: ‘brace yourself for my encouraging you’ [ibid.]. However, as Gysel had recently been appointed headmaster, it is unlikely that he would have wanted to start a new job. Moreover, there may have been political reasons, or a more suitable candidate.

After this letter, there are no more indications that Geiser tried to further Gysel’s career.

The above paragraph also illustrates Geiser’s networking skills. Not only was he on friendly terms with fellow mathematicians and university lecturers, but also with a number of schoolteachers and headmasters as well as politicians. These contacts came in very useful for his work. An example is [3j]⁴¹, in which he asks for Gysel’s help. It appears that the Kantonsschule Schaffhausen was one of the schools that concluded a treaty with the Polytechnic regarding its entry regulations for their pupils (see section 2.3). At the time of writing the letter, Geiser had not heard back from the cantonal government in Schaffhausen and asked Gysel [3j]:

Could you not make it clear to your Rector’s Office, or a member of the government, respectively, that it is in your school’s best interest to

⁴¹ In 1883 Geiser was Director of the Polytechnic, and Gysel was Deputy Headmaster of his school.

settle this business swiftly, since your new curriculum & the means needed for implementing it have already been approved.

Geiser further mentions that he would have to travel to Schaffhausen with Kappeler to discuss the situation, and that they would 'go to Frauenfeld [...] on the same mission' on the following Saturday [ibid.].

Whilst this is the only mention of Geiser and Kappeler's negotiations with secondary schools, there are a few more references to Geiser's work as Director. In several letters, he mentions that his teaching and administrative duties kept him very busy [3k, 3l, 3n]. Not only did he 'at least have to be around, even in less stressful times' [3k], but he also had to do business trips across the country. As he writes in [3n]: '[...] work is a heavy burden on me, all the more as I have to travel to Geneva next week, which will take up several days more'.

One of Geiser's more enjoyable business trips must have been the one that he mentions in [3i]:

Today I will travel to Luzern to attend the opening of the Gotthard Tunnel, to which I have been invited by the Bundesrat. Cremona will arrive there today, too, so I can be sure that I will have the most agreeable company.⁴²

In a few letters Geiser refers to a certain teacher called 'Scherer'⁴³, who seems to have been a mutual friend. His references to Scherer illustrate Geiser's influence on the educational system.

Mentioning him in [3g] for the first time, Geiser tells Gysel that 'the time has come for us to meet up with Scherer'. Unfortunately, he does not specify why

⁴² About 600 guests from across Europe were invited to attend the opening ceremony on 22-25 May 1882. Geiser was the Director of not only Switzerland's only federal university, but also its main engineering institute; naturally he would have been invited.

⁴³ Geiser does not give much information about him, but the person in question might have been the mathematics teacher and headmaster Friedrich Robert Scherrer (1854-1935) of Schaffhausen. According to obituaries of him, he was a good friend of Gysel's, and arranged Geiser's scientific estate (however, I have not been able to locate this estate, so it might not exist anymore). I have kept Geiser's spelling in case he referred to a different person. Appendix D contains a short biography of Scherrer.

they have to meet. Two years later [3i], he invites Gysel to stay at his home in Küsnacht, and asks him to invite Scherer, too. Scherer is mentioned for the last time in [3k]. Apparently he had caused some problems at work and ‘faced a crisis’ as a result. Geiser refuses to intervene at that stage, ‘unless either the authorities or Mr Scherer ask me to mediate. This would have a certain foundation in so far as I had a hand in his appointment at the time’ [ibid.]. Instead he asks Gysel to get Scherer to see reason, as Gysel is ‘closer to him with regards to age, profession, and ties of friendship, too’ [ibid.]. As the topic is not discussed any further in later letters, unfortunately we do not know whether, and how, the problem was resolved.

In a couple of letters Geiser also writes about mathematical problems. Unfortunately, they are out of context and without Gysel’s letters it is difficult to comprehend the problem Geiser refers to. In [3e] Geiser writes: ‘The other day I had reason to read your problem⁴⁴ again & have discovered that the elimination of λ and μ does not belong to the realms of impossibility after all’. He then proceeds by substituting a number of squares of variables by first order variables, which, according to Geiser, transforms ‘the three original equations’ [ibid.] into linear ones, which he solves. From these, he claims, one can then express ‘three surfaces of degrees four, six & six through a triple curve’ [ibid.]. It seems that the aim of the problem was to find centre sections of ellipsoids.

There is a second, undated⁴⁵ “mathematical letter” [3h], in which Geiser comments on five attachments that must have been enclosed, but which have not been kept with the letter. Gysel must have worked on the problem of normals for both the ellipse and the ellipsoid, and it seems that Geiser

⁴⁴ Unfortunately, I cannot reconstruct the nature of this problem. It may be that Geiser refers to Gysel’s first paper, *Contributions to Analytic Geometry of Quadratic Curves and Surfaces* (1877). Perhaps Geiser proofread it before publication and just offered an opinion on how to take the problem further?

⁴⁵ As Geiser sends his best wishes to Gysel and his family, we can assume that the letter was written after Gysel’s marriage in 1876. Some of the enclosed papers, if not all of them, seem to have concerned some aspect of normals. In 1885 Gysel published his paper *On the Orthogonal Normals of a Quadratic Surface*, so Geiser might have provided him with material for his research (although it seems to have covered surfaces of higher degrees).

proofread his solutions. Geiser's first attachment seems to have been his own solution of the problem for the ellipse, and he points out a mistake that Gysel made. In order to solve the problem for the ellipsoid, he refers Gysel to Salmon's book⁴⁶, in particular the section on finding the surface of a centre of curvature.

As an aside, on the back of some of the letters there are solutions of quadratic equations. However, as there are no references to these in the corresponding letters and as the handwriting is different, we can assume that Gysel used the free space to do rough work. As no context is given, however, regrettably it is impossible to reconstruct the problems he worked on.

As mentioned in chapter 2, Geiser edited the first volume of Steiner's *Vorlesungen über synthetische Geometrie*. A third edition of the book was published in 1887, and we learn from [3o] that Gysel revised the sixth and seventh chapters of the first volume for this. Geiser asks his friend to 'have a closer look at the figures in particular, please. Some of them might have to be replaced by better ones' [ibid.]. This request testifies to Gysel's mathematical competence – after all, Geiser knew many geometry professors, who could have acted as revisers.

Of course, Geiser also writes about such mundane things as arranging mutual visits, or sending Gysel and his family his best wishes for the New Year [3o]. He also mentions that he had to move house [3p]. Interestingly, this is the only comment about any private matters, apart from the fact that his wife also sends Gysel her best wishes on a few occasions. Neither does Geiser congratulate Gysel on his wedding, or on the births of his children⁴⁷. With regards to mutual visits, Geiser invites Gysel to stay at his house in Küsnacht on several occasions. In one of the earliest letters [3c], Geiser announces his

⁴⁶ Geiser does not specify which of Salmon's books he is referring to, he just writes 'German Salmon, Vol. 1, p. 288'. Most probably he refers to Salmon's *Analytic Geometry of Three Dimensions*, which was translated into German and edited by W Fiedler. In German, it is often referred to as "Fiedler-Salmon".

⁴⁷ Of course, he may have sent congratulations, which have since been lost.

visit in Bern. In a number of letters, Geiser also suggests that they should meet in Winterthur, or regrets that such a meeting is not possible⁴⁸.

However, something that the letters do reveal about Geiser's personal life is that he liked wine. Apparently Gysel's father-in-law was a winemaker, and he seems to have provided Geiser with wine on a regular basis: Geiser sent an empty barrel to Schaffhausen, Mr Bollinger filled it with wine and returned it to Küsnacht, where Geiser lived. Geiser refers to this wine trade in most letters; he either places an order, or mentions that he needs to settle his bills. The first mention of the trade is in 1877 [3e], but as he refers to an earlier delivery, we can assume that it had been going on for some time already.

⁴⁸ Unfortunately, I have not been able to find out why they met in Winterthur. According to A Jacob [6] of the Swiss Academy of Sciences (into which the *Schweizerische Naturforschende Gesellschaft* evolved) and U Uehlinger [8] of the *Naturforschende Gesellschaft Schaffhausen*, neither of these societies had a club house in Winterthur. Uehlinger suggested that they might have met in the Technikum Winterthur (founded in 1873, today Zürcher Hochschule für Angewandte Wissenschaften). Another possibility is that the Polytechnic or its alumni association, the GEP, owned a property in Winterthur, although I can only speculate.

Winterthur is the sixth largest city in Switzerland, and is located halfway between Zurich and Schaffhausen. It expanded considerably during the 19th century, when it became both an industrial hub and an important rail junction. Thus it would have been easily accessible from both Zurich and Schaffhausen, and Geiser and Gysel might just have met in a restaurant or pub there. For more information on Winterthur see article by M Suter in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D157.php>, accessed 14/03/2014.

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6. Rudio as a Historian of Mathematics

As mentioned in chapter 3, Ferdinand Rudio was a keen historian of mathematics. In this chapter his most important works in this area are analysed.

6.1 *Archimedes, Huygens, Lambert, Legendre*

6.1.1 Background and Motivation

Since Mr Lindemann succeeded, ten years ago now, in settling the famous problem of the quadrature of the circle by means of the rigorous proof of the transcendence of π , based on Mr Hermite's research on exponential functions, and since Mr Weierstrass derived the results by Hermite and Lindemann again in 1885, in a new and comparatively simple way, many have taken a new interest in this curious problem, whose history spans roughly four millennia and which is therefore one of the oldest problems of humankind.

On the occasion of a conclusion such as the one reached here, many like to take a look back and dwell primarily on those papers that furthered the now settled problem in a direct and distinguishable manner. Picking these papers and making them readily accessible to everybody with an interest in the historical development of mathematics did not seem like an ungrateful task to me.

[29, p. iii]

This is how Rudio begins the preface to *Archimedes, Huygens, Lambert, Legendre*¹. He continues (in this order) by stating his intentions, explaining his method of working, and outlining the contents of the book; and finishes by listing his principal sources. A considerable part of the book is devoted to a historical overview of the problem of the quadrature of the circle. In fact, it is a

¹ Henceforth referred to as *AHLL*.

historical overview of the mathematics connected to the problem and the people who contributed to its solution in one way or another. As he states himself, it is ‘a complete revision and considerable expansion’ [29, p. vi] of his paper *Das Problem von der Quadratur des Zirkels*², published two years previously. After contemplating the reasons why finding the quadrature of the circle became such a famous problem and clearly defining it, Rudio gives a comprehensive summary of quadrature attempts and approximations of π , starting with the approximation to π in Ancient Egypt and finishing with a summary of the – at the time very recent – paper by Weierstrass, where he proves that π is transcendental. This gives the historical context for the four papers that follow this overview and which Rudio considers to be ‘milestones in the historical development of the problem of the quadrature of the circle’ [29, p. iii]:

- *Κυκλου μετρησις*³ by Archimedes (287-212 BC)
- *De circuli magnitudine inventa*⁴ by Christiaan Huygens (1629-1695)
- *Vorläufige Kenntnisse für die, so die Quadratur und Rectification des Circuls suchen*⁵ by Johann Heinrich Lambert (1728-1777)
- *Éléments de géométrie. Note IV, où l'on démontre que le rapport de la circonférence au diamètre et son quarré sont des nombres irrationnels*⁶ by Adrien-Marie Legendre (1752-1833)

I will not go into those papers, as they are not Rudio’s writings. The translations are his own, though (Lambert wrote in German, therefore Rudio only edited that paper). He added some footnotes and, as he admits in the preface, corrected minor ‘spelling mistakes or calculation errors’ [29, p. v] in all but Archimedes’ treatises. Other than that, he was careful to produce translations that were as close to the original texts as possible both in content and in mathematical notation, as he points out himself. Unlike some of his

² “The Problem of the Quadrature of the Circle” [36]

³ “On the Measurement of a Circle”

⁴ “On the Discovered Magnitude of the Circle”

⁵ “Preliminary Results for Those Who Seek the Quadrature and the Rectification of the Circle”

⁶ “Elements of Geometry. Note IV, in which it is shown that the ratio of the circumference to the diameter and its square are irrational numbers”

colleagues he considered it important to preserve the original notation, rather than translating it into the modern (and much shorter) notation based on symbols [29, p. iv-v]⁷. This would preserve the ‘individual colouring’ of the text and avoid ‘giving wrong ideas about the mathematical style of the era in question’ [29, p. iv]. Furthermore, he assumed that ‘the history of mathematical language and notation is also of great interest’ [29, p. iv]⁸.

One of Rudio’s main reasons for writing *AHLL* was indeed an increased interest in the history of mathematics among both mathematicians and non-mathematicians, and he hoped that the book would contribute to raising this interest even further. Lecturing on the history of mathematics himself, he intended the book to be an introduction to the subject as well as a resource for teachers of mathematics. In particular, he writes:

Firstly, I am pleased to point out that the interest in mathematical-historical research is spreading in general, and that our peers also recognise the value of and the necessity for historical research more and more. Secondly, there could hardly be another problem, which would serve as such an excellent introduction to studying the history of mathematics, as does the problem of the quadrature of the circle. Having emanated from humble beginnings, it became linked to almost every mathematical discipline over the centuries, to the point that in order to finally solve it, the entirety of modern science had to be mobilised and deployed. Lastly, by editing these treatises, which are not widely available anymore, I hope to render a service to the teachers at secondary schools in particular. For I do not doubt that studying these papers, especially the paper by Huygens, which is of such eminent importance to mathematics teachers at secondary schools but

⁷ Rudio gives anachronistic translations of Archimedes’ treatise by Hauber (1798) and Nizze (1824) as examples here. Heath would be another example of an author who translated ancient notation into modern notation (cf. I Grattan-Guinness, chapter 7: The British Isles, in [17, p. 161-178]).

⁸ Given this assumption, Rudio would have been pleased to know that Cajori refers to *AHLL* in paragraph 397 of [11], which was first published in 1929, the year of Rudio’s death. However, Cajori’s reference [11, footnote on p. 11] concerns the use of the symbol π by Euler and the Bernoullis, and thus the historical overview in *AHLL*, not Rudio’s actual translations.

yet receives far too little attention, will greatly benefit mathematics education.

[29, p. iii-iv]

As Rudio remarks, he used original works wherever possible. His main sources on the history of mathematics were Cantor's *Vorlesungen über die Geschichte der Mathematik 1-2*, Hankel's *Zur Geschichte der Mathematik im Alterthum und Mittelalter*, and Wolf's *Handbuch der Astronomie, ihrer Geschichte und Litteratur*. Further sources included books by Montucla, Kästner, and de Heer, as well as Klügel's mathematical dictionary [cf. 29, p. vi].

6.1.2 Chapter One

Rudio begins his historical account of the quadrature of the circle by discussing the popularity of the problem [29, p. 3]:

Among all the mathematical problems that have kept mankind occupied over the course of the centuries, none has become more popular than the problem of the quadrature of the circle. Searching for the quadrature of the circle⁹ has virtually become a proverbial phrase, meaning as much as doing something that is very difficult or even impossible, and is therefore pointless. Furthermore, among all mathematical problems, none is older than the problem in question, since its history spans roughly 4,000 years and is therefore as old as the history of human civilisation¹⁰.

⁹ "die Quadratur des Kreises suchen"

¹⁰ Rudio uses the word "Kultur" here, not "Zivilisation". Given the context, "civilisation" is the more accurate translation than "culture". In the 19th and early 20th centuries especially, German scholars attempted to draw a distinction between "Kultur" and "Zivilisation". "Zivilisation" was often used to denote the technological and material progress of a society and was thus perceived as a shallower concept than "Kultur", which implied that the society had a morality. It is possible that Rudio made that distinction as well; but it should be pointed out that both terms were still used interchangeably, during Rudio's lifetime often in contrast with "barbarism" and "uncivilised societies". Furthermore, it is likely that Rudio only considered more developed civilisations such as Ancient Egypt, Mesopotamia, and the Indus Valley Civilisation, which are often called "Hochkulturen" in German. This term denotes a civilisation that possessed a sophisticated political system and religion, specialised

Rudio rightly notes that mathematicians knew (and solved) many problems, which were of greater importance to mathematics than the quadrature of the circle. He suggests that one of the reasons for the popularity of this particular problem was its simplicity, the fact that laymen could easily understand what the mathematicians attempting the quadrature tried to do. The fact that scholars repeatedly failed to solve such a (seemingly) simple problem only added to its fame. Furthermore, at various points in history it was rumoured that successfully squaring the circle would either bestow extraordinary powers on the lucky individual, or at least entail prizes awarded by various Science Academies [cf. 29, p. 3-5]. In his account, Rudio remains objective and refrains from making any judgements, but his choice of words suggest that he was rather bemused by all those superstitions that entwined around the quadrature of the circle.

Rudio defines the problem of squaring the circle in terms of constructing a circle with the same area as a given square, *using ruler and compass only*. Thus, if it were possible to square a circle, one would be able to construct (again using ruler and compass only) a line segment of length πd , which would equal the length of the circumference of a circle with its area equal to a square with side length d . However, he points out that this is not necessarily how the ancient mathematicians defined the problem [29, p. 8]. Throughout the book, Rudio reminds his readers of his definition of “construction”. In the penultimate section of his overview [29, §14, p. 61-63] he comes full circle,

occupations, towns as centres of trade and organisation, and that had developed a script and some sciences [cf. ii, vol. 10, p. 132]. I doubt that, when thinking about the history of mathematics (ignoring basic counting), Rudio would have considered early civilisations such as the Palaeolithic and Neolithic eras for example (which date back much further than 2000 BC). Thus his claim that human civilisation is 4,000 years old can be regarded as more or less correct (although modern scholars now consider the Early Dynastic Period of Ancient Egypt to have begun at about 3,100 BC; the Mesopotamian and Maya civilisations also date back to at least 3000 BC). For the German distinction between “Kultur” and “Zivilisation” cf. [i] F Braudel, *A History of Civilisations*, (translation by R Mayne), Penguin Books, New York, 1995; [ii] *Brockhaus Enzyklopädie*, 19th, completely revised edition, F. A. Brockhaus, Mannheim, 1989 (vol. 12, 580-583 for “Kultur”; vol. 24, 578-579 for “Zivilisation”); [iii] J Fisch, “Zivilisation, Kultur” in: O Brunner, W Conze, R Koselleck (eds.), *Geschichtliche Grundbegriffe. Historisches Lexikon zur politisch-sozialen Sprache in Deutschland*, Klett-Cotta, Stuttgart, 1992, vol. 7, 679-774.

explaining how this definition can be translated into algebraic expressions and how it relates to the transcendence of π .

Finally, Rudio defines the three eras into which the history of the problem of the quadrature of the circle can be divided, in his opinion¹¹:

1. From the first evidence of a quadrature in Ancient Egypt until the invention of differential and integral calculus in the second half of the 17th century: This era is characterised by attempts to approximate π , predominantly by using the method of exhaustion. Rudio highlights Archimedes and Huygens as the most important contributors to that method. This era is covered in his Chapter Two.
2. From the second half of the 17th century until 1766, when Lambert first proved the irrationality of π : This era is characterised by attempts to express π analytically, as well as by calculating π to more and more decimal places. This era is covered in his Chapter Three.
3. 1766-1882/85: During this era, mathematicians proved first that π is irrational, then that π is transcendental. Rudio calls it the 'critical' era, and discusses it in Chapter Four.

6.1.3 Chapter Two

As mentioned above, Rudio begins his historical account with the approximation of π found in Ancient Egypt, which is preserved on the Rhind Mathematical Papyrus¹². He rates the Egyptian value of $\pi=3.1604\dots$ as 'an

¹¹ Hobson uses the same division into historical eras as Rudio in his *Squaring the Circle*, 1913 [22]. The two books will be compared towards the end of this section. Klein also follows it (although more loosely) in *Famous Problems in Elementary Geometry*, chapter II, 1897 [24]. Both list *AHLL* as one of their major sources.

¹² Referring to Cantor, Rudio writes that the Papyrus was written between 2000 and 1700 BC by the scribe Ahmes (thus supporting his claim that the problem of the quadrature of the circle is as old as human civilisation; discussed above), but that it was copied from a manuscript that was 'several centuries older' [29, p. 10]. Nowadays, scholars believe that the Papyrus was written between 1650 BC and 1550 BC (dated in the 33rd year of the reign of the Hyksos king Apophis), and that the original dates back to the Twelfth Dynasty (spanning the 20th and 19th centuries BC). Cf. the website of the British Museum, where the Papyrus is kept:

already quite respectable approximation' [29, p. 1], especially when comparing it to the Babylonian value $\pi=3$, which resulted from their observation that the circumference of a circle roughly equals three times its diameter. He then cites some passages in the Bible and the Talmud where the Babylonian result is applied.

The second section [29, §5] is devoted to the mathematical achievements of the Greeks that advanced the problem in some way¹³. Rudio identifies Anaxagoras as the first Greek mathematician to “draw” a quadrature whilst being imprisoned, and explains that he cannot consider the quadratrix of Hippias as a solution to the problem, as it cannot be constructed by ruler and compass only [29, p. 12].

Then he turns to Antiphon and Bryson, explaining how Antiphon conceived of the first method of exhaustion. After a passage on Hippocrates's quadrature of lunes¹⁴, Rudio arrives at Archimedes, ‘by far the most significant mathematician of all of Antiquity’ [29, p. 14]. He states the three theorems that Archimedes proved in *Κυκλου μετρησις* and gives short summaries of the proofs, commenting how much more difficult it was to take square roots during Archimedes' lifetime [29, p. 15]. In his translation of the paper, Rudio includes additions and explanations in algebraic form, in particular the third proof. He gives Archimedes' upper and lower bounds for π , but stresses that developing the method of exhaustion was much more crucial for the history of the problem than the approximations themselves [29, p. 16].

Finally, Rudio points out Hipparchus and Ptolemy as the fathers of trigonometry, quoting Cantor to illustrate Ptolemy's importance for mathematics. As in his popular talk on mathematics in the Renaissance [37],

http://www.britishmuseum.org/explore/highlights/highlight_objects/aes/r/rhind_mathematical_papyrus.aspx, accessed 19/06/2013.

¹³ Rudio writes here that ‘we can safely assume that [Thales and Pythagoras] spent some time in Egypt and relocated Egyptian geometry to Greece’ [29, p. 12]. More recent scholars, e.g. D E Smith and M Kline, are much more cautious when discussing the influence of the Egyptians on Greek mathematics (cf. [46]; M Kline, *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, New York, 1972).

¹⁴ This was ten years before Rudio published his first paper on Simplicius's commentary and Hippocrates's lunes [31]. In *AHLL* he refers to Bretschneider.

Rudio only praises Ptolemy and fails to mention any criticism of Ptolemy's work. It is unlikely that he was not familiar with the accusations against Ptolemy raised by Tycho Brahe and by Delambre¹⁵ (whether or not they were justified should not be of importance here; on the contrary, if Rudio believed the accusations to be false they would have helped to portray Ptolemy as an eminent scholar), so we can only speculate as to why he did not mention this. Perhaps he did not consider it important, or beyond the scope of the book, or perhaps he genuinely did not know about those attacks.

The next section, on 'the Romans, Indians, and Chinese', is devoted almost entirely to (relevant) achievements by Indian mathematicians. As in his Renaissance talk, Rudio excoriates the mathematics of the Romans:

For a nation as ill disposed for scientific mathematical speculation as the Roman one, the existent approximations of π were quite sufficient for applications in real life.

[29, p. 18]

In the subsequent section, he links the non-existence of mathematical research in Europe 'from the Migration Period to the end of the 10th century' [29, p. 23] to the legacy of Roman education [cf. 29, p. 23].

He is not quite as scathing in the passage on Chinese approximations of π , but concludes that the work of Chinese mathematicians did not further the problem either [cf. 29, p. 19-20]. Other writers, e.g. Smith [46, Vol. 2, p. 309] and Hobson [22, p. 23-24], grant the Chinese a bit more credit.

'In contrast [to the Romans], the Indians were of a different mathematical rank entirely,' Rudio remarks [29, p. 18]. He gives approximations of π by Aryabhata, Bhāskara (II) and Brahmagupta (whose approximation he describes as 'a very curious' one [29, p. 18]) and explains how they were calculated. Furthermore, Rudio explains that Indian mathematicians used half

¹⁵ Cf. E F Robertson, J J O'Connor, Ptolemy biography: <http://www-history.mcs.st-andrews.ac.uk/Biographies/Ptolemy.html>, accessed 20/06/2013.

chords, as opposed to full chords, which were used by the Greeks, which led to the invention of sine tables [cf. 29, p. 19]¹⁶.

Unlike Hobson, Rudio devotes several pages of *AHLL* to the works of Arab mathematicians. Firstly, he shows how knowledge of both Greek and Indian mathematics was preserved in Arab translations, going into a little bit more detail when discussing treatises containing references to π , and the adoption of Indian numerals. Rudio also points out original work by Arab scholars in trigonometry, in particular the invention of the cotangent and the tangent by al-Battānī and Abu Al-Wafa', respectively [cf. 29, p. 20-23].

From 12th century scholar Jafir ibn Aflah¹⁷ Rudio moves on to Christian scholars in mediaeval Europe¹⁸, who in turn translated the Arabic translations of ancient Greek texts into Latin. Rudio explains how these re-translations led to the creation of the word "sine" [cf. 29, p. 24-25]. Returning to approximations of π , Rudio credits Franco of Lüttich with the first mention of the problem of the quadrature of the circle, but primarily highlights the contributions of Fibonacci, who calculated his own values using a shorter method than Archimedes.

In section eight Rudio writes about mathematical developments in the Renaissance¹⁹, which had an impact on the problem of the quadrature of the circle. He starts with Georg von Peurbach's sine tables and explains how Nicholas of Cusa was instrumental in making the problem more widely known again, but agrees with Regiomontanus's criticism of Cusa's approximations. Rudio gives a detailed summary of Regiomontanus's accomplishments in trigonometry, as Regiomontanus

[...] has earned himself such an outstanding place in the history of mathematics, and in particular in the history of trigonometry, being the

¹⁶ Rudio cites Colebrooke's *Algebra With Arithmetic and Mensuration* (London, 1817) as his major source for the passage on Indian mathematics.

¹⁷ Erroneously, Rudio places him in the 11th century.

¹⁸ Of course, many of them, such as Gerhard of Cremona, who translated Ptolemy's *Almagest*, worked in the Islamic part of Spain.

¹⁹ Here, he refers to his talk on mathematics in the Renaissance [37].

first to regard it as an independent science, that we have to stay with him for another minute.

[29, p. 28]

This assessment of Regiomontanus's contributions mirrors the importance that Rudio attributes to Regiomontanus in his talk on mathematics in the Renaissance [37] (see also section 6.3.4).

He then adds a list of mathematicians who developed trigonometry further, and sums up efforts to approximate π and square the circle that were undertaken during the Renaissance. Here, he gives a bit more detail on Oronce Fine's²⁰ work and the criticism that it evoked.

Rudio concludes the section by listing the Italian mathematicians who developed 'the theory of algebraic equations, with which our problem would become so closely linked later on' [29, p. 31], and mentions the first editions of original Greek texts, 'which had too great an impact on general mathematics education [during the Renaissance] to just be omitted here' [ibid.].

In the last section of Chapter Two, Rudio outlines the developments from the end of the Renaissance until 1654, when Huygens published *De circuli magnitudine inventa*, now concentrating on 'those events that constitute a significant advancement of our problem' [29, p. 32]. He gives the approximation to π found by Adriaan Metius before turning to François Viète, 'who may claim an exceptional place in the history of the quadrature of the circle' [29, p. 33]. Rudio explains how Viète derived an expression for the area of a circle by inscribing two polygons in a circle, thus putting Antiphon's idea into practice [cf. 29, p. 34]. From this expression, Viète obtained 'this interesting formula':

²⁰ Throughout sections eight and nine, Rudio sometimes uses German or Italian or French names, and sometimes the Latinised versions. Sometimes he also gives both names, e.g. in the cases of Regiomontanus, Metius and Vieta, but uses the Latinised version. He probably chose the more common name in each case, but this might not necessarily correlate with the choice we would make today, or even when writing in English (as opposed to writing in German). He refers to Oronce Fine by his Latinised name Orontius Finaeus, and to Willebrord Snell as Willebrord Snellius.

This curious expression is arguably not only the first exact analytic representation of π , but also the first example of representing a number in terms of an infinite product.

[29, p. 34-35]

Rudio mentions in the introduction to *AHLL* that he used original sources wherever possible, a rule from which he made no exception in the case of Viète's work²¹. As in his translations in the second part of the book, he corrected an error in Viète's expression above: 'I daresay that the factor $\frac{1}{2}$ is missing in front of each inner root in Viète's expression only by an oversight' [29, p. 34]²². He explains why this factor is necessary in a footnote. Furthermore, Rudio summarises his own proof that the infinite product in Viète's expression is absolutely convergent²³, and gives papers on a similar topic in a footnote.

He mentions the work of Adrianus Romanus in passing, before describing the work of Ludolph van Ceulen and explaining the reasons why π was also called "Ludolphsche Zahl", "Ludolphine number"²⁴. However, Rudio remains sceptical of Ceulen's achievement:

With all due respect for the mammoth industriousness and the immense patience that Ludolph evinced in these calculations, it must seem strange to us today that the number π was and still is named after a mathematician who showed relatively little originality in calculating it. His achievements do not inspire us nearly as much as does, and

²¹ Rudio used Schooten's 1646 edition of *Francisci Vietae Opera mathematica*.

²² In the 1890 version of this historical overview, Rudio mentions that Klügel pointed out the mistake before he did, but that Montucla used the erroneous expression in his works [cf. 36, p. 16]. For some reason, this footnote is omitted in *AHLL*.

²³ This proof was published in "Über die Konvergenz einer von Vieta herrührenden eigentümlichen Produktentwicklung", *Zeitschrift für Mathematik und Physik, Hist.-litt. Abt.* **36**, 1891 (no pages given). Rudio used an expression for the arc of a circle, s , in terms of sine and cosine by Euler. He believes that Euler derived it without knowledge of Viète's work [29, p. 35; 32, p. 17]. In [36] Rudio claims that Euler's expression is a generalisation of Viète's, the two being identical for $s = \pi/2$ and 'very well suited for calculating π logarithmically' [36, p. 17]. In *AHLL* he then shows how absolute convergence follows from this formula. Rudio's rigorous proof is mentioned in conjunction with Viète's expression occasionally, e.g. by Beutel [8, p. 30] and, more recently, by Weisstein [48, p. 2237].

²⁴ This expression would have been more widely known at the time of publication than it is today.

always will do, the quadrature by Archimedes, the actual creator of the methods that were in use until that point.

The beautiful theorems, with which the two great Dutch mathematicians and physicists Willebrord Snellius [...], and in particular Christiaan Huygens [...] enriched the theory of measuring the circle, are of much greater importance. Snellius and Huygens must be considered the first people who made significant changes to the method of numerical rectification developed by Archimedes, and added new ideas.

[29, p. 37]

Rudio summarises Snellius's work before stating the most important theorems in Huygens's paper, which led to a much more efficient way of calculating π . Rudio concludes the chapter by highlighting some other relevant papers by Huygens. In particular, he points out the mathematical exchange between Huygens and 'the English mathematician [James] Gregory²⁵, who was snatched from science far too early' [29, p. 41]. According to Rudio, Huygens showed that Gregory's proof, that squaring the circle was impossible, was erroneous (in the *De circuli et hyperbolae quadratura controversia*, published in Huygens's *Opera Varia I*). Rudio considers Huygens's review of Gregory's paper accurate, but does not go into the dispute between the two mathematicians in any more detail, nor does he comment on Huygens's accusations that Gregory supposedly stole his results²⁶.

Let us conclude this section with Rudio's opinion on Huygens's paper:

The aforementioned paper is not only a downright epoch-making one for measuring the circle, but it is also without a doubt one of the most beautiful and important papers in elementary geometry that have ever

²⁵ Of course, James Gregory was Scottish, not English. I would assume that Rudio made the – even today – widespread mistake of equating England with Great Britain, thus erroneously describing anything Scottish or Welsh as "English".

²⁶ Cf. E F Robertson, J J O'Connor, Gregory biography: <http://www-history.mcs.st-andrews.ac.uk/history/Biographies/Gregory.html>, accessed 11/07/2013.

been written. Like Archimedes' paper it will retain its value, even though the results contained in it can be found using a much quicker, analytical method nowadays. Looking at the rich contents of this paper in more detail here would mean doing it injustice: it belongs to those [papers] that should be read by everybody with an interest in the history of mathematics.

[29, p. 39]

6.1.4 Chapter Three

In this chapter Rudio covers the development of modern analysis as well as Euler's work, both only with regard to measuring the circle. He explains that 'gradually, the old methods of elementary geometry were replaced completely' by the development of 'analytic expressions in terms of an infinite series of operations' [29, p. 42]. Thus, the focus of the problem shifted from finding a geometric quadrature to finding an analytic expression for π .

Rudio gives such expressions by John Wallis and Lord Brouncker. He then states the expression of $\arctan(x)$ as an infinite series discovered by Gregory and Leibniz, explains how rapidly converging series can be derived from this and how they in turn can be used to calculate π (giving John Machin as an example). It is worth noting that he gives Gregory's series both in modern notation and in the original, explaining how the tangent was defined in Gregory's lifetime. Rudio concludes this section with a list of mathematicians who calculated π to more and more decimal places, but comments²⁷ that these results serve 'neither scientific nor practical purpose' [29, p. 46]. Whilst Hobson lists results by Chinese and Japanese mathematicians as well, Rudio limits himself to European ones.

The second section of this chapter [29, §11] is devoted to Euler's (relevant) achievements. First, Rudio summarises the developments with regard to the

²⁷ To illustrate his opinion, he cites Schubert's paper *Die Quadratur des Zirkels in berufenen und unberufenen Köpfen* (Hamburg, 1889).

quadrature of the circle until the mid-18th century, concluding that ‘the nature of this important and curious number [π] was still as unidentified as in Antiquity’ [29, p. 47]. Referring to the sketch of the history of trigonometry in the previous chapter, he claims that the ‘outward appearance’ [29, p. 48] of modern trigonometry stems from Euler. He quotes passages from Metius and Johann Christoph Sturm to illustrate the cumbersome, wordy expressions that mathematicians had to use before Euler introduced our modern expressions for sides and angles of shapes, for example [cf. 29, p. 48-49]. Furthermore, Rudio describes how Euler redefined trigonometric expressions as functions, and how they in turn can be expressed in terms of e . He gives Euler’s result $e^{i\pi} = -1$, which he uses later on (in §15). For now, he hints that:

This fundamental relationship between the two numbers e [...] and π [...] contains the key to solving the question whether the quadrature of the circle is possible.

[29, p. 50]

Rudio then gives some of Euler’s expressions for both π and e , referring to at the time recent work (1891) by Adolf Hurwitz, where Hurwitz derives the same expressions for e and \sqrt{e} in terms of continued fractions as Euler did. In the last passage of Chapter Three Rudio traces the use of the name “pi” or “ π ”. According to him, Euler was the first who used π to refer to the ratio of circumference to diameter, first in a paper published in 1737 and from 1739 onwards in letters to Goldbach, with the symbol being adopted by the Bernoullis shortly after [cf. 29, p. 52-53]. Compare this with the accounts by Smith [46, Vol. 2, p. 312] and Hobson [22, p. 41], who state that π was first used in its present meaning by William Jones in 1706, and that Euler (and later mathematicians) merely adopted the symbol. Today, scholars generally share this view²⁸. Neither Rudio nor Hobson refer to the – admittedly different – uses of π by William Oughtred (1647) and Duncan Gregory (1697).

²⁸ However, some scholars question whether Jones really was the first to use π in its modern meaning, as his own writings suggest that he adopted Machin’s use of the symbol. Cf. [4, p. 165-166].

6.1.5 Chapter Four

In the last chapter Rudio describes how π was first proven to be irrational and then to be transcendental, as well as explaining why the question regarding the transcendence of π is linked to the possibility of squaring the circle.

He begins with Lambert's proof that π is irrational, giving the two main theorems and a very rough outline of their proofs:

For the completion [of the proof of the irrationality of π] I [Rudio] refer to the paper itself, which is all the more worth reading as it also contains a number of interesting investigations and historic notes. Moreover, it is written in a very inventive language, as full of humour as the portrayal of this original man, which I have come to know through my esteemed colleague, Prof Wolf.

[29, p. 56]

Rudio clarifies that Lambert's proof was published in 1766 rather than in 1761, before stating that:

In order to be completely rigorous, Lambert's proof of the irrationality of π lacks a lemma concerning the irrationality of certain infinite continued fractions, which Adrien-Marie Legendre (born 1752 in Toulouse, died 1833 in Paris) added to his *Éléments de géométrie* (note 4) later on.

[29, p. 56]

Rudio states the lemma and briefly summarises the proof, as his translation of the paper is contained in *AHLL*. However, other papers suggest that Rudio was wrong here, that Lambert's proof is in fact both complete and rigorous. For a full discussion see [47], but I will give a short synopsis here: In this paper, Wallisser 'hopes to be able to revise the opinion of Rudio, primarily in the German-speaking countries' [47, p. 521], citing some papers where Rudio's view is repeated. Wallisser on the other hand believes the proof to be 'an outstanding mathematical achievement for [Lambert's] time' [47, p. 522]. He refers to two papers, by James Glaisher (1871) and Alfred Pringsheim (1898), respectively, to show this; in the latter we find an explanation as to why Rudio

deemed Legendre's lemma a necessary addition to the proof: Lambert wrote two papers in which he proved the irrationality of π . However, the paper given in *AHLL* was a popular version, whereas Rudio seems to have ignored the more rigorous *Mémoire sur quelques propriétés remarquables des quantités transcendentes circulaires et logarithmiques* (1768). In Rudio's defence, he does refer to this paper when discussing the confusion over the date of Lambert's proof [29, cf. p. 56], but for some reason he decided to disregard it. Archibald and Lehmer claim that Rudio failed to notice a numerical error in the last two entries of Lambert's table of ratios [29, p. 146-147], 'and hence made a misleading statement' [3, p. 340]. Unfortunately, they do not specify what this statement is, but it is likely that they refer to Rudio's opinion on the rigorousness of Lambert's proof. Hobson was aware of Pringsheim's paper when he wrote his *Squaring the Circle*. He writes:

It has frequently been stated that the first rigorous proof of Lambert's results is due to Legendre [...], who proved these theorems in his *Éléments de Géométrie* (1794), by the same method, and added a proof that π^2 is an irrational number. The essential rigour of Lambert's proof has however been pointed out by Pringsheim [1898], who has supplemented the investigation in respect of the convergence.

[22, p. 44]

Today, Lambert is generally credited²⁹ with proving the irrationality of π , whilst the proof of the irrationality of π^2 is attributed to Legendre or Hermite, depending on the author³⁰.

'For the sake of completeness' [29, p. 57], Rudio also outlines Fourier's proof of the irrationality of e , thus concluding section §12.

In section §13 Rudio uses Joseph Liouville's proofs that both e and e^2 cannot be roots of a quadratic with rational coefficients [cf. 29, p. 58] to introduce the notions of algebraic and transcendental numbers. He quotes from Legendre's paper to illustrate the belief spread among mathematicians that π could not be

²⁹ For example, see [7]; also [4].

³⁰ Rudio points out that both Legendre and Hermite proved the irrationality of π^2 , whereas Hobson only mentions Hermite's proof.

the root of any polynomial with rational coefficients, and the difficulties that the proof posed. Rudio indicates the method that Liouville used in 1844, when he successfully proved the existence of a transcendental number. Furthermore, he gives definitions of algebraic and transcendental numbers (using Kronecker's terminology).

Before giving Ferdinand von Lindemann's proof that π is transcendental, Rudio explains how this proof is linked to the problem of the quadrature of the circle. He repeats the assumption that he made in section §2: that 'the possibility of squaring the circle was [...] equivalent to the possibility of constructing a line segment of length πd from the line segment of length d using a ruler and compass only' [29, p. 61]. Then, turning to a finding from plane geometry, he explains how roots of a quadratic can be constructed using ruler and compass if the coefficients of this quadratic are constructible, calling these roots irrationalities of first degree. By choosing such irrationalities of first degree as coefficients of a quadratic, he continues, irrationalities of second degree can be obtained (i.e. the roots), thus leading to a series of quadratic equations for which the respective roots can be obtained. Hence, he explains, a given number is constructible if it is the root of the last quadratic in such a series. According to Rudio, this is both a necessary and a sufficient condition for constructing a given number: drawing straight lines and circles, i.e. using ruler and compass, is equivalent to solving equations of first and second degree, hence a geometric construction is equivalent to a series of quadratics [cf. 29, p. 61-62]. Stating that such a series of quadratics can be replaced by an algebraic equation with rational coefficients, Rudio gives this 'theorem, fundamental for the problem of the quadrature of the circle' [29, p. 63]:

A given number is constructible using ruler and compass if and only if it is the root of a certain algebraic equation with rational coefficients, which is equivalent to a series of quadratics of the aforementioned nature.

Rudio concludes that in order to be able to square the circle, one would have to be able to express π in terms of finitely many square roots³¹. Thus, he says, by proving that π is transcendental, one would also prove the impossibility of the quadrature of the circle.

Apart from sections §2 and §3 in the introduction, this section, §14, is the only one without a single footnote. Rudio does not give any sources for his explanations either. As he writes about the work of other mathematicians instead of presenting his latest mathematical research, it is conceivable that the information he covered in §14 could be found in textbooks and standard literature at the time.

The last section of the historical overview, §15, is entitled *The ultimate settlement of the problem of the quadrature of the circle due to the papers of Hermite, Lindemann and Weierstrass*. First, Rudio mentions Hermite's 1873 proof that e is transcendental. Using this result, he continues, Lindemann was able to show that e^z cannot be rational if z is the root of an irreducible algebraic equation with real or complex coefficients; a generalisation of Lambert's first theorem, he remarks [29, cf. p. 64]. Using Euler's result $e^{\pi i} = -1$, which Rudio mentions in §11, Lindemann proved that π is transcendental. However, Rudio comments, Lindemann:

[...] gave a much more comprehensive solution to the problem of the quadrature of the circle than the original scope of the problem suggested: Squaring the circle is not only impossible when the only means of construction are compass and ruler, but also when one is allowed to use algebraic curves and surfaces. For a construction by means of these very general resources would no longer lead to a series of quadratics, but to a series of algebraic equations all the same. This in turn would define the number to be constructed as a necessarily algebraic one. Thus, the transcendental π is barred from this possibility. [29, p. 65]

³¹ As opposed to infinitely many square roots as in Viète's expression, he adds: 'On the contrary, Viète's expression would indeed rather lead to the assumption that π does not have the properties required for squaring the circle' [29, p. 63].

Rudio then reports on Weierstrass's 1885 proof that π is transcendental in some detail. He gives the theorem that Weierstrass proved before showing that $e^x + 1 \neq 0$ for all x , where x is an algebraic number, a lemma which he needed for the proof of the transcendence of π . Rudio then summarises Weierstrass's proofs of two theorems by Lindemann concerning the transcendence of e and π , with one of them being 'the true generalisation of Lambert's theorem with regard to e' ' [29, p. 67]. He finishes with Weierstrass's work on a specific case of Lindemann's theorem, which leads to a proof of the impossibility of the quadrature of the circle.

Rudio's conclusion summarises the importance of the problem of the quadrature of the circle throughout history, and it also illustrates why studying the history of mathematics is fascinating:

A problem, not only a venerable one due to its old age, but also a most curious one, from a mathematical-historical point of view, has finally been settled by means of Lindemann's investigations. Originally, it was a purely geometric problem and of comparatively minor importance, but over the course of the centuries the question of the quadrature of the circle developed into an arithmetic problem of highest interest. It participated in all the significant transformations that mathematical opinions and methods underwent gradually. In the course of time it was transformed itself, with them and through them [i.e. the aforementioned transformations], until, eventually, the problem had been clarified and defined to the extent that a clear answer could be given. However, not only did it participate passively in these transformations, but by confronting mathematicians again and again, and in different guises, it influenced and advanced the development of mathematics considerably itself; in particular those theories that eventually led to the settlement of the problem.

[29, p. 68-69]

6.1.6 Reception

Although it does not seem that *AHLL* ever became well known or even just a standard reference book, it appears in the bibliographies of a number of books, both on the history of mathematics in general and on the history of the quadrature of the circle or π in particular. It was republished in 1971, and a reprint of the German original from 2010 is available from Amazon. A quick search on worldcat.org³² reveals that copies of either the 1892 or the 1971 editions are available at several (mostly university) libraries across Germany, Switzerland, the UK, the US, Canada, France and also some more exotic places like Brazil. The book has never been translated into English or French, to the best of my knowledge. However, it seems that a Russian translation by S Bernshtein was published in the first half of the 20th century³³.

As mentioned above, references to the book can be found in a number of later works. Some of them, such as Beutel's book *Die Quadratur des Kreises* [8], Cajori's *A History of Mathematical Notation* [11], Wallisser's paper [47], and Weisstein's paper [48], have been cited already.

In some cases, the authors of the respective books only suggest *AHLL* as further reading, such as in: C D Andriess, *Huygens: the man behind the principle*, 2005 [1]; L Berggren, J Borwein and P Borwein, *Pi: A Source Book*, 2004 [7]; A N Kolmogorov and A P Yushkevich (eds.), *Mathematics of the 19th Century*, 2001 [25]; and R Hartshorne, *Geometry: Euclid and Beyond*, 2000 [20]. Furthermore, it is listed in the bibliographies of A B Shidlovskii, *Transcendental Numbers*, 1989 [45]; and of F Chareix, *La philosophie naturelle de Christiaan Huygens*, 2006 [14].

Several authors list *AHLL* as one of their sources or references, primarily when writing about various attempts to calculate π and about how π was

³² Accessed 29/05/2013.

³³ This is based on a search on worldcat.org, where two of the search results for *AHLL* are Russian translations. One was published in 1911 in Odessa, and the translator is given as Samuil Borisovich Bernshtein [50] (the other one was published in 1934 in Moscow, and the translator's name is given as Sergei Natanovich Bernshtein [51]. Given that the information comes from library catalogues, I presume that they are just two different editions of the same translation.

proven to be transcendental. Examples include: P Beckmann, *A History of Pi*, 1971 [6]; H D Ebbinghaus et al, *Numbers*, 1990 [28]³⁴; G J Pendrick, *Antiphon the Sophist: The Fragments*, 2002 [27]³⁵; W W Rouse Ball, *Mathematical Recreations & Essays*, 1939 [5]; H Schubert, *Mathematical essays and recreations*, 1899 [42]³⁶; H-H Wörle, *Die phantastische Geschichte der Analysis*, 2012 [49]³⁷.

E W Hobson seems to have used *AHLL* as a source for a couple of his books, in particular for his *Squaring the Circle* [22], but also for [23]. As there are many similarities between Hobson's book and *AHLL* – perhaps not surprising considering that Rudio's book was one of Hobson's principal sources – I will compare the two works in some detail below.

Another author who refers to *AHLL* on several occasions is D E Smith. In his book *History of Mathematics* [46]³⁸, he cites it when writing about Lambert [46, Vol. I] and about the history of calculating π [46, Vol. II]. Further references to Rudio's works concern those with regard to Hippocrates's lunes and the method of exhaustion, as well as Eisenstein's autobiography, which he edited. Smith also refers to *AHLL* in the preface of his edition of *A Budget of Paradoxes* by A De Morgan [26]. Furthermore, in the passage where De Morgan writes about Montucla's work on the history of squaring the circle, calling it 'the [sic!] history on the subject', Smith adds a footnote stating that: 'Of course this is no longer true. The most scholarly work to-day [sic!] is that of Rudio [...]' [26, p. 159]. Archibald, writing a few years later, considers *AHLL* to be 'one of the best sketches of the history of the problem of squaring the circle', 'prior to Calò's article [on transcendental problems, in Enrique's *Fragen der Elementargeometrie*, 1907]' [2, p. 207].

³⁴ The original German edition was published in 1983. *AHLL* is listed as one of the main sources for the history of π in chapter 5 [28].

³⁵ Pendrick cites several of Rudio's books, mainly his translations of Simplicius's commentary. He refers to *AHLL* when describing how Antiphon's method of calculating π by means of inscribed polygons was used throughout the centuries, and that Lindemann proved that π is transcendental [cf. 27, p. 267].

³⁶ The first edition of *Mathematische Mußestunden* was published in 1897; however, the footnotes referring to Rudio's book seem to have been added by translator McCormack and thus appear in the English edition only. In this case, references are with regard to the papers by Archimedes and by Lambert, respectively, rather than the historical overview.

³⁷ Here, references concern the proverbial quadrature of the circle and Weierstrass's proof concerning transcendental numbers.

³⁸ The books were first published in 1923 and 1925, respectively.

F Klein gives a short historical overview of the problem of squaring the circle in his *Famous Problems in Elementary Geometry*³⁹ [24], part II, chapter II. At the beginning of chapter II (“Historical Survey of the Attempts at the Computation and Construction of π ”) Klein writes:

The following brief historical survey is based upon the excellent work of Rudio: *Archimedes, Huygens, Lambert, Legendre*. This book contains a German translation of the investigations of the authors named. While the mode of the presentation does not touch upon the modern methods here discussed, the book includes many interesting details which are of practical value in elementary teaching.

[24, p. 55]

In particular the last sentence is a high commendation of Rudio’s book, as he wrote it with the intention of providing mathematics teachers with resources (cf. chapter 6.1.1 and [29, p. iii-vi]). Klein’s survey is much shorter than Rudio’s and omits contributions by the Indians and Arabs for example. The main focus is on mathematics, and Klein includes papers by Gordan, Hilbert, and Hurwitz, which were published after 1893, i.e. after *AHLL*.

It is interesting to note that praise for *AHLL* does not only come from Rudio’s contemporaries, but also from modern (current) authors. An example, which nicely sums up what *AHLL* has to offer modern readers, is found in a footnote in [44, p. 251]:

Also⁴⁰ worth reading, though in some respects outdated [sic!], is Rudio (1892), a monograph on the measurement of the circle that prints the

³⁹ Translation by Beman and Smith, who believed that it would appeal to many who were unable to read the original *Vorträge über ausgewählte Fragen der Elementargeometrie*, first published in 1895. As Klein explains in the introduction, he gave some lectures on modern science and how they relate to elementary geometric construction (duplication of the cube, trisection of an angle, quadrature of the circle), with a view to bring university mathematics closer to mathematics in Gymnasien. I used the English translation since it is available in the University of St Andrews Library.

⁴⁰ In the same footnote, Sefrin-Weis also points out works by Heath, Knorr and Tropfke for further information about quadratures and attempts to square the circle.

major contributions by Archimedes, Huygens, Lambert, and Legendre in full, and also contains a survey on the history of the quadrature.

In most cases, authors refer to some aspect of the historical overview in *AHLL*, i.e. to Rudio's own writing. However, in C J Scriba, P Schreiber, *5000 Jahre Geometrie: Geschichte, Kulturen, Menschen*, 2009 [43], the authors use Rudio's translation of Archimedes' treatise⁴¹. Moreover, the entire translation is included in A Czwalina, *Archimedes' Werke...*, 1972 [16].

6.1.7 Comparison of Rudio's *AHLL* and Hobson's *Squaring the Circle*

Although Hobson's book was published twenty-one years after Rudio's *AHLL*, it closely follows the structure of Rudio's historical overview. It is interesting to observe what Hobson added and what he omitted, as well as what he emphasises. Several papers related to the transcendence of π were published after *AHLL* appeared in 1892; and it is not completely unfounded to assume that Rudio would have focused more on mathematicians from German-speaking countries, whereas Hobson would have had a stronger interest in British contributors to the problem.

Before we compare the two works, let us briefly highlight some similarities in the lives of the two mathematicians⁴². Both Rudio and Hobson were born in 1856 (02 August and 27 October, respectively), and Hobson died only four years after Rudio, in 1933⁴³. Both obtained very good degrees in mathematics, and were appointed to teaching positions, Hobson in 1879 and Rudio in 1881, teaching mainly elementary/introductory mathematics. Both stayed at their universities, Cambridge and the Federal Polytechnic, respectively, for the rest of their working lives. Furthermore, both were very active in

⁴¹ The publication date is given as 1925 – this is the only reference to a 1925 edition that I found.

⁴² Cf. E F Robertson, J J O'Connor, Hobson biography: <http://www-history.mcs.st-andrews.ac.uk/Biographies/Hobson.html>, accessed 17/07/2013.

⁴³ Thus, Hobson wrote his book at a much later stage of his career and life than Rudio.

mathematical/scientific societies of their respective countries. Incidentally, they both had Swiss wives. Their research tastes differed, however: Rudio was primarily interested in geometry, a field in which he made some nice, but not ground-breaking contributions; whereas Hobson published papers on analysis, making a major contribution to pure mathematics in Great Britain with his book *Theory of Functions of a Real Variable* in 1907. It seems that Rudio had a stronger interest in the history of mathematics than Hobson, and he also wrote more popular works. To many Hobson and Rudio are better known not for their mathematical work, but for their other contributions to science: in Hobson's case, these are the Gifford lectures he gave on *The domain of natural science* at the University of Aberdeen in 1921-22 (published in 1923); Rudio is primarily known for being the first chief editor of Euler's *Opera Omnia*.

It is likely that Rudio and Hobson knew each other – how well, I cannot tell⁴⁴, but it is reasonably safe to assume that they would have met at the first International Congress of Mathematicians, which Hobson attended (as one of three British participants) and Rudio helped organise (see chapter 4).

Rudio's reasons for writing *AHLL* have been discussed above, so let us now look at Hobson's motivation. In the preface he writes [22]:

In the Easter Term of the present year I delivered a short course of six Professional Lectures on the history of the problem of the quadrature of the circle, in the hope that a short account of the fortunes of this celebrated problem might not only prove interesting in itself, but might also act as a stimulant of interest in the more general history of Mathematics. It has occurred to me that, by the publication of the Lectures, they might perhaps be of use, in the same way, to a larger circle of students of Mathematics.

The account of the problem here given is not the result of any independent historical research, but the facts have been taken from the

⁴⁴ Unfortunately, there are no personal papers or any correspondence in Rudio's estate in the Archives of the ETH Library. A search on the website of the Cambridge Library archives does not reveal any correspondence between Hobson and Rudio, either.

writings of those authors who have investigated various parts of the history of the problem.

The works to which I am most indebted are the very interesting book by Prof. F. Rudio entitled “Archimedes, Huygens, Lambert, Legendre. Vier Abhandlungen über die Kreismessung” (Leipzig, 1892) and Sir T. L. Heath’s treatise “The works of Archimedes” (Cambridge, 1897).

Furthermore, Hobson consulted Cantor’s *Vorlesungen* and Colebrooke’s *Algebra with Arithmetic and Mensuration [...]* [22, p. 23, footnote], both of which Rudio used as well. Apart from these, Hobson lists more recent works than *AHLL*, including McCormack’s translation of Schubert’s *Mathematical essays & recreations*, which contains references to Rudio’s book [42].

We know that Rudio gave lectures on the history of mathematics as part of the mathematics courses he taught in the Department for Mathematics and Physics Teachers at the Federal Polytechnic, but there is no information anywhere in either *AHLL* or the first version of the historical survey to suggest that it resulted from Rudio’s teaching activities⁴⁵. Of course, he may have lectured on the history of squaring the circle, particularly as he considered it to be a good introduction to the history of mathematics. However, it is reasonably safe to assume that he would have mentioned in the preface if the book were indeed the result of a lecture series. In a footnote⁴⁶ to [36], the first publication of the historical overview, he explains that the paper resulted from

⁴⁵ Unfortunately, lecture notes of only one of Rudio’s courses survived in his estate in the ETH Library Archive, and they stem from an introductory course to differential and integral calculus.

⁴⁶ ‘Apart from individual addenda and explanations, the content of the paper at hand essentially corresponds to a talk given for the local Society for Natural Scientists on 13 January 1890. I hope that to those who are familiar with mathematical language, despite not being mathematicians themselves, the paper will offer a perhaps not unwelcome addition to the fine, popular account of the history of the problem of the quadrature of the circle in the form of Mr Schubert’s recent work; all the more as Montucla’s well-known treatise: “Histoire des recherches sur la quadrature du cercle” only covers developments up to Euler’s time. Aside from this, it can be regarded as out-dated in some respects. Perhaps I have also succeeded in rendering a small service to my peers by compiling literature of interest to mathematicians.’ [36, p. 1].

a talk he gave at a meeting of the *Naturforschende Gesellschaft Zürich* in 1890. His audience there would have primarily consisted of scholars and teachers of various (science) disciplines, as well as engineers and scientists working in industry, and not just mathematicians. Thus, the talk can be seen as one of his more popular ones, possibly for a slightly more select audience than his Town Hall talks, as he expects his readers to be familiar with ‘mathematical language’ [36, p. 1]. Note that in *AHLL* he primarily justifies publishing translations of the four treatises, whereas in his 1890 paper he hopes to supplement Montucla and Schubert’s books (cf. [36, p. 1]; footnote 46).

Hobson hoped that the book would be useful to ‘students of Mathematics’ [22, preface], whereas Rudio considered the treatises to be of use to mathematics teachers. Both authors expressed their hope that their respective works would lead to a greater interest in the history of mathematics. Rudio explicitly mentions fellow mathematicians as one of his target audiences here. Unfortunately, judging whether their books had any such influence seems like a rather arduous exercise. However, people are still interested in the history of mathematics, and both books are still worth reading nowadays.

Unlike Hobson’s book, *AHLL* is the result of independent historical research. Rudio consulted original documents wherever possible, and his translations are most certainly original work. However, Rudio also heavily drew on Cantor’s *Vorlesungen*.

As mentioned above, Hobson adopted Rudio’s three historical periods. *Squaring the Circle* generally follows the outline of the historical survey in *AHLL*, as Hobson devotes Chapter I to a ‘general account of the problem’ and the subsequent three chapters to the three eras. Even the structure of the introduction is the same as in *AHLL*, Hobson first writes about the popularity of the problem, then states it in terms of geometry and finally divides up its history into the three eras. For the latter, he uses the same characteristics as Rudio, but his summary of the three eras is more comprehensive and elaborate.

In essence, Hobson's introductory Chapter I touches upon the same points as Rudio's, although it is more elaborate and quite philosophical. Hobson writes about the history of thought in general before narrowing it down to the history of the problem in question. He agrees with Rudio that the quadrature of the circle serves as an excellent introduction to studying the history of mathematics; in fact, he chose it as the subject of his lecture series since it is such a 'good [...] opportunity of obtaining a glimpse of so many of the main phases of the development of general Mathematics' [22, p. 2]. Both describe how purported solutions to the problem appeared again and again over the centuries, and that the Paris Academy passed a resolution not to examine any more solution attempts in 1775 [cf. 29, p. 5; 22, p. 3-4]. Moreover, both conjecture why the problem gained such popularity amongst laymen. Overall, Hobson is quite derisive in this passage, whereas Rudio's tone is a bit more clement.

The section on stating the problem of the quadrature is also much longer and much more philosophical in Hobson's book. Simply put, he writes about the two distinct aspects of geometry: abstract geometry and physical geometry; and about the nature of the fundamental postulations in geometry, suggesting that Euclid's *Elements* represents an advanced rationalisation of practical geometry [22, p. 6]. Furthermore, he essentially criticises Rudio's statement of the problem, i.e. constructing the square of the circle by means of ruler and compass, claiming that 'it indicates roughly the true statement of the problem, [but it] is decidedly defective in that it entirely leaves out of account the fundamental distinction between the two aspects of Geometry [...]' [22, p. 6-7]. Hobson gives the same two fundamental postulations of Euclidean geometry as Rudio⁴⁷, but goes on to explain how to uniquely determine points [cf. 22, p. 7]. Thus, the problem becomes a more general one of Euclidean determination.

It is interesting to note that Hobson makes a clear distinction between the expressions "quadrature" and "rectification" of a circle. He admits that they describe equivalent problems, but different aspects of it. Rudio on the other

⁴⁷ Determining/constructing a unique line and a unique circle given two points *A* and *B*.

hand uses these two expressions interchangeably, without giving any explanations.

In conclusion, Hobson takes a more theoretical approach to the problem, whereas Rudio is mainly concerned with the practical side of the problem (at least at first, later on he defines it in terms of algebra). This difference might be due to personal preference and style, or to different audiences, or to different scientific training or schools.

The first passage of Hobson's Chapter II, on the earliest values for π , is very similar to the corresponding section in *AHLL*, i.e. Chapter Two. After that, however, the two books differ: although both authors follow the chronology of the various attempts to square the circle and cover the major developments, they emphasise different contributions. As with the introduction, Hobson is more interested in the mathematics itself and the development of pure science, whereas Rudio, albeit a pure mathematician as well, puts more emphasis on the practicality of the various approximations and on the "human side" of the problem. It is possible that his long career at a polytechnic influenced his approach.

Hobson's section on the Greeks is much longer than Rudio's, due to the fact that he dwells much longer on Hippias's quadratrix, Hippocrates's lunes⁴⁸, and, in particular, Archimedes' treatise on measuring the circle. He summarises their work (in the case of Archimedes, he states the theorems and summarises the proofs) using modern mathematical notation, which makes the work more accessible to modern readers, but deprives them of the possibility to learn about different (historical) mathematical styles⁴⁹. Before discussing Archimedes' treatise, Hobson digresses and reviews the history and the methods of treating limit problems by the Ancient Greeks. He concludes that their approach was a very rigorous and modern one, criticising

⁴⁸ Here, Hobson also gives more recent work on the topic, namely by Clausen and by Landau, which Rudio does not. Interestingly, Hobson does not refer to Rudio's translation of Simplicius's commentary, which had been published for a few years then.

⁴⁹ He also states Huygens's theorems in terms of mathematical notation rather than words as in the original [22, p. 28-31], but when discussing Gregory's work, he gives both original and modern notation, which is what Rudio does.

only that they did not consider that a circle might not have a definite area [cf. 22, p. 16-19]. Rudio does not include such fundamental observations; and it is natural that he does not write as much on Archimedes' work given that a translation of the full treatise is included in his book.

Moving away from the Greeks, Hobson writes more on Chinese approximations of π than Rudio does, but Rudio pays much more attention to both the Indians and the Arabs than Hobson does. Hobson does not mention any of the Indian contributions to trigonometry; furthermore, he summarises the Arab contributions in essentially two sentences. Compare this to Rudio's six pages on Arab mathematicians and the gradual spread of their translations in Europe!⁵⁰ The case is a similar one for the sections on the Renaissance: Rudio manages to fill six pages – partly by including more information on the history of trigonometry – whereas Hobson summarises the relevant developments on one page. Hobson's next sections are similar to Rudio's; he writes about Viète, Ceulen, Snellius and Huygens. Note that he gives Viète's expression for $\pi/2$ as corrected by Rudio, but without mentioning Rudio's proof that it is absolutely convergent. Moreover, he merely states Ludolph van Ceulen's efforts, refraining from any comments or criticism. In the passage on Huygens, Rudio mainly highlights the quality of Huygens's paper, but Hobson gives the 16 theorems⁵¹ instead (without proofs).

Rudio's account of the first era finishes with Huygens, but Hobson includes a few more sections, on the works of Gregory, of Descartes, and on the invention of logarithms⁵², respectively. Rudio briefly mentions both Gregory

⁵⁰ A possible explanation can be found in the political situation. As Switzerland was not a colonial power it is conceivable that Rudio was at greater liberty to praise contributions from India. On a related note, it is also possible that it simply would not have crossed Hobson's mind to consider Indian mathematics, due to the prevailing opinions, or prejudices, about nations within the British Empire. This is not meant as a criticism, Hobson was a man of his time, which saw India as inferior to Britain. Unfortunately I know neither Rudio's nor Hobson's political views, but it is likely that they were influenced by the political circumstances (at least to some degree), even if they did not have to comply with official guidelines.

⁵¹ He gives them in modern mathematical notation, not in words as Huygens did, and illustrates them with appropriate figures.

⁵² It is interesting to note that Hobson only mentions Napier here [cf. 22, p. 33], whereas Rudio credits both Napier and Bürgi with the – independent – invention of logarithms [cf. 29, p. 29], a view that is generally shared by modern scholars.

and the invention of logarithms, but completely omits Descartes's development of the process of isometries [cf. 22, p. 32]. Hobson concludes his Chapter II with some examples of the 'large number of approximate constructions for the rectification and quadrature of the circle' [22, p. 33], again given in terms of modern mathematical notation and figures, which is something that Rudio does not include either.

The structure of Hobson's Chapter III is again similar to that of Rudio's Chapter Three. First, he lists various analytic expressions for π (particularly in terms of infinite series), many of which also appear in *AHLL*. Hobson's list is the more comprehensive one, as it includes a greater number of calculations of π to decimal places and, more importantly, expressions derived by both Chinese and Japanese mathematicians in the 18th century. Rudio's list is limited to European mathematicians. Another noteworthy difference is that Hobson pays a little more attention to Wallis⁵³.

Secondly, Hobson highlights Euler's contributions to the problem. He gives a shorter summary of Euler's influence on mathematical notation and trigonometry than Rudio (but essentially on the same topics), which includes the relations between trigonometric and exponential functions. He also mentions Euler's contribution to the use of the symbol π . However, he highlights the fact that it was first used by Jones, which Rudio omits in his account (see section 6.1.4). To conclude the chapter, Hobson praises Euler's *Introductio*, something that is again not included in *AHLL*. However, Rudio gives more of Euler's results relating to either π or e .

Again following Rudio's structure, Hobson begins Chapter IV by stating the main theorems of Lambert's proof, but without giving as much detail as Rudio does. As he was well aware that Lambert's proof of the irrationality of π was rigorous enough he did not include Legendre's proof (see section 6.1.5).

⁵³ As in the case of Gregory, this difference may be due to their different nationalities. Although Rudio would have known more about Gregory and Wallis than he included in the book (and similarly, it is unlikely that Hobson did not know more about mathematicians from German-speaking countries than he mentioned), their respective audiences might have been more interested in mathematicians from their own countries (or languages).

He then gives ‘the simpler of Liouville’s methods of proving the existence of [transcendental] numbers’ [22, p. 44]. This he does in more detail than Rudio, expressing the method in terms of mathematical equalities, which require more mathematical knowledge than Rudio’s summary. Rudio explains the relationship between the quadrature of the circle and the transcendence of π primarily in terms of words, too, whereas Hobson’s account (of 4½ pages, quite a bit longer than the corresponding section in *AHLL*) is again partly in terms of equations. Overall, Hobson’s account requires more mathematical knowledge than Rudio’s; Rudio uses a more hands-on way of explaining this relationship than Hobson does. Hobson’s explanation includes a historical digression and is generally much more theoretic; he draws on principles of Euclidean geometry and more complex equations, for example.

Similarly to Rudio, Hobson gives a short summary of Hermite’s work regarding the transcendence of e before stating the theorems and main results of Lindemann’s proof. He then writes about the nature of this and subsequent proofs⁵⁴. Whilst Rudio then moves onto a proof by Weierstrass and associated results, Hobson gives Gordan’s proof of the transcendence of π from 1896. This section becomes very mathematical again, as Hobson states the necessary theorems and gives the proofs in mathematical notation.

In conclusion, both books are quite similar, but with notable differences. Rudio focuses more on mathematicians from the German-speaking countries, whereas Hobson includes more information on British ones, perhaps understandably so. Furthermore, Rudio devotes a greater portion of his account to the Indian and Arab mathematicians as well as to those of the Renaissance. Hobson on the other hand includes work by Descartes and a more recent proof of the transcendence of π . In some cases, e.g. regarding the rigour of Lambert’s proof and the first use of the symbol π , his account is more accurate.

⁵⁴ Namely proofs by Stieltjes (1890), Hilbert, Hurwitz and Gordan (1896), Mertens (1896), and Vahlen (1900). Of course, Rudio was not able to include Gordan’s proof in *AHLL*.

A helpful addition to the material are the figures in Hobson's book, although Rudio had no need for them in the main text as he included the actual papers. Hobson also uses mathematical notation much more often, which might make understanding the material easier for some, but more difficult for anyone without mathematical training. In some ways this can be interpreted as a reflection of the different audiences that they originally wrote the texts for; Hobson lectured mathematicians, whereas Rudio wrote his 1890 talk, on which the historical survey in *AHLL* is based, for an educated audience. Admittedly, most of them had a background in the sciences, but not necessarily in mathematics.

Out of the two, Rudio is the better storyteller, bringing the history of the problem to life by adding little anecdotes and thoughts. Hobson is the pure scientist and philosopher, reflecting on the nature of mathematical fields and the rigour of the mathematics involved.

Despite their different approaches and different preferences, both books make a very enjoyable read. They are packed with information, relevant to both the history of mathematics, the history of science and history in general; the mathematical concepts are explained well; and, most importantly, both Rudio and Hobson succeeded in writing entertainingly and engagingly.

6.2 The Commentary of Simplicius and Related Papers

6.2.1 Motivation

Already at secondary school Rudio showed a particular talent not only for mathematics, but also for languages. Although his biographers do not specify which languages he learned at school, we can assume that French was among them as Rudio spent a year in Paris to study mathematics; he also studied 'historic grammar of the French language' [41, p. 122] during his time in Berlin. According to Schröter, Rudio began studying Greek seriously comparatively late:

[...] in his later years, studying Greek constituted a source of enjoyment, to which he returned again and again in his idle hours. His

papers on the history of mathematics led him to these studies. Having studied Pythagoras and Hippocrates, and Archimedes, Euclid, Heron, Pappus, Diophantus, and Plato, and Aristotle and his commentators, in particular Simplicius, he concluded that it would be necessary indeed to be able to go back to the original sources so as to form a sound opinion. Thus, he began studying Greek with enthusiasm. Despite being almost in his fifties, he did not shy away from hitting the books again, attending lectures by Blümner⁵⁵, Hitzig⁵⁶, and Kaegi⁵⁷, and actively engaging in their seminars. Lucian and Aristophanes always ranked among his favourite authors.

[41, p. 122-123]

Whilst most of Rudio's work on Greek texts focuses on mathematics, he also contributed to the sixth edition of his teacher Adolf Kaegi's famous grammar textbook *Griechische Schulgrammatik* (1903; see footnote 57) and to the 13th edition of Gustav Eduard Benseler's dictionary *Griechisch-Deutsches Schulwörterbuch*, edited by Kaegi (1911). Unfortunately, I do not know what the nature of Rudio's contributions was, as more recent editions of the two books are in circulation now. His paper on Greek mathematical terminology, *Zur mathematischen Terminologie der Griechen*, was included in the compilation *Festgabe für Hugo Blümner* (1914). In addition to the classicists at the University of Zurich, Rudio corresponded with several distinguished classicists and historians of mathematics in Europe: Hermann Diels and Karl Kalbfleisch in

⁵⁵ Hugo Blümner (1844-1919), a German classicist and archaeologist; he held a professorship at the University of Zurich from 1877-1919. Cf. biography by H P Isler in *Historisches Lexikon der Schweiz*:

<http://www.hls-dhs-dss.ch/textes/d/D43584.php>, accessed 14/08/2013.

⁵⁶ Hermann Hitzig (1843-1918), a Swiss classicist; he held a professorship at the University of Zurich from 1886-1918. Cf. biography by U Dill in *Historisches Lexikon der Schweiz*:

<http://www.hls-dhs-dss.ch/textes/d/D45959.php>, accessed 14/08/2013.

⁵⁷ Adolf Kaegi (1849-1923), a Swiss Indologist and Hellenist; he held a professorship at the University of Zurich from 1893-1912. Kaegi was instrumental in reforming the teaching of Greek (grammar, in particular) in secondary schools. His textbook *Griechische Schulgrammatik*, first published in 1884, was widely used in schools until the 1970s, and is still consulted today (also in an English translation by J A Kleist). Cf. biographies by R Wachter in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D43442.php>; by G Baader in *Neue Deutsche Biographie* **10**, 1974, 723: <http://www.deutsche-biographie.de/sfz38197.html>, accessed 14/08/2013.

Germany, Paul Tannery in France, and Thomas Little Heath in the UK, to name but a few⁵⁸.

Most of Rudio's historical publications testify to his lifelong interest in ancient Greek mathematics and Greek mathematical texts:

- *The Problem of Squaring the Circle* (1890)
- *Archimedes, Huygens, Lambert, Legendre. Four Papers on Measuring the Circle. Published in German and With an Overview of the History of the Problem of Squaring the Circle* (1892)
- *Simplicius's Commentary on the Quadratures of Antiphon and Hippocrates* (1902)
- *On the Rehabilitation of Simplicius* (1903)
- *Hippocrates's Lunes* (1905)
- *Notes on Simplicius's Commentary* (1905)
- *Addendum to "Hippocrates's Lunes"* (1905)
- *Documents on the History of Mathematics in Antiquity. First Volume, in German and Greek* (1907) [= Simplicius's Commentary]
- *Sur l'histoire des conchoïdes* (1907)
- *Note on the Greek Terminology* (1908)
- *The Reputed Quadrature of the Circle by Aristophanes* (1908)

In addition, he wrote biographies or obituaries of several classicists and historians.

All of the aforementioned publications touch upon the subject of the problem of squaring the circle in some way. Whilst the main focus of his 19th-century works lies in the historical development of the problem (although *AHLL* arguably includes Rudio's first published translation of a Greek source), his 20th-century papers revolve around Simplicius's commentary on

⁵⁸ We know this because Rudio mentions corresponding with these scholars in various papers. Unfortunately, there are no letters or similar in Rudio's scientific estate in the ETH Library Archive, which could indicate which other scholars he knew.

quadratures⁵⁹. He first published his translation of the commentary⁶⁰ in *Bibliotheca Mathematica* 3 (3) in 1902. This translation constitutes the centrepiece of the 1907 edition of the commentary, but Rudio added several supporting documents and a historical overview⁶¹. The commentary includes a fragment from the Second Book of Eudemus's *Elements* on the lunes of Hippocrates. As many other classicists and historians of mathematics, Rudio tried to eliminate later annotations by Simplicius in order to attain a wholly Eudemean document. His version of the Eudemean fragment was published as *Hippocrates's Lunes* [31]. Most of the remaining papers are addenda to the aforementioned works.

Rudio was primarily a mathematician, not a classicist. On the one hand, this qualified him well to studying ancient mathematical texts, as he was able to put the mathematical contents into perspective. As Fueter notes, Rudio worked on problems from a variety of mathematical areas, including geometry, surface theory, group theory, and mechanics, which:

[...] was [a] necessary condition to allow him to perform his life's task: tackling problems in the history of mathematics. [...] It cannot be stressed enough how essential [this] condition is for delving into the history of mathematics. I might possibly be permitted to add that other scholars did not always fulfil this condition satisfactorily, to the detriment of their research. Clearly, only a mathematician who is in the thick of mathematical research is qualified to evaluate historical works properly [...]

[41, p. 125-126]

⁵⁹ The commentary constitutes part of Simplicius's Commentary on Aristotle's *Physics*. As Rudio explains in *Documents* [30, p. 3-4], this was first published in 1526, in a Latin translation by A Manutius. During the 19th century it was edited several times, e.g. by Diels. Carl Anton Bretschneider (1808-1878; a German mathematician, teacher and lawyer) was the first to alert mathematicians to the existence of the commentary on the quadratures though, when he included it in his *Die Geometrie und die Geometer vor Euklides* (1870). However, Rudio indicates on several occasions that he considers Bretschneider's translation and analysis to be unsatisfactory.

⁶⁰ Henceforth, "Commentary" refers to Simplicius's work on Aristotle's *Physics*, whilst "commentary" refers to the passage on the quadratures of Hippocrates and Antiphon contained within it. Rudio's papers only concern this passage, and "Commentary" denotes his 1902 translation of it.

⁶¹ I have only been able to access the 1907 edition, not the 1902 one.

However, some of Rudio's translations and interpretations of certain key words gave rise to controversies. Amongst those criticising some of his choices of words were fellow historians of mathematics Tannery and Heath.

Rudio's papers relating to the commentary of Simplicius will be summarised briefly in the subsequent paragraphs⁶². I will also outline the dispute surrounding his translation, and, in addition, give a few examples of references to Rudio's work by more recent scholars.

6.2.2 Overview of Relevant Papers

Simplicius's Commentary on the Quadratures of Antiphon and Hippocrates: This is Rudio's translation Simplicius's commentary, which is also included in *Documents* and will be discussed below. Rudio was one of several scholars who translated the commentary in the late 19th and early 20th centuries, but in contrast to some of his colleagues, Rudio defends Simplicius's mathematical expertise. He also continues the extraction of the Eudemian fragment from Simplicius's commentary, which was started by George Johnston Allman, Diels, and Tannery. Eneström writes in a review [18] that:

Rudio [...] has given a new translation of the commentary of Simplicius and has added elaborate comments. In doing so, he took particular pains to critically examine the passages where Simplicius seemingly exhibits a great deal of ignorance of geometry. As a result, he discovered that this ostensible unskilfulness is based partly on incomplete records, partly on inadequate understanding of accurate records.

On the Rehabilitation of Simplicius [38]: As Eneström writes in a review [19], this paper is Rudio's answer to Tannery's criticism of *Commentary* in *Bibliotheca Mathematica* 3, 1902, 342-349. Apparently Tannery concluded that Rudio's work would not change the perception of Simplicius as a 'sad dolt' [38, p. 14]. Rudio, however, thinks that he was the first scholar to have

⁶² Except for *Sur l'histoire des conchoïdes* and *The Reputed Quadrature of the Circle by Aristophanes*, as I was not able to access these papers.

accurately portrayed Simplicius's talents. He admits that it took him a while to change his mind, which was 'influenced by Bretschneider and Tannery' [ibid.], but in [38] and subsequent works he acts as an advocate for Simplicius. Tannery also objected to Rudio's use of the word $\tau\mu\eta\mu\alpha$, but Rudio insists that it can be taken to mean both "segment" and "lune". The use of this particular word is the principal bone of contention in later debates (see below). Furthermore, Rudio explains that due to lack of evidence Hippocrates cannot be seen as the first to use the method of exhaustion [38, p. 13].

In essence, the paper is a defence speech in a longer academic dispute, and Rudio makes it clear that he believes his opinion to be decisive.

Hippocrates's Lunes [31]: This paper⁶³ is basically a review of Simplicius's comments on Hippocrates's work on lunes. Rudio summarises them and comments on Simplicius's choice of sources, Eudemus of Rhodes and Alexander of Aphrodisias. According to Rudio, Simplicius preferred Eudemus to Alexander, attributing to the latter little mathematical knowledge [31, p. 181; p. 184-185]. Rudio also defends Hippocrates's work against accusations by Alexander, drawing on Eudemus and Simplicius to explain his views. In particular, he shows why Hippocrates cannot have come up with some results attributed to him (amongst others by Alexander), thus proving Simplicius's assessment of the accusations to be accurate⁶⁴. Furthermore, Rudio gives Simplicius's opinion of quadrature attempts mentioned by Alexander⁶⁵, explaining why Simplicius was right to be sceptical. As part of the discussion of various quadrature attempts, Simplicius reproduced a dialogue between

⁶³ Rudio dedicated this paper to his friend and colleague Georg Sidler, professor of mathematics at the University of Bern (see section 4.2.5.1).

⁶⁴ In essence, the point at issue is whether or not Hippocrates believed that he squared all possible lunes. Alexander claims that Hippocrates erroneously believed he did, and that he believed further that this would allow him to square the circle; a conclusion that Alexander identifies as a fallacy (but for the wrong reasons, as Rudio points out). However, Rudio shows that Eudemus and Simplicius are correct in stating that Hippocrates never believed anything of the sort. In fact, Rudio notes that the Eudemian fragment clearly states that Hippocrates only squared four types of lunes, a fact which Simplicius observes as well. Cf. [31, p. 183-184; p. 187-188; p. 196-198]. Heath is of the same opinion as Rudio, but bases his arguments on Hippocrates's calibre as a mathematician alone [cf. 21, p. 196-197: footnote 1].

⁶⁵ For instance, finding cyclic squares or dividing a circle up into lunes [cf. 31, p. 184-185].

him and his teacher, Ammonius, which Rudio includes in full (as a translation; cf. [31, p. 186-187]).

Rudio also explains Simplicius's motivation for writing this commentary in the first place, which forms part of his *Commentary on Aristotle's Physics*. According to Rudio, Simplicius wanted to find out which quadrature by means of segments Aristotle referred to in a remark regarding the philosophy of the Eleatics: 'it would be "the business of a geometer to disprove the quadrature of the circle by means of segments, but disproving [the quadrature] of Antiphon would not be the business of a geometer"' [31, p. 180]⁶⁶.

In *Commentary* and *Documents* the passages that Rudio believes to be Eudemian are set in italics, but in *Lunes* he omitted any additional comments. As he remarks:

The addenda by Simplicius (and any other distractions and distortions that have crept in over the course of the centuries by means of transcription or otherwise) have now been discarded, and we may now regard the purifying process as essentially completed. At least now there are hardly any noteworthy disputes anymore, and those that do exist concern a few isolated passages.

[31, p. 188]

Rudio summarises some of Simplicius's more interesting comments and additions in the footnotes; he also clarifies terminology and writes some of the proofs in algebraic form, thus making it easier for the modern reader to understand the original text.

Rudio concludes the paper with his opinion of Simplicius, stating that although Simplicius did not manage to answer the question whether or not Aristotle referred to Hippocrates's quadrature(s) satisfactorily, this was insignificant compared to the importance of the report itself:

[...] Simplicius, whose mathematical competence has been misjudged until the most recent times, presents himself as a scholar of extensive and solid knowledge, as a man of independent and sound verdicts.

⁶⁶ See also [21, p. 184-185]

And by providing a broad background in his report, and collecting whatever he could find on the “quadratures by the means of segments” with skill and care, he provided the history of mathematics with an invaluable service.

[31, p. 198-199]

In fact, Rudio explains the significance of the commentary already in his introduction (which, incidentally, is readable in its own right). He philosophises about the origins of geometry, very fittingly alluding to Greek mythology to illustrate his points, but concludes that whilst it might be desirable to think of Euclid’s *Elements* as the birth of geometry, it is, in truth, the result of centuries of research [cf. 31, p. 177-179]. Rudio explains that most of these ancient sources are lost, conjecturing why, and that Simplicius preserved one of the surviving fragments of Eudemus by including it in his Commentary on *Physics*.

Lunes is a nice little paper for those who want to know about Hippocrates’s lunes, an example of pre-Euclidean geometry, and Simplicius’s full Commentary. Rudio summarises Simplicius’s motives, structure and main arguments well, without being too technical or elaborate. Whilst some knowledge of geometry would be advantageous, the reader is not required to know Greek, or indeed know much about ancient history. Most importantly, this paper is another contribution to the rehabilitation of Simplicius, and should entice readers to tackle the full commentary (regardless of the ensuing academic dispute).

Addendum to “Hippocrates’s Lunes” [33]: Published in the same volume of the *Vierteljahrsschrift* as *Lunes*, this one-page note contains Hippocrates’s proof (in terms of quadratic equations) that a particular angle is obtuse, which in turn proves that the outer edge of a lune of the third type (in Eudemus) is smaller than a semicircle. As Rudio explains in *Lunes*, the original passage was corrupted, ‘and its restoration caused more trouble and debate than any other’ [31, p. 194]. He also refers to the *Addendum*, which is much more technical and

therefore not suitable to be included in *Lunes*, 'which is aimed at a greater audience' [30]. For an algebraic version of the proof see [21, p. 195].

Notes on Simplicius's Commentary [34]: In this paper Rudio reports on the progress of purifying the Eudemian fragment in Simplicius's Commentary in various translations. In particular, he lists work by Schmidt and Diels, but also comments on other publications in the period 1902-1905.

Documents on the History of Mathematics in Antiquity, Vol. I: Simplicius's Commentary on the Quadratures of Antiphon and Hippocrates, in German and Greek [30]: This is Rudio's chief work on the history of mathematics, and also the work that gets cited most often even nowadays⁶⁷. As the subheading informs us, it contains 'a historical report serving as introduction' and 'supplementary documents in the appendix, connected by a survey of the history of the problem of squaring the circle before Euclid'.

Originally Rudio and his classicist friend Wilhelm Schmidt⁶⁸ planned a series of *Documents*, but it seems that Rudio did not continue the project after Schmidt's untimely death in 1905⁶⁹. In the preface to *Documents* he explains that he felt he had to continue with the publication of the commentary, but his workload might have been too great to produce any more volumes. In addition, the Euler project slowly began to take shape, and presumably this would have taken up most of Rudio's time.

Rudio draws on a variety of sources, but primarily on a paper by Schmidt (1903) and his own papers, *Commentary* and *Lunes* in particular, sometimes verbatim. The actual translation, he informs us, is based on the 1882 edition of

⁶⁷ Or the reprint from 1968, published by Sändig, Wiesbaden, which is the edition that I used. The first edition from 1907 was published by Teubner, Leipzig. I will refer to the book as *Documents* here.

⁶⁸ Wilhelm Schmidt (1862-1905), born in Harderode, at the time in the Duchy of Brunswick, today in Lower Saxony (Germany). He studied classics at the Universities of Leipzig, Göttingen and Berlin, and obtained his PhD from Göttingen in 1893. Schmidt became a secondary teacher in 1885 and taught at various schools in the Duchy. He is primarily known for editing Heron's works, but he also wrote on the history of Greek mathematics. Rudio wrote a 33-page-long obituary on Schmidt (*Bibliotheca Mathematica* 6 (3), 1905); a review by Eneström containing the key dates of Schmidt's life is available here: <http://zbmath.org/?q=an:36.0037.01>, accessed 15/08/2013.

⁶⁹ There are no further volumes to the best of my knowledge.

Simplicius's Commentary by Diels; adding that any deviations are indicated clearly. In the preface Rudio thanks Diels and his former teacher Kaegi; the book is dedicated to Hermann Diels.

In the introduction (i.e. the historical survey) he summarises the importance of the Commentary for the history of mathematics and lists those scholars who studied it before him⁷⁰. Heath nicely puts Rudio's book into the context of the work of his predecessors:

To Bretschneider belongs the credit of having called attention to the importance of the passage of Simplicius to the historian of mathematics; Allman was the first to attempt the task of distinguishing between the actual extracts from Eudemus and Simplicius's amplifications; then came the critical text of Simplicius's commentary on the *Physics* edited by Diels (1882), who, with the help of Usener, separated out, and marked by spacing, the portions which they regarded as Eudemus's own. Tannery, who had contributed to the preface of Diels some critical observations, edited (in 1883), with a translation and notes, what he judged to be Eudemian (omitting the rest). Heiberg reviewed the whole question in 1884; and finally Rudio, after giving in the *Bibliotheca Mathematica* of 1902 a translation of the whole passage of Simplicius with elaborate notes, which again he followed up by other articles in the same journal and elsewhere in 1903 and 1905, has edited the Greek text, with a translation, introduction, notes, and appendices, and summed up the whole controversy.

[21, p. 183-184]

Next, Rudio briefly explains Simplicius's motives for writing the commentary⁷¹ before giving 'an overview of the contents of Simplicius's commentary, which shall also introduce us to the individuals appearing in it' [30, p. 6]. He begins with Simplicius, giving the key dates of his life – as far as available – and, perhaps more interestingly, quotes that illustrate his renown

⁷⁰ As he notes, a more detailed review of preceding work can be found in *Commentary*, 1902.

⁷¹ A longer version is included in [31].

among philosophers. Rudio considers Simplicius's bad reputation among mathematicians to be 'embarrassing' [30, p. 8], but concedes that the early papers on Simplicius encouraged this view.

Rudio gives short biographical sketches of Alexander, Eudemus and Hippocrates. As he returns to Simplicius's initial motivation he briefly touches upon Antiphon's work, and summarises the quadratures that Simplicius found in Alexander, giving both his own and Simplicius's opinions of them. Referring to Simplicius's dialogue with his teacher [cf. 31], Rudio gives a few biographical data of both Ammonius and Iamblichus, one of Simplicius's references in the conversation.

Similarly to *Lunes*, Rudio explains why the Eudemian fragment had to be cleared from any addenda. Moreover, he defends his interpretation of $\tau\mu\eta\mu\alpha$, acknowledging that 'people have taken offence at this' [30, p. 19]. He concludes the introduction by discussing the controversy regarding the types of lunes that Hippocrates squared. Whilst he thinks it likely that Aristotle referred to Hippocrates's fourth quadrature, he considers any objections to be unfounded. However, he tries to explain Aristotle's (supposed) view.

The better part of *Documents* is taken up by Rudio's translation of the commentary. It is given in full, both in Greek and in German. Rudio added a number of footnotes, either explaining his choice of words or deviations from Diels's translation, or commenting on the mathematics involved or Simplicius's conclusions.

The appendix, containing 'supplementary documents, connected by a survey of the history of the problem of squaring the circle before Euclid', puts the commentary and the mathematics covered in it into their historical context. At 43 pages, Rudio could even have published the appendix as a paper in its own right. In the historical survey Rudio touches upon topics covered in his first works on the history of mathematics from 1890 [36] and 1892 [29] – very fitting considering that *Documents* was his last major historical paper, at least when looking at his work today.

In an introductory section he gives an overview of the three ancient problems⁷², conjecturing that Hippocrates squared lunes in an attempt to square the circle. Rudio continues with the approximation to π found in Ancient Egypt. In comparison to *Archimedes, Huygens, Lambert, Legendre* (*AHLL*; [29]) he gives more details on the Rhind Papyrus; he even quotes a couple of relevant problems from the Papyrus as given in Eisenlohr's translation.

The main chapter of the appendix, concerning the quadratures of the Ancient Greeks, is divided into four sub-chapters, devoted to Anaxagoras, Hippocrates, Antiphon, and Iamblichus, respectively. As Rudio only covers pre-Euclidean geometry here, the chapter is naturally more comprehensive than the corresponding passage in *AHLL*.

The passage on Anaxagoras also serves as an introduction to the beginnings of Greek mathematics with regard to quadratures. As in *AHLL*, Rudio concludes that there is not enough evidence to suggest that Thales and Pythagoras worked on quadratures, but here he quotes references to their work – in Greek, giving a German translation⁷³. Moving on, Rudio remarks that some of his colleagues⁷⁴ deduced that quadratures must have been popular towards the late 5th century BC, as Aristophanes included a reference to them in his play *The Birds*. Rudio comments that although the passage in question has been misinterpreted, it can be taken as a clever pun and thus suggest that the problem was popular indeed [cf. 30, p. 90-91]. With regard to Anaxagoras, he gives a short biography, flavoured with a few quotations, and mentions that Plutarch reports that Anaxagoras drew a quadrature of the circle⁷⁵.

In the second sub-chapter Rudio gives a more detailed account of Hippocrates's life than in any of his previous publications, drawing primarily on Aristotle and Philoponus. Here we also find a particularly nice example of

⁷² i.e. squaring the circle, trisecting the angle and doubling the cube.

⁷³ In fact, all quotes from original sources are given in both Greek and German throughout the appendix.

⁷⁴ Rudio lists Montucla, Tannery and Allman here.

⁷⁵ Rudio interprets this as drawing a square in the sand, approximating the area of a circle. However, Heath disagrees with this view; he claims that Plutarch must have meant "wrote" rather than "drew" in his report [cf. 21, p. 173; 30, p. 92-93].

Rudio's wit and unique style; commenting on a passage from Aristotle he remarks:

"Hippocrates, for instance, was a skilled geometer, but he seemed to be stupid and irrational otherwise: after all, he lost a large sum of money to the publicans⁷⁶ of Byzantium during a sea voyage; purportedly out of simple-mindedness." [Quote from Aristotle]

Of course, we may add that it is not exactly compromising to be hoodwinked by shifty tax collectors. At any rate, Hippocrates would not have to be ashamed of his company if one were to gather all of his fellow sufferers right into modern times around him.

[30, p. 94]

Rudio also comments on Bretschneider's remarks, according to which the Pythagoreans in Athens ostracised Hippocrates because he received money for his teaching. Rudio investigates the relevant sources but concludes that the story is based on a later addendum [cf. 30, p. 97-100]. Finally, he gives the full passages from Aristotle that refer to quadratures and that Simplicius mentions.

The third sub-chapter is devoted to Antiphon and his quadratures. Surprisingly, Rudio does not give any biographical details, but instead quotes allusions to Antiphon from literature, in particular those concerning Antiphon's quarrels with Socrates. He then compares accounts on Antiphon's method of exhaustion written by Simplicius and Themistius. As he finds them to be very similar, both in structure and conclusion, he suggests that both authors based their accounts on the same source, most likely Eudemos's *History of Geometry*⁷⁷. In addition, Rudio introduces an account of Antiphon's quadrature by Philoponus, both in Greek and in a German translation. According to Rudio, this account had not appeared in literature before [30, p. 105], but he admits that Simplicius was in a much higher mathematical league than Philoponus. Lastly, Rudio briefly touches upon Bryson. Essentially, he

⁷⁶ i.e. tax collectors in Antiquity, e.g. as used in the Bible

⁷⁷ Rudio dismisses Tannery's view that the authors' common source was Eudemos's (lost) Commentary on Aristotle's *Physics* and claims that this common source did not specify which polygon Antiphon used as a starting point [cf. 30, p. 104-105].

agrees with Heiberg that Bryson's work is of a low standard, and argues that Bryson's (unwarranted) fame is due to a misinterpretation by Bretschneider [cf. 30, p. 108-110].

Finally, Rudio turns to Iamblichus's Commentary on Aristotle's *Categories*, or rather, Simplicius's references to it; the commentary itself is lost. Simplicius used Iamblichus's work in his commentary to report on quadrature attempts beyond Aristotle⁷⁸. According to Rudio, most of these quadratures are lost now, but notes on the quadratrix of Nicomedes are still available to us. He first explains what the quadratrix is, and gives a short overview of its history. Then, Rudio cites a couple of passages in Proclus on this curve, before giving a full translation (and the Greek original) of Pappus's paper on the quadratrix. This concludes Rudio's book.

Documents is probably Rudio's most scholarly work on a historical topic. Whilst he emphasises the importance of Simplicius's commentary and, by extension, of the book, for mathematicians, knowledge of Greek or even a profound interest in classics and Antiquity would make for more rewarding reading. Rudio does well in placing the commentary into its historic context, and the material covered in the appendix is interesting regardless of whether one has read the actual commentary⁷⁹. This additional information is certainly what makes *Documents* appealing; for mathematicians and historians of mathematics the fact that the editor and translator of the Greek text was primarily a mathematician should single out this publication from the collection of mathematical texts edited by classicists. Some of Rudio's interpretations may have been contested, but his mathematical training allowed him to judge the quality of both Hippocrates's work and Simplicius's comments.

Note on the Greek Terminology [34]: Here Rudio justifies his – contested – use of the word $\tau\eta\mu\alpha$ by presenting passages of pre-Euclidean literature where

⁷⁸ These are constructions by: Apollonius, using a 'sister of a conchoid'; Carpus; Nicomedes, using the quadratrix; and Archimedes, using his spiral [cf. 30, p. 112]. With regard to Archimedes' work, Rudio refers to Heiberg.

⁷⁹ I cannot comment on the quality of Rudio's translations.

τμήμα can be interpreted in the same way that he did. In a nutshell, Rudio claims that while τμήμα means “segment” only in Euclidean and post-Euclidean literature, it can also mean “sector”, and in some instances even “lune”, in pre-Euclidean works. If one allowed for this, he argues, then the fragment by Eudemus would be meaningful [cf. 34, p. 481]. He lists passages in both Aristotle and Simplicius’s *Commentary* of Aristotle’s books and interpretations of those passages by other classicists.

It is interesting to note that Heath advised him on a passage in Aristotle’s *De Caelo* where τμήμα is used to denote a “sector” rather than a “segment”, when he later questioned Rudio’s views in [21]. In fact, Heath dissects the passage in question, the introduction of Eudemus’s fragment, at length [21, p. 187-191]. In essence, Heath discusses whether a particular paragraph is Eudemian, as Rudio claims, or a later addition by Simplicius, as maintained by Rudio’s predecessors. Whilst Heath admits that the word τμήμα had been used to mean “sector” as well as “segment” (see above), he finds it hard to accept that it could have been ‘used in different senses in consecutive sentences without a word of explanation’ [21, p. 189], as would be the case if Rudio’s interpretation was accurate. We know that Tannery objected to Rudio’s interpretation already in 1902 [cf. 38]. As far as I know, Rudio never made a public reply to Heath’s objections – he may have been too busy with the Euler edition.

6.2.3 References to his Papers

We can find references to Rudio’s work, *Documents* in particular, not only in Heath’s *A History of Greek Mathematics* [21], but also in a variety of other works by both historians of mathematics and classicists.

Smith refers to *Documents* and *Commentary* in his *History of Mathematics* in passages on Hippocrates [46, Vol. I, p. 83] and the method of exhaustion [46, Vol. I, p. 84; Vol. II, p. 303]. Cajori gives *Commentary* as his source for the

anecdote that Aristophanes alluded to quadratures in *The Birds* [12, p. 17]. In *Classics in the History of Greek Mathematics* [15], Christianidis presents a collection of 20th-century papers on various topics concerning the history of Greek mathematics. References to *Documents* can be found in a number of papers, in all cases with regard to work by Hippocrates:

- H-J Waschkies, Introduction to *The Beginnings of Greek Mathematics* [15, p. 3-18]: references on p. 16 as well as in the bibliography
- J Mittelstrass, *Die Entdeckung der Möglichkeit von Wissenschaft*, 1962-66 [15, p. 19-44]: reference on p. 29; Mittelstrass describes *Documents* as 'excellent' here
- K von Fritz, *The Discovery of Incommensurability by Hippasus of Metapontum*, 1954 [15, p. 211-231]: footnote on p. 228
- T L Heath, *Diophantus's Methods of Solution*, 1964 [15, p. 285-327]: footnote on p. 292

Looking at more recent publications, there are references to *Documents* in the entry on Eudemus in *Lexikon des Hellenismus* [40, p. 314-315]. Boehme cites Rudio's conclusion with regard to the common source for Themistius and Simplicius in his paper *Oskar Becker, Bryson und Eudoxos* [9, p. 88]. Finally, Pendrick lists both *Commentary* and *Documents* in the bibliography of his book *Antiphon the Sophist: The Fragments* [27]; he also includes some references to *AHLL*. Most of the references to Rudio's books can be found in Pendrick's commentary on F13 [27, p. 261-275], a collection of fragments primarily on the quadrature of the circle. One of these fragments, F13(e) in Pendrick, was written by Simplicius (from his Commentary on Aristotle's *Physics*). Pendrick frequently refers to Rudio's interpretation when discussing his own translation of F13(d) by Philoponus [27, p. 268] and of F13(e) [27, p. 269-272], but does not agree with him on all occasions. He also draws on Rudio, among others, in his account of Antiphon's quadrature [27, p. 261-266].

6.3 Rudio's Popular Lectures: *Leonhard Euler and Über den Antheil der mathematischen Wissenschaften an der Kultur der Renaissance*

This chapter was published as part of S Eminger, Ferdinand Rudio's Popular Lectures, *conference volume of the History of Mathematics & Teaching of Mathematics conference, Cluj-Napoca, Romania, 21-25 May 2014*.

6.3.1 Rathausvorträge

Rudio gave these talks⁸⁰ in Zurich's town hall as part of the so-called "Rathausvorträge", "Town Hall Lectures", a series of popular lectures organised by the *Dozentenverein beider Hochschulen*⁸¹. By all accounts these lectures were very popular and a bit of an institution among Zurich's intellectuals in the second half of the 19th century. In one of the *Notizen zur schweizerischen Kulturgeschichte*⁸² in the *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich* (1902), Rudio and his co-editor Carl Schröter give an overview of the historical development of this lecture series (primarily by quoting an account by Blümner from 1893) [39], which is summarised here:

Lecturers at the University of Zurich initiated the Lecture Series in 1851, and organised themselves in a society three years later. When the Polytechnic opened in 1855, its lecturers were invited to join the society, and it became a general rule that both institutions were evenly represented on the society committee [39, p. 460-461]. Henceforth, lecturers from the two universities worked together so as to provide general lectures on a variety of topics to the general public. The lectures became an immediate success [39, p. 459], and although attendance dropped over the decades, there were still enough attendees to make the Lecture Series viable. Blümner attributes the drop in attendance partly to the fact that Zurich's intellectuals had a greater choice of lectures to attend as time progressed [39, p. 462]. However, Rudio reports that numbers remained healthy during the 1890s, in part due to people being able

⁸⁰ English translations of these talks, by the author, can be found in appendices E.3.3 and E.3.4, as well as online:

Euler: http://www-history.mcs.st-andrews.ac.uk/Extras/Rudio_Euler.html

Renaissance: http://www-history.mcs.st-andrews.ac.uk/Extras/Rudio_talk.html

⁸¹ "Society of Lecturers from both Universities", i.e. from the Polytechnic and the University of Zurich.

⁸² "Notes on Swiss Cultural History"

to buy tickets for individual lectures rather than for the entire series [39, p. 466].

The speakers were chosen by the committee, but they could speak on any topic of their choice, thus exposing the audience to all aspects of research conducted at the universities. In general, a year's Lecture Series comprised twelve lectures (six before Christmas and six thereafter), taking place on Thursday evenings at "6.15pm on the dot" [39, p. 466]. The society could use the town hall free of charge, thus most of the money raised from ticket sales was spent on academic projects, chosen by the entire society: primarily, establishing and expanding the two universities' collections of art (particularly copper engravings) and archaeological artefacts as well as commissioning oil paintings of distinguished professors [39, p. 464-465]. Both institutions got an equal share of the profits; however, the society focused on projects that would be interesting and educational for the wider public. Rudio reports that over the course of 51 years the lectures yielded a profit of approximately 60,000 Franks that were spent on academic projects [39, p. 468].

There is not much information available on the history of these Town Hall Lectures, or indeed on what happened to them after the publication of the aforementioned note. However, it seems that it was the lecturers who conceived this idea and organised the talks rather than the University's governing body. The Lecturers' Society also survived the Polytechnic's gradual liberation from the University under Kappeler's reign (see appendix B). As Blümner notes [39, p. 459], there were not many popular lectures at the time when the Lecture Series was established, and although this changed later on, both the University and the Polytechnic were instrumental in executing outreach projects, an important issue for universities today!

6.3.2 Publication

Rudio's lecture on mathematics in the Renaissance was published in 1892 [37] in volume VI (issue 142) of the new series of the German *Sammlung*

*gemeinverständlicher wissenschaftlicher Vorträge*⁸³. Founded by Rudolf Virchow⁸⁴ and Franz von Holtzendorff⁸⁵, the first issue was published in 1866. They edited the collection jointly until 1889; then Holtzendorff was succeeded by Wilhelm Wattenbach⁸⁶. From 1897-1901, when the last issue was published, Virchow was the sole editor. In 1885 the 20th volume was published, and in the following year the editors began a new series, starting again with volume I. Each volume comprises 24 issues, containing one paper each. In 1892, a one-year subscription cost 12 Mark [37, cover]. The series covers a great variety of topics, ranging from medicine, biology, and psychology to geography, history, philosophy and classics⁸⁷. A note in [37] explains that Virchow edited papers on topics in the natural sciences, whereas Wattenbach edited the papers on questions in history and literature [37, cover].

Rudio's talk on Leonhard Euler, given almost a decade before the lecture on the Renaissance, on 06 December 1883, was not published as part of the *Sammlung*. At first it was published in a "collection" by Benno Schwabe⁸⁸, but it had been out of print for quite a while before it was published again in the

⁸³ "Collection of Popular Scientific Lectures"

⁸⁴ Rudolf Ludwig Karl Virchow (1821-1902) was a German pathologist and physician at the Charité in Berlin, as well as an archaeologist and politician. In addition to his contributions to cellular pathology and cell theory, he was instrumental in improving public health in Berlin. Virchow also participated in the German March Revolution of 1848. A biography can be found at:

http://web.archive.org/web/20061207003258/http://www.charite.de/cover/de/article/rv_0.html, accessed 22/04/2014.

⁸⁵ Joachim Wilhelm Franz Philipp von Holtzendorff (1829-1889) was a German jurist and professor of jurisprudence at the universities of Berlin and Munich. Cf. biography by C Meltz in *Neue Deutsche Biographie* 9, 1972, 556-557: <http://www.deutsche-biographie.de/sfz33551.html>, accessed 22/04/2014.

⁸⁶ Wilhelm Wattenbach (1819-1897) was a German historian and professor of history at the universities of Heidelberg and Berlin. Cf. biography by C Rodenberg in *Allgemeine Deutsche Biographie*, 1898:

<http://www.deutsche-biographie.de/sfz10969.html>, accessed 22/04/2014.

⁸⁷ A list with all issues published as part of the collection can be found at http://de.wikisource.org/wiki/Sammlung_gemeinverst%C3%A4ndlicher_wissenschaftlicher_Vortr%C3%A4ge, accessed 22/04/2014. Most of the volumes have been digitised either by Google or by the HathiTrust.

⁸⁸ Benno Schwabe was a Swiss publisher (managing what is today Schwabe AG, the world's oldest publishing house); however, I have not found any information on a "collection" he published. Perhaps Rudio just means the publishing house as it focused on scientific texts and encyclopaedias (alongside literary texts). Cf. biography by S Hess in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D29882.php>, and <http://www.schwabe.ch/schwabe-ag/wir-ueber-uns/geschichte/>, both accessed 22/04/2014.

Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich (1908) [32] and also individually by Zürcher & Furrer in 1909. Rudio explains his reasons for re-publishing the talk in the *Vierteljahrsschrift*: it was to accompany the call for donations in aid of the Euler edition [32, p. 456, footnote 1].

6.3.3 Euler Talk

The talk is a concise biography of Euler aimed at non-mathematicians, but it is of interest to mathematicians nonetheless. Rudio summarises Euler's childhood, focusing on his mathematics education and interest in natural phenomena, and his studies in Basel. He explains how Euler obtained a post at the St Petersburg Academy, and also why he eventually quit it and moved to Berlin. Rudio assumes that his audience is familiar with the political developments in Russia at the time, but gives a brief overview of the history of the Berlin Academy [32, p. 460-461].

Then, Rudio moves on to Euler's contributions to mathematics and physics, first outlining the scope of his works. Rudio then explains the aim of the sciences in general: finding the laws that govern natural phenomena and connections between them, where natural phenomena are considered in terms of motion [32, p. 462-463]. He illustrates this with three examples: free fall of a stone dropped from a tower, Kepler's laws, and brightness of a sheet of paper illuminated by a source of light. Using concrete numerical examples, he states that the dependencies between the respective quantities and the square of time are proportional (in the first two cases; in the third case, brightness and distance are inversely proportional) and that these dependencies can be expressed in terms of functions [32, p. 463-464]. Rudio then sums up Euler's work on functions, highlighting his textbooks on infinitesimal as well as integral and differential calculus, and Euler's influence on mathematical formalism. He illustrates the subsequent brief outline of Euler's works in physics by explaining how Euler paved the way for achromatic optical instruments [cf. 32, p. 466]. Rudio concludes the section on Euler's works with a summary of papers concerning practical problems and a short review of Euler's *Letters to a German Princess*.

Rudio then resumes his account of Euler's life, describing his return to St Petersburg, the loss of both his second eye and his house and library, and finally his death. Rudio also outlines Euler's character, gives examples of his interests and his intellectual abilities [32, p. 468-469]. He concludes the talk by explaining the importance of Euler's works and their impact not only on mathematics, but also on everyday life.

Rudio gave the talk shortly after the centenary of Euler's death, which might have been his motivation when choosing the topic. It is by no means the only biography of Euler that was published at the time, and many more have been written since. Nevertheless, the paper can still be bought on Amazon, and it is listed in the 'further reading' section on the Euler project website⁸⁹. There is also a short reference to it in [10, p. 2]:

The first Euler anniversary event seems to have been a small seminar in Zürich on December 6, 1883, where Ferdinand Rudio delivered a short biographical talk on Euler. This seminar would probably be completely forgotten if Rudio had not published the text of his talk more than 25 years later in the wake of the 200th anniversary events in 1907⁹⁰.

Whatever his reasons, this talk is the first recorded evidence of Rudio's interest in Leonhard Euler, whose works would occupy him for more than two decades. It is the first time that he mentions a complete edition of Euler's works, although he is more than doubtful that his wish would ever come true:

If one were to publish a complete edition of his works, which, I'm sorry to say, we do not have and might never have, then this edition would comprise 40 stately quarto volumes.

[32, p. 462]

⁸⁹ <http://eulerarchive.maa.org/resources-life.html>; however, it seems that the website has not been updated in a while.

⁹⁰ In the introduction to his talk, Rudio does indeed say that 'this evening may be seen as a commemoration', but he also refers to an earlier celebration in Basel.

Rudio refers to this comment in a footnote to the 1907 re-publication of his talk, when he was actively fundraising for what became the Euler project:

And if [the talk] helps contribute to Euler's works finally being reborn in an edition worthy of the eminent mathematician, [...] a wish will come true, a wish that I had already hinted at then [...], but at the time I did not dare hope that I would witness its implementation in my lifetime.

[32, p. 456]

It would most certainly be an exaggeration to claim that the Euler project started with Rudio's 1883 talk, but it is quite interesting to see how long the idea, which at the time might have been just a pipe dream, of editing Euler's complete works seems to have occupied him.

In any case, the talk certainly gave Rudio's audience a clearer idea of the scientist Euler and of his importance. The biographical sections are flavoured with short anecdotes that make Euler seem more human; the scientific examples are kept simple and accessible for a layman audience. Rudio tried to present as thorough an account as possible, but as he mentions himself on several occasions, he did not have enough time to go into details. In the case of Euler's *Letters to a German Princess* at least, he expresses the hope that 'this evening would at least result in these Letters [...] attracting the interest that they so highly deserve among a wider circle of readers' [32, p. 467].

Rudio gave the talk as a young Privatdozent, two years after he had moved back to Zurich. However, he had not yet obtained Swiss citizenship; maybe this was a reason why he highlights Euler's Swiss background on several occasions⁹¹? Alternatively, he might have included these little remarks in order to give his audience another reason to be interested in Euler, apart from

⁹¹ An example: 'You might also be interested to learn that Euler never stopped being a Swiss, for although he lived in Berlin for 25 years and in St Petersburg for 31 years, he always used the genuine Basel vernacular with all its peculiarities, often to the amusement of those around him.' [32, p. 469]

his overall influence on mathematics and physics, and, as he explains in his conclusion, technological progress in general:

But for all that, it is an undisputed fact that [technological] progress is very closely linked to the development of mathematics, even if this connection is not always as obvious as in the case of Euler inventing the achromatic telescope. Thus, Euler's contribution to the great achievements that humanity takes both pride and delight in today is not to be underestimated, and hence his name deserves to be known and recognised even by those who have no interest in mathematics.

[32, p. 469-470]

Every now and again Rudio remarks how useful mathematics is and how many applications it can have in everyday life. However, he expressed this view more strongly in his second town hall lecture, given eight years later.

6.3.4 Renaissance Talk

In this talk, Rudio essentially gives a short historical overview of the development of modern mathematics, up to the Renaissance, 'which will always be of very particular interest [...] for those who pursue the development of mathematics and its related disciplines from a cultural-historical point of view' [37, p. 3] as 'it is the age in which the consolidating process, from which our mathematical sciences emerged as an international cultural factor, took place' [ibid.]. He reminds his audience that the Renaissance is known for new developments both in the arts and in the sciences and humanities, including mathematics. Rudio emphasises that mathematics is by no means as prosaic as it is often perceived to be, but in fact requires a great deal of creativity and imagination [37, p. 4-5]. Furthermore, mathematics has a much more profound impact on our daily lives and beliefs than most people could imagine [cf. 37, p. 5]; and Rudio gives some examples of that throughout the talk, explaining that the Renaissance is particularly suitable for making this point. Even today many people seem to be unaware of just how many aspects of our lives mathematics plays a part in, despite the

enormous technological progress and availability of its results since Rudio's time. However, for Rudio the applications of mathematical research are its by-products rather than its goal, and the result of centuries of mathematical study [ibid.].

Thus, he goes back all the way to Archimedes in order to explain the developments in the Renaissance. Rudio talks about the Greek mathematicians at the Academy in Alexandria, in particular about Ptolemy. He outlines the mathematical basics of Ptolemy's worldview in layman terms, explaining reasons why he would have invented his epicycles and deferents [37, p. 7-9]. Then, Rudio explains that whilst Greek refugees from Constantinople brought classical Greek texts to Europe in the 15th century, Greek mathematics had already reached Europe in the form of Arabic translations, which were then translated into Latin. Rudio highlights the importance of the Arab scholars in transmitting both Greek and Indian mathematics to Europe [37, p. 10-12]. Most importantly, at least from our point of view today, he emphasises that modern mathematics emerged from the 'amalgamation' of Greek and Indian mathematics [37, p. 12]. He explains that the Indians' 'approach to mathematics was completely different to that of the Greeks, but not less sophisticated' [ibid.]:

Due to their highly developed sense of aesthetics, the Greeks almost exclusively investigated mathematical problems that could easily be visualised, i.e. problems in geometry. In contrast, the Indians' exceptionally accomplished sense of numbers and an unparalleled love of calculation, spread across all social classes from ancient times, led them to dealing with problems in arithmetic and algebra for the most part.

Rudio points out that Indian mathematicians made valuable contributions to number theory and algebra, but focuses on the development of the Hindu-Arabic numerals, as this was the development most relevant to his talk and also to his audience. Apart from summarising the concept of place-value

notation, the emergence of the actual digits and their journey to Europe via Arab scholars and Fibonacci, he explains the importance of these numerals not only for mathematics, but also for every-day calculations [37, p. 13-16]. His references to the Roman numerals and the abacus, both familiar to his audience, illustrate his point [cf. 37, p. 14-15].

With the invention of the printing press Rudio arrives in the Renaissance, the era of the polymaths, as he comments [37, p. 18]. Firstly, he summarises the main contributions of mathematics to the Renaissance: the general acceptance of the Hindu-Arabic numerals and of the heliocentric system [37, p. 18]. He then goes on to illustrate how the use of mathematics enabled advances in areas such as architecture and painting. His two prime examples are the construction of the Florentine Cathedral by Brunelleschi [cf. 37, p. 19], and the theory of perspective, 'which resulted from the marriage of art and mathematics and can well and truly be called a child of the Renaissance' [37, p. 21-23]⁹². He also spends some time talking about Leonardo da Vinci, in

⁹² Rudio first explains the modern notion of perspective projection:

One puts a glass pane between the original object one wants to map and one's eye, which is situated at an arbitrary, but constant point in the space [...] If one assumes that the rays of light that travel from the points of the original in the direction of the eye, through the glass pane, leave a visible trace on the pane, then all of these traces will form an image, which is called the perspective view. The theory of perspective is simple the collection of rules according to which one can draw an accurate perspective image of a given object *without* using such a glass pane.

[37, p. 22]

In the subsequent paragraph he discusses whether or not the ancient Greek and Roman painters knew perspective. I would hazard a guess that by 'perspective', Rudio meant geometrical projection based on mathematical rules as it was developed in the Renaissance [cf. ii]. He cites Lessing and Lambert, both of whom came to the conclusion that the Ancients did not know perspective [cf. 37, p. 22]. Rudio does not disagree with this conclusion, but remarks that it was not just Leonardo da Vinci who developed the theory of perspective, as Lambert suggested, but that namely Alberti and brothers van Eyck played a vital part in establishing perspective [cf. 37, p. 22-23]. Modern scholars generally credit Alberti with first developing (modern) perspective. However, it is also generally accepted that the Ancients did know perspective. As Panofsky shows, ancient perspective represented a 'genuinely spatial view' [iii, p. 43], but it was an 'expression of a specific and fundamentally unmodern [sic!] view of space' [ibid.]. Modern perspective focuses on one single vanishing point, but ancient scholars and artists assumed that the eye sees each object separately, which resulted in a number of vanishing points and a curvilinear perspective. Ancient painters were able to create the illusion of depth and perspective, e.g. by overlapping objects and by applying a principle from optics: the fact that the intensity of colours decreases with

particular on his scientific inventions, but makes it clear that despite being a man of the Renaissance, da Vinci did not have much influence on his era [37, p. 20-21]. Rudio then moves on to his concluding topics: mathematics education and astronomy.

Rudio explains how school and university education was reformed in Germany in the 16th century, giving mathematics a more prominent position; and as he speaks to a Swiss audience, he also gives examples of a similar development in Switzerland [cf. 37, p. 24]. He then gives a biography of the German Renaissance mathematician Regiomontanus, highlighting his achievements in astronomy [cf. 37, p. 26-29], and concludes his talk with a short biography of Copernicus and summary of the Copernican system [37, p. 31-32].

Rudio's talk is very well structured and makes an interesting read, for mathematicians and non-mathematicians alike. Although he only scratches the surface of some mathematical developments and does not go into much detail, he successfully demonstrates how century-old (or even millennia-old)

increasing distance in their paintings. Famous examples include the paintings at Villa Boscoreale, at Pompeii and frescoes depicting scenes from the Odyssey originally found on the Esquiline Hill. Strong argues that the Roman artists had a 'good understanding of the use of linear and aerial perspective' [iv, p. 70] and suggests that Roman architectural schemes '[seemed] to be imitations of Greek stage sets' [iv, p. 69], with which illusions of depth could be created. Ancient scholars applied perspective in cartography and stage design [cf. i, p. 5] and studied it as part of optics. Tobin claims that Greek painters might have had a geometrical framework for creating perspective, but that the Roman painters were less interested in the mathematical foundations [v, p. 34]. In the Renaissance, these mathematical foundations were investigated thoroughly, and our modern notion of perspective was developed as well. Thus, Rudio was not wrong not to disagree with Lessing; modern perspective was established in the Renaissance, but the Renaissance painters did rediscover principles that were first established in Antiquity. It is also important to note that several of the examples of ancient perspective had not yet been studied or even excavated in 1892, meaning that Rudio (as well as Lessing and Lambert) had less evidence to draw from than modern scholars. Cf. [i] S Y Edgerton, Jr., *The Renaissance Rediscovery of Linear Perspective*, Harper & Row, New York, 1976 (Icon edition); [ii] N J Koch, "Perspektive", in: H Cancik, H Schneider (eds.), *Der Neue Pauly*, vol. 9, Verlag J. B. Metzler, Stuttgart, 2000; [iii] E Panofsky, *Perspective as Symbolic Form*, Zone Books, New York, 1991 (first German edition published in 1927); [iv] D Strong, *Roman Art*, prepared for press by J M C Toynbee, revised & annotated by R Ling, Yale University Press, New Haven, 1988 (new impression 1995); [v] R Tobin, "Ancient Perspective and Euclid's optics", *Journal of the Warburg and Courtauld Institutes* 53, 1990, 14-41.

mathematical achievements have shaped our society and continue to influence our lives today, thus justifying studying the history of mathematics. Moreover, Rudio illustrates his point that mathematics is a very versatile art with real-life applications, not just some dry subject. Popularising mathematics is still an on-going project nowadays, and Rudio's examples could easily be used today. Finally, his talk is a very interesting compilation of mathematical and historical developments that are 'still very readable' [17, p. 511] and would be of interest to contemporary readers as much as to his audience more than a hundred years ago.

Rudio's approach to Indian and Arabic mathematics is certainly surprisingly modern and farsighted for the 1890s. Highlighting their contributions to modern mathematics and stressing that we do not owe everything to the Greeks is partly why the talk is so interesting to read (even) nowadays, as his points are consistent with our current standpoint. The only section that would have to be updated is the one on Ptolemy. Rudio may have been aware of criticism against Ptolemy, but there is no way that he could have known whether or not it was justified, given that modern critics base their arguments on statistical evidence and there is still some debate as to whether accusations are justified. In any case, based on the impression I got from his works, Rudio would have mentioned any justified criticism of Ptolemy's work, but at the same time would have stressed that one would have to look at Ptolemy nonetheless as his geocentric system influenced scholars for centuries to come. Throughout his works on historical topics, Rudio maintains the view that we should not judge past events and practices based on our current conventions. An example in the Renaissance talk is the passage on astrology, which flourished in the Renaissance: although Rudio does not consider astrology to be a science, he acknowledges that:

[...] astrology often motivated and encouraged research in astronomy. Moreover, for as long as people considered the Earth to be the immovable centre of the universe, relating celestial phenomena to earthly events and asserting a causal connection between them seemed an obvious thing to do, as this was in accordance with the resulting

significance of the Earth. Changing these assumptions was only made possible by a complete reform of the entire worldview.

[37, p. 30]

Unfortunately, Rudio does not explain in his talk why he chose to talk about mathematics in the Renaissance, apart from the fact that it marks the period when ‘modern mathematics’ began [cf. 37, p. 3] and illustrates that mathematics is an art. His other historical works (ignoring the biographies here) all concern ancient Greek problems, so a talk on such a topic, for example on the history of squaring the circle, would have seemed a more natural choice. But he may have felt that such a topic would have required him to use more technical terms than he wanted, or there might have been other talks on Greek mathematics at the time. In any case, his choice was definitely a rather unusual one; I daresay that mathematics is not the first thing people would associate with the Renaissance even today. Rudio certainly felt that the role of mathematics in this era deserved to be better known, and he does a very good job at illustrating how mathematics was key to several developments and inventions not necessarily related to science at first glance, and whose effects were still felt by his audience (and, in most cases, also today). In *Archimedes, Huygens, Lambert, Legendre* he dedicates an entire section [29, chapter 2, §8] to the Renaissance, admittedly focussing on works concerning squaring the circle and calculating π , where he refers to his talk (see chapter 6.1 for an analysis of the book).

This book also gives us some indication of the sources that Rudio might have used for his Renaissance talk. In many later publications, the Indians and Arabs do not get as much recognition as in Rudio’s works⁹³, but he was not the only 19th-century author to praise the contributions of these mathematicians. In the preface to *Archimedes, Huygens, Lambert, Legendre* he lists Cantor’s *Vorlesungen über die Geschichte der Mathematik* and Hankel’s *Zur Geschichte der Mathematik im Alterthum und im Mittelalter* as well as publications by Wolf, Montucla, Kästner and Klügel [29, p. vi]. With regard to

⁹³ An example would be Smith [46]. Although he recognises that the Indians had a system of numerals that may have developed into our modern numerals, he generally brushes aside any results in arithmetic and algebra.

Indian mathematics, he also points to Colebrooke's translation of Brahmagupta's treatise *Algebra and Arithmetic with Mensuration* [29, p. 18, footnote]. Cantor included a chapter on Indian mathematics in his *Vorlesungen*, outlining not only the development of the Indian numerals, but also their contributions to algebra and arithmetic. It is likely that Rudio's views on Indian mathematics would have been influenced (at least to a degree) by Cantor's writings. In his talk for example, Rudio mentions that Buddha had to solve mathematical problems as part of courtship [37, p. 13]; the anecdote can be found in Cantor's book [13, p. 612]⁹⁴. However, whilst Cantor remains sceptical about the extent to which Indian mathematics was in fact influenced by Greek and Babylonian work [cf. 13, p. 593-660], Rudio does not imply whether such a connection existed – possibly due to time restraints. Other German authors of the 19th century who acknowledged the work of the Indian mathematicians were Alfred Arneth (1802-1858) and Franz Woepke (1826-1864). Woepke edited many Arabic texts, but also wrote on the influences of Indian and Arabic mathematics on Europe, whereas Arneth wrote on the historical development of both Greek and Indian mathematics, and on the differences between the two. As there are no references to their books in Rudio's work and no private notes by Rudio (that I know of), we can only speculate whether or not he knew of those authors⁹⁵.

Summing up, apart from being a very readable article in its own right, the talk is a good sample of Rudio's historical works. It is the least technical of them, but most of his other works were aimed at mathematicians or mathematics students, allowing him to go into more detail when it came to mathematics. In addition, he did not quote much from other publications here, but this may be due to the fact that he wrote a talk, not a paper intended for publication only. The overall style of the text is of the same quality as in his other papers though: well-structured, enjoyable to read and interspersed with

⁹⁴ Note that Cantor talks about "Bodhisattva", not "Buddha".

⁹⁵ For more details, see [17, p. 116-117], as well as biographies of Arneth and Woepke by M Cantor in *Allgemeine Deutsche Biographie*: 1875, 554-555: <http://www.deutsche-biographie.de/sfz1261.html> and 1898, 209-210: <http://www.deutsche-biographie.de/sfz86174.html>, respectively, both accessed 22/04/2014.

examples and anecdotes that illustrate his explanations and make the mathematical concepts and mathematicians more accessible. Rudio was a good storyteller, and he used this talent to make people more interested in mathematics and its history. Fueter cites praise of the talk by Rudio's contemporaries:

[The] talk represented a highlight in the Town Hall lecture series. The detailed, critical explanations, which illuminated the thorough cultural-historical knowledge of the mathematician [Rudio], inspired a congratulating colleague to call out after the talk: "You could hold a professorship in cultural history!" Meanwhile, another listener (Prof. Lunge) who was still under the spell of the excellent lecture, given without the use of any notes, remarked: "This was one of the finest talks I have ever heard in my life". After the talk had been published [...], the art historian Karl von Lützow commented in his "Zeitschrift für bildende Kunst" ["Visual Arts Journal"]: "The relationships between mathematics and art have often been discussed, but never on such a high level as here." "

[41, p. 119]

Comparing Rudio's two town hall talks, the Renaissance one is definitely of higher quality than the talk on Euler, both in terms of style and content. This may be due to the topic, or it may be due to the fact that Rudio had much more experience in both writing and public speaking by then. Anyhow, a factor that makes the Renaissance talk so enjoyable for a modern reader is its timelessness; apart from certain dated phrases it is hard to tell that it was written in the 1890s. The passages in the Euler talk concerning physics, and in particular the aether, however, give away its age. The basic data in Euler's biography still apply, though, so the talk can still be of interest to modern readers.

As a historian of mathematics, Rudio was surprisingly modern in his approach, granting Indian and Arabic mathematics the credit that they

deserve, which was not common practice in his time. Furthermore, he was interested in the bigger picture, placing mathematical developments into their historic context and thus conceiving them as part of a larger historical development. However, he cautioned against interpreting historical events from our modern point of view, as this could lead to us making unfair judgements.

In his works he managed to convey that the history of mathematics is interesting, which justifies studying it, thus defending his – at the time rather young – discipline. In addition, he showed non-mathematicians that mathematics is much more important and appealing than commonly assumed. Both of these messages, for want of a better word, still very much apply today; despite the amount of indispensable technology many people fail to grasp just how pervasive and important mathematics is in our daily lives. Common reactions to my studying mathematics are: “I was never good at maths” and “Oh gosh, that must be really hard!” (and, by implication, boring). Nor is it uncommon for people to fail to understand why one would want to study the history of mathematics. In his works, Rudin made the two disciplines accessible and interesting – there is no way of knowing, but I do hope that he inspired some of his readers to learn more.

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7. Conclusion

In conclusion, the members of the Swiss ICM organising committee achieved much more than just organising a congress. The ICMs still happen every four years, and have developed into truly international congresses with several thousand participants. With the exception of the two World Wars, they ran continuously throughout the 20th century, an indication that they were considered important enough events for some group of mathematicians to make the effort to organise them. As mentioned in chapter 4, Geiser and his colleagues could not have foreseen such a development, but in some ways it is a tribute to their work – the congresses might have been abandoned if the first few had not been such a success.

As I hope to have demonstrated, the organising committee consisted of a group of interesting men: apart from the famous mathematicians Hurwitz and Minkowski, to name but two, most left a much more subtle mark, which is much more easily overlooked. However, some of their contributions, be it to mathematics, to cadastral surveying, to the history of mathematics, or to mathematics education, warrant an interest in their lives. Due to the limitations of this thesis, and in some cases access to or limitations of sources, I was not able to delve deeper into the achievements of some individuals.

This is particularly true for my two “leading characters” Geiser and Rudio. In both cases, studying their purely mathematical papers could prove an interesting project. The same is true for the surviving notes taken during their lectures, which are kept in Zurich. Furthermore, one could also look at Rudio’s involvement in the Euler project in more detail.

Administrative duties and, even more importantly, improving education are recurring themes in Geiser’s life – he was a gifted educator and organiser. I hope to have outlined this aspect in this thesis, particularly by analysing his schoolbook and by including his letters to his schoolteacher friend Gysel.

Rudio, on the other hand, was a keen historian of mathematics, and a very talented writer. Although slightly dated, his works are very readable today, and offer a rather modern approach, which sets them apart from other books

published at a similar time. References to his books can be found in a variety of works by other scholars, historians of mathematics and classicists alike, spanning the entire 20th century and extending well into the 21st century. Surely this testifies to the quality of his work.

As mentioned above, the limitations of this thesis and restricted access or purely a lack of sources did not allow me to go into as much details as I wanted on several occasions. As with all historical studies, one is to a certain degree dependent on the range of sources available. Whilst the scientific estates of some mathematicians contain a host of valuable documents, others are distinctly devoid of information. This shaped the direction of this thesis to a certain extent. In a nutshell, however, I chose to study the aspects of the 1897 ICM, and of Geiser and Rudio, that interested me the most, in the hope that other readers would find them equally rewarding.

Appendix A – Glossary

Bundesrat: Federal Council; the Swiss government, consisting of seven members of the *Bundesversammlung*. The assembly elects them at the beginning of each legislative period, lasting four years. The president of the *Bundesrat*, who is the Swiss President, but only as a *primus inter pares*, is elected every year within the *Bundesrat*. The term also refers to a member of the Federal Council.

Bundesversammlung: Swiss Federal Assembly, the Swiss parliament. It contains two houses, the upper *Ständerat* and the lower *Nationalrat*.

Canton: see *Kanton*

Extraordinary Professorship: A professor without a chair, comparable to a reader at a British university. Typically the next stage after a *Titularprofessor* in an academic career.

Gymnasium, pl. Gymnasien: German secondary school, which prepares pupils for the *Abitur* and, ultimately, university studies. Traditionally, pupils would attend the *Gymnasium* for nine years (except for during the Third Reich, when it was cut down to eight years), although many German states are currently introducing the eight-year *Gymnasium*. In the 19th and early 20th centuries, most *Gymnasien* were “humanistisch”, classical, meaning that they focused on Latin and Greek, followed by history and mathematics (based on the humanist ideals formulated by Wilhelm von Humboldt). So-called *Realgymnasien* or *Oberrealschulen* taught primarily sciences, mathematics, and modern languages, but pupils graduating from such a school often had a limited choice of subjects they could study at university. *Gymnasium* is also a synonym for the Swiss *Kantonsschule*.

Habilitation: The highest academic examination in Germany and Switzerland, as well as in some other countries. A PhD is a necessary prerequisite for a habilitation, which examines the candidate’s ability for both independent scientific research and teaching. The mathematicians included in this thesis obtained their *venia docendi*, the right to teach, by means of a habilitation, and were then allowed to work as a *Privatdozent*. Although required for different reasons, the academic level of a Habilitation is comparable to the degree of Doctor of Science in the UK.

Industrieschule: Lower level secondary schools, originally founded in the 18th century with the intention of preparing (working class) pupils for technical or industrial jobs. During the 19th century they were gradually replaced by other types of schools. Some *Kantonsschulen* included an *Industrieschule* as their science or technical track.

Kanton: Member state of the Swiss federal state, or Confoederatio Helvetica. Currently there are 26 cantons, although six of them are so-called half cantons. There were only 25 cantons until 1979, when Jura became an independent

canton. The cantons have a certain degree of autonomy, e.g. with regard to education, but they lost most of their sovereignty, e.g. monetary, when the Swiss federal state was founded in 1848.

Kantonsrat: Cantonal parliament, but the term also refers to a member of that parliament.

Kantonsschule: Swiss higher level secondary school leading to the *Matura*. In most cantons, the *Kantonsschule* or *Gymnasium*, as it is also called, lasts four years. After six years of primary school, pupils attend a lower level *Sekundarschule* for a further two or three years, both of which are compulsory. Some *Kantonsschulen* offer entry straight after primary school, and require six or seven years. Attending the *Kantonsschule* is voluntary, but often requires an entry examination. Many *Kantonsschulen* had two tracks or branches, a classical one and a science one.

Matura: Name of the secondary school exit examinations in Switzerland (as well as in many other European countries), taken in the final year at the *Kantonsschule*. Obtaining the *Matura* is generally a necessary prerequisite for university studies.

Nationalrat: National Council. It is the lower house of the *Bundesversammlung*, and the larger of the two houses; since 1963 there are 200 members. *Nationalrat* also refers to a member of that council.

Oberrealschule: A *Gymnasium* that focused on sciences, mathematics, and modern languages. They were often called “lateinlose Schulen”, schools without Latin. Graduates generally studied sciences or engineering, as a qualification in Latin or Greek (or both) was required for a number of subjects, particularly in the humanities, but also for medicine, for example. Many *Oberrealschulen* in Switzerland prepared their pupils for studying at the Polytechnic.

Ordinary professorship: A full professor with a chair.

Privatdozent: A university lecturer. In the 19th and early 20th centuries, this was typically the first career stage after doctoral studies. At the time, many *Privatdozenten* did not receive a fixed salary, but the so-called *Hörergeld* paid by the students (i.e. a form of tuition fees). Thus, their income depended on the number of students who attended their lectures. (Professors also received the *Hörergeld*, but for them it was an addition to their regular salary).

Realgymnasium: see *Gymnasium* and *Oberrealschule*

Realschule: A lower level secondary school in Germany and Switzerland, focusing on sciences and modern languages. *Realschulen* cannot confer the *Abitur* or *Matura*, their pupils go on to apprenticeships, although some transfer to a *Gymnasium*. In some obituaries used for this thesis the term *Realschule* seems to refer either to an *Oberrealschule*, *Realgymnasium* or the science track of a *Gymnasium*.

School Board: see Schulrat

Schulrat: The governing body of the Polytechnic, based on the system at the Ecole polytechnique in Paris and directly responsible to the *Bundesrat*. Similarly to the *Bundesrat*, the members of the *Schulrat* had to represent different *cantons*, languages, and religions (and, from 1881 onwards, also different technical professions). The *Schulrat* originally had five members; from 1881 onwards this was increased to seven members. They were appointed by the *Bundesrat*. In contrast to the *Bundesrat*, however, the School Board President had a lot of powers; for example, he was responsible for appointing academic staff. Furthermore, the length of his term of office was not restricted. Nowadays, there is an Executive Board, but also a so-called "ETH-Rat" (ETH Board), which is responsible for strategic management and supervision of the ETH Zürich and the Ecole Polytechnique Fédérale in Lausanne (EPFL).

Ständerat: Council of States; the upper house of the *Bundesversammlung*. Every canton sends two representatives to the *Ständerat*.

Swiss Federal Polytechnic: "Eidgenössisches Polytechnikum" or "Eidgenössische polytechnische Schule", founded in 1854 and opened a year later as Switzerland's first federal higher education institution. Originally founded to train engineers, (fundamental) research became increasingly important over time. The Polytechnic was awarded the right to confer doctorates in 1908; in 1911 it was renamed for "Eidgenössische Technische Hochschule" (ETH Zürich), Swiss Federal Institute of Technology.

Titularprofessor: A Titularprofessor in Switzerland has similar rank and obligations as a senior lecturer at a British university. It is the next level up from a *Privatdozent*.

Appendix B – The Federal Polytechnic

This section gives a brief overview of the history of the Federal Polytechnic in the 19th and early 20th centuries. It is by no means meant to be a comprehensive historical survey and only intends to provide the backdrop to the events described in this thesis. More information on the first 150 years of the ETH can be found in the very readable book *Die Zukunftsmaschine. Konjunkturen der ETH Zürich 1855 – 2005* by D Gugerli, P Kupper and D Speich, Chronos-Verlag, Zürich, 2005.

The origins of the Federal Polytechnic in Zurich are closely linked to the foundation of modern Switzerland as a federal state in 1848. The Swiss Confederation was founded in 1291, although it has to be said that the three founding cantons Uri, Schwyz, and Unterwalden merely renewed an already existing confederacy. Not going into any details, the origins of Switzerland are often glorified by the general public. Suffice it to say that the old Confederacy had little in common with modern Switzerland. As an example, the reformation caused a rift between catholic and protestant cantons. The aftermath of this was still felt by the founding fathers of modern Switzerland, and, by extension, of the Polytechnic.

As a result of Napoleon's Helvetic campaign the Helvetic Republic replaced the old Confederacy in 1798. This republic was based on the French model and did not suit the Swiss at all. After five years of conflict Napoleon reconstituted a confederacy, but he restricted the privileges of the upper classes [4, p. 115-116]. However, after Napoleon's defeat the Swiss founded a new Confederacy, which comprised 22 cantons and was based on the pre-Napoleonic model. Most importantly, the cantons were granted full sovereignty and the ruling classes enjoyed a privileged status once again. As an aside, Switzerland chose neutrality at the same time, in 1815. Whilst the Confederacy had become neutral in 1515, it was more out of necessity than choice, as the conflicts between the different cantons did not allow for any external military engagement. However, some cantons signed capitulation treaties with European powers and Swiss mercenaries continued to fight across Europe.

Returning to the Swiss Confederation, the so-called Restoration (1814-1830) was followed by a Regeneration period. Liberal parties and societies were founded, people called for popular sovereignty, representative democracy and a separation of church and state. Furthermore, liberal groups wanted to improve education at school and university level, and introduce uniform coinage – different coinages hindered economic growth at a time when the textile industry in particular expanded rapidly [cf. 4, p. 121-122]. These developments led to a rift between liberal cantons, which were predominantly urban and protestant, and conservative cantons, which were predominantly rural and catholic. Tensions between the cantons resulted in two so-called "Freischarenzüge" in 1844 and 1845, respectively: Volunteer troops of radical Liberals attempted to overthrow the catholic government of canton Luzern and demanded that Jesuits should be banished from the country. These demands were triggered by the fact that Luzern appointed Jesuits to teach at cantonal schools in the 1840s.

The radical troops in the Freischarenzüge were defeated, but as a consequence the catholic, conservative cantons Fribourg, Luzern, Schwyz,

Unterwalden, Uri, Valais, and Zug joined forces in a so-called “Sonderbund” (“Separate Alliance”) to protect their interests. However, this violated the 1815 Federal treaty. After fruitless discussions the Tagsatzung, the predominantly liberal Swiss legislative and executive council, decided that the alliance should be disbanded and that the Federal treaty should be revised [2, p. 658-659]. In October 1847 the Tagsatzung empowered General Henri Dufour, Supreme Commander of the Federal troops, to disband the Sonderbund by means of force. The campaign lasted less than a month and was the last armed conflict on Swiss territory.

After numerous discussions the Swiss Federal Constitution was adopted on 12 September 1848, and thus Switzerland became a federal state. During the negotiations religion and languages were particular matters of dispute. As a result of the Constitution the cantons lost some of their sovereignty – as an example, customs between cantons were abolished; federal customs, a uniform currency and postal service were introduced. However, the cantons retained authority in a number of areas, including education, direct taxes, social welfare, the police, and regional infrastructure. The Confederation on the other hand was responsible for all matters of a federal nature, such as foreign affairs and defence.

These responsibilities are detailed in Articles 1-20 of the Constitution. Article 22, however, concerned higher education at a federal level: ‘The Confederation is authorised to establish a university and a polytechnic school’ [quoted in 5, p. 21]. The idea of a federal university had been around since 1798, when the Swiss Minister for Culture, Philipp Albert Stapfer, called for a Swiss polytechnic based on the Ecole Polytechnique in Paris. But, as Gugerli et al write [5, p. 22-24], there was no central policy on education in Switzerland, and the cantons had different ideas regarding a federal university. The work of the statistician Stefano Franscini, a member of the Bundesrat who collected data on aspects regarding education in Switzerland and led the project of implementing article 22, reflects this [ibid.]. However, Franscini’s colleague Alfred Escher soon became the leading figure in this project [5, p. 27].

It soon became apparent that it was quite a controversial topic, and that many people did not approve of a federal university. The reasons for this highlighted more profound issues: Gugerli et al explain that cantons in West Switzerland feared that German would become the dominant language, cantons that had their own universities feared competition and having to give up sovereignty, conservatives feared that a university would help the Bundesrat consolidate its power, and others again just disliked Escher [5, p. 29].

In August 1853 the Nationalrat Jakob Stämpfli reported that the government had a substantial budget surplus, which could be used to establish higher education institutions. A four-day long heated debate in parliament that took place in January 1854 eventually ended in a vote in favour of the project. However, the issues listed above still remained, and the Ständerat voted against a federal university, but in favour of a polytechnic that would also teach arts subjects [5, p. 33-34]. A polytechnic would not be a rival to the already established cantonal universities. It was decided that a federal polytechnic would first and foremost produce engineers, chemists and foresters, but that supplementary subjects such as mathematics, natural

sciences, modern languages, history, and politics would also be taught [5, p. 36]. The aim was to provide the thriving Swiss industry with well-educated native experts. Previously these had to be “imported” from abroad. Zurich was chosen because it had not been made Swiss capital. Incidentally, Karl Kappeler, member of the Ständerat and the future School Board President of the Polytechnic, was instrumental in speeding up discussions and taking the matter to parliament [5, p. 35].

A working group established a vision and regulations for a polytechnic. Franscini collected data from various polytechnics in Germany, France and Italy, and Escher looked for suitable models. German institutions remained the main influence during the 19th century, and Swiss secondary education was divided into classic and realist strands as it was in Germany. A number of polytechnics were founded in Germany from 1825-1836, and then again from 1860 onwards [5, p. 44]. However, the Polytechnic set itself apart by offering natural and social sciences (see also below). Many German academics applied for the first lectureships, which were advertised in October 1854. The architect Gottfried Semper, who designed the Polytechnic’s main building in the 1860s, was the first professor to be appointed to the Polytechnic. A high percentage of foreign staff and students remained an integral feature of the Polytechnic during its first few decades. In some years more than half of the student cohort came from outside Switzerland [8]. Apart from Germany, large numbers came from Austria-Hungary and Russia [ibid.]. The Polytechnic also profited from the repressive governments in Germany, which caused many academics to relocate, among them Semper.

In the autumn of 1855 the Federal Polytechnic opened its doors for students. Joseph Wolfgang von Deschwanden, a schoolteacher, became the first director, and the politician Johann Konrad Kern became the first School Board President. Despite having been built out of nothing the institution soon began to thrive and quickly established an excellent reputation, notably also abroad. It served as a model for other institutions, e.g. the polytechnic in Prague [5, p. 43]. As mentioned above, the Polytechnic valued mathematics and natural sciences, which were often regarded as auxiliary sciences elsewhere. Furthermore, the wide variety of elective subjects added a unique competitive advantage. The Polytechnic originally comprised six Departments, or Schools, that covered: Construction, Engineering, Mechanics, Chemistry, Forestry (all of these with an engineering focus), and the Arts. From 1866-1899, the period that is of most interest for this thesis, there were eight Schools:

- I. School of Civil Engineering
- II. School of Engineering
- III. School of Mechanics
- IV. School of Chemistry
- V. School of Forestry; from 1871 School of Agriculture & Forestry, comprising subdivisions for forestry, agriculture, and cultural engineering
- VI. Department for Mathematics and Science [i.e. Physics] Teachers
- VII. Department for Elective Subjects
- VIII. Preparatory Course in Mathematics, until 1881

In 1899 the Department of Military Sciences was established; before then courses in that area were taught as elective subjects. The main language of instruction was German, but in some subjects, including mathematics, native French speakers were appointed to specially created chairs. These professors taught in French only.

Departments VI and VIII are of particular importance as they are proof of the School Board's farsightedness with regard to education and to mathematics. The School Board recognised early on that well-educated teachers would produce well-educated school leavers, which in turn would become well-qualified graduates and employees. They established the Teaching Department with the aim of educating future teachers, but in fact they achieved much more. In addition to excellent schoolteachers, the department produced a number of renowned research mathematicians. The School Board, particularly its presidents Kern and Kappeler, respectively, understood the importance of mathematics as a subject in its own right. As a result, mathematics research was given a privileged position, certainly when compared to other polytechnics across Europe. During the 1890s, when a number of engineers demanded that the mathematical content of engineering degrees should be reduced, the Polytechnic continued to support the discipline, in particular pure mathematics (see appendix E.3.2 for Geiser's summary of this so-called "Ingenieurbewegung" or "anti-mathematical movement"). However, as experiments became a more integral part of engineering degrees mathematics was one of the subjects that was cut back [5, p. 87]. Nevertheless, in 1912 about 10% of Polytechnic professors were mathematicians. This high percentage is partly explained by the fact that they also taught in other Schools [3, p. 1], but nevertheless shows the importance that was attached to mathematics.

Furthermore, the School Board Presidents had a particular gift for recognising talent. Kappeler in particular is said to have attended lectures of promising candidates in order to gain an independent opinion. Guggenbühl notes that Kappeler attached great value to 'a suitable personality and teaching abilities' [6, p. 83], but asked for advice on subject-specific qualifications. Despite not being able to offer a high salary in its first decades, the Polytechnic attracted a large number of exceptionally talented staff. For many German mathematicians a teaching post at the Polytechnic served as a springboard to a prestigious chair in Göttingen, Heidelberg, or Berlin. Among the mathematicians who taught at the Polytechnic in the 19th and early 20th centuries we find, in chronological order: Joseph Ludwig Raabe, Richard Dedekind, Elwin Bruno Christoffel, Carl Theodor Reye, Hermann Amandus Schwarz, Heinrich Weber, Georg Ferdinand Frobenius, Friedrich Schottky, Adolf Hurwitz, Hermann Minkowski, Hermann Weyl, and George Pólya. The Polytechnic must have fostered research and provided a stimulating, supportive environment. As a result, future mathematics teachers were taught by some of the world's leading mathematicians.

It is interesting to note that the majority of the above mathematicians pursued research in what would today be classed as pure mathematics. This illustrates the level of support that the School Board gave to mathematics as a discipline, which was certainly not the case at other polytechnics. In other subjects this extraordinary amount of support manifested itself in the form of

research laboratories. The School Board recognised that whilst future engineers and chemists needed a solid background in theory and mathematics, experiments were a key component of their training. During the 1880s and 1890s Chemistry and Physics moved into new buildings equipped with state-of-the-art laboratories, and a mechanical engineering laboratory was established in 1900. Moreover, a laboratory for material testing was founded in 1880 as part of the Polytechnic; it has since developed into the *Swiss Federal Laboratories for Materials Science and Technology* (German acronym: EMPA). The Polytechnic was able to bridge the gap between theoretical research and applications, and in doing so became one of the world's leading technical higher education institutions.

However, the development was anything but smooth. Whilst the Polytechnic attracted large numbers of foreign applicants it struggled to admit home students. This was partly down to the low standard at Swiss secondary schools. Traditionally education had been a privilege and there were a number of old-established cathedral and private schools. School education was improved from the Helvetic period onwards, but it was very much the responsibility of individual cantons. From 1832 the Zurich School Law (*Zürcher Schulgesetz*) regulated primary and secondary education in canton Zurich. Many German-speaking cantons followed this model; in the French-speaking cantons similar laws were introduced. The 1848 Constitution granted the cantons jurisdiction in educational matters. Whilst more and more schools were founded across the country, in particular in rural areas, the standards and curricula differed considerably [see 7].

The Polytechnic maintained rigorous entry requirements and entrance examinations – too rigorous for many candidates. In order to increase the number of entrants, entry requirements were lowered to a certain degree in 1859; students were now examined in 6-7 subjects, had to write an essay in their native language and submit evidence of proficiency in the languages of instruction. Prerequisite knowledge was specified, in mathematics this included:

Full knowledge of elementary mathematics and geometry, of trigonometry and of elements of analytic geometry and algebraic analysis; elements of descriptive and practical geometry; [knowledge] of elements of physics, chemistry, mechanics and natural history; furthermore skills in drawing freehand and with a ruler.

[5, p. 63]

However, in order to meet these revised requirements secondary school education had to be improved. The Polytechnic developed two strategies: negotiations with secondary schools and, as an interim measure, establishing a Preparatory Course in 1859. The one-year-long Preparatory Course was designed to provide students with the necessary knowledge in mathematics and natural sciences, and was generally referred to as “mathematical preparatory course”. It proved very popular with students, notably also those from abroad. The aim of negotiating with secondary schools was to improve the standard of teaching and to enhance the curriculum so that pupils would fulfil the Polytechnic's entry requirements. In return pupils from vetted schools did not have to sit entrance examinations. As Gugerli et al write these

negotiations were hard work, as many schools saw them as a limitation of their rights [5, p. 64-65]. However, the Polytechnic's strategy made a significant contribution to raising the standard of Swiss school education.

As a federal institution the School Board was directly responsible to the Bundesrat. The Bundesrat appointed the School Board (whereas the School Board president appointed academic staff), and approved the Polytechnic budget. As a result, the Polytechnic was heavily dependent on government funding [see 5, p. 59]. For several decades it also shared resources – staff and rooms – with the University of Zurich, which had been established in 1833. Most of the University's staff also came from Germany, but the institution focused on arts. In the early decades of the Polytechnic there were a number of profitable agreements between the two institutions. Students from either institution could attend lectures taught at the other one – if lectures were not already taught jointly –, some staff held positions at both institutions simultaneously, and the two institutions shared their buildings. Initially the Polytechnic used rooms in the University and the Kantonsschule of Zurich. When it moved into its own building in 1864 the University was given its own area. The University only moved into its own building 50 years later, which is located right next to the Polytechnic. However, Kappeler began to withdraw his support for the collaboration with the University in the early 1860s. In his opinion the University's professors did not cater enough for engineering students. As a result the University lost talent to the Polytechnic, which offered more attractive conditions, but the Polytechnic lost the close connection to a “proper” university, which were generally seen as superior [5, p. 79].

The Polytechnic and the University also had very different understandings of academia and higher education. Most courses were compulsory; the Polytechnic's governing bodies maintained strict discipline and punished transgressions accordingly. Polytechnic students were not granted the same freedom as their University counterparts, both with regard to academic matters and expected code of conduct. As a result of these differences a rivalry between the two different cohorts developed, which even led to duels [5, p. 106-109]. In 1881, partly due to a petition by its alumni association GEP, the Polytechnic revised some of its regulations. Students were given more flexibility in their final year of study, the School Board now consisted of seven members, and academic staff could elect the Director. Furthermore, the Preparatory Course was abandoned and agreements with secondary schools were rescinded. The School Board had to re-negotiate agreements in the following decades.

More comprehensive reforms were only implemented in the early 20th century. German polytechnics had become truly academic institutions, on a par with universities. The German Emperor awarded the Institute of Technology in Charlottenburg, Berlin (today University of Technology Berlin) the right to confer doctorates in 1899, and many German polytechnics followed suit. In Zurich this happened only ten years later, after years of debates. The School Board also introduced academic freedom and separated the Polytechnic completely from the University of Zurich. To mark the end of the old ways the Polytechnic was renamed as Eidgenössische Technische Hochschule, Swiss Federal Institute of Technology. As the American William

K Tate wrote in 1913: 'The Polytechnic School at Zurich ranks among the world's greatest technical universities' [1; quoted in the original]. This is certainly still the case a century later.

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Appendix C – Publication Lists

The following lists are based on the publication lists given in L Kollros, Prof. Dr. Carl Friedrich Geiser 1843-1934; and C Schröter and R Fueter, Ferdinand Rudio 1856-1929; respectively.

I have abbreviated the titles of a few common journals as the titles of the actual publications are of more interest here.

AGM = *Abhandlungen zur Geschichte der Mathematik*

AMPA = *Annali di matematica pura ed applicata*

BM = *Bibliotheca Mathematica*

CJ = *Crelle's Journal*

MA = *Mathematische Annalen*

MBNF = *Mitteilungen der Berner Naturforschenden Gesellschaft*

NZZ = *Neue Zürcher Zeitung*

SBZ = *Schweizerische Bauzeitung*

SZ = *Schlömilchs Zeitschrift*

VSNG = *Verhandlungen der Schweizerischen Naturforschenden Gesellschaft*

VNGZ = *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich*

(DMV = *Deutsche Mathematiker-Vereinigung*)

(SNG = *Schweizerische Naturforschende Gesellschaft*)

C.1 – Geiser's Publications

1866: Beiträge zur synthetischen Geometrie, doctoral thesis, Zürich

1866: Einige geometrische Betrachtungen, VNGZ **10**

1866: Über eine geometrische Verwandtschaft zweiten Grades, MBFN **592**

1866: Über die Normalen der Kegelschnitte, CJ **65**

1866/67: J Steiner, Geometrische Betrachtungen und Lehrsätze (ed.), CJ **66**; also in K Weierstrass (ed.), *Jacob Steiner's Gesammelte Werke*, Vol. II, G. Reimer, Berlin, 1882

1867: *Jacob Steiner's Vorlesungen über synthetische Geometrie*, Vol. I: *Die Theorie der Kegelschnitte in elementarer Darstellung* (ed.), Leipzig

1867: Über zwei geometrische Probleme, CJ **67**

1868: Sopra una questione geometrica di massimo e sua estensione ad uno spazio di n -dimensioni, *Rendiconti del Istituto Lombardo*

1868: Sulle normali all'ellissoide, AMPA **2** (1)

1868: Zur Theorie der Flächen zweiten und dritten Grades, CJ **69**

1868: J Steiner, Konstruktion der Fläche zweiten Grades durch 9 Punkte (ed.), CJ **68**; also in K Weierstrass (ed.), *Jacob Steiner's Gesammelte Werke*, Vol. II, G. Reimer, Berlin, 1882

1869: *Einleitung in die synthetische Geometrie. Ein Leitfaden beim Unterrichte an höheren Realschulen und Gymnasien*, Teubner, Leipzig

1869: Über die Doppeltangenten einer ebenen Kurve vierten Grades, MA **1**

1869: Über Flächen vierten Grades, welche eine Doppelkurve zweiten Grades haben, CJ **70**

1870: Notiz über die algebraischen Minimumsflächen, MA **3**

- 1870: Über die Steinerschen Sätze von den Doppeltangenten der Kurve vierten Grades, *CJ* **72**
- 1871: Über die Fresnelsche Wellenfläche, *VSNG* **54**
- 1871: Sopra un teorema fondamentale della geometria, *AMPA* **2** (4)
- 1873: Zur Erinnerung an Jakob Steiner, *VSNG* **56**
- 1877: Zum Hauptachsenproblem der Flächen zweiten Grades, *CJ* **82**
- 1877: Die Krisis der Nordostbahn, *Die Eisenbahn* **6** (23-26)
- 1877: Über ein Problem der kinematischen Geometrie, *VSNG* **60**
- 1877: Über die quadratische Gleichung, von welcher die Hauptachsen eines Kegelschnittes im Raume abhängen, *AMPA* **2** (8)
- 1878: Sopra la teoria delle curve piane del quarto grado, *AMPA* **2** (9)
- 1881: Über die dreifachen Sekanten einer algebraischen Raumkurve. In memoriam Chelini, *Collect. mat. Cremona et Beltrami*
- 1881: Über einen fundamentalen Satz aus der kinematischen Geometrie des Raumes, *CJ* **90**
- 1884: Adresse an Professor Dr. Ludwig Schläfli in Bern, *SBZ* **3** (4)
- 1888: Rede anlässlich der Trauerfeier von Karl Kappeler, *SBZ* **12** (18)
- 1890: Rede bei der Trauerfeier für Prof. Dr. Heinrich Schneebeili, *SBZ* **15** (21)
- 1894: Bilder aus dem ersten Jahrzehnt des Polytechnikums, in: *Festschrift zur Feier des 25jährigen Bestehens der G. e. P. Zürich*, Zürich
- 1896: Das räumliche Sechseck und die Kummersche Fläche, *VNGZ* **41**
- 1898: Opening speech of the 1897 ICM, in: F Rudio (ed.), *Verhandlungen des ersten Mathematiker-Kongresses in Zürich vom 9. bis 11. August 1897*, Teubner, Leipzig, 1898
- 1898: Zur Theorie der tripelorthogonalen Flächensysteme, *VNGZ* **43**
- 1900: Zum Andenken an Johann Friedrich Peyer im Hof, *NZZ* **267-270**
- 1901: Elwin Bruno Christoffel, with L Maurer, *MA* **54**
- 1904: Zur Erzeugung von Minimalflächen durch Scharen von Kurven vorgeschriebener Art, *Sitzungsberichte der Preußischen Akademie der Wissenschaften* 1904
- 1905: Die konjugierten Kernflächen des Pentaeders, *VNGZ* **50**
- 1907: Über Systeme von Kegeln zweiten Grades, *Beiblatt zu den neuen Denkschriften der SNG* **A** (1)
- 1911: Nachruf auf Prof. Dr. U. Aeschlimann, *Gedenkblätter*, Milano
- 1918: Opere matematiche di Luigi Cremona, *VNGZ* **62**
- 1921: Zur Erinnerung an Theodor Reye, *VNGZ* **66**

C.2 – Rudio's Publications

- 1880: *Über diejenigen Flächen, deren Krümmungsmittelpunktsflächen konfokale Flächen 2. Grades sind*, doctoral thesis, Berlin
- 1883: Die geodätischen Linien auf den Flächen zweiten Grades, *VSNG* **66**
- 1883: Zur Theorie der Flächen, deren Krümmungsmittelpunktsflächen konfokale Flächen zweiten Grades sind, *CJ* **95**
- 1886: Über einige Grundbegriffe der Mechanik, *VNGZ* **31**
- 1887: Über die Bewegung dreier Punkte in einer Geraden, *CJ* **100**
- 1888: *Die Elemente der analytischen Geometrie der Ebene*, with H Ganter, Leipzig
- 1888: Über primitive Gruppen, *CJ* **102**
- 1889: Über eine spezielle Fläche vierter Ordnung mit Doppelkegelschnitt, *CJ* **104**
- 1890: Die Bauschule des eidgenössischen Polytechnikums, *SBZ* **16** (4)
- 1890: Das Problem von der Quadratur des Zirkels, *VNGZ* **35**
- 1891: *Die Elemente der analytischen Geometrie des Raumes*, Teubner, Leipzig
- 1891: Über die Konvergenz einer von Vieta herrührenden eigentümlichen Produktentwicklung, *SZ* **36**
- 1892: *Archimedes, Huygens, Lambert, Legendre. Vier Abhandlungen über die Kreismessung. Deutsch herausgegeben und mit einer Übersicht über die Geschichte des Problems von der Quadratur des Zirkels versehen*, Teubner, Leipzig
- 1892: *Über den Anteil der mathematischen Wissenschaften an der Kultur der Renaissance*, Verlagsanstalt und Druckerei, Hamburg
- 1894: Erinnerung an M. A. Stern, *VNGZ* **39**
- 1894: *Festschrift der Gesellschaft ehemaliger Studierender der eidgenössischen polytechnischen Schule in Zürich*, with A Jehgher and H Paur, Zürich
- 1894: Über den Cauchyschen Fundamentalsatz in der Theorie der algebraischen Gleichungen, *VNGZ* **39**
- 1895: Eine Autobiographie von Gotthold Eisenstein. Mit ergänzenden biographischen Notizen, *AGM* **7**
- 1895: Briefe von G. Eisenstein an M. A. Stern, with A Hurwitz, *AGM* **7**
- 1896: *Katalog der Bibliothek des eidgenössischen Polytechnikums in Zürich*, Zürich
- 1896: *Die Naturforschende Gesellschaft in Zürich 1746-1896*, Zürich
- 1896: Zur Theorie der Strahlensysteme, deren Brennflächen sich aus Flächen 2. Grades zusammensetzen, *VNGZ* **41**
- 1897: *Zum 80. Geburtstag von Friedrich Beust*, Zürich
- 1897: Über die Aufgaben und die Organisation internationaler mathematischer Kongresse, *talk at the 1897 ICM, Zurich*
- 1898: *Verhandlungen des ersten internationalen Mathematikerkongresses in Zürich, vom 9.-11. August 1897*, Teubner, Leipzig
- 1898: Über die Prinzipien der Variationsrechnung und die geodätischen Linien des n-dimensionalen Rotationsellipsoids, *VNGZ* **43**
- 1898: Zum hundertsten Neujahrsblatt der Naturforschenden Gesellschaft in Zürich, *Neujahrsblatt der NGZ*
- 1899: Die Unverzagschen Linienkoordinaten. Ein Beitrag zur Geschichte der analytischen Geometrie, *AGM* **9**
- 1901: Georg Heinrich von Wyss (1862-1900), *VNGZ* **84**
- 1902: Der Bericht des Simplicius über die Quadraturen des Antiphon und des Hippokrates, *BM* **3**

- 1902: Zur Kubatur des Rotationsparaboloides, *SZ* **47**
- 1903: Contributions to A Kaegi, *Griechische Schulgrammatik*, 6th edition, Berlin
- 1903: Zur Rehabilitation des Simplicius, *BM* **4**
- 1903: Walter Gröbli (1852-1903), *SBZ* **42** (1)
- 1905: Die Mündchen des Hippokrates, *VNGZ* **50**
- 1905: Notizen zu dem Berichte des Simplicius, *VNGZ* **50**
- 1905: Nachtrag zu der Abhandlung „Die Mündchen des Hippokrates“, *VNGZ* **50**
- 1905: Sur l’histoire des conchoïdes, *Mathesis*
- 1905: Wilhelm Schmidt, *BM* **6**
- 1907: Talk on M C P Schmitt, Kulturhistorische Beiträge zur Kenntnis des griechischen und römischen Altertums, *Berliner Philologische Wochenschrift* **27**
- 1907: *Urkunden zur Geschichte der Mathematik im Altertume 1: Der Bericht des Simplicius über die Quadraturen des Antiphon und des Hippokrates. Griechisch und Deutsch*, Teubner, Leipzig
- 1907: *Speech at the bicentenary of Euler’s birthday*, University of Basel
- 1907-08: Proposal of the Euler project to the *Schweizerische Naturforschende Gesellschaft*, *VSNG* **90** and **91**
- 1908: Die angebliche Kreisquadratur bei Aristophanes, *BM* **8**
- 1908: Leonhard Euler (reprint on 1883 talk), *VNGZ* **53**
- 1908: Fritz von Beust (1856-1908), *VSNG* **91**
- 1908: Georg Sidler (1831-1907), *VNGZ* **53**
- 1908: Friedrich Hultsch (1833-1906), *BM* **8**
- 1908: Nachruf auf Friedrich Hultsch, in: *Verhandlungen der 49. Versammlung deutscher Philologen und Schulmänner in Basel 1907*, Leipzig
- 1908: Notiz zur griechischen Terminologie, *VNGZ* **53**
- 1908: Talk on J L Heiberg and H G Zeuthen. Eine neue Schrift des Archimedes, *Deutsche Literaturzeitung*
- 1910: Die Herausgabe der sämtlichen Werke Leonhard Eulers, *Internationale Wochenschrift für Wissenschaft, Kunst und Technik* **4** (3)
- 1910: Redaktionsplan für die Eulerausgabe, with A Krazer and P Stäckel, *Jahresbericht der DMV* **19**
- 1910: Einteilung der sämtlichen Werke Leonhard Eulers, with A Krazer and P Stäckel, *Jahresbericht der DMV* **19**
- 1911: Contributions to A Kaegi, *Griechisch-Deutsches Schulwörterbuch*, 13th edition, Leipzig
- 1912: Jakob Amsler (1823-1912), with A Amsler, *VNGZ* **57**
- 1912: Mitteilungen über die Eulerausgabe, in: *Proceedings of the fifth international Congress of Mathematicians*, Vol. II, Cambridge
- 1914: Zur mathematischen Terminologie der Griechen, in: *Festgabe für Hugo Blümner*, Zürich
- 1923: Paul Stäckels Verdienste um die Gesamtausgabe der Werke von Leonhard Euler, *Jahresbericht der DMV* **32**
- 1925: Die Petersburger Akademie und die Schweiz, *NZZ* **1348**

Furthermore:

1901-1921: Notizen zur schweizerischen Kulturgeschichte, with C Schröter, *VNGZ* **1-54**

1905-1914: Kleine Bemerkungen zu Cantors „Vorlesungen über die Geschichte der Mathematik“, *BM* 6, 7, 8, 9, 14; *Jahresbericht der DMV* 31

From 1907 onwards: Die Eulerausgabe (reports on the progress of the Euler project), *VNGZ*

Numerous articles in *NZZ*, *SBZ*, *VSNG*, etc.

Numerous articles, letters, and pamphlets regarding the Euler project

Editor of the following volumes of *Leonhardi Euleri Opera omnia*:

1915: *Commentationes arithmeticae*, Vol. I, Lipsiae

1917: *Commentationes arithmeticae*, Vol. II, Lipsiae

1921: *Commentationes algebraicae ad theoriam aequationum pertinentes*, with A Krazer and P Stäckel, Lipsiae

1922: *Introductio in analysim infinitorum*, with A Krazer, Lipsiae

1925: *Commentationes physicae* I, with E Bernoulli, R Bernoulli, and A Speiser, Lipsiae

Appendix D – Friedrich Robert Scherrer (1854 – 1935)

Friedrich Robert Scherrer was born on 16 May 1854 in Schaffhausen. He was the oldest son of Johann Friedrich Scherrer, who owned a soap factory, and Pauline Knubler. Scherrer grew up in his hometown with five siblings, and attended school there. After two years in the Realschule he attended the classical track of the Gymnasium Schaffhausen for two further years, before switching to the science track, ‘so as to make faster progress in the exact sciences’ [2].

In 1871 he matriculated at the Polytechnic in Zurich, more precisely at the Department for Mathematics and Physics Teachers. Two years later he moved to Strasbourg, continuing his studies at the university¹ there. As he writes in his résumé, he particularly enjoyed ‘Christoffel’s lectures on Abelian functions and their applications’ [2].

Scherrer returned to Switzerland in 1876 as he was appointed as a mathematics teacher at the Kantonsschule in Thurgau. He also had an offer to teach at a Realschule in Strasbourg, but ‘could not resolve to swear an oath of loyalty to the German Emperor’ [2]. In 1899 he took up a mathematics post at the teachers’ college in Küsnacht, where he stayed for the remainder of his working life. He served as the college’s deputy director from 1900-1911, and as director from 1922-1926.

Scherrer also pursued a respectable military career, which culminated in a position as commanding officer of the mobile artillery along the Gotthard. In addition, he held a lectureship in the Polytechnic’s Military Department from 1893-1896.

He wrote a number of papers, mainly on topics related to his teaching duties. Examples include *Die Fassung des Begriffes der Wurzel im Schulunterricht* (1903), *Die Struktur der Heronschen Dreiecke* (1916), and *Zur Methodik der Kreisberechnung* (1929)², all published in education journals. Some of his papers appeared in more research-focused journals, e.g. *Über ternäre biquadratische Formen* (1881), *Die Wurfbewegung im leeren Raum, synthetisch behandelt* (1931), and *Die Kreis- und die Hyperfunktionen. Eine Vergleichung auf geometrischer Ebene* (1932)³. In addition, he published collections of mathematics problems, for example with R Gerlach (1930). For the International Commission on Mathematical Instruction, founded at the Rome ICM in 1908, Scherrer wrote a report *Der mathematische Unterricht an den Lehrer- und Lehrerinnenseminarien der*

¹ In Scherrer’s obituaries, the university is described as ‘newly founded’. In fact, the University of Strasbourg has a long history, having received university status already in 1621. After the Franco-Prussian War however, it was re-founded as a German university in 1872 and renamed Kaiser-Wilhelms-Universität Strassburg in 1877. After the First World War it became a French university again.

² *The Definition of a Root in School Education* (1903), *The Structure of Heron’s Triangles* (1916), and *On the Methodology of Circle Computation* (1929)

³ *On Ternary Quartics* (1881), *The Pitch Movement in Empty Space – a Synthetic Approach* (1931), and *Circle Functions and Hyperfunctions. A Geometric Comparison* (1932)

Schweiz (1912)⁴. Furthermore, Scherrer edited Vol. 14 on ballistics of the second edition of Euler's *Opera Omnia* (1922).

In 1924 the University of Zurich awarded Scherrer an honorary doctorate in recognition of his achievements.

After his retirement, Scherrer 'devoted [his] remaining time and strength to sorting the estate of the former university professor Dr Geiser, a task that he was able to finish' [1]. Furthermore, whenever he was in Schaffhausen, 'he insisted on visiting his old friend and colleague, Professor Dr Gysel, and they spent blessed hours of not only true friendship, but also vivid mathematical discussion together' [ibid.].

Friedrich Robert Scherrer died on 01 January 1935.

References:

[1] Obituary of F R Scherrer in *Schaffhauser Tagblatt* **8**, 11 January 1935

[2] F R Scherrer, *Lebenslauf von Friedrich Robert Scherrer von Schaffhausen und Neunkirch (Kt. Schaffhausen)*, transcription of an autography submitted to the University of Zurich on 28 March 1932

⁴ *Mathematics Instruction at Teachers' Colleges in Switzerland* (1912)

Appendix E – Translations

E.1 Material relating to the 1897 ICM¹

E.1.1 Letter from C F Geiser to fellow mathematicians in Zurich²

Zurich, 16th July 1896

Dear Sir!

As you will know, it has already been suggested several times to unite the mathematicians of different countries at an international congress, which would have to be repeated at appropriate intervals. Recently, it has been proposed specifically (notably by Messrs Weber in Strasbourg and Klein in Göttingen) that a first such meeting should be held in Zurich in 1897.

The executive committees of the German Mathematical Society and the Société mathématique de France approved this project & the presidents of the aforementioned societies contacted me to that effect.

I hereupon arranged for the potentially necessary introductory steps to be taken by the Zurich mathematicians & invite you to attend a preliminary meeting on Tuesday, 21st July, 5pm, in the conference room 10^C of the Polytechnic.

Yours respectfully
Geiser

E.1.2 Welcoming Speech by Adolf Hurwitz

Sunday 08 August 1897, 21:00, Tonhalle Zurich

Dear foreign colleagues!

Please allow me to cordially welcome you with a few words on behalf of the mathematicians in Zurich. Many of you have rushed here from afar, following the call that we have sent out to all countries in which mathematical hearts beat. We are exhilarated by the strong response to our call: close to 200 peers followed our invitation and have gathered here for joint serious work and merry, comfortable get-togethers.

It is true that the great thoughts of our science were created and developed in a scholar's quiet room in most cases; no other science, with the exception of

¹ The German originals of the speeches were published in the congress proceedings: Hurwitz, p. 22-23; Geiser, p. 24-28 and p. 60; Rudio, p. 31-37.

² In Hs 637: 1, part 2, ETH Library Archive

philosophy, has such a brooding and hermitic character to it like mathematics. But still, a mathematician also feels the need to communicate and discuss with peers. Surely each one of us has already experienced what an inspiring power is inherent in personal scientific communication.

May this inspiring power of personal contact prove itself in these days as well, where we are offered so many and diverse opportunities for scientific discussion.

May we also enjoy the cheerful and informal company of our peers, enhanced by the knowledge that representatives of various nations feel connected in peace and friendship by the most ideal interests.

Once again, my dear peers, I call out to you:

Welcome to Zurich!

E.1.3 Opening Speech by Carl Friedrich Geiser

Monday 09 August 1897, Auditorium of the Polytechnic

Dear attendees!

I warmly welcome you all on behalf of the society formed by peers from various countries, which issued the invitation to the first international congress of mathematicians. In particular, I am delighted to welcome you on behalf of my colleagues from Zurich. To them, the fact that so many of you have appeared guarantees that you have met our request to meet in our town with kind and approving acceptance. Admittedly, we were very apprehensive when it was first suggested that we take on the congress. However, we told ourselves that the location of Zurich at the crossing point of the big routes from Paris to Vienna and from Berlin to Rome would forward the success of this endeavour considerably. Moreover, we chose these festive days to be in a time in which Switzerland already is a major gathering point of those who seek tranquillity and recreation, courage and strength for new chores. Thus it will be tempting for you to spend a few days or weeks in the invigorating proximity of our cascading brooks and rustling fir trees, in the tranquil view of our blue lakes and green mountain pastures or right amongst the rough rocks and cold glaciers of our high mountains after the efforts of our joint work.

According to the simple customs of this country and the – after all still basic – conditions of the town, we cannot offer you much in terms of exterior decoration and glamour at our meetings. So, from this point of view, we should scarcely have dared to inaugurate the series of the international congresses of mathematicians. Therefore, please do not misinterpret it as immodesty that instead we commemorate the part Switzerland has played in the development of the exact sciences in the last few centuries in the artistic

decoration of the identification card³. As it were, by showing you our great mathematicians we put our meeting under the protection of these powerful spirits.

You can see the three greatest of the wonderful Bernoulli family. In the centre we have Jakob, on whose withdrawn and forceful features there still seems to be a remote and belated reflection of the iron-filled and gunpowder-blackened times of the Thirty Years' War. To his right Johannes, who turns away from brother and son in the proud self-confidence of a *Roi soleil* of science. To the left Daniel, whose suave and likeable features confirm everything his contemporaries convey to us about his modesty and his courteousness. It has been said about Jakob and Johannes that they have contributed more to differentiation and integration than their creators. The history of the kinetic theory of gases, of the mechanic theory of heat, of the principle of the conservation of energy mentions Daniel among the greatest mathematical physicists of all times.

Next is Leonhard Euler, who in the middle of the last century took a universal position in our sciences, similar to Voltaire's position in literature. His significance cannot be illustrated better than by the fact that the Parisian Academy of Sciences quite extraordinarily elected him as one of its external members in 1755. This was at a point when all of the eight positions designated by the constitution were filled. "L'extrême rareté de ces sortes d'arrangements est une distinction trop marquée pour ne pas Vous en faire l'observation", Minister d'Argenson wrote to him. Let us bring to mind that, when the eight *Associés étrangers* were nominated for the first time in 1699, the two Bernoulli brothers were elected alongside Newton and Leibniz; and let us add that later on Euler was one of these foreign academics at the same time as Daniel Bernoulli and Albrecht von Haller. Then we may regard it as a consoling turn of fate that, in times of the inexorably proceeding political decline of the old Confederation, the scientific significance of Switzerland was on an unequalled high – admittedly a high that since has not been reached even from afar.

Our century is represented by Jakob Steiner, the sovereign in the realm of synthetic geometry. In today's meeting I can spot men who sat at the feet of the master; and one of my most important personal memories is having stepped into the magical circle of this unforgettable man. Yet his figure is already surrounded by a legendary sheen, as if he was separated from us by centuries. In our memory he lives on as the bold shepherd lad who successfully mingles with the greatest minds of science. Thus he appears to be a worthy son of the people, of which it says in Tasso's *Gerusalemme liberata*:

³ The identification or congress card was issued to all congress attendees upon registration and served as a "ticket" to lectures and talks (the organising committee issued special coupons for the social events and dinners). The front side showed the congress poster that Geiser describes in his speech, with portraits of the three Bernoullis, Euler, and Steiner above a picture of the Polytechnic, all adorned by alpine flowers. The sub-committees were listed on the back of the card, with the names of the respective presidents, who wore coloured ribbons/badges for identification: Geiser (organising committee): red and white ribbon; Hurwitz (reception committee): white ribbon; Rudio (board and lodging committee): blue and white ribbon; Herzog (amusement and decoration committee): green ribbon; Gröbli (finance committee): yellow ribbon [Hs 637: 1, part 3, ETH Library Archive].

E con la man che guidò rozzi armenti
Par che I regi sfidar nulla paventi.

One of the noblest creations of Gottfried Semper, the central block of the Polytechnic, forms the architectural completion of our series of portraits. Not only do we want to provide you with a memento of the place where your scientific and business discussions will take place. At the same time we would like to draw your attention to the importance of the polytechnics for mathematics and its applications nowadays. In a report on the development of mathematics at the German universities, which was written for the world exhibition in Chicago, the influence that the foundation of the Parisian Polytechnic School had on research and teaching is described. It is furthermore pointed out how, since the middle of our century, scientifically brilliant mathematicians have been appointed to German-speaking technical educational establishments. Announcements of this kind let us hope that old, unsubstantiated prejudices would disappear little by little and that the full equality of all institutions of higher education would be accepted more and more. But more recent movements show that, in certain circles, the duties of higher technical education do not seem to be fully clarified and standardised yet. On one side, there are loud voices from the practical side: mathematics is granted too much importance. The other side demands, no less resolutely, that the final and highest education of technicians should be reserved for universities.

We gratefully acknowledge that the congress wants to dedicate some of its work to these important questions. A distinguished technician will rise to speak about this topic⁴, and without a doubt we will hear a most qualified theorist talk about these questions as well. But whatever the conclusions in talks and discussions may turn out to be: affirmative, restrictive or dismissive – they will not vitally influence the natural way that the polytechnics will have to follow. For students and teachers, for the practising technician and the scientist in the area of pure science there is only one choice: a lasting success can only be achieved by him who tirelessly strives for the highest goal with all his soul.

Dear assembly!

You will not expect that in my opening speech I talk about the duties and the uses of mathematical congresses in great detail; this will still happen today. Just allow me a short concluding remark.

Surely none of us will believe that in future the solution of great problems in science will be the result of such meetings. Though they may seem to be thoroughly objective truths, the highest accomplishments in all intellectual fields bear a quite personal stamp, which is only blurred and damaged by external interference. Who doesn't think about legendary treasures that have to be silently retrieved by innocent hands when thinking about Riemann's creations, developed in tranquil solitude? And don't Weierstrass's

⁴ Geiser refers to the talk of his colleague Aurel Stodola, *Über die Beziehungen der Technik zur Mathematik* ("On the Relationships Between Engineering and Mathematics").

fundamental papers reflect the man's magnificent simplicity, independence and completeness?

But extensive and abundant fields remain accessible to collaborative work; areas of research that can be cultivated purposefully and successfully only through the simultaneous utilisation of numerous forces. And the effects of such gatherings do not remain constricted to the inner circle of the immediate participants. The noble competition in selfless dedication to an ideal goal also encourages others to similar efforts.

Especially in our country, the representatives of intellectual life are open to and grateful for any suggestions. Every day reminds us how small our regional borders are. When the sun descends into the depths of the eastbound valleys of Grisons in the morning, it already lights up the ravines of the Jura, through which the Rhone pushes when leaving Geneva. And when the last ray of sunlight disappears from the peaks of the Bernina group, then the giant snow-capped mountains that surround the high valley of Zermatt go pale, too. By engrossing our minds in your papers and their results with respectful attention, we liberate ourselves from spatial and temporal boundaries. We gain an intellectual citizenship in an empire of infinite dimensions: it is the empire of science, of which it can be said in a higher and nobler sense than of the empire of Karl V, that in it the sun never sets.

E.1.4 Closing Speech by Carl Friedrich Geiser

Wednesday, 11 August 1897, Auditorium of the Polytechnic

Dear attendees!

We have exhausted our agenda and no further items have been registered for consideration. Thus, there is nothing left for me to do but to close today's second general meeting and hence the official part of our congress on behalf of the committee appointed by you. Indeed, a few hours of merry sociability will still provide us with manifold occasions to amicably exchange ideas about the success of our work. However, please allow me to joyfully express the thought that is currently on all our minds already now: the possibility of uniting the mathematicians from many different countries for interesting and fruitful meetings as well as for lively personal contact has been displayed and thus the future of the international congresses of mathematicians is secured. And if, at the end of this lovely day, I call out a cordial farewell to you all on behalf of my colleagues in Zurich, then I may also assume to speak in accordance with the kind invitation of our peers from France when I add:

Auf Wiedersehen in Paris – See you in Paris!

E.1.5 Über die Aufgaben und die Organisation internationaler mathematischer Kongresse by Ferdinand Rudio

Monday 09 August 1897, Auditorium of the Polytechnic

Dear Assembly!

I have the honour to address you with a few words on the duties and the organisation of international congresses of mathematicians on behalf of the organising committee. Of course, you will not expect that we stand here before you with an elaborately worked out programme already now. After all, today's task is to lay the foundations of a project, and the future will show which fruits this project will bear. However, we may look forward to that future with confidence. We are entitled to do so by the great interest with which the invitations to an international congress of mathematicians were met by peers of various countries. In particular, we are entitled to do so by the grand assembly that has gathered for joint work in the auditorium of the Federal Polytechnic today.

Dear attendees, please allow me to draw your attention to a few organisational questions first. In your hands you are holding "regulations" devised by the committee, as well as "resolutions" that will be subjected to your judgement. The articles treating organisation in the regulations concern mainly the rules of procedure of this year's congress and need not be touched at this instance. Therefore, I will immediately turn to the resolutions, which I want to present to you one by one.

Resolutions of the International Congress of Mathematicians in Zurich, 1897

I. Henceforth, international mathematical congresses shall be held in intervals of 3-5 years and in due consideration of the various countries.

II. In the closing ceremony of each congress, the dates and place of the next congress as well as its organising and inviting bodies shall be designated.

III. Should any circumstances make it impossible to hold a congress on the designated dates and in the designated place, the executive committee of the previous congress is authorised to make the necessary arrangements for calling a new congress, as may be the case. For this purpose, it will also contact the bodies defined in resolution II.

IV. For tasks of an international nature, whose solution requires a fixed organisation, each congress may nominate a permanent sub-committee, whose period of office lasts until the next congress.

The responsibilities and liabilities of such sub-committees shall be determined every time such a sub-committee is nominated.

V. The next congress shall be held in Paris in 1900. The Société Mathématique de France is commissioned with its preparation and organisation.

Dear attendees, these are essentially the standards with which the next international congresses of mathematicians shall comply. They are kept as

simple and transparent as possible on purpose, and should suffice for the beginning.

But now, what are the problems that can be expected to be solved by these international congresses? A sketch, but really just a sketch of these problems is contained by article 1 of the regulations with which you have been presented. To begin with, it says there:

“The congress has the purpose of furthering the personal relations between the mathematicians of various countries.”

Dear attendees! If you glance at the programme, if you have a look around in this hall, then you cannot help but thinking that the international congresses of mathematicians would have a right to exist even if their only purpose was to bring the mathematicians of all countries of the Earth closer together, to give them opportunities to exchange ideas with each other; but also opportunities to communicate in a friendly way as it is brought about by the pursuit of common ideals. Fostering personal relations and the resulting direct and indirect advancement of science will always form an essential point in the programmes of national and international science societies.

But let us not remain here. In article 1 of the regulations it continues as follows:

“The congress has the purpose of providing, in the talks of the main assemblies and of the section sessions, an overview of the current state of the various fields of mathematical sciences and their applications, as well as the treatment of individual problems of particular importance.”

Ladies and Gentlemen! Shall I explicitly point out how completely different the spoken word appears compared to the written or printed word, how a display only gains shape, colour, warmth, in a nutshell: life, through the personality of the speaker? It should be unnecessary to linger on at this point and in front of this audience.

But precisely the talks that can be expected for the main assemblies of our congresses, already give reason to very specific, well-defined areas for international activity. These talks will, in most cases, naturally be clearly arranged presentations on the historic development and the current state of individual areas of science. Now should it not be possible to group these presentations together according to certain aspects and distribute them in a suitable manner among the mathematicians of various nations so that they can work on them systematically? All we would do would be to follow the example which has been given by the German Society of Mathematicians for several years, and which is represented by the works of Messrs Brill and Noether, Franz Meyer and others, on an international level. By doing this, we would attain a systematic sequence of historic monographs within a short amount of time. At the same time – I am following an idea of Mr Eneström here – we would arrive at a methodical sequel of the grand opus that Mr Moritz Cantor is about to finish with the year 1759, and whose continuation should be far beyond the strength of one individual.

Viribus unitis! shall be our watchword. With united forces it shall be possible to solve problems, which could not even have been attempted until now due to a lack of cooperation. When asked to give an example, please give

me, on Swiss soil, credit for thinking about publishing the works of Leonhard Euler, for example. This is an obligation of honour, which until now could not have been fulfilled by the mathematical world. As is generally known, an important precondition for setting to work on this mammoth task has been fulfilled now, after our American colleague Hagen published a complete list of Euler's works last year. You also know from the communications of Mr Hagen that publishing these works no longer counts as a utopia; in fact, that maybe all it needs is international moral support.

Of course, I wanted to mention only one example for joint literary undertakings here. Further examples, though of a completely different nature to the one mentioned, could be added by using the manifold suggestions with which the committee was approached from various sides. I will mention, purely objectively and without giving my personal opinion: the possibly annual publication of an address book of all mathematicians in the world, including a declaration of their particular research interest; the publication of a biographical-literary dictionary of all currently living mathematicians with their portraits; the publication of a mathematical literature journal.

Furthermore, one could think of holding international scientific expositions, for example following the model of the nice exposition that took place under the aegis of Mr Dyck in Munich in 1893.

Among the literary undertakings, one has to be pointed out in particular. Article 7 of the regulations mentions the printing of the proceedings of the congress. There is no doubt that this and corresponding similar publications will contribute significantly to furthering our collaboration and to raising the feeling of togetherness among the mathematicians.

Also, we received suggestions regarding questions of terminology and of international agreement on the choice of certain mathematical units. Similar to international meetings where people came to mutual consent regarding the most important physical units like Volt, Ampère, Ohm; one should aim for an international agreement on the partition of angles, for example. As is well known, it was attempted in Germany and France to switch over to decimal partition of angles, which of course provides significant advantages for calculations. But now this has created an inequality: some keep the old degrees and divide only them decimally, others want to decimally and centesimally divide only the quadrants, and again others the entire periphery. It is therefore said to be a task of international communication to eliminate the disparity prevalent in more recent tables by deciding on uniform angular dimensions.

Of course, it cannot be the intention of my presentation to get lost in details. Therefore, I will confine myself to discussing only one more point in particular; though it should currently be by far the most important one. The most important one, because it concerns a question that has become urgent and that demands to be tackled energetically. I mean the question of mathematical bibliography.

Completely leaving aside for now what has been done in this area so far and what is still being done, and deliberately not mentioning which institutes currently work on these papers for the time being, I just want to quickly characterise the goal that should be aimed for.

What is of the utmost importance to any science, not just to mathematics, due to today's enormous productivity and the associated literary fragmentation, is an efficient and continuous bibliography.

The purpose of such a bibliography is, amongst others, to give anyone who is interested in any field, say number theory, information about everything that has been published in this field anywhere in the world, not just in the last few years, but in the last few months and weeks. This is done by exactly copying the titles of the publications. Such a bibliography for a certain science can only be provided by an international institution, i.e. through international collaboration. This can only be done successfully when a universally approved classification for the respective science exists. Now, dear attendees, unfortunately we do not yet have such a universally approved classification accepted by all peers in mathematics. However, we do have a number of classifications that all work excellently in their own ways. May I remind you of the classification of the Parisian bibliographic congress that was held in 1889 under the presidency of Mr Poincaré, whose absence we regret so much today. May I remind you of the classification of the annual on the progress of mathematics, published by Mr Lampe, of the classification of the universally oriented Dewey decimal system, and of several others. But we will reach the ideal of a bibliography that uniformly satisfies the needs of both scholars and librarians in the whole world only by a uniform, universally approved classification.

Dear Assembly! If anywhere, then we have an important and grateful task of international communication at hand here. Besides, this task is already alleviated by the fact that two powerful and highly prestigious institutes have already turned their attention towards this problem: the Institut International de Bibliographie in Brussels and the Royal Society in London. The former recently held its second international conference. The international congress of mathematicians cannot and must not watch the work of these institutes idly and indifferently. It still has the opportunity to participate in this, and may I add that this participation will only be welcomed by both institutes. But even if the international congresses of mathematicians would want to proceed independently and create their own institute, which would not be particularly difficult, they would not, in principle, rival the two mentioned institutes for some time yet. A proof for this is given by the international Concilium bibliographicum, which has been thriving in Zurich for two years and which implements the mentioned ideals in zoology under the direction of Mr Field.

Dear attendees, I can abstain from dwelling further on this topic or even going into details, as Mr Eneström will give a talk on the latest mathematical-bibliographical developments in the section for history and bibliography. As I learned just recently, representatives of the Institut International de Bibliographie and of the Royal Society will also attend this session. We can therefore assume that a certain proposal concerning mathematical bibliography will emerge from this section, which would be put to you in the second main meeting.

Dear Assembly! I have come to the end of my presentation. As I have remarked already, it cannot claim to be complete in any way or direction. I simply wanted to show that alongside the great interest that the international meetings of mathematicians have in themselves, there also exist tasks that are worth a joint effort. The more firmly the ties that shall unite us from now on

are established, the more such tasks will naturally present themselves over the course of time.

May the work, for which we lay the foundation here in Zurich today, be a worthy member in the series of great international creations! May it contribute alongside them to unite not only the scholars of all nations, but also the nations themselves for joint cultural endeavours!

E. 2 Letters

Spellings, underscores and dates as in the originals.

E.2.1 Letters from Carl Friedrich Geiser to Julius Gysel (1874 – 1890)¹

My dear Mr Gisel [sic!],

I am returning your dissertation, which I find very neat, apart from a few minor editorial details. A shorter version wouldn't do any harm – but it will only be later that you will learn, by practising, how to be succinct in your writing without becoming incomprehensible.

I am not making any comments on purpose; the dissertation should be nothing but your own work. And Schläfli will probably pour the lye of his criticism over it.

Now hurry up with your doctorate so that people can see your competence and, if I may say so myself, that you have learnt something from me in particular. I am very pleased indeed that you have been my student & have now stepped forward with such a neat achievement.

Your
Geiser

Best regards to Schläfli.

4 June 74

Kind regards from my wife.

Dear Mr Gysel,

Although I am not actually working, I have, nevertheless, so many things to think about that until now I have always failed to answer your letter of the 8th of this month. If you haven't acted of your own accord since then, contact Olivier immediately so that he can tell you all the necessary steps required by the faculty. As for printing the paper, this is the job of Professor Wolf who edits the Zurich *Vierteljahrsschrift*. But why do you not want the paper to be presented to the Bernese *Naturforschende Gesellschaft* by Schläfli? After all, the society should be glad to add such a paper to their communications.

I am extremely happy that you get along so well with Schläfli. If you increasingly get the feeling that there is an infinite amount to learn from him, then this will be good for your self-awareness. Give him my warmest regards & best regards to yourself from my wife & me.

Your

¹ In D I.02.521*.04/0155

C. F. Geiser

Zurich, 28 June 1874

My dear Mr Gysel,

I have refrained from answering your cry for help, as I will come to Bern myself next Saturday in order to cheer up your frightened heart. Admittedly, for the time being you have to try to get back your post in Schaffhausen. If that is not possible, then the path indicated by Schläfli is such a honourable one that you definitely have to follow it. I will enquire of the Director of Education if it wouldn't be possible find something for you in the event of your habilitation. Incidentally, the old god is still alive & will not leave such a successful Schaffhausen Babbitt like you high and dry.

Your old
Geiser

Zurich, 24 Febr. 75

On Saturday, I will arrive in Bern on the morning express train.

My dear Mr Gysel,

In an advertisement in the N.Z.Z.² I have read about the vacancy of the mathematical professorship at the Academie in Lausanne (with a salary of 3600 fr.). Wouldn't you fancy putting your name forward? Even if you don't have a chance of success, people will become aware of your name & you won't lack glowing recommendations from Schläfli & myself. As for the little bit of French, you will easily learn that if you get accepted. And if you succeed, you have taken an important step forwards. (If you want to talk to me on this subject, meet me on Sunday here in Zurich).

In great haste
Your
C. F. Geiser

2nd July 75

NB: May I ask you to do the correction as quickly as possible.

² *Neue Zürcher Zeitung*

My dear Mr Gysel,

The other day I had an occasion to read your problem again & have found out that the elimination of λ and μ does not belong to the realms of impossibility after all. If you put

$$x^2 = \xi, y^2 = \eta, z^2 = \zeta, a^2 = \alpha, b^2 = \beta, c^2 = \gamma$$

$$(\beta - \gamma)(\gamma - \alpha)(\alpha - \beta) = D, \lambda + \mu = s, \lambda\mu = p$$

Then the three original equations become linear equations in ξ, η, ζ , which if you solve them become:

$$\xi = -\frac{\beta - \gamma}{DN} (p + \alpha s + \alpha^2)^2 \{ps + 2(\beta + \gamma)p + \beta\gamma s\}$$

$$\eta = -\frac{\gamma - \alpha}{DN} (p + \beta s + \beta^2)^2 \{ps + 2(\gamma + \alpha)p + \gamma\alpha s\}$$

$$\zeta = -\frac{\alpha - \beta}{DN} (p + \gamma s + \gamma^2)^2 \{ps + 2(\alpha + \beta)p + \alpha\beta s\}$$

where $N = 2p^2 + (\alpha + \beta + \gamma)ps + 2(\beta\gamma + \gamma\alpha + \alpha\beta)p + \alpha\beta\gamma s$.

Now we can construct the left hand sides of the equations, which represent the three surfaces of degrees four, six & six through the triple curve (you've still got my paper on this). This has to result in expressions in $p+s$ from which the elimination of these quantities becomes possible. You can also do everything homogeneously, i.e. work with four coordinates. Here we would be well advised to introduce a homogeneous variable for $p+s$. Possibly the easiest way to do this is by putting

$$p + \alpha s + \alpha^2 = u, p + \beta s + \beta^2 = v, p + \gamma s + \gamma^2 = w$$

In any case we can now easily find curves that belong to the locus that we were looking for (which is of degree at most 32, according to the equations for ξ, η, ζ). Put $p = 0$ in order to get four central sections of the original ellipsoid. For $s = 0$ we get four central sections on another ellipsoid that belong to the surface of degree 32. In infinity there are 4 straight lines. For $u = 0, v = 0, w = 0$ we can find parts of the cut with the principal planes etc. etc. Incidentally, we can find the cut with the original ellipsoid by direct elimination. There exists, as well as the degree of the triple curve an old, admittedly feeble glimmer of hope that the surface may not exceed degree 12. –

The wine from your father-in-law, Mr Bollinger, hasn't arrived yet. However, when reading his letter I noticed to my horror that he has filled the very barrel, which, according to the waybill of the last delivery, has a broken stave. As for the superior variety, Mr Bollinger wanted to send a keg of 60-70 litres along with it. May you be so kind as to ask him on my behalf that he despatches such a wine.

With best regards to you & your family

Your
C. F. Geiser

Zurich, 16th March 1877

Zurich, 6th March 1878

My dear Mr Gysel,

Since you didn't seem to be very happy with your revenues as teacher at the Gymnasium Schaffhausen recently, I have seized the opportunity when it presented itself and recommended you as teacher of physics, mechanics and descriptive geometry at the Kantonsschule in Pruntrut. This has happened in a way that I have reason to hope (although it isn't absolutely certain) that you will be approached.

Thus, the opportunity is provided that you will either improve [your salary] in Schaffhausen or move on to a better-paid position. In their letters they mentioned only 3400 fr., but I talked to them in such a resolute manner that you will not be taken for immodest if you ask for 4000 fr.

I wanted to inform you of this circumstance as quickly as possible, but may I ask you to keep this confidential. After all, it would be possible that the Bernese government won't take the bait.

Your
C.F. Geiser

Dear Gysel,

Today I will send an empty barrel back to your Mr Father-in-law & ask him through you to please send it back to me with a suitable kind of wine, the choice of which I leave to him (60-70 cs. per litre). The empty barrel was left in the cellar longer than intended. It therefore requires a very thorough cleaning, all the more as the last wine got a taste of wood sooner than the other deliveries.

Furthermore, I would like to suggest that you deliver the by now quite swollen bill of your father-in-law in person & receive the money for it. You have never been here in Küßnacht & you really have to get to know our current residence. Since the time when we have to meet Scherer (whom I haven't seen since his Parisian adventures) "has come" anyway, [meeting in] Küßnacht or Winterthur will be pretty much the same at the end of the day.

Take the train that arrives in Zurich at 11.44 am; you'll just arrive in time to take the boat at 12.05 pm which I will take myself in order to get home for lunch. I leave it to you to decide whether you want to come on Saturday in 8 days or in 14 days (4th or 11th Dec.; next Saturday, 27th Nov., would suit me, too). Though may I ask you to inform Scherer and myself by correspondence card in due time.

With warmest regards to you and all of your family from myself & my family

Your
C.F. Geiser

Küssnach-Zurich, 22nd Nov. 1880

My dear Mr Gysel,

Attachment I Concerning the problem of normals for the ellipse, it has been solved accurately, as shown by my earlier treatment which I have attached. Only the remark that G_∞ is touched in the points on the circle is wrong. They simply belong to the curve, but in the points of the ellipse at ∞ distance are double points as well as in the centre (4). Which are the other singularities?

Attachment II You have spotted correctly that the problem is harder when it comes to the ellipsoid. Sheet II shows how everything is reduced to the elimination of two unknowns from three equations. The problem has a certain similarity with the problem of finding the surface through the centre of curvature. [German Salmon, Vol. 1 p 288.] So we are well advised to find the sections in the principal planes first, which are of degree 12 (?). The intersection with the ellipsoid itself is of degree 24 (?), therefore we have sufficient reasons to conclude that the surface is of degree twelve (?). What is it for the spheroid in particular?

As there are two (or ∞ many) normals of the ellipsoid in each plane, we can look for those planes in which these normals enclose a right angle. The surface enclosed by them is (?) identical to the one we looked for & of degree four, therefore easier to examine.

Attachment III If we want three normals to be perpendicular to one another, the point of intersection also has to be on a surface of degree 6, which cuts out a triple curve on the previous surface.

In general, search for as many distinct elements as possible first & treat the question of degree and class last. (For example, we can easily find points on each normal).

Attachment IV For the altitude hyperboloid the theorem in the attachment is fundamental. With its help the problem has been solved. An analytic attempt, which hasn't been completed though and which may contain some mistakes,

Attachment V at least contains the key to the secret is enclosed as well.

Hereby you now have all the decipherable papers contained in my documents. If you would like to come here for a day during the Christmas holidays, I will prepare myself to give you some more material – right now I am lacking the time as I have very urgent business to attend to.

With best regards to you & your family

Your
C.F. Geiser

Wednesday noon

On an extra sheet of paper:

In order to know if the location of the points, from which two perpendicular normals extend to an ellipsoid, is identical to the envelope of the planes, in which there exist two orthogonal normals, we note that the equation of the envelope in the planar coordinates can easily be constructed. Let u_0, v_0, w_0 be the coordinates of such a plane. Then the osculation point can be found in the same way as the tangent plane in point coordinates. Now if this osculation point coincides with the point of intersection of the normals, which lie in the tangent plane, the conjecture is correct – otherwise not.

Dear Mr Gysel,

Within the next few days an empty wine barrel will arrive for your father-in-law. I leave it to his discretion to send me something similar in case he does not have any wine of the quality purchased by me last time anymore. Although the prices may rise as a result of the frost, I don't want to pay more than 70-75, at most 80 cs. per litre.

The debt register has now grown so much that I deem it absolutely necessary to pay again & I think it would be best if you came here with the receipted bill & collected the various balances. Actually, I want to see you again most urgently & therefore I suggest either next Saturday or Saturday in a week. You can stay the night here & we will then have the opportunity to go on a little outing from here. If you advise Scherer as well, he can come, too, we can put him up as well.

Today I will travel to Luzern to attend the opening of the Gotthard Tunnel, to which I have been invited by the Bundesrat. Cremona will arrive there today, too, so I can be sure that I will have the most agreeable company.

With best regards to you & your family from all of us

Your
C.F. Geiser

Küssnacht-Zurich
21st May 1882

Küssnach-Zurich 20th March 83

My dear friend,

We are still waiting in vain for any announcement by the Schaffhausen government regarding the conclusion of a contract that concerns admission of your pupils to the Polytechnic. Since, in my opinion, there are no fundamental

difficulties anymore & and it is of value to sort out some of the conventions by October this year, we would like to speed this matter up a bit: Couldn't you explain to your rector's office or to a member of the government respectively that it is in your school's interest to settle this business swiftly and definitively, as your new curriculum & the means needed for its implementation have already been regularised. Hopefully the preliminary discussions may provide the opportunity for me to come to Schaffhausen with Mr Kappeler and to spend some time with you – otherwise hardly any opportunities arise, as my official functions give me a lot of work. (On Saturday I will go to Frauenfeld with Mr Kappeler on the same mission.)

Will you please tell your father-in-law that the empty barrel will arrive in the next couple of days and should be sent back filled with a similar quality as last time, if possible.

With best regards to you & your family

Your
C.F. Geiser

My dear Mr Gysel,

Within the next few days your father-in-law will receive the well-known barrel, which he may return filled with a suitable wine (according to my taste & wallet). May I ask that the bill of lading is issued to Küsnacht (per N.O.B. steam navigation) explicitly, or else I will have to have the barrel being picked up especially from Zürich railway station & this only causes troublesome & unnecessary annoyance for me.

Speaking of annoyance, what you write to me about our unfortunate friend has unfortunately been communicated to me from elsewhere as well. However, I don't understand what I am supposed to do in this matter, unless either the authorities or Mr Scherer ask me to mediate. This would have a certain foundation in so far as I had a hand in his appointment at the time. Anyhow, you are closer to him with regards to age, profession, and ties of friendship, too, than I am & you can show him seriously & authoritatively the crisis that he is facing in its full dangerousness. If he then grumbles & rants about the Bierphilisten³ without coming to his senses, there is still time to call on me. Thus, write to him or arrange a meeting with him, which I shall not attend; together we can still preach him reason in Wintherthur [sic!] later on. Incidentally, he should understand on his own that a young husband is tied to a position more than a bachelor and therefore has to comply sometimes, even if he does not like to do this.

With best regards and congratulations that I don't hear any complaints about you at least I remain

³ Literally, "Beer Philistines", but I assume that the term has nothing to do with the Biblical Philistines. A "Philister" or "Alter Herr" ("Old Sir") is a member of a student corporation who has completed his studies, but still retains ties with the corporation. The term "Philister" is rather derogatory, implying an old-fashioned attitude.

Your
C.F. Geiser

NB: Do not believe that I have forgotten you. I very often had the need to see you and to talk to you, but the post gives me a lot to do and demands that at least I am around, also in less stressful times. Hopefully we will meet in Winterthur this month though, if you will just take the necessary steps.

Küssnach [sic!], 6th Nov. 83

Küssnach [sic!] 9th July 1884

Dear Mr Gysel,

I am so piled with work, lectures and business matters that I cannot even think of escaping to Winterthur. However it would be nice if we could see each other again & so I propose that you come to Zurich. You now have holidays & surely will be glad to drink a pint among friends & have a gentle conversation before the summer heat will arrive. Will you let know Scherer, too, please & come next Wednesday. It will be best if you pick me up at my office. Please bring your father-in-law's bill along with you as well; the debts have been troubling me for a long time now.

With best regards to you & your family

Your
C. F. Geiser

Thursday, 21 Jan. 1886

Highly confidential!!!

Dear Mr Gysel,

At the Gymnasium Zurich the post as mathematician, coordinated by Gröbli, is to be filled. In today's meeting, the supervisory board commissioned me to ask you if, in the event of a possible offer, you would be inclined to accept the same? Well-understood, this is just an enquiry that is mutually non-committal as various hitches are still possible. May I now ask you to come to Zurich on Saturday in order to discuss the matter with me face-to-face. Don't harbour any illusions whatsoever, but brace yourself for my encouraging you. A telegram to my office in Zurich on Saturday morning shall indicate when you will come and pick me up in order to go to Küssnacht for a meal. So see you the day after tomorrow.

Please bring your father-in-law's receipted bill with you, & apologise to him on my behalf for the delay. I will give you remaining sum to take with you.

Warmest regards to you & your family

Your
Geiser

On the margin: Don't speak to anybody about this matter!

Dear Mr Gysel,

Today an empty barrel will be dispatched to your father-in-law's address. Would you please ask him to have the same filled and returned to my address. It will probably be necessary that the cooper apply special care when cleaning it, in that a certain aftertaste of the last delivery might have possibly stemmed from the barrel.

I would like to propose another get-together in W'thur⁴ during the holidays, but work is a heavy burden on me, all the more as I have to travel to Geneva next week, which will again imbibe several days. Hopefully a suitable occasion for a reunion will arise at the beginning of the new semester.

Best regards to you & your family

Yours,
C. F. Geiser

K. Z. 26th March 1886

My dear Mr Gysel,

I won't be going to Schaffhausen for the New Year & therefore have to inform you in writing that you have been recommended the third section (chapters 6 and 7) of Vol. I of the Steiner lectures for revision. This is for the third edition. Would you have a closer look at the figures in particular, please, some of which may perhaps have to be replaced by better ones.

Now please accept our warmest regards & best wishes for the New Year for your family and yourself. In particular, please share my wish that henceforward we will see each other more often than was the case in the last two years.

Yours truly,
C. F. Geiser

NB: Maybe you are interested in the enclosed paper, which I ask to be returned on occasion.

Zollikon 26 Dec. 86

⁴ Winterthur

My dear Mr Gysel,

The first two sections of Steiner I are now revised completely & I am now starting to work on the third one, the one for which you had kindly promised your assistance. May I ask you now to provide me with your material (I already have § 22), or, alternatively, to forward it to me successively, as it is completed. As I want to go on a journey as soon as possible after the end of the semester, which takes place on the 19th, but have to send the manuscript to Leipzig before that, you'll excuse me for urging a bit.

With best regards to you & your family

Yours truly,
C. F. Geiser

Director of the Swiss Federal Polytechnic

Zurich 7th June 1887

Dear Mr Gysel,

Don't you want to bring me the paper with the analy. proofs of Steiner's theorems to W'thur on Thursday, please? I need it for the seminar on Saturday.

Goodbye.

Yours,
C. F. Geiser

Dear Mr Gysel,

Although my barrel probably knows the route to Schaffhausen by heart by now, I nevertheless ask you again, out of habit, to arrange for your Mr Father-in-law to send the same back filled to Zollikon steamboat landing. He knows the appropriate variety.

Are we going to see each other in Zurich or W'thur in the near future? But not next week though, as I have to be away from Zurich for the last three days.

With best regards to you & your family
C. F. Geiser

Zollikon 11 April 90

Zollikon 30 Oct. 90

Dear Mr Gysel,

The familiar empty barrel has been dispatched to your Mr Father-in-law again & shall return to me later with an appropriate filling. But admittedly probably only in spring, as the lease for the flat has been terminated with effect from 1st April & I currently have absolutely no idea where fate will take us. Hopefully Mr Bollinger won't be cross with me for preferring to leave the barrel at his instead of keeping it here.

When shall we see each other again in W'thur? With best regards to you & your family

C. F. Geiser

E.2.2 Letters from Carl Friedrich Geiser to Fritz Bützberger (1895 – 1907)¹

Hs 194: 158

Postcard addressed to Bützberger in "im Forst bei Bützberg (Ct. Bern)"²

Küsnacht-Zürich 3 Oct. 1895

I am at your disposal all day next Sunday.

Geiser

Hs 194: 159

Küsnacht 10. Nov. 95

Dear Mr Bützberger,

A few days ago I received the enclosed letter, which informs you about the events planned in Switzerland, in consideration of the forthcoming Pestalozzi celebration. I would be pleased to confer with Mr Fritschi on the subject of the suggestion that was put forward, should you wish so. Should you have reservations about entrusting your fine work to a non-mathem[atical] journal, contact Prof. Cantor in Heidelberg, so that the *Zeitschrift für Mathematik & Physik* includes the article. Do not hesitate to say that I asked you to send the paper if you think that this would have an impact.

Another possibility is that you had it published in the Pestalozzi celebratory issue first, & then arranged for a reprint in Hofmann's [sic!] *Zeitschrift für mathem. & naturw. Unterricht*.

Yours truly,
C. F. Geiser

¹ In: **Hs 194: 158-163**

² Italics in E.2.2 by the author

Hs 194: 160

Mr Fritschi accepts your paper for the Pestalozzi issue with pleasure & thanks. Please get in touch with him directly to discuss further details. His address is: Mr F. secondary school teacher, Neumünster-Zürich

Küsnacht 20 Nov. 95

C. F. Geiser

Hs 194: 161

Küsnacht-Zürich 23 Feb. 96

Dear Mr Bützberger,

In my reply to the enclosed card I explained to Mr Hofmann [sic!] that I could not provide him with the desired article. I further wrote that he should contact you in order to receive the paper on Steiner's education in Yverdon. I leave it to your discretion whether you want to wait for Hoffmann's request, or, for the sake of convenience, send him your paper directly.

Yours truly,
C. F. Geiser

Letter from Hoffmann to Geiser:

To Mr Prof Dr Geiser, Küsnacht-Zürich
Leipzig 18 II 96
Neustadt, Eisenbahnstrasse 57 I

Dear Sir,

Due to the centenary of the great Math. J. Steiner on 18th March [this year] I am approaching you with the request to write a short article for my journal, in order to renew the memory of this intellectual hero. However, should you not be able to do so, could you indicate a person who could effect this worthily. Or should I, in connection with your brochure *Zur Erinnerung an J. Steiner*, repeat the main passages of the same? In my opinion, this would be almost too prosaic!

I am looking forward to a favourable reply
Yours sincerely
Prof I. C. V. Hoffmann [sic!]

Hs 194: 162

Mrs Anna Barbara Begert née Steiner died in 1870. We do not know the more specific date.
K.-Z. 3 XI 07

G

At the end of the letter of 25. II. 55 to Schläfli (p. 128), Steiner writes: "Schlatter (=Post-Heiri)"³

Hs 194: 163

Tomorrow, Saturday, at 5 pm, you will find me in the reading room of the Polytechnic's Library.
Küsnacht-Zürich 12. XI. 07 C. F. Geiser

E.2.3 Letters from Ludwig Schläfli to Julius Gysel (1874 – 1888)⁴

21 August 1874, Bern

In the morning of the 22nd I will take the express train (at 6.30 am) to Zurich.
Kind regards

L. Schläfli

August 1874 [???

I will depart in the morning of 30 August (*remark on the letter/card: to Gravedona 1874*).

Kind regards to you and your parents

Yours, L. Schläfli

Prof Enneper from Göttingen, who spent about two days in Bern and whom I talked to about your dissertation, expressed the wish to receive a copy of the same.

10 September 1874, Gravedona (Como)

Dear friend!

Having arrived in Gravedona already on 3 September, I have refrained from writing to you so far. I appreciated your kind hospitality, of which I have fond memories. I assume that you will have received my letter from Bern. During

³ Whilst I have not been able to reconstruct what Steiner wrote about, the Postheiri was an independent journal (1847-1875), widely known for its caricatures. Cf. T Gürber, in *Historisches Lexikon der Schweiz*:
<http://www.hls-dhs-dss.ch/textes/d/D24777.php>, accessed 25/03/2014.

⁴ In: **DI.02.521*.04/0153: SCHLAEFLI Ludwig**

the voyage and in the first week of my stay here I had continuously nice weather; now it's raining here. Have you received my manuscript on spherical harmonics, and were you able to read it?

I hope that you will be as fit as a fiddle when you receive these lines. With kind regards to you and your parents

Your obedient servant L. Schläfli

(in casa di Sig^a Dubini)

Wednesday 7 July 1875, Bern

Dear friend!

Thank you very much indeed for your kind invitation to visit you. I have agreed with Mr Graf to meet him on Tuesday 13 July in Waldshut and then go to Wilchingen from there. I really am glad that I don't have to be in Zurich already on Sunday.

Best wishes,

Yours, L. Schläfli (Café Gräf)

21 September 1875, Rezzonico

Dear friend,

Mr Graf talked to me about a visit that I should pay to him and you in Luzern when I come back. This could happen in the first half, though probably not until the second half of October. Only I'd like to know the weekdays or any other convenient time when a visit would be possible. As the exchange of letters back and forth will take a good six days, I am writing already now in order to hear convenient days from you.

I have read through the manuscript on Bessel functions that you have had thoroughly and have found several incorrect bits in it. The small hamlet where I live, Molvedo, is situated a good hour above Menaggio, a quarter of an hour below the nearest coaching inn, Rezzonico. Prof Casorati has rented the ground and first floors of a house that belongs to the landlord Dell'Era, of the inn Ponzone in Milano (the physician of this region lives on the second (uppermost) floor). He [Casorati] has given me the only big room in the Stöckli⁵ next to the house. Below the house there is a garden reaching down to the lake; above it there is an orchard and a vineyard. In the morning I work, in the afternoon I participate in the collective walks on the cobbled paths along the flanks of the hills. The air is good and the views are lovely. The way of life of the people here is possibly more basic than would be the case in Switzerland given the same abundance of nature.

⁵ A "Stöckli" is a smaller multi-purpose house on Swiss farms; traditionally the parents would live in there after having passed the farm on to their children.

Address letters to casa di Sig prof. Casorati
Rezzonico
(sul lago di Como)

With best wishes to you and Mr Graf
Your obedient servant L. Schläfli

Saturday 16 October 1875, Rezzonico

Dear friend!

I am pleased that you are getting married, and send you my best wishes. If nothing crops up, I will arrange my affairs such that I will arrive in Luzern in the morning of the 24 October. It depends on the condition of the road across the Gotthard; if there is a lot of fresh snow, the coaches can be delayed by a day. Here the weather has been very good for a while. Only recently there were a few days when the rain came pouring down, the temperatures have dropped considerably; there is snow on the Monte Legnone.

Best wishes to you and Mr Graf

Yours, L. Schläfli

27 August 1877, Bern

Dear friend!

I have a reason to accuse myself because of my long silence. Thank you very much indeed for your letter. I was very pleased to meet your compatriot Mr Vögeli. At the moment of writing I don't know exactly how I should reply to your kind invitation. I should visit Prof Casorati in the canton Grisons; as he has been there already since the middle of August, it will have to happen soon. If you are on holidays from the second half of September until the beginning of October, then there would be the possibility to visit Gubler in Zurich and make a detour to Schaffhausen from there. Though as I will be lumbered with a lot of work this winter I do not wish to stay long. I was very surprised and very glad to see Mr Beck from Riga here.

If you could possibly find the time to reply to me, you could write to my address in Bern anyway, I will arrange for the letters to be sent on.

Best wishes to you and your wife

Your obedient servant L Schläfli

25 September 1877, Rezzonico

Dear friend!

Mr Bach told me about your father-in-law's accident. In my last letter I kept silent because I was in a hurry to take the letter to town with me. Since 15 September I live in Rezzonico by lake Como. Apart from short sections where I could walk on foot due to the slope, I always travelled by coach or by ship; e.g. I travelled from Andermatt to Como in one day, without having any time anywhere to enjoy anything (except for a cup of coffee in Airolo). The voyage and the great heat caused an aggravation of my condition, which has persisted until now. I am glad that I will be back in Bern on 7 October, when your holidays will begin; I don't have any time for travelling. Admittedly it would be very nice if Mr Graf would come as well, and we could go for a wander in the forest by Neunkirch. But it's more beneficial for me if I don't abandon my habits and my routine. Thank you very much for your offer concerning the planes of third order; I also have notes on that topic at home. Given the mood I am currently in, I'd be very happy if I could decline your kind invitation. As always, you will remain in my thoughts.

Best wishes,

Your obedient servant L. Schläfli

10 October 1880, Bern

Dear friend!

I am sorry that I couldn't meet you during your last visit in Bern. How long do your holidays last? Would it be at all possible to pay you a visit on one of the next nice days?

Best wishes,

Yours, L. Schläfli

22 October 1880

Dear friend!

Unfortunately I missed the most convenient time, and the weather has since been so unsettled that I could not possibly have hoped that you would have been so kind as to go on an outing with me. I also would have liked to talk you anyway.

Kind regards

Yours, L. Schläfli

26 March 1882, Bern

Dear friend!

Would you be so kind as to tell me the address of Mr Amsler junior, so that I can return one of his earlier letters.

Kind regards,

Yours, L. Schläfli

21 April 1882, Bern, Bruckfeld 237^c

Dear friend!

A disruption caused by moving has prevented me from writing to you sooner. I thank you for your kind reply and hope that you have fully recovered from the illness that affected you. You will share my happiness when I tell you that the Academy of Sciences in Christiania has presented me with the complete published works of Abel. Moreover, the Academy of Sciences in Paris has awarded me the complete works of Cauchy, probably at Hermite's suggestion. Yesterday I received the first of the four volumes, a grand book. I don't know how to explain this occurrence; my programme aimed against the late Heine, which I had sent to Hermite, possibly contributed to my recognition. I was prepared for attacks when I penned the programme, but most people will probably not take much notice of it, as it does not lie within the domain of prevalent ideas.

If your free time coincides with mine, I accept your kind invitation with pleasure.

Best wishes,

Your obedient servant L. Schläfli

17 July 1882, Bern, Bruckfeld, House Brägger (no. 237c)

My dear friend!

Today I have my last lecture. I did not expect to be done so soon, but believed that I could only take off the last days of this month. Because of the uncertainty I haven't replied to your friendly letter yet. I want to stay here until the 22nd; you may decide what to do in the remaining time at your discretion. I plan to stay in Rietwyl for a day on my way to Zurich.

I am really glad to have some free time. Although I had an audience of only three and 15 hours per week, my workload was consistently too high and often I was not well.

If I may suggest that I depart from here next Sunday, the 23rd; I might possibly be in Zurich already on Monday morning. But I am not determined; you are entitled to commanding time.

Many thanks for your kind invitation and with best wishes

Your obedient friend L. Schläfli

1884 [January or February?]

Dear friend!

First of all I have to apologise that I reply to your kind letter only that late. I had decided to write to you already at the beginning of this year but had not yet executed my decision. I like to reminisce about the time we spent together. As regards the letter of congratulations arranged by Geiser (signed by 18 lecturers, including the three Swiss Sidler, Wild and Wolf, who are all of the same age as me), I was surprised that Geiser thought of this. I suspect that Regierungsrat⁶ Affolter initiated this: I visited him shortly before Christmas at his behest, spent a few happy hours in his and others' company and told him that I would turn 70 the following month. Not only did Geiser post the address of the 18 Zurichers in the *Bauzeitung*, but, as I surmise, he also sent copies of the newspaper to Beltrami and Cremona. I was showered with congratulations. I am ashamed that, in comparison to others, I am always too short when expressing goodwill.

The unexpected death of Mr Schönholzer is a loss for the municipal Gymnasium, which shall replace the former Kantonsschule. He influenced his pupils to such an extent that the classical languages teachers became jealous. They thwarted him by demanding of the pupils such unreasonable efforts like performing a play in Greek. At the Gymnasium, his position was not filled again, but at the university his tasks were assigned to two new lecturers, Mr Huber from Ramsen and Mr Leuch from Bern. In the last few years, Mr Schönholzer and I didn't see each other very often, since the habitual meeting over a pint of beer was lacking due to the fact that he was no longer allowed to drink beer. But he thought about me more than I could possibly have imagined.

I would be very pleased if you would like to come and visit me in Bern. My flat is in Bruckfeld 14, or, as it is called now, in Neubruckstraße. It's the second house off the street, where a hollow way leads across to Enge.

Please give my best wishes to your wife and your children, but I will not promise that I will teach any of them. I wish you all happiness and good health.

Your obedient friend L. Schläfli

30 December 1884

Dear friend!

This year you have sent me such kind congratulations that I believe that you will be pleased to accept the enclosed picture.

I wish you and your family health and happiness in the New Year.

Yours truly, L. Schläfli

⁶ Member of the cantonal government

06 January 1885, Bern
Dear friend!

Thank you very much for the delight you have given me by sending me your excellent photograph. I hope that you will come to Bern at some point.

Kind regards,

Your obedient servant L. Schläfli

01 April 1885, Bern

Honoured friend!

Please accept my thanks for the valuable paper that you were so kind as to send to me. I have started to read it and will continue with great pleasure.

Best wishes,

Your obedient servant L. Schläfli

16 November 1888, Bern, Bolligenstraße 18

Dear friend!

I am very pleased that you are coming to Bern. May I suggest designating the Müller Inn in the Gerechtigkeitsgasse [...] as our meeting place, I will arrive there at 7pm (20th Nov.). Not only my hearing, but also my vision has degraded.

Kind regards,

Yours, L. Schläfli

Remark [by Gysel?]: I saw Schläfli for the last time in Bern in May 1894.

[undated]

Dear friend!

Tomorrow, 26 October, I will arrive in Schaffhausen at 12.08pm.

Schläfli

E.3 Papers

E.3.1 C F Geiser: *In Memoriam Jakob Steiner*

Verhandlungen der Schweizerischen Naturforschenden Gesellschaft, Vol. 56, 1873, 215-251.
Footnotes by the author.

Anyone who has ever had – how shall I put it? – the delight, the pleasure, the honour of attending the public session of an Academy of Sciences, will remember the venerable impression that such a body conveys for a long time afterwards. The men, for the most part older, greying figures, who one is used to regard as the heads of scholarship of their country, sit calmly at a wide green table. The more or less prominent merit of each individual is indicated in the most fortunate manner by the proportionate abundance of star-shaped orders, just like the number of growth rings suggests the age of the trees in the forest.

A nostalgic mood sets on the assembly when the academy commemorates its deceased, and in particular when the memory of a luminary of science is benevolently honoured by an elaborate eulogy. Which setting would be more suitable for such a commemoration than the circle in which the deceased has spent his last years and in which his papers have seen the light of day? Not only his scholarly activity, but also his personal appearance vividly persist here, and many a relation, which otherwise would have remained undiscovered, can be uncovered in this place.

Unfortunately, these academic eulogies also have their Achilles' heel; they are not called "Éloges" in France for nothing. The cool, refined style demands reticence in all doubtful points and discrepancies that cannot be evaded are narrated in as subtle a manner as the smile of the Baron Munchhausen in Immermann's novel, who, as is well-known, smiled so subtly that nobody could notice it anymore. Thus, such encomiums often resemble modern images of saints, for which the most magnificent aniline colours have been used in order to enhance the picturesque drapery of crimson and azure; it goes without saying that the long, blonde curls then appear in the most delicate arrangement.

Not every head is suited to be a model for such an image – for example, where would one find the comb that would be required for curling Jakob Steiner's wild mane of hair into academic shapes? This is probably the reason why our great fellow countryman, who had been one of the most magnificent adornments of the Berlin Academy for almost thirty years, has not yet been honoured by a eulogy. For more than ten years already he has been buried in native soil, the memory of his extraordinary personality is gradually starting to adhere in the memory of his contemporaries, and so far no friend willing to describe his life and work has been found. Will I be reprimanded, because I now attempt to draw a delicate outline of his figure based on family ties, personal and scientific relationships and the memories they evoke? Considerations of the kind mentioned above cannot be taken in our society, one is free to distribute light and shadow at one's own discretion, and true deference for the deceased is not derogated if the faults of his appearance

come up as well as the merits. Now he who does not approve of the keynote of my portrayal, maybe even thinks that the publican, who had the whole scientific community buzzing a while ago, has been joined by the sinner, may take comfort in the thought that this fugacious paper will have been long gone with the wind, while Steiner's masterpiece will awake the admiration of future generations in increasing splendiddness.

Jakob Steiner was born on 18 March 1796; on a day¹, which in later years, when a grim mood usurped it, afforded him an opportunity to quarrel with a fate that did not even let him celebrate his birthday in Berlin without appearing as a seditious democrat to the regime of the Nachmärz². He spent most of his boyhood in his native village Utzenstorf, situated about half-way between Burgdorf and Solothurn, but not as an bucolic Gessnerean³ shepherd boy: as soon as possible, he worked diligently and strenuously on his parents' little farm, together with his siblings. Apart from the most simple school subjects, which mainly consisted of memorising the Heidelberg Catechism and the hymnbook, he found no intellectual stimuli in the farming village. Not even among his kinsfolk, as he was "the first case in our family" and no relative had enjoyed tuition beyond the general elementary education before him. Even at an old age he complained that he learned how to write only at the age of fourteen, because the village priest withheld any further instruction until the pupils knew the catechism book forwards and backwards. This is also the reason why Steiner had a heavy hand throughout the rest of his life and did not like to do the mechanical work of writing.

If we want to get a lively image of his life up until the age of seventeen, then we have to rely on the writings of his boyhood friend Bitzcius. Under the name of Jeremias Gotthelf⁴, Bitzcius portrays the conventions and the states of things of the canton Bern as they have persisted almost unchanged for such a long time with unrivalled mastery. He constantly spent time in the fresh air of the fields and the forests, which strengthened his body and enhanced his senses. He often proudly highlighted how he could see the cows move on the pastures of the distant Jura, shining resplendent in a blue shimmer. Not

¹ On 18 March 1848, a gathering outside the Stadtschloss in Berlin turned into an armed fight between citizens and the troops of the Prussian king Friedrich Wilhelm IV after two shots had been fired. According to the authorities, 303 people lost their lives in the subsequent street battles. The revolutionists fought for democratic rights such as freedom of speech, freedom of the press and the voting right.

² Historic period after the March Revolution of 1848 in the German states.

³ Here Geiser probably refers to the Swiss poet and painter Salomon Gessner (1730-1788), who specialised in idylls and pastoral landscapes. Cf. biography by B Weber in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D11825.php>, accessed 09/02/2012. Incidentally, Gessner was the great-grandfather of Geiser's wife Emma (see chapter 2.1).

⁴ Jeremias Gotthelf was the pseudonym of Albert Bitzcius (1797-1854), a famous Swiss author and priest. He described the conditions of rural life very realistically. Cf. biography by H P Holl in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D11835.php>, accessed 09/02/2012.

without ironic satisfaction he used to add that at a more advanced age he was still able to identify a "Rindvieh"⁵ even from a distance.

Of course, Steiner never was what you would call a child prodigy, i.e. a boy who speaks Latin fluently already at the age of four and who, after having recited his little speech at a party, never picks up the slice of cake designated for him without having been encouraged to do so. However there must have been a point in time when the thought came to him that he was destined for a more superior career than cultivating his father's land and producing the material for the family's stockings by tending to the little sheep business. Some of his friends seized this idea and developed it further. How often may he have pondered how to blaze a trail for his inner drive, when he was sitting behind books he hardly understood instead of driving out to the field with horse and wagon in order to bring in the hay? Now the circumstances of his time played to his most ardent wish in the best possible way, as they opened up a way for him that could not have been more favourable to the development of his character. Therefore I will attempt to depict this way in a few words here.

By the time Steiner reached adolescence, the aftermath of the French revolution, in whose most thunderous storm his cradle had stood, had faded to such an extent that the leading statesmen of our native country could think about rebuilding what had been destroyed. On the other hand, its thunder was still rumbling so audibly across the continent that even in the tranquil Utzenstorf people attentively watched all steps taken by the local authorities. Thus, the efforts realised in Iferten⁶ with the help of Swiss governments by the great people's educator Pestalozzi, who at the beginning of the century had worked in the neighbouring Burgdorf with the moral support of the French authorities, caused a lively discussion in the secluded village. The older members of the population were ill disposed towards everything reminding of the revolution and the Helvetic Republic. Some of the experienced, wealthy men who expressed themselves in this spirit, might have been there in the year ninety-seven, when the General Bonaparte was stopped by the Bernese farmers with the cry: "du donners Schelm, en jiedere Schelm blib i sym Land"⁷ when he was travelling through Switzerland⁸. But the younger generation held a different view, not least out of a desire for opposition. Apart from that, the touching yet at the same time powerful figure of the reformer of educational theory had an irresistible appeal to all conative minds.

Perhaps the young Steiner reached the conclusion that the educational establishment in Iferten could educate him to a lesser extent by means of plain consideration but rather due to his gut instinct. However, once he had come to this conviction, he put the all of the stubborn doggedness of a Bernese of the old stamp into carrying his idea into effect. I know of the fierce arguments that

⁵ The word "Rindvieh" translates to cattle, but it is also used as a term for 'fool'.

⁶ The old German name for Yverdon-les-Bains in the canton Vaud where Pestalozzi had his famous school.

⁷ "You maledict scoundrel, every scoundrel shall stay in his own country" (many archaic words in the original quote).

⁸ In November 1797, on the occasion of travelling to the Second Congress of Rastatt, Napoleon Bonaparte undertook a reconnaissance trip through Switzerland, coming to the conclusion that Switzerland was ready for the revolution.

preceded the family's decision on this important matter due to his very own narration. A streak of excessive canniness, which also vested in the son, caused the father to refuse the means for the education of the young man; even more so as he was very reluctant to miss the extremely diligent and economically minded worker in the house and a younger brother did not qualify for fulfilling similar hopes. Luckily, Steiner had saved a modest sum from his business, the father eventually added a negligible amount of money and dismissed the son. Henceforth, he was the master of his own destiny and turned to Pestalozzi in good spirits, admittedly sacrificing many a beloved interest.

The famous establishment was already on the decline by the time Steiner arrived there. Amongst the sounds of the trumpets⁹ with which Pestalozzi's staff kept the world up to date about their activities, one could very often hear a wistful tone, which heralded that the great philanthropist saw his ideals vanish little by little, losing a piece of his breaking heart with each of them.

It is hard to say how many benefits Steiner took with him from his training in Iferten, since really thorough scientific lessons were not given. As far as I know, Steiner was the only pupil of this institute apart from the geographer Karl Ritter, who reached an outstanding position in the Republic of Letters. The indirect gain was all the greater. Pestalozzi set great store by commencing lessons with the translation of geometric concepts into numbers. On this occasion, the creative imagination of the inquisitive pupil stirred for the first time. Soon he felt far superior to his teachers and successfully applied the Master's fundamental principle of encouraging pupils to use their own initiative to the then teaching assistants by forcing them to study, so that they would understand what he, the novice, could see in his mind. Already back then he always went for general considerations, as is shown by his first geometric discovery, which, however small and insignificant it may appear, deserves to be mentioned here. His teacher had told him that a solid triangle was formed by three planes, whereupon Steiner, whose clothes still smelled of his father's byre, immediately chipped in: but of course there are eight of them.

The way in which he acquired the elements of science became crucial to all of his later work. He recognised even the most modest success if it emerged from someone's own efforts and individual thinking, whereas he regarded the most extensive amount of information only with distrust and disdain if the information had not been understood properly and was not presented in an original way. Even the school in Iferten could not comply with his desire to process all knowledge and all skills independently: Once he complained that he was a respectable drawer before he went to the school, but after he had been taught how to draw, he was no longer able to put the church in his native village on paper as clearly and stately as before.

The Socratic teaching method, which checks how much the student understands every moment, was one of Steiner's peculiarities and a result of Pestalozzi's influence. In later years, it added a particular charm to his university lectures. Since the limited funds of the pupil were not enough to

⁹ Geiser uses the word "Posaunenstösse", which could be translated as "sounds of the trombones". However, as the "trumpets of Jericho" are trombones in the German translation of the idiom, I chose the word "trumpets" here.

cover the full amount of the tuition fees, he had to commit to work as a teaching assistant after having completed his studies. The great educator felt that the prospective teacher was a promising young man and helped him, giving him advice as often as possible. As a result, Steiner kept a fond memory of him [Pestalozzi] and remembered him with unlimited reverence and admiration even when doom befell him and the noble man became everybody's laughing stock. Just as Steiner predicted the immediate downfall of the establishment in Iferten with a sharp eye, he never let go of the belief that Pestalozzi's star would brightly re-emerge from the dark.

Steiner had left his parents' house under lightning and thunder in order to follow a superior desire and become the pupil of an intellectually outstanding and stimulating man. At the time, he did not yet know the proper purpose of his life; maybe he was driven only by the thought of training as a teacher. Now he left this man at the point when the house that had become his second home faced its collapse in a bigger storm, with the clearly identified objective of devoting his life to mathematics and to geometry in particular. He turned to Heidelberg to continue his studies, admittedly with the prospect of having to earn his living by giving private tuition. Indeed, he did make ends meet during the three years of his stay there (1818-21); however, in recognition of the individuals concerned it has to be mentioned explicitly that several of his more wealthy fellow countrymen and fellow students attended his lessons only in order to support their ambitious older friend. Even in the most dismal situations of his old age, he kept fond and happy memories of that time and of his friends back then, of which I will only mention our Casimir Pfyffer¹⁰ and the Federal Councillor Näf¹¹.

The state of mathematics at almost all German universities of that time, in which admittedly Gauss, outshining all of his contemporaries in solitary and formidable greatness, was the pride and glory of the university in Göttingen, is known well enough so that the meagreness of the Heidelberg lectures do not have to be described in more detail. Steiner soon fell out with his main teacher, professor Schweins¹². Later on, using an admittedly rather cheap pun, he named the geometry presented to him at university after this lecturer¹³. He found his private studies more inspiring. They were conducted with little

¹⁰ Kasimir Pfyffer of Altshofen (1794-1875) was a Swiss politician, lawyer and publicist. He was one of the leaders of the liberals in Lucerne in the 19th century and member of the National Council of Switzerland from 1848-1863. Cf. biography by H Bossard-Borner in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D5258.php>, accessed 09/02/2012.

¹¹ Probably Wilhelm Matthias Naeff (1802-1881), Swiss politician and one of the seven members of the very first Federal Council (elected in 1848), a post he held for 27 years. Cf. biography by M Kaiser in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D4044.php>, accessed 09/02/2012.

¹² Franz Ferdinand Schweins (1780-1856), geometer who studied at Göttingen. He was Moritz Cantor's advisor at Heidelberg. Cf. biography by M Cantor in *Allgemeine Deutsche Biographie*, 1891, 364: <http://www.deutsche-biographie.de/sfz79732.html>, accessed 09/02/2012.

¹³ Steiner's pun would have been "Schweinegeometrie" ("pigs' geometry"), which roughly translates to "ruddy geometry" or, to use a less strong phrase, "useless geometry".

literary means, but with all the more intense intellectual power. He certainly laid the foundations for his first papers already in Heidelberg.

After completing his studies, he accepted a post as a teacher at a private educational establishment in Berlin. I wonder if the budding mathematician guessed that he would spend more than forty years, i.e. with short intermissions the rest of his life, in this city? Probably not, since things did not go to plan in the beginning: He left his job and worked as a private teacher for some time. An attempt to get a permanent position at the Friedrich-Werdersches Gymnasium¹⁴, where Dove¹⁵ had just been employed, failed due to the Swiss's awkwardness when exposed to the still unfamiliar North German and specifically Berlin elements. He was already thinking about leaving again, but in the meantime his reputation as a private teacher had risen steadily and in a crucial moment, fate led him into the house of Wilhelm von Humboldt where he was to teach Humboldt's eldest son. It was here where Steiner, due to a series of fortunate events, could show his full potential and gain a firm foothold at the beginning of an actual scientific career.

When Prussia started to gather strength after the disastrous campaign of 1806¹⁶, it did not find itself in the lucky situation of today's France: In miraculous apparitions, the Mother of God augurs France a bright future without any efforts whatsoever. The protestant state had identified general conscription and improving general education as the main instruments for regaining the power they had lost. Precisely on Humboldt's prompting, a number of junior Prussian education experts had visited Pestalozzi's school at public expenses and they had given glowing reports to the relevant authorities. Thus, a suitable start to an animated conversation between the former minister and the young teacher easily presented itself. To his own surprise, Steiner had presented himself to his best advantage in the pupil's examination. Suddenly Steiner saw himself being drawn into the circle of the prominent men who frequented the famous house. Very soon his relations became genial, amiable and jovial ones, since Humboldt had lived in Bern for a few years as the Prussian ambassador and liked to speak about Switzerland, in which he still took a keen interest. When Her Excellency¹⁷ even handed him a cup of coffee and said: "Herr Steiner, weit er öppe-n-e chli Nidle zum Gaffeh"¹⁸ at a larger social gathering to which the Bernese private teacher had been invited, his adoration for this superb couple became limitless.

¹⁴ A classical Gymnasium in Berlin; in the 19th century one of the best secondary schools in Prussia.

¹⁵ Heinrich Wilhelm Dove (1803-1879) was a German physicist and meteorologist. Cf. biography by R Scherhag in *Neue Deutsche Biographie* 4, 1959, 92-93: <http://www.deutsche-biographie.de/sfz11761.html>, accessed 09/02/2012.

¹⁶ First campaign in the War of the Fourth Coalition; it 'ended' with the capture of Berlin and Napoleon reaching Warsaw.

¹⁷ In the German Empire, only ministers, envoys, ambassadors and privy councillors were entitled to the title "His Excellency". At the same time however, a man's title would also be used to address his wife (e.g. Herr Doktor Müller and Frau Doktor), which is probably the reason why Geiser uses Humboldt's title for his wife as well ("Frau Excellenza").

¹⁸ Mrs Humboldt speaks Swiss German here: "Mr Steiner, would you like a bit of cream with your coffee?"

But also His Excellency himself saw to it that our fellow countryman, who was now generally accepted as an excellent teacher and a promising mathematician, did not lack the cream on his coffee anymore. He found a position at the Gewerbeschule, though not as quickly as he had hoped for. There the opportunity arose to make amends for the failed attempt at the Werder-Gymnasium; and what was even more important: A benevolent eye regarded his efforts with favour; an eye that followed many an aspiring talent and made sure that in each case the necessary support and encouragement were not missing. Do I really need to add that I am talking about Alexander von Humboldt, to whom German scholarship is deeply indebted, not least for Dirichlet and Jacobi? The great polymath remained Steiner's loyal guardian until his death. In one of his last letters he touchingly wrote that having seen the younger academic colleague for the first time in the house of his long deceased brother was a wistful yet dear memory of his.

But he did not just find protectors, but also colleagues and friends in this time that was full of work and hope. The Senior Government Building Officer Adam Ludwig Crelle took a medial position. Despite his patronising air, which he dressed in a fine and superior smile, he had a tremendous respect for the cloddish Swiss. But Crelle was very good friends with the slightly younger Abel, who did his further studies in Berlin.¹⁹ Steiner gave a very humorous account of the first meeting with Abel and of how each of them still secretly pitied the other one because of his lack of social skills in later years. Trusting in the productivity of the two young mathematicians, the Senior Government Building Officer founded the famous journal²⁰. Often you could see him on a walk with his protégés, so that the confidants called to each other: "There walks Adam with his two sons Cain and Abel", to Steiner's great annoyance. Admittedly, he did lose out on this joke.

Almost simultaneously, a very close friendship with Jacobi developed. Jacobi's extraordinary knowledge and skills had become noticeable already early on, and Jacobi was of great use to the older, but more prudent and one-sided Steiner. Of course, the most important thing remained his scientific motivation, which became a treasure trove of new, worthwhile brainwork every time the two friends met. Even after the newly rising mathematician had moved to Königsberg he showed a continuous support for the friend who remained behind. After *Systematische Entwicklung*²¹ was published, Jacobi got him an honorary doctorate of the University of Königsberg in honour of his scientific achievements. Likewise, it is due to the joint efforts of Jacobi [sic!] and Humboldt that an extraordinary professorship at the University of Berlin was created for Steiner in 1834, while at the same time the Royal Prussian Academy of Sciences elected him as a member.

Now the time has come to talk about Steiner's work as well, and surely I will not be reprimanded for looking at the entirety of his achievements straightaway, alongside the papers published up to the year mentioned above.

¹⁹ Geiser is not very accurate in this paragraph on Crelle!

²⁰ It seems that "Adam Ludwig Crelle" is in fact August Leopold Crelle (1780-1855), founder of *Crelle's Journal*.

²¹ *Systematische Entwicklung der Abhängigkeit geometrischer Gestalten voneinander*

This allows us to abandon the chronological order and to treat related problems at the same time. Even so, the task of portraying important scientific activities is difficult enough, as everyone who knows the papers in question already values their quality, while to the ignorant, all the explanations about things he knows nothing about must be worthless. May I ask my peers in particular to forgive me for the uncertainty of the subsequent opinions and explanations. When a man, who asserts his own degree of importance among the finest mathematicians of all times, has expressed the difficulty of giving an account of the achievements of a close colleague with grand modesty, then it is easy to see that it is only with a tentative hand that a gleaner can attempt to sketch the good crop yielded on the large field cultivated by Steiner.

Steiner's earliest papers, which were published partly in *Gergonne's Annalen*, partly in the first volumes of *Crelle's Journal*, do not yet reveal the pioneering genius, but they already manifest the master of looking at given simple shapes from different angles. Some of the papers, notably the comprehensive *geometrische Betrachtungen* in the first volume of *Crelle's Journal*, are an exemplary treatment of the properties of circles and spheres. The second chapter of the later published ingenious little paper *Die geometrischen Constructionen, ausgeführt mittelst der geraden Linie und eines festen Kreises* may serve as an introduction to this, in a manner of speaking. The fact that he immersed himself in problems much more deeply than his colleagues already back then, is shown by the solution (published without proof) to Steiner's generalisation of Malfatti's problem, in addition to his tackling of many a new problem. More recent papers, following his chosen path, served as a splendid vindication of Steiner's in answer to grumbling by other mathematicians.

His paper *Verwandlung und Theilung sphärischer Figuren* was deservedly acknowledged, though admittedly he confessed later that at that time too great an importance was attached to spherical geometry. – May I take this opportunity to remind us of the proof of Euler's theorem on polyhedrons, which stands out due to its great simplicity and clarity, regardless of the fact that it does not solely emanate from the given elements of the shape but requires angular measurements.

In addition to these papers there are others, which already contain a wealth of results on geometric points, leading on to conics and planes of second degree. Among these papers *Développement d'une série de théorèmes relatifs aux sections coniques*, published in volume 19 of *Gergonne's Annalen*, holds a prominent position. He later summarised the main results of his research based on this paper under the familiar title *Populäre Kegelschnitte* and repeatedly talked about them in his university lectures. The main appeal of *Populäre Kegelschnitte* lies in the fact that the finest theorems relating to conics, namely their foci, are derived almost effortlessly from the solution to a very basic problem of maxima and minima. Steiner talked about this method of obtaining extensive results by means of the most basic examinations with great delight. Once he said: "When I am given a plate full of cherries, then I deliberately take the inconspicuous ones first and with relish I eat the finest ones last". Another time he called out to his students, who wanted to crack the geometrical nuts²² given to them with steam engines: "If you don't become as innocent as little children, then you won't go to the Kingdom of Heaven". The

²² i.e. brainteasers

way in which he related an equilateral hyperbola to a triangle with an inscribed circle and a parabola to a triangle with a circumscribed circle, and derived the associated theorems by “standing on the spot, only turning on the heels”, is of particular beauty.

He could never finish work in this area; year after year he revised his notes on the popular conics and added new material. He also published parts of them, such as for example the papers *Elementare Lösung einer geometrischen Aufgabe etc.* (Crelle, vol. 37) and *Ueber eine neue Erzeugungsart der Kegelschnitte* (vol. 45), in which the idea of replacing the foci by doubly tangent circles is explained in particular. The paper *Aufgaben und Lehrsätze*, issued in various time periods, also belongs here. Out of these, the paper in vol. 55 is worth mentioning specifically, as are the subsequent *Geometrischen Betrachtungen und Lehrsätze* (vol. 66). Studying these publications may be recommended explicitly to the younger generation, which wants to take wing for the first time and is looking for nice aims that are not too difficult.

Before I will attempt to characterise Steiner’s most voluminous creation *Systematische Entwicklung der Abhängigkeit geometrischer Gestalten voneinander* (1832), I will give a short reminder of the efforts of his predecessors. In doing so, I will repeat well known and often repeated historic facts. Due to Euler, Lagrange and Laplace, the superiority of the analytic, arithmetic methods over the synthetic, observing methods seemed to have been established in such a way that geometry had almost come to a halt for the most part of the 18th century. This was also criticised by Gauss several times.

Then Monge paved a new way for geometry with his *Géométrie descriptive* and the *Applications d’Analyse à la Géométrie*, by adding new results from different angles and by opening up new methods. Descriptive geometry, which will be linked to Monge’s name for eternity, taught us to identify observation as the basis of all problems set in space. It showed how the most basic construction rules and lead to a long series of geometric truths. As it permitted a multitude of practical applications, it sparked an interest in this new discipline in many circles. No less important were the *Applications*, by showing that analysis and synthesis do not have to face each other like enemy troops. On the contrary, only when they are united they unlock the deepest secrets of mathematics, which they would never have reached on their own. Although the way in which the material is arranged in the book leaves a lot to be desired and the methods of proof do not always meet the expectations, the book is guaranteed a lasting position in the history of mathematics, as a successful union of analytic brilliancy and geometric ingenuity.

Once the pioneering work had been done, the disciples continued the master’s work. Among these disciples, Dupin and Brianchon, Gergonne and Bobillier were particularly successful. Poncelet appeared a bit later, but with extraordinary brilliancy. Far away by the river Volga, in Russian war captivity, and without any external resources, he completed the preliminary work for a book that guaranteed him the leading position among the French scholars of his time.

The masterpiece *Traité des propriétés projectives des figures*, which “the prisoner of Saratov”²³ published in 1822, contained the very first successful attempt to reduce the enormous number of theorems on linear shapes, circles

²³ i.e. Poncelet

and conics that had been derived over the centuries to a few fundamental theorems and basic principles. It is well-known what a wealth of actual results the book could offer by combining the methods resulting from Monge's *Géométrie descriptive* by means of observation and from Carnot's *Géométrie de Position* by means of calculations.

Poncelet is the first to realise and describe the full significance of three great geometric principles that up until then had been used only occasionally. In doing so, he elevated proposals by Monge and Brianchon in particular to fundamental tools in geometry. To begin with, he looks at the properties of planar shapes that remain unchanged by perspective projection. Using these properties, he transfers the most important properties of a circle straight onto conics. Then he shows that by interchanging points and straight lines, points on a line and lines through a point, etc., a theorem can be opposed to every projective theorem in planar geometry. This is done on the basis of the theory of poles and polars with respect to a conic. Moreover, he constantly mentions that in the whole area considered here the accuracy of a geometric theorem depends in no way on whether the auxiliary shapes required for its proof are real or imaginary. Of the many new results of the book, let me highlight the theorems on polygons that are inscribed in a conic and simultaneously circumscribe another conic, because of their numerously discussed connection to seemingly remote areas of mathematics. Finally, may I also point out the supplement, which contains the attempt to migrate from the plane into space and in particular shows for the first time that a sheaf of surfaces of second degree contains four cones, whose apices are very closely connected to the harmonic properties of the surfaces considered here.

Poncelet could not rejoice in his ingenious achievement properly, as an unedifying polemic on the content of the book began as soon as the book was published. This polemic was probably one of the main reasons why the great geometer turned to applied mathematics and practical mechanics, where he would win no less everlasting laurels. Looking back at the bygone days of agitated debate today, one will willingly accept that Poncelet's irritability, which reveals itself in the second volume of *Traité des propriétés projectives* (1866), can be attributed to the not always loyally run campaign, in which Gergonne tried to push away the theory of reciprocal polars by means of his admittedly more comprehensive but not fully justified principle of duality. Furthermore, Cauchy, resenting the imperfect strictness of the proofs, attacked the principle of continuity with undeniable ill will.

But soon afterwards Möbius pondered the endless flood of animated questions in the classic *Der Barycentrische Calcul* (1827). His thoughts could not but reveal a shrewd master of geometry to the skilled eye. However, the imaginative book was left to stand offside the big highway of accepted opinions. Hardly anyone took notice of it until Steiner entered the battle arena in the book that we have to review. He took stock of the argument and literally presented new ways for this science, which to this day it has not yet examined completely.

If we want to envision the full significance of the new basis of geometry created by Steiner, then we have to remind ourselves of how the projective property of the cross-ratio of four points on a line and of four rays through a point has already been recognised comprehensively by Möbius. Again and again this cross-ratio figured in geometric papers, from the times of Pappus

up to Brianchon's *Mémoire sur les lignes de second ordre* and to Poncelet's work. By assigning a range of points²⁴ and a sheaf of rays to each other projectively, in their onefold infinitely many elements, Steiner succeeds in giving definite definitions of the true basic shapes from which conics can be generated and in exhausting their most fundamental properties. He based his work on the examinations of barycentric calculus, unfortunately without keeping its consistent consideration of the signs of line segments and angles. If you also consider that the transfer into space, or the use of a straight cone respectively, is not a vital component of the theory, then it is easy to show improvement over Poncelet.

This improvement only revealed itself in full in the second volume of Steiner's lectures (1867) published after his death, but the comments dating to 24 May 1836 that can be found in the preface of the same, together with various indications to the "systematic development" (on page 167 amongst others), show how early he had completed the structure in essence. First of all, the principle of duality openly enters the examinations. One can generate conics projectively, without having to leave the plane; and not only do the few elements one uses reveal the true nature of involution, they also lead to the theory of poles on polars most easily. Eventually the principle of continuity becomes redundant. In Poncelet's work, it obscures the true geometric view so often, and it divides Pascal's theorem into two cases that are fundamentally the same, for example.

The basic shapes with a onefold infinite number of elements are as important in the theory of planar surfaces of second degree as they are to conics. Steiner described a range of fundamental developments of these surfaces. In addition, he used the basic shapes to prove many theorems and lemmas concerning polygons and polyhedrons in the plane and in space, and to linearly reducing the necessary constructions to his famous identification of the superimposed corresponding elements of two projective ranges of points.

If one adds the fact that the implementation of "the introductory terms" into complete parts of the book, as was originally intended, should naturally have led to the portrayal of the theory of general surfaces of second degree, as well as to the theory of space curves of third and fourth degrees, (maybe also to the surfaces of third degree according to Grassmann's way of generating them), then one will be sorry that Steiner never realised his grand plan as he had sketched it. Perhaps Seydewitz's²⁵ fine papers, which emerge from the endless sand desert of Grunert's *Archiv*²⁶ like green oases, will at least convey

²⁴ The German term is "Punktreihe". It is the set of all points that have a relation of incidence with a straight line.

²⁵ Franz Seydewitz (1807-1852): German mathematician and mathematics teacher at the Gymnasium in Heiligenstadt. He published numerous papers on projective geometry, most of them in Grunert's *Archiv*. Cf. biography by M Cantor in *Allgemeine Deutsche Biographie*, 1892, 92: <http://www.deutsche-biographie.de/sfz80109.html>, accessed 09/02/2012.

²⁶ Johann August Grunert (1797-1872): German mathematician and mathematics teacher; he was appointed to an ordinary professorship in mathematics at the University of Greifswald in 1833. In 1841 he founded the journal *Archiv der Mathematik und Physik*. His intention was to bridge the gap between school and university mathematics and to give schoolteachers the opportunity of keeping up to date with their science after having finished their university studies. Until his death

an approximate picture of some of the intentions that the great geometer may have envisioned.

This discussion would not be complete if I did not point out that Steiner was fully aware of what a lasting benefit he had given to science. Besides two passages in the text itself (pages 128 and 140) the wonderful preface, from which I quote the following sentences, has to be considered in particular:

“Current writing has tried to uncover the organism through which the most diverse phenomena in the space world are connected to each other. There are a small number of very simple fundamental relations. In these, the schematism from which the remaining theorems can be derived consequentially and without any difficulties reveals itself. By dint of proper acquirement of the few basic principles one makes oneself master of the entire subject. Order is established in chaos and one can see how all components engage naturally and form a line in apple-pie order, and how related sets amalgamate to well-defined ones. That way one acquires the naturally fundamental elements so to speak, in order to be able to bestow infinitely many properties upon the shapes as economically as possible and in the most straightforward manner.”

The author rightly deemed this masterpiece, which uncovers the way in which nature generates its geometric shapes and creations in a manner of speaking, worthy enough of being dedicated to the man he considered the most ingenious he had ever met and to whom he was greatly indebted. Hence the dedication page now bears the name “Wilhelm von Humboldt” and will bring to mind for a long time in the future that two important people whose careers took such different directions were good friends at one point in their lives.

Already very early on Steiner looked at algebraic curves of degree greater than two from time to time. Partly they are connected to the theory of conics; partly they can easily be generated geometrically, independent of conics. For instance, at the very beginning of his examinations of parabolas inscribed in a triangle, a curious curve of third class and fourth degree appears as the envelope of the vertex tangents. Steiner published a range of nice properties of this curve, though rather late. Moreover, almost all papers that I have classed as belonging to the “popular conics” contain outlooks on higher curves.

This area became even more relevant after he had found the new geometric methods as is shown by the appendix of the *Systematic Development*, which contains several problems and theorems referring to his. Section § 59 also contains the first example of a higher transformation of planar shapes and may therefore also be regarded as the starting point of more recent important papers. By the way, let me also mention that Steiner knew transformations on reciprocal radii and used them ingeniously to derive surprising theorems relating to families of circles and families of spheres.

His papers on curves of third degrees extend further; unfortunately he did not publish them at the time when all of them would have been novel. On one

54 volumes of the *Archiv* were published. Cf. biography by O Volk in *Neue Deutsche Biographie* 7, 1966, 231: <http://www.deutsche-biographie.de/sfz24369.html>, accessed 09/02/2012.

hand he used Maclaurin's and Poncelet's finest theorems; on the other hand he examined the curve from new angles, by representing it as the triple curve of a net of conics. He was also familiar with their perspective relation to the basic curve of a sheaf of surfaces of second degree. Anyhow, several of the results he published are rather valuable, in particular those on inscribed polygons and those referring to conics that touch in six points.

But the short paper *Allgemeine Eigenschaften der algebraischen Curven*, read in August 1848 at the Berlin Academy and later published in the 47th volume of *Crelle's Journal*, is of a much more permanent importance. In this paper, the various polars of a point with respect to a curve of n^{th} degree are defined using Bobillier's algorithm and the polar envelopes are studied. The generation of algebraic curves from projective sheaves of lesser order is developed for the first time, and the singularities of the core curves are compiled. Finally, Cramer's paradox is explained in its most general form. Looking at Cremona's *Introduzione*, which treats these results and many more in detail, is the best thing we can do in order to put the full significance of Steiner's paper in perspective.

The usefulness of these general properties has proved itself by two examples: by the problem of the normals of algebraic curves and by the problem of the double tangents of a curve of fourth degree. Steiner solved the principal points of the latter (*Crelle* vol. 49) at the same time as Hesse, but he used completely different methods than the famous analyst. The hints he gives in volumes 45 and 47 of the named journal do not only indicate just how early he tackled the problem; they can also be used to create "the intricate and unorthodox combinations of the given elements" that lead to the problem's solution. In addition to the results on the curves of fourth degree the paper also contains nice theorems on curves of third degree and their core curves as well as the curves of third class now named after Cayley.

Let me also mention the grand paper *Ueber solche algebraische Curven, welche Mittelpunkte haben etc.* as it also belongs to this area. Steiner's mastery of an area of mathematics, whose quintessence, completely alien to him, lies in the theory of algebraic equations (as is clearly shown by the connected theory of certain Abelian functions), splendidly stood the test in the paper on double tangents. In the paper mentioned above, he was no less successful in his attempt to make his general geometric methods useable for problems tying in with the notion of measure. His confession that several of the theorems are not proved sufficiently does not lessen the big value of the work in this difficult field.

In addition to algebraic curves he also had to look at algebraic surfaces. Unfortunately, nothing that could give a sufficient clue to Steiner's results relating to the general case of surfaces of n^{th} degree has been published. His brilliant paper on surfaces of third degree makes up for this instead. The research for this paper began a long time before others started to work on this topic and was presented to the Academy in January 1856. At this point Steiner did not know enough about the papers of English mathematicians that had been published in the meantime. The paper's main point of interest lies in the various ways of generating curves, "whereby in future the same can be treated almost as easily as, until now, the surfaces of second degree". But also the results referring to the core surface as well as the theory on polars in

general attest to the inventive mind of the author, even though the English were particularly successful in this field.

Steiner immediately added observations on normals on algebraic surfaces (notably for those of second degree) to the examinations on algebraic curves mentioned above. Apart from these two papers there is hardly any printed evidence of the work relevant here, unless we count the note on Roman surfaces, which was made by others. Perhaps this remark is of interest: that Steiner claimed to have assumed the right angle in the semi-circle. Indeed, Schröter made the theorem from involution theory that Steiner was referring to, or its extension into space respectively, the basis of this famous paper on this topic.

Of specific interest are furthermore the papers that treat infinitesimal geometry, if I may say it like this. The first one to mention is the paper on the centre of curvature of planar curves. It derives the fundamental theorem on the area of the base point curves in a very simple way and, in addition, reveals the reciprocal relations of curves that roll on top of each other. The geometric maxima and minima already turn out to be of particular interest when working on these questions. After a number of smaller, related papers, Steiner devoted his famous paper *Sur le maximum et le minimum des figures* (Crelle vol. 24) to extrema.

One of today's most sharp-witted mathematicians proposed in his dissertation that: "mathematics is an art as much as it is a science". This caused Eisenstein²⁷ to reply that: "mathematics is indeed an art, but it is not a science". This paradox could not be defended more impressively than by pointing out the paper mentioned above. Without a doubt Steiner can offer achievements that are more important and more valuable to science than these examinations, yet I do not hesitate to declare them the most magnificent achievement of his overflowing genius with regard to style and content. He knows how to pour a bright light over the smallest things; a light that makes them interesting as one can see them being connected to higher structures. And vice versa, he effortlessly reduces problems that seemed to be unsolvable before he came along to very basic theorems. It is here in particular where his attempt to constantly move geometric shapes so as to be able to overhear their properties proved itself – he never lets them get stiff and cold, he always keeps them in a warm flow.

Should one wish to convince oneself of how astonishing his methods of proof seem, then one is advised to compare Legendre's proof of the theorem: "that out of two regular polygons the one that has more sides than the other has the bigger area" with Steiner's proof. And should one wish to convince oneself how far Steiner exceeds his predecessor in this field, L'Huilier, whom he ungrudgingly acknowledged, then one is advised to check particularly those problems, in which "nature mocks the boundary conditions imposed upon her by us". It was long after Steiner and using his methods that calculus

²⁷ Gotthold Eisenstein (1823-1852): German mathematician whose main interests were in number theory and elliptic functions. Cf. biography by J J O'Connor and E F Robertson:

<http://www-history.mcs.st->

[andrews.ac.uk/history/Mathematicians/Eisenstein.html](http://www-history.mcs.st-andrews.ac.uk/history/Mathematicians/Eisenstein.html), accessed 09/02/2012.

of variation managed to find means to follow synthesis in the solution of this kind of problems.

Dirichlet was full of praise for the paper and explained its importance vividly to the great physiologist Johannes Müller, who had been wondering why such simple things were treated in the Academy. It is symptomatic that at the time he made the objection that Steiner's proof of the fundamental theorem: "the circle has the biggest area out of all shapes of same circumference" assumed the existence of a maximum, when instead an infinite asymptotic convergence to a limit that would not have to be reached would be possible, from the standpoint of outmost rigour.

Steiner was a synthetist to all intents and purposes, with the result that people often but wrongly said that he opposed the analytic methods. However he allocated them, admittedly with reservations, a honourable position, like in the preface of *Systematic Development* and in the introduction of *Maximum and Minimum* for example. Others in turn believed that he knew more about analysis than he wanted to admit and joked that he secretly counted constants behind closed doors. The way I see it, roughly speaking the situation is like this: In the first period of his career, which also includes the two great achievements just mentioned above in particular, he followed his own mind entirely. Later however, when he engaged in the theory of algebraic curves and surfaces of higher degree, he sometimes took a number of simple theorems from analysis and algebra (which he then used with great ingenuity), without being able to verify their accuracy from a purely geometric viewpoint. Sometimes, when more remote questions came up, he asked mathematicians whom he was friends with for advice. Particularly in the unpublished papers one can occasionally find evidence of the successful support granted by Jacobi, Aronhold and Schläfli in particular, and on which Steiner always left a mark of his originality. He was more than happy to accept this, however once in a comical way, such as when he honoured Jacobi, who solved various important problems for him by means of the arbitrary parameter in a sheaf of curves, by admiringly talking about the usefulness of the Jewish Coefficient²⁸ in a lecture. Or when, with an indescribable look of suspiciousness, oddly mixed with appreciation, he called out to a younger mathematician who could assist him once with a theorem from the theory of determinants: "So-o-o, the rascal also understands determinants?"

Jacobi and Dirichlet, who were friends with Steiner for a long time, knew that he possessed not only a truly unbounded imagination, but also a rare power of deduction. Hoping that other branches of mathematics would benefit from these great qualities as well they prompted Steiner to dabble in number theory and in mechanics (particularly in the problem of the three solids). However it turned out that there were certain limits to his talents after all, as none of the efforts to that effect were rewarded by any noteworthy success.

Steiner's activities as an academic teacher may lead us away from our portrayal of his scientific achievements and back to his path of life. It has already been mentioned that he trained as a teacher under the guidance of

²⁸ I guess that Steiner referred to a result by Jacobi, who was Jewish, here.

Pestalozzi and that he also retained Pestalozzi's method, which is related to the Socratic method, later on. In doing so, he gained the invaluable advantage of being able to acknowledge the pupils' point of view and adjusting his lecturing style to their needs at any point. He always started at very basic things, at which he looked from a higher angle in a surprising manner, such that after some thinking the audience was swiftly introduced to more general theories. This was not done by treating dry theorems abstractly, but with the help of well-chosen, descriptive and clear examples. However it was essential that the pupils worked individually; merely listening and taking notes was not sufficient. In this regard Steiner used to resort to the parable: "Not all of those who say to me Lord, Lord, will enter the Kingdom of Heaven".

What made his lectures particularly interesting was his original diction, which made everything he taught very graphic. Therefore pupils did not have to draw out shapes, in particular three-dimensional ones; instead they could see them in their mind's eye purely because of the description. This was even more important as Steiner stuck to his principle, which he had expressed early on: "Stereometric considerations are understood properly only when one looks at them in a pristine manner, without any illustrations, and only in one's imagination." His lectures were an excellent teaching material in itself, which Jacobi in particular valued highly, as he advised his students that they should first attend Steiner's lectures before attending his. The power of geometric imagination gave his whole personality an individual trait even beyond his scientific and pedagogic activities. It highly qualified Steiner for evaluating three-dimensional artwork, which became apparent on his Italian voyage in particular. When looking at damaged statues he was able to identify the sculptor's intentions and assess the accuracy of the restorations that had been done with an ingenuity that was highly acknowledged by the experts. And he had not been given any instructions! Even in literary works he mainly looked for how they displayed the art of descriptive presentation. This explains the fact that he greatly admired the brute force and realistic narration of Gotthelf although he was a political opponent of this writer.

In his own writings he would never stop editing until he had found the right expression for his thoughts everywhere; each of his papers was carefully re-written twice or three times before being subjected to its final editing. This gave him the great advantage that not only the results obtained by him, but also the methods he applied and the whole description in general were given a lasting quality. All too often the writings even of important mathematicians obtain their full lasting quality only after other researchers have rephrased and reshaped them. Who does not remember how his quest to give life, drive and warmth to the language for the seemingly cold and dead subject matter already debouches in the new terms he chose. These felicitous terms always describe something taken from the nature of the object, such as for example²⁹ "*Strahlbüschel, Kegelschnittschaar, Schaar-Schaar, Kerncurve, Flächengebüsch, Voll- und Theilcurve etc.*". How sad do our recent times look in comparison: Either there is an exaggerated cult of names, which at least could be justified to a certain extent; or a few curves are introduced in dismal dullness as "Pippian"

²⁹ I could not find proper English expressions for most of these terms. Equivalent expressions would perhaps be: sheaf of rays, family of conics, family-family, basic curve, set of surfaces, full curve and partial curve.

or “Quippian” just because their discoverer denoted them by P and Q in his paper.

For as long as the verve of youth and the power of his prime of life sufficed, the main purpose of Steiner’s life was to work tirelessly. And for just as long his ultimate enjoyment of life was the pure joy at his discoveries. Like hardly any other he confirmed the validity of the saying that the true genius also requires the most intensive work. Alas, this changed sure enough as he advanced in years. His health was affected due to the extraordinary intellectual efforts. Whilst he successfully fought the imaginary ghost in the plane and in space before, to use Hesse’s expression, he now had to enter an unfortunate battle against the real ghosts in his abdomen. From this point onwards a distressing change happened to his entire appearance. He could feel how his creativity slackened and his memory deteriorated; yet at the same time he faced a whole range of problems that called for the whole man. At the same time he began to occupy himself with his position in science more than was necessary; he wanted to make sure that every little title of his fame was enshrined, although he by no means lacked recognition. The Parisian Academy for example had elected him a correspondent member with just one dissentient vote, after he had been suggested for election “au premier rang et hors de ligne”. In the last years of his life he was on the list of candidates for the position of a foreign member of the Academy. Even in his home canton people tried to prove how highly they valued him by wanting to create a position for him at the University of Bern. This position would have been more beneficial to him than his Berlin professorship in several respects.

But now the geometers are a quaint little bunch: their mathematical work demands a big dose of imagination in order for it to be successful; and when this imagination transmits from abstract shapes in space to the concrete reality of life it can cause unpleasant situations. An earlier example of this is the unedifying argument between the French geometers that has been mentioned earlier on, as well as when Poncelet’s last papers appeared, in which even the just deceased Steiner, who had been close friends with him, was struck with a vigorous handshake. This fact also revealed itself no less vividly when one heard Mr Dronke’s³⁰ reports of how the private councillor Plücker³¹ poured out his spectral-analytic-geometrically developed heart over a cup of weak tea – also when we saw a second edition of the battle with the dragon happen among those who were still alive. Who knows, in future my memories of Steiner might be quoted as a new illustration of this general remark above?

Sure enough Steiner was clever enough not to have anything printed about his secret complaints, albeit he prepared an “elucidation pamphlet” then. But he who wanted to listen to him could follow his flow of words for entire summer’s nights while going for walks, or half winter’s nights stuck in wine taverns, without an ending in sight. The manner of speaking that he used on

³⁰ Adolf Dronke (1837-1898), a German mathematics teacher, published an essay on Julius Plücker in 1871 based on documents supplied by Plücker himself. Cf. <http://www.rlb.de/cgi-bin/wwwalleg/srchrnam.pl?db=rnam&recnums=0009303>, accessed 09/02/2012.

³¹ Julius Plücker (1801-1868) was a German mathematician and physicist. Cf. biography by J J O’Connor and E F Robertson: <http://www-history.mcs.st-andrews.ac.uk/history/Mathematicians/Plucker.html>, accessed 09/02/2012.

these occasions one would generally call ranting, and he was almost even better at that than at geometry. I have had the pleasure of meeting men who take an excellent position among those living today when it comes to ranting, but I have to confess, without wanting to insult anyone, that none of them even remotely compared to old Steiner. His original expressions, his graphic and drastic expressivity served him excellently with this: once He got started, the heights of the Olympus quaked and the sun, the moon and the stars hid their light behind the rumbling thunderheads that approached. I devoutly believe that, had Steiner lived a couple of centuries ago, we would now worship him as one of the greatest theologians of his time.

It goes without saying that his social position in the circles of the university and the academy was badly damaged under such circumstances. But I have to go into his relationship to Jacobi, which is traditionally discussed in many cases and which gave rise to many anecdotes, in a bit more detail so that he does not appear as the partial culprit. The cordial relations between the two men began very early on and were further reinforced when Dirichlet entered the scene. Dirichlet's noble and gentle personality could counterbalance the brusqueness of the other two. This was necessary in view of the fact that Jacobi's legitimate ego asserted itself once too often than convenient just then for example, and Steiner on the other hand was all too happy to forget all the help he had been given. When at the occasion of such collisions the first one fidgeted with the sharp-edged blade of his Berlin wit right under the other one's nose, then this one in turn knocked him silly with the formidable flail of his innate Bernese crudeness.

Soon enough Jacobi, who, as one may suspect, rendered the meticulous references in the *Systematic Development* possible, had to discover one of Steiner's main weaknesses. Unfortunately, Steiner had also been Pestalozzi's student when it came to imperfections, who³² once boasted that he had not read a book for thirty years. Now although one would not want to condemn the fact that Steiner used references sparingly compared to the modern citation mania (which now even begins to quote those for whom one would not have an opportunity to quote them) per se, individual cases where the work of others was not mentioned are far too conspicuous. I am thinking specifically about the strange way in which he implements Plücker's formulae, the most immortal expression of their discoverer's fame, in his papers. Admittedly, Plücker was not an iota better in that respect.

It goes without saying that Jacobi used such incidents in order to frame Steiner. Curiously enough, we owe the knowledge of an important paper of Abel to such a collision of minds. Although Steiner could not follow all of his ideas, Jacobi loved to summarise the content of his papers for him and make it accessible. Once it happened that the slightly irritated synthetist told the analyst: "come off it, Abel told me that years ago and he also informed the Parisian Academy". The conversation that evolved from this comment is said to have been the first indication of the existence of the *Mémoire sur une classe très étendue de fonctions*, which resurfaced only after Jacobi made resolute queries.

³² Although the German original suggests that Geiser means Pestalozzi here, the grammatical construction used does not make it entirely clear whether this anecdote refers to Pestalozzi or to Steiner.

I do not want to portray the demise of the great geometer in every detail, I do not want to describe how he, “a burnt out crater”, wandered from resort to resort in summer, “a burden to himself and to others”, for years. In winter he struggled to take up his teaching activities, but every now and again teaching gave him a few pleasant hours for himself and his students in all his misery. It is heart-wrenching to see how unfortunate fate and own faults ensnared him more and more, how real and imaginary illness permeated and weakened his body, how unhappiness and distrust overshadowed his soul. Enough of this, he died on 1 April 1863 and sadly he could be certain of having died a natural death, as he even drove the doctors away from his sickbed.

The bitter memory that has been left by the imperfections of his personality will probably have faded soon and we will remember kindly only his likeable and great qualities, his enormous manpower and his creative mind, paired with the devotedness to his homeland and its good old customs. Even in the last years of his life he did not tire of letting children he did not know tell him all about their family background when he was out and about on the fields and country roads of his native village, or of helping an old little farmer to tow their cart out of the mud if it was stuck. And indeed, the Steiner prize of the Berlin Academy will attest to his passionate devotion to science for a long time. Meanwhile in the friendly Utzenstorf, every year the prize for mental arithmetic that he donated to the primary school will also commemorate its loyal son.

Having come to the end of these memories, whose colourful content is certainly not sufficient for spanning Steiner’s life and work, I experience a certain difficulty when trying to express the entire value of this man in a succinct comparison. When von Staudt died, people in Munich felt that it was as if the modern Euclid had returned home. Another geometer, who is still alive and shares the surprising peculiarity of not having read the German papers in *Crelle’s Journal* with the great mathematician of antiquity, has been called the Archimedes of our century. Resorting to antiquity for a comparison, as is popular on such occasions, has essentially already been anticipated for me. Therefore, let me turn to the future and tell you: when in a future era a geometer outshines all his peers and fellow researchers with regard to the richness of his imaginativeness and the mastery of his descriptions, then it will be said that Jakob Steiner has risen again.

E.3.2 C F Geiser: *In Memoriam Theodor Reye*

Talk at the Mathematical Colloquium in Zurich on 26 October 1920; published in *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich* **66**, 1921, 158-180. Footnotes by Geiser, endnotes by the author.

On 2 July 1919, Theodor Reye, professor at the University of Strasbourg, died in Würzburg. From 1863-1870, the excellent teacher and researcher acted first as *Privatdozent*, then as *Titularprofessor* at the Federal Polytechnic. His main work "Projective Geometry" (*Geometrie der Lage*) was designed as a scientific foundation for Culmann's lectures on graphic statics in the first edition, and it still provided excellent services to the students of the ingenious engineer after the author's departure. For the students of our department for teachers in mathematics and physics, as long as they prefer a geometric direction, the extended new editions of the book (which were accompanied and followed by numerous reviews in journals) still serve as a reliable and inspiring guide even today. The *Schweizerische Bauzeitung*¹ therefore wished to revive the memory of Reye among its readers by an obituary and invited me to write it.

The need for giving an idea of the scientific importance of this researcher that covers at least the main points; the wish to discuss the questions concerning the "theoretical" instruction at technical colleges as well, questions that tie in with the activities of the teacher and that continued to raise an interest in him when they were discussed extensively later on, even when he was not personally affected by them anymore; and above all the want for depicting people's fates in connection with the events of the day², have of course led me far beyond the scope of the *Bauzeitung*. – May today's talk be kindly accepted at least by a select audience.

I.

Carl Theodor Reye was born on 20 June 1838 in Ritzebüttel (Hamburg). He attended the Johanneum and the academic Gymnasium. Afterwards, he studied mathematics, mechanics and engineering at the polytechnic school Hannover for three years. A short interlude of practical work was followed from autumn 1859 onwards by two semesters at the Zurich Polytechnic, where Clausius lectured on mathematical physics and analytic mechanics in particular at the time. The completion of his studies was in the form of a year in Göttingen, where he found the opportunity to attend lectures on partial differential equations and their applications on physical problems by Riemann. In this period he wrote his doctoral thesis (1861).

The thesis on "The mechanical theory of heat and the potential law of gases" (*Die mechanische Wärmetheorie und das Spannungsgesetz der Gase*) probably originated from encouragement that Reye got from Clausius, the creator of this theory. The thesis shows that he was intimately acquainted with the famous results of Regnault's experiments: it contains this sentence as one of its main conclusions: "While Regnault's experiments seemed to revoke one of the basic principles of the mechanical theory of heat in Mariotte's law, the

¹ Journal of the Society of former students of the Federal Institute of Technology.

² I am greatly indebted to Mr Prof Lasius, who was a close friend of Reye's for more than a century, for valuable biographical notes.

more precise potential law of gases, which is based on the mechanical theory of heat, not only confirms the conclusions drawn from Mariotte's law, apart from minor corrections, but also it led to new results that are consistent with experience and that can be regarded as as many new proofs of the mechanical theory of heat."

A second paper on this very topic treats "The expansion of the atmospheric air during the formation of clouds" (*Die Ausdehnung der atmosphärischen Luft bei der Wolkenbildung*) (1863). The paper turns against an explanation of the formation of hail given by F. Mohr. It is interesting as one of the first applications of the mechanical theory of heat on meteorology backed up by precise numerical calculations. May I remind you of the fact that the formation of the Föhn wind was vividly discussed at the annual meeting of the Swiss Society of Natural Scientists in 1864, without leading to any certain results. Only a few years later a correct explanation was given, which was also based on the theorems used by Reye.¹

Reye began his academic teaching career in autumn 1861 in Hannover, but already at Easter 1863 he relocated to the Federal Polytechnic as a Privatdozent, where he announced a course on applications of differential equations on mathematical physics for the first semester.

II.

Due to Karl Culmann, the creator of graphic statics, a reputation has devolved upon the Federal Polytechnic, which admittedly has been reverberating as a historic reputation only for a while already. The basic principles have been preserved, but for the most part, the executing methods have been replaced by others. Anyhow, may I be allowed to talk about this man and his achievements in more detail here, as Reye's transition from mathematical physics to geometry ties in with this. This transition was a move that turned out to be of crucial importance for the rest of his life.

Culmann begins the preface of the first edition of his book (1865) with the following sentence: "What to do with all those theories to which the different branches of engineering have given rise ... is a question that Poncelet without doubt had in mind when he strived to conceive geometrical solutions to the various problems presenting themselves in engineering." One can explain the reference to Poncelet by the fact that Culmann came to Metz in 1837 in order to prepare for the Ecole polytechnique. Poncelet had been a professor at the Artillery School in Metz until 1834 and remained in constant contact with the town (his home town) despite having been posted to Paris. Since an uncle of Culmann's also taught at the Artillery School, the seventeen-year-old boy had the opportunity to hear many a personal fact about the famous mathematician and engineer. Furthermore, he was able to see the first drawbridge ("Pont-levis à la Poncelet") and the first "Poncelet-wheels" in action. This was later supplemented by studying the master's papers, so that to us Poncelet appears as a forerunner and direct guide for Culmann with respect to graphic statics, which is based on modern geometry.

¹ Reye also evinced his interest in meteorology by the book "Hurricanes, Tornadoes and Meteorological Columns" (*Die Wirbelstürme, Tornados und Wettersäulen*), published in 1872.

Culmann's intention of sitting the entry exams for the Parisian School afterwards was not put into practice, as a matter of fact. Soon after having arrived in Metz, he came down with typhus, whose after-effects lasted for more than a year. As is mentioned in a *Curriculum vitae*, his parents (his father was a priest in Bergzabern, his mother was Alsatian) saw this as a "sign of God" that their son was not destined for France, but for Germany, and sent him to study at the Polytechnic School in Karlsruhe.

As a railway engineer in the Bavarian civil service (1841-48) and on a big study trip to England and North America (1849-51), Culmann gained insight into the practical problems that were to be solved theoretically. There was a report on his study trip in *Försters Bauzeitung*: "An account of the latest progress in the construction of bridges, railways and river steamboats in England and the United States of North America" (*Darstellung der neuesten Fortschritte im Brücken-, Eisenbahn- und Flussdampfschiffbau in England und den Vereingten Staaten Nordamerikas*). On the occasion of appointment negotiations, he wrote about this report to the Swiss School Council: "I imagine that in the same [report] I first of all clearly demonstrated how the various forces act in compound bridges and how their dimensions have to be calculated accordingly." This already indicated the main direction of the future professor's research; who introduced graphic statics as a special course in 1860. But because he perceived the students' insufficient geometrical preparatory training as a major drawback in the first few years, Reye took on an introductory course on "Projective Geometry" in 1864. By doing this, he entered a new field that soon demanded all of his scientifically productive activities.

III.

From the transition period we have a paper dating back to 1865, published in *Schlömilchs Zeitschrift*: "Contribution to the theory of moments of inertia" (*Beitrag zur Lehre von den Trägheitsmomenten*). If a system M of physical points of the masses m_1, \dots, m_k is applied to a perpendicular coordinate system in

space, such that the coordinates x_i, y_i, z_i belong to m_i , then $T_x = \sum_{i=1}^k m_i x_i^2$ is the

moment of inertia of M with respect to the YZ -plane. Similarly, $T_y = \sum_{i=1}^k m_i y_i^2$ is

the moment of inertia with respect to the ZX -plane, and $T = \sum_{i=1}^k m_i (x_i^2 + y_i^2)$ is

the moment of inertia with respect to the Z -axis. Therefore, we have $T = T_x + T_y$. Hence, the moment of inertia of a system of mass M with respect to a straight line G equals the sum of the moments of inertia of M with respect to an arbitrary orthogonal pair of planes EE' , whose line of intersection coincides with G . Thus, one can deduce the moments of inertia of M with respect to all straight lines from the moments of inertia with respect to all planes in the space; also, one can derive the properties of the former from the properties of the latter.

Reye now shows that an arbitrarily given system M can be replaced by another M' in infinitely many ways. This M' consists of only four points, whose locations and masses are to be determined in such a way that the

corresponding moments of inertia of M and M' with respect to all planes (or straight lines) coincide. Each set of four such points is a "pole tetrahedron" (quadruple of harmonic points) with respect to a surface of second degree, B , which is linked to the so-called central ellipsoid C of the system of mass M in a simple way. Hesse has shown that the surface B , which cannot be real under the assumption that the masses of all points in the system are all positive (it is therefore called "the imaginary image of the system"), is enveloped by all planes that produce the moment of inertia Zero with respect to M . Therefore, B is also called the "zero surface" of the moments of inertia of M . In the algebraic part of the paper, the transformation of the whole homogeneous function of second degree of four variables into a sum of four squares of linear functions plays a role; thus a relationship to the orthogonal substitutions of four dimensions is established.

Almost simultaneously Reye published a paper "On geometric relations of second degree" (*Über geometrische Verwandtschaften 2. Grades*), which has a quite synthetic approach. It concerns problems that have been treated in parts by Steiner and Seydewitz already.

Reye's absolute mastery of the new field of research emerged magnificently in his talks "Projective Geometry"; the first part thereof was published in 1866. Initially, they were meant to serve as an introduction to Culmann's lectures, but then their purpose was to open up and facilitate the understanding of Staudt's book of the same title (1847) in general. Reye explains that he would have preferred the methods of this mathematician above all others, even if he had not been asked to follow his work.

Turning projective geometry into a full science as Staudt tried to do is of course only possible insofar as one excludes all metric (with respect to angles and line segments) properties of figures, i.e. "measure geometry" from it. And yet it does not seem natural to conduct a rigorous partition since Poncelet and Steiner showed how closely the two directions can be connected to each other and can support one another. Who would treat the right angle only in connection with the involution; who would introduce circles and spheres only as special cases of conics and surfaces of second degree? In this spirit Cremona comments (*Opere matematiche I. 35*) in the review of "Contributions to projective geometry" (*Beiträge zur Geometrie der Lage*) that Staudt had published as a sequel to *Projective Geometry* (1856/57).

In addition, the basic structures and prerequisites are not sufficiently defined, or justified, respectively, throughout. For example, §2 begins: "A plane is an angular surface of first order, in which every point can be regarded as the centre. Thus, if a straight line goes through two points of a plane, it lies in the plane completely."¹ In §3 it says: "Two straight lines that lie in two different planes without intersecting are called parallel to each other. ... Through each point that does not lie on one of the lines there goes a straight line that is parallel to the first line."²

May I also remind you of the introduction of the projective relation between uniform structures (§9): "Two uniform basic structures are called projective to each other if they are related to each other in such a way that for

¹ In Gauss's works VIII., 194, the definition and the theorem are treated with the help of metric expressions.

² The quoted Gauss-volume offers abundant material on the theory of parallels.

each harmonic structure in one structure there exists a corresponding harmonic structure in the other one." And: "If one wants to relate two uniform basic structures projectively to each other, then for three elements of the first structure one can arbitrarily choose three elements of the other one, which have to correspond to the first three elements. But thereupon each element of one structure is assigned to an element of the other one." Concerns have been raised over this description, but in the end it was possible to resolve them.¹

I have to add that Staudt's abstract presentation that reduces everything to an absolute minimum makes studying his corresponding papers very tedious. Felix Klein, who has worked on geometrical topics so often and so successfully, gratefully remembers that during his first months as a lecturer a friend "made Staudt's hard-to-access chains of thoughts on the foundation of a projective geometry free of measure relations accessible" to him.

The concerns of a scientific nature have now been eliminated, in particular as it was possible to present the possibility of a consistent and complete geometry without using measure expressions, due to amendments by the successors. But Reye fully satisfies the pedagogical demands. Occasionally he himself explained that he managed to fully comprehend Staudt's train of thoughts and transform the almost skeleton-like sentences into vivid expressions only under greatest efforts. As a reward for his tireless work he succeeded in producing a book that offers a systematic introduction into Staudt's writings² and into the "newer" geometry in general, which is of an exquisite clarity and vividness.

In accordance with the immediate practical intention, he does not open his lectures with a strictly systematic development of the basic principles, but instead presupposes only an at all sufficient geometrical preparatory training, on which he bases the theorems and methods of proof that are particularly important to graphic statics in the form given by Staudt. In the process, he points out the metric relations (which after all are sometimes considered by Staudt, too) in appendices and individual special lectures. – The first part, published in 1866, was followed by a second one in 1868. The second part advances far now and included some of his own very valuable analyses besides the known results by coeval researchers. But the preface, dated to 5 October 1867, informs us: "Unfortunately, I am now denied to contribute to the propagation of my favourite science as a teacher in the same way as before; because I was ruthlessly deprived of my course of lectures on projective geometry recently, so that it could be assigned to the newly appointed professor for descriptive geometry at his request."³

Reye was hit all the harder as he had acted as Prof Deschwenden's assistant and had lectured on descriptive geometry as his replacement for a longer period of time after Deschwenden's illness. In addition, Culmann had pointed

¹ Cf. notes by Klein and by Darboux, *Math. Annalen*, vol. XVII.

² Klein, too, explicitly acknowledges that Staudt's observations are presented more clearly in Reye's *Projective Geometry*.

³ The preface of the second edition of the second volume contains the by all means legitimate complaint against an Italian who had published a *Projective Geometry*, 9/10 of which had been taken from Reye's book without any reference. The plagiarism had already experienced a translation into French.

out the advantages of combining descriptive and the newer geometry in the preamble of graphic statics, and Reye, who in turn also indicates this in the first lecture of *Projective Geometry*, rightfully believed to absolutely be up to such a combination. He was relieved from his resulting embitterment by his appointment as professor for geometry and graphic statics at the newly founded Polytechnic in Aachen in 1870.

IV.

In Zurich, the new Professor Fiedler gained the utmost recognition as an excellent lecturer from his first appearance onwards. Reasons for this were the aplomb of his thoughts and the formal fluency of his lectures, the extraordinary talent for quick and well-arranged drawings on the blackboard – moreover, the students felt a firm and consistent willpower directed towards an important goal. Beyond the Polytechnic, the new direction of his lectures also got a lot of approval and, to some extent, succession, after the completed and enhanced systematic foundation of the lectures had been published in 1871 as a big textbook.¹

Perhaps one best appreciates the fundamental importance of this book by comparing it to de la Gournerie's "Traité de Géométrie descriptive" (1860-1864). In a sense, this is a summary of the descriptive geometry that had developed in France since Monge. In many cases, it complements the constructive parts by interesting analytic developments of an algebraic or infinitesimal nature. Moreover, the meticulously edited text is accompanied by an atlas, whose figures are first-rate in terms of clarity and beauty. But compared to such displays of a bygone era, the big fundamentally executed recreations, though being partial, appear in a fresher wreath of fame after all.

Despite all the recognition, students soon started complaining about the burden of the workload that was expected of them and about the increasing difficulty of understanding the subject matter as it progressed.² Also, amongst the staff it was argued that, from an organisational point of view, it was not practical to attach so much importance to one single subject. Architects and engineers complained that Fiedler's problems and the drawing techniques necessary to solve these problems did not consider practical applications at all.³ That way, discords arose, which then led to unedifying incidents, without creating the necessary equilibrium. It was not until the needs of the individual departments were shown more consideration and the specific methods of Culmann's concerning graphic statics became less important, that the curriculum and the students' timetables were reduced, to general satisfaction.⁴

¹ The second edition of "Descriptive Geometry" (*Die darstellende Geometrie*) gained the amendment: "in organic connection with projective geometry" (*in organischer Verbindung mit der Geometrie der Lage*). This expresses the basic principles of the book explicitly.

² In contrast, Culmann (who opposed any "popular" treatment of his own publications and lectures, too) attests in the preface of the second edition of *Graphic Statics* (1875) that the students possessed the knowledge necessary for the geometric part since Prof Fiedler was preparing them.

³ Fiedler says in the preface of his book: "Actual technical examples and applications are excluded, as they are not of universal value for science."

⁴ For the current subject matter of the course, cf. the little handbooks by Fiedler's successors:

V.

If I have talked about these events that happened a long time ago, then because they mark a characteristic moment in the big and continuously wavering discussion on the subject of the appropriate contents and extent of “theoretical” instruction at technical colleges. Naturally, this mainly concerns mathematics, descriptive geometry and mechanics here. An excellent historic account of these issues can be found in Paul Stäckel’s book “The mathematical education ... at the German technical colleges” (*Die mathematische Ausbildung ... an den deutschen technischen Hochschulen*) (1915)*, in which Fiedler’s importance for descriptive geometry is discussed as well. But much more important and of lasting interest is the description of the big “Engineers Movement”. Under the leadership of Professor Riedler from Berlin, it strove for a radical reform of theoretical education from 1895-1900 and achieved it to a big extent.¹

Riedler’s attack was supported by resolutions of the Society of German Engineers (*Verein deutscher Ingenieure*) (1895), whereupon a statement by all professors of mathematics, descriptive geometry and mechanics justified their deviating opinion (1896). This certainly called for a counterstatement, on the majority signed by “applied” professors (1897) and which was of the following content: “The present-day instruction course for mathematicians does not qualify them for properly recognising the needs of technology, of which they overestimate the mathematical side. Therefore, we have to enlist teachers whose training had a significant technical basis for teaching mathematics. A two-year degree course at a technical college cannot create this basis, it can only be gained in the engineering departments.” “The technical departments have to be entitled to a significant influence on ... the appointments of mathematics teachers.” “Teaching in all fields of mechanics must be assigned to engineers only.” “The beginning of in-depth teaching of mechanics has to be at the start of studying and must not depend on achieving a certain level of mathematical education.”

As far as the harsh verdict on mathematicians refers to professors who designed their teaching style at a technical college according to the style of university lectures, it is by all means legitimate. But there have always been lecturers who, although they had studied at universities only, knew how to

Descriptive Geometry by M. Grossmann (1915)

Géométrie descriptive by Kollros (1918)

¹ Stäckel cites Riedler’s respective main writings – they go beyond the period of the “engineers movement” without having lost any of their polemic power, as is shown by the following quote (Stäckel, p. 83):

“A. Riedler. “Off the walk in single file” (*Abseits vom Gänsemarsch*), Berlin 1914. Preface p. 1. Here, in a vignette, a theoretician is depicted as a hound that fancies chickens, while a hen sits on its tail.” The latest information that I heard (since the publication of Stäckel’s book), is the leaflet: “Reality deniers in science and technology by A. Riedler” (*Wirklichkeitsblinde in Wissenschaft und Technik von A. Riedler*) (Berlin 1919). It is a polemic pamphlet with the most abusive rants against individuals and corporations – [Eugen Meyer-Charlottenburg directed a vigorous defence at these attacks (1920)] – but it also treats general questions on education at technical colleges. The pugnacious nature of the author comes to light in an abundance of exaggerated descriptions and bold statements, but in many cases one will agree with the verdicts and suggestions of this ruthless man anyway.

adapt to the needs of the technicians most splendidly. Although more than a century has passed since then, I can still see the untarnished memory of Christoffel¹, who as an incomparable teacher used the persistent interest and the ungrudging work of the future technicians for his lectures at the Zurich Polytechnic.

The fact that the degree programme is not an absolutely definite ruling for the lecturing style of a lecturer is shown by the different ways in which Reye and Fiedler treated descriptive geometry, although both of them had thorough technical training. As Deschwenden's replacement, Reye maintained the habitual lecturing style (beyond Deschwenden's death) and would have, had he become his successor, connected the topic more closely with projective geometry, without neglecting the demands of the practitioners. Fiedler, who had to take on descriptive geometry when he was a teacher at the business school in Chemnitz, recorded his thoughts on a reform already then, in the paper "Central projection as a geometric science" (*Zentralprojektion als geometrische Wissenschaft*) (1859). In Zurich, he did repress "practical applications" in favour of "pure science".

The most important question that the "engineers movement" dealt with concerns teaching in mechanics. In order to avoid any biased partisanship, may I mention two opinions that are completely outwith the argument. In the preface of the book *Natural Philosophy* ("Handbuch der theoretischen Physik") from July 1867, written in collaboration with P. G. Tait, W. Thomson says: "Nothing can be more disastrous for progress than too great a trust in mathematical symbols, because the student is only too inclined to take the most convenient line and to regard the formula, not the fact as physical reality."² And almost half a century later, H. Poincaré begins the chapter VI "La Mécanique classique" in the book "La Science et l'Hypothèse" with the following words: "Les anglais enseignent la mécanique comme une science expérimentale; sur le continent on l'expose toujours plus ou moins comme une science deductive et à priori. Ce sont les Anglais qui ont raison, cela va sans dire."ⁱ

In the period between these two verdicts, laboratories for physics have been established at the technical colleges. Besides, research centres for various technical areas have been created, so that the students are offered many occasions where they can learn about the relationship between theoretical development and experimental verification of mechanical processes in lectures and tutorials. But beyond those more practical points, may I point out the big value mathematical studies have on general education, also for technicians. A reason for this is given by Riedler's statement: "Almightily and intolerantly reigns an educational system that achieves the lowliest results at the biggest efforts. The scholarly, unfruitful theory flies out of sight of the real world and over the clouds to Abel and Riemann, where the theta-functions

¹ Cf. the biographical note on Christoffel in vol. I of his complete works (Teubner, 1910). At the business academy in Berlin, Aronhold (who had studied at the University of Königsberg) represented mathematics with great success since 1861 (Stäckel, p. 28).

² Cf. what Helmholtz says about "the accentuation of physical coherence in contrast to the elegance of mathematical methods" in the preface of the German translation of the book (p. XI).

disappear, where the specific term dimension is replaced by the general term manifold and where one can then do gymnastics in a world of four or more dimensions." (Stäckel, p. 34).

In order to identify the lack of judgement of this sentence, one only has to think of Albert Einstein's "physical principles of gravitational theory". They emanated from a generalisation of the theory of relativity proposed by Lorenz and Einstein, whose mathematical expression leads into a region of four dimensions (just like analytic mechanics can be considered as four-dimensional [x, y, z, t : space coordinates and time], too). In the theory of relativity the expression $c^2 dt^2 - dx^2 - dy^2 - dz^2$ (where c stands for speed of light) is of particular importance; it is a full homogeneous function of second degree in the four differentials dt, dx, dy, dz . By introducing new variables, it can be expressed in the form $dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$. Now if the x_1, x_2, x_3, x_4 are introduced as orthogonal coordinates in a space of four dimensions, analogous to x, y, z as Cartesian coordinates in a space of three dimensions, then $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$ is called the square of the line element in the space (x_1, x_2, x_3, x_4) . Now, in lieu of x_1, x_2, x_3, x_4 , let us introduce new variables x'_1, x'_2, x'_3, x'_4 , which are connected with the original ones by a linear substitution. Thus, from ds^2 we get a general function of second degree of the dx_i , which reduces to $dx_1'^2 + dx_2'^2 + dx_3'^2 + dx_4'^2$ for orthogonal substitutions. Strangely enough, this relates to Lorenz's physical theories, which is why the respective transformations are named after him.¹

Now the generalisation performed by Einstein consists in expressing the square of the line element as $ds^2 = \sum_{i,k} g_{ik} dx_i dx_k$, where i and k run from 1 ... 4

and where the g_{ik} are functions of $x_1 \dots x_4$.² From a mathematical point of view, the generalisation is associated with Christoffel's fundamental paper "On the transformation of homogeneous differential expressions of second degree" (*Über die Transformation der homogenen Differentialausdrücke zweiten Grades*) (*Crelle*, vol. 70). This in turn refers to Riemann's habilitation treatise: "On the hypotheses that form the basis of geometry" (*Über die Hypothesen*,

¹ A purely mathematical representation of utmost symmetry is given by Minkowski: "The fundamental equations for the electromagnetic processes in solids in motion" (*Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern*). (*Nachrichten der k. Gesellschaft der Wissensch. zu Göttingen*, 21st Dec 1907). Also cf. Minkowski's talk: "Space and Time" (*Raum und Zeit*). Meeting of the German Natural Scientists, Cologne, 21st Sept 1908.

² Einstein's respective papers are:

"Draft of a generalised theory of relativity and a theory of gravitation" (*Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation*). *Schlömilch's Zeitschrift für Math. u. Phys.*, vol. 62.

"Covariance properties of field equations..." (*Kovarianzeigenschaften der Feldgleichungen*). *Schlömilch*, vol. 63.

"Physical principles of a theory of gravitation" (*Physikalische Grundlagen einer Gravitationstheorie*). *Vierteljahrsschrift der Naturf. Gesellsch. Zch.*, vol. 58. Each of the papers is followed by a mathematical development by M. Grossmann.

Einstein's thoughts got their final form in the paper: "The formal principle of the general theory of relativity" (*Die formale Grundlage der allgemeinen Relativitätstheorie*). *Berliner Sitzungsberichte phys. math. Klasse*, 29th Oct 1914.

welche der Geometrie zu Grunde liegen). With those two papers one has of course been moved into a “world of more than four dimensions”.¹

The physical conclusions that Einstein drew from his theory also apply to an explanation of the well-known anomaly of the perihelion movement of Merkur (1915). Le Verrier wanted to accommodate this movement to Newton’s gravitational theory by means of a new planet “Volcano”, whose alleged discovery by Lescarbault was not confirmed, of course. The talented W. Ritz², who unfortunately deceased at such a young age, aimed at a new solution based on the general theory of electrodynamics (where the velocity of propagation of gravitation is assumed to be equal to the one of light) – though without getting a satisfactory result.³

But Einstein’s prediction about the “curvature of rays of light in a gravitational field, which amounts to 0.84 arcseconds for a ray of light passing the sun and therefore is not inaccessible for experimental verification” is of a much bigger fundamental importance. This deflection, deduced from a comprehensive theory, was confirmed at the solar eclipse on 25. V. 19. It means that with the utmost probability, Newton’s law is not absolutely accurate, but only an approximation of reality (though an extraordinary one). In order to understand the full value of this finding, please think of the end of the eulogy that Bertrand held in honour of Le Verrier in the Parisian Academy: “Le consentement unanime assure à l’astronomie entre toutes les sciences le premier rang. Seule elle a révélé une règle invariable et précise qui explique tout. Si l’étude du ciel apportait une restriction, si petite qu’elle fût à la loi de Newton, l’astronomie aurait perdu sa couronne. Le Verrier la lui a conservée.”ⁱⁱⁱ [Of course, Gauss and Riemann would not have been likely to agree with this opinion.]

Now if, according to Prof Weyl’s announcements, our mathematically adequately trained students are to be introduced to the marvellous thoughts in Riemann’s habilitation treatise and if in addition the more advanced scientific interests of our future engineers will be catered for in general lectures on the theory of relativity during the current semester, then our technical college [where Einstein was a student and then a lecturer, after all] may confidently borrow Riedler’s expression and claim to be a first-rate “gymnastics school for four or more dimensions or manifolds”.

I will return once more to Stäckel’s book, which has afforded the opportunity to discuss key issues of higher technical education and from which I have repeatedly taken examples of fierce polemics. As an idyllic quiet point in the argument, may I quote a verdict by the art historian Gurlitt from Dresden (Stäckel p. 60), which probably found and always will find the

¹ The relationship between these two papers really becomes evident in the analysis that R. Dedekind added on to Riemann’s “Parisian prize script” (Riemann’s works, first edition, p. 384).

² W. Ritz, Oeuvres (Paris 1911) XVII Sur l’Electrodynamique générale. 2^{ème} Partie § 16 Gravitation p. 419. [Ritz was a student at the Federal Polytechnic from 1897-1900].

³ Minkowski’s aforementioned paper from 1907 contains an appendix: “Mechanics and the postulate of relativity” (*Mechanik und Relativitätspostulat*), in which propagation of gravitation with the speed of light is assumed as well. At the end, it is mentioned that it should not be possible to educe an objection against the proposed modified mechanics in favour of Newton’s law from astronomical observations.

jubilant approval of the entire student body involved: "A young architect who wants to design a villa will not benefit from having studied mathematics if he does not know the ways of life of the occupants of a villa; a dinner in a sophisticated household teaches him more important skills for his life-task of building residences for highbred people than a semester of higher algebra."

VI.

Reye had scarcely had the time to settle in Aachen, when he was appointed as an ordinary professor for geometry and applied mathematics at the new Kaiser-Wilhems-Universität in Strasbourg, mainly at Christoffel's instigation, who knew and valued him from Zurich. He now lectured primarily on synthetic (and supplementary on analytic) geometry, then, on his own choice, alternately on analytic mechanics, elasticity of solids or potential theory. So, his teaching activities were completely in the same fields in which he did research or which he had studied thoroughly with lasting interest since his student days. (For example, whilst being a lecturer in Zurich, he devotedly attended Christoffel's relevant lectures and wrote them up in parts.) Admittedly he only had one single student and two guest students as his regular audience when the lecture course ran for the first time (summer 1872), but already in winter 1876/77 his audience consisted of 27 students, which may have been in accordance with the average of later days. About the success of these lectures a former student, Prof Timerding, says the following¹: "Reye was a born teacher, who is interested in each pupil's personality and wants to understand their character right from the beginning. His seminars were exemplary, not just in the way they were run and in their success, but also in the way in which he treated his students and their sensibility with care. His lectures on synthetic geometry were prepared to perfection. His ability to conjure up three-dimensional figures in the mind's eye of his audience through his talks was unmatched."

In mentioning Reye's scientific achievements during his time in Strasbourg, which comprised more than half of his long life, I have to limit myself to the key starting points of his main papers.² Firstly, let me mention his papers on mass geometry, which tie in with the aforementioned paper on the moment of inertia. The line of thought is as follows: An arbitrary system of mass M , which is assumed to be constant in space, generates a static moment with respect to an arbitrary plane E . The static moment is formed as the sum of all the products of each element of mass with its distance to E . All planes E , whose corresponding static moment equals zero, pass through the centre of gravity S of the system M . Analytically this follows from the fact that the so-called plane coordinates of this E satisfy an equation of first degree (which is called the equation of S); accordingly the point S , as an epitome of all planes

¹ This opinion, which refers to the period after 1890, can be found in the book by W. Lorey: "Studying mathematics at German universities" (*Das Studium der Mathematik an den deutschen Universitäten*), p. 158. On the same page there is a verdict about Christoffel.

² The majority of Reye's papers were published in Crelle's "Journal for pure and applied mathematics" (*Journal für die reine und angewandte Mathematik*). They are easy to find by means of the summarising indexes, each at the end of a series of ten volumes. Here I have to abstain from adding more quotes.

going through it, is now called a surface of first class. – But if the elements of mass are multiplied with the squares of their distances from a plane and if one then forms the sum, then the moment of inertia is generated, and the plane coordinates for all E for which it is zero satisfy an equation of second degree; the E envelop a surface of second class (the zero surface or the imaginary image of the system). If one forms the sum of the products of the elements of mass with the cubes of their distances, then a moment of higher rank of the system of mass with respect to a plane is generated. All planes, for which this moment = 0, envelop a surface of third class. Therefore, one distinguishes a 1., 2., 3., ... moment with respect to a plane and in each case the respective 1., 2., 3., ... zero surface of 1., 2., 3., ... class for any given system of mass. Inversely, any arbitrary surface of class n can be represented as the n^{th} zero surface, whose polynomial can in turn be represented as the sum of n^{th} powers. Thus we have established a simple and fruitful relationship between the theory of algebraic surfaces and mass geometry. An interesting example is the following: There exist infinitely many different systems of mass, all of which have a prescribed surface of third class as their third zero surface. Amongst them there is one that consists of only five mass points, i.e. the equation of each surface of third class can be expressed in such a way that the sum of five cubes (of linear expressions in plane coordinates) has to equal zero. By using the principle of duality, this gives the mass geometrical proof of Sylvester's theorem at the same time. This theorem belongs to the front row of the theorems with which the surfaces of third degree can now be treated almost as easily as the surfaces of second degree. (The general surface of fourth class [or degree] requires at least ten quartics.)

In a sense, Reye completed his works on mass geometry by developing the general term of "non-polar" surfaces. The origin is the fundamental definition: Let an arbitrary surface of class n Φ^n be represented as an n^{th} zero surface of a system of mass $m_i(x_i, y_i, z_i)$, where $i = 1, 2, 3, \dots$, by the equation $\Phi^n = \sum m_i(\alpha x_i + \beta y_i + \gamma z_i - p)^n$, where α, β, γ, p are plane coordinates. Furthermore, let a surface of degree k be represented by $F^k(x, y, z) = 0$. Using the elements $M_i = m_i \cdot F^k(x_i, y_i, z_i)$ one can generate a new system of mass and by means of it the equation $\Pi^{n-k} = \sum M_i \cdot (\alpha x_i + \beta y_i + \gamma z_i - p)^{n-k} = 0$. This surface of class $(n-k)$ is called the polar from F^k to Φ^n . In general, Π^{n-k} is determined by Φ^n and F^k , but it can happen that all coefficients in the polynomial Π^{n-k} are equal to zero when it is arranged with regard to the variables α, β, γ, p . The result is that a certain polar from F^k to Φ^n does not exist anymore. In this case, F^k is called non-polar to Φ^n . For the simple case that $n = 2, k = 2$, the non-polarity depends on the only condition $\sum m_i \cdot F^2(x_i, y_i, z_i) = 0$. This is identical to the disappearance of the simultaneous bilinear invariant of the polynomials Φ^2 and F^2 , which first appeared in Hesse's theory of pole tetrahedrons of the surfaces of second degree and is of fundamental importance there (cf. lectures on analytic space geometry, 1st edition, p. 153).

Another sequel to the work on moments of inertia emanated from the theorems: With respect to a given system of mass M each point O in the space generates an ellipsoid of inertia E , whose centre is O ; the principal planes and

principal axes of E shall also be referred to as the principal planes and principal axes of O (for the centroid S of the system in particular they are called central planes and central axes). A straight line g is a principal axis only if it is perpendicular to its polar with respect to the “imaginary image” of the system of mass. Infinitely many principal axes pass through an arbitrary point P in space, which form a cone of second degree. In an arbitrary plane there are infinitely many principal axes that envelop a conic.

The second part of *Projective Geometry* built on that, and the new ideas of Plücker on the geometry of straight lines, published in English journals in 1865¹, were implemented. The lines previously referred to as main axes form a (∞^3) manifold, which is denoted a ray complex of second degree.² It can be defined directly, without any relationship to the system of mass, as the epitome of all straight lines that are perpendicular to their polars, with respect to a given surface of second degree. The complex is determined by the surface, but the converse is not true, the surface is not determined by the complex. This then leads to theorems on the normals of a system of concentric-homothetic surfaces of second degree and to theorems on the normals of a confocal system. – Much later Reye occupied himself with finding a classification of the complexes of second degree given by the most general equations – analogous to dividing the surfaces of second degree given by the most general equations into 1. real F_2 [a) with real, b) with imaginary lines], 2. imaginary F_2 . He ended up with 8 different types, of which 3 are split into 2 subcategories each.³

Reye studied an analogue to the ray complexes by considering all spheres in space that form (∞^4) manifolds and by extracting (∞^3) manifolds because of analytic or geometric conditions. He called these (∞^3) manifolds “sphere complexes”.⁴ These structures are much easier to deal with than the ray complexes [which, as (∞^3) manifolds are created, from the ∞^4 lines in space]. If the equation of a sphere A is given in Cartesian coordinates by $A = \alpha_0(x^2 + y^2 + z^2) - 2\alpha_1x - 2\alpha_2y - 2\alpha_3z + \alpha_4 = 0$, then $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are called the homogeneous coordinates of A . Now let $X = \xi_0(x^2 + y^2 + z^2) - 2\xi_1x - 2\xi_2y - 2\xi_3z + \xi_4 = 0$, then we see that for all spheres X , which intersect with A in a right angle, their homogeneous coordinates satisfy the equation $\alpha_4\xi_0 - 2\alpha_1\xi_1 - 2\alpha_2\xi_2 - 2\alpha_3\xi_3 + \alpha_0\xi_4 = 0$. As this equation is linear with the ξ , we say that the X form a linear sphere complex. To express it more generally: Let the angle of intersection of A and X be φ , then

$$\cos^2 \varphi \cdot (\alpha_1^2 + \alpha_2^2 + \alpha_3^2 - \alpha_0\alpha_4) (\xi_1^2 + \xi_2^2 + \xi_3^2 - \xi_0\xi_4) = \frac{1}{4} (\alpha_4\xi_0 - 2\alpha_1\xi_1 - 2\alpha_2\xi_2 - 2\alpha_3\xi_3 + \alpha_0\xi_4)^2$$

¹ Plücker’s main paper on this topic “New geometry of space, based on observations of the straight line as an element in space” (*Neue Geometrie des Raumes, gegründet auf die Betrachtung der geraden Linie als Raumelement*) was published only after his death 1868/69, i.e. later than Reye’s book.

² It is a very special complex of second degree; before Reye, Chasles already treated it.

³ For both the complexes and for the F_2 it is assumed that the equations contain real coefficients only.

⁴ Sophus Lie had already given a theoretical relationship between ray geometry and sphere geometry before Reye’s works were published.

Hence, the spheres X that intersect with A in a given angle φ form a complex of second degree. For $\varphi = 90^\circ$ we get the complex of first degree that we have found already twice; for $\varphi = 0$ or $\varphi = 180^\circ$ we get the complex of second degree, whose spheres touch A . The equation $\xi_1^2 + \xi_2^2 + \xi_3^2 - \xi_0\xi_4 = r^2\xi_0^2$ gives the spheres with radius r , i.e. for $r = 0$ the complex of zero spheres.¹ From these few basic terms and their future development we can easily derive a number of theorems on intersection and touch of spheres analytically. (Cf. Reye's little book "Synthetic Geometry of Spheres and Linear Systems of Spheres" (*Synthetische Geometrie der Kugeln und linearen Kugelsysteme*) as well as the paper "On Quadratic Sphere Complexes" (*Über quadratische Kugelcomplexe*), *Crelle*, vol. 99).

Connected to the research on ray complexes and on sphere complexes, both of which are created as (∞^3) manifolds on (∞^4) manifolds, are the six papers on "linear manifolds of projective basic structures" (*über lineare Mannigfaltigkeiten projektiver Grundgebilde*) (*Crelle*, vol. 104-108), as the possibility of a purely geometric identification of a manifold of three or more dimensions is discussed in the introduction to the fourth paper (vol. 107, p. 162).

At the end of the indications about the different directions of Reye's scientific work we will point out his work on curves of intersection and systems of point of intersection of algebraic surfaces. He examines Jacobi's theorems and gives interesting results particularly on the intersections of surfaces of second degree. His geometric (linear) construction of the eight point of intersection of three F_2 when the other seven points are given is a very fine piece of work; it is arguably the simplest solution to this often examined problem. (Cf. *Crelles Journal*, vol. 100. Volume 99 contains solutions by Hesse [Caspary], Schröter, Sturm, Zehnter; of which the first one is analytic and based on the properties of the orthogonal transformations of a homogeneous function of second degree with four variables).

VII.

At the age of 70, Reye retired from his post as a lecturer. On this occasion, he could proudly say that he had achieved a lasting success in his tireless ideal striving for his "favourite science". This was recognised over the course of time; the Gesellschaft der Wissenschaften in Göttingen (Science Society Göttingen) and the Academy in Bologna appointed him a corresponding, the Accademia dei Lincei in Rome appointed him an external member. Furthermore, he was also one of the honorary members of the Zürcher Naturforschende Gesellschaft (Zurich Society for Nature Scientists), not to mention other distinctions. But more important to him than these visible gestures was the knowledge that he fulfilled his post true to the view that he put into modest and non-personal words at some later point: "We took it for granted that we professors were appointed to Strasbourg in 1872 in order to nourish and distribute science. Everybody tried hard to do his best without talking much about it."

¹ Since the zero sphere is identical to the cone that stands above the imaginary circle K_∞ in space at infinite distance, its apex being the centre of the sphere, the zero sphere complex simultaneously is the ray complex of the lines that intersect K_∞ .

This part of a letter taken from the aforementioned book by Lorey, published in 1916, already leads us into the period of the big world-changing events and now sounds like a wistful obituary on the university to whose rise Reye contributed and whose downfall he had to witness. But beginning and end were determined by the outcome of wars, which shall be mentioned here as in doing so, individual people in our presentation appear on an important historic background.

On 24 December 1867, at Poncelet's funeral, the Académie des Sciences, the Faculté des Sciences (where he established the "Course in physical and experimental mechanics" (*Cours de Mécanique, physique et expérimentale*) that became famous far beyond the boundaries of the lecture theatre) and the Comité du Génie paid their last respects to this glorious mathematician, technician and combat engineer officer. The first speaker was the famous geometer and smart politician Charles Dupin; at the beginning of his speech he referred to the native town of the deceased: "Metz où tout respire à la fois la science et la guerre – devant laquelle se brisaient autrefois les efforts de Charles-Quint, et devant laquelle se briseraient encore les efforts de quelque empereur improvisé des bords du Rhin et de la Moselle."ⁱⁱⁱ And yet, the prophet (born in 1784) had to witness that Metz fell and that the "empereur improvisé" was greeted with his new title for the first time in the Hall of Mirrors in the Palace of Versailles.

After the annexation of Alsace-Lorraine to Germany, Metz developed into a stronghold against France. In Strasbourg, the peaceful task of the university, equipped with the most abundant funds and named after Kaiser Wilhelm, became important, in addition to the provision for military protection. The objective of the university was to channel the intellectual interests of the newly gained citizens towards the empire to which they were now annexed, resulting in supporting a lasting affiliation to the Empire.

Reye, who was a good German¹, but in no way of a jingoistic nature, and whose agreeable, modest manner surely gained him the vivid sympathy of his Alsatian students, too, may have believed that the harmonisation and assimilation gradually made more and more progress. We have reason to suspect that because of an almost incidental paragraph in his rectorship speech from 1. V. 86 on "The Synthetic Geometry in Antiquity and in the Modern Age" (*Die synthetische Geometrie im Alterthum und in der Neuzeit*). It says there that an important, demonstration-based method for transferring fundamental theorems from circles onto conics and cones was not developed until the 19th century. "We owe this method of perspective projection mainly to a son of our Empire, Poncelet, born in Metz in 1788." What would the citizen so devoted to his hometown, the brave soldier and fervent patriot have said about this citizenship devolved upon him?

Reye's words were by all means meant to be inoffensive, but some sentences in the speeches held at the centenary of the technical college in Berlin-Charlottenburg (18./21. X. 99) and at two subsequent ceremonies

¹ Reye decided in 1863 to join a planned Freischarenkorps [a volunteer military unit] to assist with the liberation of Schleswig-Holstein from Danish dominance (which of course never came about due to changed political circumstances).

shortly afterwards (January 1900) sound much more self-confident and fierce.¹ The engineering professor Riedler was elected as rector for this jubilee year, at first probably because of his excellent aptitude for representing, but then also in recognition of his successful contribution to the re-organisation of the college² and, by extension, to higher technical education in general. He offered the foreign guests of honour the opportunity to get to know him at a splendid dinner already on the day before the main ceremony. At the end of this dinner, even the present “pure” mathematicians cheerfully joined the tribute proposed to the host, although the genius of “higher algebra” dolorously veiled his head.

On the day of the main ceremony, firstly the monuments of Werner Siemens and Alfred Krupp, the two world-renowned representatives of German engineering and industrial organisation, were unveiled on the forecourt in front of the outer staircase of the college. And now, as the highlight of the entire celebration, it was proclaimed that the Prussian technical colleges should henceforth have the right to “award a doctorate to graduate engineers^{iv} by virtue of an examination so that they become Doctors of Engineering (shortened notation, and in German font, in fact: Dr. Ing.).”

Surrounded by his colleagues and the most high-ranking civil servants in his department, the Minister of Education read out the certificate, then the rector stepped “towards the steps of the throne” in order to express the colleges’ gratitude for this proof of grace (as well as for the appointment of representatives of the same to the [Prussian] House of Lords). But the highlight of the highlight was the Emperor’s speech, “which will gleam in golden letters in the history of technical sciences and their colleges in perpetuity ... It was as if the spirit of the age itself spoke, as if the wing beat of a great future whooshed above these crowds gathered in front of their imperial master.”

One can imagine how on this Byzantine gold-leaf ground the speeches of the celebration (of which, by virtue of his position, a fair number fell to the rector) proved to be an exceptional expression³ of the sentiments of power and sure victory of a great nation, which was in the process of resounding material advancement: “The end of the century witnesses the downfall of the Romanic people, while the Germanic culture is about to conquer the world. It sees Germany as the leading political and economic power, shielding its productive work with a steel-clad fist.” – “Germany’s future lies on the sea ... The organisation of the German naval fleet is the next big task of the next century, of the German Empire and of German technology.” – “Wars have become the more seldom the more perfect the methods used have become: The fear of these methods forces love of peace, and the Empire that is armed the best offers peace.”⁴

¹ Cf. “Essays in honour of the centenary” (*Festschrift zur Jahrhundertfeier*) and the “Chronicle of the College 1799-1899” (*Chronik der Hochschule 1799-1899*).

² *Chronik*, p. 179-198. Riedler also made personal sacrifices so as to implement his ideas by donating machinery amounting to 120,000 and 24,000 Mark for the new laboratory of which he was in charge.

³ Which is even enhanced by the immediate succession of sentences here.

⁴ How petty bourgeois did the congratulations presented by the Federal Polytechnic appear compared to these sentences. The congratulations concluded with the

The future revealed itself to be completely different to how these glowing speeches seemed to announce it to be. The big battle for world domination, which flickered through these speeches like sheet lightning in the sky, has come to a temporary end. And neither the military skills of Hindenburg-Ludendorff, nor the immense force and the most pertinacious resistance of the armies of millions of soldiers, nor the powerful naval fleet and the most perfect technical means of war were able to exact the decision in accordance with the fanfares at the time.

In a paper on the death of the governor field marshal Manteuffel (1885), a great historian wrote the oddly convoluted sentence, "that Alsace was torn off France and tied to Germany by the decision of weapons, so as the old believed and the new assure, by God's will." The imperial territories are re-united with France, under circumstances, for which one could give a similarly justified explanation. But we can still hear the warning words of old Moltke (on 14 May 1890): "If this war, which has been hovering above our heads like the sword of Damocles for more than ten years, if this war breaks out, then its duration as well as its end will not be foreseeable. The greatest powers in Europe, armed like never before, will enter a war against each other; none of them can be crushed so thoroughly in one or two campaigns that it would declare itself defeated, that it would make peace accepting rigorous conditions, that it would not come back in full force, albeit after a year's time, in order to revive the battle. It can turn into a seven years' war, it can turn into a thirty years' war." Whether the League of Nations, which will have its first meeting in Geneva within a few weeks, will succeed in averting a return of the murdering and the ravage we had to endure during the years of war – who can tell?

After having given up his lectures, Reye, as professor emeritus, still attended the meetings of the lecturing staff; a peaceful and intellectually stimulating existence in familiar surroundings seemed secured until the natural ending. But the war also created grave sorrows in his family: a son who had been under arms for the entire duration of the war came back home safe, but a grandson was killed in action. In the autumn of 1918, Reye decided to return to the old Germany, hoping that he would be able to spend the last days of his life in peace and quiet. Unfortunately, the relocation created various difficulties, which, when an order was rescinded by a counter-order, turned into the most repugnant delays and eventually forced the eighty-year-old to conduct the complicated relocation without the help of his son that he had hoped for. At least he could eventually settle down in the comfortable home in Würzburg, which a son-in-law had chosen for him. In May 1919 he was even able to celebrate his golden wedding; but the shocks, disappointments and the agitation of the last months had such strong after-effects that he fell victim to them a few weeks after the anniversary.

following words: "May this new period that begins for your college widen and increase its benedictory activities; may it remain a blazing site of joyful work on unifying creations, arts and sciences of peace also in its second century."

* The full title of Stäckel's book is "The Mathematical Education of Architects, Chemists and Engineers at German Technical Colleges" (*Die mathematische Ausbildung der Architekten, Chemiker und Ingenieure an den deutschen technischen Hochschulen*)

ⁱ Translation: "The English teach mechanics as an experimental science; on the continent it is always more or less presented as a science that is deductive and a priori. It goes without saying that it's the English who are right."

ⁱⁱ Translation: "The unanimous consent guarantees that astronomy holds the first place among all the sciences. Only astronomy has displayed an invariable and precise rule that explains everything. If studying the skies were to bring about a restriction to Newton's law, as small as it may be, then astronomy would lose its crown. Le Verrier preserved it for himself."

ⁱⁱⁱ Translation: "Metz where everything exhales science and war at the same time – in front of which the efforts of Charles V have failed in the past, and in front of which the efforts of some improvised emperor from the banks of the Rhine or the Mosel will fail, too."

^{iv} The German university degree Diplom-Ingenieur (can be translated as graduate engineer) stands academically between today's B.Sc. (Eng) and the M.Eng. A Doctor of Engineering is called Doktor-Ingenieur.

E.3.3 F Rudio: *Leonhard Euler*

Lecture given in the Town Hall of Zurich, 06 December 1883¹; *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich* **53**, 1908, 456-470. Footnotes by Rudio.

The translation is also available on the MacTutor History of Mathematics website: http://www-history.mcs.st-andrews.ac.uk/Extras/Rudio_Euler.html.

Ladies and Gentlemen!

A few weeks ago we witnessed the centenary of the day when mathematics lost one of its most outstanding representatives, a man whose name no mathematician will mention without a feeling of utmost admiration even today: Leonhard Euler from Basel. Under these circumstances and at this meeting in particular I will surely need no further justification for giving you a broad overview over the life and works of this distinguished mathematician.

The human intellect, ladies and gentlemen, manifests itself in the most diverse shapes and forms, but despite this diversity few are blessed with the ability to set themselves apart from their fellow human beings, and fewer still have the privilege of leaving lasting marks of their lives. Thus, when we know of a man, characterised by outstanding intelligence or an unusual artistic aptitude, and see that he employs his talents for the benefit and enjoyment of his fellow human beings, then, although we will not be able to follow all of his ways, we will have sympathy for him from a human point of view alone because we see a bit of human perfection embodied in him, to which we aspire ourselves, though often in vain.

And just imagine to what extent our interest is awakened when we talk about a man whose tremendous genius exceeded the ordinary to the extent that he left a mark of his intellect on an entire science for a whole century and beyond.

Leonhard Euler was such a man.

Lately an academic celebration commemorating Euler took place in his native town. The purpose of this celebration was to attest to the fact that we fully appreciate the great legacy that Leonhard Euler left us. With this in mind, this evening may be seen as a commemoration, which we dedicate, in grateful acknowledgement and admiration, to the Manes of a scientist whom you may regard as one of the greatest prides of your home country Switzerland.

¹ This lecture, first published in the well-known collection by Benno Schwabe, has been out of print for many years now. Although demand never ceased, I would never have considered to re-publish it, had it not been for the fact that just now, as we are about to launch a fundraising campaign in aid of an edition of Euler's works, people expressed the wish that the call for donations would be accompanied by a short biography of Euler aimed at the general public. Thus, the lecture that I gave more than a quarter of a century ago may set off into the world again. And if it helps contribute to Euler's works finally being reborn in an edition worthy of the eminent mathematician, then the reprint is not uncalled-for after all. And at the same time a wish will come true, a wish that I had already hinted at then (see p. 462), but at the time I did not dare hope that I would witness its implementation in my lifetime.

Leonhard Euler² was born on 15 April 1707 in Basel. His father, Paul Euler, was a preacher at St Jakob; his mother, Margarethe, came from the Brucker family. Euler spent his first childhood years not in Basel, but in the nearby village Riehen, whereto his father had been appointed as preacher already in 1708. The humble rural conditions of Leonhard Euler's upbringing surely contributed to his simple, modest attitude, as well as his impartiality, which he managed to preserve up to old age. There is an amusing anecdote from when he was four years old. Living in the countryside, the young Leonhard naturally had many opportunities to see how hens hatch eggs and thus produce their young. This natural process must have strongly impressed the young boy: one day he went missing, but after a long search he was eventually found in the henhouse, sat on top of a large pile of eggs that he had collected. To the bewildered question what on earth he was doing there he replied, with childlike earnestness, that he wanted to hatch chickens.

Leonhard's first teacher was his father, who prepared him for entry to the secondary schools in Basel. Note that Paul Euler liked to study mathematics when he was young; he had been regarded as a talented student of the great Basel mathematician Jakob Bernoulli. Thus it is not surprising that mathematics had a special place in his teaching. However, he did this not because he wanted to make a mathematician of his son; in fact, for him it was a matter of course that Leonhard would become a preacher, too, some day and possibly even his successor in Riehen. He appreciated mathematics as a useful training of the mind and because he saw it as the basis of any solid scientific education.

Leonhard Euler was a man of many talents, which meant that it was not too difficult for him to obey his father's wish. Thus, when he matriculated at the University of Basel later on, he did join the faculty of theology and studied oriental languages with great enthusiasm. But Euler's readiness of mind and incredible memory allowed him to engage in deep mathematical speculations as well and in particular to attend lectures by Johann Bernoulli, the brother of Jakob Bernoulli. Soon he attracted the attention of his teacher to such an extent that Europe's most famous mathematician at the time did not consider it below him to grace this youth of barely sixteen years with a more personal relationship.

Fortunately, around about that time Leonhard succeeded in finally getting his father's permission to fully devote his studies to his favourite subject, mathematics. Paul Euler had understood that his son was not born to live the contemplative life of a humble country preacher, but that he was destined to take the lead in mathematics some day, as a worthy successor of the great Bernoullis.

After Euler had gained all common academic qualifications, he competed for a prize that the Paris Academy awarded for the best paper on the rigging of ships at the young age of nineteen. Admittedly, he was awarded only second place, but the young mathematician, who had never left Basel and hence had never seen a big ship, had the satisfaction of having been defeated

² For the biographical part, I used the commemorative speeches by Condorcet and Fuss, the *Correspondance mathématique* published by Fuss, as well as the *Biographies in Switzerland's Cultural History* by Prof. R Wolf.

only by a nautical engineer, who had been considered an authority both in theory and in practice for many years.

Around about the same time, in spring 1727, Euler applied for the vacant professorship of physics in Basel. At the time there was a curious custom at the University of Basel: the successful applicant was chosen from among the approved candidates by lot. The outcome was to Euler's disadvantage and at the same time influenced the whole of his living conditions.

Two years previously, the two brothers Daniel and Nikolaus Bernoulli, sons of Johann Bernoulli and friends of Euler, had been appointed to posts at the St Petersburg Academy then founded by Catherine I of Russia. Before they had left Basel, they had promised their young friend that if at all possible they would get him a position at the St Petersburg Academy, too. Now Euler received a message saying that a suitable vacancy had opened up, as long as he would be happy to lecture on physiology rather than mathematics. "Come to St Petersburg as soon as you can and show the Academy that, although I have told them many good things about you, I have not told them everything by far. I would argue that by your appointment I render a much greater service to our academy than to yourself", Daniel Bernoulli wrote to the then nineteen-year-old Euler.

The prospect of getting a job at such a big academy was too tempting for Euler to be put off by the condition mentioned above. He had a remarkable knowledge of the sciences as well and had already written a theory of sound. After having familiarised himself with anatomy and physiology with great enthusiasm and success, he left his fatherland that very year, 1727, at the young age of twenty. He never returned to his home country.

Upon his arrival in St Petersburg he was appointed assistant at the mathematical institute at the Academy straightaway. Strangely enough, nobody ever mentioned physiology anymore. Euler had the privilege of working alongside his friend Daniel Bernoulli for six years. The lively rivalry that developed between the two great mathematicians and that persisted until Daniel Bernoulli's death in 1782 was very important to mathematics. At this point I would like to emphasise that their friendship was never tainted by jealousy; a factor that adds a particular zest to reading their extensive correspondence, published by Fuss.

Daniel Bernoulli returned to Basel in 1733 since he could not tolerate the climate in St Petersburg, to which his brother Nikolaus had fallen victim already a few years previously. Although Euler was a mere 26 years old then, his scientific importance was already so widely recognised that nobody had any concerns about appointing him Daniel Bernoulli's successor.

In 1735 Euler gave a truly startling demonstration of the astonishing effortlessness with which he solved even the most complicated problems, but unfortunately it had a most catastrophic outcome for him. The Academy had been commissioned to make some astronomical calculations that had to be carried out in the shortest amount of time possible. All the other mathematicians at the Academy said that they would need several months for these calculations. Euler completed them in the space of three days. But what a sacrifice he made to science! The – one would almost say superhuman – strains to which he subjected himself during those three days led to a dangerous illness, which in turn resulted in the loss of his right eye. For any other person, the loss of such an important organ would have been a very

good reason to take it easy, but Euler's industriousness increased rather than decreased because of the misfortune that befell him.

Meanwhile, the political circumstances in Russia had become unbearable for any intelligent human being, and surely I may add that this was the case for a Swiss in particular. You are all too familiar with the favouritism that spread under the successors of Peter the Great – some of them incompetent and some of them despotic – that I do not have to go into more detail about these unfortunate conditions, which are summarised in this sentence: "The Russian constitution is despotic, albeit alleviated by assassination."

Thus it is understandable how happy Euler was to accept the brilliant offer that Friedrich the Great made him in 1741.

It is well known that Friedrich I, the grandfather of Friedrich the Great, set up the Berlin Academy of Sciences in 1700; its first president was Leibniz. Under his successor, Friedrich Wilhelm I, whose well-known preference for soldiers did not allow for a proper understanding of scientific ambitions, the Academy decayed. When Friedrich the Great ascended to the Prussian throne in 1740 he did so with the ambition of putting his country in a respectable position, not just with regards to politics, but also to social issues and science.

To that end he first attempted to resurrect the Berlin Academy by appointing the most distinguished scholars in Europe.

Ladies and gentlemen, it must make you proud indeed to learn that among all the mathematicians alive then, the 34-year old Leonhard Euler from Basel was regarded as the most worthy one to head the series of many distinguished names that have since adorned this famous institute. Euler came to Berlin in 1741 and was appointed director of the Academy's mathematical institute immediately. He held this post for 25 years, until 1766; and he was, alongside Voltaire, undoubtedly the most prominent representative of the select circle that gathered around Friedrich the Great at the time.

The following anecdote gives us an impression of the pressure under which Euler must have lived during his last months in St Petersburg: The Queen Mother once wondered about Euler's conspicuous reticence, which he had no reason for whatsoever as she had always treated him most kindly. Euler did not fail to give the necessary explanation. "I come from a country where one gets hanged when one speaks", he replied.

Ladies and gentlemen, I have arrived at a period in Euler's life where it would be suitable to have a look at the scientific works of this marvellous man.

Now, you could not possibly expect me to cover the progress that is linked to Leonhard Euler in great detail. The nature of this lecture bans me from doing this, but Euler's immense productivity, possibly unparalleled in the history of all sciences, does so even more.

If I wanted to quickly read out only the titles of his works, I would need more time than you would want to grant me, as the index of all of his works alone fills more than 60 printed pages. This index lists more than 800 scientific publications, among them many that fill thick volumes. If one were to publish a complete edition of his works, which, I'm sorry to say, we do not have and might never have, then this edition would comprise 40 stately quarto volumes. After having returned to St Petersburg later on, Euler claimed on several occasions that he would be able to write so many mathematical papers that they would last the Memoirs of the Academy for 20 years after his death.

And he did more than he had promised: His papers adorned the Memoirs of the St Petersburg Academy until 1823, i.e. 40 years after his death, and the papers that had been left in the archive were published in 1830. Furthermore, when his works were compiled in 1843, i.e. 60 years after his death, and it was believed that his mammoth legacy had finally been conquered, all of a sudden more than 50 further unpublished papers were found, which had been missed all the same.

Ladies and gentlemen, you are amazed already having just heard this dry listing; how much more would you marvel if I were able to acquaint you with the content of Euler's papers, too. But you will rightfully ask me to give you a broad overview over the area where Euler has achieved so many great things. I believe that I will accomplish this most easily by starting with a few introductory remarks on the relationship between mathematics and the sciences.

The task of the sciences is to find the laws that govern the physical world, i.e. to fathom the dependencies among the individual phenomena. Whether these connections are logically necessary, i.e. not possible in any other way, is an idle question. To us, the – admittedly empirical – certainty with which we can deduce the appearance of one phenomenon from the appearance of another phenomenon suffices and must suffice.

In modern sciences, people aim to consider all phenomena in terms of motion: the nature of sound consists in the vibrations of a sounding body and sound is transmitted to us by means of oscillations through the air around us; the nature of light consists in the oscillations of this extremely fine, weightless matter that is called aether and permeates all bodies, according to the wave theory of light established by Huygens and Euler and developed by Fresnel and Thomas Young; the nature of heat consists in a more or less intensive motion of the smallest particles of the heated body, according to the principles of thermodynamics. I deliberately mention these examples because the name Euler is linked to all of them, as we will see shortly.

Ultimately, the laws that govern these phenomena of motion are expressed in terms of numbers. May I use this opportunity to point out to you how accurate a feeling the Pythagoreans had developed more than 2000 years ago, in considering numbers to be the ultimate underlying principle of all being. Our modern point of view differs from the Pythagorean one only insofar as we have actually conducted experiments on a range of natural phenomena and have shown that ultimately, their nature can indeed be expressed in terms of numerical ratios.

Please allow me to pick some of the above examples and use them to derive the terms that one should know in order to be able to get an impression of Euler's research areas.

First of all, imagine that you are on top of a high tower and drop a stone. The laws that govern the motion of the falling stone have first been laid down by Galilei and can be summarised as follows: If you measure the time that the stone needs in order to cover a certain distance, it will cover the exact same distance in the same amount of time, no matter how often you repeat the experiment. If the time is doubled, the stone will cover a distance that is 2×2 or four times as long; if it's tripled, the distance is 3×3 or nine times as long; if the time is ten times as long, then the distance covered is 10×10 or 100 times as long, etc. Thus, if you have measured that the stone covers a distance of 5

metres in one second, you are now able to calculate the distance that the stone will cover in an arbitrary amount of time. For example, in 4 seconds it will cover a distance that is 4×4 , i.e. 16 times as long as the distance it covers in one second, i.e. in 4 seconds it will cover a distance of 16×5 or 80 metres. Of course, you have to count the seconds from the point onwards when you drop the stone.

You can see that the distance that an object covers in free fall in a given time can be deduced mathematically. We say that the distance covered is dependent on the length of time or that it is a function of time, and since 2×2 is called the square of 2, 3×3 the square of 3, and 10×10 the square of 10, we say that the distance increases proportionally to the squares of the given lengths of time.

Let us move on to a second example: to Kepler's laws. Kepler discovered that the Earth orbits the Sun in an ellipse, with the Sun being one of its foci. In more popular terms, the Earth rotates around the Sun on a circle-like line and the Sun is located almost at the centre of this circle. Now imagine a line drawn from the Sun to the Earth. We will call this line a radius vector. As the Earth moves around the Sun, this radius vector moves around the Sun and sweeps out a portion of the circle-like area during a given interval of time. We call such a portion a sector. Kepler's second law now states that the Earth orbits the Sun in such a way that the radius vector sweeps out equal sectors during equal intervals of time. Thus, if you have found by observation how large a sector the radius vector sweeps out in, say, one hour, then the sector swept out in two hours will be twice as large, the one swept out in three hours will be three times as large, the one swept out in ten hours will be ten times as large, etc. We say that the sector is dependent on time, or a function of time. Moreover, the sector grows proportionally to time.

Let us move on to the third example. Imagine a point of light and, at some distance from it, a sheet of white paper. Then the point of light illuminates the sheet to a certain degree. The brightness of the sheet increases the closer the sheet is to the point of light, and decreases the further away it is from the point. Now if you measure the sheet's brightness at a given distance, then the brightness will decrease by a factor of 2×2 or 4 as the distance is doubled, by a factor of 3×3 or 9 if the distance is three times as big, by a factor of 10×10 or 100 if the distance is ten times as big, etc. We say that the brightness is dependent on distance, or that it is a function of distance. In particular, we say that the brightness decreases proportionally as the squares of distance increase.

Having heard these examples, you will now understand what we mean by saying that the aim of the sciences is to express the interdependencies of individual phenomena in terms of mathematical functions, in that we consider a function to be the dependency of two quantities expressed in terms of numbers. Since the natural phenomena are dependent on each other in the most varied ways, there are infinitely many mathematical functions – but please do not believe that these dependencies and hence the corresponding functions are always as simple as in the examples I mentioned above. There are some highly complex cases around. We will now refer to mathematics as the language with which natural phenomena can be described most simply and at the same time most thoroughly. As an example, it would not be possible to describe the motions of Earth around the Sun in a more simple and comprehensive manner than by Kepler's laws.

We have now arrived right at the centre of Euler's research area. It is one of Euler's main achievements to have studied the myriad of functions that were either offered to him directly by nature or that his ingenuity first had to derive, in extenso for the first time: he investigated their properties and identified the source of these properties, and he grouped them together according to common features. Furthermore, he ascribed functions that had been regarded as distinct, such as the so-called trigonometric and exponential functions for example, to each other. He dedicated two particular major works to these investigations: his *Introduction to Infinitesimal Calculus* and his *Manual for Differential and Integral Calculus*. Even today, more than a hundred years later, these books are still the most readable of all textbooks on higher analysis. Although many books have been written on this topic since, almost all of them are more or less variations of the area studied by Euler.

But I cannot move on from reviewing Euler's mathematical work without having considered an important factor. I have said that mathematics is a language in which natural phenomena can be described in the simplest and most comprehensive manner. With this in mind, you will understand how important it is to express mathematical thoughts themselves as concisely and clearly as possible. In this respect, Euler's work was epoch-making. We can be safe to say that the whole form of modern mathematical thinking has been created by Euler. If you read any author immediately before Euler, it is very difficult indeed to understand his terminology, as he has not yet learned how to let the formulas speak for themselves. This art was not taught until Euler came along.

But Euler was not just a great mathematician, he was also a great physicist and astronomer. He wrote several rather major works on the motion of celestial bodies and was the first to write on analytic mechanics. He might have been the only one in his century to have a correct idea of the nature of heat: he taught that there is no special heat matter, but that the nature of heat consists in the motion of the smallest particles of the heated body. Furthermore, and in opposition even to an authority like Newton, he supported the theory first expressed by Huygens that the nature of light does not consist in a special light matter, but in the oscillations of the aether filling the universe.

Please allow me to point out a special achievement of Euler's in optics. You will all have noticed at some point that, when you look through cut glass, i.e. a glass prism or a glass lens, the objects you see appear to have not only contorted shapes, but also coloured fringes. These coloured fringes are caused by the varied refrangibility of the individual colours that are the components of colourless light. But when using optical instruments, such chromatic fringes are highly distracting – you can see this for yourselves by using a bad lorgnette, for example. Euler discovered that these chromatic distortions are absent in the human eye, which, at the end of the day, is an optical instrument, too. The reason for their absence is that light passes through several substances of different refractivity inside the eye, so that the various chromatic distortions cancel out. Inspired by this discovery, he calculated which combinations of lenses one has to use in order to construct achromatic instruments, i.e. instruments free from those chromatic distortions. When Euler published his results, he was attacked most severely from all directions; in particular by the English physicist Dollond, who referred to Newton's

explanation that achromatic instruments were considered impossible. But Euler was so certain that his calculations were correct, that he did not budge until his opponent Dollond himself constructed the first achromatic telescope using a combination of flint and crown glasses in 1758. In doing so, he validated Euler's results most splendidly. Dollond is generally referred to as the creator of this seminal invention for optical instruments in physics textbooks, but it would only be an act of justice if Euler were mentioned alongside Dollond, as indeed he is the intellectual creator of this important invention.

Although Euler was without a doubt the most important mathematician of the last century (and perhaps of all centuries), he still found time to delve into studying a range of purely practical problems. I will only mention that we owe a comprehensive treatment of artillery sciences to him, in which he developed a complete theory of the motion of thrown objects. Furthermore, he made a range of valuable contributions to shipbuilding by developing the theory of floating bodies and deducing which shapes ships need to have in order to be as manoeuvrable as possible whilst also being as stable as possible. These works caused greatest sensation at the time and have been translated into almost all European languages.

I have now reached an area in this short overview over Euler's merits that might be of particular interest to you: his popular works.

I will only talk about one of them, Euler's Letters to a German Princess. These letters are addressed to a niece of Friedrich the Great and are the continuation of the lessons that she received from Euler. He covers the most important aspects of astronomy, of mathematical and physical geography, of physics, and of philosophy in 234 mostly very short letters, using such a clear, lucid language – I would almost go as far as to say pleasant – that the letters may still be considered an exemplary popular presentation. Time bars me from looking at them in more detail so I have to limit myself to pointing out those letters to you. However, I would count myself lucky if this evening would at least result in these Letters to a German Princess by Euler attracting the interest that they so highly deserve among a wider circle of readers.

Euler left Berlin in 1766, i.e. in the 60th year of his life, and returned to St Petersburg. The motivations for this change were in part some differences with the Berlin Academy, but in particular the splendid offers made by Empress Catherine II and which Euler, who had a very big family, did not dare reject. Having barely arrived in St Petersburg, he was taken severely ill. Although he recovered in time, he lost also his second eye entirely. Thus, Euler was completely blind during the last 17 years of his life. However, now Euler's unusual mental abilities truly showed themselves: henceforth, the totally blind old man developed an almost frenetic activity; almost half of his entire production dates to the years when Euler was bereft of a scholar's most valuable organ.

Shortly afterwards, he was hit by another misfortune, which, given the circumstances, must have affected him particularly severely. His house, a gift from the Empress, fell prey to a big fire. His library and part of his manuscripts were burnt. He himself would have died in the fire had it not been for a man called Grimm, who was from Basel but lived in St Petersburg. He saw the danger that his famous compatriot was in, entered the burning

house at the risk of his own life and carried the blind old man out of the flames across his shoulders.

We truly have to admire the exceptional calmness and serenity of mind that Euler must have possessed to be able to return to his scientific research again and again after such severe strokes of fortune. Admittedly, his incredible imagination and a downright phenomenal memory certainly helped him in this. Euler belonged to those mathematicians who had the entirety of their science at their command at every moment.

The subsequent details might offer an indication of his mental abilities. In a sleepless night, Euler, aged 75, calculated the first six powers of the first 20 numbers and recited them forwards and backwards for several days. In his old age he still knew the entire Aeneid by heart; in fact, he could state the first and last verse on every page of the edition that he used in his youth.

Euler possessed what we today call a general education to a very high degree. He had a thorough knowledge of classic Antiquity, of history and of literature. He knew more about medicine and the sciences than most people; we have heard that he was appointed a physiologist at the St Petersburg Academy at the mere age of twenty. He dedicated his leisure time to the musical arts, but he revealed himself as a mathematician at the piano, too, he even wrote a theory of musical arts.

Euler was an excellent person, not something that you can say about every great man. He was unusually kind-hearted and almost naively pious. You might also be interested to learn that Euler never stopped being a Swiss, for although he lived in Berlin for 25 years and in St Petersburg for 31 years, he always used the genuine Basel vernacular with all its peculiarities, often to the amusement of those around him.

Euler's death was worthy of a great scholar; he passed away whilst being engaged in research. Even on 18 September 1783 he studied the motion of balloons, which had just started to emerge, with the same vivid interest with which he approached every new invention. He had mastered a complicated calculation and talked about it with a friend, when he suddenly fell back and his quill fell out of his hand: Euler had ceased to calculate and to live.

A century has passed since then, a century full of progress in mathematics. But as great as the brilliant discoveries by Lagrange, Gauss, Jacobi, are, we are still under the dominant influence of this formidable person that was Euler. We do not read his works out of historical interest or to learn what people thought about this or that difficult problem last century. We recognise him as our teacher, to whose guidance we still submit ourselves today, full of the humility and admiration that his intellectual superiority inspires in us.

However, I want to conclude this remembrance of Euler by looking at a different angle. The century that separates us from Euler is abundant, perhaps over-abundant in technological progress. But for all that, it is an undisputed fact that this progress is very closely linked to the development of mathematics, even if this connection is not always as obvious as in the case of Euler inventing the achromatic telescope. Thus, Euler's contribution to the great achievements that humanity takes both pride and delight in today is not to be underestimated, and hence his name deserves to be known and recognised even by those who have no interest in mathematics.

At the St Petersburg cemetery, a mighty block made from Finnish granite, with the inscription "Leonardo Eulero Academia Petropolitana", reminds the

wanderer that he is at the same place where the mortal remains of this outstanding mathematician are buried. It is possible that, in thousands and thousands of years, the stone will have been removed due to events of some kind, that its inscription will be weathered and its significance will have been forgotten. But the name Leonhard Euler will live on as a symbol of intellectual perfection for as long as there is civilisation, for he himself has raised himself a monument that is greater, more sublime and more imperishable than any man-made structure: his immortal works.

E.3.4 F Rudio: *On the Contribution of Mathematics to the Culture of the Renaissance*

Talk given in the Town Hall Zurich on 05 February 1891; *Sammlung gemeinverständlicher Vorträge* 6 (142), Verlagsanstalt und Druckerei A. G., Hamburg, 1892. Footnotes by the author, not by Rudio.

The translation is also available on the MacTutor History of Mathematics website: http://www-history.mcs.st-andrews.ac.uk/Extras/Rudio_talk.html.

Ladies and Gentlemen!

For those who pursue the development of mathematics and its related disciplines from a cultural-historical point of view, the age of the Renaissance will always be of very particular interest: it is the age in which the consolidating process, from which our modern mathematical sciences emerged as an international cultural factor, took place.

The Renaissance! The rebirth of the arts and the sciences! This word conjures up such a wealth of images in our soul! Before our inner eye the grand masters of Italian art rise up; we imagine seeing the works of Leonardo da Vinci, of Raphael, of Michelangelo, which have been constituting a common and inexhaustible source of the most pure and noble pleasure for civilised people across the globe for centuries. We feel transported to the illustrious courts of the art-loving Italian princes, in particular the Medici, and partake in the literary efforts that tie in with the Italian national literature established by Dante, Petrarch, and Boccaccio.

And then again we remember that at the same time the age of the Renaissance is the age where sciences, in particular classical studies, flourished again; that it coincides with the enlightening and liberating activities of Humanism and the Reformation. We let our gaze wander northwards, away from Italy, and meet, apart from the stalwart figures Zwingli and Luther, the great humanist Melanchthon, Erasmus of Rotterdam, and Reuchlin. In particular, we fondly remember the appearance of the unrelenting fighter³ whose glorious and prolific life ended on the close-by

³ Rudio refers to the German humanist and reformer Ulrich von Hutten (1488-1523), who died on Ufenau, an island in Lake Zurich. Cf. biography by H U Bächtold in *Historisches Lexikon der Schweiz*: <http://www.hls-dhs-dss.ch/textes/d/D21507.php>, accessed 22/04/2013.

Ufenau, and to whom we have taken such a liking due to David Friedrich Strauss and not least due to the heartfelt poetry of Conrad Ferdinand Meyer. And by surrendering to these imaginations, we suddenly find ourselves in the middle of the struggle that liberated the people from the spells of the Mediaeval Ages, right in the centre of the great, magnificent humanist movement. Everywhere the spirits stir, everywhere fresh scientific activities blossom – it seems as if a warm spring breeze is wafting across the country, as if we can hear the shout of joy with which Ulrich von Hutten concluded his ever memorable letter to Willibald Pirckheimer: “Oh century! Oh sciences! It is a joy to live!”

And, I hear you ask, does mathematics have a share in all this glory? Mathematics: this science, which is always regarded with respect, out of consideration for the largely impressive advantages that it warrants modern technology alone, but not always with sympathy; this science, which is so often characterised, and so unjustly so, as a dry, prosaic one, which addresses the intellect, and analysis, but not the mind, nor imagination; this science, which one, therefore, so likes to put in a certain conflict with creativity and art, or indeed with the most ideal pursuit of all⁴!

He who would go through the trouble of dispelling such prejudices would only have to point out Plato, one of the most qualified – and at the same time most visionary – representatives of Antiquity, who began his lectures in the Academy with the sentence: “No-one ignorant of geometry may enter my house!” Plato loved and appreciated mathematics so much not because of any material advantages, but precisely because it is particularly well suited to removing one’s mind from the world of sensations and render it amenable for philosophy, due to its inherent power of abstraction. And for those who do not wish to let Antiquity convince them that mathematics has a part in the grand, ideal challenges of humanity that is not to be underestimated, that conviction will suggest itself during thorough and comprehensive study of the cultural activities of the Renaissance. Today I will briefly sketch out some of these activities for you.

Ladies and gentlemen, it hardly seems possible to appreciate the contribution that a science as abstract as mathematics has made to the culture of any era in any other way than by putting it in context with the overall historical development of this discipline. Mathematics, like in fact any pure scientific research, sees its purpose not so much in the applicability of its results, but much more in satisfying very particular philosophical questions posed by the intellect. Incidentally, with regard to results, it is very hard to predict when, in what form and to what extent they will influence the civilisation process at some future point. Hence, you will expect me to present the cultural factors of the Renaissance that we are interested in specifically not in an isolated manner, but as the products of a historical development. And if I go back a little further than would seem necessary at first glance, then please do not assume that the reason for this is the difficulty of speaking about mathematics without being able to assume any special knowledge. Rather, although the questions I will consider first are very distant to the Renaissance

⁴ Given the context and the time of writing, Rudio probably means philosophy.

in time, they are linked directly to it in contents and are therefore vital in order to understand these factors.

Mathematics had been flourishing in Italy already once before. This was at the time of the Greek colonies in Lower Italy and in Sicily. And the last Greek mathematician on Italian soil, Archimedes of Syracuse, was also the most brilliant of mathematicians in Antiquity. He received his deathblow, deeply immersed in his studies and not worrying about anything other than “do not disturb my circles” in the face of an assailing warrior when the Romans conquered his hometown in 212 BC. This put the final nail in the coffin for the science that he represented so brilliantly; along with him, mathematical research in Italy was wiped out for more than a millennium! For over the centuries, the great Roman nation did not produce a single noteworthy mathematician, at least by Greek standards. Indeed, we can hardly regard it as a coincidence that this nation, which showed little originality and was wholly dependent on the Greeks with respect to arts and literature, also showed such an incredibly poor predisposition for any mathematical speculation.

In the last centuries BC and the first centuries AD, it was almost exclusively the scholars of the Academy in Alexandria who cultivated mathematics. Already under the first Ptolemy, at around 300 BC, Euclid had written his famous “Elements”, which you all know, here. Since the Renaissance they have been forming the basis of the teaching of geometry, even today. Here lived Eratosthenes, the geographer and chronologer, who mathematicians know due to the so-called “sieve of Eratosthenes”; and Apollonius, the great geometer who conducted in-depth studies of the theory of conic sections and introduced the terms ellipse, hyperbola, and parabola. Here Hipparchus, the actual originator of scientific astronomy to whom we owe the introduction of longitude and latitude in order to determine the position of a point on a sphere, conducted his famous observations of the moon in about the year 150 BC. And it was here, in Alexandria, where Claudius Ptolemy wrote his immortal work “Μεγάλη Σύνταξις”, the Great Treatise, about 300 years later.

The name of Ptolemy, the founder of the worldview named after him, is connected with the culture of an era spanning almost one and a half millennia, and particularly with the culture of the Renaissance so profoundly that I cannot but dwell on this magnificent figure for a while.

You all know that the Ptolemaic system, which was regarded and adored like a Gospel until the mid-16th century, is based on the geocentric principle, i.e. on the fundamental view that the Earth hovers motionless in the centre of the universe and that the celestial bodies, including the Sun, rotate around his centre. But this view alone, which, after all, eventually suggests itself to any naïve observer, did not suffice to establish the everlasting fame of Ptolemy. In fact, he deserves credit for describing the movements of the celestial bodies as exhaustively as possible, using trigonometric tools that he largely crafted himself and whose perfection remained unsurpassed for centuries.

Hipparchus had already observed that the seasons are of different durations 300 years before Ptolemy. The only explanation for this phenomenon that he could find was that although the Sun rotated uniformly around the Earth in a circle, the Earth was not in the exact centre of this circle. Now, Hipparchus could give a sufficient explanation for the apparent motion of the Sun based on this so-called eccentric circle, but the much more involved motions of the Moon and the planets presented insurmountable difficulties.

The apparent, i.e. as seen from Earth, annual motion of the planets is characterised by a loop: although the planets move in one particular main direction, i.e. from West to East on the whole, they occasionally slow down, stop, move backwards for a short period of time, stop again, and continue to move in the original direction. Now, how should one go about explaining this complicated motion without infringing the fundamental view of Antiquity that only allowed the use of uniform motion on a circle in order to explain celestial motions, and to which people adhered partly due to metaphysical reasons? Ptolemy solved this problem by assuming that every planetary motion was composed of two circular motions. According to Ptolemy, every planet initially moves uniformly along a circle, the so-called epicycle, whose centre, in turn, moves along a second circle, the so-called deferent, which is circumscribed eccentrically around the Earth. By describing the planetary orbits as so-called epicycloids, he could explain the planetary motion, complicated greatly by antecedence in particular, without violating the fundamental principle mentioned above.

This is a brief sketch of the basis of Ptolemy's famous theory of epicycloids, which testifies to commendable ingenuity. But now I have also touched upon the mathematical content of the worldview, which was the predominant one for most of the Renaissance and which started to give way to a more sophisticated understanding of the worldview only towards the end of the era. I will explain later how this significant change happened.

There is a widespread belief that after the Arabs conquered Alexandria in 641 Greek culture found a new home in Constantinople, where many Alexandrian scholars had moved, and that here, in the capital of the Eastern Roman emperors and protected by them, Greek science was maintained and developed throughout the Mediaeval Ages. When scholarly Greek refugees brought classic manuscripts to safety to Italy, due to the Turks' advance and the conquest of Constantinople in 1453, the Occident became acquainted with the precious treasure of Greek culture, which initiated the renaissance of the sciences.

As far as mathematics is concerned, this belief is inaccurate. I do not want to disavow the ardour and the strong impulse caused by those invaluable manuscripts. A tale according to which the well-known Italian humanist Pico della Mirandola paid for a single Livius with an entire estate illustrates the enthusiasm with which those manuscripts were received, and how badly the intellectuals wanted to possess old Greek and also Roman manuscripts! But knowledge of Greek mathematics had already been brought to the civilised people of Europe via a different route. In addition, and just to mention it straight away: Greek mathematics only formed one arm of the gigantic stream of mathematical thoughts that flooded the Occident towards the end of the Mediaeval Ages.

The Arabs adopted the legacy of Antiquity initially. As is well known, they created an empire that stretched towards the Indus in the East and the Ebro in the West within an incredibly short amount of time. However, what we marvel at the most is the flexible minds that this people must have possessed, so as to enable them to develop a primitive nomadic lifestyle into a civilisation such as under the splendid governance of Harun al-Rashid⁵ in just

⁵ I have used the modern transcriptions of Arabic names here.

under 150 years. It was under this ruler, with whom we have been so familiar since our childhood through the tales of "Arabian Nights", but even more so under the Caliph Al-Ma'mun that a fruitful period of translating began, due to which we know many a Greek writing that might have been lost otherwise. The first Greek manuscripts that were translated to Arabic were Ptolemy's *Σύνταξις*, Euclid's *Elements*, Apollonius's conic sections and Archimedes' treatises on measuring the circle and on the sphere and cylinder. The name "Almagest", which we use for Ptolemy's work even today, stems from this period. It derives from the Arabic article *al* and the Greek superlative *μεγίστη*, "greatest", which *Μεγάλη* ("great") *Σύνταξις* had gradually turned into.

Ladies and gentlemen, human history is full of peculiar discrepancies! Christendom trembled when the Arabs triumphantly conquered all of Spain at the beginning of the 8th century and even advanced into the Frankish Empire across the Pyrenees; they hailed Karl Martell after the victorious battle between Tours and Poitiers for having heroically liberated them from the Barbarians. And under the same Barbarians, particularly under the Umayyad dynasty, sciences and arts gradually reached their heyday, their golden age, in Spain. Moreover, Christendom received the most precious contribution of its own intellectuality from the very same people whom they considered had obliterated all civilisation!

In the 12th and 13th centuries scholars from all over Europe flocked to the academies in Toledo, Seville, Cordoba and Granada to study the Greek classics and, most importantly, to translate them from Arabic to Latin. That way, and in particular due to the work of diligent translators such as Gerhard of Cremona, Adelard of Bath and others, the Christian Occident gradually gained an insight into the sophisticated mathematics of Antiquity.

But I have already mentioned that Greek mathematics only formed *one* of the sources that were to amalgamate and develop into our modern mathematics. Indeed, the significance of the Arabs for civilisation, who primarily communicated the ideas of different peoples, was of a much more universal nature than was assumed previously. For, apart from knowledge of Antiquity, we also owe to them a first insight into the intellectual life of a people whose approach to mathematics was completely different to that of the Greeks, but not less sophisticated, and which complemented it very well: I am talking about the Indians.

Due to their highly developed sense of aesthetics, the Greek almost exclusively investigated mathematical problems that could easily be visualised, i.e. problems in geometry. In contrast, the Indians' exceptionally accomplished sense of numbers and an unparalleled love of calculation, spread across all social classes from ancient times, led them to dealing with problems in arithmetic and algebra for the most part.

After all, India is the home of chess, the arithmetic game *par excellence*; poetic musing and dreaming was connected with mathematical speculation so intimately that poets were fond of indulging in charming intellectual games, and, conversely, mathematicians liked to verse their treatise. Arithmetic riddles and competitions formed part of the Indians' social amusements; in fact, it is even reported that Buddha, when courting a girl, had to sit an arithmetic exam in order to beat his rival in love!

We owe an invention to our congeners at the banks of the Ganges, whose sophisticated mathematics was brought to us by the Arabs at the same time as that of the Greeks. This invention has been in the possession of European scholars since the beginning of the 13th century, but its results started to benefit the whole human race only very gradually and only since the intellectual upsurge that took place towards the end of the 15th century. Despite the classic simplicity of this invention – or perhaps precisely because of it – we may add it to the collection of influential cultural factors that shaped the Renaissance: I am referring to the invention of Indian numerals and the Indian place-value system.

Contrary to popular belief, the nature of the Indian numeral system does not rely on the fact that the number 10 forms the basis of the system. Both the Greeks and the Romans, in fact, all Indo-Germanic peoples, used the decimal system. In addition, according to studies by Alexander von Humboldt and other scholars we may argue that all the peoples of the world, except for a few isolated cases, use 5, 10 or 20 as a base for counting, with respect to the arrangement of the human body. Rather, the Indian numeral system is characterised by the fact that one can express any arbitrarily large number by using no more than ten symbols and positioning them next to one another. One of these symbols, zero, denotes absence, whereas the other symbols denote the numbers from 1 to 9. In order to express any arbitrary number it suffices to simply state that in addition to a numerical value, each of these symbols also has a certain place-value depending on its position. This is done in such a way that, reading from right to left, the first digit always represents the ones, the second digit represents the tens, the third one represents the hundreds, etc.

Ladies and gentlemen, you may think that it can hardly be of any importance to speak about such elementary things that each and every one of us knows from school. I do not even wish to remind you that this simple assumption, which forms the basis of our calculation today, has escaped a mathematically gifted people such as the Greeks. We have to travel back only a few centuries to find completely unfamiliar conditions, which we would barely be able to cope with. Indeed, it is only with history's help that we can conceive the major benefit of this Indian invention today, which has such a profound impact on all aspects of scientific and everyday life.

For instance, it was impossible for the Romans, using their cumbersome numerals that you are all familiar with, to do arithmetic in a similar manner to us today, using our digits. You can easily see this for yourselves by adding or even multiplying two arbitrary large numbers written in Roman numerals. Admittedly it would prove difficult to invent a more uncritical and more awkward notation for numbers than the Roman one. The Greek notation on the other hand is in a different league entirely. They had a specific symbol for each one, each ten and each hundred: the letters of their alphabet (using a few letters of an outdated alphabet). In arranging these symbols they already tried out the place-value numeral system. But even so, doing arithmetic by our standards was not really possible with these numerals either, especially when dealing with large numbers, possibly except for exceptionally gifted intellectuals like Archimedes and Apollonius. Thus, the people of Antiquity had to rely on using their fingers to do calculations; incidentally, particularly when doing complicated computations this requires a certain subtlety and

dexterity, which the Italians display in the Morra game nowadays. Alternatively, they had to resort to computing on a so-called abacus, a tool that is basically identical to the calculating frame that you are all familiar with from our elementary schools. Apart from a few modifications, which I will not mention here, these are essentially the computation methods that were used throughout the Mediaeval Ages.

In light of these primitive tools the question suggests itself: Would our whole modern culture, all of our scientific and social life, and our polymorphic infrastructure even be conceivable without this inconspicuous invention that is the Indian numerals?

These remarks will give you a rough idea of how important calculating with digits is in the historical development of civilisation. Now please allow me to make a few comments referring to the origins of our numerals. Nowadays there are no more doubts that they originated in India – including the zero, without which a place-value numeral system would not be possible. They definitely existed in the second century AD, and in all likelihood they emanated from the first letters of the Sanskrit names for the respective nine numerals. The invention of zero is likely to be more recent, but its first appearance can be traced back to 400 AD after all. The Arabs learned this new computation method from the Indians; specifically, after Muhammad ibn Musa al-Khwarizmi, a mathematician who lived at the beginning of the ninth century under the Caliph Al-Ma'mun, had made Indian arithmetic accessible to his fellow countrymen in several treatises. As has been shown by Reinaud and Boncompagni, the sobriquet al-Khwarizmi, meaning "he who comes from Khwarizmi" (today's Khiva) has been preserved in the words "algorism" and "algorithm", which is the name for all regular calculation procedures in mathematics.

We already know to some extent how the Indian numerals eventually reached the Occident in the 12th century. In particular, I have to highlight the outstanding contribution of Leonardo Pisano, called Fibonacci, undoubtedly the most important mathematician of the entire Christian Mediaeval Ages. He deserves credit for introducing the Indian digits in his seminal work *Liber abaci*, written in 1202. The fact that we know of the Indian numerals due to the work of the Arabs later often led to the erroneous term "Arabic numerals". However, both the Arabs themselves and also the Italians in the Renaissance were well aware of the Indian origins of their arithmetic.

Thus, at the beginning of the 13th century we see the Christian Occident being in possession of the mathematics of two highly gifted people; one of which, the Greeks, represents the mathematics of Antiquity, and the other one, the Indians, represent the mathematics of the Mediaeval Ages. I am happy to add that the contribution of the Indians to mathematics is by no means limited to elementary arithmetic. In fact, their most outstanding achievements can be found in algebra and higher number theory (especially relating to indefinite equations of first and second degree). Mathematicians like Aryabhata, Brahmagupta and Bhaskara are rightly placed among the most distinguished number theorists of all times.

One should be right in thinking that such great incitements, particularly the magnificent example of Fibonacci, would have led to a tremendous intellectual upsurge, an epoch of greatest mathematical productivity. However, this was hardly the case until the middle of the 15th century. One

gets the impression that the nations were almost overwhelmed by the scores of new intellectual material opened up to them, and that they could process it only little by little.

But the situation radically changed in the middle of the 15th century: the invention of the printing press, which caused unequalled social and intellectual upheavals, joined the momentous stimulation originating in Constantinople, which we have already met. It was only now that mathematics had a more profound impact on Western culture, and that we can talk about a revival of the Greek and Indian spirit. The wisdom of the Brahmins now begins to bear fruit. The Indian place-value system becomes widely accepted – I will explain how – and henceforth forms an integral part of the intellectual heritage of all civilised people. On the other hand we notice that Greek geometry, culminating in Ptolemy's theories, is booming, particularly in Germany and Italy; and we marvel at the global scientific process that comes to a close in the mid-16th century, when the Ptolemaic worldview gives way to our modern one. As long as we look at the grand scheme of things only, we may regard these two developments – the acceptance and permanent absorption of the Indian place-value system into our modern culture, and the fading of the Ptolemaic system and the foundation of a new worldview that emanated from it – as the most important influences of mathematics on the culture of the Renaissance.

But let us ignore these two major areas of mathematical activity for a moment. There is a wealth of developments that testify to the extent of which the whole culture of the Renaissance was steeped in revived mathematical thinking. However, this can hardly come as a surprise given that we are talking about an age that was characterised by the notion of universality more than any other era afterwards. In fact, this does not only mean that people devotedly cultivated and advanced the individual sciences, and moreover, that the arts experienced a most glorious heyday, but also that this all-round, harmonic development of all human capacities recurred in certain *individuals*. These polymaths, perfect representatives of their era, illustrate the spirit of the Renaissance in a similar manner.

But it was not *solely* the pursuit of universal learning that made mathematics so compelling and attractive to the great masters of the era, like Brunelleschi, Leonardo da Vinci, Raphael, Michelangelo and, in particular, Albrecht Dürer. They were fully aware of the fact that notwithstanding the freedom of one's imagination, art was also subject to the law of necessity and conversely, that mathematics was also subject to the law of beauty, despite the rigour of logical reasoning. Thus they found mathematics to be related to their art and appreciated the benefits that it gained from their studying mathematics. The year 1420, justly given as the date for the renaissance of architecture, illustrates this well. The Florentine Cathedral had been completed, except for its dome. Architects from across the world gathered at a congress to study and solve the problem of constructing the dome. The most bizarre ideas were proposed; the suggestion to build the entire dome out of pumice, as this would reduce the strain, not even being the most foolish among them. Then Brunelleschi stepped forward, with the unheard-of proposal to close the enormous opening with a free cupola, without any sort of scaffolding. The suggestion was ridiculed, but Brunelleschi did not budge until the project was assigned to him, and he executed it closely following the

plan he had submitted. Thus he solved a problem that required an experienced architect and mathematician. The slender dome of Santa Maria del Fiore rises in a beautiful elliptical curve: an everlasting monument to its ingenious constructor.

As tempting as it may be for the mathematician to follow the footsteps of those great masters of the Renaissance, I do believe that, with respect to the topic I have chosen, the only way to do this age justice is by restricting myself to outlining the essence and the historic development of the great leading ideas, of which there was no shortage in this era. I will certainly not find it easy to pass over a man whose magical and powerful figure involuntarily reminds one of Goethe and whose mathematical ingenuity is comparable to that of Archimedes. But if I have to withstand appreciating the mathematician Leonardo da Vinci in more detail here, then I truly do not do this due to a lack of admiration for this all-embracing polymath, but because his grand scientific ideas, hidden away in handwritten notes that were not destined for publication, were virtually unknown to his contemporaries. Therefore, they unfortunately remained without any influence whatsoever on his era. Only our century had the privilege of discovering Leonardo to have been one of the most significant scholars, and certainly the most brilliant mechanic and physicist of his time. I will give you only a few random and incoherent examples: Leonardo knew the laws of free fall before Galilei did; he was the first to study, both in theory and in practice, the influence of friction resistance on motion; he clearly stated that a *perpetuum mobile* and squaring the circle are impossible. He discovered capillary action; invented the *camera obscura*, and in doing so laid the foundations for an important area of optics; he invented an instrument to determine humidity; he identified scientific experiments as the true source of natural philosophy a whole century before Francis Bacon. Most remarkably perhaps, he developed a wave theory almost two centuries before Huygens, which he used to explain phenomena of sound and even of light! And all these scientific achievements are not just jotted down, but written out in full, in a clear, beautiful language and accompanied by excellent drawings. His notebooks are located in Milan and Paris. Furthermore, we learn about his work as an engineer, building fortifications and hydraulic structures; we learn that he built the great canal of Martesana, which connects Ticino with the river Adda and irrigates about 80,000 morgen of land⁶; that he invented and constructed a variety of machines for various purposes, among them various aircrafts and even a steam-powered ship. And then we suddenly remember that this very same man created The Last Supper and was such a divine artist – and we automatically remember the words of Jakob Burckhardt: “ It will only ever be possible to catch a glimpse of the formidable outline of Leonardo’s character!”

But there is *one* achievement, which resulted from the marriage of art and mathematics and can well and truly be called a child of the Renaissance, that I have to explain in a bit more detail: the foundation and the development of the theory of perspective. This is how one could go about understanding what it means to have an accurate perspective image of an arbitrary object: One puts a glass pane between the original object one wants to map and one’s eye,

⁶ Morgen – unit of measurement used in Germany and some other states, used until the 20th century. The size of a morgen varied across the regions, ½ acre to 2 ½ acres.

which is situated at an arbitrary, but constant point in the space, the so-called point of view. The other eye is closed. If one assumes that the rays of light that travel from the points of the original in the direction of the eye, through the glass pane, leave a visible trace on the pane, then all of these traces will form an image, which is called the perspective view. The theory of perspective is simply the collection of rules according to which one can draw an accurate perspective image of a given object *without* using such a glass pane.

Many have debated whether or not the ancient Greek and Roman painters knew the art of perspective. Lessing has provided us with an in-depth study of this question in "Laocoon"⁷ and "Letters of Antiquarian Contents". His conclusion is to the disadvantage of the ancient painters, as "this part of art was completely unknown to the Ancients." His contemporary Johann Heinrich Lambert, the famous mathematician and architect at the court of Friedrich the Great, continued this historical critique in his treatise "Free Perspective", published here in Zurich. He claimed that one would have to consider Leonardo da Vinci as "the first to think of the true refinement of painting and of perspective." Admittedly, today we no longer associate only one single name with establishing perspective and consciously introducing it to painting and are happy to honour the great contributions of brothers Johann and Hubert van Eyck and, in particular, of the well-rounded Leon Battista Alberti in Italy, which were all made *before* Leonardo's time. However, everyone who knows a little bit about the development of painting will clearly recognise the milestone in the history of art, with regard to the use of linear perspective and particularly of the so-called aerial perspective, which is marked by the name of this great Florentine.

His great contemporary Albrecht Dürer, equipped with sound geometric knowledge, continued the work begun by Leonardo and his predecessors. Like Leonardo, Dürer has earned himself a place of honour in the history of mathematics. His fundamental work "Instructions for Measuring with Compass and Ruler", published in Nuremberg in 1525 and dedicated to his friend and benefactor Willibald Pirckheimer, was of the utmost importance for the development of art, and specifically for applied arts.

There is another reason why the year 1525 is of particular interest to us, which also leads me back to a topic I touched upon earlier. This year marks the beginning of mathematics teaching at German elementary schools. It is Luther to whom the Germans owe this important reform of elementary education. In the "Letter to the Councilmen of all German towns so that they may establish and maintain Christian schools", published at the end of the year 1524, he states: "I am speaking for myself: If I had any children and I could afford it, they would not only have to study languages and history, but also learn how to sing and make music and all of mathematics." And Luther did not talk in vain. In the 15th century, the view that mathematics was not for public, but only for private education, still prevailed. Children in public schools were not even taught simple calculations with digits. But this radically changed when Luther appeared on the scene.

It fell to the re-organised primary schools of the 16th century to solve a cultural problem: they were given the task of replacing the old methods of calculating and instead making the use of Indian digits such as we know it

⁷ Published in 1766

today an integral part of each and everybody's education. We gladly remember the men who successfully performed this significant task. Among them Adam Ries, whose name you are all familiar with and whose strenuous life came to an end in 1559, is entitled to be called a teacher of the German nation.

But radical reforms took place not only in the primary schools, but also in the secondary schools and universities during the first half of the 16th century: at all such institutions, mathematics was adopted as a regular subject and independent professorships in mathematics were established. Those reforms happened not only in Germany, but also here in Switzerland, due to the tireless work of Zwingli. As early as in the 1520s, the humanist Myconius, a friend of Glarean, taught mathematics alongside classical languages at the school by the Fraumünster. Konrad Gessner, the future polymath and famous bibliographer, and Joachim Rheticus, future professor at Wittenberg and friend of Copernicus, were among his pupils. In Germany, the upswing of mathematics education in secondary schools and universities is closely linked to Melanchthon. An excellent mathematician himself, he deserves high credit for his efforts in spreading mathematics both orally and in writing. It was due to him, for example, that when the Gymnasium was founded in Nuremberg in 1525, a teaching post for mathematics was created straightaway as well; the first of its kind in Germany. It is easy to see why it was Nuremberg in particular that set an example when one realises that this town was not only the centre of German art in the 15th and 16th centuries, but also led the way among German cities with respect to trade, industry, science, and literature. The convivial house of the one and only Willibald Pirckheimer, whose memory is honoured in the magnificent letter by Hutten, alone, was comparable to an academy of arts and sciences, which promoted extensive international scholarly intercourse and actively facilitated all of the ideal efforts of the time!

The lively interest in mathematics, which characterised not only Pirckheimer's circle of artists and scholars, but also the citizenry of Nuremberg, was passed on in this town by tradition. In order to get an impression of this, just check the comprehensive "Historische Nachricht von den Nürnbergischen Mathematicis und Künstlern, welche fast von dreyen *seculis* her durch ihre Schriften und Kunst-Bemühungen die Mathematic und mehreste Künste in Nürnberg vor andern trefflich befördert und sich um solche sehr wohl verdient gemacht"⁸, compiled by Gabriel Doppelmayr in 1730. Already on the first page of this interesting compilation we stumble across the name of the man who laid the foundations to Nuremberg's mathematical fame and has justly been called the most active reformer of exact sciences in the 15th century: Regiomontanus. In drawing your attention, ladies and gentlemen, to this distinguished figure, I have also arrived at the topic that will conclude my talk and which I have therefore looked at in some detail at the beginning (not at all coincidentally): astronomy.

⁸ "Historic news about mathematicians and artists from Nuremberg, who have rendered outstanding services to mathematics and several arts in Nuremberg, splendidly promoting them through their writings and artistic efforts, for almost three generations."

Born in 1436 in the Franconian town Königsberg, Johannes Müller, known as Regiomontanus in the scientific world, went to Vienna already as a fifteen-year old youth, in order to learn mathematics and astronomy from Peurbach. At the time, Peurbach was the most famous astronomer around and an exquisite humanist, who owed his excellent reputation partly due to an outstanding textbook on planetary theory, of course in the Ptolemaic style. By working together, the relationship between teacher and pupil soon became a very intimate one. This was exemplified during a stay in Vienna in 1460, when the scholarly Cardinal Bessarion, one of the Greek refugees who had come to Italy from Constantinople, invited Peurbach to continue his astronomic studies in Rome. Peurbach insisted that his young friend be allowed to accompany him, a condition to which Bessarion gladly agreed. Unfortunately, Peurbach, not yet 38 years old, died before they could set out on their journey. Even so, Bessarion held open his invitation, and when he returned to Rome in 1461 Regiomontanus was allowed to go with him. Regiomontanus spent seven busy years in Italy, keeping an active scientific correspondence with the local scholars, in particular with the astronomer Bianchini who had been a teacher of Peurbach's. He plunged into the study of the Greek mathematicians with enthusiasm; as we know, these had only been known through translations from Arabic to Latin up until then, but now they were available to him in the original. When he left Italy in 1468, he was in possession of a whole collection of valuable Greek manuscripts; among them the manuscript of Ptolemy's $\sigma\upsilon\nu\tau\alpha\acute{\xi}\iota\varsigma$ in particular, a gift from Bessarion. After sojourns in Vienna and Ofen, Regiomontanus chose to settle permanently in Nuremberg. There, Bernhard Walther, a man characterised not only by his wealth, but also by his great interest in science, set up an observatory, a mechanics workshop for producing scientific instruments, and even a private printing press for him. Here in Nuremberg Regiomontanus displayed positively astonishing scientific activities; marvellous plans occupied his mind. For example, he wanted to publish printed, accurate original editions of all the Greek manuscripts that he had brought with him from Italy. However, in 1475 he was honoured by an invitation from the Pope, who bestowed upon him the title of Bishop of Regensburg and appointed him to a post in Rome, where he was assigned the task of carrying out the absolutely necessary calendar reform. Regiomontanus only reluctantly complied with this request, which would prove to be so fatal for both him and science. Having barely arrived in Rome, he died of the plague, at the mere age of 40.

This is not the place to talk about Regiomontanus's scientific works or indeed demonstrate how he paved the way for Copernicus by means of improving trigonometric tools and the existing astronomical tables. I can only mention those achievements that played a direct part in the progress of civilisation of the time. Among those, publishing the calendar and the so-called ephemerides rank among the most important ones, alongside the credit he deserves for spreading mathematics. Admittedly, hand-written catalogues where all the days of the year were separated by weeks and months and from which people could gather the dates of movable feasts or the beginning of an eclipse for example, existed before Regiomontanus's time. However, arranging the calendar in the perfected form that we know today and that we cannot do without anymore is due to Regiomontanus. He published the first printed German calendar in Nuremberg in 1474.

Publishing his ephemerides was even more significant. One can call them a kind of chronicle of the heavens, where Regiomontanus had listed consecutively all the phenomena that could be observed on the skies, i.e. the respective positions of the heavenly bodies, for the period from 1475 to 1506 – done in advance and based on meticulous calculation. The ephemerides promptly caused a stir all over Europe, and became positively epoch-making in nautical sciences. Regiomontanus's ephemerides, together with the tools for astronomical observations that Regiomontanus had designed himself as well (the best of their kind before the invention of the telescope) now allowed sailors to determine the position of their ship out on the open sea according to the positions of the stars. The glory of Bartholomew Diaz, of Vasco da Gama, and of Christopher Columbus in particular, is not derogated by the fact that these audacious men had Regiomontanus's ephemerides on board their ships, which had been introduced to the Portuguese navy by the famous seafarer and cosmographer Martin Behaim from Nuremberg. However, by considering this fact, the great discoveries of the 15th century no longer appear as isolated events or as foolhardy endeavours, but as the results of continuous, purposeful efforts of the mind, which had their origins in the renaissance of the exact sciences.

Another work of Regiomontanus's, admittedly published long after his death, might be of interest to us: his paper on comets. Regiomontanus was the first to include comets in scientific studies. Unconcerned by the superstition that had been linked to the comets as dreaded celestial phenomena since time immemorial and inspired by the comet of 1472, Regiomontanus came up with the for his time completely novel idea of treating it like any other celestial body for once and carry out astronomical measurements. He was of the opinion that it was appropriate for those who wanted to talk about a comet to first know its position, its dimension, its distance from Earth, etc. Regiomontanus's views with regard to the physical composition of comets have also only been verified afterwards.

In order to get a complete and generally accurate picture of mathematics in the Renaissance, one cannot ignore another aspect, which is related to the superstitious beliefs about comets and which has often been referred to as an illness of those times. For the Renaissance was also a heyday of astrology; the alleged art of predicting events in the future based on the position of the stars. The belief in mysterious relationships between the stars and the destinies of human beings was such a widespread one that, up to the 17th century, even the astronomers at universities could not elude its influence on their teaching. Very few were brave enough to campaign against astrology, but of course they were unsuccessful. However, we may count the frequently misjudged Swiss physician Theophrastus Paracelsus, unjustly nicknamed Bombastus, amongst them, alongside Italian scholars such as Paolo Toscanelli, Pico della Mirandola and others. A reason for this would be his beautiful sentence, which deserves to be preserved for future generations due to its classy simplicity: "A child needs neither stars nor planets, its mother is its planet and its star."

But we do not want to reproach this era because of astrology. In many cases astrology was just an expression of the boisterous quest for knowledge, of the immense, albeit not always properly channelled thirst for knowledge and education of the time, which is represented in the legend about Faust, for

example. And having said that, astrology often also motivated and encouraged research in astronomy. Moreover, for as long as people considered the Earth to be the immovable centre of the universe, relating celestial phenomena to earthly events and asserting a causal connection between them seemed an obvious thing to do, as this was in accordance with the resulting significance of the Earth. Changing these assumptions was only made possible by a complete reform of the entire worldview.

You all know the name of the immortal man to whom we owe this truly grand achievement. However, I can barely hope that I will be able to do the merits of Nicolaus Copernicus justice in the short amount of time available.

Copernicus was born on 19 February 1473 in Thorn, Western Prussia. After having completed his school education he matriculated at the University of Cracow, where he studied humanities, mathematics and medicine, thus gaining a solid and eclectic education. In addition, he was well versed in arts, in drawing, painting and in music. At the age of 23 he went to Italy, in order to prepare for the post as Canon in Frauenburg, which his uncle, the future Bishop of Ermland intended for him, by studying theology and medicine in Bologna. Among the most famous teachers at the University of Bologna at the time was the astronomer Domenico Maria di Novara. Copernicus, whose favourite subject had always been astronomy, met this fine man, and soon a relationship akin to the one between Regiomontanus and Peurbach developed between them. Surely we can look for the origins of the bold ideas to which we owe our modern worldview in the stimulating scientific exchange that Copernicus experienced in Bologna and later on in Rome, Padua and Ferrara. However, we will attribute the unusual sense of aesthetics and beauty that Copernicus displayed in formulating his system to the influence of Italian art, which the young astronomer greatly appreciated.

Copernicus returned from Italy around the year 1505, filled with scientific and artistic impressions. Soon after he took on the post of Canon in Frauenburg, which he held until his death on 24 May 1543. His calm and serene life was filled only with the duties of his priesthood, the affiliated hospital for the poor, and his scientific studies, to which he devoted all of his leisure time. It was only late in his twilight years, and at the urging of his friends, that he decided to publish the ripe fruit of these studies: his great work "On the Revolutions", which he had kept back, to use his own words, not for *nine* years, but for *four times nine* years. When the first printed sheets arrived in Frauenburg, Copernicus was already in mortal agony. He is reputed to still have touched and looked at them before passing away.

Alexander von Humboldt says that: "The founder of our current world view was characterised by his courage and the confidence that he displayed to an almost higher degree than by his knowledge. He highly deserves the fine praise from Kepler, who called him the *man of free spirits*."

Indeed, imagine what a powerful scientific conviction was needed in order to confront the Ptolemaic tradition, held sacred for fourteen centuries! *The Earth rotates!* She is a planet like the others, like Mercury, Venus, Mars, Jupiter, and Saturn! *All planets, and hence the Earth, too, rotate around a common fixed centre, the Sun!* Moreover, in addition to the annual revolution around the Sun, the Earth also rotates around its axis daily! These are the phrases with which Copernicus caused a stir all over the world at the time, and which have been engrained in our brains so thoroughly nowadays, that we regard them as

part of our intellectual identity. The Ptolemaic system was no doubt cleverly devised, as long as one would wish to base it on the geocentric principle, i.e. relating the movements of the celestial bodies to the Earth as a fixed centre. But it was precisely the assumption of this principle that had led to the greatest complications and insalubrities when trying to explain the motion of the planets. Order, simplicity and harmony appeared the moment that Copernicus replaced the geocentric principle with the heliocentric one. "I have not been able to find a more beautiful symmetry of the universe and a more harmonic link between the orbits by any other alignment, than by placing the Light of the World, the Sun, governing the whole family of rotating bodies, on a regal throne in the centre of the beautiful Temple of Nature!" he said enthusiastically.

Ladies and gentlemen! I will have to conclude my talk with the foundation of the Copernican system, which signified the beginning of a new, intellectually more liberal world in humanity's awareness of science. Of course, my talk can make no claims to be complete as the Renaissance was far too prosperous for that, also with regards to mathematics.

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