Personalized Pricing and Distribution Channels*

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2 April 2020

Abstract

This paper examines the effects of personalized pricing on distribution channels. We explore whether a brand manufacturer prefers to sell through its own direct channel only (mono distribution) or through an independent retailer as well (dual distribution). Compared with uniform pricing, personalized pricing allows for higher rent extraction but also leads to fiercer intra-brand competition. The latter can however be mitigated through the wholesale contract. As a result, if the two channels are horizontally differentiated, or vertically differentiated with the independent retailer offering the high-end product, dual distribution is optimal under both personalized and uniform pricing. By contrast, if the manufacturer offers the high-end product, mono distribution can be optimal under personalized pricing even if the retailer broadens the demand of the manufacturer’s product. Instead, with uniform pricing, selling through both channels is always optimal. We also show that industry profits may be the largest in a hybrid pricing regime, in which the manufacturer forgoes the use of personalized pricing and only the retailer charges personalized prices. Our results are able to explain the distribution channels observed in different industries.

Keywords: personalized pricing, distribution channels, dual distribution, vertical contracting, downstream competition.

*We thank Juanjuan Zhang (the editor), an associate editor, and three anonymous referees for very helpful comments and suggestions that greatly improved the paper. We also thank Nicolas Schutz and seminar participants at the University of Bergamo, University of Cologne, University of Grenoble, University of Porto, Télécom Paris, Toulouse School of Economics, and the MaCCI Annual Conference 2018 (Mannheim) for helpful comments and suggestions. Financial support of the European Research Council (ERC), under the Seventh Framework Programme (FP7/2007-2013) (grant Agreement No 340903) and the Horizon 2020 research and innovation programme (grant agreement No 670494), and the Agence Nationale de la Recherche under “Investing for the Future program” (grant ANR-17-EURE-0010) is gratefully acknowledged.

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1 Introduction

The growing use of the Internet and advances in information technologies enable firms to gather unprecedented volumes of consumer data. This has led to important changes in their pricing policies, by allowing them to practice price discrimination at finely-tuned levels. For example, firms tailor their prices according to consumers’ purchase history, their physical location, the device they are using, their online search behavior, their social network activity, and so on.1 Tanner (2014) reports that buyers using a discount site, such as Nextag.com, receive prices as much as 23% lower than direct visitors. Large Internet stores, such as Amazon and Staples, vary their prices according to customers’ geographic locations by up to 166%. Firms often implement this differentiated pricing through coupons and specific promotions, thereby moving closer to personalized pricing.

At the same time, technological advances have made possible the entry of new online retail companies. For example, in the apparel industry, the British company Asos (founded in 2000) sells several thousand products with a revenue of around £2.5 bn in 2018. In the food industry, iHerb (founded in 1996) distributes over 30,000 nutritional and organic food products. A core question for brand manufacturers is whether or not to sell their products through these independent retail outlets. For example, although Asos sells more than 850 brands, Prada or Louis Vuitton products cannot be found there. Similarly, iHerb sells coffee of several brands but the organic coffee and food producer Equal Exchange sells exclusively through its own stores. This question is relevant beyond online markets. For example, the advancement of wireless technology has allowed mobile virtual network operators, such as Ting or Lycamobile, to enter the market for mobile phone services without rolling out their own networks. Established mobile operators were confronted with the issue of whether or not to grant these virtual operators access to their networks.2

These independent firms bring value to the industry by broadening the customer base. For example, the manufacturer and the independent retailers may offer different services (e.g., professional advice vs. lower transaction costs). This generates horizontal differentiation, as different consumer groups may favor different services. Alternatively, the firms may be vertically differentiated. In some cases, the manufacturer caters foremost to the high-end segment. For example, luxury-goods brand manufac-

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1For example, according to TRUSTe—a consulting firm on privacy and technological tools—the 100 most widely used sites on the Internet are monitored by more than 1,300 firms. Also, established companies, such as Bloomberg and Axiom or more recent ones such as PubMatic or Freshplum, which specialize in developing machine learning algorithms, act as data brokers and help firms to predict a consumer’s willingness-to-pay (The Economist, 2014).

2The telephone industry is one of first industries where firms used customized pricing to a large extent; see, e.g., Chen and Iyer (2002).
urers often provide a better shopping experience than independent retailers. In the case of small and unknown manufacturers selling their products through established retail outlets, the retailer can instead be the one catering to high-valuation consumers.

Independent retailers however also compete with the manufacturer’s own retail outlets, and this intra-brand competition may dissipate profits. Both the benefit of dual distribution, through increased demand, and its costs, through increased competition, are affected by the possibility of price discrimination. Targeted prices allow for larger rent extraction, therefore increasing the benefit of demand expansion, but they may also intensify competition.

These observations suggest that personalized pricing and consumer tracking may not only affect firms’ pricing strategies but also their distribution strategies. Yet, even though personalized pricing has received substantial attention in the literature (see e.g., Shaffer and Zhang, 1995, 2002; Choudhary et al., 2005; Ghose and Huang, 2009), its interaction with other marketing decisions is not well understood.³

Building on these considerations, the objective of this paper is to identify the implications of price discrimination for manufacturers’ distribution strategies. Does personalized pricing change the incentives of a manufacturer to sell through an independent retailer? Is this decision influenced by the type of differentiation between the two firms? Does the trade-off between fiercer competition and higher rent extraction affect the wholesale contract? Can a firm benefit from forgoing the use of personalized pricing?

To answer these questions, we set up a simple model with one brand manufacturer selling directly to final consumers and one independent retailer. The retailer competes with the manufacturer in the downstream market but also adds value to the industry. Specifically, the two firms are either horizontally or vertically differentiated, and in the latter case either the manufacturer or the retailer offers the high-end product.

For each demand pattern, we consider four different scenarios. In the first scenario, the manufacturer and the retailer offer uniform prices to final consumers. This represents a market in which consumer tracking is not possible. In the second scenario, both firms engage in personalized pricing. This reflects the situation in which the two firms have highly-frequented (e.g., online) stores allowing them to gather very precise consumer data.⁴ In the third scenario, only the manufacturer can set personalized prices. This represents for example a situation in which, thanks to past purchases, the manufacturer has better consumer data than the retailer. In the fourth scenario, only the retailer can set personalized prices. This reflects the situation in which a large

³We provide a detailed literature overview in the next section.
⁴Although personalized pricing is an extreme form, it allows us to highlight the effect of price discrimination on distribution choices in a clear way.
retailer offers many products and is thereby able to collect more consumer data than a brand manufacturer. Our analysis therefore captures that new technologies enable firms to use personalized pricing as a trend but there are different capabilities of doing so, both at the market and at the firm level. This allows us to answer the question of how personalized pricing affects channel decisions.

We also consider an extended scenario in which the pricing regime is endogenous and negotiated by the firms—that is, personalized pricing is available to both firms, and they negotiate whether each of them adopts it or not. This case is particularly relevant if the bilateral relationship is a substantial part of the business of both firms, and not just one in a broad portfolio of relationships.

When the pricing regime is given, we first show that dual distribution is always optimal in the case of horizontal differentiation, and in the case of vertical differentiation when the retailer offers the high-end product. In these scenarios, the firms can find a wholesale price that is high enough to mitigate retail competition, and still allows the industry to benefit from the value brought by the retailer. For example, in the case of horizontal differentiation, a wholesale price equal to the willingness-to-pay of the consumer indifferent between the two firms enables them to segment the market: the retailer cannot profitably serve the manufacturer’s core market, and conversely the manufacturer, for which the wholesale price constitutes the opportunity cost of selling directly to consumers, has no incentive to serve the retailer’s core market. Furthermore, personalized pricing by both firms then leads to a higher industry profit than any other pricing regime. This result is markedly different from the one obtained for inter-brand competition between independent firms (e.g., Thisse and Vives, 1988, and Shaffer and Zhang, 1995), according to which firms are trapped in a prisoner’s dilemma and personalized pricing reduces industry profits. In the case of intra-brand between distribution channels, an appropriate wholesale contract reverses this result.

We then focus on vertical differentiation with the manufacturer offering the high-end product. We first show that dual distribution remains optimal as long as the manufacturer offers a uniform retail price—regardless of whether the retailer also sets a uniform price or charges personalized prices. There again, the manufacturer can use the wholesale price to control the intensity of intra-brand competition to a sufficient extent, and yet allow the retailer to expand sales in the low-end segment. This enables the manufacturer to raise its own price and extract more surplus from the high-valuation consumers.

By contrast, if the manufacturer charges personalized prices, then it may favor mono distribution (i.e., sell only through its direct channel)— both when the retailer

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5The firms can contract on uniform pricing, for instance, by adopting privacy or fair treatment policies.
charges a uniform price and when it charges personalized ones. Specifically, relying exclusively on direct distribution is optimal when the retailer does not substantially expand demand, as the effect of increased intra-brand competition then dominates the benefit of expanding demand. For example, when both firms can price discriminate, they can price aggressively in each other’s strong segment without sacrificing margins in their own core business. As a result, it becomes more difficult to control intra-brand competition without impeding market expansion. Mono-distribution is moreover more likely to be optimal when the retailer charges a uniform price, as this reduces its added value. This finding suggests that channel design and wholesale contracting are crucial for the profitability of personalized pricing.

Our insights can explain why brand manufacturers adopt different channel strategies across industries. For example, luxury-goods manufacturers such as Prada or Louis Vuitton often eschew retailers and mostly sell their products in their own stores. Similarly, established mobile network operators have often been reluctant to grant MVNOs access to their network—prompting regulators to impose such access. These markets fit the demand pattern of manufacturers catering to consumers with a high willingness-to-pay and independent retailers catering to those with a lower willingness-to-pay. By contrast, in industries like the apparel industry, retailers (e.g., Asos and Zalando) offer services such as next day delivery and/or free return, whereas manufacturers (e.g., Burberry or Marc O’Polo) offer professional advice and trying on in their stores. Their services are thus differentiated horizontally rather than vertically and, in line with our analysis, manufacturers use both channels.

We then endogenize firms’ pricing policies. Interestingly, we find that it can actually be profitable for the manufacturer not to use personalized pricing, even if it has the ability to do so. The industry profit can indeed be largest in a hybrid pricing regime, in which only the retailer charges personalized prices. Restricting the manufacturer’s pricing policy induces it to focus on its core market, thereby dampening the competitive pressure and allowing the retailer to extract more surplus. Hence, this hybrid pricing regime can achieve the right balance between rent extraction (within each channel) and the avoidance of fierce competition (between channels).

Finally, we show that our main insights carry over to a situation in which the manufacturer can shut down its direct channel. Then, mono distribution by the retailer is more likely to be optimal if it can charge personalized prices, as it is more valuable in that case.

We discuss in the conclusion the lessons from our analysis, which may provide guidance for contracting with pure retailers. A key insight is that price discrimination and consumer addressability—which is feasible in many modern industries—affects not only the pricing strategy, but also the optimal distribution network. Indeed, mono
distribution may then be optimal even when a retailer adds value to the market. The reason is that competition for final consumers can destroy profits in the manufacturer’s core market.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the model and considers the demand patterns of horizontal differentiation and of vertical differentiation in which the retailer offers the high-end product. Section 4 analyzes the demand pattern in which the manufacturer offers the high-end product. Section 5 endogenizes the choice of the pricing regime. Section 6 extends the analysis by allowing the manufacturer to shut down its direct channel, and Section 7 concludes. Formal proofs are provided in the Online Appendix.

2 Related Literature

The literature on competition with price discrimination has almost exclusively focused on “inter-brand” competition between fully independent firms.\(^6\) This literature usually distinguishes between models of horizontal and vertical differentiation. In their seminal paper, Thisse and Vives (1988) analyze the effects of price discrimination for horizontally differentiated firms competing on a Hotelling line. They demonstrate that this leads to a prisoner’s dilemma: firms adopt price discrimination but profits fall due to increased competition.\(^7\) Shaffer and Zhang (1995) highlight a similar prisoner’s dilemma when firms discriminate through coupon targeting and consumers differ in the cost of redeeming coupons. Chen and Iyer (2002) allow firms to choose the proportion of consumers for whom they acquire information. In this case, firms may benefit from consumer addressability and may refrain from acquiring full information. Chen et al. (2020) allow consumers to bypass price discrimination and buy at a uniform price. They show that this possibility can collectively harm consumers and allow firms to benefit from price discrimination.\(^8\) Montes et al. (2019) obtain a similar result in a model in which consumers, by incurring a cost for keeping privacy, can prevent firms from exploiting information about their preferences. They show that a decrease in this cost may harm consumers and benefit firms.

Choudhary et al. (2005), in one of the first papers introducing the expression “personalized pricing”, consider instead competition between vertically differentiated firms, and find that pricing strategies can be non-monotonic in consumer valuations. In ad-

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\(^6\)See Stole (2007) and Zhang (2009) for an overview of different forms of price discrimination and targeted pricing and how they affect competitive outcomes.

\(^7\)Liu and Serfes (2013) extend the framework of Thisse and Vives (1988) by studying the effects of price discrimination in two-sided markets. Matsumura and Matsushima (2015) show instead that firms may choose not to price discriminate in order to limit rivals’ incentives to engage in cost reduction.

\(^8\)Shaffer and Zhang (2000) consider asymmetric customer bases and provide conditions under which a firm may offer a lower price to its own customer base.
dition, they show that personalized pricing can lead to an increase or a decrease in quality levels. Two papers combine vertical and horizontal differentiation. Shaffer and Zhang (2002) show that firms offering higher quality may benefit from personalized pricing, even though competition is fiercer. This is due to a gain in market share, which dominates the effect of lower prices. Ghose and Huang (2009) consider the case in which firms can customize product quality according to consumer preferences, and find that this can make personalized pricing profitable.\footnote{For empirical papers on how firms can implement personalized pricing, see, for example, Wertenbroch and Skiera (2002) or Elsner et al. (2004). Rossi et al. (1996), Dubé and Misra (2017), and Shiller (2020), among others, provide estimates for the profitability of personalized pricing relative to uniform pricing in different set-ups.}

Our paper contributes to this literature by studying the implications of personalized pricing on intra-brand competition, wholesale contracting, and on the choice of distribution channels. To the best of our knowledge, the only paper that also analyzes the interplay between personalized pricing and the distribution strategy is Liu and Zhang (2006). They consider a setting in which the retailer has access to personalized pricing and the manufacturer—who charges a linear wholesale tariff—can enter through direct marketing with a uniform price. They show that the adoption of personalized pricing harms the retailer by inducing the manufacturer to charge a higher wholesale price, but can nevertheless be profitable by deterring the manufacturer from entering the downstream market. Their focus is on the retailer’s pricing strategy and its implications for downstream entry by the manufacturer. We focus instead on an integrated manufacturer’s decision to allow a retailer to enter the market, and study the implications of pricing strategies on this decision. We also allow for personalized pricing by both firms and consider a non-linear wholesale tariff.

Our paper also contributes to the literature on Internet channel entry (Chiang et al., 2003; Yoo and Lee, 2011) by determining the conditions under which an incumbent who directly markets its products deters entry by a pure retailer.\footnote{The literature on channel coordination (McGuire and Staelin, 1983; Moorthy, 1987, 1988; Rey and Stiglitz 1988, 1995) usually focuses on channel coordination without suppliers being able to sell directly to final consumers.} On a broader level, our paper establishes that the possibility of price discrimination not only has short-run effects on competition but also influences other important marketing decisions. It is therefore in line with Jing (2016) and Li (2018): Jing (2016) shows that behavior-based pricing affects quality differentiation between firms, and Li (2018) determines how behavior-based pricing shapes competition between manufacturer-retailer channels, showing that this crucially depends on whether only retailers can adopt behavior-based pricing or manufacturers can do so as well.\footnote{Behavior-based price discrimination refers to the practice of charging consumers different prices dependent on their purchase history. For papers analyzing how this affects dynamic pricing, see Acquisti and Varian (2005) for the monopoly case and, for example, Villas-Boas (1999), Fudenberg and Tirole 1998, 1999).}
Finally, our paper also contributes to the literature on market foreclosure. Several papers show that a vertically integrated firm has the incentive to raise wholesale prices to a non-integrated downstream rival to dampen price competition (see e.g., Salinger, 1988, Ordover et al., 1990, Hart and Tirole, 1990, Chen, 2001, and Bourreau et al., 2011). However, if the rival adds value to the industry, for example, by offering a differentiated product, foreclosure takes place only partially, as the integrated firm benefits from entry through the wholesale revenue. By contrast, our paper shows that an integrated firm may fully deny access to its products if price discrimination downstream is feasible.

3 The Model

3.1 Setting

Supply. A monopoly brand manufacturer, firm A, sells its good through a direct distribution channel and can also rely on an independent retailer, firm B. In order to highlight the strategic motive for mono or dual distribution, we assume away any fixed costs of opening a new distribution channel. For simplicity, variable costs are assumed to be linear and normalized to zero.

Demand. Consumers have heterogeneous preferences over the firms’ offerings. Specifically, a consumer of type \( x \in [0, X] \) has a willingness-to-pay for the offering of firm \( i = A, B \) (“product i”, thereafter) given by:

\[
 u_i(x) = \max \{ r_i - s_i x, 0 \} .
\]

Without loss of generality, we suppose that consumers are ranked by decreasing order of preference for product A: \( s_A > 0 \), but allow \( s_B \) to be positive or negative. We do assume that both products play an effective role. Specifically, letting:

\[
 \tilde{x} \equiv \frac{r_A - r_B}{s_A - s_B}
\]

(2000), Zhang (2011), Rhee and Thomadsen (2017), and Choe et al. (2018) for the case of (imperfect) competition. Rey and Tirole (2007) provide an overview of this literature. An exception is Weeds (2016) who shows that vertically integrated content providers may fully foreclose rival distributors due to dynamic considerations. If consumers have switching costs, exclusivity confers a market share advantage, which is beneficial in the future. In Section 6, we also consider the possibility that the manufacturer shuts down its direct channel and distributes only through the independent retailer. \( r_i \) can be interpreted as consumers’ intrinsic utility for product \( i \), net of the unit costs of production.
denote the consumer type who receives the same value from both products, and:

\[ \hat{u} \equiv \frac{s_A r_B - s_B r_A}{s_A - s_B} \]

denote the corresponding utility, we maintain the following assumptions:

\[ \hat{x} \in (0, X) \text{ and } \hat{u} > 0. \]  \hspace{1cm} (1)

The first assumption ensures that no product “dominates” the other. The second assumption ensures that both products offer value to some consumers who have a positive willingness-to-pay (namely, those with a type close to \( \hat{x} \)). Together, these assumptions ensure that the two products compete for those consumers.

There are thus three possible demand configurations:

- If \( s_B < 0 \), consumers’ utilities are \textit{negatively} correlated: consumers who are more attracted by one product are less attracted by the other one (see Figure 1a). This corresponds to the pattern commonly observed in classic models of horizontal differentiation.\(^{16}\)

- If instead \( s_B > 0 \), consumers’ utilities are \textit{positively} correlated: consumers who are more attracted by one product are also more attracted by the other. This corresponds to the pattern commonly observed in classic models of vertical differentiation, in which consumers have heterogeneous tastes for quality, and consumers who value quality more typically derive greater utility from both products. Consider for example the classic model of Shaked and Sutton (1982), and suppose that the firm offering the product of higher quality also has higher unit costs. Then, high-valuation consumers are more profitable for the high-quality firm, and low-valuation consumers are more profitable for the low-quality firm.\(^{17}\) Two cases can then arise:

  - product A is the “high-end” product, that is, it is favored by high-valuation consumers. This case occurs when \( s_B \in (0, s_A) \) (see Figure 1b)—our assumptions in (1) then imply \( r_A > r_B \).

  - product B is instead the “high-end” product; this case occurs when \( s_B > s_A \) (see Figure 1c), implying \( r_B > r_A \).

\(^{16}\)For example, the standard Hotelling model in which consumers with reservation value \( v \) are uniformly distributed along a unit-length segment and face a transportation cost \( t \) corresponds to \( r_A = v, r_B = v - t, s_A = t \) and \( s_B = -t \).

\(^{17}\)For instance, denote by \( q_i \) the quality supplied by firm \( i \) and the corresponding unit costs \( c_i \). Consumers’ valuations are \( \theta q_i \), where \( \theta \) is uniformly distributed over \([0, 1]\). Using firm \( i \)’s margin \( m_i = p_i - c_i \) as its strategic pricing decision, a classic model of vertical differentiation corresponds to \( r_i = q_i - c_i, x = 1 - \theta, \) and \( s_i = q_i \). Indeed, the net utility of consumer type \( \theta \) when buying from firm \( i \) is \( \theta q_i - p_i = u_i(\theta) - m_i \), where \( u_i(\theta) = r_i - s_i \theta x. \)
These demand configurations reflect the fact that the brand manufacturer and the retailer usually provide different offers, and consumers’ valuations for these offers are heterogeneous. Brick-and-mortar stores and, sometimes, the websites of brand manufacturers allow customers to obtain professional advice from trained salespeople and free testing of the product. Online retailers offer instead lower transaction costs (e.g., due to one-click shopping), free return policies, and allow customers to see valuable user feedback (see e.g., Acquisti and Varian, 2005). The various configurations discussed above correspond to different industries. Horizontal differentiation arises when customers with a similar willingness-to-pay for the manufacturer’s good have heterogeneous preferences regarding these services. This is the case, for example, in the apparel or consumer electronics industry, in which some customers value more trying on a Marc O’Polo shirt or testing Sony headphones than generous return policies and recommendations, whereas for others it is the reverse. Vertical differentiation with the manufacturer offering the high-end product is more relevant for markets such as fragrances or high-end clothes, where manufacturers’ stores often provide an aura of luxury, which is particularly attractive to high-valuation consumers. By contrast, consumers with a lower valuation are likely to prefer the retailer’s channel, where they can find more variety. Vertical differentiation with the retailer offering the high-end product can instead arise in markets in which the manufacturer is relatively small or unknown. An example is the market for organic food, where retailers such as iHerb or Wholefoods offer a large variety and have been established for years, whereas many brands sold there specialize on a particular kind of product, and only low-valuation customer make the effort of visiting the manufacturer’s website.

Retail competition. $A$ and $B$ compete in prices for consumers. For each firm, we consider two types of pricing policies: uniform pricing (non-discrimination), in which the firm charges the same price to all consumers, and personalized pricing, in which the
firm can perfectly discriminate consumers according to their types. Firm $i$’s price is denoted by $p_i$ under uniform pricing, and by $p_i(x)$ under personalized pricing. Combining the pricing policies of the firms, there are in total four different pricing regimes: two symmetric regimes (i.e., both firms charge a uniform price, and both firms charge personalized prices), and two hybrid regimes (i.e., $A$ charges a uniform price and $B$ personalized prices, and vice versa).

**Wholesale contracting.** We consider two-part tariffs of the form $T(q) = F + wq$, where $F$ denotes the fixed fee and $w$ the uniform wholesale price paid by $B$ to $A$, and $q$ is the quantity bought by $B$.\(^{18}\) We suppose that the two firms adopt the Nash bargaining solution to negotiate about $w$ and $F$, with bargaining power $\alpha$ for $A$ and $1 - \alpha$ for $B$.

**Timing.** As discussed in the Introduction, the pricing regime is driven by technology and data considerations: personalized pricing may be available in some industries, and not in others. In addition, the bilateral relationship on which we are focusing here can be part of a broader portfolio of relationships, in which case the pricing policy may be based on the overall portfolio, and not on the particular bilateral relationship. This leads us to first treat the pricing regime as exogenous. However, in other instances, the pricing policy may be tailored to the specific relationship. For this reason, we consider in Section 5 an extended version of our set-up in which we allow firms to contract on a wholesale tariff and on their retail pricing policies.

Regarding the interplay between upstream and downstream pricing decisions, we follow the literature and assume that wholesale contracting precedes retail pricing. This reflects the fact that wholesale negotiations are relatively less frequent than retail price adjustments. This leads us to consider the following timing:

- **Stage 1:** $A$ and $B$ negotiate the wholesale contract.
- **Stage 2:** Active firms set their retail prices. Consumers then observe all retail prices and decide whether or not to buy, and from which firm to buy. If active, $B$ then orders the quantity from $A$ to satisfy its demand.

In the first stage, firms can share their joint profit through the fixed fee; hence, they seek to maximize the industry profit. Dual distribution is thus optimal if the resulting industry profit is larger than the one with mono distribution. For the sake of exposition, we will then say that dual distribution is optimal.

In the second stage, for symmetric pricing regimes, firms simultaneously set their prices. For asymmetric pricing regimes, we follow Thisse and Vives (1988), Liu and Zhang (2006), and Choe et al. (2018) in assuming that the firm charging a uniform price can perfectly discriminate consumers according to their types. Firm $i$’s price is denoted by $p_i$ under uniform pricing, and by $p_i(x)$ under personalized pricing. Combining the pricing policies of the firms, there are in total four different pricing regimes: two symmetric regimes (i.e., both firms charge a uniform price, and both firms charge personalized prices), and two hybrid regimes (i.e., $A$ charges a uniform price and $B$ personalized prices, and vice versa).

\(^{18}\)In Online Appendix E, we show that our main results also hold with a linear wholesale contract.
price, say firm \( i \), acts as a price leader: it sets \( p_i \) before the competitor sets its personalized prices \( p_{-i}(x) \). This assumption ensures the existence of a pure-strategy Nash equilibrium. As pointed out by Thissae and Vives (1988), it is natural for asymmetric regimes, as \( i \) can announce and advertise its uniform price in advance, whereas this may be too complex or costly for the competitor. In addition, as noted by Choe et al. (2018) and Chen et al. (2018), the adjustment of a uniform price is a higher-level managerial decision, that is relatively slower in practice than the adjustment of personalized prices.

**Solution concept.** Our solution concept is subgame perfection.\(^{19}\) In the case of price discrimination, asymmetric Bertrand competition for each consumer \( x \) is known to generate multiple equilibria. Following the literature, we focus on the equilibrium in which the firm offering the lower value prices at cost.\(^{20}\)

**Remark: wholesale personalized pricing.** We focus on the case in which personalized pricing may be possible at the retail but not at the wholesale level. That is, the wholesale contract cannot be conditioned on consumers’ types. While this would allow the firms to maximize the industry profit, it is usually infeasible. Manufacturers are often unable to monitor which consumers their distributors are selling to; and even if they could obtain that information, it would be difficult to verify it in a court of law.

### 3.2 Optimality of Dual Distribution

Dual distribution expands the product range, which enables the firms to bring more value to consumers, but it creates competition from \( B \), which may dissipate profits. This concern can however be mitigated by raising the wholesale price paid by \( B \). The next proposition shows that this instrument is indeed particularly effective in the case of horizontal differentiation, as well as in the case of vertical differentiation when firm \( B \) offers the high-end product.

**Proposition 1:** In all pricing regimes, dual distribution is optimal in the case of horizontal differentiation (i.e., \( s_B < 0 \)) and, in the case of vertical differentiation, with \( B \) offering the high-end product (i.e., \( s_B > s_A \)).

**Proof:** See Online Appendix A.

In the case of horizontal differentiation, consumers \( x \leq \hat{x} \) constitute \( A \)'s “core mar-

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\(^{19}\)Formally, subgame perfection applies from stage 2 onwards. In stage 1, Nash bargaining could be also be achieved as the equilibrium of a non-cooperative random-proposer game in which each firm gets to make a take-it-or-leave-it offer with a probability reflecting its bargaining power. To obtain a deterministic outcome, it suffices to introduce a preliminary stage in which one firm (either one) makes an offer, with the random-proposer game acting as default option.

\(^{20}\)This is the unique Coalition-Proof Nash equilibrium (in particular, it is the Pareto-dominant equilibrium from the firms’ standpoint) and is also the unique trembling-hand perfect equilibrium.
ket”, whereas consumers $x \geq \hat{x}$ constitute $B$’s core market. Firm can then avoid competition by agreeing on, say, $w = \hat{u}$: $B$ cannot profitably sell in $A$’s core market, as these consumers have a willingness-to-pay for $B$’s product which is below $\hat{u}$. Conversely, as $w$ constitutes $A$’s opportunity cost when competing against $B$, $A$ has no incentive to serve $B$’s core market. However, $B$ sells to consumers in its core market at a higher price than $A$ can do with mono distribution. As a consequence, the industry profit with dual distribution is higher than with mono distribution.

In particular, if both firms can charge personalized prices, setting $w = \hat{u}$ enables them to extract the entire consumer surplus: $A$ sells to consumers $x \leq \hat{x}$ at personalized prices $u_A(x)$ and $B$ sells to $x \geq \hat{x}$ at personalized prices $u_B(x)$. The equilibrium industry profit is thus higher whenever both firms charge personalized prices than in any other pricing regime. This result is in sharp contrast to the one obtained in the classic papers on personalized pricing (e.g., Thisse and Vives, 1988, and Shaffer and Zhang, 1995), which consider inter-brand competition between independent firms. The possibility of personalized pricing then leads to a prisoner’s dilemma in which firms choose personalized pricing but the industry profit is lower than with uniform pricing. Our analysis shows that in the case of intra-brand competition between distribution channels, the result reverses, and a well-designed wholesale contract allows firms to even extract the entire consumer surplus.

Consider now the case of vertical differentiation in which $B$’s core market includes the high-valuation consumers. If $A$ charges a uniform price in the retail market, negotiating a wholesale price equal to $A$’s monopoly price under mono distribution raises $A$’s profit: $A$’s margin when $B$ serves a customer under dual distribution is then the same as that under mono distribution, and $A$’s can still charge the same price as before to the other customers. As $B$ can charge a higher price to high-valuation consumers, the industry profit increases compared to mono distribution. The argument carries over when $A$ charges personalized prices in the retail market: setting the wholesale price to the highest of $A$’s monopoly prices, that is, $w = r_A$, strictly increases the industry profit. (In this case, $A$ is strictly better off if $B$ makes a sale.)

4 Vertical differentiation with $A$ offering the high-end product

Having shown that dual distribution is optimal with horizontal differentiation and when $B$ offers the high-end product, from now on we turn to the remaining scenario

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21 $A$ receives a margin of $w$ whenever $B$ sells to a consumer; $w$ thus represents $A$’s opportunity cost of serving the customer itself.
in which \( A \) offers the high-end product (i.e., \( s_A > s_B > 0 \)). As already noted, the assumptions \( \hat{x} > 0 \) and \( \hat{a} > 0 \) then imply \( r_A > r_B > 0 \) and:

\[
\rho \equiv \frac{r_B}{r_A} > \frac{s_B}{s_A} \equiv \sigma.
\]

To simplify the exposition, we restrict attention to the case in which no firm can serve all consumers. That is, letting:

\[
x_i \equiv \frac{r_i}{s_i},
\]

denote firm \( i \)'s marginal demand, we focus on the case \( (\bar{x}_A < \bar{x}_B \leq X) \). None of our qualitative results hinges on this assumption, but it helps to convey our insights in a concise way.

4.1 Equilibrium Analysis

We proceed by first stating our main results of this section, which show that the optimality of dual distribution crucially depends on whether \( A \) can charge personalized prices or not. We then sketch the arguments behind the results for each of the four pricing regimes:

Proposition 2: In the case of vertical differentiation, when \( A \) offers the high-end product (i.e., \( s_A > s_B > 0 \)):

(i) If \( A \) charges a uniform price, then dual distribution is optimal, regardless of \( B \)'s pricing regime.

(ii) If \( A \) charges personalized prices, then mono distribution is optimal if and only if:

\[
\rho < \sigma \frac{2 + \delta(\sigma)\sqrt{1 - \sigma}}{1 + \sigma},
\]

where \( \delta(\sigma) = 1 \) if \( B \) charges personalized prices as well and \( \delta(\sigma) = \sqrt{\frac{2\sigma + \sigma^2}{1 + 2\sigma}} > 1 \) if \( B \) charges a uniform price.

Proof: See Online Appendix B.

We now sketch the arguments underlying Proposition 2.

Part (i)

We start with the situation in which both firms charge a uniform price. If only \( A \) is active, it faces the monopoly demand \( (r_A - p_A) / s_A \); it thus charges the monopoly price \( p_A^m = r_A / 2 \), serves consumers \( x \leq x_A^m = r_A / (2s_A) \), and obtains a profit of (the subscript
$U$ stands for Uniform pricing:

$$\Pi^m_U = \frac{r_A^2}{4s_A}.$$ 

If instead $A$ and $B$ are both active, they charge retail prices $p_A$ and $p_B$ such that some consumers favor $A$ whereas others favor $B$. Let $x_{AB} > 0$ denote the consumer indifferent between buying from $A$ or $B$, and $x_B > 0$ denote the consumer indifferent between buying from $B$ and not buying:

$$x_{AB}(p_A, p_B) = \frac{r_A - p_A - r_B + p_B}{s_A - s_B} \quad \text{and} \quad x_B(p_B) = \frac{r_B - p_B}{s_B}.$$ 

Any consumer $x < x_{AB}$ prefers $A$ to $B$; hence, the demands for $A$ and $B$ are, respectively, $x_{AB}$ and $x - x_{AB}$. The profit functions of the two firms (gross of the fixed fee) are then $\Pi_A = x_{AB}(p_A, p_B)p_A + [x_B(p_B) - x_{AB}(p_A, p_B)] w$ and $\Pi_B = [x_B(p_B) - x_{AB}(p_A, p_B)] (p_B - w)$.

In the first stage, $A$ and $B$ negotiate over $w$ and $F$, following the Nash bargaining solution, taking into account that, in the second stage, each firm sets its retail price so as to maximize its own profit. This implies that the firms set $w$ to maximize the industry profit and use $F$ to share it according to their bargaining powers and outside options. Denoting the equilibrium prices at the retail stage by $p_i(w)$, the maximization problem with respect to $w$ is:

$$\max_w x_{AB}(p_A(w), p_B(w))p_A(w) + [x_B(p_B(w)) - x_{AB}(p_A(w), p_B(w))] p_B(w)$$

Dual distribution is optimal if the resulting industry profit is larger than $A$’s profit from mono distribution, $\Pi^m_U$. As shown in Proposition 2, this is indeed always the case under uniform pricing.

We illustrate the intuition by Figure 2. To see that dual distribution leads to a higher industry profit than mono distribution, note first that the firms can replicate the outcome of mono distribution by negotiating a wholesale price $w^m = u_B(x^m_A)$. This induces $A$ to charge the monopoly price $p^m_A$ and prevents $B$, which must charge at least $w^m$, from profitably attracting any consumer. Indeed, consumers with $x > x^m_A$ are not willing to pay $w^m$, and those with $x < x^m_A$ prefer $A$’s product at price $p^m_A = u_A(x^m_A)$ to $B$’s product at any price $p_B \geq w^m = u_B(x^m_A)$. Consider now a small reduction in the wholesale price, $w < w^m$, that generates a retail equilibrium in which $B$ serves some consumers at price $p_B = w^m - dp$. In this retail equilibrium, $B$ cannot obtain a negative profit and $A$ cannot obtain less than what it would earn by charging $\hat{p}_A = p^m_A - dp$, so as to maintain its market share, $x_A = x^m_A$. Doing so would lead $B$ to sell a

---

22Specifically, $A$’s outside option is its profit from mono distribution whereas $B$’s outside option is zero.
quantity $dx_B$ implicitly given by $dp = -u_B'(x^m_A) dx_B$. Hence, the industry profit cannot be lower than $\pi_A + \pi_B \geq [(p_A^m - dp) x^m_A + wdx_B] + 0 \approx \Pi^m_U + u_B(x^m_A)dx_B - x^m_A dp = \Pi^m_U + \frac{d}{dx} [u_B(x) x]_{x=x^m_A} dx_B$, which exceeds $\Pi^m_U$: as $B$ faces a more elastic monopolistic demand (that is, $|u_B'(x)|/u_B(x) < |u_A'(x)|/u_A(x))$, its monopolistic output exceeds $x^m_A$ (that is, $\frac{d}{dx} [u_B(x) x]_{x=x^m_A} > 0$).

This shows that the industry profit is always larger if $B$ is marginally active. This insight does not hinge on the demand being linear; it holds more generally as long as $B$’s monopolistic output exceeds that of $A$.\textsuperscript{24} Note however that the equilibrium wholesale price may be substantially lower than $w^m$, so that $B$’s market share may also be substantial.

We now consider the scenario in which $A$ still charges a uniform price but $B$ can offer personalized prices. The intuition why dual distribution is optimal in this case is illustrated by Figure 3, which depicts the equilibrium prices under uniform pricing, $p_A^*$ and $p_B^*$, and the retail prices of $B$ that would emerge if the two firms opted for dual distribution and set $w = p_B^*$ and $p_A = p_A^*$ (i.e., the equilibrium prices under uniform pricing).\textsuperscript{25}

\textsuperscript{23}This is due the fact that $\rho > \sigma$.

\textsuperscript{24}We provide a proof of this statement in Online Appendix I.

\textsuperscript{25}Remember that $A$ acts as a price leader in this regime: $w$ and $p_A$ are chosen before $B$ sets its retail prices.
A then serves consumers \( x < x_A^* \) (for whom \( u_A(x) - u_B(x) = w \)), whereas \( B \) serves consumers between \( x_A^* \) and \( x_B^* \) (for whom \( u_B(x) = w \)), and charges them prices equal to \( \min \{ p_A^* + u_B(x) - u_A(x), u_B(x) \} \). The resulting industry profit is larger than under uniform pricing: in the segment served by \( A \), the profit is the same because \( p_A = p_A^* \); by contrast, in the segment served by \( B \), \( B \) charges a strictly higher price than \( p_B^* \). Because opting for dual distribution was already optimal with uniform pricing, and yields even more profits in the regime with personalized pricing by \( B \), it also dominates mono distribution in the latter regime.

![Figure 3: Profit with personalized pricing by \( B \) only if \( w = p_B^* \)](image)

**Part (ii)**

We now turn to the scenarios in which \( A \) charges personalized prices. We start with the symmetric regime, in which \( B \) charges personalized prices as well. If \( B \) is not active, then \( A \) charges each consumer \( x \) a price equal to her utility \( u_A(x) \), and thus obtains a profit of (the subscript \( P \) stands for *Personalized pricing*):

\[
\Pi_P = \int_{x_A^*}^{x_B^*} (r_A - s_A x) \, dx = \frac{r_A^2}{2s_A^2}.
\]

We now turn to equilibria in which \( A \) and \( B \) are both active, starting with the retail stage.

*Retail competition.* As firms now compete for each consumer \( x \), three cases can arise.
If \( u_B(x) < w \), then \( B \) cannot offer a positive value to consumers without incurring a loss; \( A \) then charges the monopoly price \( p_A = u_A(x) \).

If instead \( u_A(x) < w \leq u_B(x) \), \( A \) would have to price below \( w \) to win the consumer, and is therefore better off letting \( B \) serve this consumer; hence, \( B \) wins the competition by charging the monopoly price \( u_B(x) \) (and \( A \) charges a price equal to its opportunity cost \( w \), or any other price exceeding \( u_A(x) \)).

The most interesting case occurs when \( w \leq u_A(x) , u_B(x) \). As \( w \) constitutes \( A \)'s opportunity cost from serving the consumer, and \( u_B(x) \) is \( B \)'s real cost, a standard Bertrand argument applies: for consumers \( x \) with \( u_i(x) > u_j(x) \), for \( i \neq j \in \{ A, B \} \), firm \( i \) wins the competition and sells to the consumer at price \( p_i(x) = w + u_i(x) - u_j(x) \), whereas the other firm sets \( p_j(x) = w \).

**Wholesale negotiation.** We now turn to the determination of the wholesale contract. We first note that for any wholesale price \( w \) above \( \hat{u} \), \( B \) is inactive in equilibrium: it is dominated by \( A \) in the consumer segment \( x < \hat{x} \), and cannot offer a positive value at a profitable price in the segment \( x > \hat{x} \). The profit thus cannot exceed \( \Pi_B^0 \).

If the firms negotiate a wholesale price \( w \leq \hat{u} \), they are both active in the continuation equilibrium. Let:

\[
\hat{x}_i(w) \equiv \frac{r_i - w}{s_i}, \tag{4}
\]

denote the marginal consumer willing to buy product \( i \) at price \( w \). The profits of the two firms at the retail stage can be expressed as \( \Pi_A \) and \( \Pi_B \), where:\n
\[
\Pi_A = \int_0^{\hat{x}} [w + u_A(x) - u_B(x)] \, dx + w [\hat{x}_B(w) - \hat{x}],
\]

and:

\[
\Pi_B = \int_{\hat{x}}^{\hat{x}_A(w)} [u_B(x) - u_A(x)] \, dx + \int_{\hat{x}_A(w)}^{\hat{x}_A(w)} [u_B(x) - w] \, dx.
\]

These profit functions are illustrated by Figure 4, where the hatched area represents the industry profit. The first term in \( A \)'s profit comes from consumers \( x < \hat{x} \) (first region in Figure 4): \( A \) offers them a higher value, and serves them at price \( w + u_A(x) - u_B(x) \). The second term in \( A \)'s profit reflects the wholesale revenue generated by consumers served by \( B \) (the other two rectangles in the figure). \( B \)'s profit comes from consumers for whom it offers a higher value, and can also be split in two parts. The first term corresponds to consumers \( \hat{x} < x < \hat{x}_A(w) \) (second triangle), for whom both firms compete, and so \( B \) only earns a margin \( u_B(x) - u_A(x) \). The second

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\(^{26}\)Both firms set a price of \( w \) to consumer \( \hat{x} \).

\(^{27}\)As we show in Online Appendix B, conditional on reaching an agreement (i.e., \( w < \hat{u} \)), the firms negotiate a wholesale price so that \( B \) expands potential demand; that is, \( B \) sells to consumers who would not be interested in buying from \( A \) at any positive price (i.e., \( w \) is sufficiently low that \( \hat{x}_B(w) > \hat{x}_A \)).
term corresponds to consumers \( x_A(w) < x < x_B(w) \) (third triangle), to whom \( A \) offers a lower value than \( w \), and so \( B \) can extract the full value and earn a margin \( u_B(x) - w \).

Figure 4: Industry profit under personalized pricing

In the negotiation in the first stage, the firms maximize the industry profit:

\[
\Pi(w) = \int_0^{\hat{x}_A(w)} [w + |u_B(x) - u_A(x)|] \, dx + \int_{\hat{x}_A(w)}^{\hat{x}_B(w)} u_B(x) \, dx.
\]

Taking the derivative with respect to \( w \) (and using \( u_i(\hat{x}_i(w)) = w \) for \( i = A, B \)) yields:

\[
\Pi'(w) = \hat{x}_A(w) + w\hat{x}_B'(w).
\]

When negotiating on the wholesale price \( w \), firms face the following trade-off. By increasing \( w \), the firms obtain a higher benefit from the inframarginal consumers in the range \( x < \hat{x}_A(w) \): as the two firms compete for these consumers, an increase in \( w \) increases the final consumer price by the same amount. However, increasing \( w \) has also a negative effect on the marginal consumer, \( \hat{x}_B(w) \), for whom \( B \) can charge the full value, \( u_B(\hat{x}_B(w)) = w \). By contrast, the revenue from consumers between \( \hat{x}_A(w) \) and \( \hat{x}_B(w) \) is unchanged, as these consumers continue buying from \( B \) and pay their reservation price.
Using (4), the first-order condition \( \tilde{x}_A(w) + w\tilde{x}'_A(w) = 0 \) yields:\(^{28}\)

\[
w = \frac{s_{B}r_{A}}{s_{A} + s_{B}}.
\]

The associated industry profit is (recalling the notation \( \rho \equiv r_B/r_A \in (0, 1) \) and \( \sigma \equiv s_B/s_A \in (0, \rho) \)):

\[
\Pi_P = \frac{r_A^2}{2s_A} \frac{\sigma (1 + 3\sigma) - 4\sigma (1 + \sigma) \rho + (1 + \sigma)^2 \rho^2}{\sigma (1 - \sigma^2)},
\]

which is smaller than the monopoly profit \( \Pi_P^m \) given by (3) if and only if (2) holds, with \( \delta(\sigma) = 1 \).

In contrast to the case with uniform pricing by \( A \), when price discrimination at the retail stage is possible, mono distribution may indeed occur. This happens if \( \rho \) is sufficiently small.\(^{29}\) This result holds despite the fact that, with personalized pricing, the two firms share the market efficiently: consumers \( x < \tilde{x} \) (resp., \( x > \tilde{x} \)) buy from \( A \) (resp., \( B \)). This was not true under uniform pricing, as \( B \) then sets a lower price and therefore also sells to consumers who have a relative preference for \( A \)’s product. However, personalized pricing also allows a firm to lower the price charged to marginal consumers down to marginal cost without sacrificing profit on inframarginal consumers. This has two implications: first, \( B \) can serve additional consumers, and thereby expand the market, and second, \( B \) prices more aggressively in \( A \)’s core market. This in turn makes \( A \) more aggressive. Competition is thus more intense, which dissipates profits; whenever this effect prevails, the firms do not reach an agreement, resulting in mono distribution. Instead, under uniform pricing, firms tend to focus on exploiting their market power over consumers in their respective core markets, which leads to relatively high prices, and dual distribution.

The equilibrium configurations under personalized-pricing are depicted in Figure 5, in which the abbreviation \((PP)\) refers to the regime of personalized pricing by both firms. Note first that under uniform pricing, dual distribution is optimal in the whole range \( \rho > \sigma \) (i.e., the range in which \( B \) adds value to the industry).\(^{30}\) By contrast, the optimal distribution choice under personalized pricing depends on the specific values of \( \rho \) and \( \sigma \). The condition stated in Proposition 2 (\( ii \)) shows that mono distribution is more likely to be optimal, the lower the net additional value being brought by \( B \), namely, the lower the relative intercept of \( B \)’s demand function (as measured by \( \rho \))

\(^{28}\)The profit function is concave as \( \Pi''(w) = - (1/s_A + 1/s_B) < 0 \).

\(^{29}\)In Online Appendix F, we show that a qualitatively similar result emerges if firms can negotiate \( A \)’s retail prices in the wholesale contract, in addition to specifying \( w \) and \( F \).

\(^{30}\)The figure also depicts the range \( 0 \leq \rho < \sigma \), in which \( B \) does not add value, and, hence, mono distribution is always optimal.
and the steeper the relative slope (as measured by $\sigma$). In fact, for $\delta(\sigma) = 1$, condition (2) always holds if $\sigma \geq (\sqrt{5} - 1)/2 \approx 0.618$, as the right-hand side is then larger than 1. For $\sigma < (\sqrt{5} - 1)/2$, the right-hand side is strictly increasing in $\sigma$.

![Figure 5: Equilibrium configurations in the regimes (PP) and (PU)](image)

Finally, we turn to the situation in which only $A$ can charge personalized prices (regime (PU) in Figure 5). As stated in Proposition 2, mono distribution is again optimal if $\rho$ is sufficiently small. The intuition is similar to the case with personalized pricing by both firms—i.e., dual distribution leads to competition and destroys profit from the high-valuation consumers to a sufficiently large extent that the industry profit is higher with mono distribution. However, as $\delta(\sigma)$ then exceeds 1, mono distribution is optimal for a larger range in the hybrid regime. In particular, as illustrated in Figure 5, mono distribution is then always optimal if $\sigma \geq 0.473$. This result emerges despite the fact that competition is less intense in the hybrid regime due to the fact that $B$ can only charge a uniform price. The intuition behind the result is as follows: with symmetric personalized pricing, the firms can benefit from $B$’s ability to extract consumer rent, particularly so if $\sigma$ is relatively small. By contrast, if $B$ cannot charge personalized prices, this ability is reduced. In addition, $B$ always demands a mark-up on the wholesale price, which implies that $B$ serves fewer low-valuation consumers even if wholesale prices in both regimes were the same. Hence, the industry profit with dual distribution is lower in the hybrid regime, which leads to a larger range in which mono
distribution is optimal.

We conclude this section by noting that wholesale prices can be ranked across pricing regimes:

\[ w_{UU}^* < w_{UP}^* < w_{PP}^* < w_{PU}^*, \]

where the subscripts refer to A’s and B’s pricing policies. This shows that wholesale prices are higher when A can charge personalized prices as compared to a uniform one.

### 4.2 Discussion

We now discuss several lessons from our analysis. A first insight is that the impact of personalized pricing on competition and profits may affect channel decisions: dual distribution, which is always optimal under uniform pricing, need not be so anymore under personalized pricing. However, the way in which the pricing regime affects the channel decision departs from common wisdom. For example, in the case of horizontal differentiation, personalized pricing intensifies inter-brand competition and dissipates profits (see e.g., Thisse and Vives (1988) and Shaffer and Zhang (1995)), suggesting that dual distribution is less attractive. But in the case of intra-brand competition considered here, firms can use the wholesale contract to alleviate competition, and can actually maximize total industry profit; as a result, dual distribution is always optimal.

In the case of vertical differentiation, the impact of personalized pricing on inter-brand competition is less clear-cut. As we show in the Appendix, the profit of one firm (either one) or even both firms may actually increase. Yet, this provides little guidance on the impact of personalized pricing on the channel decision. If the manufacturer offers the low-end product, then, as shown by Proposition 1, dual distribution is always optimal, regardless of the pricing regime’s impact on inter-brand competition and profits. By contrast, if the manufacturer offers the high-end product then, as shown by Proposition 2, mono distribution can be optimal, regardless of how personalized pricing would affect inter-brand competition and profits. In particular, mono distribution can be optimal even if personalized pricing would increase both firms’ profits in the case of inter-brand competition—the reason is that it would increase even more the profit from mono distribution.

A second insight concerns firms’ market shares in the case of vertical differentiation with A offering the high-end product. Under uniform pricing, dual distribution is always optimal. However, B’s market share is small if, for example, \( \rho \) is close to 0.

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31 We show this result in Online Appendix G.

32 The case of inter-brand competition can be interpreted as imposing to charge a wholesale price \( w \) equal to marginal cost.
or $\sigma$ close to 1. By contrast, under personalized pricing, dual distribution is not optimal when $B$ would have a small market share. The reason is that $B$ would then add little value to the industry but still compete for high-valuation consumers. As a consequence, $A$ opts for dual distribution only when the independent retailer serves a sufficiently large share of the market. Table 1 illustrates this with a numerical example. We set $\sigma = 0.4$ and then report $B$’s market shares for different values of $\rho$ under uniform pricing by both firms and personalized pricing by both firms. Notice that $B$’s market share is higher with personalized pricing than with uniform pricing whenever $\rho$ is sufficiently high, so that dual distribution occurs under personalized pricing.  

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Uniform Pricing</th>
<th>Personalized Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.27</td>
<td>0</td>
</tr>
<tr>
<td>0.6</td>
<td>0.45</td>
<td>0</td>
</tr>
<tr>
<td>0.7</td>
<td>0.59</td>
<td>0</td>
</tr>
<tr>
<td>0.8</td>
<td>0.69</td>
<td>0.74</td>
</tr>
<tr>
<td>0.9</td>
<td>0.77</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Table 1: Market shares of firm $B$ in the different regimes

Finally, our results are in line with the distribution structures observed in different industries. Specifically, our model predicts that the mode of differentiation—i.e., horizontal versus vertical—and whether the manufacturers or the retailer offers the high-end product are key determinants of the optimal distribution system. In case of vertical differentiation, when the manufacturer offers the high-end product, mono distribution may occur. This matches well with the distribution systems in markets for luxury products, such as perfume, high-end apparel, or jewelry, in which brand manufacturers usually eschew independent retailers and prefer to distribute only through the direct channel. For example, several recent antitrust cases center around the theme that high-end brand suppliers—e.g., the perfume seller Coty or the sport shoe manufacturer Asics—fortbid sales through through third-party websites or other independent distributors. Our model provides an explanation for this result, and predicts that this problem gets more severe if price discrimination becomes more finely tuned. Instead, for more mainstream products, such as low-end clothes, electronic equipment, or certain kinds of food, manufacturers usually seek to distribute through independent retailers; hence, these products are available through multiple channels. For these products, consumers view manufacturers and retailers usually as horizontally differentiated or high-valuation consumers prefer the retailer for relatively small brands. Consistent with the observed channel structures, our model predicts that dual distribution is optimal in these cases.

\[^{33}\text{A similar comparison can be made when considering the hybrid regimes.}\]
5 Endogenous Pricing Regime

In this section, we extend our model by considering the situation in which both pricing policies (i.e., personalized pricing and uniform pricing) are available to both firms, and firms endogenously decide on the pricing regime in their negotiation.

When being able to negotiate the pricing regime, the contract at the wholesale stage consists of four elements: the per-unit price \( w \), the fixed fee \( F \), \( A \)'s pricing policy, and \( B \)'s pricing policy. The firms choose these variables to maximize the industry profit, conditional on retail prices being chosen individually by each firm later. Throughout the section, we are particularly interested in the question whether or not firms benefit from not using personalized pricing.

The next proposition characterizes the equilibrium pricing regimes that occur for different parameter constellations:

**Proposition 3:** If firms can contract on their pricing policies, then:

(i) dual distribution together with personalized pricing by both firms is optimal if and only if:

\[
\sigma \frac{2 + \sqrt{1 - \sigma}}{1 + \sigma} \leq \rho \leq \frac{1 + 4\sigma}{3 + 2\sigma};
\]

(ii) dual distribution together with uniform pricing by \( A \) and personalized pricing by \( B \) is optimal if and only if:

\[
\rho \geq \max \left\{ \frac{1 + 4\sigma}{3 + 2\sigma}, \sigma + \sqrt{\frac{\sigma (1 - \sigma^2)}{2 + \sigma}} \right\};
\]

(iii) for all other parameter combinations, mono distribution is optimal.

**Proof:** See Online Appendix C.

Figure 6 illustrates these insights. If distributing via \( B \) does not expand demand significantly (\( \sigma \) large and/or \( \rho \) small), then mono distribution maximizes the industry profit, as it avoids downstream competition and allows to better exploit price discrimination. When instead the independent retailer brings enough value, the firms opt for dual distribution. In this case, the firms may benefit from restricting \( A \) to a uniform price, but prefer \( B \) to charge personalized prices. Specifically, restricting \( A \)'s pricing policy is optimal when \( B \) is a relatively strong competitor for high-valuation consumers (i.e., \( \rho \) high enough); this is represented by the lower-right area in the figure. To see why this may occur, note that restricting \( A \)'s pricing policy leads it to focus on its core market, which dampens the competitive pressure on \( B \), allowing it to extract more

\[\text{As the figure shows, the three thresholds of the proposition coincide at } \sigma = \tilde{\sigma} \approx 0.248, \text{ where } \tilde{\sigma} \text{ is the unique solution to the equation } \sqrt{1 - \sigma} = \sigma (3 + 2\sigma) \text{ in the range } (0, (\sqrt{5} - 1)/2). \text{ Dual distribution together with personalized pricing by both firms can only be optimal if } \sigma \leq \tilde{\sigma}, \text{ as otherwise (5) cannot hold. Similarly, dual distribution together with uniform pricing by } A \text{ and personalized pricing by } B \text{ can only be optimal if } \sigma < (\sqrt{5} - 1)/2, \text{ as otherwise } \sigma + \sqrt{(\sigma (1 - \sigma^2))/(2 + \sigma)} \text{ would be larger than 1.}\]
surplus from medium-range consumers—i.e., those consumers at the margin between buying from A or from B. This however comes at a cost, as A’s ability to extract rents from the high-valuation consumers is impeded. Therefore, if B is only a weak competitor for high-valuation consumers but expands demand significantly (i.e., \( \rho \) and \( \sigma \) are relatively small), personalized pricing by both firms is optimal.

Figure 6: Endogenous pricing regime

6 Distribution by the retailer only

We assumed so far that A always uses its direct channel. In this section, we extend our model by allowing firms to opt for mono distribution by B. By the same logic as before, the firms will find this optimal when doing so maximizes the industry profit. Letting \( R^B_{XY} \) denote the range of values of \( \rho \) and \( \sigma \) for which mono distribution by B is optimal in the pricing regime \( XY \), the next proposition shows that the ranges can be clearly ordered:

**Proposition 4:** In each of the four pricing regimes, there exist values of \( \rho \) and \( \sigma \) for which mono distribution by B is optimal. The range of these parameters can be ordered as follows:

\[
R^B_{PU} \subset R^B_{UU} \subset R^B_{PP} \subset R^B_{UP}.
\]

**Proof:** See Online Appendix D.
In contrast to mono distribution by $A$, which can only be optimal if $A$ charges personalized prices, mono distribution by $B$ can be optimal in each pricing regime. The intuition is as follows: in all four pricing regimes, mono distribution by $B$ is optimal if $\sigma$ is relatively small, which implies that $B$ increases demand substantially. Therefore, via using only $B$’s channel, firms avoid competition, which allows $B$ to profit most from its large demand. By contrast, $A$’s relative advantage is not to broaden demand but to deliver a higher utility to high-valuation consumers. If $A$ can only set a uniform price, its rent extraction possibility is limited, and mono distribution by $A$ is never optimal.

In addition, the ranges in which mono distribution by $B$ is optimal in the different pricing regimes has a clear order. This range is larger if $B$ charges personalized prices rather than a uniform price, and, given $B$’s pricing policy, the range is larger if $A$ can only set a uniform price. This is obvious when comparing mono distribution by $A$ with mono distribution by $B$, as the profit achieved by a firm under mono distribution is maximal when it can extract the entire surplus through personalized pricing. The proposition shows that the insight carries over when considering dual distribution as well.

Finally, we consider the extent of mono distribution by either firm in the different pricing regimes. The resulting ranking is not clear-cut; for example, there exists a range of parameter values in which mono distribution by one of the two firms is optimal in the regime $PP$ but not in the regime $UP$ and another range in which the opposite holds. However, both firms switching from uniform to personalized pricing strictly expands the scope for mono distribution (by one or the other firm): letting $R_{XY}^{A/B}$ denote the parameter range in which mono distribution is optimal, we have:35

$$R_{UU}^{A/B} \subset R_{PP}^{A/B}.$$  

**7 Conclusion**

This paper analyzes the effects of personalized pricing on the incentives of a brand manufacturer to opt for dual distribution. Adding an independent distribution channel enables the manufacturer to reach out to different consumer groups but triggers intra-brand competition with its own distribution channel. We show that the profitability of dual distribution crucially depends on the demand pattern. If the two channels are horizontally differentiated, or if the independent retailer offers the high-end product, then dual distribution is optimal regardless of whether firms can set personalized prices or only a uniform one. By contrast, if the manufacturer offers the high-end

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35The details of the computation can be found in Online Appendix H.
product, this result only holds if the manufacturer charges a uniform price to consumers. If instead the manufacturer can offer personalized prices, then the two channels compete more intensely for each type of consumer; this dissipates profits to such an extent that the manufacturer opts for dual distribution only when the independent retailer expands demand significantly. These insights differ substantially from those obtained for inter-brand competition between independent firms, but are in line with the channel structures observed in different industries. This shows that accounting for the possibility of influencing intra-brand competition through the wholesale contract is important for assessing the effects of personalized pricing.

We also find that a hybrid regime—in which only the retailer can offer personalized prices—may lead to the highest industry profit. The manufacturer then extracts less surplus from high-valuation consumers, but benefits from reduced intra-brand competition. This implies that the manufacturer may optimally forgo charging personalized prices even if it has the possibility of doing so.

Our most important implication for marketing managers is that the extent to which price discrimination is feasible not only affects the pricing strategy but also the optimal distribution network. With prices becoming more and more finely tuned to consumer tastes, brand manufacturers risk fierce competition with pure retailers, even if these retailers may appeal to different consumer groups. In case a manufacturer caters foremost to the high-end segment, this channel conflict calls for a cautious use of new distribution channels when price discrimination is possible at a finely-grained level. This holds particularly for products sold online, for which consumer data and purchase history is available. It can then be more profitable to rely on mono distribution by the manufacturer or by the retailer, in order to avoid intra-brand competition. By contrast, dual distribution is beneficial if price discrimination is hard to achieve.

Another implication is that adopting a non-discriminatory pricing policy can be a profitable strategy for manufacturers. This is particularly true for companies facing the opportunity of distributing their products through a data-intensive retailer, which can perform price discrimination and broaden demand substantially. In that case, not using consumer data for its own distribution channel can achieve the right balance between rent extraction (by the retailer) and the avoidance of fierce intra-brand competition.

We conclude by briefly discussing two interesting avenues for future research emerging from our model. First, we considered a situation in which a firm can set personalized prices to all of its consumers, given that personalized pricing is possible. However, firms may have access to data only from consumers who have previously bought from the firm. A dynamic extension of our model in which firms set a uniform price in early periods but can charge personalized prices to its previous customers in later
periods can shed light on how price setting to learn about consumer preferences shapes channel design. Second, our model assumes that the brand manufacturer does not face competition from rival manufacturers. This is a reasonable assumption in markets in which brands are strongly differentiated and helps singling out the effects at work in a clear way. Analyzing whether new effects emerge with competition between manufacturers, and the resulting implications for wholesale contracts in such an extended scenario, constitutes a fruitful direction for future research.

Appendix

Inter-brand versus intra-brand competition

This appendix emphasizes that the classic insights from the literature on inter-brand competition provide little guidance for the choice of the distribution channels. In the case of horizontal differentiation à la Hotelling, personalized pricing dissipates profits under inter-brand competition, and maximizes instead the industry profit achieved under intra-brand competition (i.e., dual distribution). In the case of vertical differentiation, the following proposition shows that personalized pricing can actually increase industry profit:

Proposition 5: Suppose \( s_B > 0 \) (vertical differentiation). Under inter-brand competition:

- if \( \sigma \leq \sigma^* \equiv 2(3 - 2\sqrt{2}) \approx 0.343 \), personalized pricing increases industry profit;
- if instead \( \sigma \geq \sigma^* \equiv 5 - \sqrt{17} \approx 0.877 \), personalized pricing decreases industry profit;
- finally, if \( \sigma < \sigma^* < \sigma \), there exist \( \rho(\sigma) > \sigma \) and \( \bar{\rho}(\sigma) \in (\rho(\sigma), 1) \) such that personalized pricing increases industry profit if and only if \( \rho < \rho(\sigma) \) or \( \rho > \bar{\rho}(\sigma) \).

Proof. The case of inter-brand competition between independent firms is equivalent to that of intra-brand competition in which the wholesale price \( w \) equals zero. Using the same steps as in the proof of Proposition 2, the profits of the two firms are given by:

\[
\Pi^A_I(w = 0) = \frac{r_A^2}{s_A} \frac{(2 - \rho - \sigma)^2}{(4 - \sigma)^2(1 - \sigma)} \quad \text{and} \quad \Pi^B_I(w = 0) = \frac{r_A^2}{s_A} \frac{(2\rho - \sigma(1 + \rho))^2}{\sigma(4 - \sigma)^2(1 - \sigma)}
\]

for uniform pricing, and by:

\[
\Pi^A_P(w = 0) = \frac{r_A^2}{s_A} \frac{(1 - \rho)^2}{2(1 - \sigma)} \quad \text{and} \quad \Pi^B_P(w = 0) = \frac{r_A^2}{s_A} \frac{(\rho - \sigma)^2}{2\sigma(1 - \sigma)}
\]
for personalized pricing. It follows that $A$ benefits from personalized pricing if and only if:

$$\rho \leq \rho_A(\sigma) \equiv \frac{(\sqrt{2} - 1) (2\sqrt{2} - \sigma)}{4 - \sqrt{2} - \sigma},$$

whereas $B$ benefits from personalized pricing if and only if:

$$\rho \geq \rho_B(\sigma) \equiv \frac{\sigma (\sqrt{2} + 1) (4 - \sqrt{2} - \sigma)}{2\sqrt{2} + \sigma},$$

where $\rho_A(\sigma) > \rho_B(\sigma)$ when $\sigma < \sigma_r$ in which case both firms benefit from personalized pricing. Turning to the joint profits, the difference between the industry profit with personalized and uniform pricing is given by:

$$\frac{r_A^2}{2s_A} \left( 8 + 14\sigma - 9\sigma^2 + \sigma^3 \right) (\rho^2 + \sigma) - 4\rho \sigma (12 - 6\sigma + \sigma^2) \frac{(1 - \sigma)(4 - \sigma)^2}{(1 - \sigma)(4 - \sigma)^2}.$$ (6)

The numerator is a convex quadratic polynomial of $\rho$ with the two roots:

$$\rho(\sigma) \equiv \frac{2\sigma (12 - 6\sigma + \sigma^2) -(1 - \sigma)(4 - \sigma)\sqrt{\sigma(12\sigma - 4 - \sigma^2)}}{8 + 14\sigma - 9\sigma^2 + \sigma^3},$$

and

$$\overline{\rho}(\sigma) \equiv \frac{2\sigma (12 - 6\sigma + \sigma^2) + (1 - \sigma)(4 - \sigma)\sqrt{\sigma(12\sigma - 4 - \sigma^2)}}{8 + 14\sigma - 9\sigma^2 + \sigma^3},$$

whereas the denominator is positive. It is straightforward to check that for $\sigma < \sigma_r$, the numerator of (6) is always positive, implying that personalized prices increases industry profit. If instead $\sigma \geq \sigma_r$, personalized pricing increases industry profit if $\rho \leq \rho(\sigma)$ or $\rho \geq \overline{\rho}(\sigma)$. The threshold $\overline{\rho}(\sigma)$ is increasing in $\sigma$ and reaches 1 at $\sigma = \sigma_r$. Similarly, $\rho(\sigma)$ is decreasing in $\sigma$ and reaches the lower bound for $\rho$, which is $\sigma$, also at $\sigma = \sigma_r$. It follows that for $\sigma > \sigma_r$, personalized pricing decreases industry profit in the entire admissible range for $\rho$. ■

This proposition shows that personalized has a different impact on inter-brand competition in situations of vertical and horizontal differentiation. In the context of a Hotelling model, Thisse and Vives (1988) and Shaffer and Zhang (1995) show that personalized pricing unambiguously lowers industry profit. The proposition shows instead that, with vertical differentiation, this is not necessarily true: there is indeed a sizable range in which the opposite obtains.

The impact of personalized pricing on inter-brand competition moreover provides little guidance for the question whether or not dual distribution is optimal in case of intra-brand competition. First, as shown by Proposition 1, if the manufacturer offers the low-end product, then dual distribution is always optimal in case of intra-brand...
competition, regardless of the pricing regime. Therefore, the optimal distribution regime in this case does not depend on whether personalized pricing increases or decreases profits under inter-brand competition. Second, as shown by Proposition 2, if the manufacturer offers the high-end product, then mono distribution is optimal under intra-brand competition if $\rho$ is close to $\sigma$—i.e., if $\rho < \tilde{\rho}(\sigma) \equiv \sigma (2 + \sqrt{1 - \sigma}) / (1 + \sigma)$. By contrast, as shown by Proposition 5, with inter-brand competition and $\sigma \leq \sigma \approx 0.877$, personalized pricing increases the industry profit whenever $\rho$ is close to $\sigma$. Mono distribution can actually be optimal even if personalized pricing increases the profits of both firms in the case of inter-brand competition. Indeed, it is straightforward to check that $\tilde{\rho}(\sigma) > \rho_B(\sigma)$ for $\sigma \in (0, 1)$ and $\tilde{\rho}(\sigma) < \rho_A(\sigma)$ for $\sigma \leq 0.230$; hence, this situation occurs for $\sigma \in (0, 0.230)$ if $\rho_B(\sigma) \leq \rho \leq \tilde{\rho}(\sigma)$.

References


Online Appendix A: Proof of Proposition 1

A.1: Horizontal differentiation

A.1.1: Uniform pricing by $A$

Let $p_m^A$ define firm $A$’s optimal (uniform) price under mono distribution, and $\Pi_m^A$ the associated profit. To show that dual distribution is optimal, it suffices to show that an appropriate two-part tariff leads to a continuation equilibrium in which both firms earn greater profit (and one firm does strictly so) than under mono distribution. To see this, suppose that the firms agreed instead on the wholesale tariff $F = 0$ and $w = \hat{u}$.

Assume first that $B$ also charges a uniform price. In the continuation equilibrium, $B$ then charges a price $p_B > w = \hat{u}$, such that it obtains a positive share the market: as consumers close to $X$ have a higher valuation for $B$’s product than for $A$’s product, there exists an $\hat{x}_B \in (\hat{x}, X)$ such that consumers $x > \hat{x}_B$ buy from $B$ in the continuation equilibrium. Suppose now that $A$ charges the mono-distribution price $p_m^A$. If $p_m^A \geq \hat{u}$, then $B$ still attracts all consumers $x > \hat{x}_B$; hence, firm $A$’s profit becomes $\Pi_m^A + \hat{u} (X - \hat{x}_B) > \Pi_m^A$. If instead $p_m^A < \hat{u}$, then $B$ still attracts those consumers that are close enough to $X$ hence, compared to mono distribution, total sales can only increase, and $A$ earns a greater margin ($\hat{u} > p_m^A$) on the consumers served by $B$. We thus have that, by charging $p_m^A$, $A$ obtains a greater profit than $\Pi_m^A$. It follows that, in the continuation equilibrium (where it best responds to $p_B$), $A$ a fortiori obtains a profit higher than $\Pi_m^A$. In other words, the tariff $F = 0$ and $w = \hat{u}$ yields a continuation equilibrium that strictly increases each firm’s profit.

The same reasoning applies to the case in which $B$ charges personalized prices. In the continuation equilibrium, $B$ attracts some consumers, and by charging $p_m^A$ in the retail stage, $A$ obtains a higher profit than $\Pi_m^A$, either by expanding the market served or by earning a higher margin on existing customers.

A.1.2: Personalized pricing by $A$

Under mono distribution, firm $A$ charges each consumer a price $u_A(x)$. Suppose now that the firms agree on the two-part tariff $F = 0$ and $w = \hat{u}$.

If both firms charge personalized prices, $A$ still charges $u_A(x)$ to consumers $x < \hat{x}$.

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36 In the case of mono distribution, either $A$ covers part of the market, or it covers all the market but leaves no net utility to the marginal consumer $x = X$. In both cases, $B$ attracts consumers close enough to $X$ with the price $p_B$. 

33
(as $B$ cannot offer them any positive value at any price $p_B(x) \geq w = \hat{u}$), and now obtains $w > u_A(x)$ on consumers $x > \hat{x}$ (which are served by $B$ at price $p_B(x) = u_B(x)$).

Consider now the case in which $B$ charges a uniform price. In the continuation equilibrium, $A$ still charges $p_A(x) = u_A(x)$ to consumers $x < \hat{x}$, for the same reason as above. For consumers $\hat{x} < x \leq X$, $B$ charges the price $p_B$ that maximizes its monopoly profit, given the wholesale price $w = \hat{u}$, and attracts some consumers $x > \hat{x}_B$, where $\hat{x}_B \in (\hat{x}, X)$. To see this, note first that $A$ charges $p_A(x) = u_A(x)$ to all consumers $x$, with $u_B(x) < p_B$, because these consumers obtain a negative utility when buying from $B$, which implies that $A$ can extract the entire consumer surplus. Instead, for consumers who obtain a positive utility when buying from $B$, $A$ can earn a higher margin—i.e., $\hat{u} = u_A(\hat{x}) > u_A(x)$—when $B$ serves these consumers than when serving these consumers itself. Therefore, in the continuation equilibrium, $A$ charges these consumers any price larger than or equal to $u_A(x)$, and $B$ serves these consumers. It follows that, again, $A$ earns a higher margin on the consumers served by $B$, and obtains a higher profit compared to mono distribution.

A.2: Vertical differentiation

We now turn to the case of vertical differentiation, with $B$ being more attractive for high-valuation consumers (i.e., $s_B > s_A$ and $r_B > r_A$).

A.2.1: Uniform pricing by $A$

Suppose that the firms agreed on the wholesale tariff $F = 0$ and $w = p^m_A$.

Assume first that $B$ also charges a uniform price. In the continuation equilibrium, $B$ then charges a price $p_B > w = p^m_A$ on all consumers served and obtains a positive share the market, whereas $A$ charges some price $p^+_A$. Suppose now that $A$ deviates and charges the mono-distribution price $p^m_A$. Following this deviation, total demand (weakly) exceeds that of the mono-distribution outcome (as consumers have more choice) and $A$ obtains a margin $p^m_A$ (either directly through $p_A$ or indirectly through $w$).

Recall that, as usual, we focus on the Pareto efficient equilibrium, in which $A$ does not sell below its opportunity cost (given here by $w = \hat{u}$).

To see this, suppose instead that $B$ does not attract any consumer. In that case, $B$ prices at cost (i.e., $p_B = w = p^m_A$) and $A$ prices so as to attract consumers with type $x = 0$ (i.e., $p_A \leq p^m_A - (r_B - r_A) = p^m_A$), and obtains a profit lower than $\Pi^A$ (as it charges $p_A \neq p^m_A$, and $B$ attracts no additional consumer). But then, $A$ would profitably deviate by charging $p'_A = p^m_A$: compared with the mono distribution outcome, this would (weakly) expand demand (as consumers can now buy from both firms) and $A$ would obtain the same margin on each customer (either directly through $p'_A$ or indirectly through $w$); hence, the deviation brings a profit of at least $\Pi^m_A$.

Compared with mono distribution, $B$ now charges a “lower” price (the mono distribution outcome can be interpreted as $B$ charging $p_B \geq r_B$), and in response $A$ also lowers its own price.
It follows that $B$’s profit is strictly positive after the deviation, and $A$’s profit is weakly larger. Therefore, in the continuation equilibrium in which $A$ best responds to $p_B$, the same result must hold. As a consequence, the tariff $F = 0$ and $w = p_A^m$ yields a continuation equilibrium that strictly increases $B$’s profit and weakly increases $A$’s profit.

The same reasoning applies to the case in which $B$ charges personalized prices. In the continuation equilibrium, after firms agreed on a tariff $F = 0$ and $w = p_A^m$, $B$’s demand is positive, and $A$ charges some price $p_A^{++}$. A deviation to $p_A = p_A^m$ then gives $A$ a profit that is weakly larger than $\Pi_A^m$ due to the fact that it obtains the same price $p_A^m$ on all consumers served and demand weakly exceeds that with mono distribution.

### A.2.2: Personalized pricing by $A$

Under mono distribution, $A$ charges each consumer a price $u_A(x)$. Suppose now that the firms agree on the two-part tariff $F = 0$ and $w = r_A$. In the continuation equilibrium, $B$ obtains a positive demand from consumers close enough to $x = 0$: these consumers have a valuation for $B$’s product equal to $u_B(x)$, which is larger than $B$’s wholesale price $w = r_A$, and $A$ earns a higher margin by letting $B$ serve these consumers, as $w = r_A > u_A(x)$. Therefore, $A$ charges prices larger than or equal to $w$ to the consumers served by $B$, and obtains a higher margin compared to mono distribution. This holds regardless of whether $B$ charges personalized prices or a uniform price. In addition, $B$ does not offer a positive net utility to consumers it does not serve: if it did, $B$ would serve these consumers because $A$ is better off by letting $B$ serve these consumers and get a margin $w > u_A(x)$ instead of serving the consumers itself. It follows that $A$ can still charge a price of $p_A(x) = u_A(x)$ to these consumers. Hence, $A$ obtains the same profit as under mono distribution from consumers that it serves but a strictly larger profit from consumers served by $B$. Therefore, the two-part tariff $F = 0$ and $w = r_A$ increases the profits of both firms.

### Online Appendix B: Proof of Proposition 2

#### B.1: Uniform pricing by $A$

##### B.1.1: Uniform pricing by $B$

We start with part (i) and first analyze the situation of uniform pricing by both firms. To solve for the subgame-perfect equilibrium, we proceed by backward induction and first determine the reaction functions in the downstream stage. To simplify the exposition, we proceed under the assumption that both demands are positive in equilibrium and verify later that this is in fact true. The linearity of the demand functions ensures
that firms’ profit functions are strictly concave in their prices; hence, firms’ reaction functions are characterized by the first-order conditions, which yield:

\[
\begin{align*}
p_A(p_B; w) &= \frac{r_A - r_B + p_B + w}{2}, \\
p_B(p_A; w) &= \frac{r_B + w}{2} + s_B \frac{(p_A - r_A)}{2s_A}.
\end{align*}
\]

Combining these reaction functions yields the equilibrium retail prices, as a function of the wholesale price \(w\):

\[
\begin{align*}
p_A(w) &= \frac{r_A(2s_A - s_B) + s_A(3w - r_B)}{4s_A - s_B}, \\
p_B(w) &= \frac{r_B(2s_A - s_B) + w(2s_A + s_B) - r_A s_B}{4s_A - s_B}.
\end{align*}
\]

The associated demands are \(D_A(w) = x_{AB}(p_A(w), p_B(w))\) and \(D_B(w) = x_B(p_B(w)) - x_{AB}(p_A(w), p_B(w))\).

We now turn to the first stage. In the negotiation stage, firms choose \(w\) so as to maximize the industry profit, \(\Pi(w) = p_A(w)D_A(w) + p_B(w)D_B(w)\). This profit is again a strictly concave function of \(w\), as its second-order derivative is given by:

\[
\Pi''(w) = -\frac{2s_A(4s_A + 5s_B)}{s_B(4s_A - s_B)^2} < 0.
\]

Hence, the equilibrium wholesale price is characterized by the first-order condition, leading to:

\[
w^*_U = \frac{s_B(4s_A(r_A + r_B) + r_A s_B)}{2s_A(4s_A + 5s_B)}.
\]

Inserting the equilibrium prices into the demand functions \(D_A\) and \(D_B\) yields:

\[
\begin{align*}
D_A^* &= \frac{2s_A^2(2r_A - r_B) + s_B(3s_A r_B - 4s_A r_B - s_B r_A)}{2s_A(4s_A + 5s_B)(s_A - s_B)}, \\
D_B^* &= \frac{(2s_A + s_B)(r_B s_A - r_A s_B)}{s_B(4s_A + 5s_B)(s_A - s_B)}.
\end{align*}
\]

The assumption that the two demand functions intersect at a positive valuation (i.e., \(r_A/s_A < r_B/s_B\)) ensures that both equilibrium demands are positive. Indeed, \(D_A^*\) is strictly falling in \(r_B\) and is equal to \(r_A(2s_A + s_B)/(2s_A(4s_A + 5s_B)) > 0\) at the highest possible value of \(r_B\), which is \(r_B = r_A\). Direct inspection of \(D_B\) shows that it is positive for \(r_A/s_A < r_B/s_B\). As \(w^*\) constitutes a global maximum in the relevant range, and achieving \(D_B = 0\) is feasible with a high enough \(w\), it follows that in equilibrium it is optimal for the firms to generate positive sales for \(B\). Indeed, the resulting profit, equal
to:
\[
\Pi_U^* = \frac{r_A^2 s_B \left(5 s_A s_B + 4 s_A^2 - s_B^2\right) + 4 s_A r_B \left(s_A s_B + s_B\right) \left(s_A r_B - 2 s_B r_A\right)}{4 s_A s_B \left(s_A - s_B\right) \left(4 s_A + 5 s_B\right)},
\]

exceeds the monopoly profit that A can obtain with mono distribution, \(\Pi_U^m\):
\[
\Pi_U^* - \Pi_U^m = \frac{(s_A + s_B)\left(s_A r_B - r_A s_B\right)^2}{s_A s_B \left(4 s_A + 5 s_B\right) \left(s_A - s_B\right)} > 0.
\]

B.1.2: Personalized pricing by B

We next analyze the case in which B can charge personalized prices (and A still a uniform one). Given \(w\) and \(p_A\), B’s price response is such that consumers \(x\) with \(u_A(x) - p_A > u_B(x) - w\), or:
\[
x < \bar{x}(w, p_A) = \frac{r_A - p_A - r_B + w}{s_A - s_B},
\]
end-up buying from A. Instead, consumers \(\bar{x}(w, p_A) < x < \bar{x}_B(w)\) end-up buying from B at price \(p_B(x) = u_B(x) - \max\{u_A(x) - p_A, 0\}\). A’s variable profit (gross of the fee \(F\)) is therefore given by:
\[
p_A \bar{x}(w, p_A) + w [\bar{x}_B(w) - \bar{x}(w, p_A)].
\]
Optimizing this with respect to \(p_A\) yields:
\[
p_A(w) = w + \frac{r_A - r_B}{2}.
\]

We now turn to the wholesale stage. The two firms seek to maximize the industry profit given by:
\[
\Pi = p_A(w) \bar{x}(w) + \int_{\bar{x}(w)}^{\bar{x}(w)} [p_A(w) + u_B(x) - u_A(x)] dx + \int_{\bar{x}(w)}^{\bar{x}_B(w)} u_B(x) dx,
\]
where
\[
p_A(w) = w + (r_A - r_B)/2, \quad \bar{x}(w) = (r_A - p_A - r_B + w) / (s_A - s_B), \quad \bar{x}(w) = (r_A + r_B) / (2 s_A) - w / s_A, \quad \text{and} \quad \bar{x}_B(w) = (r_B - w) / s_B.
\]
Maximizing this profit with respect to \(w\) yields (the subscript \(UP\) stands for the pricing regime in which A sets a Uniform price and B Personalized prices):
\[
w_{UP}^* = \frac{s_B (r_A + r_B)}{2 (s_A + s_B)}.
\]

\(^{40}\)It is straightforward to check that the industry profit is a concave function of \(w\).
Inserting \( w = w_{UP}^* \) into (9), we obtain that the industry profit is given by:

\[
\Pi_{UP} = \frac{r_A^2 s_{AB} + 2r_A^2 s_{AB} - 4r_A r_B s_{ASB} - 2r_A r_B s_{BSB} + 2r_B^2 s_{A} + r_B^2 s_{ASB}}{4s_A^2 s_B - 4s_B^3} = \frac{r_A^2 \sigma + 2\sigma^2 - 4\rho \sigma - 2\rho \sigma^2 + 2\rho^2 + \rho^2 \sigma}{s_A 4\sigma (1 - \sigma^2)}.
\]

We can now show that this profit exceeds the profit obtained under uniform pricing, which, from (8), can be written as:

\[
\Pi_U^* = \frac{r_A^2 4\sigma + 5\sigma^2 - \sigma^3 - 8\rho \sigma - 8\rho \sigma^2 + 4\rho^2 + 4\rho^2 \sigma}{4\sigma (1 - \sigma) (4 + 5\sigma)}.
\]

We have:

\[
\Pi_{UP}^* - \Pi_U^* = \frac{(\sigma - \rho)^2 r_A^2}{4\sigma s_A} \frac{4 + 6\sigma + \sigma^2}{(4 + 5\sigma) (1 - \sigma^2)} > 0.
\]

This establishes that the profit in the regime with uniform pricing by \( A \) and personalized pricing by \( B \) is larger than under uniform pricing by both firms.

We know from above that dual distribution is optimal in case both firms set uniform prices. Because the industry profit in the regime in which only \( B \) can charge personalized prices (i.e., \( \Pi_{UP}^* \)) is larger than \( \Pi_U^* \), it must also be larger than the profit with mono distribution.

**B.2: Personalized pricing by \( A \)**

**B.2.1: Personalized pricing by \( B \)**

We now turn to part (ii), and start with the situation of personalized pricing by both firms. As noted in the main text, if the wholesale price \( w \) is such that \( w \geq \hat{u} \), \( B \) will be inactive;\(^41\) hence, the industry profit cannot be larger than \( \Pi_B^* \). Using the notation \( \rho \equiv r_B/r_A \in (0, 1) \) and \( \sigma \equiv s_B/s_A \in (0, \rho) \), the threshold \( \hat{u} \) is:

\[
\hat{u} \equiv \frac{s_A r_B - s_B r_A}{s_A - s_B} = r_A \frac{\rho - \sigma}{1 - \sigma}.
\]

We now focus on \( w \leq \hat{u} \). We need to distinguish whether or not \( B \) finds it profitable to supply (some) consumers uninterested in \( A \)'s product. From Figure 3, such consumers exist if and only if \( \bar{x}_B(w) > \bar{x}_A \). The latter inequality can only hold if \( w \) is sufficiently low, that is:

\[
w < w \equiv r_B - \frac{s_B}{s_A} r_A = r_A (\rho - \sigma).
\]

\(^41\)Recall that \( \hat{u} = u_A(\hat{x}) = u_B(\hat{x}) \).
Note that \( w = (1 - \sigma) \hat{u} < \hat{u} \).

**Region** \( w \leq \hat{w} \)

In this region, in which \( \tilde{x}_B(w) \geq \tilde{x}_A \), as shown in the text, the industry profit is given by:

\[
\Pi(w) = \int_0^{\tilde{x}_A(w)} [w + |u_B(x) - u_A(x)|] \, dx + \int_{\tilde{x}_A(w)}^{\tilde{x}_B(w)} u_B(x) \, dx.
\]

It is strictly concave in \( w \): using \( u_A(\tilde{x}_A(w)) = u_B(\tilde{x}_B(w)) = w \), we have:

\[
\Pi'(w) = \tilde{x}_A(w) + w \frac{d\tilde{x}_B}{dw}(w) = \frac{r_A - w}{s_A} - \frac{w}{s_B} = \frac{r_A}{s_A} \left( 1 - \frac{1 + \sigma w}{\sigma r_A} \right),
\]

and thus (as \( \tilde{x}_A(w) \) and \( \tilde{x}_B(w) \) are both linear and strictly decreasing in \( w \)):

\[
\Pi''(w) = \frac{d\tilde{x}_A}{dw}(w) + \frac{d\tilde{x}_B}{dw}(w) < 0.
\]

**Region** \( w < \hat{w} \leq \hat{u} \)

If instead \( w > \hat{w} \), the industry profit includes an additional term, as illustrated by Figure A.1. This term corresponds to consumers in the region \( \tilde{x}_B(w) < x \leq \tilde{x}_A \): \( B \) does not find it profitable to supply these consumers (as \( u_B(x) < w \)), but they are still willing to buy from \( A \), which can extract their full surplus. The industry profit can then be written as:

\[
\Pi(w) = \int_0^{\tilde{x}_A(w)} [w + |u_B(x) - u_A(x)|] \, dx + \int_{\tilde{x}_A(w)}^{\tilde{x}_B(w)} u_B(x) \, dx + \int_{\tilde{x}_B(w)}^{\tilde{x}_A} u_A(x) \, dx.
\]

The first-order derivative is equal to:

\[
\Pi'(w) = \tilde{x}_A(w) + [w - u_A(\tilde{x}_B(w))] \frac{d\tilde{x}_B}{dw}(w)
\]

\[
= (r_A - w) \left( \frac{1}{s_A} + \frac{1}{s_B} \right) - (r_B - w) \frac{s_A}{s_B^2}
\]

\[
= r_A \sigma^2 + \sigma - \rho + (1 - \sigma - \sigma^2) \frac{w}{r_A}.
\]
Hence:

\[
\Pi_-' (\hat{u}) = \frac{r_A}{s_A} \frac{1 - \rho}{1 - \sigma}.
\]

\[
\Pi_+' (w) = \frac{r_A}{s_A} \frac{1 + \sigma}{\sigma} \left( \frac{2 + \sigma}{1 + \sigma} - \rho \right),
\]

\[
\Pi_'' (w) = \frac{1 - \sigma - \sigma^2}{s_A \sigma^2}.
\]

It follows that \( \Pi (w) \) is strictly concave in \( w \) if:

\[
\sigma > \hat{\sigma} = \frac{\sqrt{5} - 1}{2} \simeq 0.618,
\]

and is instead weakly convex if \( \sigma \leq \hat{\sigma} \); in addition, \( \Pi' (\hat{u}) > 0 \) whereas \( \Pi' (w) \geq 0 \) if and only if:

\[
\rho \leq \hat{\rho} (\sigma) \equiv \sigma \frac{2 + \sigma}{1 + \sigma},
\]

where \( \hat{\rho} (\sigma) \) increases with \( \sigma \) and exceeds 1 for \( \sigma \geq \hat{\sigma} \). Furthermore, not only is the profit function \( \Pi (w) \) continuous at \( w = \underline{w} \), its derivative \( \Pi' (w) \) is also continuous:

\[
\Pi_-' (w) = \frac{r_A}{s_A} \left( 1 - \frac{1 + \sigma}{\sigma} \frac{w}{s_A} \right) \bigg|_{w=r_A(\rho-\sigma)} = \frac{r_A}{s_A} \frac{1 + \sigma}{\sigma} \left( \frac{2 + \sigma}{1 + \sigma} - \rho \right) = \Pi_+' (w).
\]
**Optimal distribution policy**

As long as \( w \geq \hat{u} \), \( B \) cannot attract any consumer at any profitable price: hence, doing so cannot be more profitable than mono distribution. Furthermore, if \( \rho \leq \hat{\rho}(\sigma) \), then \( \Pi'(w) \geq 0 \), implying that dual distribution cannot be more profitable than mono distribution:

- in the range \( w \leq w \leq \hat{u} \), the profit function \( \Pi(w) \) is increasing, as it is quadratic and its derivative is non-negative at both ends of the range (namely, \( \Pi'(w) \geq 0 \) and \( \Pi'(\hat{u}) > 0 \));

- in the range \( w \leq w \), the profit function \( \Pi(w) \) is again increasing, as it is concave and its derivative is non-negative at the upper end of the range (namely, \( \Pi'(w) \geq 0 \));

- it follows that the profit achieved under dual distribution cannot exceed \( \Pi(\hat{u}) \), which is less profitable than mono distribution.

As already noted, \( \hat{\rho}(\sigma) \) is increasing in \( \sigma \) in the range \( \sigma \in [0, 1] \), and satisfies \( \hat{\rho}(\sigma) \geq 1 \) for \( \sigma \geq \hat{\sigma} \). It follows that, if \( \sigma \geq \hat{\sigma} \), then dual distribution cannot be more profitable than mono distribution, as we then have \( \hat{\rho}(\sigma) \geq 1 (> \rho) \).

If instead \( \sigma < \hat{\sigma} \) and \( \rho > \hat{\rho}(\sigma) \), then \( \Pi'(w) < 0 \). From the analysis for the region \( w \leq w \) above, the first-order condition \( \Pi'(w) = 0 \) then determines the candidate optimal wholesale price, which is given by:

\[
w = w^*_p = \frac{s_B r_A}{s_A + s_B} \equiv \frac{\sigma r_A}{1 + \sigma} \in (0, w).
\]

The corresponding profit is:

\[
\Pi^*_p = \frac{r^2_A}{2s_A} \frac{\sigma (1 + 3\sigma) - 4\sigma (1 + \sigma) \rho + (1 + \sigma)^2 \rho^2}{\sigma (1 - \sigma^2)}.
\]

Compared with the profit from mono distribution, \( \Pi^*_p \), dual distribution introduces a change in profit equal to:

\[
\frac{r^2_A}{2s_A} \frac{\sigma^2 (\sigma + 3) - 4\sigma (1 + \sigma) \rho + (1 + \sigma)^2 \rho^2}{\sigma (1 - \sigma^2)}.
\]

The numerator of this expression is a convex quadratic polynomial of \( \rho \) and its roots are:

\[
\sigma \frac{2 - \sqrt{1 - \sigma}}{1 + \sigma} \text{ and } \sigma \frac{2 + \sqrt{1 - \sigma}}{1 + \sigma}.
\]
Furthermore, \( \hat{\rho}(\sigma) \) lies between these two roots in the relevant range \( \sigma < \hat{\sigma} \):

\[
\frac{\sigma^{2-\sqrt{1-\sigma}}}{1+\sigma} \quad \text{and} \quad \frac{\sigma^{2+\sqrt{1-\sigma}}}{1+\sigma}
\]

where the last inequality stems from \( \sqrt{1-\sigma} > \sigma \) in the relevant range \( \sigma < \hat{\sigma} \). It follows that dual distribution is more profitable than mono distribution if and only if \( \sigma < \hat{\sigma} \) and \( \rho \) exceeds the larger root, that is, if:

\[
\rho > \hat{\rho}(\sigma) \equiv \sigma \frac{2+\sqrt{1-\sigma}}{1+\sigma}
\]

Note that \( \hat{\rho}(\sigma) \) is increasing in \( \sigma \) in the range \( \sigma \leq \hat{\sigma} \), and exceeds 1 in the range \( \sigma \geq \hat{\sigma} \). Hence, as \( \rho < 1 \), the condition \( \rho > \hat{\rho}(\sigma) \) implies \( \sigma < \hat{\sigma} \). Finally, \( \hat{\rho}(\sigma) \) is equivalent to the right-hand side of (2) with \( \delta(\sigma) = 1 \).

**B.2.2: Uniform pricing by \( B \)**

We now move to the hybrid regime in which \( A \) charges personalized prices and \( B \) a uniform one. Again, we solve the game by backward induction. Consider first \( A \)'s price response to a given \( w \) and \( p_B \). Two market share configurations can occur, dependent on the value of \( p_B \). When \( p_B \) is relatively small, the marginal consumer indifferent between buying from \( B \) and not buying, \( x_B = (r_B - p_B)/s_B \), exceeds \( \bar{x}_A \). \( A \)'s best response is to serve consumers \( x \) with \( u_A(x) - w > u_B(x) - p_B \), or:

\[
x < x_{AB}(w) \equiv \frac{r_A - w - r_B + p_B}{s_A - s_B},
\]

whereas consumers \( x \) with \( x_{AB}(w) < x < x_B \) end-up buying from \( B \).

Instead, when \( p_B \) is high enough so that \( x_B < \bar{x}_A \), a third demand region exists between \( x_B \) and \( \bar{x}_A \), in which consumers end-up buying from \( A \). The thresholds for the first two demand regions are the same as in the market share configuration above.

We start with the first case. If \( B \) serves consumers \( x \) with \( x_{AB}(w) < x < x_B \), its profit is:

\[
(p_B - w) \left( \frac{r_B - p_B}{s_B} - \frac{r_A - w - r_B + p_B}{s_A - s_B} \right).
\]

Maximizing with respect to \( p_B \) yields:

\[
p_B(w) = \frac{s_A(r_B + w) - s_B(r_A - w)}{2s_A}.
\]

(10)
We now turn to the negotiation at the wholesale stage. Because $A$ charges to each consumer $x$ a personalized price of $r_A - s_A x - r_B + s_B x + p_B(w)$, the industry profit is:

$$
\int_0^{r_A-w-r_B+p_B(w)} [r_A - s_A x - r_B + s_B x + p_B(w)] \, dx + p_B(w) \left( \frac{r_B - p_B(w)}{s_B} - \frac{r_A - w - r_B + p_B(w)}{s_A - s_B} \right),
$$

with $p_B(w)$ given by (10). Maximizing (11) with respect to $w$ yields (the subscript $PU$ stands for the pricing regime in which $A$ sets personalized prices and $B$ a uniform price)

$$
w_{pu}^* = \frac{s_B [r_A(2s_A + s_B) + r_B s_A]}{2s_A^2 + 5s_A s_B + s_B^2}.
$$

This market configuration is only valid if $x_B \geq \bar{x}_A$. Comparing the two thresholds at the equilibrium values, we obtain that the inequality is fulfilled if and only if:

$$
r_B \geq \frac{r_A s_B (2s_A^2 + 4s_A s_B + s_B^2)}{s_A^2 (s_A + 2s_B)}
$$
or, equivalently,

$$
\rho \geq \frac{\sigma (2 + 4 \sigma + \sigma^2)}{1 + 2 \sigma}.
$$

As $\rho$ is bounded above by 1, this inequality can only be satisfied if $\sigma^2 (4 + \sigma) - 1 \leq 0$ (which is approximately equivalent to $\sigma \leq 0.473$). Inserting $w = w_{pu}^*$ in (11), the resulting industry profit is:

$$
\Pi_{pu}^* = \frac{s_A^2 r_B^2 + s_A s_B (2r_A^2 + 3r_B^2 - 4r_A r_B) + s_A s_B^2 (5r_A^2 + 2r_B^2 - 8r_A r_B) - s_B^2 r_A^2}{2s_B (s_A - s_B) (2s_A^2 + 5s_A s_B + s_B^2)}
$$

$$
= \frac{r_A^2 \rho^2 + \sigma (2 - 4 \rho + 3 \rho^2) + \sigma^2 (5 - 8 \rho + 2 \rho^2) - \sigma^3}{s_A \frac{2 \sigma (1 - \sigma) (2 + 5 \sigma + \sigma^2)}{1 + 2 \sigma}}.
$$

Compared with the profit from mono distribution, $\Pi_{pu}^*$, the profit from dual distribution is larger if and only if: \(^{43}\)

$$
\rho \geq \hat{\rho} (\sigma) \equiv \frac{2 + \sqrt{(1 - \sigma)(2 + 5 \sigma + \sigma^2)}}{1 + 2 \sigma}.
$$

Note that $\hat{\rho} (\sigma)$ is increasing in $\sigma$ in the range $\sigma \in (0, 0.473)$ and exceeds 1 for $\sigma > 0.473$. As $\rho < 1$, the condition $\rho \geq \hat{\rho} (\sigma)$ implies $\sigma < 0.473$. Moreover, $\hat{\rho} (\sigma)$ is indeed larger than the right-hand side of (12) for $\sigma < 0.473$. Hence, if the first market configuration is valid, both firms are active if and only if (13) holds.

We now turn to the second market configuration. In this case, the industry profit

\(^{42}\)The maximization problem is strictly concave.

\(^{43}\)Because $\Pi_{pu}^*$ is a convex quadratic polynomial in $\rho$, the equation $\Pi_{pu}^* - \Pi_{pu}^2 = 0$ has two roots. The lower one is below zero, and the larger one is $\hat{\rho} (\sigma)$. 

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is:

\[ \int_0^{r_A-w-r_B+p_B(w)} \frac{r_A - s_A x - r_B + s_B x + p_B(w)}{s_A - s_B} dx \]

\[ + p_B(w) \left( \frac{r_B - p_B}{s_B} - \frac{r_A - w - r_B + p_B(w)}{s_A - s_B} \right) + \int_{r_B-p_B}^{r_A} \frac{r_A - s_A x}{s_B} dx, \]

with \( p_B(w) \) again given by (10). Maximizing with respect to \( w \), we obtain that the second-order condition for an interior solution is fulfilled if and only if \( \sigma^2(4+\sigma) - 1 > 0 \), resulting in a wholesale price of:

\[ r_A s_B^3 + s_A s_B^2 \left( 3r_A + r_B + s_A^2 s_B (r_A - r_B) - r_B s_A^3 \right) \]

\[ s_B^3 + 4s_B^2 s_A - s_A^3. \]

However, at this wholesale price, \( B \)'s demand is negative for \( \sigma^2(4+\sigma) - 1 > 0 \). As a consequence, in case the maximization problem is concave, mono distribution is optimal. Instead, if \( \sigma^2(4+\sigma) - 1 \leq 0 \), the maximization problem (14) is convex. It follows that \( w \) is optimally either set so high that \( B \) is not active, which results in mono distribution, or that \( w \) is set so low that \( x_B \geq \bar{x}_A \).

In the latter case, the first market configuration is valid if (12) holds. Instead, if (12) is not fulfilled, the optimal \( w \) is set such that \( x_B \) exactly equals \( \bar{x}_A \), or \( (r_B-p_B(w))/s_B = r_A/s_A \), with \( p_B(w) \) given by (10). Solving the last equation for \( w \) yields \( w = (s_A r_B - s_B r_A)/(s_A + s_B) \). The resulting industry profit is:

\[ \frac{r_A^2}{s_A} \frac{1 + \rho^2 - 2\sigma \rho (1 - \rho) - \sigma^2 (1 + 4\rho) + 2\sigma^3 + \sigma^4}{2(1 - \sigma)(1 + \sigma)^2}. \]

Compared with \( \Pi_P^m \), the profit with dual distribution is larger if and only if:

\[ \rho \geq \sigma + (1 + \sigma) \sqrt{\frac{\sigma(1 - \sigma)}{1 + 2\sigma}}. \]  

However, the right-hand side of (15) is larger than the right-hand side of (12) for all values of \( \sigma \), with \( \sigma^2(4+\sigma) - 1 \leq 0 \). Since the profit function with \( x_B = \bar{x}_A \) is only valid if (12) does not hold, (15) cannot be fulfilled for any admissible value of \( \sigma \). Hence, mono distribution is optimal is this case.

As a consequence, dual distribution is more profitable than mono distribution if and only if \( \rho \geq \tilde{\rho}(\sigma) \), which can only hold if \( \sigma < 0.473 \). Note that \( \tilde{\rho}(\sigma) \) is equivalent to the right-hand side of (2) with \( \delta(\sigma) = \sqrt{\frac{2+5\sigma + \sigma^2}{1+2\sigma}} \). Since:

\[ \sqrt{\frac{2+5\sigma + \sigma^2}{1+2\sigma}} > 1, \]

mono distribution is optimal for a larger range in the hybrid regime—i.e., the regime
in which only $B$ charges a uniform price—compared to the symmetric regime in which both firms charge personalized prices.

**Online Appendix C: Proof of Proposition 3**

If firms can negotiate the pricing regime at the wholesale stage, they chose the one that maximizes the industry profit. As shown in the proof of Proposition 2, $\Pi_{UP} > \Pi_U$, regardless of the values of $\rho$ and $\sigma$. Therefore, firms will never choose the pricing regime of uniform pricing.

We next compare the two regimes in which $A$ charges personalized prices—i.e., the regime in which $B$ charges personalized prices as well (denoted by the subscript $P$) and the regime in which $B$ only charges a uniform price (denoted by the subscript $PU$). If dual distribution is optimal in each of the two regimes, the respective profits are:

$$\Pi_P^* = \frac{r_A^2 \sigma (1 + 3\sigma) - 4\sigma (1 + \sigma) \rho + (1 + \sigma)^2 \rho^2}{\sigma (1 - \sigma^2)}$$

and:

$$\Pi_{PU}^* = \frac{r_A^2 \rho + \sigma(2 - 4\rho + 3\rho^2) + \sigma^2(5 - 8\rho + 2\rho^2) - \sigma^3}{2\sigma(1 - \sigma)(2 + 5\sigma + \sigma^2)}.$$

Taking the difference yields:

$$\Pi_P^* - \Pi_{PU}^* = \frac{r_A^2 (1 + 3\sigma + \sigma^2) (\rho(1 + \sigma) - 2\sigma^2)}{2\sigma(1 - \sigma^2)(2 + 5\sigma + \sigma^2)} > 0.$$

Moreover, as shown in Proposition 2, dual distribution is optimal for a larger range in the regime with personalized pricing of both firms than in the regime in which only $A$ can charge personalized prices. Because the profit from mono distribution is the same in both regimes and the profit from dual distribution is higher in case dual distribution is optimal, it follows that the regime with personalized pricing by both firms (weakly) dominates the hybrid regime. Therefore, firms do not choose the latter regime.

The preceding arguments imply that firms will either choose the regime in which they both charge personalized prices or the one in which only $B$ charges personalized prices. In the latter regime, the industry profit is:

$$\Pi_{UP}^* = \frac{r_A^2 \sigma + 2\sigma^2 - 4\rho\sigma - 2\rho\sigma^2 + 2\rho^2 + \rho^2\sigma}{4\sigma (1 - \sigma^2)}.$$

Instead, whenever both firms offer personalized prices, dual distribution is optimal if:

$$\rho > \tilde{\rho}(\sigma) \equiv \frac{\sigma (2 + \sqrt{1 - \sigma})}{(1 + \sigma)}.$$
which, together with $\rho < 1$, implies $\sigma < \hat{\sigma} = (\sqrt{5} - 1)/2$. The industry profit from dual
distribution is then equal to:

$$
\Pi_p^* = \frac{r_A^2 (1 + 3\sigma) - 4\rho\sigma (1 + \sigma) + \rho^2 (1 + \sigma)^2}{2\sigma (1 - \sigma^2)}.
$$

Therefore:

$$
\Pi_p^* > \Pi_{UP}^* \iff \frac{r_A^2 (1 + 3\sigma) - 4\rho\sigma (1 + \sigma) + \rho^2 (1 + \sigma)^2}{2\sigma (1 - \sigma^2)} > \frac{r_A^2 (1 + 3\sigma) - 4\rho\sigma (1 + \sigma) + \rho^2 (1 + \sigma)^2}{2\sigma (1 - \sigma^2)} > \frac{r_A^2 (1 + 3\sigma) - 4\rho\sigma (1 + \sigma) + \rho^2 (1 + \sigma)^2}{2\sigma (1 - \sigma^2)}.
$$

It follows that dual distribution together with personalized pricing by both firms is optimal if and only if:

$$
\frac{\sigma (2 + \sqrt{1 - \sigma})}{1 + \sigma} \leq \rho \leq \frac{1 + 4\sigma}{3 + 2\sigma}.
$$

If instead $\rho \leq \hat{\rho}(\sigma)$, then the industry profit with personalized pricing is the mono
distribution profit $\Pi_p^m = r_A^2/(2s_A)$, and thus:

$$
\Pi_{UP}^m > \Pi_p^m \iff \frac{r_A^2 (1 + 3\sigma) - 4\rho\sigma (1 + \sigma) + \rho^2 (1 + \sigma)^2}{2\sigma (1 - \sigma^2)} > \frac{r_A^2 (1 + 3\sigma) - 4\rho\sigma (1 + \sigma) + \rho^2 (1 + \sigma)^2}{2\sigma (1 - \sigma^2)} > \frac{r_A^2 (1 + 3\sigma) - 4\rho\sigma (1 + \sigma) + \rho^2 (1 + \sigma)^2}{2\sigma (1 - \sigma^2)}.
$$

Because $\rho > \sigma$, the only relevant case is:

$$
\rho > h(\sigma) \equiv \sigma + \sqrt{\frac{\sigma (1 - \sigma^2)}{2 + \sigma}}.
$$

It is easy to check that $\sigma (2 + \sqrt{1 - \sigma})/(1 + \sigma) \leq h(\sigma) < g(\sigma)$ (resp., $\sigma (2 + \sqrt{1 - \sigma})/(1 + \sigma) > h(\sigma) > g(\sigma)$) for $\sigma < \hat{\sigma}$ (resp., $\sigma > \hat{\sigma}$), where $\hat{\sigma} \approx 0.248$ is the unique solution in $(0, \hat{\sigma})$ to:

$$
\sqrt{1 - \sigma} = \hat{\sigma} (3 + 2\hat{\sigma}).
$$

It follows that dual distribution together with uniform pricing by $A$ and dual distribu-
tion by $B$ is optimal if and only if:

$$
\rho > \max \{g(\sigma), h(\sigma)\} = \begin{cases} 
g(\sigma) & \text{if } \sigma \leq \hat{\sigma}, 
h(\sigma) & \text{if } \hat{\sigma} < \sigma < \hat{\sigma}.
\end{cases}
$$

It also follows from the preceding analysis that mono distribution is optimal if and
only if:

\[ \rho < \min \left\{ \frac{\sigma (2 + \sqrt{1 - \sigma})}{1 + \sigma}, h(\sigma), 1 \right\} = \begin{cases} \frac{\sigma (2 + \sqrt{1 - \sigma})}{1 + \sigma} & \text{if } \sigma \leq \tilde{\sigma}, \\ h(\sigma) & \text{if } \tilde{\sigma} < \sigma < \hat{\sigma}, \\ 1 & \text{if } \hat{\sigma} < \sigma < 1. \end{cases} \]

**Online Appendix D: Proof of Proposition 4**

We start the analysis with the regime in which both firms set uniform prices. If \( A \)'s channel is shut down, \( B \) sets the monopoly price in the retail market, equal to \( r_B \), and the industry profit is \( r_B^2/(4s_B) \). Comparing this profit with the industry profit under dual distribution, which is given \( \Pi_U \), yields that mono distribution by \( B \) gives a higher industry profit if and only if:

\[ \rho \geq \rho_{UU} \equiv \frac{4(1 + \sigma) - (1 - \sigma)\sqrt{4 + 5\sigma}}{3 + 5\sigma}. \]

It is straightforward to check that this inequality always holds at \( \rho = 1 \) but is never fulfilled at \( \rho = \sigma \). In addition, \( \rho_{UU} \in (0, 1) \) for all \( \sigma \in (0, 1) \), that is \( \rho_{UU} \) is in the interior of the admissible range.

Second, we analyze the pricing regime \((UP)\). Without the presence of \( A \)'s channel, \( B \) extracts the entire surplus in the retail market, which implies that the industry profit is \( r_B^2/(2s_B) \). Instead, with dual distribution, the industry profit is \( \Pi_{UP} \). Comparing the two profits yields:

\[ \frac{r_B^2}{2s_B} \geq \Pi_{UP} \iff \rho \geq \rho_{UP} \equiv \frac{2 + \sigma - \sqrt{3(1 + \sigma)(1 - \sigma)}}{1 + 2\sigma}. \]  \hspace{1cm} (16)

This inequality always holds at the upper bound \( \rho = 1 \). At the lower bound \( \rho = \sigma \), the inequality is also fulfilled if \( \sigma \geq 1/2 \). Hence, mono distribution by \( B \) is optimal in this regime for \( \sigma \geq 1/2 \), and, if (16) holds, also for \( \sigma < 1/2 \).

Third, we turn to the regime \((PU)\). Without \( A \)'s channel, the industry profit is \( r_B^2/(4s_B) \) because \( B \) can only charge a uniform price. Instead, if \( A \)'s channel is open, dual distribution is optimal if and only if:

\[ \rho \geq \sigma + \frac{2 + \sqrt{(1 - \sigma)(2 + 5\sigma + \sigma^2)}}{1 + 2\sigma} \]  \hspace{1cm} (17)

and leads to a profit of \( \Pi_{PU} \); otherwise, mono distribution by \( A \) is optimal with a profit of \( \Pi_{PU} = r_A^2/(2s_A) \). Comparing \( \Pi_{PU} \) with the profit from mono distribution by \( B \) (i.e., \( r_B^2/(4s_B) \)) yields that the latter is larger if and only if \( \rho \geq \sqrt{2\sigma} \). Because this comparison is only relevant if (17) does not hold, we need to check if \( \sqrt{2\sigma} \) is smaller than the right-
hand side of (17). Thus is true if and only if $0.357 \lesssim \sigma$.

If instead dual distribution is optimal in case $A$’s channel is open, we obtain:

$$\frac{r_B^2}{4s_B} \geq \Pi_{PU} \iff \rho \geq \rho_{PU} \equiv \frac{(1 + 2\sigma) - (1 - \sigma)\sqrt{2(2 + 5\sigma + \sigma^2)}}{3 + 8\sigma + \sigma^2}. \quad (18)$$

This threshold is larger than the one of the right-hand side of (17) if and only if $0 \leq \sigma < 0.357$, approximately. Hence, mono distribution by $B$ is optimal if $\rho \geq \sqrt{2\sigma}$ in case $0.357 < \sigma \leq 1$, and if (18) holds in case $0 \leq \sigma < 0.357$.

Finally, proceeding in the same way for the regime $(PP)$, we obtain that the industry profit with mono distribution by $B$ is higher than with mono distribution by $A$ or dual distribution if $\rho \geq \sqrt{\sigma}$ for $\sigma > 0.157$, and if

$$\rho \geq \rho_{PP} \equiv 1 - \sqrt{\frac{1 - \sigma}{2(1 + \sigma)}}$$

for $0 \leq \sigma < 0.157$. This shows that in all pricing regimes, mono distribution by $B$ is optimal for some parameters.

We now compare the ranges for which shutting down $A$’s channel is optimal in the four pricing regimes. We start with a comparison of the regimes $(PU)$ with $(UU)$. In the latter, mono distribution is optimal if $\rho \geq \rho_{UU}$ holds, whereas in the former mono distribution by $B$ is optimal if $\rho \geq \sqrt{2\sigma}$ in case $0.357 < \sigma \leq 1$, and, if (18) holds in case $0 \leq \sigma < 0.357$. We start with the case $0 \leq \sigma < 0.357$. The difference:

$$\rho_{PU} - \rho_{UU} \iff \frac{4(1 + 2\sigma) - (1 - \sigma)\sqrt{2(2 + 5\sigma + \sigma^2)}}{3 + 8\sigma + \sigma^2} - \frac{4(1 + \sigma) - (1 - \sigma)\sqrt{4 + 5\sigma}}{3 + 5\sigma}$$

equals 0 at the lower bound $\sigma = 0$. Instead, at the upper bound it is approximately equal to 0.034. The difference is also increasing in $\sigma$, which implies that it is positive for all $\sigma$ between 0 and 0.357. In the range, $0.357 < \sigma \leq 1$, the relevant comparison is:

$$\sqrt{2\sigma} - \frac{4(1 + \sigma) - (1 - \sigma)\sqrt{4 + 5\sigma}}{3 + 5\sigma}.$$

It is easy to check that this difference is again equal to 0.034 at the lower bound. At the upper bound, it is equal to $\sqrt{2} - 1 > 0$. It is also increasing for all values of $\sigma$ in the range between 0.357 and 1. It follows that the range in which mono distribution by $B$ is optimal in the regime $(PU)$ is a subset of the one in the regime $(UU)$, that is, $R_{BU} \subset R_{UU}$.

Next, we compare $R_{UU}$ with $R_{PP}$. In the regime $(PP)$, mono distribution by $B$ is optimal if $\rho \geq \sqrt{\sigma}$ for $0.157 < \sigma \leq 1$, and if $\rho \geq \rho_{PP}$ for $0 \leq \sigma < 0.157$. We start again
with the latter case. The difference:

\[
\rho_{UU} - \rho_{PP} = \frac{4(1 + \sigma) - (1 - \sigma)\sqrt{4 + 5\sigma}}{3 + 5\sigma} - 1 - \sqrt{\frac{1 - \sigma}{2(1 + \sigma)}}
\]

equals \(1/\sqrt{2} - 1/3 > 0\) at \(\sigma = 0\), it is approximately equal to 0.339 at \(\sigma = 0.157\), and increasing for all values of \(\sigma\) between 0 and 0.157. Turning to the range \(0.157 < \sigma \leq 1\), the relevant comparison is:

\[
\frac{4(1 + \sigma) - (1 - \sigma)\sqrt{4 + 5\sigma}}{3 + 5\sigma} - \sqrt{\sigma},
\]

which is approximately equal to 0.339 at \(\sigma = 0.157\), decreasing in \(\sigma\) for \(\sigma \in (0.157, 1]\), and equal to 0 at \(\sigma = 1\). It follows that the thresholds above which mono distribution by \(B\) is optimal, are strictly lower in the regime \((PP)\) than the threshold in the regime \((PU)\); hence, \(R^B_{UU} \subset R^B_{PP}\).

Finally, we compare \(R^B_{PP}\) with \(R^B_{UP}\). In the regime \((UP)\), mono distribution by \(B\) maximizes the industry profit if \(\rho \geq \rho_{UP}\), which always holds for \(\sigma \geq 1/2\). We start with the range \(0 \leq \sigma < 0.157\). At the lower bound \(\sigma = 0\), the difference \(\rho_{PP} - \rho_{UP}\) equals \(\sqrt{3} - 1 - 1/\sqrt{2} = 0.025 > 0\), and the upper bound \(\sigma = 0.157\), this difference is approximately 0.056. The difference is also increasing in \(\sigma \in [0, 0.157]\), which implies that it is positive in the entire range. Turning to the range \(0.157 < \sigma \leq 1\), the relevant comparison is \(\sqrt{\sigma} - \rho_{UP}\). This difference is approximately equal to 0.056 at the lower bound, equals 0 at the upper bound \(\sigma = 1\), and is decreasing in \(\sigma\) in the relevant range. It follows that \(R^B_{PP} \subset R^B_{UP}\).

**Online Appendix E: Linear wholesale tariff**

In this appendix, we show that our main results carry over to the case with a linear wholesale tariff. In contrast to the two-part tariff analyzed in the main model, linear tariffs create double marginalization problems and tend to generate inefficiently high prices. Yet, because of their simplicity or for fairness reasons,\(^{44}\) linear tariffs are sometimes used in practice.\(^{45}\) We restrict our attention to the two situations in which both firms can charge only uniform prices and in which both firms can charge personalized prices. We also simplify the exposition by allocating all bargaining power at the wholesale stage to \(A\) (i.e., we assume that \(A\) makes the wholesale contract offer).\(^{46}\) We then

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\(^{44}\)Cui *et al.* (2007) show that a linear wholesale price contract can be efficient if the retailer is inequity averse when comparing its profit with that of the manufacturer.

\(^{45}\)This is, for example, the case of the U.S. pay-TV industry; see Crawford and Yurukoglu (2012) and Crawford *et al.* (2018).

\(^{46}\)The qualitative result are similar if instead \(B\) made the offer.
obtain the following proposition:

**Proposition:** Suppose that wholesale contracts are restricted to a wholesale price offered by $A$. Dual distribution is optimal in case both firms can charge only a uniform retail price. By contrast, when both firms offer personalized prices, mono distribution is optimal if and only if:

$$\rho \leq \sigma \frac{2 + \sqrt{2(1 - \sigma)}}{1 + \sigma}.$$  

The proposition shows that our main insights carry over when considering linear wholesale prices instead of two-part tariffs. The intuition is the same as before. Dual distribution expands demand in the low-end segment but triggers competition with the manufacturer’s own distribution channel. As long as that channel charges a uniform price, this competition is not too fierce and can be sufficiently mitigated through an appropriate wholesale price. Dual distribution is therefore optimal. When instead both firms offer personalized prices, competition is tougher; mono distribution is then optimal if $B$ does not add enough value to the industry. Compared to the setting in which the wholesale contract consists of a two-part tariff, the range in which mono distribution is optimal is now even larger, as a linear wholesale price contract does not allow firms to share their joint profits in any way they wish to, which implies that $A$ obtains a smaller part of $B$’s profit than with two-part tariffs.

**Proof of the proposition:**

With dual distribution, the second stage of the game leads, as before, to downstream prices given by (7). We now consider the first stage for the two pricing regimes.

Under uniform pricing, the profit function of $A$ is now $\Pi_A = D_A p_A + D_B w$. Inserting the corresponding demand functions, $p_A$ and $p_B$ from (7), and maximizing with respect to $w$, we obtain that the equilibrium wholesale price is (using “***” to distinguish from the equilibrium that arises with two-part tariffs):  

$$w_U^{**} = \frac{r_A s_B^2 + 8r_B s_A^2}{2s_A (8s_A + s_B)} = \frac{r_A (8\rho + \sigma^2)}{2(8 + \sigma)}.$$  

Inserting $w_U^{**}$ into the profit yields the equilibrium profit with dual distribution:

$$\Pi_U^{**} = \frac{4s_A r_B^2 + 8s_A s_B r_A (r_A - r_B) - r_A^2 s_B^2 (3s_A + s_B)}{4s_A s_B (8s_A + s_B) (s_A - s_B)}$$

$$= \frac{r_A^2}{4s_A} \frac{4\rho^2 + 8\sigma (1 - \rho) - \sigma^2 (3 + \sigma)}{\sigma (1 - \sigma) (8 + \sigma)}.$$  

As in Section 4, it can be checked that demands $D_A$ and $D_B$ are both positive at $w = w_U^{**},$

---

47The second-order condition is $-2s_A (8s_A + s_B)/(s_B (4s_A - s_B)^2) < 0$, implying that the profit function is concave.
implying that dual distribution is optimal. Indeed, comparing $\Pi_{U}^{**}$ with $\Pi_{U}^{m} = r_{A}^{2}/4s_{A}$, yields:

$$\Pi_{U}^{**} - \Pi_{U}^{m} = \Pi_{U}^{m} - \frac{4(\sigma - \rho)^{2}}{\sigma(1 - \sigma)(8 + \sigma)} > 0.$$ 

We now turn to personalized pricing. As in the case of two-part tariffs, in the range $w \geq \hat{u}$, $B$ is inactive and so $A$ cannot obtain more than the mono distribution profit $\Pi_{P}^{m}$. We thus focus on $w \leq \hat{u}$, distinguishing again between $w \leq \bar{w} = r_{A}(\rho - \sigma)$ and $w > \bar{w}$. We start with the former case. With linear tariffs, the profit function is:

$$\Pi_{A}(w) = \int_{0}^{\hat{x}} [w + u_{A}(x) - u_{B}(x)] dx + \int_{\hat{x}}^{\hat{x}_{B}(w)} w dx,$$

which is strictly concave in $w$:

$$\Pi'_{A}(w) = \hat{x}_{B}(w) + w \frac{dx_{B}(w)}{dw} = \frac{r_{B} - w}{s_{B}} - \frac{w}{s_{B}} = \frac{r_{B} - 2w}{s_{B}},$$

and thus $\Pi_{A}^{m} = -2/s_{B} < 0$. When instead $w > \bar{w}$, $A$’s profit can be written as:

$$\Pi_{A}(w) = \int_{0}^{\hat{x}} w + u_{A}(x) - u_{B}(x)dx + \int_{\hat{x}}^{\hat{x}_{B}(w)} w dx + \int_{\hat{x}_{B}(w)}^{\hat{x}_{A}(w)} u_{A}(x)dx.$$ 

The first derivative is equal to:

$$\Pi'_{A}(w) = \hat{x}_{B}(w) + w \frac{dx_{B}(w)}{dw} (w) = \frac{r_{B} - 2w + r_{A} - u_{A}(\hat{x}_{B}(w))}{s_{B}} \frac{dx_{B}(w)}{dw} (w) = \frac{r_{B} - 2w + r_{A}}{s_{B}} - \frac{u_{A}(\hat{x}_{B}(w))}{s_{B}} = \frac{r_{A}}{s_{A}\sigma^{2}} \left[ \sigma - \rho (1 - \sigma) + (1 - 2\sigma) \frac{w}{r_{A}} \right].$$

Hence:

$$\Pi'_{A-}(\hat{u}) = \frac{r_{A} 1 - \rho}{s_{A} 1 - \sigma},$$

$$\Pi'_{A+}(\bar{w}) = \Pi'_{A}(\bar{w}) = \frac{r_{A}}{s_{A}\sigma} (2\sigma - \rho),$$

$$\Pi'_{A}(w) = \frac{1 - 2\sigma}{s_{A}\sigma^{2}}.$$ 

It follows that $\Pi_{A}(w)$ is strictly concave in $w$ if $\sigma > \hat{\sigma}^{**} = 1/2$ and is weakly convex otherwise; in addition, $\Pi'_{A}(\hat{u}) > 0$ whereas $\Pi'_{A}(\bar{w}) \geq 0$ if and only if:

$$\rho \leq \hat{\rho}^{**}(\sigma) \equiv 2\sigma,$$

where $\hat{\rho}^{**}(\sigma)$ increases with $\sigma$ and exceeds 1 for $\sigma \geq \hat{\sigma}^{**}$. Furthermore, the profit
function \(\Pi_A(w)\) and its derivative \(\Pi'_A(w)\) are both continuous at \(w = w\).

As mentioned above, as long as \(w \geq \hat{w}\), \(A\) cannot obtain a higher profit than with mono distribution. Furthermore, if \(\rho \leq \hat{\rho}^{**}(\sigma)\), then \(\Pi'_A(w) \geq 0\), implying that dual distribution cannot be more profitable than mono distribution:

- in the range \(w \leq w \leq \hat{w}\), the profit function \(\Pi_A(w)\) is increasing, as it is quadratic and its derivative is non-negative at both ends of the range (namely, \(\Pi'_A(w) \geq 0\) and \(\Pi'_A(\hat{w}) > 0\));
- in the range \(w \leq w\), the profit function \(\Pi_A(w)\) is again increasing, as it is concave and its derivative is non-negative at the upper end of the range (namely, \(\Pi'_A(w) \geq 0\));
- it follows that the profit achieved under dual distribution cannot exceed \(\Pi_A(\hat{w})\), which is less profitable than mono distribution.

As already noted, \(\hat{\rho}^{**}(\sigma)\) is increasing in \(\sigma\), and satisfies \(\hat{\rho}^{**}(\sigma) \geq 1\) for \(\sigma \geq \hat{\sigma}^{**}\). It follows that, if \(\sigma \geq \hat{\sigma}^{**}\), then dual distribution cannot be more profitable than mono distribution, as we then have \(\hat{\rho}^{**}(\sigma) \geq 1 (\geq \rho)\).

If instead \(\sigma < \hat{\sigma}^{**}\) and \(\rho > \hat{\rho}^{**}(\sigma)\), then \(\Pi'_A(w) < 0\) and, in the range \(w \leq w\), from (19), \(\Pi_A(w)\) is maximal for \(w^{**} = r_B/2\), which lies below \(w\) and yields a profit equal to:

\[
\Pi^{**}_p = \frac{2r^2_A s_B + r^2_B(s_A + s_B) - 4r_A r_B s_B}{4s_B(s_A - s_B)} = \frac{r^2_A 2\sigma + \rho^2(1 + \sigma) - 4\rho \sigma}{4s_A \sigma(1 - \sigma)}.
\]

Compared with the profit from mono distribution, \(\Pi^m_p = r^2_A/2s_A\), dual distribution introduces a change in profit equal to:

\[
\Pi^*_p - \Pi^m_p = \Pi^m_p \left(\frac{2\sigma - 4\rho \sigma + \rho^2(1 + \sigma)}{2\sigma (1 - \sigma)} - 1\right) = \frac{2\sigma^2 - 4\rho \sigma + (1 + \sigma) \rho^2}{2\sigma (1 - \sigma)}.
\]

The numerator of this expression is a convex quadratic polynomial of \(\rho\) and its roots are:

\[
\sigma \frac{2 - \sqrt{2(1 - \sigma)} + \sqrt{2(1 - \sigma)}}{1 + \sigma} \quad \text{and} \quad \sigma \frac{2 + \sqrt{2(1 - \sigma)} + \sqrt{2(1 - \sigma)}}{1 + \sigma}.
\]

Furthermore, \(\hat{\rho}^{**}(\sigma)\) lies between these two roots in the relevant range \(\sigma < \hat{\sigma}^{**}\):

\[
\frac{\sigma^{2 - \sqrt{2(1 - \sigma)}}}{1 + \sigma} \cdot \frac{1}{\hat{\rho}^{**}(\sigma)} = \frac{2 - \sqrt{2(1 - \sigma)}}{2(1 + \sigma)} < 1,
\]

\[
\frac{\sigma^{2 + \sqrt{2(1 - \sigma)}}}{1 + \sigma} \cdot \frac{1}{\hat{\rho}^{**}(\sigma)} = \frac{2 + \sqrt{2(1 - \sigma)}}{2(1 + \sigma)} > 1,
\]

where the last inequality stems from \(\sqrt{2(1 - \sigma)} > 2\sigma\) in the relevant range \(\sigma < \hat{\sigma}^{**}\). It
follows that dual-distribution is more profitable than mono-distribution if and only if \( \sigma < \hat{\sigma}^{**} \) and \( \rho \) exceeds the larger root, that is, if:

\[
\rho > \hat{\rho}^{**}(\sigma) \equiv \frac{2 + \sqrt{2(1-\sigma)}}{1+\sigma}.
\]

Note that \( \hat{\rho}^{**}(\sigma) \) is increasing in \( \sigma \) in the range \( \sigma \leq \hat{\sigma}^{**} \), and exceeds 1 in the range \( \sigma \geq \hat{\sigma}^{**} \). Hence, as \( \rho < 1 \), the condition \( \rho > \hat{\rho}^{**}(\sigma) \) implies \( \sigma < \hat{\sigma}^{**} \).

**Online Appendix F: Wholesale contract including A’s retail price**

In this appendix, we show that our main results carry over to a scenario in which the manufacturer and the retailer do not only negotiate about the wholesale tariff (i.e., the wholesale price \( w \) and the fixed fee \( F \)) but can also negotiate the manufacturer’s retail price(s) in the contract. Although this scenario is perhaps less realistic than the one in our main model—i.e., the scenario requires commitment by the manufacturer to its retail prices, which may not be credible as retail prices can be changed relatively quickly and may be difficult to monitor—it is an important robustness check for our results. The reason is that commitment to A’s retail price(s) allows firms to achieve higher profits in case of dual distribution due to the fact that competition is less fierce than with simultaneous retail price setting. Therefore, one may wonder whether our result that mono distribution is optimal under personalized pricing may hinge on this assumption. The next proposition shows that this is not the case. As in Appendix E, we focus on the two cases in which both firms either set a uniform price or set personalized prices.

**Proposition:** Suppose that firms can contract on A’s retail price, in case of uniform pricing, or its retail prices, in case of personalized pricing. If both firms can charge only a uniform retail price, dual distribution is optimal. By contrast, if both firms offer personalized prices, mono distribution is optimal if and only if:

\[
\rho \leq \sqrt{\sigma} \quad \text{for} \quad \sigma > 3 - 2\sqrt{2} \approx 0.172;
\]
\[
\rho \leq \frac{\sigma(3 - \sigma)}{1 + \sigma} \quad \text{for} \quad \sigma \leq 3 - 2\sqrt{2} \approx 0.172.
\]

As in our main model, dual distribution is optimal in the entire parameter range if both firms charge a uniform price but mono distribution may be optimal if both firms charge personalized prices. In both scenarios, the profit with dual distribution is higher than in the case in which retail prices of both firms are set after the wholesale contract is negotiated. This is due to the fact that the possibility of commitment to the retail price by one firm relaxes downstream competition, and leads to higher prices.
and profits: the two firms could negotiate the same retail prices as \( A \) would set in case retail prices are chosen simultaneously by the two firms. Because \( B \)'s best response is the same as in the simultaneous game, this leads to the same industry profit. Therefore, if firms negotiate different retail prices, a revealed-preference argument implies that the industry profit must be higher. However, the proposition shows that this effect is not large enough to overturn our main result: mono distribution is still optimal for \( \rho \) low enough if both firms can set personalized prices.

**Proof of the proposition:**
In case of uniform pricing, \( B \)'s best response at the retail stage is the same as the one with simultaneous choice of the retail prices. From the proof of Proposition 2, it is given by:

\[
p_B(p_A; w) = \frac{r_B + w}{2} + \frac{s_B(p_A - r_A)}{2s_A}.
\]

In the negotiation stage, the two firms maximize the objective function \( p_A D_A(p_A, w) + p_B(p_A, w) D_B(p_A, w) \) with respect to \( w \) and \( p_A \). Doing so, we obtain that the maximization problem is strictly concave and the solution is:

\[
w = \frac{r_A s_B}{2s_A} \quad \text{and} \quad p_A = \frac{r_A}{2}.
\]

The resulting industry profit is:

\[
\frac{r_A^2 s_B + r_B^2 s_A - 2r_A r_B s_B}{4 s_B (s_A - s_B)}.
\]

Comparing this profit with \( \Pi^*_U \) from the proof of Proposition 2 yields that the former is larger than the latter for all parameter combinations. Since \( \Pi^*_U \) was already larger than the profit from mono distribution, we obtain that dual distribution is also optimal if \( A \)'s retail price can be negotiated in the wholesale contract.

We now turn to personalized pricing by both firms. In this case, the industry profit under mono distribution by \( A \) is \( \frac{r_A^2}{2s_A} \). If \( A \)'s personalized prices can be negotiated in the wholesale contract, the firms can, in addition to choosing mono distribution, negotiate prices such that either both firms are active or only \( B \) receives a positive demand. We start with the latter case. By setting \( p_A(x) \geq u_A(x) \) for all \( x \) and \( w = 0 \), the industry profit is equivalent to the profit that occurs if only \( B \) sells the product and charges personalized prices—i.e., \( \frac{r_B^2}{2s_B} \). Comparing this with the profit from mono distribution by \( A \), we obtain that mono distribution by \( A \) is more profitable if and only if \( \rho \leq \sqrt{s} \).

We next consider the scenario in which both \( A \) and \( B \) are active. For any \( w \) and \( p_A(x) \), \( B \)'s best response in the retail stage is to set \( p_B(x) = \min [p_A(x) + u_B(x) - u_A(x), u_B(x)] \)
if $p_B(x) \geq w$ and $p_B(x) = w$ otherwise. If both firms are active, the industry profit can be higher than in the two extreme cases of mono distribution by either $A$ or $B$ only if $w \in (0, \hat{u})$. If $w > \hat{u}$, $B$ can never add value compared to the case of mono distribution by $A$ because it cannot sell in the segment $\hat{x} < x \leq \bar{x}$ in which it adds value. Instead, $w = 0$ can only be optimal if $B$ gets the entire demand. For any $w \in (0, \hat{u})$, $A$’s optimal personalized prices are $p_A(x) \geq u_A(x)$ for all $x > \hat{x}$ and $p_A(x) = w + u_A(x) - u_B(x)$ for all $x \leq \hat{x}$. These prices allow $B$ to extract the entire consumer surplus from consumers who obtain a higher value when buying from $B$ than when buying from $A$ but also allow $A$ to sell to all consumers who have a higher valuation for $A$’s product that for that of $B$. In addition, $A$ charges the highest prices, given competition from $B$ at the retail stage. Given these personalized prices, in the negotiation stage the two firms choose $w$ so as to maximize:
\[
\int_{0}^{\hat{x}(w)} [w + u_A(x) - u_B(x)] dx + \int_{\hat{x}(w)}^{\bar{x}(w)} u_B(x) dx.
\]
This problem is strictly concave, and the unique solution is $w = s_B(r_A - r_B)/(s_A - s_B)$, leading to an industry profit of:
\[
\frac{r_A^2 \rho^2(1 + \sigma) - \sigma(4\rho - 1 - \sigma)}{2\sigma(1 - \sigma)^2}.
\]
Comparing this profit with the one from mono distribution by $A$ yields that it is larger if and only if:
\[
\rho > \frac{\sigma(3 - \sigma)}{1 + \sigma}.
\]
It is easy to check that for these values of $\rho$, the optimal $w$ is indeed below $\hat{u}$.

Finally, comparing the profit given by (20) with the one in which only $B$ is active, we obtain that the former is larger than the latter if $\rho \leq (1 + \sigma)/(3 - \sigma)$. Solving the expression $\sigma(3 - \sigma)/(1 + \sigma) = (1 + \sigma)/(3 - \sigma)$ for $\sigma$ yields $\sigma = 3 - 2\sqrt{2} \approx 0.172$.\footnote{This threshold can also be obtained by equalizing $\sigma(3 - \sigma)/(1 + \sigma)$ and $\sqrt{\sigma}$.}

The result of the proposition follows.

**Online Appendix G: Comparison of wholesale prices across regimes**

Under uniform pricing, the equilibrium wholesale price is given by:
\[
w^*_{UU} = \frac{s_B}{2s_A} \frac{4s_A(r_A + r_B) + r_A s_B}{4s_A + 5s_B} = \frac{r_A \sigma}{2} \frac{4(1 + \rho) + \sigma}{4 + 5\sigma},
\]
whereas in the hybrid regime in which only $B$ can charge personalized prices the wholesale price is given by:

$$w_{UP}^* = \frac{s_B r_A + r_B}{2} = \frac{r_A \sigma}{2} \frac{1 + \rho}{1 + \sigma}.$$

Hence:

$$w_{UP}^* - w_{UU}^* = \frac{r_A \sigma^2 (\rho - \sigma)}{2 (1 + \sigma) (4 + 5 \sigma)} > 0,$$

where the inequality follows from $\rho > \sigma$.

When both firms charge personalized prices, the wholesale price is either so large that $B$ does not serve any consumer, or equal to:

$$w_{PP}^* = \frac{s_B r_A}{s_A + s_B} = \frac{r_A \sigma}{1 + \sigma},$$

which satisfies:

$$w_{PP}^* - w_{UP}^* = \frac{r_A \sigma}{1 + \sigma} - \frac{r_A}{2} \frac{1 + \rho}{1 + \sigma} = \frac{r_A \sigma}{2} \frac{1 - \rho}{1 + \sigma} > 0.$$

Therefore, regardless of whether mono or dual distribution is optimal when both firms charge personalized prices, the wholesale price in this symmetric regime is higher than in the regime in which only $B$ charges personalized prices.

Finally, we compare the wholesale prices in the symmetric regime in which both firms charge personalized prices with the ones in the hybrid regime in which only $A$ charges personalized prices. From Proposition 2, we know that in the hybrid pricing regime, mono distribution is optimal for a larger range than in the symmetric pricing regime. Hence, if dual distribution is optimal in the symmetric regime but mono distribution in the hybrid regime, the wholesale price in the latter regime is larger. If instead mono distribution is optimal in both regimes, the wholesale price is the same.

It remains to compare the wholesale prices in case dual distribution is optimal in both regimes. If dual distribution is optimal in the hybrid regime, the equilibrium wholesale price is:

$$w_{PU}^* = \frac{r_A \sigma (2 + \sigma + \rho)}{(2 + 5 \sigma + \sigma^2)}.$$

Taking the difference between $w_{PU}^*$ and $w_{PP}^*$ yields:

$$w_{PU}^* - w_{PP}^* = \frac{\sigma (\rho (1 + \sigma) - 2 \sigma)}{(1 + \sigma) (2 + 5 \sigma + \sigma^2)},$$

which is strictly increasing in $\rho$. Inserting the lower bound for $\rho$ given by $\tilde{\rho}(\sigma)$ into the
right-hand side of the last equation yields:

\[
\frac{\sigma^2}{\sqrt{(1 + \sigma)(1 + 2\sigma)(2 + 5\sigma + \sigma^2)}}.
\]

which is strictly positive. Therefore, \(w^*_\text{PU} - w^*_p > 0\) for all admissible parameters at which these prices occur in equilibrium.

As a consequence, we obtain a clear ranking of the wholesale prices:

\[w^*_\text{UU} < w^*_\text{UP} < w^*_\text{PP} \leq w^*_\text{PU}.\]

**Online Appendix H: Size ranges for mono distribution by either firm**

In this appendix, we compare the parameter range in which mono distribution by either of the two firms is optimal for the pricing regimes of uniform pricing by both firms and personalized pricing by both firms. We start with the former regime. From Proposition 2, mono distribution by \(A\) is never optimal in this regime, and from the proof of Proposition 4, mono distribution by \(B\) is optimal if:

\[
4(1 + \sigma) - (1 - \sigma)\sqrt{4 + 5\sigma} < 3 + 5\sigma.
\]

Therefore, dual distribution is optimal if (21) does not hold. Instead, in the regime in which both firms charge personalized prices, we obtain by combining the results of Proposition 2 and the proof of Proposition 4, that dual distribution is optimal only if \(\sigma \lesssim 0.157\) and:

\[
\frac{\sigma(2 + \sqrt{1 - \sigma})}{1 + \sigma} < \rho < 1 - \sqrt{\frac{1 - \sigma}{2(1 + \sigma)}}.
\]

Taking the difference between the right-hand sides of (21) and (22) yields:

\[
\frac{4(1 + \sigma) - (1 - \sigma)\sqrt{4 + 5\sigma}}{3 + 5\sigma} - \left(1 - \sqrt{\frac{1 - \sigma}{2(1 + \sigma)}}\right) = \frac{(3 + 5\sigma)\sqrt{2(1 - \sigma^2)} - 2(1 - \sigma^2)\left(\sqrt{4 + 5\sigma} - 1\right)}{2(1 - \sigma)(3 + 5\sigma)},
\]

which is positive for all \(\sigma \lesssim 0.157\). Therefore, dual distribution occurs for a larger parameter range in the regime with uniform pricing by both firms. Conversely, the parameter range in which mono distribution by either firm occurs is larger in the regime with personalized by both firms—i.e., \(R_{\text{UU}}^{A/B} \subset R_{\text{PP}}^{A/B}\).
Online Appendix I: Generalization of Proposition 2, Part (i)

In this appendix, we generalize part (i) of Proposition 2 to an extended setting in which consumers with unit demand have valuations \( u_A(x) \) and \( u_B(x) \) for the products of the two firms, where \( u_A(\cdot) \) and \( u_B(\cdot) \) are both twice continuously differentiable, \( x \) is distributed according to a twice continuously differentiable c.d.f. \( G(x) \) over \( \mathbb{R}_+ \) and:

- \( \forall x \in \mathbb{R}_+, u'_A(x) < u'_B(x) < 0; \)
- \( u_i(\bar{x}_i) = 0 \) for some \( \bar{x}_i > 0; \) and
- \( u_A(\hat{x}) = u_B(\hat{x}) > 0 \) for some \( \hat{x} > 0. \)

This implies that, as in our baseline model, the curves \( u_A(\hat{x}) \) and \( u_B(\hat{x}) \) intersect exactly once, and this intersection occurs in the positive quadrant.

Let:

\[
D_m^i(p_i) \equiv G\left(u_{i}^{-1}(p_i)\right),
\]

denote the monopolistic demand for firm \( i \)'s product:

\[
p_i^m \equiv \arg \max_{p_i} p_i \, D_m^i(p_i),
\]

denote firm \( i \)'s monopoly price:

\[
x_i^m \equiv u_{i}^{-1}(p_i^m),
\]

denote the location of the associated marginal consumer, and:

\[
q_i^m \equiv D_m^i(p_i^m) = G\left(x_i^m\right) \quad \text{and} \quad \pi_i^m \equiv p_i^m \, q_i^m,
\]

denote the monopoly output and profit, respectively. Our working assumption is that \( B \) would seek to serve more consumers than \( A \) in these monopoly situations:

**Assumption A:** \( B \)'s monopoly profit function is strictly quasi-concave and \( q_B^m > q_A^m. \)

Let \( w^m \equiv u_B(x_A^m) \). For \( w \geq w^m \), there exists a continuation equilibrium in which \( A \) charges its monopoly price, \( p_A^m \), and \( B \) does not serve any consumer (e.g., by charging \( p_B = w^m \)). If instead \( w < w^m \), both firms can obtain a positive market share: \( A \) then faces a demand:

\[
D_A(p_A, p_B) \equiv G\left(\Delta^{-1}(p_A - p_B)\right),
\]

where:

\[
\Delta(x) \equiv u_A(x) - u_B(x),
\]

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whereas $B$ faces a demand given by:

$$D_B(p_A, p_B) \equiv D_B^m(p_B) - D_A(p_A, p_B).$$

For the sake of exposition, we will assume that there then exists an equilibrium where both firms obtain a positive market share, which is moreover “well-behaved”:

**Assumptions B:** For any $w \leq w^m$, there exists a unique downstream equilibrium, $(p^*_A(w), p^*_B(w))$, where $p^*_A(w)$ and $p^*_B(w)$ are continuous and increasing in $w$, and such that $p^*_A(w^m) = p^*_A$ and $p^*_B(w^m) = w^m$.

We have:

**Proposition:** Under Assumptions A and B, dual distribution is the unique optimal distribution strategy under uniform pricing.

**Proof:** We first consider the regime in which both firms charge a uniform price. Starting from a situation in which the firms negotiate $w = w^m$, and thus $A$ obtains $\Pi_A^*$, consider a small reduction in the wholesale price from $w^m$ to $w < w^m$, together with a fixed fee, $F(w)$, designed to appropriate $B$’s profit (or almost all of it, to ensure acceptance). $A$ then obtains (almost all of) the industry profit, which can be expressed as:

$$\Pi(w) = \Pi_A(w) + \Pi_B(w),$$

where:

$$\Pi_A(w) = p^*_A(w) D_A(p^*_A(w), p^*_B(w)) + w D_B(p^*_A(w), p^*_B(w)) + F(w),$$

$$\Pi_B(w) = [p^*_B(w) - w] D_B(p^*_A(w), p^*_B(w)) - F(w).$$

By deviating from the downstream equilibrium and charging:

$$\hat{p}_A(w) = p^*_B(w) - u_B(x^m_A) + u_A(x^m_A) = p^*_A + p^*_B(w) - w^m,$$

$A$ would maintain its output of $q^m_A$, and generate an output $\hat{q}_B = D_B^m(p^*_B(w)) - q^m_A$ for $B$. Therefore:

$$\Pi_A(w) \geq \hat{p}_A(w) D_A(\hat{p}_A(w), p^*_B(w)) + w D_B(\hat{p}_A(w), p^*_B(w)) + F(w)$$

$$= [p^*_A + p^*_B(w) - w^m] q^m_A + w [D_B^m(p^*_B(w)) - q^m_A] + F(w)$$

$$= \pi_A^m + [p^*_B(w) - w - w^m] q^m_A + w D_B^m(p^*_B(w)) + F(w).$$
Likewise, noting that $B$ could always choose to deviate from the downstream equilibrium and charge $p_B = w$, we have:

$$
\Pi_B (w) \geq -F(w).
$$

Adding these two inequalities yields (recalling that $\Pi (w) = \Pi_A (w) + \Pi_B (w)$):

$$
\Pi (w) - \pi^m_A \geq \phi (w) \equiv [p_B^r (w) - w - w^m] q_A^m + w D_B^m (p_B^r (w)).
$$

Note that $\phi (w^m) = 0$ because $p_B^r (w^m) = w^m$ and $D_B^m (w^m) = G (x_A^m) = q_A^m$. Taking the derivative of $\phi (w)$ and evaluating it at $w = w^m$, we obtain (again using $p_B^r (w^m) = w^m$ and $q_A^m = D_B^m (w^m)$):

$$
\phi' (w^m) = \left[ \frac{dp_B^r}{dw} (w) - 1 \right] q_A^m + D_B^m (w^m) + w \frac{dD_B^m}{dp_B^r} (p_B^r (w)) \frac{dp_B^r}{dw} (w)
$$

$$
= \frac{dp_B^r}{dw} (w) \left[ D_B^m (w^m) + w^m \frac{dD_B^m}{dp_B^r} (w^m) \right],
$$

where the expression within bracket is negative from Assumption A.\(^{49}\) It follows that a reduction of $w$ below $w^m$ is strictly profitable, implying that dual distribution is the unique optimal mode of distribution.

Turning to the hybrid regime in which $B$ charges personalized prices, the same logic as in the main text can be applied. In particular, setting $p_A = p_A^*$ and $w = p_B^*$, where $p_A^*$ and $p_B^*$ are the equilibrium retail prices under uniform pricing, delivers a higher industry profit than dual distribution with uniform pricing, and therefore also a higher profit than $\Pi_U^m$.

References


\(^{49}\)Indeed, Assumption A implies that firm $B$’s optimal monopoly demand is strictly larger than $q_A^m = D_B^m (w^m)$; hence, firm $B$’s monopoly price is below $w^m$, which implies that the first-order condition evaluated at $w^m$ is negative.