Effect of multipath and antenna diversity in MIMO-OFDM systems with imperfect channel estimation and phase noise compensation[☆]

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The effect of phase noise in multiple-input–multiple-output systems employing orthogonal frequency division multiplexing is analyzed in a realistic scenario where channel estimation is not perfect, and the phase noise effects are only partially compensated. In particular, the degradation in terms of SNR is derived and the effects of the receiver and channel parameters are considered, showing that the penalty is different for different receiver schemes. Moreover it depends on the channel characteristics and on the channel estimation error. An analytical expression is used to evaluate the residual inter-channel interference variance and therefore the degradation. The effects of multipath and antenna diversity are shown to be different for the two types of linear receivers considered, the zero-forcing scheme and the minimum mean squared error receiver.

1. Introduction

The combination of orthogonal frequency division multiplexing (OFDM) and multiple-input–multiple-output (MIMO) is an already established technique, due to the lower implementation complexity of OFDM with respect to other modulation formats when combined with MIMO in multipath channels.

In order to get close to the capacity improvements predicted by the theoretical studies, one common approach is to employ spatial multiplexing, where independent information streams are transmitted from the antennas. However these systems suffer from impairments, which strongly affect the system performance, such as the phase noise, introducing a loss of orthogonality among sub-carriers which gives rise to inter-channel interference (ICI), or the channel estimation errors.

Typically in OFDM and MIMO-OFDM the effects of phase noise are considered separately from the channel estimation [1–7], in other words, the phase noise correction and ICI cancelation schemes proposed in the literature generally assume a perfect channel knowledge. On the other hand, the evaluation of the estimation error effects is done in the absence of phase noise.

Among the works where the phase noise is considered in OFDM and MIMO-OFDM, in [2,3] a cancelation scheme removing the common phase error (CPE) is proposed, in a case where different oscillators can be employed on different antennas. In [4,5] an iterative technique is introduced to cancel successively the terms of ICI. The degradation is evaluated before the actual receiver.

In [6] the effect of phase noise is analyzed for different configurations of transmit and receive antennas in a zero-forcing receiver. An analytical expression for the degradation is obtained resorting to an approximation of the phase noise exponential by its first-order component in the Taylor series expansion. However, an ideal channel estimation is considered to show the penalty caused by phase noise.

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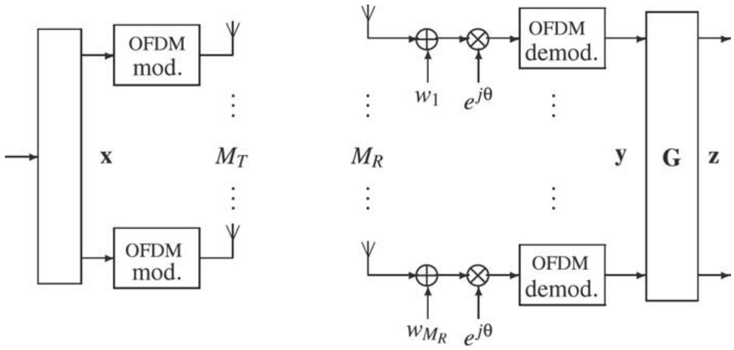

Fig. 1. Scheme of the MIMO-OFDM transmission system.

Note however that the combined effect of phase noise and channel estimation, as we will show, can be different for different types of receivers, so that the same signal degradation prior to the receiver is not having the same effect on different schemes.

On the other hand, the effect of the channel estimation error on the performance of a ZF receiver scheme is considered in [7] for a flat fading channel without phase noise, showing the dependence of the bit error rate (BER) performance on the antenna configurations.

Here we address the joint problem of phase noise and channel estimation for a frequency-selective channel, presenting some results which compare the sensitivity of linear receivers, zero-forcing (ZF) and minimum mean squared error (MMSE), to the effects of phase noise and channel estimation errors. Some preliminary results were presented in [8], however resorting mainly to simulations to estimate the penalty of these systems, in terms of SNR, under different configurations of transmit and receive antennas. Some approximate analytical result was used in the limit of small phase noise. In the present work, the use of analytical expressions of the residual ICI variance after CPE compensation allows a deeper insight into the effects of the antenna diversity and the multipath characteristics on the SNR degradation. Moreover, the analysis does not rely on any approximation of the phase noise contribution. As it is shown in detail in the following, the combined effects of phase noise and estimation errors will have a different impact on the ZF and on the MMSE receiver, with a strong dependence on the channel and antenna diversity conditions.

The paper is organized as follows: In Section 2 the system model is presented, defining the main parameters which will influence the performance. In Section 3 the linear receiver characteristics are presented together with the model for the channel estimation error. In Section 4 the analytical derivation of the variance of the residual ICI is outlined. Finally, in Section 5 the results are compared and discussed, giving some design guidelines, in order to account and combat the effects of phase noise and estimation errors.

2. MIMO-OFDM system

The MIMO-OFDM transmission system employing spatial multiplexing is depicted in Fig. 1.

We assume that the data symbols, belonging to a QAM constellation, are OFDM modulated over N sub-carriers. The MIMO system has M_T transmit antennas where independent information streams are transmitted and M_R receive antennas. We define $\mathbf{x}$ the vector of transmitted symbols $\mathbf{x} = \left[\mathbf{x}_0^T, \ldots, \mathbf{x}_{N-1}^T\right]^T$, where each of the component vectors $\mathbf{x}_n$ groups the symbols transmitted on the nth sub-carrier on all the antennas $\mathbf{x}_n = [x_{n,1}, \ldots, x_{n,M_T}]^T$.

2.1. Channel and phase noise model

The discrete-time equivalent channel model for each antenna pair (i, j) with $i = 1, \ldots, M_R$ and $j = 1, \ldots, M_T$ is a multipath channel with impulse response $h_m[i, j]$ contained within the cyclic prefix employed by the OFDM modulation. Then, grouping all the transmit–receive antenna pairs, we have the $M_R \times M_T$ channel matrices $\mathbf{h}_m$ $m = 0, \ldots, N_{Ch}$, where N_{Ch} is the length of the channel impulse response.

The phase noise is introduced at the receiver [6] and is modeled by a Wiener random walk process [9] $\theta(t)$, sampled at times kT, namely $\theta_k = \theta(kT)$. The system performance is related to the 3-dB bandwidth of the Lorentzian power density spectrum of the carrier B, leading to a phase noise increment variance σ_θ^2 over the period T, given by $\sigma_\theta^2 = 2\pi BT$. In the following results, in Section 5, the amount of phase noise is accounted by the normalized parameter $B_\theta = BNT$. Since typically the major contribution to the overall phase noise is due to the high-frequency oscillators, we assume that the phase noise contribution is common to all the antennas, deriving from the common down-conversion performed by a single oscillator at the receiver, before the actual OFDM demodulation.

2.2. Received signal

We define the received signal collected by all the antennas, after the receiver DFT, $\mathbf{y} = \left[\mathbf{y}_0^T, \ldots, \mathbf{y}_{N-1}^T\right]^T$, where each component $\mathbf{y}_n$ groups the signals on the nth sub-carrier on all the antennas $\mathbf{y}_n = [y_{n,1}, \ldots, y_{n,M_R}]^T$. Then $\mathbf{y}$ can be expressed as

$$\mathbf{y} = \mathbf{QHx} + \mathbf{W}. \tag{1}$$

The matrix $\mathbf{Q}$ represents the phase noise contribution, while $\mathbf{H}$ is the $M_R N \times M_T N$ channel frequency response matrix, namely the block diagonal matrix given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_0 & & & \\ & \mathbf{H}_1 & & \\ & & \ddots & \\ & & & \mathbf{H}_{N-1} \end{bmatrix} \tag{2}$$

where each block is the nth component of the DFT of the channel matrix, affecting the nth sub-carrier,

$$\mathbf{H}_n = \sum_{m=0}^{N-1} \mathbf{h}_m e^{-j2\pi \frac{mn}{N}}. \tag{3}$$

We assume that, for each of $M_R \times M_T$ channel matrices $\mathbf{h}_m$, $h_m[i,j]$ is modeled by independent zero-mean gaussian random variables with power equal to the power delay profile, so that $E[\mathbf{H}\mathbf{H}^H] = M_T \mathbf{I}_{M_R}$, with $\mathbf{I}_{M_R}$ denoting the $M_R \times M_R$ unit matrix. The term $\mathbf{W}$ represents the AWGN contribution, where, due to circular symmetry, the effect of phase noise on the additive noise is neglected. The phase noise matrix $\mathbf{Q}$ is given by

$$\mathbf{Q} = \begin{bmatrix} \Theta_0 & \Theta_{N-1} & \cdots & \Theta_1 \\ \Theta_1 & \Theta_0 & \cdots & \Theta_2 \\ \vdots & \vdots & \ddots & \vdots \\ \Theta_{N-1} & \Theta_{N-2} & \cdots & \Theta_0 \end{bmatrix} \otimes \mathbf{I}_{M_R}, \tag{4}$$

where $\otimes$ denotes the Kronecker product and Θ_n is the nth component of the DFT of the phase noise vector

$$\Theta_n = \sum_{k=0}^{N-1} e^{j\theta_k} e^{-j2\pi \frac{kn}{N}}. \tag{5}$$

The term along the diagonal of the matrix $\mathbf{Q}$, namely Θ_0, represents the CPE. Note that the overall effect is given by an equivalent matrix $\mathbf{H}_{eq} = \mathbf{QH}$ comprising the effects of both the phase noise and the channel, giving the received signal

$$\mathbf{y} = \mathbf{H}_{eq}\mathbf{x} + \mathbf{W}. \tag{6}$$

3. Receiver schemes

We will analyze linear receivers, where the recovered signal is obtained by

$$\mathbf{z} = \mathbf{G}\mathbf{y}. \tag{7}$$

In particular we consider a zero-forcing and a minimum mean squared error approach to determine the matrix $\mathbf{G}$.

3.1. Zero-forcing (ZF)

The ZF approach gives the filter matrix $\mathbf{G}_{\text{ZF}}$ which removes the interference between received streams, at the expense of enhancing the additive noise, leading to

$$\mathbf{G}_{\text{ZF}} = \mathbf{H}_{eq}^{\dagger}, \tag{8}$$

where $(\cdot)^{\dagger}$ denotes the Moore–Penrose pseudo-inverse.

In practice, we can assume a solution where an estimated version of $\mathbf{H}_{eq}$ is available, namely $\tilde{\mathbf{H}}_{eq} = \tilde{\mathbf{H}}\,\tilde{\mathbf{Q}}$.

3.2. Minimum Mean Squared Error (MMSE)

The MMSE approach gives a filter matrix $\mathbf{G}_{\text{MMSE}}$ which balances the amount of interference with the noise enhancement, leading to

$$\mathbf{G}_{\text{MMSE}} = \left(\mathbf{H}_{eq}^H \mathbf{H}_{eq} + \frac{1}{\text{SNR}} \mathbf{I}_{M_R} \right)^{-1} \mathbf{H}_{eq}^H, \tag{9}$$

where $(\cdot)^H$ denotes the conjugate transpose. Note that, again, in a practical implementation, the matrix $\mathbf{H}_{eq}$ is substituted by its estimated version $\tilde{\mathbf{H}}_{eq}$. In this case, as shown in the following, the estimation error will affect also the balance with the noise, expressed by the term $1/\text{SNR}$ in (9).

3.3. Channel estimation

For both linear receivers we assume that first the CPE is compensated by multiplication of the received signal by the matrix $(\Theta_0)^{-1}\mathbf{I}$, in other words $\mathbf{Q}$ is approximated by its diagonal elements $\tilde{\mathbf{Q}} = \tilde{\Theta}_0\,\mathbf{I}$ and $\tilde{\mathbf{H}}_{eq} = \tilde{\Theta}_0\,\tilde{\mathbf{H}}$.

The CPE can be estimated for example by means of a number N_p of pilot sub-carriers, which are usually inserted in the OFDM symbol in positions p_i, $i = 1, \ldots, N_p$ on the antenna streams M_j, $j = 1, \ldots, M_p$. It is estimated as the average phase displacement with respect to the expected symbol [10,11], evaluating the average

$$\tilde{\Theta}_0 = \frac{1}{M_p}\frac{1}{N_p}\sum_{i=1}^{N_p}\sum_{j=1}^{M_p}\frac{y_{p_i,M_j}}{|y_{p_i,M_j}|}. \tag{10}$$

Then, the effect of the channel estimation inaccuracy on $\mathbf{H}$ is modeled by an additive estimation error, so that the actual channel matrix is expressed as $\mathbf{H} = \tilde{\mathbf{H}} + \mathbf{Z}$ where $\mathbf{Z}$ is the estimation error matrix, assumed independent of the channel matrix $\mathbf{H}$ and with independent Gaussian elements $z[i,j]$ with zero mean and variance σ_{est}^2 equal to the mean squared error (MSE) achieved in the channel estimation.

4. Derivation of the ICI variance

From (1) we can express the ICI term before the application of the linear equalizer $\mathbf{G}$ as

$$\mathbf{u} = \left[(\mathbf{Q} - \Theta_0 \mathbf{I}_N) \otimes \mathbf{I}_{M_R}\right]\mathbf{Hx}. \tag{11}$$

The ICI term $\mathbf{v}$ at the decision point, after equalization, is

$$\mathbf{v} = \mathbf{G}\left[(\mathbf{Q} - \Theta_0 \mathbf{I}_N) \otimes \mathbf{I}_{M_R}\right]\mathbf{Hx}. \tag{12}$$

If we consider the component on the nth sub-carrier, we have

$$\mathbf{v}_n = \mathbf{G}_n \sum_{\substack{i=0 \\ i \neq n}}^{N-1} \Theta_{n-i}\,\mathbf{H}_i \mathbf{x}_i \tag{13}$$

where $\mathbf{G}_n$ is the nth diagonal block of $\mathbf{G}$, as well as $\mathbf{H}_i$ is the ith diagonal block of $\mathbf{H}$. The variance of the ICI on the nth sub-carrier, averaged over the transmit antennas M_T, is then

$$\sigma_{v_n}^2 = \sigma_x^2 \frac{1}{M_T} \sum_{\substack{i=0 \\ i \neq n}}^{N-1} E\left[|\Theta_{n-i}|^2\right] \text{Tr}\left\{E\left[\mathbf{H}_i^H\,\mathbf{G}_n^H\,\mathbf{G}_n \mathbf{H}_i\right]\right\} \tag{14}$$

where the symbols $\mathbf{x}_i$ have been assumed independent with $E[\mathbf{x}_i\mathbf{x}_i^H] = \sigma_x^2 \mathbf{I}_{M_T}$.

Let us examine separately the different terms of (14) in the following sub-sections.

4.1. Power of the phase noise terms

The evaluation of $E[|\Theta_m|^2]$ in (14) can be obtained by considering the spectral characteristic of the phase noise. In particular $E\left[|\Theta_m|^2\right]$ represents the power spectral density (PDF) of the sampled phase noise process $\mathscr{P}_\theta^{(s)}$, evaluated at the mth sub-carrier frequency, $m\Delta f$, with $\Delta f = \frac{1}{NT}$, which is related to the continuous time phase noise PDF $\mathscr{P}_\theta(f)$ by the periodic repetition

$$E\left[|\Theta_m|^2\right] = \mathscr{P}_\theta^{(s)}(m\Delta f) = \sum_{k=0}^{+\infty} \mathscr{P}_\theta\left(m\Delta f + kN\Delta f\right). \quad (15)$$

Since the power spectral density of the noisy carrier corresponding to the Wiener model is a Lorentzian line, we have

$$\mathscr{P}_\theta(f) = \frac{2\pi B_\theta}{1 + \left(\frac{fT}{B_\theta}\right)^2}. \quad (16)$$

4.2. ZF receiver

In the expression of the ICI variance (14) we have

$$\mathrm{Tr}\left\{E\left[\mathbf{H}_i^H\,\mathbf{G}_n^H\,\mathbf{G}_n\mathbf{H}_i\right]\right\} = \mathrm{Tr}\left\{E\left[\mathbf{H}_i^H\,\left(\mathbf{H}_n^\dagger\right)^H\,\mathbf{H}_n^\dagger\mathbf{H}_i\right]\right\} + \mathrm{Tr}\left\{E\left[\mathbf{Z}_i^H\,\left(\mathbf{H}_n^\dagger\right)^H\,\mathbf{H}_n^\dagger\mathbf{Z}_i\right]\right\} \quad (17)$$

where, due to the independence of the matrix $\mathbf{Z}$ and $\mathbf{H}$ and the fact that $E\left[\mathbf{Z}_i\mathbf{Z}_i^H\right] = \sigma_{\mathrm{est}}^2\mathbf{I}$ we have

$$\mathrm{Tr}\left\{E\left[\mathbf{Z}_i^H\,\left(\mathbf{H}_n^\dagger\right)^H\,\mathbf{H}_n^\dagger\mathbf{Z}_i\right]\right\} = \sigma_{\mathrm{est}}^2\frac{M_T}{M_R - M_T}, \quad (18)$$

for $M_R > M_T$, being the expected value of the trace of an inverse Whishart matrix [12,13].

The first right hand side term of (17) can be expressed as a function of the correlation ρ among the sub-carrier components of the channel frequency response. For an exponential power-delay-profile, we have a correlation between the frequency response on the n and i sub-carriers given by $\rho = \frac{1}{1+j2\pi(n-i)\Delta f T_{\mathrm{rms}}}$, where Δf is again the sub-carrier frequency spacing and T_{rms} represents the r.m.s. value of the power delay profile. We assume the same power delay profile for all the spatial channels $\mathbf{h}_{[i,j]}$, $i = 1, \ldots, M_R, j = 1, \ldots, M_T$.

A detailed derivation has been presented in [14], by using the properties of the Whishart and the inverse Whishart matrices.

For $M_R > M_T$ we have

$$\mathrm{Tr}\left\{E\left[\mathbf{H}_i^H\,(\mathbf{H}_n^\dagger)^H\,\mathbf{H}_n^\dagger\mathbf{H}_i\right]\right\} = |\rho|^2 M_T + (1-|\rho|^2) \times \frac{M_T^2}{M_R - M_T}. \quad (19)$$

We can note that in (19) there is a balance between two terms, depending on the correlation factor ρ, where the antenna diversity $(M_R - M_T)$ has a dominating impact on the degradation or not. We will show in the results section that this leads to a crossing point in the degradation curves corresponding to different antenna configurations. This crossing point occurs when

$$|\rho|^2 = (1-|\rho|^2)\frac{M_T}{M_R - M_T}. \quad (20)$$

In other words, for values of ρ such that

$$|\rho|^2 > \frac{M_T}{M_R}, \quad (21)$$

the advantage of increasing the diversity becomes lower. Thus, if the channel multipath model becomes closer to a flat fading and T_{rms} gets smaller, the SNR degradation due to the ICI is not reduced by increasing the diversity.

4.3. MMSE receiver

In the MMSE receiver, the receiver matrix $\mathbf{G}$ is given by

$$\mathbf{G} = \left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} + \frac{1}{\mathrm{SNR}}\mathbf{I}_{M_T}\right)^{-1}\tilde{\mathbf{H}}^H. \quad (22)$$

The variance of the ICI term is given again by (14) and if we consider the trace of the inner matrix in (14), we can separate the effect of the estimation error from the effect of the sub-carrier correlation of the channel response, as done in the ZF case,

$$\mathrm{Tr}\left\{E\left[\mathbf{H}_i^H\,\mathbf{G}_n^H\,\mathbf{G}_n\mathbf{H}_i\right]\right\} = \sigma_{\mathrm{est}}^2\,E\left[\mathrm{Tr}\left\{\mathbf{G}_n^H\mathbf{G}_n\right\}\right] + \mathrm{Tr}\left\{E\left[\mathbf{H}_i^H\,\mathbf{H}_n\left\{\left(\mathbf{H}_n^H\mathbf{H}_n + \frac{1}{\mathrm{SNR}}\mathbf{I}_{M_T}\right)^{-1}\right\}^H \times \left(\mathbf{H}_n^H\mathbf{H}_n + \frac{1}{\mathrm{SNR}}\mathbf{I}_{M_T}\right)^{-1}\mathbf{H}_n^H\mathbf{H}_i\right]\right\}. \quad (23)$$

In this case, as it can be seen from (23), the derivation in the general case would be quite complex. However, the most significant analysis for the MMSE receiver is for low SNR values, since for high values, as known, the MMSE receiver is equivalent to the ZF. Then, in the case of low SNR, the matrix is $\mathbf{G}_n = \mathrm{SNR}\,\mathbf{H}_n^H$, so that the first term of (23) is

$$E\left[\mathrm{Tr}\left\{\mathbf{G}_n^H\mathbf{G}_n\right\}\right] = \mathrm{SNR}^2 E\left[\mathrm{Tr}\left\{\mathbf{H}_n^H\mathbf{H}_n\right\}\right] = \mathrm{SNR}^2\,M_T M_R, \quad (24)$$

being the expected trace of a Whishart matrix [12,13], and the second term of (23) is

$$\mathrm{SNR}^2\,\mathrm{Tr}\,\left\{E\left[\mathbf{H}_i^H\,\mathbf{H}_n\mathbf{H}_n^H\mathbf{H}_i\right]\right\} = \mathrm{SNR}^2\,\left[|\rho|^2\,M_T M_R(M_T + M_R) + (1-|\rho|^2)\,M_T^2 M_R\right]. \quad (25)$$

Again, for the detailed derivation, the reader is deferred to [14].

As we saw in (19) for the ZF receiver, also for the MMSE receiver, from expression (25) we can see that the contribution of the residual ICI shows a balance between two terms, with the prevalence of one or the other, depending on the characterization of the multipath channel, expressed by the correlation factor ρ among the channel frequency response over different sub-carriers. In this case, however, (25) does not contain a term with the diversity $(M_R - M_T)$ and the number of antennas M_T and M_R is not having different effects for different diversity values, as shown in the following results.

5. Performance comparison and discussion

We present some performance results for a system where OFDM employs $N = 64$ sub-carriers with symbols

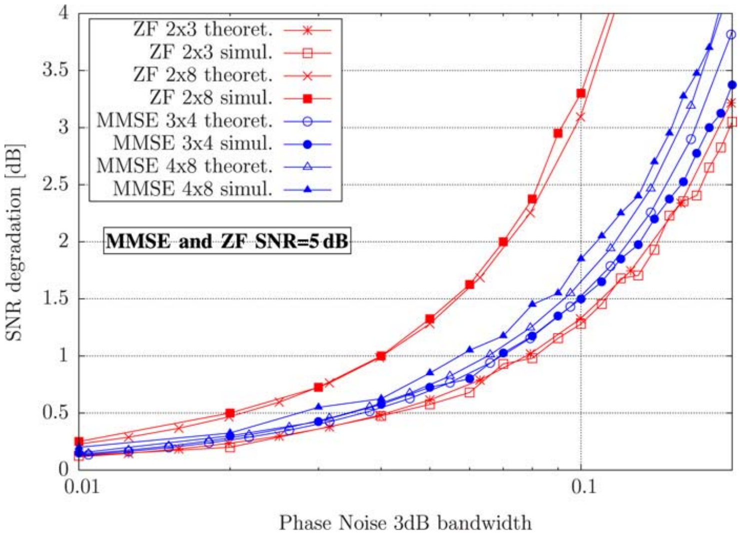

Fig. 2. Comparison of the SNR degradation obtained by simulation and of MSSE and ZF receivers with: SNR $= 5$ dB, channel estimation error MSE $= -30$ dB and $T_{\mathrm{rms}} = 0.1$.

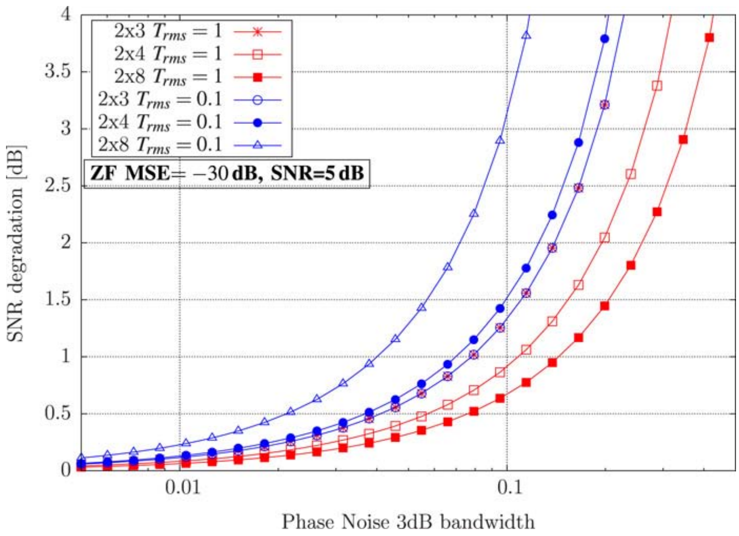

Fig. 3. Degradation of ZF receiver with: SNR $= 5$ dB, channel estimation error MSE $= -30$ dB and two values of T_{rms}.

belonging to a 4-QAM constellation, while different configurations are used for the number of transmit M_T and receive antennas M_R. The multipath power delay profile is exponential characterized in terms of its delay spread T_{rms}, normalized to the OFDM period NT.

In order to compare the performance of different schemes and point out the joint effect of the phase noise and of the channel estimation error, the SNR degradation is introduced. The SNR degradation is defined as the increment in the SNR value needed to achieve the same error probability as in the case of no phase noise and ideal channel estimation. Note that in the reference conditions of no phase noise and ideal channel estimation the performance may be different for different receivers and different channel delay spread T_{rms}. What we present is therefore the additional degradation due to phase noise and channel estimation errors.

We first show some simulation results to prove the accuracy of our analysis. Then, analytical results are discussed and compared, in different channel conditions and antenna configurations.

5.1. *Accuracy of the theoretical analysis*

First we present a comparison between the simulation and the theoretical analysis results in Fig. 2. The results refer to an SNR of 5 dB, a channel estimation error with MSE $= \sigma^2_{\mathrm{est}} = -30$ dB and a multipath channel with $T_{\mathrm{rms}} = 0.1$. We can observe a very good matching between the simulation and the theoretical results even for high phase noise. In fact, as explained in Section 4, the typical approximation $e^{j\theta_k} \approx 1 + j\theta_k$, valid for small phase noise, is not used here to evaluate the phase noise power.

5.2. *ZF*

5.2.1. *Effect of phase noise*

In Fig. 3, we show the SNR degradation for the ZF receiver in the low-SNR region, as a function of phase noise normalized bandwidth, with a channel estimation that can be considered almost ideal, with and estimation error MSE of −30 dB. It can be seen that, when considering different values for the number of transmit and receive antennas,

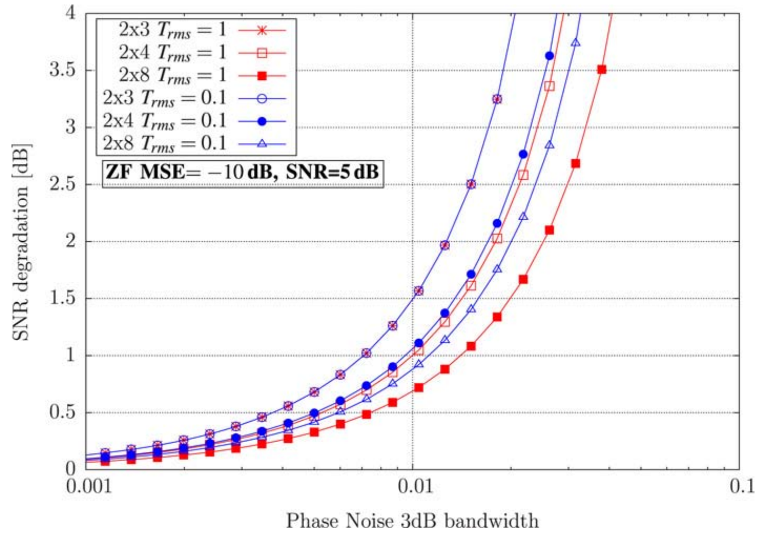

Fig. 4. Degradation of ZF receiver with: SNR = 5 dB, channel estimation error MSE = −10 dB and two values of T_{rms}.

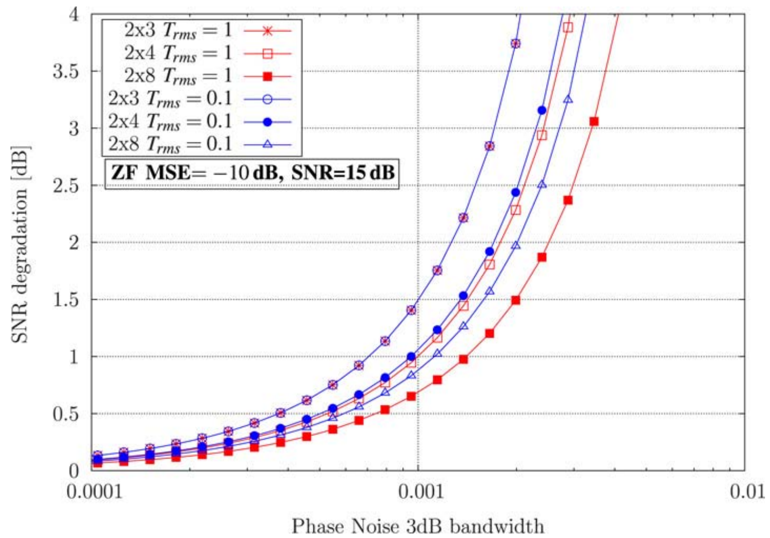

Fig. 5. Degradation of ZF receiver with: SNR = 15 dB, channel estimation error MSE = −10 dB and two values of T_{rms}.

the effect of the antenna diversity ($M_R - M_T$) on the degradation due to phase noise is very different depending on the multipath characteristics, that determine the degree of correlation among the frequency components of the channel response. In fact, for a channel where the fading can be approximated closer by a flat fading over all the sub-carriers an increased diversity leads to a higher degradation, for the same phase noise, while the opposite occurs if the T_{rms} value increases and the correlation between the frequency response on different sub-carriers decreases. In that case, the use of higher antenna diversity order gives a lower SNR degradation due to phase noise.

In Fig. 4, we show again the SNR degradation as a function of the phase noise amount, in this case accounting for the effect of an imperfect channel estimation, by an error with MSE of −10 dB. Together with a shift of the degradation towards smaller values of phase noise, in other words a worse performance due to the degradation introduced by the channel estimation error, we can notice that a different behavior appears in terms of diversity. In fact, when the channel estimation error becomes non-negligible, increasing the antenna diversity leads to a lower sensitivity to the phase noise effect, independently of the multipath channel correlation.

The same results in terms of SNR degradation are now presented in the medium-high SNR region, for SNR = 15 dB, for the ZF receiver. In Fig. 5 the SNR degradation with a channel estimation error with MSE = −10 dB is shown in the same multipath conditions and the same antenna configurations of Fig. 4. It can be clearly seen by comparing Figs. 4 and 5 that, by changing the reference SNR, the degradation is simply shifted to higher values, but the same behavior can be observed in terms of diversity, multipath channel spread T_{rms} and channel estimation MSE. The higher sensitivity to phase noise for higher SNR is expected, since we are considering lower error probabilities, where the effects of phase noise or estimation error would be more noticeable. Based on this observation, in the following results we will concentrate mostly on the low SNR region, keeping in mind the fact that just a higher degradation is observed if higher reference SNR values are considered.

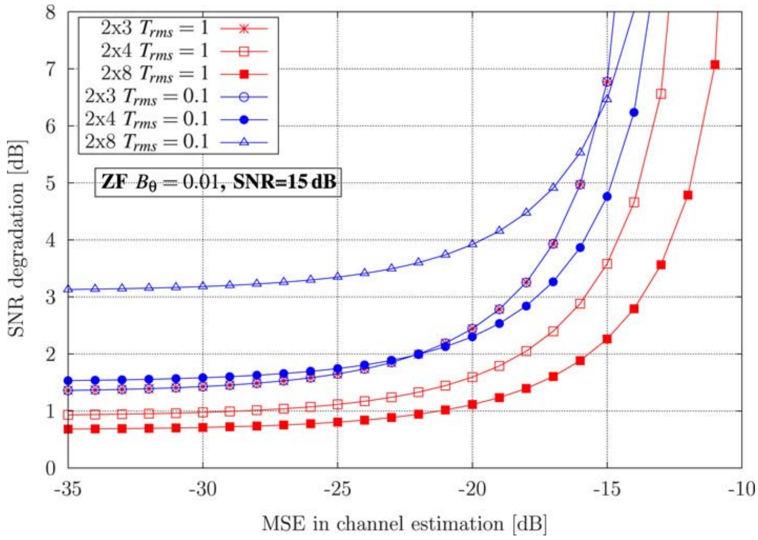

Fig. 6. Degradation of ZF receiver with: SNR $= 5$ dB, $B_\theta = 0.01$, and two values of T_{rms}.

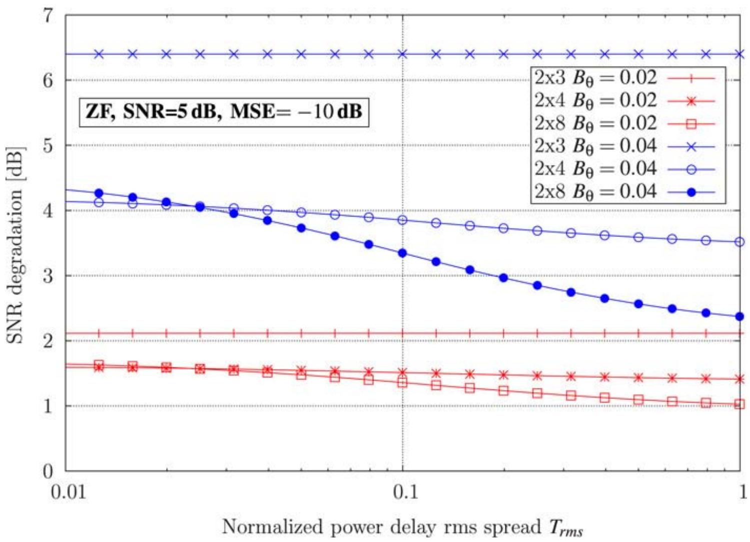

Fig. 7. Degradation of ZF receiver with: SNR $= 5$ dB, estimation error MSE $= -10$ dB and two values of the phase noise normalized bandwidth B_θ.

5.2.2. Effect of channel estimation error

In Fig. 6 the degradation is shown as a function of the channel estimation error MSE, for a fixed amount of phase noise, and two values of T_{rms}, for the low SNR value of 5 dB. By fixing the phase noise, clearly the SNR degradation increases by increasing the error introduced by the channel estimation algorithm, up to a point where the same performance achieved in the absence of phase noise and estimation error cannot be reached, in other words the degradation would be infinite. However, what is more interesting to note is again the combined effects of diversity and multipath delay spread on the degradation. We can see in effect a crossing point between the curves corresponding to different antenna diversity configurations. That is, for small values of the channel estimation error, it is not convenient to increase the diversity order, while if the channel estimation error becomes large, then antenna diversity helps to reduce the penalty. The occurrence of this crossing point, however, depends on the value of the channel delay spread.

5.2.3. Effect of channel delay spread

In Fig. 7 the degradation is shown as a function of the normalized channel rms delay spread T_{rms}, for two values of the phase noise normalized bandwidth B_θ, a reference SNR $= 5$ dB and a fixed channel estimation error, with MSE $= -10$ dB. Again what is more relevant to notice is the effect of the antenna diversity with respect to different values of the correlation. We can note that the point of crossing between the degradation lines does not depend on the phase noise amount, which only shifts the degradation to higher or lower values.

5.3. MMSE

The results previously obtained for the ZF receiver are now presented in the case of the MMSE receiver.

5.3.1. Effect of phase noise

If we now consider the MMSE receiver, we first show in Fig. 8 the SNR degradation for increasing phase noise,

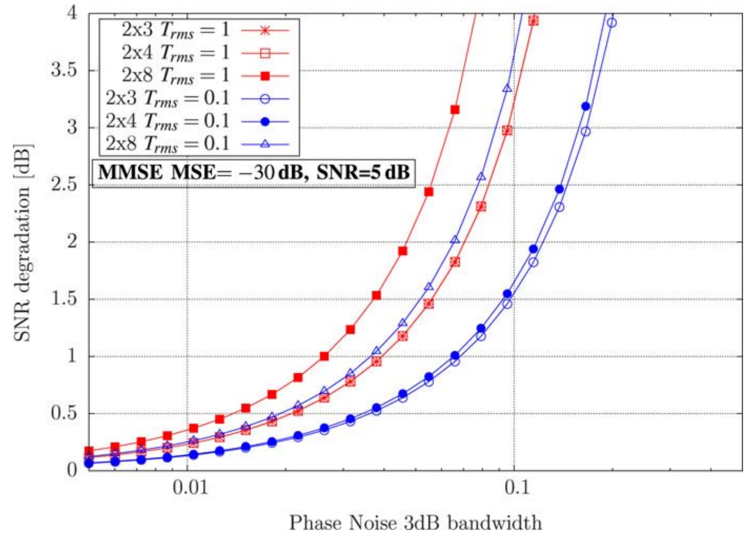

Fig. 8. Degradation of MMSE receiver with: SNR = 5 dB, channel estimation error MSE = −30 dB and two values of T_{rms}.

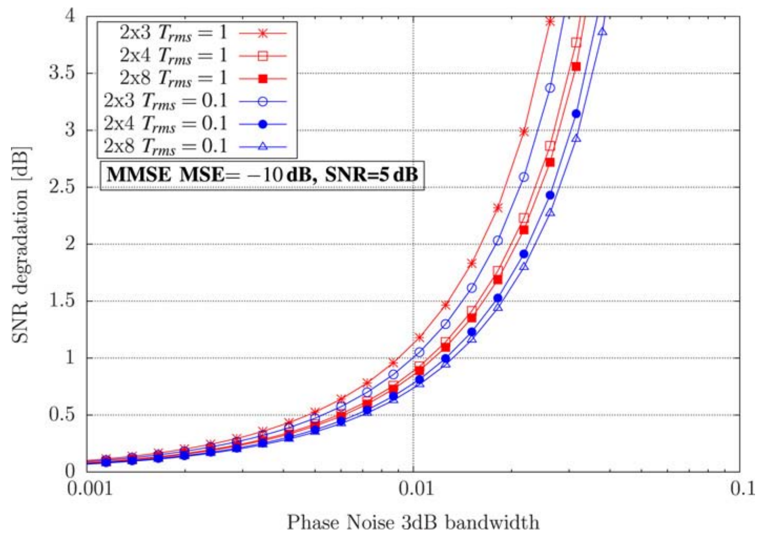

Fig. 9. Degradation of MMSE receiver with: SNR = 5 dB, channel estimation error MSE = −10 dB and two values of T_{rms}.

in the low-SNR region, with a channel estimation that can be considered almost ideal, characterized by an estimation error with MSE of −30 dB. In this case we have a different effect of the antenna diversity on the SNR degradation produced by phase noise, in the case of an almost ideal channel estimation. In fact, by increasing the diversity order we have a higher sensitivity to the phase noise, which is more evident if the correlation among the frequency response in different carriers increases, that is the delay spread decreases and the channel gets closer to a flat fading channel.

In Fig. 9, we show again the SNR degradation as a function of the phase noise amount, while accounting for the effect of an imperfect channel estimation, represented by an estimation error with MSE of −10 dB. In this case the effect of increasing the diversity is favorable in order to reduce the SNR degradation introduced by phase noise, since the channel estimation error contribution is reduced. On the other hand the variation is quite small.

Note that in this case the degradation curves do not show a different behavior for different channel delay spread values, with respect to the antenna diversity order, as occurs in the ZF case.

5.3.2. *Effect of channel estimation error*

To get a better insight into the differences between the two receivers for low SNRs, in Fig. 10 the degradation is shown as a function of the channel estimation error MSE, for a fixed amount of phase noise, and two values of T_{rms}. In this case, two different regions, where the effect of the antenna diversity is different, can be appreciated, which is similar to the case of ZF. Note however that in this case, the occurrence of this crossing point does not depend on T_{rms}. In fact the dependence of the SNR degradation on T_{rms} is very small.

5.3.3. *Effect of channel delay spread*

Fig. 11 corroborates this last affirmation, where the effect of the multipath channel delay spread on the degradation is shown as a function of T_{rms}, for a fixed channel estimation error. We can see that the dependence is moderate and, compared to Fig. 7, we have no crossing

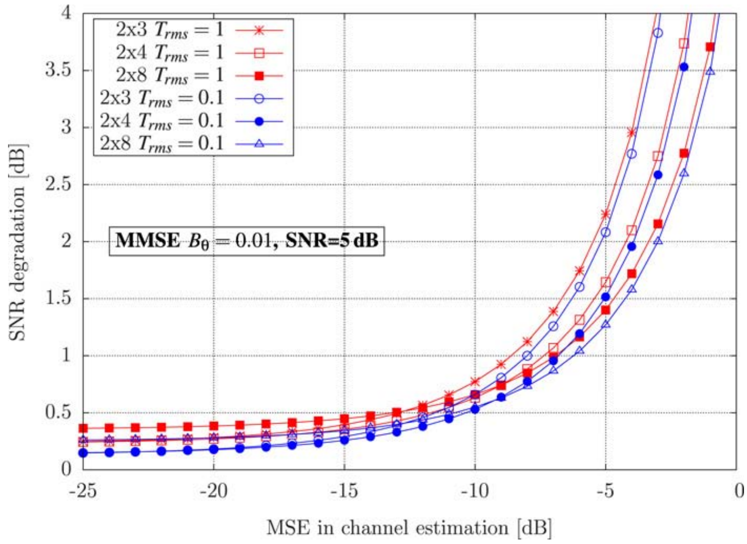

Fig. 10. Degradation of MMSE receiver with: SNR = 5 dB, $B_\theta = 0.01$, and two values of T_{rms}.

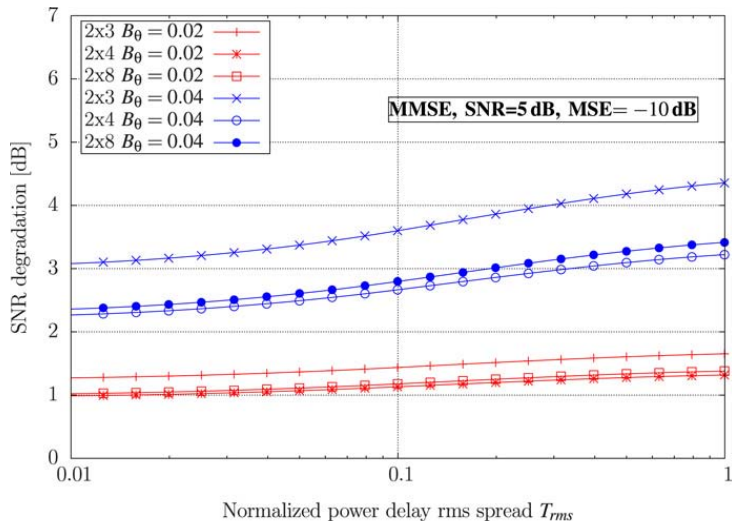

Fig. 11. Degradation of MMSE receiver with: SNR = 5 dB, estimation error MSE = −10 dB, two values of the normalized phase noise bandwidth B_θ.

point where the effect of the antenna diversity becomes different.

6. Conclusions

The effect of phase noise has been analyzed in MIMO-OFDM systems together with the effect of a non-ideal channel estimation with linear receivers, namely, ZF and MMSE. The penalty caused by phase noise and imperfect channel estimation is expressed in terms of SNR degradation using an analytical expression of the residual ICI variance, valid for a general multipath channel. In the case of MMSE, this expression is suitable for the low SNR region, where MMSE differs more from ZF. It can be seen that the effect of phase noise depends on the receiver scheme, both under ideal channel conditions and if a realistic channel estimation is considered, which necessarily introduces an estimation error. In particular, we can see a balance between the phase noise induced degradation and the contribution of the channel estimation error, which is different for the two types of linear receivers. As a "rule of thumb" we could say that in the low-SNR region the phase noise degradation becomes noticeable only if the value of the normalized phase noise bandwidth becomes $B_\theta > 0.01$. For these values of phase noise the degradation becomes also sensitive to different antenna configurations. On the other hand, we can see that the channel estimation error "threshold", that gives a noticeable degradation and different effects of the antenna diversity/configuration, corresponds to a MSE about −10 dB. Moreover, for the ZF receiver, a different effect of the antenna diversity is appreciable for different multipath channel conditions, that is for different power delay spread values. This gives a crossing point, where an increased diversity produces a lower degradation, which is almost independent of the phase noise level, and occurs at an estimation error between −25 and −15 dB, depending on the transmitter antenna number M_T.

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