UNIVERZITET CRNE GORE
UNIVERSITY OF MONTENEGRO

GRAĐEVINSKI FAKULTET U PODGORICI
FACULTY OF CIVIL ENGINEERING, PODGORICA

ŠESTI INTERNACIONALNI NAUČNO-STRUČNI SKUP

GRAĐEVINARSTVO - NAUKA I PRAKSA

THE SIXTH INTERNATIONAL CONFERENCE

CIVIL ENGINEERING - SCIENCE & PRACTICE

ZBORNIK RADOVA

PROCEEDINGS

ŽABLJAK, 7-11. 03. 2016.

ISBN 978-86-82707-30-1

ZBORNIK RADOVA GNP 2016 / GNP 2016 PROCEEDINGS

Izdavač / Publisher

UNIVERZITET CRNE GORE / UNIVERSITY OF MONTENEGRO
GRAĐEVINSKI FAKULTET / FACULTY OF CIVIL ENGINEERING

Za izdavača / For Publisher

Prof. Miloš Knežević, Dr-Ing.

Urednici / Editors

Prof. Miloš Knežević, Dr-Ing.

Assist.Prof. Biljana Šćepanović, Dr-Ing.

Tehnički urednik / Technical editor

Goran Pavlović, Dipl.Ing.

Tiraž / Number of copies

400

CIP - Каталогизација у публикацији
Национална библиотека Црне Горе, Цетиње

ISBN 978-86-82707-30-1
COBISS.CG-ID 29599504

6TH INTERNATIONAL CONFERENCE
CIVIL ENGINEERING - SCIENCE AND PRACTICE

ŽABLJAK, 7-11 MARCH 2016

Đorđe Vuksanović[1], Miroslav Marjanović[2], Dragan Kovačević[3]

FINITE ELEMENT MODELING OF FREE VIBRATION PROBLEM OF DELAMINATED COMPOSITE PLATES USING ABAQUS CAE

Summary

The aim of this paper is to discuss different finite element models of laminated composite plates with delaminations using the commercial software Abaqus CAE. Three-dimensional (solid), as well as two-dimensional (shell) models in Abaqus CAE, are considered. For comparison, the previously derived and validated numerical model of both intact and delaminated composite plates, based on the Generalized Layerwise Plate Theory of Reddy, is used. Influences of the finite element types and the mesh density on the free vibration response are analysed. Excellent agreement is achieved using the shell model in Abaqus CAE, while the limitations of solid models are noted.

Key words

Finite element, Abaqus, laminated composite plate, delamination, free vibrations

MODELIRANJE PROBLEMA SLOBODNIH VIBRACIJA LAMINATNIH KOMPOZITNIH PLOČA SA DELAMINACIJAMA PRIMENOM METODE KONAČNIH ELEMENATA U ABAQUS-U

Rezime

Cilj ovog rada je razmatranje različitih numeričkih modela laminatnih kompozitnih ploča sa delaminacijama primenom metode konačnih elemenata u komercijalnom programskom paketu Abaqus CAE. Razmatrani su trodimenzionalni (solid), kao i dvodimenzionalni (shell) modeli u Abaqus-u. Prethodno izveden i potvrđen numerički model za neoštećene i oštećene laminatne kompozitne ploče, baziran na Reddy-evoj Opštoj laminatnoj teoriji ploča, korišćen je za poređenje rezultata. Analizirani su uticaji tipa konačnog elementa i gustine mreže na slobodne vibracije. Odlično poklapanje rezultata dobijeno je primenom shell modela, a istaknuta su i ograničenja solid modela.

Ključne reči

Konačni element, Abaqus, laminatna kompozitna ploča, delaminacija, slobodne vibracije

[1] Full Professor, Faculty of Civil Engineering, University of Belgrade, Bulevar kralja Aleksandra 73, 11000 Belgrade, Serbia, george@grf.bg.ac.rs
[2] Teaching Assistant, Faculty of Civil Engineering, University of Belgrade, mmarjanovic@grf.bg.ac.rs
[3] Research Assistant, Faculty of Civil Engineering, University of Belgrade, dkovacevic252@gmail.com

1. INTRODUCTION

Laminated composite plates are widely used in many engineering disciplines. In order to predict their behavior in dynamic loading environments, the fundamental dynamic characteristics (such as natural frequencies) of the laminated structure have to be predicted accurately. For this purpose, different numerical solutions are derived so far, based on the Equivalent-Single-Layer (ESL) plate theories [1-4]. However, these models are primarily intended for thin plates and they are not completely adequate for thick plate situations for two reasons: (1) simplifications related to the transverse shear deformation and (2) inability to account for jumps in transverse shear strains at layer interfaces.

Delamination is the most common form of damage in laminated composite plates. Consideration of existing interfacial damage often is relevant for the assessment of the residual lifetime of such structures [5-6]. This requires adequate computational models able to consider delamination of plates in dynamic loading environments. In case of dynamic loading conditions, it is of particular importance to understand the effects of delamination on fundamental dynamic characteristics such as natural frequencies and mode shapes.

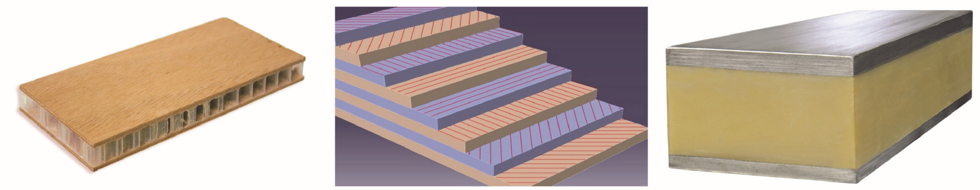

Figure 1. Different types of laminar composites

Previously explained ESL plate theories are unable to describe the delamination kinematics, because they do not treat the individual motion of every layer independently. To overcome this, the extended Generalized Laminated Plate Theory (GLPT) of Reddy [7-9] is used here to analyze the free vibration response of laminated composite plate with embedded delamination. The layered finite element model based on the GLPT is used for the validation of results for natural frequencies of laminated composite plate obtained using different finite element models in Abaqus CAE [10], which will be explained in the next section.

2. LAYERED FINITE ELEMENT MODEL

For comparison, the layered finite element model based on the GLPT is used [5,6,9]. Piece-wise linear variation of in-plane displacements is imposed, as well as the constant transverse displacement. It is assumed that C_0 continuity through the plate thickness is satisfied, thus the degrees of freedom are only translation components in three orthogonal directions. Linearly elastic orthotropic material is used, as well as geometrically linear kinematic relations. In-plane displacement field is interpolated using the *2D* classical Lagrangian interpolations, while the jump discontinuities in the displacement field to represent three modes of delamination are imposed using Heaviside's step functions. The following nodal variables (degrees of freedom) are used: (u,v,w) as mid-plane absolute displacements, (u^I, v^I) as relative displacements in I^{th} node through the thickness and (U^I, V^I, W^I) as displacement jumps in the I^{th} delaminated interface. It is obvious that the proposed model allows arbitrary number of delaminations to be modeled, while the cross sectional warping is also included by using of the layerwise expansion of in-plane displacements. The finite element model is coded in original MATLAB [11] program. For the generation of models and post-processing, GiD Pre/Post Processor developed in CIMNE, Barcelona, is used [12]. Reduced integration is used to avoid shear locking.

3. ABAQUS FE MODELS OF LAMINATED COMPOSITE PLATE

Using the Abaqus CAE, intact and delaminated rectangular composite plates have been modeled. Two types of the 3D (solid) models and a single 2D (shell) model for intact plate have been considered. Behavior of delaminated plate has been analyzed using one solid model and one shell model. The material taken in this analysis is orthotropic and linearly elastic and has thre planes of material symmetry. It is implemented in the model by assigning the nine independent Engineering constants.

3.1. 2D (SHELL) MODELS

Shell model of the intact plate has been composed of one shell-like part with composite shell section. Composite shell section consists of four layers made of the orthotropic material with different orientations. Shell model of the delaminated plate has been composed of two different shells-like parts. Taking that delamination is placed between layers 3 and 4, the upper shell contains one layer, while the lower shell contains three layers which are assigned as composite shell sections (see Figure 4). The perfect bonding between the adjacent surfaces has been achieved using Tie constraint option in Abaqus CAE. Finally, the ofset of the sections are set from the upper surface to the bottom of the lower shell part, and from the bottom surface to the top of the upper shell part.

Finite elements used in this model are **S4R** finite elements. These are 4-node doubly curved quadrilateral elements with reduced integration. Boundary conditions are prescribed along the clamped edges by restraining all degrees of freedom in edge nodes.

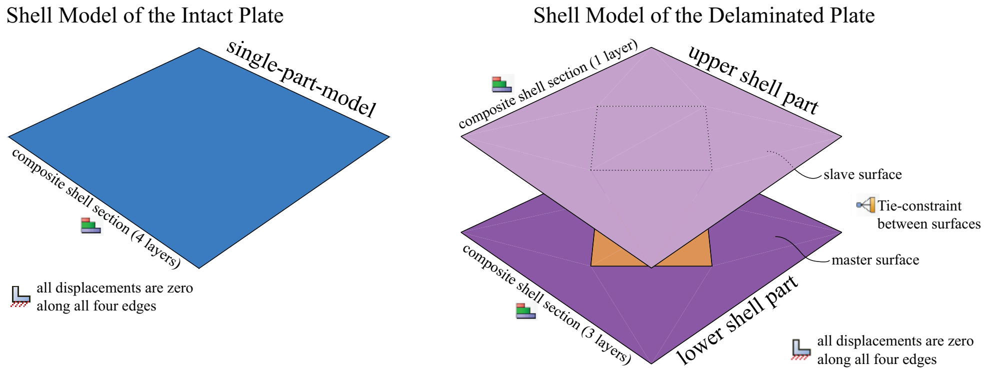

Figure 2. 2D (shell) models of the laminated composite plate in ABAQUS CAE

3.2. 3D (SOLID) MODELS

Regarding the intact plate, the first solid model is composed of four different parts, taking that one part represents a single orthotropic layer. Homogeneous solid section has been assigned to all parts, with the appropriate material orientation. Connection between different layers has been achieved by constraining the adjacent surfaces using Tie constraint option in Abaqus CAE. Tie constraint secures that bonded nodes have the same displacement components (Figure 3). In general case, lower layer represents master surface, while upper layer represents slave surface. The second solid model considered in the analysis of intact plate is composed of a single part. To obtain composite plate behavior, solid composite section has been assigned to the part (Figure 3). This section contains four different layers which have been distributed over part. Regarding the delaminated plate, only the solid model of four different parts is considered. Delamination has been modeled by omitting the Tie constraint between the adjacent surfaces over the delaminated zone.

Finite elements used in solid models are **C3D8R** hexahedral finite elements. These are 3D stress elements with 8 nodes and reduced integration. Boundary conditions are assigned along clamped boundaries of both solid models by restraining all translational degrees of freedom in edge nodes.

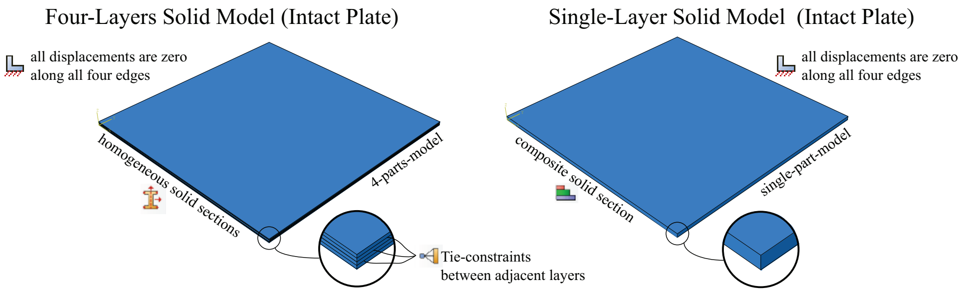

Figure 3. 3D (solid) models of the laminated composite plate in ABAQUS CAE

4. NUMERICAL EXAMPLE AND DISCUSSION

In this benchmark example, the influence of the imposed delamination on the free vibration response of laminated composite plate is investigated numerically. A four layers clamped (CCCC) square plate with a side length $L = 600mm$ and a total thickness $h = 10mm$ ($h/L = 0.0333$) is considered (see Figure 4). All layers are of equal thickness $h = 2.5mm$ and they are composed in the symmetric stacking sequence with the fibers orientations (0/90/90/0) relatively to the global coordinate system xyz. The material parameters for all layers correspond to the carbon/epoxy: $E_1 = 109.34GPa$, $E_2 = E_3 = 8.82GPa$, $G_{12} = G_{13} = 4.32GPa$, G_{23}=3.20GPa, $\nu_{12} = \nu_{13} = 0.342$, $\nu_{23} = 0.520$, $\rho = 1500kg/m^3$. In all calculations, the plate is discretized using three different mesh densities, to test the convergence. The element size varies from 15-30mm. The intact plate models are meshed using the structured meshes of rectangular elements, while the delaminated plate models are discretized using the unstructured meshes of quadrilateral or hexahedral elements. In solid models in Abaqus four elements through the plate thickness (one element per layer) are used.

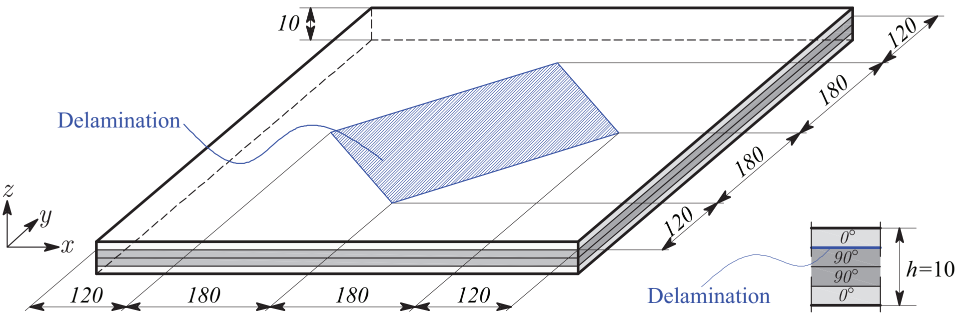

Figure 4. The geometry of the laminated composite plates with embedded delamination

The free vibration response is calculated for the intact plate using all previously described numerical models, as well as for the plate with the embedded delamination between layers 3 and 4, as shown in Figure 4. The results for the intact plate are elaborated in Table 1, while the results for the delaminated plate are provided in Table 2. These results are graphically interpreted in Figures 5-6. Illustrations of the first four mode shapes plotted using the GLPT model in GiD and, for comparison, using the shell model in Abaqus CAE, for the intact and delaminated plates and two mesh densities, are given in Figures 7-9.

Table 1. Natural frequencies of the 4-layers intact laminated composite plate (0/90/90/0) calculated using four different numerical models and three different mesh densities

Mode	GLPT Model			Abaqus CAE Shell Model			Abaqus CAE Solid Model (1 layer)			Abaqus CAE Solid Model (4 layers)		
	30 *mm*	20 *mm*	15 *mm*	30 *mm*	20 *mm*	15 *mm*	30 *mm*	20 *mm*	15 *mm*	30 *mm*	20 *mm*	15 *mm*
1	259.34	258.49	258.19	259.11	258.25	257.96	320.28	274.40	261.97	289.40	258.59	252.71
2	394.57	391.83	390.88	394.10	391.35	390.40	446.80	355.08	333.63	425.17	343.08	326.44
3	642.18	635.89	633.72	640.91	634.64	632.47	633.16	506.10	477.98	617.66	497.71	472.96
4	660.55	649.07	645.15	659.21	647.75	643.84	793.71	702.54	675.20	704.30	658.39	649.54
5	723.48	717.70	715.70	722.10	716.28	714.26	877.19	726.78	694.15	865.55	720.82	690.62
6	915.51	905.28	901.76	913.52	903.16	899.60	947.75	772.47	728.50	875.80	733.00	704.96
7	1051.09	1017.90	1006.76	1047.73	1014.75	1003.68	1171.16	899.69	835.64	1115.03	866.48	815.35
8	1225.59	1202.08	1194.03	1221.41	1198.08	1190.09	1181.66	1013.08	976.05	1172.54	1008.59	973.42
9	1245.09	1217.50	1208.20	1241.33	1213.63	1204.31	1448.16	1091.26	1009.22	1298.86	1064.28	992.51
10	1281.08	1260.21	1253.05	1276.97	1256.12	1248.96	1470.34	1324.86	1253.26	1403.94	1241.66	1229.82
11	1413.62	1393.75	1386.88	1409.37	1389.20	1382.24	1551.68	1349.49	1278.02	1485.14	1310.62	1239.72
12	1567.39	1491.31	1466.20	1560.12	1484.81	1459.95	1633.89	1362.40	1318.97	1544.47	1327.70	1277.12

Table 2. Natural frequencies of the 4-layers delaminated composite plate (0/90/90-del-0) calculated using three different numerical models and three different mesh densities

Mode	GLPT Model			Abaqus CAE Shell Model			Abaqus CAE Solid Model (4 layers)		
	30 *mm*	20 *mm*	15 *mm*	30 *mm*	20 *mm*	15 *mm*	30 *mm*	20 *mm*	15 *mm*
1	239.53	237.21	236.47	240.41	238.38	237.89	278.51	229.23	167.48
2	336.24	329.40	326.53	344.51	336.10	334.00	454.08	339.78	271.90
3	382.50	376.18	374.26	380.60	376.18	375.08	554.07	372.62	319.23
4	450.49	445.61	444.16	452.81	446.78	445.36	758.94	435.90	331.21
5	576.27	539.78	526.49	592.60	548.25	538.10	777.06	602.22	407.26
6	656.51	642.58	638.58	653.44	642.26	639.60	813.86	618.88	466.15
7	669.49	653.77	649.42	661.94	651.61	648.94	1052.34	629.34	502.63
8	763.71	725.33	711.77	785.06	739.46	728.20	1067.50	639.58	518.18
9	846.17	789.97	762.70	854.36	797.32	778.90	1195.42	813.35	591.07
10	883.07	847.91	839.66	906.56	851.67	843.47	1302.11	863.00	636.30
11	924.31	892.13	883.60	917.07	889.38	883.63	1431.75	882.12	685.23
12	1060.51	1014.92	997.77	1038.41	1006.77	995.72	1502.03	892.30	686.96

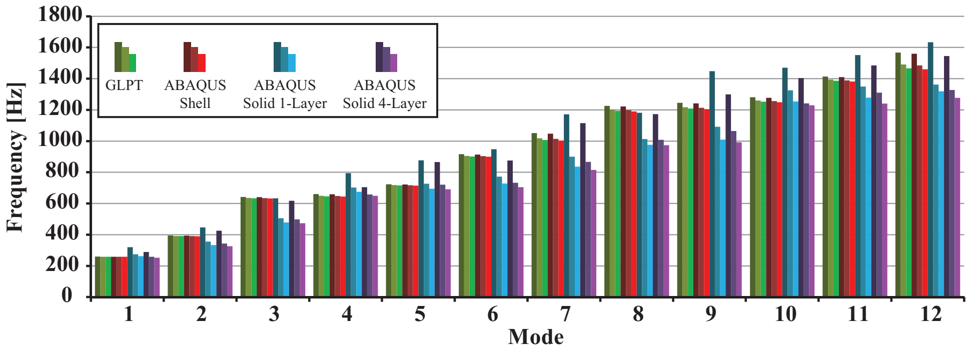

Figure 5. Natural frequencies of the 4-layers intact laminated composite plate (0/90/90/0) calculated using four different numerical models and three different mesh densities

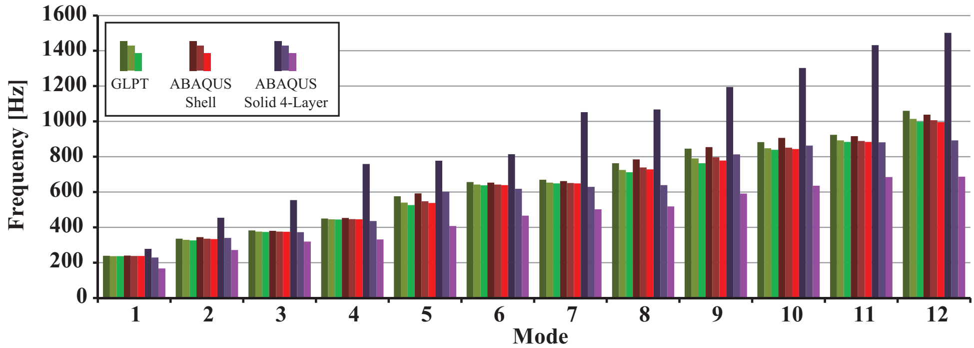

Figure 6. Natural frequencies of the 4-layers delaminated composite plate (0/90/90-del-0) calculated using three different numerical models and three different mesh densities

As shown in Tables 1-2, excellent agreement between results of the GLPT for the intact plate and Abaqus CAE intact shell model has been obtained. It has been excpected, regarding that both models are 2D models, used to interpret three-dimensional behaviour of a composite plate. Results obtained for delaminated plate for these two models are also very close. Only marginal deviation in the frequency can be noticed, especially for higher modes of vibration. Both models show similar covergence trend, which is illustrated in Figures 5-6 using several groups of bars in different color. Difference between lower natural frequencies differs negligible by changing the size of the finite elements, while that difference is slightly higher for higher modes of vibration. However, the difference between results obtained using the GLPT model and Abaqus CAE solid models is obvious, both for intact and delaminated plate models. Solid models manifest huge dependence on the size of the finite element mesh. It has been noticed that by increasing number of finite elements in solid models, local nature of eigen modes dominates the global behaviour. If one solid model ought to be chosen to model composite plate, that would be solid model containing four parts. Its behaviour is closer to the GLPT model, but still significantly different. Regarding the thickness of the analyzed plate, it is more natural to model this plate using shell elements. This is the reason, why the results between GLPT model and solid models disagree. The other reason is that solid model implies the 3D stress state, while all considered shell models assume the plane stress state which involve the inextensibility of the transverse normal during the deformation.

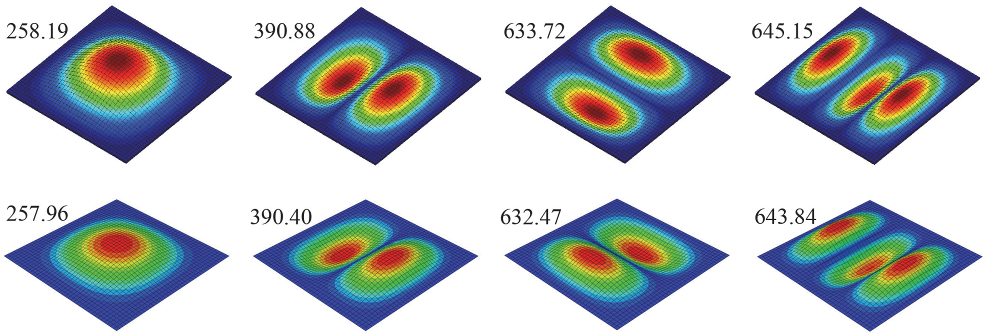

Figure 7. First four natural frequencies [Hz] and the corresponding mode shapes of the intact laminated composite plate (upper line – GLPT model in GiD, lower line – intact shell model in Abaqus CAE), for the element size of 15mm

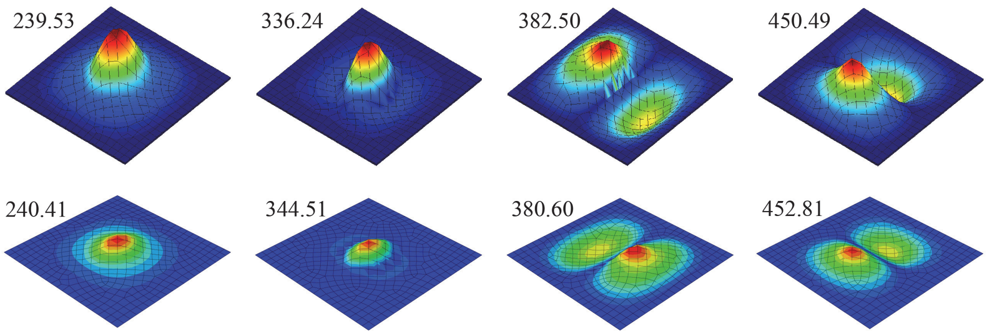

Figure 8. First four natural frequencies [Hz] and the corresponding mode shapes of the delaminated composite plate (upper line – GLPT model in GiD, lower line – delaminated shell model in Abaqus CAE), for the element size of 30mm

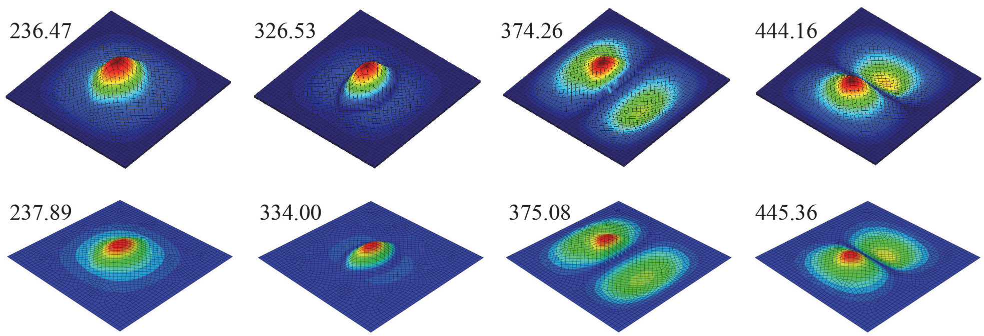

Figure 9. First four natural frequencies [Hz] and the corresponding mode shapes of the delaminated composite plate (upper line – GLPT model in GiD, lower line – delaminated shell model in Abaqus CAE), for the element size of 15mm

5. CONCLUSIONS

Different finite element models (Abaqus CAE) of delaminated composite plates are presented and applied in the free vibration analysis of laminatedcomposite plates. The considered shell model in Abaqus CAE is capable to accurately predict the natural frequencies both for the intact and delaminated plates, while the solid model shown to be less accurate and more sensitive to the mesh refinement, leading to much stiffer response for the coarse mesh and very soft response for fine meshes. The same trend is detected both for intact and delaminated composite plates. The results from Abaqus CAE are validated against the previously derived layered finite element model based on the extended GLPT. Using of reduced integration was necessary for avoiding of shear locking, because of the very low thickness of the layers. All considered models are capable to capture the local mode shape and frequency of the delaminated segment which oscillates independently.

ACKNOWLEDGMENTS

The authors are thankful for the financial support received by Ministry of Education and Science of the Republic of Serbia, through the Projects TR 36048 and TR 36046.

LITERATURE

[1] A. K. Noor: "Free vibrations of multilayered composite plates", AIAA Journal, Vol.11, 1973, p.1038-1039.
[2] D. R. J. Owen, Z. H. Li: "A refined analysis of laminated plates by finite element displacement methods – II. Vibration and stability". Computers & Structures, Vol.26, No.6, 1987, p.915-923.
[3] Dj. Vuksanović: "Linear analysis of laminated composite plates using single layer higher-order discrete models", Composite Structures, Vol.48, 2000, p.205-211.
[4] J. N. Reddy: "Mechanics of laminated composite plates and shells: theory and analysis", CRC Press, 2004.
[5] M. Marjanović, Dj. Vuksanović: "Layerwise solution of free vibrations and buckling of laminated composite and sandwich plates with embedded delaminations", Composite Structures, Vol. 122, 2015, p.67-81.
[6] M. Marjanović, Dj. Vuksanović, G. Meschke: "Geometrically nonlinear transient analysis of delaminated composite and sandwich plates using a layerwise displacement model with contact conditions", Composite Structures, Vol. 108, 2014, p.9-20.
[7] J. N. Reddy: "A plate bending element based on a generalized laminated plate theory", International Journal for Numerical Methods in Engineering, Vol.28, 1989, p.2275-2292.
[8] E. J. Barbero, J. N. Reddy: "Modeling of delamination in composite laminates using a layer-wise plate theory". International Journal of Solids and Structures, Vol.28, No. 3, 1991, p.373-388.
[9] M. Ćetković, Dj. Vuksanović: "Bending, free vibrations and buckling of laminated composite and sandwich plates using a layerwise displacement model", Composite Structures, Vol.88, 2000, p.219-227.
[10] ABAQUS User Manual. Version 6.9. Providence, RI, USA: DS SIMULIA Corp; 2009.
[11] MATLAB, MathWorks Inc. The Language of Technical Computing, MATLAB 2011b, 2011.
[12] Web reference: http://gid.cimne.upc.es