Nonlinear robust integral sliding super-twisting sliding mode control application in autonomous underwater glider

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The design of a robust controller is a challenging task due to nonlinear behaviour of the glider and surround environment. This paper presents design and simulation of nonlinear robust integral super-twisting sliding mode control for controlling the longitudinal plane of an autonomous underwater glider (AUG). The controller is designed for trajectory tracking problem in existence of external disturbance and parameter variations for pitching angle and net buoyancy of the longitudinal plane of an AUG. The algorithm is designed based on integral sliding mode control and super-twisting sliding mode control. The performance of the proposed controller is compared to original integral sliding mode and original super-twisting algorithm. The simulation results have shown that the proposed controller demonstrates satisfactory performance and also reduces the chattering effect and control effort.

[Keywords: Autonomous underwater glider (AUG); Integral sliding mode control; Super-twisting sliding mode control; Chattering reduction]

Introduction

The autonomous underwater glider (AUG) was first initiated by Henry Stommel in 19891. The Stommel’s idea inspired many other researchers to involve in this area of research. Finally in 2001, three operational AUGs were developed and tested: Slocum2, Spray3 and Seaglider4, which are currently being used by many agencies and research groups for oceanography data collection5,6. Also many laboratory-scale AUGs were developed for research purposes, such as robotic gliding fish (Michigan State University)7, USM glider8 (University Science Malaysia), FOLAGA9 (Join effort between IMEDEA, ISME and University of Genova, Italy) ALEX10 (Osaka Perfecture University), and ROGUE11 (University of Princetone). The AUG dives under water by actuating its internal sliding mass horizontally, vertically or cylindrically and pumping the ballast back and forth.

The gliders are multi-input-multi-output (MIMO) nonlinear systems. AUG is considered as an under-actuated system, highly nonlinear, time-varying dynamic behaviour in nature, uncertainties in hydrodynamic coefficients, and also disturbances by ocean currents12. Several control techniques have been proposed to control the motion of the AUG. The survey done by Ullah et al.13 reviewed the control strategies ranging from classical control, proportional-integral-derivative (PID), optimal control linear quadratic regulator (LQR) up to intelligent control such as neural network and fuzzy logic. The PID controller proposed14,15 is the most popular used due to its simple architecture and less tuning parameters. The LQR11,15,16,17,18,19 also offers simple architecture, wherein only two tuning parameters need to be varied to achieve the desired performance. Both PID and LQR provide good performance. However, since the model is linearized about the equilibrium point, the performance of the controller is only effective in a small neighbourhood of the equilibrium.

The model predictive control (MPC)20,21 was designed to control the attitude of Slocum glider. The
control architecture was divided into higher-level and lower-level controllers for controlling the internal configuration of the glider and was made the actuator to execute actions for maintaining the imposed internal configurations. The MPC\textsuperscript{22} used in conjunction with the path-following technique for online tuning of the desired vehicle velocity along with the trajectory and thus validated the 3D motion dynamics of the Slocum glider. Yuan Shan and Zheng Yan\textsuperscript{21} designed the MPC using one-layer recurrent neural network to improve the computational problem in MPC to control the longitudinal plane of AUG.

The intelligent control\textsuperscript{23,24,8} do not need a precise mathematical model of the plant; however, it will suffer from high computational time and need high tuning effort to attain a good performance. The sliding mode control (SMC) is another technique used\textsuperscript{25,26}. The boundary layer SMC was proposed\textsuperscript{25,27} for 1 degree of freedom (DOF) and 2 DOF internal movable sliding mass, respectively. The Taylor’s series expansion method is used in obtaining the linearised model of AUG. Hai Yang and Jie Ma\textsuperscript{28,29} proposed the SMC for nonlinear system of longitudinal plane of AUG. The reaching law is designed based on rapid-smooth reaching law. The performance is improved using inverse system method where the output equations are differentiated repeatedly until the input appeared in the equations, then the control law are designed based on that equations\textsuperscript{28,29}. Mat-Noh et al.\textsuperscript{26} proposed SMC to control the pitching and the net buoyancy of the longitudinal plane system. The control law is designed based on super-twisting sliding mode control (STSMC). The standard STSMC composed only discontinuous part, however the control law consists of equivalent and discontinuous parts\textsuperscript{26}. The intelligent technique had been proposed\textsuperscript{23}, wherein the neural network is used to control the horizontal and vertical plane of the AUG. The controller was designed based on the linearized model.

This paper proposes the combination of two types of SMC control strategies, namely, integral sliding mode and super-twisting sliding mode called integral super-twisting sliding mode control. In the basic study of this algorithm\textsuperscript{30}, the controller was designed for nominal system and system with input disturbance and the performance of ISTSMC was compared to integral SMC. However, in this paper, the proposed controller algorithm is designed for nominal system, system with input disturbance and system with parameter variations. The performance of the proposed controller is compared to the performance of STSMC and integral SMC (ISMС). With this study, the robustness of the proposed controller is tested and benchmarked with the performance of the original ISМС and STSMC. This paper discusses the mathematical model of the longitudinal plane of the AUG and the detail derivation of control for the proposed controller, ISМС and STSMC (Table 1).

### Approach and Methods

#### Dynamic model of an AUG

In this paper, the work is based on the work done by Graver, wherein detailed derivation of the motion equation can be found\textsuperscript{11}. Here, only the longitudinal plane is considered. The dynamics of the longitudinal plane is controlled using internal movable sliding mass and the variable ballast mass. The rudder is fixed to stabilize the glider straight motion in longitudinal plane; therefore, the lateral dynamics can be ignored. Hence all the lateral components are equal to zero except for the pitching component. The glider’s reference frame is shown in Figure 1. The notation of the glider is given in Table 2.

A glide path is specified by a desired path angle, $\xi_d$ and desired speed $V_d$.

\[ \dot{\xi} = \theta - \alpha \]  

where $\theta$ = pitching angle, $\alpha$ = angle of attack

\[ V = \sqrt{v_x^2 + v_z^2} \]

The initial coordinates $(x', z')$ such that $x'$ is the position along the desired path and is defined as

\[ \begin{pmatrix} x' \\ z' \end{pmatrix} = \begin{pmatrix} \cos \xi_d & -\sin \xi_d \\ \sin \xi_d & \cos \xi_d \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} \]

$z'$ measures the position of the vehicle in the direction perpendicular to the desired path. The dynamics of the $z'$ is given in Eq. (4)

\[ z' = \sin \xi_d x + \cos \xi_d z \]

In most applications, the internal movable mass of the underwater glider only moves along x-axis\textsuperscript{7} and together with ballast pumping rate will make the glider dive in water column. Therefore, in this paper
Table 1 — Summary of Control Applications In Aug

<table>
<thead>
<tr>
<th>Control Techniques</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
</table>
| PID                | • Simple architecture  
                    • Easy implementation | • Requires linearised model  
                    • Depends on model accuracy  
                    • Not robust to disturbances  
                    • Based on SISO system |
| LQR                | • Is easily solved using numerical method  
                    • Involves only two tuning parameters. Q and R are the design parameters which are positive definite symmetric matrices that measure the control accuracy and expenditure of energy on the control effort. | • Requires linearised model  
                    • Requires discrete model  
                    • Requires higher computational time due to optimisation process of receding horizon.  
                    • Robustness against uncertainties is still questionable |
| Model Predictive Control | • Prediction capability allows solving optimal control problems on line,  
                           • The predicted output and desired reference is minimised over a future horizon | |
| SMC                | • Reduces order compensated dynamics  
                    • Robust to parameter uncertainties, model nonlinearities and external disturbances | • Chattering effect exists in the control signal  
                    • Requires upper bounds of uncertain parameters and disturbances  
                    • Controller is not tested with parameter variation  
                    • Computationally intensive  
                    • Requires large effort on parameter tuning  
                    • Linear model is obtained using linmod function to develop the controller |
| Intelligent        | • Robust to parameter uncertainties, model nonlinearities and external disturbances | |

Table 2 — The notation used for the glider

<table>
<thead>
<tr>
<th>No.</th>
<th>Motion Axis</th>
<th>Linear and angular velocity</th>
<th>Position and orientation</th>
</tr>
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<tbody>
<tr>
<td>1.</td>
<td>Motion in the x-direction (surge)</td>
<td>$v_1 \ (m/s)$</td>
<td>$x$</td>
</tr>
<tr>
<td>2.</td>
<td>Motion in the y-direction (sway)</td>
<td>$v_2 \ (m/s)$</td>
<td>$y$</td>
</tr>
<tr>
<td>3.</td>
<td>Motion in the z-direction (heave)</td>
<td>$v_3 \ (m/s)$</td>
<td>$z$</td>
</tr>
<tr>
<td>4.</td>
<td>Rotation about the x-axis (roll)</td>
<td>$\phi_1 \ (rad/s)$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>5.</td>
<td>Rotation about the y-axis (pitching)</td>
<td>$\phi_2 \ (rad/s)$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>6.</td>
<td>Rotation about the z-axis (yaw)</td>
<td>$\phi_3 \ (rad/s)$</td>
<td>$\psi$</td>
</tr>
</tbody>
</table>

$\dot{\mathbf{x}} = \frac{1}{2} \left( \frac{(m_p + m_1) (m_p + m_3)}{m_p} \right) \left( \frac{(m_p + m_1) Y - m_p m_3 (m_p + m_1) r_1 r_2 \omega_1}{m_p (m_p + m_3) r_2^2 X_1 + m_p (m_p + m_1) r_2 r_3 X_3 - m_p m_2 (m_p + m_3) r_2 r_3 u_1^2} \right)$  
(6)

$\dot{\psi}_1 = \frac{1}{\alpha} \left( -m_p (m_p + m_3) r_2^2 Y + m_p m_2 \left( (m_p + m_3) r_2 r_3 + m_p \right) \left( m_p + m_3 \right) r_2^2 u_1^2 \right)$  
(7)

$\dot{\psi}_3 = \frac{1}{\alpha} \left( m_p \left( J_2 (m_p + m_1) + m_p \left( m_p + m_1 \right) r_3^2 \right) r_1^2 \omega_2 - m_p^2 r_1 r_3 X_1 \right)$  

Fig. 1 — The reference frame of the glider

the 1 degree of freedom (DOF) internal movable mass is considered where the mass moves along the x-axis.

The original motion equations\(^\text{11}\) need to be rewritten with $r_{p3}$ are fixed at one position. The motion equations for 1 DOF internal movable mass are written in Eq. (5-11)

$$\dot{\theta} = \omega_2$$  
(5)
\[ a = I_2(m_p + m_1)(m_p + m_3) + m_p m_3 (m_p + m_1)r_p^2 + m_p m_3 (m_p + m_3)r_p^3 \]  
\[ X_1 = -m_3 v_2 \omega_2 - P_{g2} \omega_2 - m_{em} g \sin \theta + \text{Leina} - \text{Dfosa} \]  
\[ X_3 = m_1 v_2 \omega_2 + P_{g2} \omega_2 + m_{em} g \cos \theta - \text{Leosa} - \text{Dfosa} \]  
\[ Y = (m_{f2} - m_{f1})v_1 v_2 - [r_p^2 P_{f1} + r_p m_p (v_2 - r_p \omega_2)] \]  
\[ \dot{\theta} = [u_1, u_2]^T \]  
\[ x = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]^T = [\theta, \omega_2, v_3, r_p, r_p^2, m_b]^T \]  
\[ y_1 = x_1 \]  
\[ y_2 = m_{em} = m_h + m_p + m_b - m_{df} \]  

Controller design

This section discusses the methodology of the controller design. The proposed controller is a combination of two SMC strategies, namely, ISMC and STSMC. Therefore, here three controllers will be designed, namely, the proposed controller, ISMC and STSMC.

Before designing the controller, the motion equations in Eq. (5-11) are rewritten in the general form of nonlinear equation as given in Eq. (26).

\[ \dot{x} = f_k(x, t) + g_k(x, t)u_k + \varphi_k(x, t) \]  

where, \( x \in R^n \) and \( u \in R^m \) are defined as state and input vectors, \( \varphi_k(x, t) \) represent the bounded matched perturbations, \( k = 1, 2, ..., n \) and \( t = 1, 2, ..., m \)

\[ \varphi_k(x, t) \] is bounded with a known norm upper bound,

\[ |\varphi_k(x, t)| \leq \rho_k(x, t) \]  

where, \( \rho_{kl} \geq 0 \) for \( k = 1, 2, ..., n \).
where \( X_k(x, u, t) = f_k(x, t) + (g_k(x, t) - 1)u_t \) (28)

The equations for the selected output are rewritten in the form of Eq. (28) as given in Eqs. (29-31).

\[
X_1 = X_2 
\]

(29)

\[
X_2 = f_1(x, t) - g_1(x, t)u_1 - g_1(x, t)\delta_1(x, t) 
\]

(30)

\[
X_7 = u_2 + \delta_2(x, t) 
\]

(31)

where \( \delta_1 \) and \( \delta_2 \) are the external disturbances. The controllers are designed for the tracking problems. The errors of the selected outputs are defined in Eqs. (32) and (33).

\[
\begin{align*}
\varepsilon_1 &= X_1 - X_{1i} \\
\varepsilon_2 &= X_7 - X_{7i}
\end{align*}
\]

(32)

\[
\begin{align*}
\varepsilon_1 &= X_1 - X_{1i} \\
\varepsilon_2 &= X_7 - X_{7i}
\end{align*}
\]

(33)

\[ A. \quad \text{Integral Sliding Mode Control (ISMC):} \]

The integral sliding mode control (ISMC) was proposed by Utkin and Shi in 1996\(^{31} \). The advantage of ISMC is that the sliding surface is enforced from the beginning and thus eliminates the reaching phase which is also called ‘no reaching phase’ SMC. The control law is defined as

\[
\begin{align*}
\dot{u}_1 &= u_{1i} + \eta_1 \\
\dot{u}_2 &= u_{2i} + \eta_2
\end{align*}
\]

(34)

The system without perturbation can be designed using any method such as PID, pole-placement, LQR, MPC, etc. and \( u_{1i} \) is nonlinear (discontinuous) control design to reject the perturbations, \( \delta_1(x, t) \).

The sliding manifold is defined as

\[
S = S_0 + z 
\]

(35)

where \( S_0 \) is the conventional sliding surface, and \( z \) is the integral term which can be determined.

The underwater glider is controlled by two control inputs. Thus, two control laws and two sliding surfaces are designed. The ISMC control law and sliding surface are given in Eqs. (36) and (37), respectively.

\[
\begin{align*}
\dot{u}_1 &= u_{110} + u_{111} \\
\dot{u}_2 &= u_{210} + u_{211}
\end{align*}
\]

(36)

\[
S_{12} = S_{110} + S_{111} \\
S_{22} = S_{210} + S_{211}
\]

(37)

The ideal controls, \( u_{1i0} \) and \( u_{2i0} \) are defined based on a linear control and those are written in Eqs (38) and (39) as

\[
\begin{align*}
\dot{u}_{1i} &= -(u_{111} - u_{110}) - g_1(x, t) \\
\dot{u}_{2i} &= -(u_{211} - u_{210})
\end{align*}
\]

(38)

(39)

The conventional sliding surfaces \( S_{1i0} \) and \( S_{2i0} \) are defined as

\[
\begin{align*}
S_{110} &= \varepsilon_1 + \tilde{\varepsilon}_1 \\
S_{210} &= \varepsilon_2
\end{align*}
\]

(40)

Differentiating Eqs (37) and (40) with respect to time

\[
\begin{align*}
\dot{\varepsilon}_1 &= \dot{\varepsilon}_{1i0} + \dot{\varepsilon}_1 \dot{\varepsilon}_1 + \dot{\varepsilon}_1 \tilde{\varepsilon}_1 = c_1 \tilde{\varepsilon}_1 + \chi_1(x, u, t) \\
\dot{\varepsilon}_2 &= \dot{\varepsilon}_{2i0} + \dot{\varepsilon}_2 \varepsilon_1 + \dot{\varepsilon}_2 \varepsilon_2 + \varepsilon_2
\end{align*}
\]

(41)

(42)

where \( \dot{\varepsilon}_1 \) and \( \dot{\varepsilon}_2 \) are chosen as

\[
\begin{align*}
\dot{\varepsilon}_1 &= u_{1i0} - c_1 \varepsilon_1 \\
\dot{\varepsilon}_2 &= -u_{2i0}
\end{align*}
\]

(43)

(44)

Substitute \( \dot{\varepsilon}_1 \) into Eqs. (41) and (42), then \( \dot{\varepsilon}_1 \) and \( \dot{\varepsilon}_2 \) reduces to Eqs. (45) and (46)

\[
\begin{align*}
\dot{\varepsilon}_1 &= \chi_1(x, u, t) - u_{111} + g_1 \delta_1(x, t) \\
\dot{\varepsilon}_2 &= u_{211} + \delta_2(x, t)
\end{align*}
\]

(45)

(46)

The equivalent controls \( u_{1i1e} \) and \( u_{2i1e} \) are determined as \( \dot{\varepsilon}_1 = 0 \) and \( \dot{\varepsilon}_2 = 0 \) are written in Eqs. (47) and (48)

\[
\begin{align*}
\dot{u}_{1i1e} &= \frac{1}{g_1} \left[ \varepsilon_1 + (-g_1 + 1)(u_{110}) + g_1 \delta_1(x, t) \right] \\
\dot{u}_{2i1e} &= -\delta_2(x, t)
\end{align*}
\]

(47)

(48)

and the reachability conditions are chosen as
\[ u_{11_{\text{dis}}} = -M_1 \text{sign}(s_1) \]  

(49) 

\[ u_{21_{\text{dis}}} = -M_2 \text{sign}(s_2) \]  

(50) 

then the nonlinear controls, \( u_{11} \) and \( u_{21} \) are written as 

\[ u_{11} = u_{11_{eq}} + u_{11_{\text{dis}}} = \frac{1}{g_1} \{ f_1 + (-g_1 + 1) \} (u_{10}) + g_1 \delta_1(x,t) - M_1 \text{sign}(s_1) \]  

(51) 

\[ u_{21} = -\delta_2(x,t) - M_2 \text{sign}(s_2) \]  

(52) 

The ISMC control law to track the pitching angle and net buoyancy are given in Eqs. (53) and (54) 

\[ u_1 = k_1 e_1 + k_{21} e_2 + \frac{1}{g_1} \{ f_1 + (-g_1 + 1) \} (u_{10}) + g_1 \delta_1(x,t) - M_1 \text{sign}(s_1) \]  

(53) 

\[ u_2 = -k_{21} e_2 - \delta_2(x,t) - M_2 \text{sign}(s_2) \]  

(54) 

C. Super-Twisting Sliding Mode Control (STSMC): The super twisting SMC (STSMC) was introduced by Levant in 1993\(^{12}\). It is a viable alternative to the conventional first order sliding mode control for the systems with relative degree-one. The trajectories of the super-twisting are characterized by twisting around the origin of the phase portrait of the sliding variable as shown in Figure 2. 

This algorithm is also known as model free SMC because it contains only the discontinuous control part and the control law is free from the system parameters. However, here the STSMC is designed using conventional SMC, from where the equivalent control law is derived. Then the discontinuous control law is designed using super-twisting algorithm. Therefore, the control law for tracking the pitching angle and the net buoyancy are written in Eqs. (55) and (56) respectively 

\[ u_1 = u_{1_{eq}} + u_{1_{\text{STSMC}}} \]  

(55) 

\[ u_2 = u_{2_{eq}} + u_{2_{\text{STSMC}}} \]  

(56) 

The sliding surfaces and their derivatives are defined for tracking pitching angle and the net buoyancy as written in Eqs. (57), (58), (59) and (60) respectively.

\[ s_1 = c_1 e_1 + \dot{e}_1 \]  

(57) 

\[ s_2 = \dot{e}_2 \]  

(58) 

and 

\[ \dot{s}_1 = c_1 \dot{e}_1 + \ddot{e}_1 \]  

(59) 

\[ \dot{s}_2 = \ddot{e}_2 \]  

(60) 

The equivalent control laws are defined as \( \dot{s}_1 = 0 \) and \( \dot{s}_2 = 0 \) 

\[ u_{1_{eq}} = \frac{1}{g_1} \{ f_1 + g d_1 + c_1 \dot{e}_1 - \dddot{x}_{1d} \} \]  

(61) 

\[ u_{2_{eq}} = -\delta_2(x,t) \]  

(62) 

The reachability conditions are chosen as super-twisting SMC given as 

\[ u_{1_{\text{dis}}} = -\beta_{11} \{ s_1 \text{sign}(s_1) - \beta_{12} \int_0^{t} \text{sign}(s_1) dt \} \]  

(63) 

\[ u_{2_{\text{dis}}} = -\beta_{21} \{ s_2 \text{sign}(s_2) - \beta_{22} \int_0^{t} \text{sign}(s_2) dt \} \]  

(64) 

Finally the control laws are written in Eqs. (65) and (66). 

\[ u_1 = u_{1_{eq}} + u_{1_{\text{dis}}} = \frac{1}{g_1} \{ f_1 + g d_1 + c_1 \dot{e}_1 - \dddot{x}_{1d} \} - \beta_{11} \{ s_1 \text{sign}(s_1) - \beta_{12} \int_0^{t} \text{sign}(s_1) dt \} \]  

(65) 

\[ u_2 = u_{2_{eq}} + u_{2_{\text{dis}}} = -\delta_2(x,t) - \beta_{21} \{ s_2 \text{sign}(s_2) - \beta_{22} \int_0^{t} \text{sign}(s_2) dt \} \]  

(66)
D. **Integral Super-Twisting SMC (ISTSMC):** The proposed control law of ISTSMC is defined based on ISMC control as written in Eqs. (67) and (68).

\[ U_1 = u_{10} - u_{11} \]  \hspace{1cm} (67)

\[ U_2 = u_{20} + u_{21} \]  \hspace{1cm} (68)

The linear control laws were previously defined in Eqs. (38) and (39); the sliding surfaces, and equivalent controls \( \left( u_{11eq}, u_{21eq} \right) \) are similar to the one defined in Eqs. (37), (47) and (48). However, the discontinuous controls \( \left( u_{11dis}, u_{21dis} \right) \) are defined based on the super-twisting SMC as in Eqs. (63) and (64). Therefore, the nonlinear control laws for ISTSMC are written in Eqs. (69) and (70).

\[ u_{11} = \frac{1}{g_1} \left\{ f_1 + (-g_1 + 1)(u_{10}) + g_1 \delta_1(x,t) \right\} - \beta_{11} |s_1|^p \text{sign}(s_1) - \beta_{12} \int_0^t \text{sign}(s_1) \, dt \]  \hspace{1cm} (69)

\[ u_{21} = -\delta_2(x,t) - \beta_{21} |s_2|^p \text{sign}(s_2) - \beta_{22} \int_0^t \text{sign}(s_2) \, dt \]  \hspace{1cm} (70)

Finally, the ISTSMC control laws are defined in Eqs. (71) and (72).

\[ U_1 = h_{11} s_1 + h_{12} s_2 + \frac{1}{g_2} \left\{ f_2 + (-g_2 + 1)(u_{10}) + g_2 \delta_2(x,t) \right\} - \beta_{11} |s_1|^p \text{sign}(s_1) - \beta_{12} \int_0^t \text{sign}(s_1) \, dt \]  \hspace{1cm} (71)

\[ U_2 = -h_{21} s_2 - \delta_2(x,t) - \beta_{21} |s_2|^p \text{sign}(s_2) - \beta_{22} \int_0^t \text{sign}(s_2) \, dt \]  \hspace{1cm} (72)

The proposed control laws in Eqs. (71) and (72) gains from the advantage of no reaching phase of ISMC algorithm and the sliding surfaces convergence in finite time as the main advantage of STSMC.

F. **Stability Analysis:** The stability analysis of the proposed controller algorithm ISTSMC is discussed here. To ensure the convergence of the controlled parameters of the plant stabilized at the desired value, the sliding mode must be first ensured. Therefore the analysis of stability is made to ensure sliding mode and the output convergence. This is done using lyapunov stability theorem.

**Theorem 1:** Consider the nonlinear system in Eq. (28) subjected to bounded uncertainty in Eq. (27) with assumptions, the system is proper \( (m = p) \) and minimum phase where the zero dynamic of the system is asymptotically stable. If the sliding manifolds \( (s_j) \) as written in Eq. (37) the dynamics of integral terms \( (z_i) \) as written in Eqs. (43) and (44) and the discontinuous controls \( (u_{1i}) \) as written in Eqs. (63) and (64), then the convergence conditions are satisfied.

**Proof:** Let us consider the lyapunov functions and their time derivatives in Eqs. (73), (74), (75) and (76), respectively.

\[ V_1(s_1) = \frac{1}{2} s_1^2 \]  \hspace{1cm} (73)

\[ V_2(s_2) = \frac{1}{2} s_2^2 \]  \hspace{1cm} (74)

\[ \bar{V}_1(s_1) = s_1 \left( \gamma_1(x,u_1,t) - u_{11} + g_1 \delta_1(x,t) \right) \]  \hspace{1cm} (75)

\[ \bar{V}_2(s_2) = s_2 \left( \gamma_2(x,u_1,t) \right) \]  \hspace{1cm} (76)

Substitute the Eq. (37) along with Eqs. (43) and (44) into Eqs. (75) and (76), to produce Eqs. (77) and (78).

\[ \bar{V}_1(s_1) = s_1 \left( \gamma_1(x,u_1,t) - u_{11} + g_1 \delta_1(x,t) \right) \]  \hspace{1cm} (77)

\[ \bar{V}_2(s_2) = s_2 \left( \gamma_2(x,u_1,t) \right) \]  \hspace{1cm} (78)

Now substitute the Eq. (69) into Eq. (77), and Eq. (70) into Eq. (78) which gives

\[ \bar{V}_1(s_1) = s_1 \left[ -\beta_{11} s_1 \tilde{E} \text{sign}(s_1) - \beta_{12} \int_0^t \text{sign}(s_1) \, dt \right] \]  \hspace{1cm} (79)

\[ \bar{V}_2(s_2) = s_2 \left[ -\beta_{21} s_2 \tilde{E} \text{sign}(s_2) - \beta_{22} \int_0^t \text{sign}(s_2) \, dt \right] \]  \hspace{1cm} (80)

The following sufficient conditions for finite time convergence must be satisfied\(^{33,34}\):

\[ \beta_{11} \geq \frac{4pE_{min}(\tilde{E})}{E_{max}(\tilde{E} - c_0)} \]  \hspace{1cm} (81)

\[ \beta_{21} \geq \frac{E_{min}(\tilde{E})}{E_{max}(\tilde{E} - c_0)} \]  \hspace{1cm} (82)

\[ 0 < p \leq 0.5 \]  \hspace{1cm} (83)

**Results and Discussion**

Here, the performance of the proposed controller ISTSMC is compared to the performance of the ISMC and STSMC. All the designed controllers are
evaluated for the nominal system, system with input disturbance and system with parameter variations. The controllers were evaluated using the parameters adopted from Graver’s work\(^5\). All the parameters are depicted in Table 3.

The controllers were simulated for the glide angle switching from 25° downward to 25° upward. The responses of the nominal system are shown in Figures 3 to 8.

All the controllers are stabilized in the vicinity of the desired values. From Figures 3 and 4, ISTSMC shows the fastest tracking at about \(t=10\) s for pitching angle and less than 1 s for net buoyancy. ISMC gives the highest tracking about 15 s for pitching angle and about more than 1 second for net buoyancy. The ISMC provides the largest control efforts while the ISTSMC provides the smallest efforts. The chattering exhibit in control input, \(u_1\). However, the responses are in downward trend. The ISTSMC provides the smallest chattering in both control inputs.

Here, the water current is considered as input matched external disturbance. The input matched external disturbance of \(\delta_1(x, t) = 5x_1\sin(\pi t)\) and \(\delta_2(x, t) = 0.1x_7\sin(\pi t)\) were induced to input channels 1 and 2, respectively. The simulation results for disturbance case are shown in Figures 9-14.

All the controllers are stabilized in the vicinity of the desired values. The STSMC shows the highest overshoot and the ISMC provide the largest oscillation. The ISMC shows the largest control effort in \(u_1\) and smallest effort in \(u_2\). The ISTSMC provides slightly smaller efforts than STSMC in both control inputs. The sliding surfaces of all controllers are stabilized in the vicinity of origin with ISTSMC providing the smallest errors. The ISTSMC improved the steady state error for more than 100% as compared to ISMC and STSMC.

The parameter variations are applied to the system time \(t=25\) s. The parameters were increased by 30% of the original values. The increment of parameters is

Table 3 — Parameter values of the glider

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hull mass, (m_h)</td>
<td>40</td>
<td>kg</td>
</tr>
<tr>
<td>Internal sliding mass, (m_p)</td>
<td>9</td>
<td>kg</td>
</tr>
<tr>
<td>Displaced fluid mass, (m_{af})</td>
<td>50</td>
<td>kg</td>
</tr>
<tr>
<td>Added mass, (m_{f1}, m_{f2}, m_{f3})</td>
<td>50, 60, 70</td>
<td>Kg(\cdot)m(^2)</td>
</tr>
<tr>
<td>Inertia, (J_1, J_2, J_3)</td>
<td>4, 12, 11</td>
<td>-</td>
</tr>
<tr>
<td>Lift coefficient, (K_{L1}, K_{L2})</td>
<td>0, 132.5</td>
<td>-</td>
</tr>
<tr>
<td>Drag coefficient, (K_{D1}, K_{D2})</td>
<td>2.15, 25</td>
<td>-</td>
</tr>
<tr>
<td>Moment coefficient, (K_{MO}, K_{M2})</td>
<td>0, -100</td>
<td>-</td>
</tr>
<tr>
<td>Constant coefficient, (K_{01}, K_{02})</td>
<td>50, 50</td>
<td>-</td>
</tr>
</tbody>
</table>

![Fig. 3 — Pitching angle \(\theta\) (without disturbance)](image)

![Fig. 4 — Net buoyancy \(m_{em}\) (without disturbance)](image)

![Fig. 5 — Control inputs \(u_1\) (without disturbance)](image)
depicted in Table 4. The simulation results are shown in Figures 15-20.

The simulation results are shown in Figures 15-20. All the controllers are stabilized at the vicinity of the desired values even after 30% increment in parameter values. The pitching angle of ISMC shows the highest deviation, while ISTSMC shows the smallest deviation at time $t=25$. However, increment does not
affect the net buoyancy of all controllers since the incremented parameters did not appear in the equation.

Table 4 — Increment of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal</th>
<th>Increased (30%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{f1}$</td>
<td>5</td>
<td>6.50</td>
</tr>
<tr>
<td>$m_{f3}$</td>
<td>70</td>
<td>91</td>
</tr>
<tr>
<td>$J_2$</td>
<td>12</td>
<td>15.60</td>
</tr>
<tr>
<td>$K_{L0}$</td>
<td>0</td>
<td>0.30</td>
</tr>
<tr>
<td>$K_L$</td>
<td>132.5</td>
<td>172.25</td>
</tr>
<tr>
<td>$K_{DO}$</td>
<td>2.15</td>
<td>2.80</td>
</tr>
<tr>
<td>$K_D$</td>
<td>25</td>
<td>32.50</td>
</tr>
<tr>
<td>$K_{MO}$</td>
<td>0</td>
<td>0.30</td>
</tr>
<tr>
<td>$K_M$</td>
<td>-100</td>
<td>-130</td>
</tr>
<tr>
<td>$K_{a3}, K_{a2}$</td>
<td>50, 50</td>
<td>65, 65</td>
</tr>
</tbody>
</table>

Fig. 12 — Control inputs $u_2$ (with disturbance)

Fig. 13 — Sliding surfaces $s_1$ (with disturbance)

Fig. 14 — Sliding surfaces $s_2$ (with disturbance)

Fig. 15 — Pitching angle $\Theta$ (parameter variation)

Fig. 16 — Net buoyancy $m_{em}$ (parameter variation)

for the net buoyancy. The sliding surfaces of all controllers are stabilized in the vicinity of origin with ISTSMC providing the smallest error and ISMC showing the largest. With the combination the ISMC and STSMC, the proposed controller has the advantages of both controllers. The controller gains of the proposed controller for all cases are summarised in Table 5.
Fig. 17 — Control inputs $u_1$ (parameter variation)

Fig. 18 — Control inputs $u_2$ (parameter variation)

Fig. 19 — Sliding surfaces $s_1$ (parameter variation)

Fig. 20 — Sliding surfaces $s_2$ (parameter variation)

<table>
<thead>
<tr>
<th>Table 5 — The ISTSMC controller gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal System</td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>$m_{cem}$</td>
</tr>
<tr>
<td>With Disturbance</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>$m_{cem}$</td>
</tr>
<tr>
<td>With Parameter Variations</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>$m_{cem}$</td>
</tr>
</tbody>
</table>

**Conclusion**

In this paper, the nonlinear robust ISTSMC is designed and proposed for robust tracking and robust rejection against disturbance and parameter variations of an autonomous underwater glider. The simulation results have shown that the proposed controller provides a good performance under existence of disturbance and parameter variations and also the chattering is reduced. Simulation results have also shown that the ISTSMC can be further improved by introducing optimization method for tuning parameters of the controller which will directly improve the controller performance. This will become the next research work in future.

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References