Ergodicity of pumping tests

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[1] Standard interpretations of pumping tests in heterogeneous formations rely on effective representations of porous media, which replace spatially varying hydraulic properties with their constant counterparts averaged over the support volume of a test. Rigorous approaches for deriving representative (effective, apparent, upscaled, etc.) parameters employ either ensemble or spatial averaging. We derive a set of conditions under which these two paradigms yield identical results. We refer to them as conditions for the ergodicity of pumping tests. This allows one to use stochastic approaches to estimate the statistics of the spatial variability of hydraulic parameters on scales smaller than the support volume of a pumping test.

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1. Introduction

[2] Pumping tests are routinely used to infer hydraulic parameters on the field scale. While most pumping tests are conducted in heterogeneous aquifers, their standard interpretations (e.g., Theis' solution) assume homogeneity. This apparent dichotomy can be overcome by treating the region of a heterogeneous aquifer affected by the pumping test as its support volume, thus ignoring all spatial variability on smaller scales. Such an approach is conceptually analogous to Darcy's experiments that assign a constant hydraulic conductivity to (heterogeneous) soil samples.

[3] While routinely used in practice, these approaches are often insufficient when one is interested in the so-called subgrid variability of hydraulic parameters, i.e., in their variability on scales smaller than the support volume of a pumping test. Recently, *Copty and Findikakis* [2004] and *Neuman et al.* [2004] have proposed alternative methods for extracting statistical information about the subgrid variability of hydraulic conductivity (e.g., its geometric mean, variance, and correlation length) from pumping tests. Both approaches rely on a stochastic representation of hydraulic conductivity at the point support volume, and result in a set of type curves for statistical moments (mean and variance) of hydraulic head. Statistical properties of the subgrid variability of hydraulic conductivity are obtained by superimposing hydraulic head measurements on the type curves.

[4] A conceptual difficulty inherent in these and other similar approaches lies in the fact that pumping test data are obtained from an aquifer that is treated as a single realization, while the type curves represent averages over the ensemble of many realizations. This is justified if the spacing between the pumping and observation wells is sufficiently large, so that the signal samples (travels through) all the relevant heterogeneities of the medium. Such a line of reasoning explicitly invokes the concept of ergodicity, which postulates the equivalence between the ensemble and spatial averages. Our main goals are to establish conditions under which these two averages are equivalent in the context of convergent flow induced by groundwater pumping, and to analyze whether these conditions might be met in real systems.

[5] An extensive body of literature dealing with representative parameters for convergent flow in heterogeneous aquifers (see, e.g., a recent review by *Sanchez-Vila et al.* [2006]) can be divided into two groups. This classification is broadly based on a mathematical framework (stochastic or deterministic) that is used to derive representative parameters. The definitions of, and the relationship between, effective, equivalent, apparent, and interpreted hydraulic parameters are given by *Sanchez-Vila et al.* [2006].

[6] Stochastic derivations of representative parameters for convergent flow date back to the pioneering work of *Shvidler* [1962] and *Matheron* [1967]. These authors demonstrated that the ensemble averaged Darcy's law is nonlocal, i.e., that the mean Darcy's flux $\langle \mathbf{q} \rangle$ at a point $\mathbf{r}$ depends on the values of the mean hydraulic head gradient $\nabla \langle h \rangle$ at points other than $\mathbf{r}$. This led *Shvidler* [1962] to conclude that the effective transmissivity T_{eff}, defined as the negative ratio between the ensemble averages of flux and head gradient, does not generally exist. In other words, it is generally impossible to represent the mean Darcy's law in a familiar form $\langle \mathbf{q}(\mathbf{r}) \rangle = -T_{\text{eff}} \nabla \langle h(\mathbf{r}) \rangle$ without a localization approximation. Numerous subsequent studies [e.g., *Ababou and Wood*, 1990; *Indelman et al.*, 1996; *Sanchez-Vila*, 1997; *Noetinger and Gautier*, 1998; *Riva et al.*, 2001; *Indelman*, 2001, 2003; *Guadagnini et al.*, 2003] provided either nonlocal or localized expressions for pseudoeffective (also called apparent by different authors) hydraulic conductivity or transmissivity, defined as the (variable in space) tensor that relates the expected values of flux and head gradient. When localized, this property depends on the distance to the pumping well, and is influenced by boundaries.

[7] Deterministic analyses are based on spatial averaging and date back to the study by *Cardwell and Parsons* [1945],

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who considered convergent flow in a heterogeneous aquifer and derived general bounds for the equivalent conductivity T_{eq}. The latter is defined as a constant that is assigned to the heterogeneous aquifer's region affected by pumping in a way that preserves the pumping rate and the drawdowns in the pumping and observation wells. As such, T_{eq} is a global parameter which characterizes the ability of the medium to convey fluid and, in fact, is a quantity that is inferred from (steady state or pseudosteady state) pumping tests in heterogeneous aquifers. *Desbarats* [1992, 1994] and *Durlofsky* [2000] used empirical and numerical approaches to obtain equivalent hydraulic parameters. *Sanchez-Vila et al.* [1999a] used a perturbation analysis to derive an approximate analytical expression for equivalent transmissivity under steady state convergent flow conditions. Other approaches to spatial averaging represent the subgrid variability either by a set of inclusions of given shape embedded into an otherwise homogeneous aquifer or by a realization of the random transmissivity field with given statistics (see the recent review by *Raghavan* [2004, and references therein]).

[8] Most of the available studies focus on the vicinity of a well, where flow is singular, often affected by the skin effect and, as discussed above, clearly nonergodic. We are concerned with the derivation of representative transmissivity for large support volumes defined by the distance between the pumping and observation wells. The problem is formulated in section 2 and is analyzed both deterministically and stochastically in sections 3 and 4, respectively. The correspondence between the two solutions provides ergodicity conditions, which are established in section 5, and discussed in detail in section 6.

2. Problem Formulation

[9] Consider a steady state pumping test conducted with a constant flow rate Q in a heterogeneous confined aquifer. For a fully penetrating well of negligible radius that is located at the coordinate origin $\mathbf{r} = 0$, flow in the aquifer is described by two-dimensional Darcy's law and mass conservation,

$$\mathbf{q} = -T\nabla h, \qquad -\nabla \cdot \mathbf{q} + Q\delta(\mathbf{r}) = 0, \tag{1}$$

where $\mathbf{q}(\mathbf{r})$ is the Darcian flux, $T(\mathbf{r})$ is the transmissivity, $h(\mathbf{r})$ is the hydraulic head (or drawdown), and $\delta(\mathbf{r})$ is the Dirac delta function. This equation is subject to the usual boundary condition at $\mathbf{r} = 0$, and the condition that drawdown is negligible ($h = 0$) at a distance sufficiently far away from both the well and the observation points ($\mathbf{r} \to \infty$).

[10] Let Ω denote the support volume of a pumping test corresponding to the distance between the pumping and the observation wells. The boundary $\partial\Omega$ of the support volume Ω is an (unknown) equipotential line $h = h_o$, where h_o is the hydraulic head in the observation well (for homogeneous aquifers, Ω is a circle of radius R, whose area is $\|\Omega\| = \pi R^2$). The spatial average of a function $\mathcal{A}(\mathbf{r})$ over the support volume Ω is defined as

$$\overline{\mathcal{A}} = \frac{1}{\|\Omega\|}\int_\Omega \mathcal{A}(\mathbf{r})d\mathbf{r}, \tag{2}$$

where $d\mathbf{r} = r dr d\theta$ is the expression of the differential in polar coordinates. A deterministic version of Reynolds' decomposition allows one to represent $\mathcal{A}(\mathbf{r})$ as the sum $\mathcal{A}(\mathbf{r}) = \overline{\mathcal{A}} + \mathcal{A}'(\mathbf{r})$ of its spatial average $\overline{\mathcal{A}}$ and fluctuations $\mathcal{A}'(\mathbf{r})$. By definition, the spatial average of the fluctuations is $\overline{\mathcal{A}(\mathbf{r})'} \equiv 0$. Then we can write (1) as

$$\nabla^2 h + \nabla \cdot (\epsilon\nabla h) + \frac{Q}{\overline{T}}\delta(\mathbf{r}) = 0, \tag{3}$$

where $\epsilon(\mathbf{r}) = T'(\mathbf{r})/\overline{T}$ represents the variability of the transmissivity T at some predefined length scale. The definition of ϵ implies that

$$\overline{\epsilon(\mathbf{r})} \equiv 0. \tag{4}$$

The presence of the second term in (3) makes it obvious that the spatial average of transmissivity $\overline{T}$ does not directly represent an equivalent transmissivity T_{eq}. Approximate expressions for T_{eq} are derived in section 3.

3. Spatial Averaging

[11] Let us introduce a Green's function $G(\mathbf{r}, \mathbf{r}_1)$ as a solution of the following deterministic problem

$$\nabla^2_{\mathbf{r}} G + \delta(\mathbf{r} - \mathbf{r}_1) = 0, \qquad \mathbf{r}, \mathbf{r}_1 \in \Omega \tag{5}$$

subject to the homogeneous boundary condition $G(\mathbf{r} \in \partial\Omega, \mathbf{r}_1) = 0$. Then (3) can be recast as an integrodifferential equation

$$h(\mathbf{r}) = h_o - \int_\Omega \epsilon(\mathbf{r_1})\nabla_{\mathbf{r_1}} h \cdot \nabla_{\mathbf{r_1}} G(\mathbf{r}, \mathbf{r_1})d\mathbf{r_1} + \frac{Q}{\overline{T}}\mathrm{G}(\mathbf{r}, \mathbf{0}). \tag{6}$$

Note that if $\epsilon(\mathbf{r}) = 0$, (6) represents the hydraulic head response to pumping at $\mathbf{r} = \mathbf{0}$ in a homogeneous aquifer with transmissivity $\overline{T}$.

[12] Since $\mathbf{q} = -T\nabla h = -\overline{T}\,(1 + \epsilon)\nabla h$, it follows from (6) that

$$\begin{aligned}\frac{\mathbf{q}}{\overline{T}} = &-\nabla h + \int_\Omega \epsilon(\mathbf{r})\epsilon(\mathbf{r_1})\nabla_{\mathbf{r}}\nabla^T_{\mathbf{r_1}} G(\mathbf{r}, \mathbf{r_1})\nabla_{\mathbf{r_1}} h d\mathbf{r}_1 \\ &- \frac{Q}{\overline{T}}\epsilon(\mathbf{r})\nabla_{\mathbf{r}} G(\mathbf{r}, \mathbf{0}).\end{aligned} \tag{7}$$

We define two new variables that correspond to the weighted averages of hydraulic head gradient $\mathbf{J}_{av}$ and flux $\mathbf{q}_{av}$,

$$\mathbf{J}_{av} = \frac{1}{\|\Omega\|}\int_\Omega r\nabla h d\mathbf{r}, \qquad \mathbf{q}_{av} \equiv \frac{1}{\|\Omega\|}\int_\Omega r\mathbf{q}d\mathbf{r}. \tag{8}$$

The weight for each point $\mathbf{r}$ equals its distance to the well, $r = |\mathbf{r}|$. This choice of weights is due to the fact that in homogeneous media both the head gradient and the flux are inversely proportional to r, and so $r dh/dr$ and $r q_r$ are constant. It should be noted the dimensions of $\mathbf{J}_{av}$ and $\mathbf{q}_{av}$ are $[L]$ and $[L^2T^{-1}]$, respectively. For radially symmetric domains Ω, it follows from (5) that $r\ \nabla_{\mathbf{r}}\ G(\mathbf{r},\mathbf{0}) = \mathbf{const}$, and the weighted spatial averaging of (7) yields

$$\frac{\mathbf{q}_{av}}{\overline{T}} = -\mathbf{J}_{av} + \frac{1}{\|\Omega\|}\int_\Omega r\int_\Omega \epsilon(\mathbf{r})\epsilon(\mathbf{r_1})\nabla_{\mathbf{r}}\nabla^{\mathrm{T}}_{\mathbf{r_1}}\mathbf{G}(\mathbf{r}, \mathbf{r_1})\nabla_{\mathbf{r_1}} h d\mathbf{r_1}d\mathbf{r}. \tag{9}$$

Equation (9) provides a spatially averaged Darcy's equation for flow to a well. This equation is nonlocal, which mirrors the behavior of its stochastically averaged counterpart. Hence the effective (upscaled, equivalent) transmissivity does not generally exist.

[13] Our analysis below will rigorously demonstrate that pumping tests are ergodic, i.e., that their stochastic interpretations are valid in any given realization of the $T(\mathbf{r})$ field, if the following assumptions are met.

[14] 1. The support volume Ω of a test is large enough to ensure that the Green's function for an infinite domain, $G_\infty(\mathbf{r}_1,\mathbf{r}_2) = -(2\pi)^{-1} \ln \rho$ (with $\rho = |\mathbf{r}_1 - \mathbf{r}_2|$ being the distance between the points $\mathbf{r}_1$ and $\mathbf{r}_2$), is an accurate approximation (in the weak sense) of the Green's function G in (5). This requirement can be easily understood from the physical point of view, since a large support volume allows the flow to "sample," i.e., average over, the aquifer's small-scale heterogeneities.

[15] 2. The flow is predominantly radial in the weak sense (integrated over Ω). The weak sense is crucial, since this assumption clearly does not hold pointwise in the vicinity of the well, wherein flow is controlled by the local T values.

[16] 3. Away from the well, the hydraulic head gradient varies slowly in space, mainly driven by the distance to the well, i.e., $r\nabla h \approx$ constant.

[17] Note that the conditions of quasi-steady flow in an infinite, mildly heterogeneous (the small variance of T) aquifer are sufficient to meet the requirements 1–3, but they are not necessary conditions. We further discuss the validity of these three assumptions in section 5.

[18] The first assumption yields the trace of the tensor

$$\nabla_{\mathbf{r}}\nabla_{\mathbf{r}'}^T G(\mathbf{r},\mathbf{r}') = \begin{pmatrix} \dfrac{\partial^2 G}{\partial r \partial r'} & \dfrac{1}{r} & \dfrac{\partial^2 G}{\partial r' \partial \theta} \\ \dfrac{1}{r'} & \dfrac{\partial^2 G}{\partial r \partial \theta'} & \dfrac{1}{rr'} & \dfrac{\partial^2 G}{\partial \theta \partial \theta'} \end{pmatrix} \tag{10}$$

as

$$\mathrm{Tr}\left[\nabla_{\mathbf{r}}\nabla_{\mathbf{r}'}^T G(\mathbf{r},\mathbf{r}')\right] = -\frac{r}{r'}\nabla_{\mathbf{r}}^2 G(\mathbf{r},\mathbf{r}') = \frac{r}{r'}\delta(\mathbf{r}-\mathbf{r}'). \tag{11}$$

Indeed, for large Ω, $G(\mathbf{r},\mathbf{r}_1) \approx G_\infty(\mathbf{r}, \mathbf{r}_1) \equiv G_\infty(\mathbf{r} - \mathbf{r}_1)$, so that

$$\frac{\partial^2 G}{\partial r \partial r'} = -\frac{1}{r'}\frac{\partial}{\partial r}\left(r\frac{\partial G}{\partial r}\right) \quad \text{and} \quad \frac{\partial G}{\partial \theta'} = -\frac{\partial G}{\partial \theta}. \tag{12}$$

The second assumption implies that the vector $\nabla_{\mathbf{r}}\nabla_{\mathbf{r}'}^T G(\mathbf{r}, \mathbf{r}')\, \nabla_{\mathbf{r}'}h$ is collinear with the vector $\nabla_{\mathbf{r}'}h$. This allows one to replace the tensor $\nabla_{\mathbf{r}}\nabla_{\mathbf{r}_1}^T G$ under the integral in (9) with its trace (11), i.e.,

$$\begin{aligned} &\int_\Omega \epsilon(\mathbf{r}_1)\nabla_{\mathbf{r}}\nabla_{\mathbf{r}_1}^T G(\mathbf{r},\mathbf{r}_1)\nabla_{\mathbf{r}_1}h\mathrm{d}\mathbf{r}_1 \\ &\approx \frac{1}{2}\int_\Omega \epsilon(\mathbf{r}_1)\mathrm{Tr}\left[\nabla_{\mathbf{r}}\nabla_{\mathbf{r}_1}^T G(\mathbf{r},\mathbf{r}_1)\right]\nabla_{\mathbf{r}_1}h\mathrm{d}\mathbf{r}_1, \end{aligned} \tag{13}$$

so that (9) becomes

$$\frac{\mathbf{q}_{\mathrm{av}}}{\overline{T}} = -\mathbf{J}_{\mathrm{av}} + \frac{1}{2\parallel\Omega\parallel}\int_\Omega r\epsilon^2(\mathbf{r})\nabla_r h\mathrm{d}\mathbf{r}. \tag{14}$$

The third assumption allows one to localize (14), which leads to an effective Darcy's equation

$$\mathbf{q}_{\mathrm{av}} = -T_{\mathrm{eq}}\mathbf{J}_{\mathrm{av}}, \tag{15}$$

where the equivalent transmissivity is given by

$$T_{\mathrm{eq}} = \overline{T}\left[1 - \frac{1}{2\parallel\Omega\parallel}\int_\Omega \epsilon^2(\mathbf{r})\mathrm{d}\mathbf{r}\right]. \tag{16}$$

This is the transmissivity that is inferred from pumping tests in heterogeneous aquifers, provided that the pumping and observation wells are sufficiently far apart.

4. Stochastic Averaging

[19] The stochastic framework provides an alternative approach to derive the effective transmissivity of heterogeneous aquifers by treating the point transmissivity $T(\mathbf{x})$ as a random field that is characterized by a joint probability density function $p_T(\mathcal{T}, \mathbf{x})$. The ensemble mean of the local transmissivity is then defined as

$$\langle T\rangle \equiv \int \mathcal{T} p_T(\mathcal{T},\mathbf{x})\mathrm{d}\mathcal{T}, \tag{17}$$

and the ensemble means of dependent variables, including the hydraulic head gradient $\mathbf{J}$ and flux $\mathbf{q}$, are given by

$$\langle J_i\rangle \equiv \int \mathcal{J}_i p_{J_i}(\mathcal{J}_i,\mathbf{x})\mathrm{d}\mathcal{J}_i = \int J_i(\mathcal{T})p_T(\mathcal{T},\mathbf{x})\mathrm{d}\mathcal{T} \tag{18}$$

and

$$\langle q_i\rangle \equiv \int \mathcal{Q}_i p_{q_i}(\mathcal{Q}_i,\mathbf{x})\mathrm{d}\mathcal{Q}_i = \int q_i(\mathcal{T})p_T(\mathcal{T},\mathbf{x})\mathrm{d}\mathcal{T}. \tag{19}$$

Here $p_{Ji}(\mathcal{J}_i, \mathbf{x})$ and $p_{qi}(\mathcal{Q}_i, \mathbf{x})$ are the marginal probability density functions of the ith components (i = 1, 2) of the vectors $\mathbf{J}$ and $\mathbf{q}$, respectively. If the transmissivity T is a stationary random field, i.e., $p_T(\mathcal{T}, \mathbf{x}) \equiv p_T(\mathcal{T})$, the stochastic averaging of Darcy's law (1) gives rise to an approximate effective equation [e.g., *Sanchez-Vila*, 1997; *Indelman and Abramovich*, 1994]

$$\langle\mathbf{q}\rangle \approx -T_{\mathrm{eff}}\langle\mathbf{J}\rangle. \tag{20}$$

The stochastically derived effective transmissivity T_{eff} is given by

$$T_{\mathrm{eff}} \approx \langle T\rangle\left[1 - \frac{\sigma_\epsilon^2}{2}\right], \tag{21}$$

where σ_ϵ^2 denotes the variance of ϵ. The derivation of (20)–(21) relies on the assumptions analogous to those used in section 3, i.e., it requires a flow domain to be infinite and an observation point to be sufficiently far from the well.

[20] The comparison of the equivalent (effective) transmissivities derived by means of the spatial averaging (16) and stochastic averaging (21) reveals that the two are identical if, in addition to the assumptions of section 3, the following conditions are met.

[21] 1. The underlying stochastic field of transmissivity T must be second-order stationary; that is, both $\langle T \rangle$ and σ_T^2 must be constant.

[22] 2. The underlying stochastic field of transmissivity T must be second-order ergodic; that is, its mean and variance obtained by the spatial and ensemble averages must be interchangeable.

[23] Hence the second-order ergodicity of the local transmissivity is a necessary (but not sufficient) condition for the ergodicity of pumping tests in heterogeneous aquifers.

5. Validity of Modeling Assumptions

[24] A closer look at the derivation of the spatially and stochastically averaged Darcy's laws (15)–(16) and (20)–(21), respectively, sheds light on the validity of the corresponding modeling assumptions.

[25] The first assumption concerns the size of the support volume Ω. This volume is the only conduit through which aquifer's heterogeneity impacts the Green's function G in (5) [*Tartakovsky and Winter*, 2002]. When Ω is large enough for its boundary $\partial\Omega$ to be approximated by a circle of radius R, the Green's function $G(\mathbf{r}, \mathbf{r}_1)$ can be accurately approximated by its infinite counterpart G_∞. The analogy between the spatial and stochastic averaging helps one to answer an important question of how large is large enough. The stochastic analysis of *Neuman et al.* [2004] showed that the degree of the deviation of $\partial\Omega$ from a circle is determined by the degree of heterogeneity as quantified by the correlation length λ of the random field $T(\mathbf{r})$.

[26] While the equivalent transmissivity obtained by the spatial averaging in section 3 is a constant, its counterpart derived by the stochastic averaging in section 4 can, in general, vary with r [see, e.g., *Sanchez-Vila* 1997]. The latter dependence diminishes with the size of Ω (i.e., with R), and becomes negligible for R larger than a few correlation lengths λ [*Neuman et al.*, 2004]. For mean uniform flow, the boundary effects dissipate for $R > 3\ \lambda$ [*Paleologos et al.*, 1996]. For the convergent flow considered in this study, the spatial variability of the effective transmissivity T_{eff} in (20) becomes negligible when the Green's function $G(\mathbf{r}, \mathbf{r}_1)$ can be accurately approximated by its infinite counterpart G_∞ (see the discussion above). This observation provides a low bound on the size of the support volume Ω that insures the ergodicity of pumping tests.

[27] The effective transmissivity (21) represents a first-order (in σ_ϵ^2) perturbation expansion of the stochastically averaged Darcy's law, which limits its applicability to mildly heterogeneous formations with $\sigma_\epsilon^2 < 1$. To make this result applicable to highly heterogeneous media, it is common [e.g., *Paleologos et al.* 1996] to invoke the so-called Landau-Lifshitz conjecture, which replaces (21) with $T_{\text{eff}} = \langle\ T\ \rangle \exp(-\sigma_\epsilon^2/2)$. While the spatial averaging procedure we used to derive (16) is perturbation free, the established analogy between the spatial and stochastic averaging allows us to generalize (16) as

$$T_{\text{eq}} = \overline{T} \exp\left[-\frac{1}{2 \parallel \Omega \parallel} \int_\Omega \epsilon^2(\mathbf{r}) \mathrm{d}\mathbf{r}\right]. \qquad (22)$$

6. Discussion

[28] Our analysis provides a new insight into the concept of interpreted transmissivity T_{int} introduced recently by *Sanchez-Vila et al.* [1999b]. Their analytical study revealed that long-duration pumping tests in a synthetic heterogeneous aquifer with stationary transmissivity T yield the drawdown versus log-time curves, whose slope is nearly identical at all observation points. This slope was used to define T_{int}, which should equal the T_{eq} obtained from either (16) or (22) under the steady state conditions. In other words, $T_{\text{int}} = T_{\text{eq}} = T_{\text{eff}}$, provided the transmissivity field is second-order ergodic and stationary and the assumptions 1–3 of section 3 are met.

[29] Moreover, equation (16) confirms the finding of *Copty and Findikakis* [2004], who argue that in multi-Gaussian fields T_{eff} must be slightly smaller than the geometric mean T_{g}. If the conditions on T are not met (e.g., if the connectivity among the largest local T values differs from that of the smallest local T values), T_{int} could be larger than T_{g}, as shown numerically by *Meier et al.* [1998].

[30] Up to this point we have focused on the representative parameters far from the well. Close to the well, (9) remains valid, while its subsequent localization does not. One can show that the leading term in the equation describing the local flow close to the well (obtained by disregarding products of ϵ) can be written as a generalization of Darcy's law,

$$\mathbf{q}_w = -T_w \nabla h_w, \qquad (23)$$

where the subscript $_w$ denotes the values at the well. As a consequence, the representative transmissivity equals the value at the well, and thus the drawdown measured at the well is mainly controlled by the local conditions at the well [e.g., *Meier et al.* 1999]. Therefore ergodic conditions would never apply at the well.

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