JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 22, No. 3, May–June 1999

Dynamic System Characterization via Eigenvalue Orbits

J. Marczyk,* J. Rodellar,† and A. H. Barbat‡
Technical University of Catalonia, 08034 Barcelona, Spain

A new model-free approach for the description of general dynamical systems with unknown structure, order, and excitation is introduced. The approach is based on the new concept of eigenvalue orbit. The eigenorbits are obtained by building an associated linear time-variant system through a matrix that relates the output measurements in a moving horizon window and viewing the trajectories of its time-varying eigenvalues. How the eigenorbits may be computed from the measurements and used for the characterization of the original system is shown. The basic properties of the eigenorbits are presented via a series of theorems for the case of a discrete-time, linear time-invariant system. A set of examples are included to illustrate these properties for more general classes of systems and to suggest some practical issues that can be drawn from the orbits.

Introduction

CONSIDERABLE progress has been made in the fields of modeling and control of dynamic systems, and both disciplines have reached a high level of maturity and sophistication. The abundant literature on these subjects, however, is based on the unstated implication that the solution of a real control problem is the superposition of two separate tasks, namely, that of modeling and control. It has been shown by Skelton[1] that these two processes are, in reality, not separable. Models, no matter how sophisticated and elaborate, are always incomplete and never exactly describe the physical phenomena. Models can be built based on a set of assumptions, decided on by the analyst, that form the idealization of the system and known physical laws to formulate a mathematical model of the idealized system.

An alternative way to build a model is the use of identification methods relating input/output data.[2] An important contribution to the identification of modal properties of linear systems under free-response conditions has been given by Juang and Pappa[3] in their work on the eigensystem realization algorithm (ERA). The linearization properties of ERA have been employed in identifying approximate linear models for simple nonlinear systems by Horta and Juang.[4] An interesting method based on ERA for identifying nonlinear interactions in structures is presented by Balachandran et al.[5] According to the authors, using the ERA in conjunction with a sliding time-windowing technique reveals oscillating damping coefficients when nonlinear coupling (interaction) between structural modes is present. This method shows how linear identification algorithms may be used to capture nonlinear phenomena within a dynamical process. Examples of quadratically and cubically coupled pairs of oscillators illustrate the performance of the algorithm.

It is clear that the majority of the modern control-oriented work is actually based on some sort of model. Sometimes, the plant model is augmented with a disturbance model so as to take into account the information that is available about the environment while synthesizing the controller. On other occasions, the uncertainties present within the system are explicitly taken into account, and a wide variety of methods have been developed to cope with stochasticity, time dependence, parameter jumps, etc. However, no matter how sophisticated the model, or its treatment, the model is nearly always the central issue of practically any control problem.

In recent years, approaches not relying on concepts of the described classes have been proposed for the description and control of dynamic systems in a model-free perspective. Neural networks and fuzzy logic are nowadays well known within this context; see Refs. 6 and 7.

This paper proposes a novel and general model-free approach to the problem of general process characterization and control of dynamical systems. This description is based on the concept of eigenvalue orbits. These orbits are built by graphing the paths of the eigenvalues of a dynamic matrix, which relates the output measurements supplied by a set of sensors within a moving-time window. No assumptions on system linearity, order, parameter uncertainties, or operating environment shall be formulated or used toward the end of model synthesis. The concept of model is, in the framework of the present work, left as such. The only information that shall be used for the purpose of providing process description is supplied by the sampled sensor readings.

The approach presented herein for system characterization shares certain common points and analogies with the ERA.[5] In fact, it can be also considered as a kind of linearization method attempting to capture information from nonlinear systems by linear tools. However, in a different vein, this paper is not concerned with explicitly identifying structural parametrical properties of the system but just with drawing the trajectories of the eigenvalues of a time-variant matrix practically built from sensor readings. These trajectories, as is shown in the following sections, contain interesting information on the system being described.

The theoretical framework of the eigenvalue orbits is related to such concepts as phase-portraits, Lyapunov exponents, and Lyapunov transformations.[8] A more in-depth treatment of these issues is covered in Refs. 9 and 10. However, it appears that in the fields of dynamic systems, system identification, or model-free control practically no literature on eigenvalue orbits exists.

The main objective of this paper is to introduce the concept of eigenvalue orbit as a model-free descriptor of dynamic systems. Next, the basic eigenorbit theorems are demonstrated in the case of discrete-time, linear time-invariant (DLTI) systems. Finally, a set of numerical experiments are presented with the scope of highlighting the basic properties of the orbits and suggesting some practical features of the system behavior that can be drawn from the orbits.

Background Concepts

Consider a general nonlinear system of the type

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, t), \qquad \boldsymbol{y} = g(\boldsymbol{x}, t) \tag{1}$$

with $\boldsymbol{x} \in R^n$ and $\boldsymbol{y} \in R^q$, where n is the number of states and q the number of measurement channels, and where the structure of both f and g is supposed to be unknown, together with the order of $\boldsymbol{x}$. We base the central idea behind the schemes proposed on the measurements $\boldsymbol{y}$ being seen as exact and direct images of the states of some unknown dynamical system. The sensor outputs are a superposition

Received Nov. 17, 1997; revision received Sept. 25, 1998; accepted for publication Nov. 1, 1998.

*Researcher, Department of Structural Mechanics, School of Civil Engineering, Campus Nord UPC, Gran Capitán s/n. Senior Member AIAA.

†Professor, Department of Applied Mathematics III, School of Civil Engineering, Campus Nord UPC, Gran Capitán s/n.

‡Professor, Department of Structural Mechanics, School of Civil Engineering, Campus Nord UPC, Gran Capitán s/n.