

# Preparing arbitrary pure states of spatial qudits with a single phase-only spatial light modulator

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**Abstract.** We present a new method for preparing multidimensional spatial qudits by means of a single phase-only spatial light modulator (SLM). This method improves previous ones that use two SLMs, one working in amplitude regime and the other in phase regime. To that end, we addressed diffraction gratings on the slits that define the state and then we performed a spatial filtering in the Fourier plane. The amplitude of the coefficients of the quantum state are determined by the modulation deep of the diffraction gratings, and the relative phase is the mean phase value of the diffraction gratings. This encoding result to be more compact, less expensive and use the photons more efficiently.

Quantum systems are the information carriers in quantum information processing and computing protocols. While qubits (two-level systems) are the usual and most basic systems to carry out such tasks, qudits [ $D$ -level systems ( $D > 2$ )] have been attracting growing interest due to their greater potential for those applications [1]. In particular, single photons are the natural choice for communications since they are easily transportable and have several degrees of freedom (DOFs) to encode information. In the case of using the transverse momentum-position encoding [2, 3], the simplest approach is achieved by discretizing the one-dimensional transverse modes of the photons when they are made to pass through an aperture with  $D$  slits which sets the dimension of these so-called *spatial qudits*.

Spatial qudits are relatively simple to generate and offer the possibility of working in high dimensions without cumbersome optical setups. Recently, this encoding has drawn interest for miscellaneous applications such as quantum information protocols [4], quantum games [5], and quantum key distribution [6]. Therefore, the ability to prepare arbitrary pure states of such systems represents an important step toward realizations of quantum optics experiments based on it. Static amplitude and phase masks could be used for this purpose, but in practice it would be extremely difficult and time-consuming as each state intended to be prepared would require to set up the corresponding masks in the apparatus. Programmable spatial light modulators (SLMs) can dramatically simplify this process, allowing real-time manipulations of the state coefficients without physically aligning any optical components. In this regard, it has been



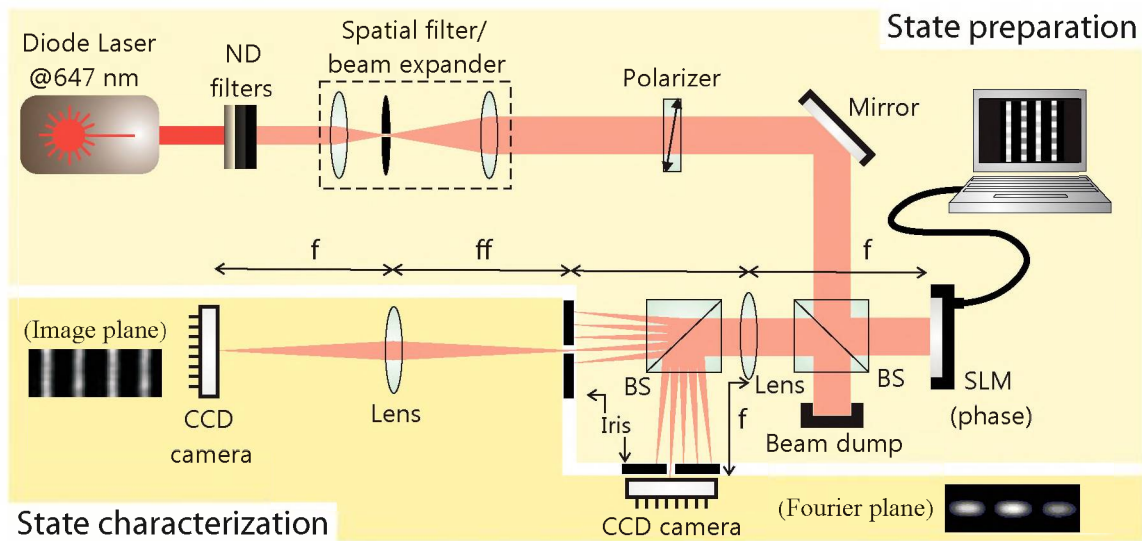


Figure 1. Experimental setup

shown [7] that by imaging the output beam of an amplitude-only SLM onto a phase-only SLM allows one to get complete and independent control of the amplitude and phase of the complex coefficients that define the qudit state. However, the use of two SLMs, besides being more expensive, entails two drawbacks: (i) the overall diffraction efficiency is very low, and (ii) in order to avoid even more losses, the image of the first SLM must match the second one pixel by pixel, which is difficult in practice.

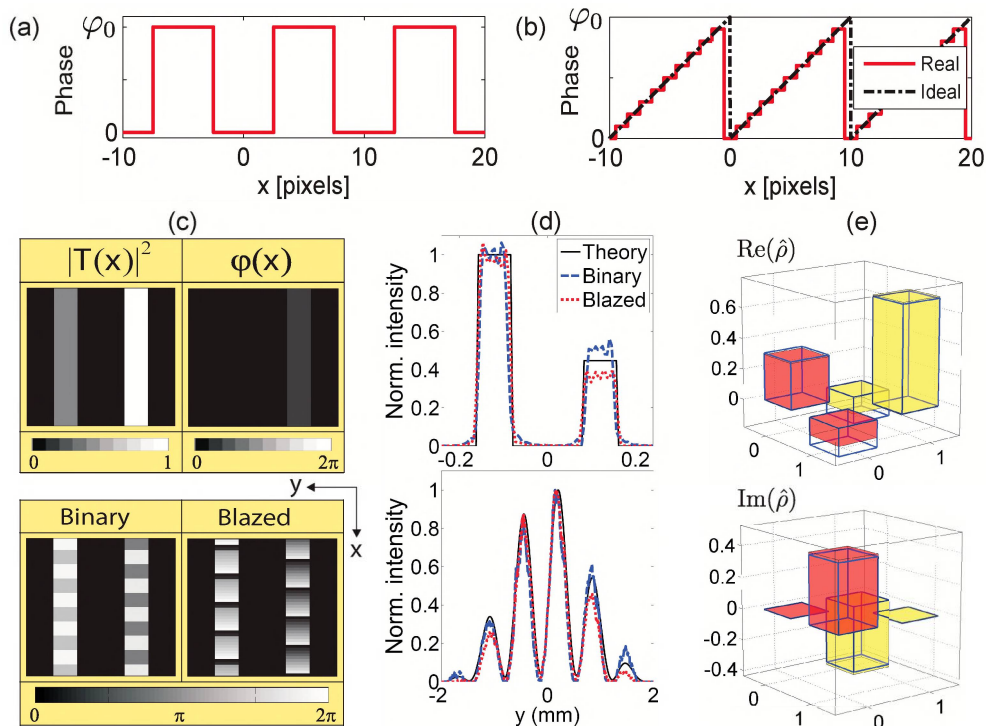
Here, we present a demonstration of a method for preparing arbitrary pure states of spatial qudits with a single phase-only SLM [8], which has a much higher diffraction efficiency and does not require imaging systems. Among the alternative methods to represent a complex function in a single SLM [9, 10] we choose, for simplicity, a technique where the amplitude information is encoded in a phase-only filter [11, 12]. On one hand, the required amplitude is achieved by programming a phase grating, with an appropriate modulation into the slit regions. On the other hand, the required phase value is obtained by adding a constant phase value to the phase grating. Then, the first diffraction order is selected at the Fourier plane and the filtered information is antitransformed on the final plane (Fig. 1). The result is a light distribution corresponding to an image of the slits containing the complete complex modulation. Regarding the type of selected grating, either binary or blazed, the overall quality of the prepared states is very similar. However, blazed gratings provide better diffraction efficiency than binary ones. This enhanced efficiency can be important to increase the signal-to-noise ratio when working with single photon sources.

We have analyzed the performance of the proposed method by preparing a large number of arbitrary pure qubit states uniformly distributed on the surface of the Bloch sphere. To quantify the quality of the preparation we used the fidelity between the state intended to be prepared and the density matrix of the state actually prepared which is reconstructed by tomography (Fig. 2). The tomographic process is performed by measurements at different transverse positions in the near and in the far field [13]. In both cases, either binary or blazed grating, we obtained fidelities above 96% and average fidelities of 99.6%. We have found for  $D = 3$  and  $D = 7$  that the fidelities were above 94%. All these relevant results are summarized in the Table 1.

We compared the luminous efficiency of our setup with the one that employed two SLMs [7]. In the case of using a blazed grating, our scheme is  $\frac{1}{\eta}$  times more efficient, where  $\eta$  represents the light attenuation due to polarizing elements needed to obtain pure amplitude modulation

**Table 1.** Average fidelities of preparation for  $D$ -level qudits.

$D$	2	3	4	5	7
# of states/grating	561	24	70	25	94
$\bar{F}_{\text{bin}}$	0.996	0.995	0.985	0.968	0.970
$\bar{F}_{\text{bla}}$	0.996	0.996	0.991	0.971	0.977



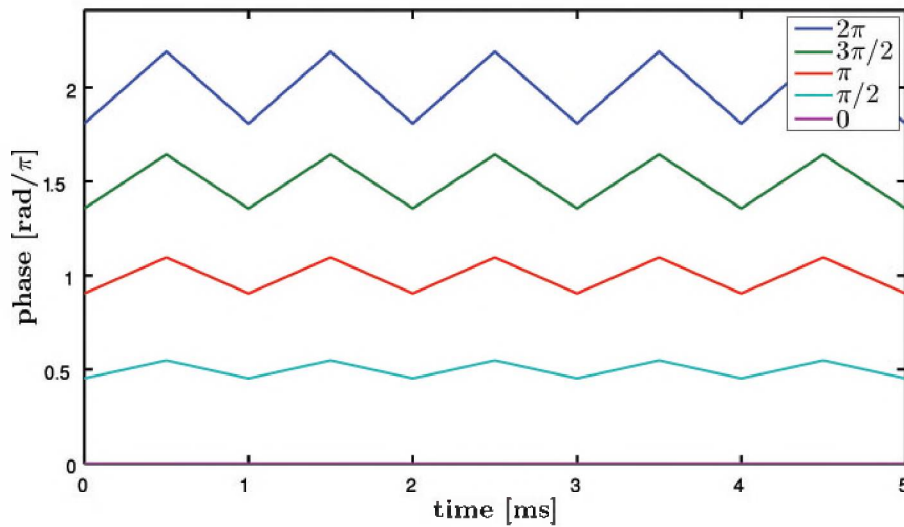
**Figure 2.** Ten-pixels period gratings: (a) binary and (b) blazed. (c) (Upper panels) amplitude and phase components of the aperture  $T(\mathbf{x})$  for preparing the state  $0.56|0\rangle + 0.83e^{i0.63\pi}|1\rangle$ . (Lower panels) corresponding phase masks for preparing this state at the first diffraction order. (d) Normalized intensity distributions and theoretical prediction for measurements of this state at near (upper panel) and far field (lower panel). (e) Real and imaginary parts of the experimentally reconstructed density matrix using the blazed grating. The ideal density matrix is shown as a blue wire grid and the obtained fidelity is  $F = 0.996$ .

with a twisted nematic SLM [15, 16]. The factor  $\eta$  depends on the parameters involved in each particular case (wavelength, twist angle, birefringence of the SLM, etc.), but a typical value is about 0.1. Therefore, the method presented here appears as a valuable tool for experiments based on this encoding. In addition, it can be useful for assisting the measurement of high-dimensional photonic qudits, also including mixed states, enabling not only the storing of information but also the read out of it. It enables the implementation of the strategy developed in [14] by projecting the image of the state to be measured at SLM and addressing at this device the phase grating function corresponding to the measurement state. A detection at the center of the filtered diffraction order at the Fourier plane will accomplish the process allowing one to obtain the statistics of arbitrary observables.

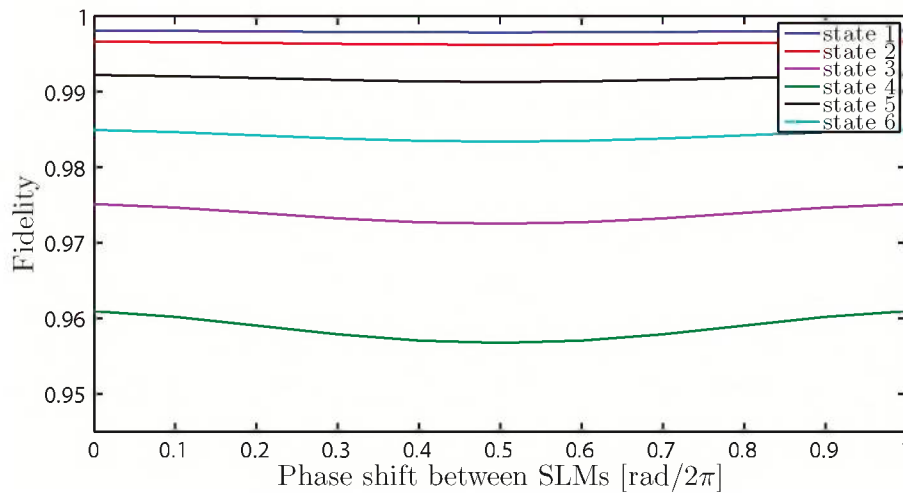
Finally, it is important to remark that although we have obtained a good quality of

**Table 2.** Selected arbitrary states

state 1	$ \psi_1\rangle = 0.70 0\rangle + 0.70e^{i1.06\pi} 1\rangle$
state 2	$ \psi_2\rangle = 0.70 0\rangle + 0.70e^{i0.23\pi} 1\rangle$
state 3	$ \psi_3\rangle = -0.69 0\rangle + 0.73e^{i1.35\pi} 1\rangle$
state 4	$ \psi_4\rangle = -0.92 0\rangle + 0.40e^{i1.90\pi} 1\rangle$
state 5	$ \psi_5\rangle = 0.48 0\rangle + 0.88e^{i0.87\pi} 1\rangle$
state 6	$ \psi_6\rangle = -0.48 0\rangle + 0.88e^{i1.55\pi} 1\rangle$



**Figure 3.** Temporal phase fluctuation model for the SLM.



**Figure 4.** Fidelity vs. phase shift between SLMs, obtained by means of numerical simulation for six arbitrary states ( $D = 2$ ) using the phase addition method.

preparation, the method presents an inherent phase fluctuation associated to the used SLMs, in our case liquid crystal on silicon displays (LCoS). We have selected LCoSs due to their high

spatial resolution and very high light efficiency. Nevertheless, these displays may lead to flicker in the optical beam because of their addressing scheme (pulse width modulation) which introduces temporal phase fluctuations [17, 18] that affect the quality of the encoded state. In order to study this effect we have analyzed the performance of the proposed method by simulating the preparation of some selected arbitrary states in one SLM and their tomographic reconstruction in a second SLM. For this purpose we have used a simplified model for the typical temporal fluctuations of the SLMs that consists on a triangular shaped fluctuation as is depicted in Fig. 3. Given that the optical configuration contain two similar SLMs that could be desynchronized, we have considered different phase shift between both modulators. Numerical simulations of the fidelity vs. the phase shift are shown in Fig. 4 for six arbitrary states. These states are shown in Table 2 As it can be noted fidelities are almost independent of the phase shift between the phase fluctuations in each SLM. This would indicate that it is not necessary to deal with the temporal synchronization of the modulators.

## Conclusions

We have shown that a single phase-only SLM enables the preparation of arbitrary pure states of spatial qudits. Mean values of fidelities of preparation above 94% were obtained and a higher efficiency (at least 10 times) with respect to other methods is expected. It is important to note that this method can be useful not only for preparation but also for assisting the measurement of such systems. It enables the implementation of a strategy based on imaging the state represented on the first SLM onto the second SLM. In this case the first SLM is used to represent the state to be analyzed and the second one to represent the reconstruction basis. Therefore, the method presented here may become a valuable tool for experiments using spatial qudits.

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