



Computing, a powerful tool for improving the parameters simulation quality in flood prediction

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Abstract

Floods have caused widespread damage throughout the world. Modelling and simulation provide solutions and tools which enable us to forecast and make necessary steps toward prevention. One problem that must be handled by physical systems simulators is the parameters uncertainty and their impact on output results, causing prediction errors. In this paper, we address input parameter uncertainty toward providing a methodology to tune a flood simulator and achieve lower error between simulated and observed results. The tuning methodology, through a parametric simulation technique, implements a first stage to find an adjusted set of critical parameters which will be used to validate the predictive capability of the simulator in order to reduce the disagreement between observed data and simulated results. We concentrate our experiments in three significant monitoring stations, located at the lower basin of the Paraná River in Argentina, and the percentage of improvement over the original simulator values ranges from 33 to 60%.

Keywords: Parametric simulation, tuning simulation, flood prediction, flood simulation improvement, high performance computing in flood simulation

1 Introduction

In recent decades, computational fluid simulation has emerged as an area of intense work with an increasing demand for higher precision. Developments of hydraulic simulation models to perform flood routing evolved from manual systems in the middle of the 20th century to current computer-based systems. Predictions of flood inundation extent have been made possible by advances in numerical modelling techniques and increases in computer power [8] [2].

The decision making to reduce the injuries and deaths can be improved if it is assisted by technical measures and evaluation methodologies that enhance the understanding of flood risk or vulnerability [1]. Computational models are used to estimate flood depth and inundation extent. These models are being increasingly used to make predictions, despite there being a series of limitations which cause a lack of accuracy in forecasting [7]. The sources of uncertainty are those derived from uncertainties stemming from the mathematical models, from approximations in numerical solutions and from the difficulty of providing the model with accurate input values. Imprecision in the values of input parameters is the most common source of this problem [17] [13].

The study of hydrodynamic processes in surface waters, such as rivers, has played a pioneering role in the development of numerical models for use in hydraulic engineering. This has led to the production of relatively sophisticated software in accounting for a large amount of runoff details: watercourse levels, associated flows and floodplains occupation [18]. Hydrodynamic modelling of a fluvial channel involves defining certain parameters as input variables which, for various reasons, may incorporate uncertainties in the results. Firstly, these parameters are measured or estimated at specific sectors, which is necessary to interpolate values in the whole domain. Secondly, the parameters' measurement is not direct, as it involves an estimation error associated with the estimation methodology [2].

The main idea of this work is to minimize the differences that exists between real and simulated results. The purpose of this goal is to alert as early as possible and certainly on the occurrence of floods and very low river water at the river basin in study. To overcome this problem, we use a computational tuning methodology to enhance a flood simulation program, thus minimizing the effect of parameter uncertainty on the simulated results. We use a parametric simulation in order to find the best set of parameters, or adjusted set, which will be used as the input set for the underlying flood simulator emulating an "ideal" flood simulator as much as possible. The parametric simulation results in a large number of scenarios carrying out the search for the optimal set of input parameters. This process requires a huge amount of computation and it is on possible with resources in parallel programming and high performance computing.

This paper is organized as follows: After the introduction in Section 2 we present the computational system EZEIZA V and the main features of the river model whose flood routing is simulated using Ezeiza. In Section 3 we introduce the tuning methodology to reduce output errors overcoming the input-parameter uncertainty problem. In Section 4, the experimentation implemented is described. Finally, a discussion on results obtained and a final conclusion is drawn in Section 5.

2 The flood wave propagation simulator: EZEIZA V

Our work starts using a computer model of a real system such as flood events. The computational model is the conceptual model implemented on a computer, and the conceptual model is the mathematical representation of the physical system to be modeled [15]. We represent the idea in Fig. 1

To perform this work we selected the software EZEIZA V(Ezeiza), currently used as one of the tools of the Sistema de Información y Alerta Hidrológico (SIyAH) of the Instituto Nacional del Agua (INA) at Buenos Aires, Argentina, in order to alert as early as possible on the occurrence of extreme water level events at La Plata Basin in South America [9].

Flood routing models approaches are hydrologic and hydraulic. Hydrologic routing uses the conservation of mass equation, but makes simplifying assumptions. On the other hand,

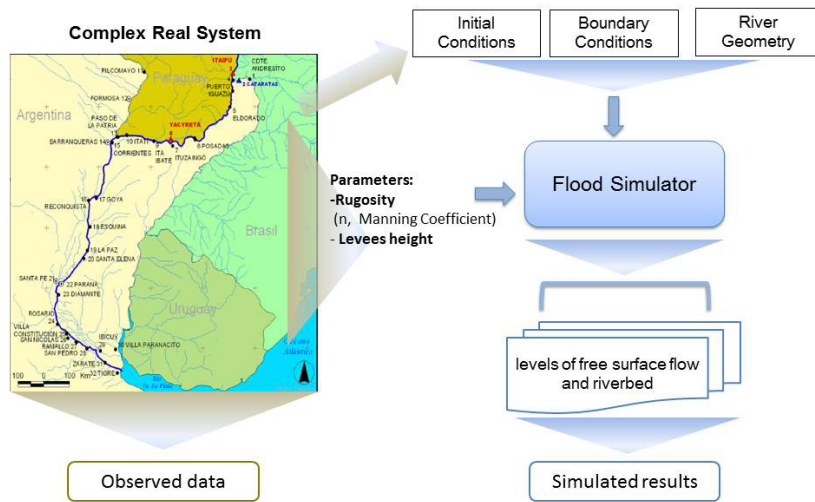


Figure 1: The real system Paraná Basin and the simulation process.

hydraulic routing uses both the conservation of mass and the conservation of momentum equations, but requires much more topographic and flow information. Because of the complicated numerical methods used, hydraulic routing equations can only be solved by using computer software. Ezeiza is based on a hydraulic approach. Ezeiza simulator software and Paraná River model are defined in the next section.

2.1 The computational system Ezeiza

Ezeiza is a computational implementation of a one-dimensional hydrodynamic model based on the Saint Venant equations, which were developed by Laboratorio de Hidráulica Computacional of INA [11]. Ezeiza software family started to be developed in the 70s. This simulator was chosen because:

- It exports results to files that may be processed by statistical and/or mathematical software.
- It makes possible parametric simulations by changing the parameters values in the input files. This is essential to address the tuning process.

Computational model validation is introduced in [9] and it was performed with the Paraná River model we use in this work. An exhaustive performance study was carried out later by Ing. Latessa [10] at INA, who stated the need to improve Ezeiza simulated results. The study of Latessa was aimed to improve the modelling of the Paraná River and provided more certainty to the input data, because the input parameters were measured decades ago. As shown in the report, the accuracy of the model was quite improved. However, differences between the measured values and the output data of the simulator in monitoring stations can still be

<i>Watercourse</i>	<i>Length(Km)</i>	<i>Sections</i>	<i>Boundary Upstream</i>	<i>Boundary Downstream</i>
Paraná	1083	76	Flow Q	Height H
Paraguay	376	77	Height H	-----

Table 1: Data of Rivers System: Paraná-Paraguay Model

detected. These differences are those that we attempt to minimize in this paper, conducting a study of the errors in the predicted results.

2.2 The Paraná River model

La Plata basin is one of the most important rivers systems in the world. The Paraná River is one of the main rivers that form the basin. This is the second longest of South American rivers and it has a length of 4000 km alongside its major tributary, the Paraguay River (2550 km), which occupies a region with a benign climate, fertile lands, easy access and few pollution problems. The Paraná River boasts an extensive floodplain, which in some sections exceeds 50 km [3].

The stretch of the Paraná River simulated by Ezeiza extends between the Yacyretá dam (Corrientes) to Villa Constitución (Santa Fe), both in Argentina. The Paraguay River runs from Puerto Pilcomayo (Formosa) to its confluence with the Paraná. Both river basins were divided into a number of sections, to measure rivers flow and height in each of them. This data were used to model development, and these are shown in Table 1.

The calibration parameters, and those of major impact on the results, are the rugosity coefficient, or Manning values, and the levees height. Manning values for flood plains can be quite different from values for channels; therefore, Manning values for flood plains are determined independently from channel values. The friction coefficient represents a quantification of the hydraulic resistance and is determined by the roughness of the riverbed [12]. The most important factors that affect the selection of Manning values for riverbeds are the type and size of the materials that compose the bed and banks of the channel and the shape of the channel. The model requires specifying the roughness parameters for all river sections [13] [14]. There is a need for extensive data to describe the model domain, as we describe in Section 3.

Daily average flows (in Paraná River, Yacyretá dam) and daily average water levels (in Paraguay River, Puerto Pilcomayo) are the boundary conditions upstream. Downstream boundary conditions are represented by daily hydrometric heights registered in Villa Constitución. The model’s output is a time series whose points correspond to level values, which are calculated over the system domain.

3 Tuning Methodology

Flood propagation on channels is done by numerically solving the Saint Venant equations, either with one-dimensional or two-dimensional approximations. Ezeiza is a computer simulator based upon a one-dimensional approach. These models, in comparison to higher dimensional models, require a minimal amount of input data to define the system behavior, are simpler to use and demand less computer power [6] [14]. However, this results in less accurate flood propagation data, although as we explained before, the uncertainty of input parameters is the main source

of output errors [2].

The tuning methodology implements a first stage to finding an adjusted set of model parameters which will be used in a next stage to validate the predictive capability of the simulator. In this paper we are focused on achieving the first objective, which is key carrying out the second stage. Here we present the tuning method to find a set of adjusted parameters, in order to handle the disagreement between observed data and simulated results.

3.1 The Parametric Simulation

The process of parametric simulation consists of changing the values of the internal input parameters of the simulator and launching as many simulation as different combinations of parameters values are possible. This technique reduces the search space fixing the less sensitive parameters, as the parameters values used in the current model of Paraná River by INA, in order to find the best simulation scenario. That is, the scenario which allows us to reach the shortest distance between observed data and simulated results [5] [4]. For this purpose, we combined the Manning values for each section along river model domain, by measuring a similarity index at every scenario. This similarity index and an improvement rate measure allow us to identify the best scenario and consequently the adjusted parameters used for simulator tuning.

Ezeiza is used as a black box regardless of how realistic the simulator is. To carry on the tuning phase we need to run the simulator loading the full system input data into the simulator, which represents the real extent of river system. The data required to define the modeled river system is as follows:

- Initial conditions: levels and flow at every point of the river's domain.
- Boundary conditions: time series of rivers levels and flow at upstream and downstream points.
- Geometry data: data on the topography of the system.
- Input Parameters: Manning values and levees height, at every river sections.
- Observed data: water heights of Paraná River measured at each monitoring station.

This information was provided by INA, including the model improvements we mentioned before [11]. The observed data include an extended period of time (1994-2011), whose values are daily heights measured at 20 monitoring stations placed along the Paraná River basin.

3.2 Scenarios Generation

We can easily change the Manning values in every section along the riverbed. This is possible thanks to the set of input files used to launch each simulation with Ezeiza, which makes it easier to implement the parametric simulation. A particular setting of the set of parameters defines an individual scenario.

The number of possible scenarios is determined by the cardinality, C_i , for each of the N parameters considered. For each parameter i we define an associated interval and an increment value, which are used to move throughout the interval. For example, given the parameter i we define the associated domain and step values with the tuple:

$$\langle [Limit_{inf}, Limit_{sup}], Step_i \rangle \quad (1)$$

where the interval is bounded by $Limit_{inf-i}$ and $Limit_{sup-i}$ and the values are scanned in $Step_i$ steps.

$$\#Scenarios = \prod_{i=1}^N C_i \tag{2}$$

$$C_i = ((Limit_{sup} - Limit_{inf}) + Step_i) / Step_i \tag{3}$$

We show in Eq.3 the cardinality expression for parameter i where $\#Scenarios$ is the calculation of the total number of scenarios that we obtain after performing all the possible combinations of parameters values. As we perform an exhaustive parametric simulation in this phase, by simulation we will modify a single parameter, leaving the other fixed. In this way, we create each scenario.

3.3 Tuning Implementation

Paraná River model divides the river domain into 76 sections. Each section has a Manning value for the floodplain, a Manning value for the riverbed and the levees' height:

- Manning values for floodplain are within the [0.1, 0.2] range, with an ideal step of 0.01.
- Manning values for riverbed are within the [0.01, 0.04] range, with an ideal step of 0.005.
- Levees height is within the 5m to 50m range with a step of 5m. (The step value is set according to the local geography)

The large size of the search space doesn't allow small steps to cover the intervals. The implementation of the method necessitates reducing the search space taking a greater step, as we shall see later.

We used the root mean square error (RMSE) to calculate the similarity index to evaluate the simulator response for each simulation scenario launched with Ezeiza. The observed data is a time series of daily heights at each monitoring station. We use 15 monitoring stations located along the Paraná river basin.

The use of RMSE turns out to be an adequate indicator of overall agreement between the shape of the graphs of simulated and observed levels through time. A greater value of RMSE is related to a higher bias between the simulated and the observed values. Its range starts at 0, this is the ideal case [16]. The RMSE expression, for a determined monitoring station after launching a determined scenario, is the following:

$$RMSE_j = \sqrt{1/n \sum_{i=1}^n (Q_{sim}^i - Q_{obs}^i)_j^2} \tag{4}$$

where Q_{sim}^i and Q_{obs}^i are the simulated and observed heights values at station j for each n days of simulation.

Our similarity index was calculated using the *RMSE average* by averaging RMSE values of 15 monitoring stations after the simulation time. To calculate the similarity index for scenario k , $Index_k$, we used the next expression:

$$Index_k = \frac{\sum_{j=1}^{15} RMSE_j}{15} \tag{5}$$

\widehat{Scen} represent the best index value resulting from tuning process:

$$\widehat{Scen} = Min_k(Index_k) \tag{6}$$

INA provided us with the Ezeiza simulator and the scenario data, which are currently operational to provide their daily forecasts. We call it the *INA's scenario*. To estimate our prediction reliability, we compared our error results, when running our experimentation scenarios, to those errors obtained when running INA's scenario.

To carry on the parametric simulation, we created a set of scenarios, as we explained in Section 3. Each one of the generated scenarios was fed into Ezeiza simulator, as well as the Paraná model input data. We calculated each similarity index value, with the aim of finding an adjusted set of input parameters to improve the simulator prediction.

It's not possible to improve the prediction for every monitoring station but it is decisive to choose those stations that are significant for each simulation. The impact of changing a parameter value occurs at stations located in the amended section's surroundings. Sometimes this seems to be a task subject to the skill of looking at hydrographs, but finding the way to do this automatically is one of this paper's goals. The parametric simulation offered a huge number of scenarios. These results' (simulated data) misalignment in relation to the observed data were compared to the misalignment of simulated data provided by the INA's scenario, in order to measure the improvement achieved with each of our experiences launched.

At the same time, the process finds the stations with minimum RMSE in a specific scenario. Then we measure the adjustment ratio between simulated results and observed data along the complete time series in each selected station, then we repeat this method for INA simulated data to compare both ratios and to calculate the percentage improvement.

$Error_{Scenario,Station}$ is the station error for a selected scenario:

$$Error_{Scenario,Station} = 1/dias \sum_{i=1}^{dias} abs(Q_{sim}^i - Q_{obs}^i) \tag{7}$$

$Improvement_{Station}$ is the improvement achieved at a selected station in regard to INA results:

$$Improvement_{Station} = abs(Error_{INA,Station} - Error_{Scenario,Station})/Error_{INA,station} \tag{8}$$

4 Experimental Results

The parametric experimentation is characterized by a huge search space. In this work we combine only the Manning values, leaving aside levees heights for now. On this basis, if we implement an exhaustive search of the adjusted parameters we must launch 112^{76} simulations, this means that Ezeiza must be executed $10E155$ times. We had to reduce the search space taking into account the following considerations.

We implemented a test parametric simulation in the lower section of the Paraná River, combining the possible Manning values in sections 70–72–74 and 76. The Manning cardinality is shown in Table 2.

The number of scenarios arising from the previous analysis in Table 2 is:

$$\#Scenarios = (2 * 4)^4 = 4096 \tag{9}$$

The simulation was implemented in the 1/01/1999 - 31/12/1999 period. We selected this period because it covers both high and low streamflow measurements over Paraná basin.

<i>Manning</i>	<i>Interval</i>	<i>Cardinality</i>	<i>C value</i>
<i>Floodplain</i>	$\langle [0.1, 0.2], 0.1 \rangle$	$((0.2 - 0.1) + 0.1)/0.1$	2
<i>Riverbed</i>	$\langle [0.010, 0.035], 0.005 \rangle$	$((0.04 - 0.01) + 0.01)/0.01$	4

Table 2: Domain cardinality for Manning values.

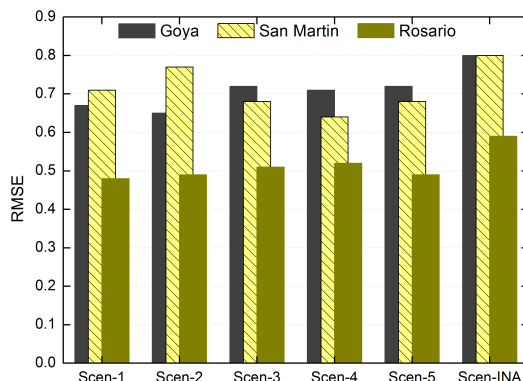


Figure 2: RMSE for three stations selected from the best scenarios resulting from the parametric simulation.

After performing the parametric simulation, we can detect the best scenarios using Eq. 1 and Eq. 3. Once the best scenarios were selected, we carry out the tuning simulator process. The tuning methodology analyzes the prediction accuracy achieved in each case.

We inspected the calculated RMSE for each monitoring station and for a determined scenario. This analysis is built on the results obtained when running INA’s scenario. We enhanced INA’s simulated results looking for those stations whose RMSE were less than the RSME reached with INA’s scenario. We chose 5 scenarios under these conditions, in other words, with the minimum similarity index and the greater number of monitoring stations whose RMSE were lower than the RMSE achieved with INA’s scenario. This selection is shown in Fig. 2

Ezeiza run was carried out on a CPU Intel 8 Core i7-2600 - 3.4 GHz. Running a full simulation under the conditions set lasted 2 minutes. When we run 4096 scenarios, the execution time is 8192 minutes or 137 hours. We used parallel processing resources to reduce the processing time. A master-worker architecture is suitable to parallelize the process because a main processor can calculate each combination of parameters and send them to a set of workers. Simulations are distributed among the nodes in a collaborative way, on the other hand, the execution time in the master is negligible. This system has been developed on a PC LINUX cluster (16 nodes) using MPI as a message passing library. The parallel implementation in a 16-workers system lasted 512 minutes.

As mentioned, improving the prediction for every monitoring station is unlikely. We selected 5 sections located in the lower Paraná basin to combine their Manning values. The improvement was evident in those stations close to the modified, we mean those sections located in the lower reaches of Paraná too. The stations we present in this work, are Goya, Rosario and San Martín,

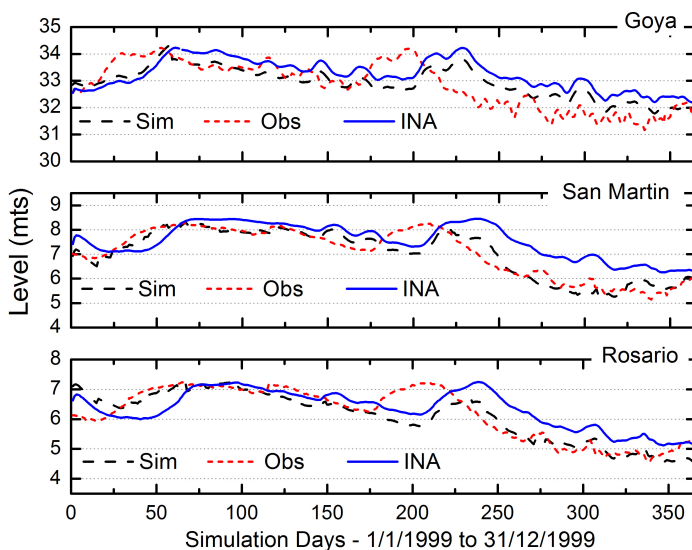


Figure 3: Goya, Rosario and San Martín time series heights related to observed data along simulation period.

Monitoring station	Average difference Observed - Simulated	Average difference Observed - INA	Percentage improvement regarding INA
GOYA	0.472	0.782	40%
ROSARIO	0.327	0.485	33%
SAN MARTIN	0.274	0.686	60%

Table 3: Improvement rate - Goya, Rosario and San Martín with the best adjustment results.

located in the lower Paraná. These stations have achieved a significant improvement, as we can see in Fig. 3

Fig. 3 compares the observed and simulated data for the three monitoring stations. We can see in these graphs (hydrograph) the adjustment achieved in the levels values (in meters) along the 365 simulation days. Our process calculates this adjustment rate to compare fit between observed and calculated data, both for the best scenario resulting from our tuning process as for the scenario currently used by INA. We used Eq. 7 and Eq. 8 to calculate the average distance and improvement rate. These results are in the Table 3

5 Conclusions and Future Work

In this paper we described every step of the tuning process to enhance the prediction performed by the runoff simulator Ezeiza. We have described in detail the parametric simulation technique and we have presented the results obtained in this first stage of our work.

The simulator is a black box, which is executed as a tool into the software package developed

to carry out this experimentation. We focused our efforts on the tuning method to get a better prediction. The search for the best set of parameters was carried out through an exhaustive search technique, which implies a lot of search time. Even though we limited the number of simulations bounding the parameter's domain, we were able to find encouraging results.

The implementation of the parametric simulation technique, along with parallel computing, allowed us find the best simulating scenarios to improve outcomes and reduce the differences between simulated and observed values. The improvement percentage of our simulation experiences, regarding simulated results come from INA's scenario, ranges from 33 to 60% in the best predicted stations. They are Goya, Rosario and San Martín. These stations were chosen as a result of a better similarity index and the improvement rates achieved.

The time series obtained at these stations shows a disarrangement in respect to observed data precisely in those time periods when a sudden level change takes place, which happens when water level raises or falls more rapidly than the rest of the period. This reflects the underlying model, but it's not our goal to change the simulator kernel, for this reason we are seeking to handle this situation by improving the tuning method. Furthermore, since the parametric simulation needs a large amount of computation time, we have used the parallel scheme in a master-worker programming paradigm to improve the computation time. We take advantage of this scheme because our processes can be distributed among processors, without significant communications overhead.

Our future work is focused on considering longer periods of simulation to tune the system for successive periods of flood and downspout. Consequently, we will seek to establish validity ranges for the tuning method, when the simulator performs prediction in the future. We are working toward applying heuristics that minimize the tuning time and make it possible to handle the huge search space, and so to develop a fitness function to evaluate the system's response automatically. We will certainly continue resorting to the techniques of parallel computing to implement this phase.

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