



# Nonlocal SU(3) chiral quark models at finite temperature: The role of the Polyakov loop

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## Abstract

We analyze the role played by the Polyakov loop in the description of the chiral phase transition within the framework of nonlocal SU(3) chiral models with flavor mixing. We show that its presence provides a substantial enhancement of the predicted critical temperature, bringing it to a better agreement with the most recent results of lattice calculations.

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The detailed knowledge of the phase diagram for strongly interacting matter has become an issue of great interest in recent years, both from the theoretical and experimental points of view. On the theoretical side, even if a significant progress has been made on the development of ab initio calculations such as lattice QCD [1–3], these are not yet able to provide a full understanding of the QCD phase diagram due to the well-known difficulties of dealing with finite chemical potentials. In this situation, it is important to develop effective models that show consistency with lattice results and can be extrapolated into regions not accessible by lattice calculation techniques. In previous works [4–7] the study of the phase diagram of SU(2) chiral quark models that include nonlocal interactions [8] has been undertaken. These theories can be viewed as nonlocal extensions of the widely studied Nambu–Jona-Lasinio model [9]. In fact, nonlocality arises naturally in the context of several successful approaches to low-energy quark dynamics as, for example,

the instanton liquid model [10] and the Schwinger–Dyson resummation techniques [11]. Lattice QCD calculations [12] also indicate that quark interactions should act over a certain range in momentum space. Moreover, several studies [13,14] have shown that nonlocal chiral quark models provide a satisfactory description of hadron properties at zero temperature and density. On the other hand, when looking at the description of the chiral phase transition, it has been noticed that for zero chemical potential these SU(2) models lead to a rather low critical temperature  $T_{cr}^0$  in comparison with lattice results [4,5]. The aim of the present work is to go one step beyond these previous analyses, studying the finite temperature behavior of nonlocal chiral models that include mixing with active strangeness degrees of freedom, and taking care of the effect of gauge interactions by coupling the quarks with the Polyakov loop. The inclusion of the Polyakov loop has been considered recently in the context of NJL-like models (so-called PNJL models) [15–19], serving as an order parameter for the deconfinement transition. In particular, this has been done in Ref. [20] in the framework of a nonlocal two-flavor model, focusing on the analysis of mesonic correlations. Here we show that within nonlocal SU(3) effective models with flavor mixing the coupling to the Polyakov

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loop leads to a significant increase of the chiral phase transition temperature  $T_{\text{cr}}^0$ , which is otherwise as low as in SU(2) symmetric models. As stated, this is a desired effect in order to be in better agreement with lattice expectations [21].

We deal here with the nonlocal covariant SU(3) quark model described in Ref. [22], including now the coupling to the Polyakov loop. The corresponding Euclidean effective action is given by

$$S_E = \int d^4x \left\{ \bar{\psi}(x) [-i\gamma_\mu D_\mu + \hat{m}] \psi(x) - \frac{G}{2} [j_a^S(x) j_a^S(x) + j_a^P(x) j_a^P(x)] - \frac{H}{4} T_{abc} [j_a^S(x) j_b^S(x) j_c^S(x) - 3 j_a^S(x) j_b^P(x) j_c^P(x)] + \mathcal{U}(\Phi[A(x)]) \right\}, \quad (1)$$

where  $\psi$  is a chiral  $U(3)$  vector that includes the light quark fields,  $\psi \equiv (uds)^T$ , and  $\hat{m} = \text{diag}(m_u, m_d, m_s)$  stands for the current quark mass matrix. For simplicity we consider the isospin symmetry limit, in which  $m_u = m_d = \bar{m}$ . The currents  $j_a^{S,P}(x)$  are given by

$$j_a^S(x) = \int d^4y d^4z r(y-x)r(x-z) \bar{\psi}(y) \lambda_a \psi(z), \quad (2)$$

$$j_a^P(x) = \int d^4y d^4z r(y-x)r(x-z) \bar{\psi}(y) i\gamma_5 \lambda_a \psi(z), \quad (3)$$

where the form factor  $r(x-y)$  is local in momentum space, namely

$$r(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-i(x-y)p} r(p), \quad (4)$$

and the matrices  $\lambda_a$ , with  $a = 0, \dots, 8$ , are the usual eight Gell-Mann  $3 \times 3$  matrices—generators of SU(3)—plus  $\lambda_0 = \sqrt{2/3} \mathbb{1}_{3 \times 3}$ . The constants  $T_{abc}$  in the 't Hooft term accounting for flavor-mixing are defined by

$$T_{abc} = \frac{1}{3!} \epsilon_{ijk} \epsilon_{mnl} (\lambda_a)_{im} (\lambda_b)_{jn} (\lambda_c)_{kl}. \quad (5)$$

The coupling to the Polyakov loop has been implemented in Eq. (1) through the covariant derivative  $D_\mu \equiv \partial_\mu - iA_\mu$ . In what follows, we assume that the quarks move in a background color gauge field  $\phi = iA_0 = ig\delta_{\mu 0} G_a^\mu \lambda^a / 2$ , where  $G_a^\mu$  are the SU(3) color gauge fields. Then the traced Polyakov loop, which is taken as order parameter of confinement, is given by  $\Phi = \frac{1}{3} \text{Tr exp}(i\phi)$ . We will work in the so-called Polyakov gauge, in which the matrix  $\phi$  is given a diagonal representation  $\phi = \phi^3 \lambda^3 + \phi^8 \lambda^8$ , which leaves only two independent variables,  $\phi^3$  and  $\phi^8$ . Finally, the action (1) also includes an effective potential  $\mathcal{U}$ , which accounts for the gauge field self-interactions.

To proceed we consider the grand canonical thermodynamical potential of the model within the mean field approximation. Using the standard Matsubara formalism we get

$$\Omega_{\text{MFA}}(T) = -2T \sum_{f,c} \int \frac{d^3p}{(2\pi)^3} \sum_{n=-\infty}^{\infty} \text{Tr} \ln [\omega_{nc}^2 + \Sigma_{fc}^2(\omega_{nc}^2)] - \frac{1}{2} \left[ \sum_f \left( \bar{\sigma}_f \bar{S}_f + \frac{G}{2} \bar{S}_f^2 \right) + \frac{H}{2} \bar{S}_u \bar{S}_d \bar{S}_s \right] + \mathcal{U}(\bar{\Phi}, T), \quad (6)$$

where  $f = (u, d, s)$ ,  $c = (r, g, b)$ , and we have used the definition  $\omega_{nc}^2 = (\omega_n - \phi_c)^2 + \vec{p}^2$ ,  $\omega_n = (2n+1)\pi T$  being the usual Matsubara frequencies. The quantities  $\phi_c$  are defined by the relation  $\phi = \text{diag}(\phi_r, \phi_g, \phi_b)$ . The constituent masses  $\Sigma_{fc}$  are here momentum-dependent quantities, given by

$$\Sigma_{fc}(\omega_{nc}^2) = m_f + \bar{\sigma}_f r^2(\omega_{nc}^2). \quad (7)$$

Within the stationary phase approximation, the mean field values of the auxiliary fields  $\bar{S}_f$  turn out to be related with the mean field values of the scalar fields  $\bar{\sigma}_f$  by [22]

$$\bar{\sigma}_u + G \bar{S}_u + \frac{H}{2} \bar{S}_u \bar{S}_s = 0, \\ \bar{\sigma}_s + G \bar{S}_s + \frac{H}{2} \bar{S}_u^2 = 0. \quad (8)$$

The mean field effective potential  $\mathcal{U}(\bar{\Phi}, T)$ , which accounts for the Polyakov loop dynamics, can be fitted by taking into account group theory constraints together with lattice results, from which one can estimate the temperature dependence. Following Ref. [19] we take

$$\mathcal{U}(\Phi, T) = \left[ -\frac{1}{2} a(T) \Phi^2 + b(T) \ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4) \right] T^4, \quad (9)$$

with the corresponding definitions of  $a(T)$  and  $b(T)$ . Owing to the charge conjugation properties of the QCD Lagrangian [23], the mean field value of the Polyakov loop field  $\Phi$  is expected to be a real quantity. Assuming that  $\phi^3$  and  $\phi^8$  are real-valued fields [19], this implies  $\bar{\phi}^8 = 0$ ,  $\bar{\Phi} = [2 \cos(\bar{\phi}^3/T) + 1]/3$ .

For finite current quark masses the quark contribution to  $\Omega_{\text{MFA}}(T)$  turns out to be divergent. To regularize it we follow the same prescription as in previous works [6]. Namely, we subtract from  $\Omega_{\text{MFA}}(T)$  the quark contribution in the absence of fermion interactions, and then we add it in a regularized form, i.e., after the subtraction of an infinite,  $T$ -independent contribution. From the minimization of this regularized thermodynamical potential, it is possible now to obtain a set of three coupled “gap” equations that determine the mean field values  $\bar{\sigma}_u$ ,  $\bar{\sigma}_s$  and  $\bar{\phi}^3$  at a given temperature.

We are also interested in the estimation of chiral condensates, which are given by the vacuum expectation values  $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle$  and  $\langle \bar{s}s \rangle$ . As usual, they can be obtained by varying  $\Omega_{\text{MFA}}$  with respect to the corresponding current quark masses. The explicit regularized expression for a quark condensate  $\langle \bar{f}f \rangle$  reads

$$\langle \bar{f}f \rangle = 4 \sum_c \int \frac{d^3 p}{(2\pi)^3} \left\{ -T \sum_{n=-\infty}^{\infty} \left[ \frac{\Sigma_{fc}(\omega_{nc}^2)}{\omega_{nc}^2 + \Sigma_{fc}^2(\omega_{nc}^2)} - \frac{m_f}{\omega_{nc}^2 + m_f^2} \right] + \frac{m_f}{E_f} (n_{fc}^+ + n_{fc}^-) \right\}, \quad (10)$$

where  $n_{fc}^{\pm} = \{1 + \exp[(E_f \pm i\bar{\phi}_c)/T]\}^{-1}$ , with  $E_f = \sqrt{\vec{p}^2 + m_f^2}$ .

In what follows we analyze the chiral phase transition for a definite form factor, taking into account the temperature dependence of effective masses, chiral condensates and susceptibilities. For simplicity, we have considered a Gaussian form factor

$$r(p^2) = \exp(-p^2/2\Lambda^2), \quad (11)$$

where  $\Lambda$  is a free parameter of the model, playing the role of an ultraviolet cut-off momentum scale. This parameter, as well as quark current masses and couplings in Eq. (1), can be chosen so as to reproduce the empirical values of meson properties at  $T = 0$ . We take into account the analysis in Ref. [22], where the parameters are fixed to obtain the empirical values of meson masses  $m_\pi$ ,  $m_K$  and  $m_{\eta'}$ , together with the pion decay constant  $f_\pi$ . As shown in that work, in this way one can get a good description of the light pseudoscalar meson phenomenology at zero temperature. For definiteness we will work here with the parameter set GI in Ref. [22]. Namely, we use

$$\begin{aligned} \bar{m} &= 8.5 \text{ MeV}, & m_s &= 223 \text{ MeV}, & \Lambda &= 709 \text{ MeV}, \\ G\Lambda^2 &= 10.99, & H\Lambda^5 &= -295.3. \end{aligned} \quad (12)$$

The corresponding numerical results are given in Fig. 1, where we show the behavior of various physical magnitudes as functions of the temperature within the described nonlocal SU(3) model. Left (right) panels correspond to the model without (with) the inclusion of the Polyakov loop. In the upper panels we show the behavior of the mean field values  $\bar{\sigma}_u$  and  $\bar{\sigma}_s$  (solid and dotted lines, respectively). For the sake of comparison, the curves have been normalized to the respective values at  $T = 0$ , which are found to be  $\bar{\sigma}_u^0 = 304 \text{ MeV}$  and  $\bar{\sigma}_s^0 = 427 \text{ MeV}$  (notice that at  $T = 0$  there is no effect of the Polyakov loop). It is clearly seen that in both cases the SU(2) chiral restoration proceeds as a smooth crossover, whereas there is an enhancement of the order of 80 MeV in the corresponding critical temperature when the effect of the Polyakov loop is taken into account. In order to properly define the values of these critical temperatures, we consider the chiral susceptibility  $\chi_q$ , defined as

$$\chi_q = \left( \frac{\partial \langle \bar{q}q \rangle}{\partial m_q} \right)_{T=\text{cte}}. \quad (13)$$

The phase transition temperature can be defined as the point where the susceptibility shows a peak, the sharpness of this peak serving as a measure of the steepness of the crossover [6]. The curves showing the behavior of the susceptibilities in the nonlocal SU(3) models are plotted in the central panels of Fig. 1. As already mentioned, it is seen that the inclusion of the Polyakov loop leads to an enhancement in the critical SU(2) chiral restoration temperature, which brings it to a better agreement with the most recent lattice values  $T_{\text{cr}}^{0(\text{Lat})} =$

160–200 MeV [21]. Our results are (see solid lines in the central panels of Fig. 1)

Nonlocal SU(3) model without Polyakov loop:

$$T_{\text{cr}}^0 = 114 \text{ MeV}.$$

Nonlocal SU(3) model including Polyakov loop:

$$T_{\text{cr}}^0 = 198 \text{ MeV}.$$

In addition, it is seen that after the inclusion of the Polyakov loop there is an enhancement in the sharpness of the peak. The restoration of the SU(2) chiral symmetry can be also observed when looking to the behavior of the chiral condensates with the temperature, which is shown in the lower panels of Fig. 1. At  $T = 0$  we find  $\langle \bar{u}u \rangle^{1/3} = -211 \text{ MeV}$  and  $\langle \bar{s}s \rangle^{1/3} = -186 \text{ MeV}$ . As usual, the restoration of the SU(3) chiral symmetry is much less pronounced, due to the larger current mass of the strange quark. In fact, both in absence or presence of the Polyakov loop a broad peak is observed in the corresponding chiral susceptibility  $\chi_s$ . This is shown by the dotted lines in the central panels of Fig. 1, where the values of  $\chi_s$  have been normalized by a factor 20 to be comparable with the corresponding values of  $\chi_u$ . The peaks in  $\chi_s$  are found to be placed at about 190 (230) MeV for the model without (with) the Polyakov loop. It is interesting to notice that due to flavor mixing effects the light quark condensates  $\langle \bar{u}u \rangle$  exhibit a second drop at those temperatures. In addition, owing to flavor mixing some signal of the SU(2) chiral restoration can be noticed in the behavior of  $\chi_s$ : one can observe a shoulder at about 115 MeV for the model without Polyakov loop (barely noticeable in the figure), and a small peak at about 200 MeV for the model with Polyakov loop. Finally, it is also important to analyze the behavior of the mean field value of the traced Polyakov loop  $\bar{\Phi}$ . As stated, the latter can be interpreted as an order parameter of the deconfinement transition,  $\bar{\Phi} = 0$  and  $\bar{\Phi} = 1$  being related to confined and deconfined quarks, respectively. Correspondingly, a susceptibility associated with the Polyakov loop can be defined as  $\chi_\Phi = d\bar{\Phi}/dT$ . In the nonlocal SU(3) theory under consideration, the behavior of  $\bar{\Phi}$  and  $\chi_\Phi$  with the temperature are shown by the dashed curves in the upper and central right panels of Fig. 1, respectively. By comparison with the behavior of the mean field value  $\bar{\sigma}_u$ , it can be seen that even if the deconfinement transition goes smoother than the SU(2) chiral restoration, both transitions are found to occur at the same temperature region, located at about  $T = 200 \text{ MeV}$ .

The analysis presented above corresponds to the specific case of a covariant nonlocal SU(3) model, in which we have considered a separable four-fermion interaction. Such an interaction is motivated by the instanton liquid model [10], where it arises from the nonlocal structure of the QCD vacuum. For simplicity we have used here a Gaussian-like form factor, choosing a definite parameter set to get a good agreement with meson phenomenology at zero temperature. In order to determine the range of validity of the conclusions extracted from our analysis, we have also considered different parameter sets (within the range allowed by meson phenomenology), and different shapes for the form factors such as, e.g., Lorentzian functions. We have also analyzed an alternative way of including

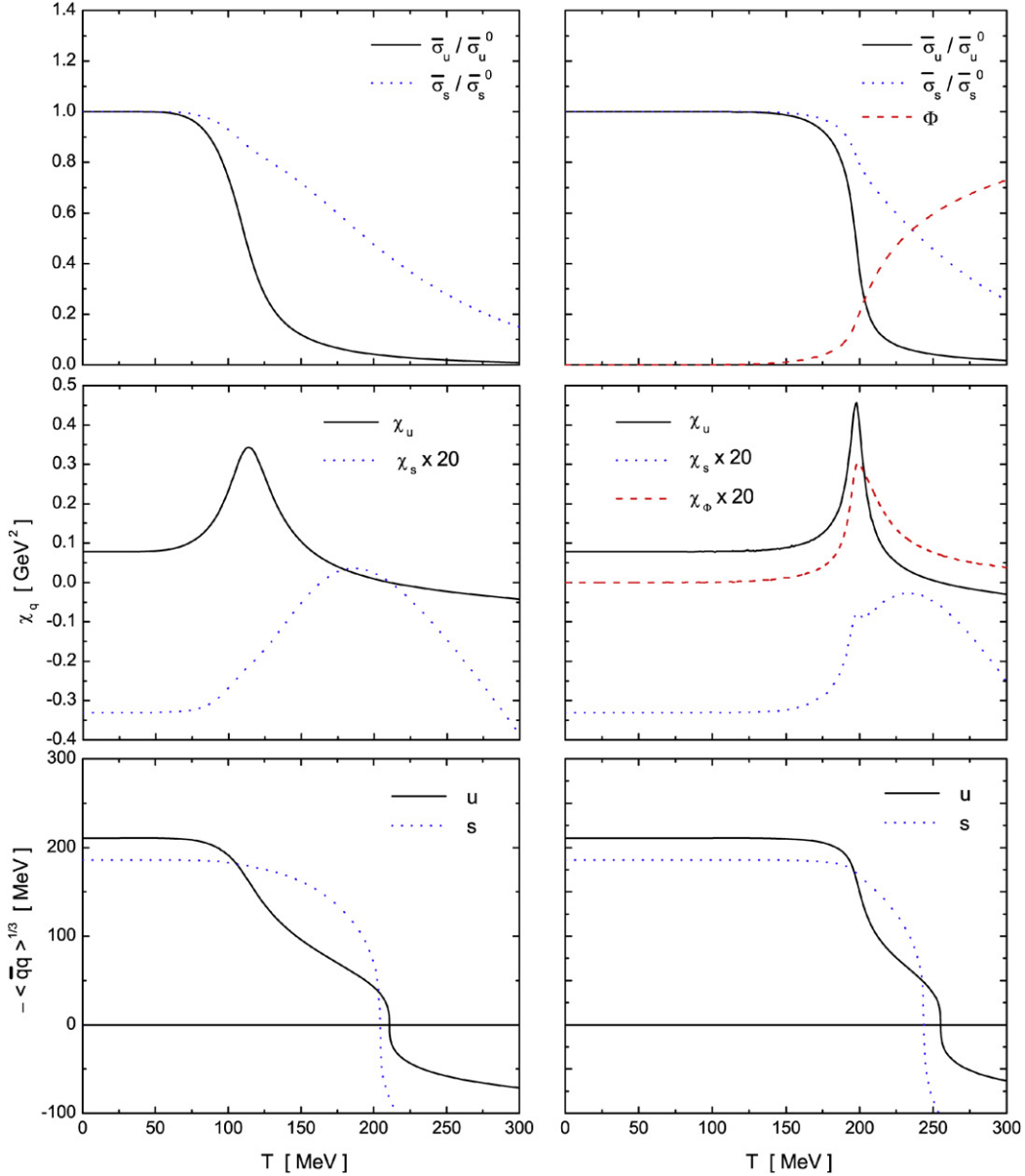


Fig. 1. Behavior of various physical magnitudes as functions of the temperature. Left (right) panels correspond to the model in the absence (presence) of the coupling to the Polyakov loop. The upper panels display the behavior of the mean field values  $\bar{\sigma}_q$  (normalized to their value at  $T = 0$ ) and the traced Polyakov loop  $\bar{\Phi}$ , while central and bottom panels show the behavior of the susceptibilities  $\chi_q$ ,  $\chi_\Phi$  and the quark–antiquark condensates, respectively. For large values of  $T$ , the behavior of condensates corresponds to that of a system of free fermions with nonvanishing mass.

the nonlocality based on an effective one-gluon exchange picture of strong interactions (see Ref. [24] for a detailed comparison of model features in the SU(2) case at  $T = 0$ ). It comes out that the results are qualitatively similar in all these cases.

To summarize, we have analyzed the role played by the Polyakov loop in the description of the chiral phase transition within the framework of nonlocal SU(3) chiral models with flavor mixing. We have found that its presence provides a substantial enhancement of the predicted critical temperature, bringing it to a better agreement with the most recent results of lattice calculations. Another interesting effect of the coupling to the Polyakov loop is that the phase transition becomes steeper, showing a sharper peak in the chiral susceptibility. We notice

that, as it was done for the two-flavor case in Ref. [20], it would be of great interest to extend the present SU(3) model analysis beyond the mean field approximation, considering the effect of mesonic correlations. Other tasks to be addressed are the analysis of the behavior of meson properties with the temperature, and the extension of all these studies in the presence of finite chemical potentials. We hope to report on these issues in forthcoming publications.

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