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A consistent dynamical model for pion-photoproduction

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Abstract

Pion-photoproduction amplitude is calculated within a consistent isobar model already used to fix the $\Delta^{++}(1232\ \text{MeV})$ parameters in pion–nucleon scattering and bremsstrahlung. This amplitude is expressed in terms of (physical) on-shell quantities and off-shell contributions coming from final state interactions. These latter are isolated in principal value integrals on the non-polar part of the pion–nucleon K-matrix, being the main effect of them to dress the bare form factors present in the $\gamma N \to \Delta$ vertex. First, we hide the dressing into effective form factors getting at $k_\gamma^2 = 0$, $G_M = 2.97 \pm 0.08$ and $G_E = 0.055 \pm 0.010$ in full consistence with recent chiral effective field theory calculations. Then, different models to regularize the mentioned integrals are analyzed. We choose those better describing the non-resonant multipoles, in order to get model independent results for the resonant ones. Finally, we try to predict the bare form factors getting $G_M^0 = 1.69 \pm 0.02$ and $G_E^0 = 0.028 \pm 0.008$, which are consistent with recent lattice quark calculations and other more sophisticated dynamical models.

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The pion-photoproduction reaction is one of the most suitable mechanisms to study the nucleon structure and nucleon resonances. During the last thirty years, several models have been proposed to describe the $\gamma N \to N'\pi$ amplitude and observables. These differ essentially in how final state interactions (FSI) are incorporated and how intermediate resonances are treated, being electromagnetic gauge invariance and unitarity guiding basic principles to built up amplitudes. Quantum chromodynamics (QCD) is considered the basic theory of strong interactions from which the hadron structure and interactions should be described. Nevertheless, at intermediate energies a perturbation approach is not adequate, and then we must relay on effective treatment in terms of baryons and mesons.

Within the developed models we have Breit–Wigner treatment of resonances plus non-resonant tree-level amplitudes, being unitarity incorporated through adjustable phases to satisfy the Watson theorem [1]. Also, we have models where the amplitude is expressed in terms of the K-matrix and pion–nucleon scattering phase shifts [2,3]. Other approaches are based on isobar models where the resonant amplitude is built using the Feynman rules coming from effective Lagrangians, divided in two types of models:

(i) Those where the amplitude remains on-shell and the FSI are directly dropped or treated effectively through the usage of form factors and adjustable phases to get unitarity [4–6], known as effective Lagrangian approaches (ELA);

(ii) Those where FSI are generated dynamically through loop integrals present in the amplitude which account for off-shell effects [7–9], known as dynamical models.

One of the main objectives of these studies, was the determination of the nucleon deformation. As it is well-known the $\Delta^+(1232\ \text{MeV})$, lowest energy nucleon resonance, is excited through the $\gamma p(S^\pi = 1/2^+) \to \Delta^+(S^\pi = 3/2^+)$ transition where the magnetic dipole ($M1$) and electric quadrupole ($E2$) multipolarities would participate. In the symmetric quark model ($p, \Delta^+ \equiv uud$) this corresponds to a spin-flip picture where if both p and Δ^+ have a spherical $L = 0$ radial wave function, $E2$ should be forbidden. Nevertheless, a D admixture in the quarks wave function produced by the tensor force [10] (P admixture in πN states) leads to a no vanishing $E2$ and consequently, to a deformed nucleon. This picture becomes

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complicated by the emission of virtual pions coupling to the photon (pion cloud effect) increasing $M1$ and dominating $E2$, as consequence of breaking of chiral symmetry.

From other point of view, a model should be able of describing consistently reactions produced by both electromagnetic and hadronic probes. Recently, we have studied the Δ^{++} contribution to elastic and radiative $\pi^+ p$ scattering within an effective Lagrangian model including Δ, N, π, ρ and σ degrees of freedom. We adopted a description of the Δ^{++} and its interactions that fulfills electromagnetic gauge invariance and invariance under contact transformations, when finite width effects are incorporated through a complex mass scheme. The total $\pi^+ p$ scattering cross section was used to fix the mass, width and strong coupling of the Δ^{++} resonance [11], while the differential one was found in very good agreement with experimental data. Then, from data for the radiative $\pi^+ p$ scattering, we carried the latest determination of the magnetic dipole moment $\mu_{\Delta^{++}} = (6.14 \pm 0.51)e/2m_p$, in agreement with predictions coming from the phenomenological quark model [12,13]. Consequently, the aim of this work will be to get a new determination for the magnetic (G_M) and electric (G_E) form factors in "full consistency" with our previous calculations, using now data coming from the $\gamma p \to \pi^0 p$ and $\gamma p \to \pi^+ n$ processes. In the present model we introduce some refinements, required in the photoproduction case, but the Δ mass, width, and $\pi N \Delta$ coupling will be the same as before.

The pion-photproduction amplitude produced by a photon with polarization $\epsilon(\lambda)$ reads

$$M(\lambda) = \mathcal{N}\bar{u}(B_\mu + TGB_\mu)\epsilon^\mu(\lambda)u$$
$$\equiv \mathcal{N}\bar{u}\left(B_\mu + i\int \frac{dq^4}{(2\pi)^4} T(q)G(q)B_\mu(q)\right)\epsilon^\mu(\lambda)u, \quad (1)$$

being B the $\gamma(k_\gamma)N(q_N) \to \pi(q_\pi)N(q'_N)$ transition potential, u the nucleon spinor, $\mathcal{N} \equiv m_N/2(2\pi)^3\sqrt{E_\pi E_\gamma E_N E'_N}$,[1] T the scattering-matrix operator taking into account πN FSI to all orders, and G stands for the πN propagator. B consists of s (pole), u (cross), and t (exchange)-channel and c (contact) contributions, shown in Fig. 1, and can be split in its pole (P) and non-pole (NP) contributions as $B = B^{NP} + B^P$. B^{NP} encloses the nucleon Born terms (graphs in the first row of Fig. 1) that together lead to a gauge invariant amplitude, plus the ρ and ω-exchange contributions (graphs in the second row of Fig. 1) and the Δ-cross term (second graph in the third row of Fig. 1), being these two last self-invariant by construction. Within B^P we include only the Δ-pole contribution (first graph in the third row of Fig. 1)

$$B^P \equiv B^{(s)}_\Delta = i f^{0\dagger}_{\Delta N\pi} g^0_\Delta f^0_{\Delta N\gamma}, \quad (2)$$

where $f^0_{\Delta N\gamma}$ denotes the bare $\gamma N \to \Delta$ vertex, $f^0_{\Delta N\pi}$ the bare $\pi N \to \Delta$ vertex, and g^0_Δ the bare Δ propagator. This contribution is also self gauge invariant since $f^0_{\Delta N\gamma} \cdot k_\gamma = 0$ by construction, and the Δ-cross contribution to B^{NP} comes from

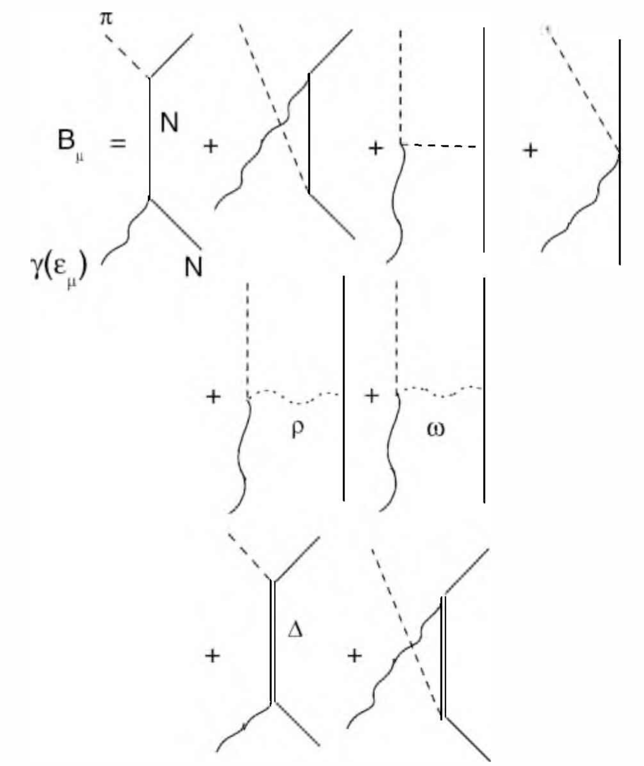

Fig. 1. In the first row we show the pole, cross, pion-in-fligth and contact nucleon contributions respectively (here $B \equiv \mathcal{N}\bar{u}Bu$). In the second row the ρ and ω-exchange are shown. Finally, in the third row we have the pole and Δ-cross contributions respectively.

Eq. (2) by exchanging $f^0_{\Delta N\gamma}$ with $f^{0\dagger}_{\Delta N\pi}$ and incoming (outgoing) pions (photons) with outgoing (incoming) ones. The effective Lagrangians to construct nucleon Born and exchange terms in B^{NP} are the usual ones [7] and we do not repeat here, nevertheless those related with the Δ require further explanation being detailed below. Note that the N-pole Born contribution should be included in B^P and built with bare vertexes and propagators as $B^{(s)}_\Delta$ in Eq. (2), then dressed dynamically by FSI [7]. However, maintain gauge invariance would be difficult under such scheme since dressed (physical) coupling constants and masses are used in the other Born contributions. We have rather chosen to keep the nucleon pole term intact using dressed masses and coupling constants m_N, $f_{NN\pi}$, etc. Since $(q_N + k_\gamma)^2 = m_N^2 + 2m_N E_\gamma$, where $q_N = (m_N, \mathbf{0})$ and $k_\gamma = (E_\gamma \gtrsim 150\text{ MeV}, \mathbf{k})$, it is clear that it does not develop a pole being a smooth varying function of E_γ.

Next we make the same splitting $T = T^P + T^{NP}$ [7], where T^{NP} satisfies the equation

$$T^{NP} = V^{NP} + V^{NP}GT^{NP}, \quad (3)$$

being V^{NP} the contribution to the πN scattering potential involving intermediate nucleons, Δ-cross, and the ρ and σ meson-exchanges shown in Fig. 2, they providing a smoothly-varying background around the resonance region [11]. As we did with the N-pole contribution, we adopt physical masses and couplings. On the other hand, T^P is built as [7]

$$T^P = \bar{f}^{\dagger}_{\Delta N\pi} g_\Delta \bar{f}_{\Delta N\pi} \quad (4)$$

[1] E are the usual relativistic energies.

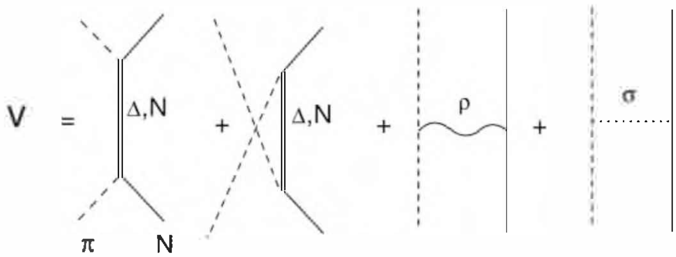

Fig. 2. The first diagram corresponds to N or Δ-pole contribution, while the second one indicates N or Δ-cross contributions (here $V \equiv \mathcal{N}(E_\gamma \to E_\pi)\bar{u}Vu$). The last two ones indicate the ρ and σ meson-exchange contributions.

where

$$\tilde{f}_{\Delta N\pi} = f^0_{\Delta N\pi}\left(1 + GT^{NP}\right), \tag{5}$$

and

$$g_\Delta = \left[\left(g^0_\Delta\right)^{-1} - \Sigma_\Delta\right]^{-1}, \quad \Sigma_\Delta = f^0_{\Delta N\pi} G \tilde{f}^{\dagger}_{\Delta N\pi}, \tag{6}$$

are respectively the $\pi N \to \Delta$ vertex and Δ propagator, dressed by the NP interaction. Finally, by substitution of Eqs. (3) to (5) in (1), the photoproduction amplitude can be cast into the form

$$M(\lambda) = \mathcal{N}\bar{u}\left[\left(B^{NP}_\mu + T^{NP} G B^{NP}_\mu\right) + \tilde{f}^{\dagger}_{\Delta N\pi} g_\Delta \tilde{f}_{\Delta N\gamma\mu}\right]\epsilon^\mu(\lambda)u, \tag{7}$$

being,

$$\tilde{f}_{\Delta N\gamma\mu} = f^0_{\Delta N\gamma\mu} + \tilde{f}_{\Delta N\pi} G B^{NP}_\mu \tag{8}$$

the $\gamma N \to \Delta$ dressed vertex. $M(\lambda)$ in Eq. (7) has background (first term in brackets) plus resonant (second term in brackets) contributions. In order to deal with effective *real* coupling constants fulfilling unitarity at the same time, it is convenient to put T in terms of the real K-matrix operator [14], which satisfies

$$K = V + \mathcal{P}[VGK]. \tag{9}$$

From here we work in the πN CM-system where $q_\pi \equiv (E_\pi(\mathbf{q}), \mathbf{q})$ and $q_N = (E_N(-\mathbf{q}), -\mathbf{q})$, and within the Thompson three-dimensional reduction (TTR) prescription [15,16] where four-dimensional momentum integrals are reduced to three-dimensional ones, replacing G by the Thompson propagator

$$G_{TH}(\sqrt{s}, \mathbf{q}) = \frac{1}{\sqrt{s} - H_0(\mathbf{q}) + i\eta} = -i\pi\delta\left(\sqrt{s} - H_0(\mathbf{q})\right) + \mathcal{P}\frac{1}{\sqrt{s} - H_0(\mathbf{q})}, \tag{10}$$

being $H_0(\mathbf{q})$ the umperturbed πN positive-energies sector Hamiltonian and $\sqrt{s}$ the total energy of the πN system. After a partial wave expansion of the amplitude [14] we get in terms of the multipole index $\tilde{\alpha} \equiv \alpha J_\gamma$, where $\alpha \equiv TLJ$ indicates the isospin, orbital angular momentum and total angular momentum of the final πN state respectively and $J_\gamma = L, L \pm 1$ the photon total angular momentum,

$$M^{\tilde{\alpha}} = \cos\delta_\alpha e^{i\delta_\alpha}\left(B^{NP} \cdot \epsilon + \mathcal{P}\left[V^{NP} G_{TH} B^{NP} \cdot \epsilon\right]\right)^{\tilde{\alpha}} + \frac{e^{-i\phi_\alpha}}{\sqrt{1 + (\pi\rho V^{NP\alpha})^2}}\left[\tilde{B}^P \cdot \epsilon\right]^{\tilde{\alpha}}, \tag{11}$$

being

$$\tilde{B}^P_\mu = f^{\dagger}_{\Delta N\pi\, on} g_\Delta\left(f^0_{\Delta N\gamma\mu} + \mathcal{P}\left[f_{\Delta N\pi} G_{TH} B^{NP}_\mu\right]\right),$$
$$f_{\Delta N\pi} \equiv \left(f^0_{\Delta N\pi} + \mathcal{P}\left[f^0_{\Delta N\pi} G_{TH} V^{NP}\right]\right), \tag{12}$$

$\rho(\mathrm{q})$ the phase space density, and $\phi_\alpha \equiv \mathrm{arctg}(\pi\rho V^{NP\alpha})$. "on $\equiv$ on-shell" and indicates that q $\equiv |\mathbf{q}|$ is determined by $\sqrt{s}$, $A^{\tilde{\alpha}} \equiv (\mathcal{N}\bar{u}Au)^{\tilde{\alpha}}$, and δ_α are the πN phase-shifts appearing as consequence of accounting for FSI. We should solve Eq. (9) to get K^{NP} (non-pole contribution to K) what is never free of ambiguities coming for the inclusion of form factors, in place of this we have approximated $K^{NP} \cong V^{NP}$ to get Eqs. (11) and (12). It is worth to note that we have expressed the photoproduction amplitude in terms of physical phase-shifts and principal value (PV) integrals involving V^{NP}.

Before giving numerical results, we shortly review how the unstable character of the Δ is introduced. As mentioned in Ref. [12], the Lagrangian densities $\hat{\mathcal{L}}_\Delta(A)$ (kinetic term) and $\hat{\mathcal{L}}_{\Delta N\pi}(A)$ (interaction term) are invariant under the contact transformation on the ψ^μ_Δ field (see Ref. [17])

$$\psi^\mu_\Delta \to \psi^\mu_\Delta + a\gamma^\mu\gamma_\alpha\psi^\alpha_\Delta, \qquad A \to A' = \frac{A - 2a}{1 + 4a}, \tag{13}$$

where A and a are arbitrary parameters. This ensures that physical amplitudes involving the Δ resonance are independent of A [18,19] and that spurious spin-1/2 components are removed from the field describing an on-shell Δ-particle. The $\hat{\mathcal{L}}_{\Delta N\gamma}(A)$ Lagrangian reads

$$\hat{\mathcal{L}}_{\Delta N\gamma}(x) = ie\bar{\psi}_{\Delta\nu}(x)\Lambda^{\nu\nu'}(A)\Gamma_{\nu'\mu}\mathbf{T}^{\dagger}_3\psi(x)A^\mu(x) + \text{h.c.},$$

being

$$\Lambda^{\nu\mu}(A) = g^{\nu\mu} + \frac{1}{2}(1 + 3A)\gamma^\nu\gamma^\mu,$$

and

$$\Gamma_{\nu\mu} = G^0_M K^M_{\nu\mu} + G^0_E K^E_{\nu\mu}, \tag{14}$$

where $K^M_{\nu\mu}$ and $K^E_{\nu\mu}$, are define in Ref. [20]. Here G^0_M and G^0_E are the bare form factors at $k^2_\gamma = 0$, which are dressed by FSI through the PV integrals, while m_N and m_Δ are the physical N and Δ masses respectively.

As it was proved in Ref. [17], in the case of elastic and radiative $\pi^+ p$ scattering, the A-dependent Feynman rules involving the Δ can be replaced by a set of A-independent vertices and propagators called *reduced* Feynman rules. Also, in the $\gamma p \to N\pi$ resonant amplitude this will be true, being the pole-term contribution ($f^0_{\Delta N\pi} \equiv \frac{\mathrm{f}^0_{\Delta N\pi}}{m_\pi} q_{\pi\mu}$, $g^0_\Delta \equiv G^{\mu\nu}$, $f^0_{\Delta N\gamma\mu} \equiv e\Gamma_{\mu',\mu}$)

$$B^P_\mu = i\mathcal{N}\left(\frac{\mathrm{f}^0_{\Delta N\pi}}{m_\pi}\right)e\bar{u}(q'_N)q_{\pi\nu}G^{\nu\mu'}(P = q_N + q_\pi)\Gamma_{\mu',\mu}u(q_N), \tag{15}$$

where the reduced form of the Δ^+ propagator has been given in Ref. [12], and where we have omitted isospin–spin indexes. Amplitude (15) blows up when $s = P^2 = (m^0_\Delta)^2$, nevertheless it is the dressed propagator (6) which enters in Eq. (12), being solution of

$$\{(g_\Delta^0)^{-1} - \Sigma_\Delta\}g_\Delta = 1. \tag{16}$$

Σ_Δ could be calculated, nevertheless by simplicity we choose to make an expansion of Σ_Δ on grounds of Lorentz-invariance [21], that in the CM system ($\mathbf{P}=0$) reduces only to terms with $g_{\mu\nu}$ and $\gamma_\mu\gamma_\nu$. Consequently, we parameterize $\Sigma_\Delta \equiv \Sigma_{\mu\nu}$ as

$$\Sigma_{\mu\nu} = \left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu\right)[m_\Delta - m_\Delta^0 - i\Gamma_\Delta/2], \tag{17}$$

where m_Δ and Γ_Δ are parameters to fit (see [12]) and then by inversion of (16) we get [22]

$$g_\Delta = G^{\mu\nu}(m_\Delta^0 \to m_\Delta - i\Gamma_\Delta/2). \tag{18}$$

This approximation is known as the complex mass scheme (CMS) and is well justified for the amplitudes involving intermediate Z^0 and $W^\pm$ gauge boson [23,24] and the $\rho^\pm$ meson resonances [25]. We have used successfully the CMS to treat the Δ^{++} resonance in the π^+p scattering, keeping at the same time gauge invariance in the radiative case [12]. Note that as consequence of (18), $\tilde{B}^{P\tilde{\alpha}}$ is a *complex* quantity that should behave as a real amplitude times $\cos\delta_\alpha e^{i(\delta_\alpha+\phi_\alpha)}$, in order to satisfy the Watson theorem [7,26]. This is the case, as will be seen from the numerical results.

In what follows we show numerical results obtained with Eq. (11). For consistency with our previous calculation on π^+p scattering [12] we adopt the same parameter values in V^{NP} as before and the values $m_\Delta = 1211.7$ MeV, $\Gamma_\Delta = 92.2$ MeV and $\frac{f^2_{\Delta N\pi}}{4\pi} = 0.317$ (physical coupling), imposing isospin symmetry (now we have Δ^+ or Δ^0 in place of Δ^{++}), and assuming that $f_{\Delta N\pi} \cong \frac{f_{\Delta N\pi}}{m_\pi} q_{\pi\mu}$. To built B^{NP} also we need the coupling constants $g_{\omega pp} = 3g_{\rho pp}$ and $\kappa_\omega = \kappa_p - \kappa_n$, which are obtained from a vector dominance model, and $g_{\omega\pi\gamma} = 3g_{\rho\pi\gamma} = 0.32e$. In conclusion, the only free parameters of the model are G_M^0 and G_E^0, which will be fixed by fitting the $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ multipole moments,[2] extracted from the photoproduction amplitudes [7,22] using the isospin decomposition

$$M_{L\pm} = \begin{cases} M_{L\pm}^{1/2} + \frac{2}{3}M_{L\pm}^{3/2}, & \text{for } N\pi = p\pi^0, \\ -\sqrt{2}(M_{L\pm}^{1/2} - \frac{1}{3}M_{L\pm}^{3/2}), & \text{for } N\pi = n\pi^+. \end{cases} \tag{19}$$

to the experimental analysis.

In a first step, to avoid the calculus of PV integrals we parameterized the $\gamma p \to \Delta$ vertex in Eq. (12) as

$$\begin{aligned} f_{\Delta N\gamma\mu} &\equiv f^0_{\Delta N\gamma\mu} + \mathcal{P}(f^\dagger_{\Delta N\pi} G_{TH} B^{NP}_\mu) \\ &= f^0_{\Delta N\gamma\mu}(G^0_{M,E} \to G_{M,E}) \\ &= G_M K^M_{\nu\mu} + G_E K^E_{\nu\mu}, \end{aligned} \tag{20}$$

where G_M and G_E will be considered "effective" form factors, to be fitted. The PV contributions in the first term of Eq. (11) are dropped getting

$$M^{\tilde{\alpha}} \cong \cos\delta_\alpha e^{i\delta_\alpha}(B^{NP}\cdot\epsilon)^{\tilde{\alpha}} + \frac{e^{-i\phi_\alpha}}{\sqrt{1+(\pi\rho V^{NP\alpha})^2}}(f^\dagger_{\Delta on} g_\Delta f_{\Delta N\gamma}\cdot\epsilon)^{\tilde{\alpha}}, \tag{21}$$

which defines the so-called "consistent isobar model" (CIM), which as we will see presents some refinements as regards the used for π^+p scattering.

The fits to $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ are shown in Fig. 3. We get $G_M = 2.97 \pm 0.06$ and $G_E = 0.055 \pm 0.005$ with an acceptable description of these multipoles, which can be extended to $E_{0+}^{3/2}$ and other ones, as will see below. Within the CIM we have only two free parameters G_M and G_E to fit. We avoid the use of any ad hoc form factor to improve the fit, following the philosophy of effective Lagrangian models in the description of low-energy hadron interactions (here elastic π^+p scattering and pion-photoproduction in the Δ resonance region), where one must incorporate only the structureless relevant degrees of freedom. The effective values $G_M = 2.97$ and $G_E = 0.055$ are fully consistent with those obtained recently in chiral effective field theory (χ EFT) calculations [29] $G_M = 2.95$ and $G_E = 0.070$, what indicates that our effective form factors include the pionic loop corrections to the $\gamma N \to \Delta$ vertex.

When we set $\delta_\alpha = \phi_\alpha = 0$ in (21), i.e., dropping FSI, we stand in our *improved Born approximation* applied previously to describe the elastic and radiative π^+p scattering where $T \approx V^{NP} + f^\dagger_{\Delta N\pi on} g_\Delta f_{\Delta N\pi on}$. Results are shown in Fig. 3 as "non-unitary" since as consequence of the absence of FSI the unitarity is severely violated, specially in the $E_{1+}^{3/2}$ multipole where the background contribution (first term in Eq. (21)) is quite dominant. The lacking of unitarity comes from the fact that these contributions are real, nevertheless we showed in π^+p scattering as was possible to recover it by multiplying the amplitude by a fixed phase $e^{-i\phi}$, being ϕ adjusted to satisfy the condition $\mathrm{Im}\, M = -1/2|M|^2$ [12]. The direct consequence of unitarity in the case of photoproduction is the Watson theorem, asserting that the phase of each multipole should correspond to the pion–nucleon phase shift in the same channel. ϕ_α, known in other models as unitarization phases [1,6,9], here have a clear origin ($\phi_\alpha = \mathrm{arctg}(\pi\rho V^{NP\alpha})$) and play the same role as ϕ, being adjusted (by simplicity in place of being calculated) so that $e^{-i\phi_\alpha}\tilde{B}^{P\tilde{\alpha}} \sim \cos\delta_\alpha e^{i\delta_\alpha} \times$ real amplitude. These are shown in Fig. 4 for the $M_{1+}^{3/2}$, $E_{1+}^{3/2}$, together with values for $E_{0+}^{3/2}$, which will be used below. We made $\frac{1}{\sqrt{1+(\pi\rho V^{NP\alpha})^2}} \cong 1$ since $\phi_\alpha = \mathrm{arctg}(\pi\rho V^{NP\alpha}) \leqslant 20^0 = 0.3$ rad, accounting for a change of only $\sim 5\%$ in the amplitude (21).

If now we try to estimate bare form factors G_M^0, G_E^0 in Eq. (14) for confronting with quark models, it is necessary to substitute them in the bare vertex of Eq. (12) and repeat the fits to $M_{1+}^{3/2}$, $E_{1+}^{3/2}$ with Eq. (11). Nevertheless, now we need to make a dynamically dressing of the bare form factors through the PV integrals, in place of treat them effectively as before. These three-dimensional integrals, diverge as consequence of the off-shell behavior of the intermediate particles momenta,

[2] $M^{\tilde{\alpha}} = M^T_{L\pm}, E^T_{L\pm}$ have $J = L \pm 1/2$ and correspond to $J_\gamma = L$ and $L \pm 1$, respectively.

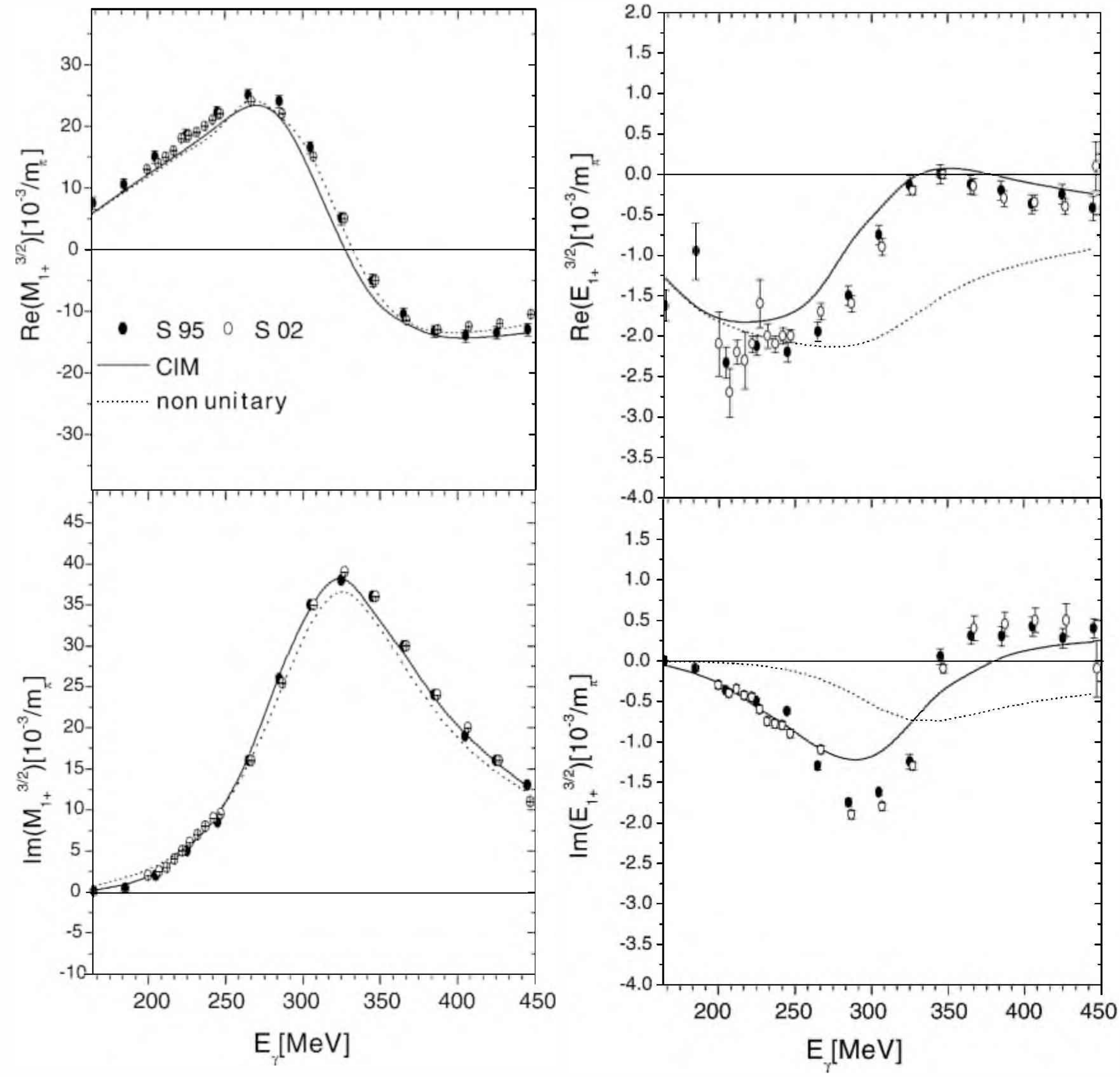

Fig. 3. $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ multipole amplitudes obtained with the CIM described by Eq. (21). Full lines, results when unitarization phases are included, and dotted lines when we make $\delta_\alpha = \phi_\alpha = 0$. The fits are done with the data analysis (S95) taken form Ref. [27] (filled circles). We also show the data analysis (S02) from Ref. [28] (open circles).

regularization form factors (RFF) being necessary to account for the composite nature of the intermediate hadrons. In order to make model-independent predictions, we analyze three different sets of functional forms (with their corresponding cutoff values), already used in previous works:

Model I. RRF used in [9] where for the intermediate hadrons in G_{TH} a global one $(\Lambda^2 + q_{on}^2/\Lambda^2 + q^2)^2$ was adopted, with $\Lambda = 400$ MeV. The corresponding RRF for the V^{NP} and B_μ driven terms are from Refs. [1] and [30], respectively, to have consistence with Ref. [9];

Model II. RFF adopted in Ref. [31], with $F_B(q) = (2\Lambda_B^4/2\Lambda_B^4 + (q_{on}^2 - m_B^2)^2)^2$ for each $B = N, \Delta$ leg, $F_M(t) = \Lambda_M^2/(\Lambda_M^2 + t_{on} - m_M^2)$ for $M = \rho, \omega, \sigma$ legs, and $F_\pi(q) = \Lambda_\pi^2 - m_\pi^2/(\Lambda_\pi^2 - (q_{on}^2 - m_\pi^2))$ for the pion leg, where $p_{on} \equiv (\sqrt{p_{on}^2 + m^2}, \mathbf{p})$. Here for the intermediate hadrons we adopt $\Lambda_\pi^4/(\Lambda_\pi^4 + sq^2)$, which accounts for (see [31]) the effect of the higher-mass states on the high-energy behavior of the πN propagator. The momentum dependence of the RFF in the nucleon born terms of B^{NP} causes a lack of the gauge invariance. To overcome this shortcoming we adopt a "very" simplifying approximation adopting $F(q) = (F_\pi(q_\pi)F_N(q_N)F_N(q'_N))^{1/3}$ as RFF for the inner lines of these terms, and assuming for the contact term the same factor as the nucleon-pole or cross one;

Model III. In the Model I we have different RFF for V^{NP} and B_μ, to avoid this difference we take for each leg that enters in a hadronic vertex $F_H(q) = (2\Lambda_H^4/2\Lambda_H^4 + (q_{on}^2 - m_H^2)^2)^2$ [32] both, for baryons and mesons. For intermediate hadrons we use the same RFF as in Model I and we assume the same prescription of the Model II to keep gauge invariance.

Still, we have only two free parameters G_M^0, G_E^0 to fit once choosed a RFF model. We found a strong dependence of the fitting when we change from one to another model, specially for G_E^0 that until it ends up changing sign depending on the case, as can be seen in Table 1 where G_M^0, G_E^0 and the bare $R_{EM}^0 \equiv -G_E^0/G_M^0$ ratio (which should be contrasted with

Table 1
$G^0_{M,E}$ fits to the $M^{3/2}_{1+}$, $E^{3/2}_{1+}$ multipoles using different RFF models in the calculation of PV contributions. Also we show values (%) for the bare R^0_{EM} and physical $R_{EM} = \mathrm{Im}\, E^{3/2}_{1+} / \mathrm{Im}\, M^{3/2}_{1+}$ (at E_γ such that $\mathrm{Re}[M^{3/2}_{1+}, E^{3/2}_{1+}] = 0$) ratios and for the helicity amplitudes $A_{1/2,3/2}$ in units of 10^{-3} GeV$^{-1/2}$

Model	G^0_M	G^0_E	R^0_{EM}	R_{EM}	$A_{1/2}$	$A_{3/2}$
I	1.30 ± 0.02	-0.014 ± 0.004	1.10 ± 0.30	-3.20 ± 0.32	-127 ± 1	-251 ± 3
II	1.69 ± 0.02	0.028 ± 0.008	-1.67 ± 0.45	-2.91 ± 0.25	-135 ± 1	-265 ± 3
III	1.29 ± 0.02	-0.038 ± 0.011	2.95 ± 0.80	-3.94 ± 0.40	-125 ± 1	-256 ± 3
CIM	2.97 ± 0.08	0.055 ± 0.010	-1.85 ± 0.33	-2.10 ± 0.34	-129 ± 3	-244 ± 6
χEFT	2.95	0.070	-2.39	–	–	–
PDG	–	–	–	-2.50 ± 0.50	-135 ± 6	-255 ± 8

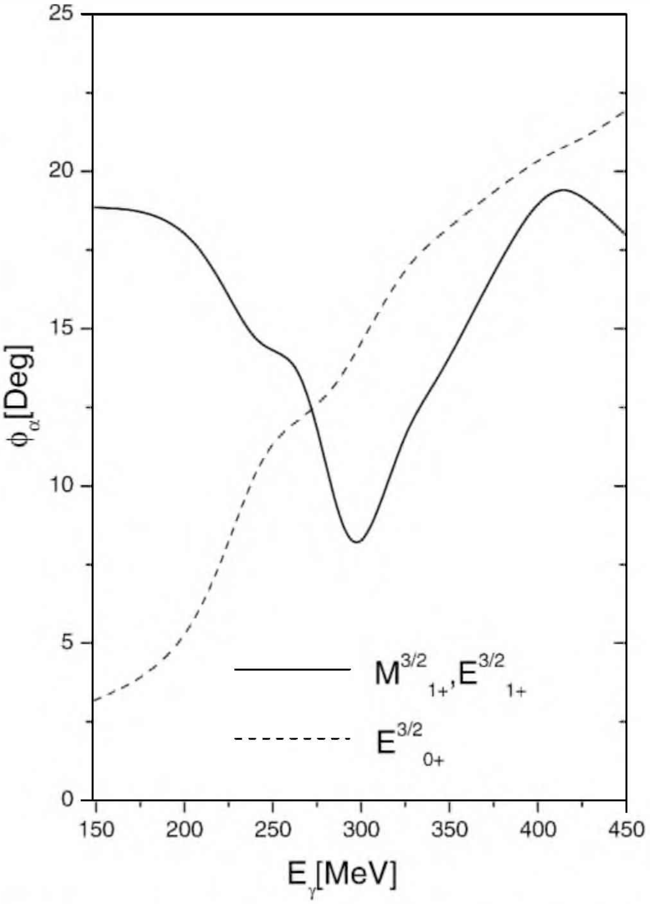

Fig. 4. Calculus of the $\Phi_\alpha = \mathrm{arctg}(\pi\rho V^{NP\alpha})$ for the $M^{3/2}_{1+}$, $E^{3/2}_{1+}$ and $E^{3/2}_{0+}$ multipoles.

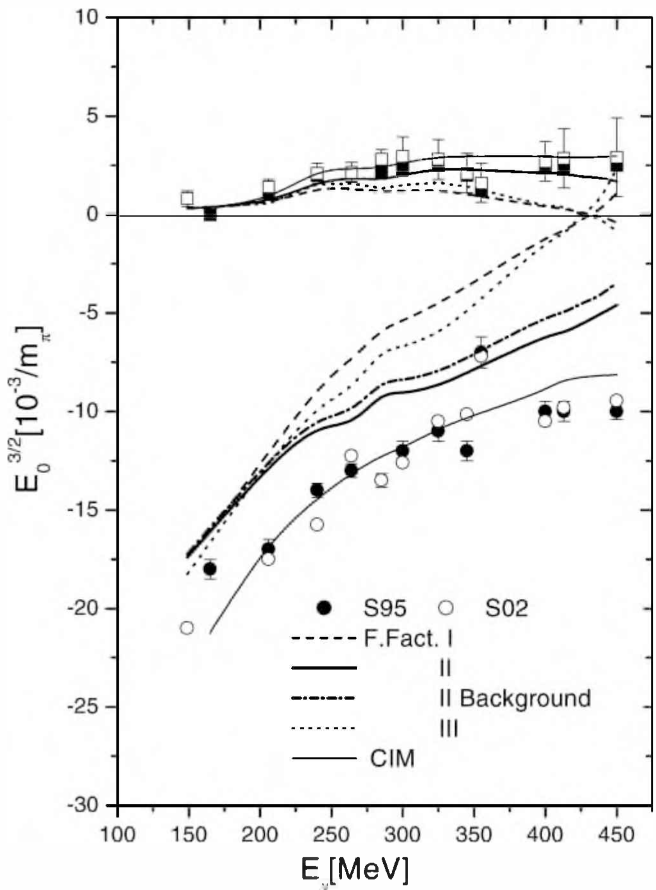

Fig. 5. Calculus of the $E^{3/2}_{0+}$ multipole with different RFF models. The line labeled with "background" in the Model II is obtained from Eq. (11) dropping the second term. The shown data are the same as in Fig. 3, we indicating with squares the imaginary parts.

quark models) are shown together with other usual observables. We show also G_M, G_E and these observables (in the same column as for the bare case) obtained with the CIM, χEFT [29] and from PDG [13].

For selecting the more appropriated RFF model, farther than to look for coincidence with the experimental predictions and previous theoretical calculations of $M^{3/2}_{1+}$ and $E^{3/2}_{1+}$, we guide by two criteria. Firstly, the evaluation of the so-called non-resonant multipoles, which are rough exhausted by the background contribution to the amplitude. They weakly dependent on G^0_M and G^0_E since B^{NP} is independent on the resonant amplitude, consequently departures from the data should be ascribed to the overestimation of the PV integrals as can be seen in detail for the $E^{3/2}_{0+}$ multipole in Fig. 5. Here the Model II leads to the best approximation, which still being the closest to the CIM results does not give a so good data description as it, since we are not fitting the cutoff parameters to get a coincidence with the experiment. The pattern followed by the different RFF models in Fig. 5 for $E^{3/2}_{0+}$ rough repeats for the others S and P non-resonant multipoles whose we have available data in the region of interest of our analysis, as shown in Fig. 6. The CIM agreement and the difference between the models in the $E^{1/2}_{2-}$ case are no so evident, since as has been pointed out [6] this multipole is quite dominated by the contribution of the $N(1520)$ resonance, not included in our model.

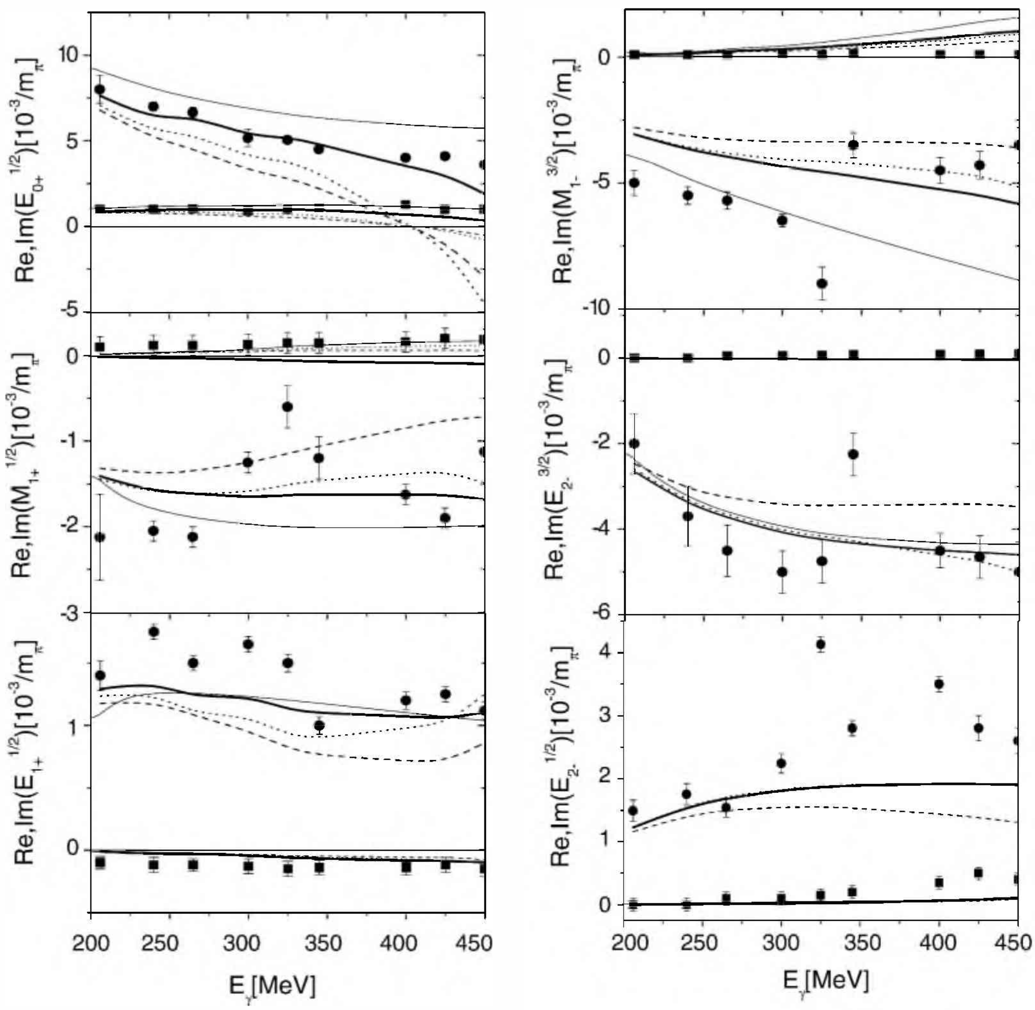

Fig. 6. Same calculation and lines convention as in Fig. 5 but for other S and P non-resonant multipoles ($M^{1/2}, E^{1/2} \equiv M_p^{1/2}, E_p^{1/2}$) with available data in the region of interest of our analysis. Results are compared with the S95 data analysis.

The second criteria is to analyze the RFF depending on if they lead to a positive or negative value for G_E^0. From Eq. (12) we see that the PV off-shell integral dresses the bare $f^0_{\Delta N\gamma\mu}$ vertex. It can be seen [22] that this contribution adds coherently with $f^0_{\Delta N\gamma\mu}$ both, in the $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ amplitudes, for $G_E^0 > 0$ while when $G_E^0 < 0$ they interfere destructively only in the $E_{1+}^{3/2}$ case and the PV contribution (larger than $f^0_{\Delta N\gamma\mu}$) "sweeps" the bare vertex amplitude, which is not physically sound. As can be seen from the Table 1, only in Model II we get a positive G_E^0.

Results obtained with the Model I are consistent with Ref. [9] although our value $G_M^0 = 1.30 \pm 0.02$ is smaller than their 1.65 ± 0.02, may be due to the different treatment of the Δ we do, being G_E^0 also negative. Results with the Model III although a little bit different, also are consistent with that reference. Negative values for G_E^0 and positive values for R_{EM}^0, have been also reported in more recent calculations [31]. In the Model II we get different results in spite of using similar form factors, may be for the approximation we include them and the different treatment of the Δ propagator we do, in order to be consistent with our previous calculations [12]. Our results with the Model II are fully consistent with those reported by Sato [8], who got $G_M^0 = 1.85 \pm 0.05$ and $G_E^0 = 0.025$. In Fig. 7 we show results for $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ with the Model II. The agreement with data is comparable with the CIM case, showing a notable consistence between the dynamical dressing of $G_{M,E}^0$ and the effective procedure. Here we also show curves corresponding to $G_E^0 = -0.028$, showing how a negative value affects the results. Moreover, it worth to note the results for $E_{0+}^{3/2}$ shown in Fig. 5, get worse.

Now, we shortly compare our values for G_M^0, G_E^0 and R_{EM}^0, with those obtained in different quark models in Table 2. As the majority these models do not include the pion cloud contribution, the reported values for G_M^0 are above us, but Ref. [35] where we get a close value for G_M^0 and the half for G_E^0. As matter of fact, at the moment of comparing, not only R_{EM}^0 should be taken into account but also G_M^0 and G_E^0. This was not observed in recent comparisons with quark models [39]. We see that our values are consistent with the quark model in Ref. [33] and with the more recent lattice quenched (pion cloud contribution not included) calculations [37]. In recent preliminary unquenched [38] calculations G_M^0, get closer to our result. Nevertheless, we stress that *all*, G_M^0, G_E^0, and R_{EM}^0 in our model are consistent with quark lattice calculations. This, together with

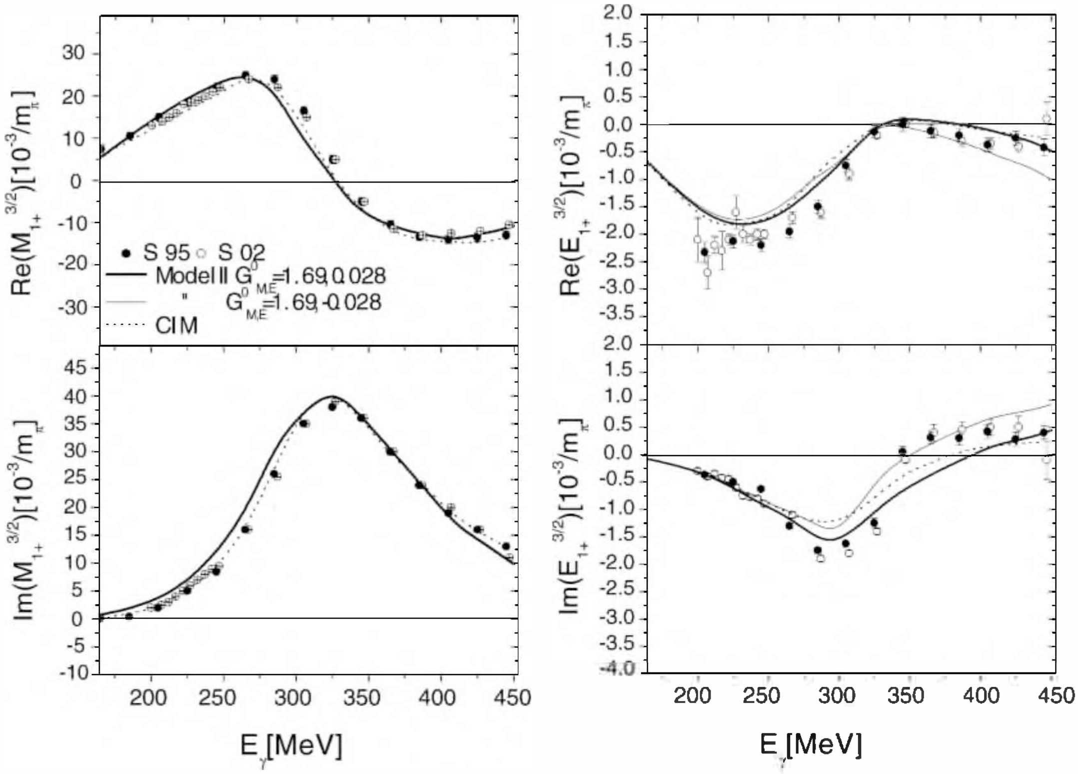

Fig. 7. With "Model II" we indicate the calculus of the $M^{3/2}_{1+}$ and $E^{3/2}_{1+}$ using the amplitude of Eq. (11) with $\tilde{B}_\mu^{P(s)}$ given in Eq. (12), and using the Model II to calculate the form factors involved in the calculus of the PV integrals. Data are the same as in Fig. 3.

Table 2
Comparison of our vales for G^0_M, G^0_E and R^0_{EM} with those obtained within different quark models. Here $q \equiv$ quenched and $u \equiv$ unquenched

Model	G^0_M	G^0_E	R^0_{EM} (%)
Form factor II	1.69 ± 0.02	0.028 ± 0.008	-1.67 ± 0.45
Light-front framework CQM [33]	2.30	0.019	−0.83
Algebraic CQM [34]	2.25	–	–
Quark + diquark CQM [35]	1.13–1.66	0.062–0.056	−5.49–(−3.37)
Non relat. CQM [36]	3.0	0.105	−3.5
Lattice QCD [37,38]			
($Q^2 = 0.1$ GeV^2, $m_\pi = 0$, q)	2.40 ± 0.12	0.045 ± 0.020	-1.93 ± 0.94
($Q^2 = 0.1$ GeV^2, $m_\pi = 370$, q)	2.70 ± 0.10	0.038 ± 0.014	-1.40 ± 0.60
($Q^2 = 0.02$ GeV^2, $m_\pi = 364$, u)	2.25 ± 0.60	–	–

the agreement of the CIM effective approximation with χEFT results suggest that within our model for pion-photoproduction, based on an approach developed previously for $\pi^+ p$ scattering and bremsstrahlung, the bridge established between physical and bare form factors is achieved consistently. In future we shall try to extend the model to describe pion-weak production.

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