Using a regulator for a servomotor model designed for a prosthetic joint

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Abstract. Through the modern control theory it is possible to face any problem situation using state equations without considering mathematical rules used from recursive algorithms. The simplification of the various developments will be reflected with new findings, giving values to new concepts such as controllability and stability. This paper seeks to find new work items of the servomotor system by using a regulator, allowing a more controllable and stable performance. The servomotor presented here is designed to generate joint movements of a robotic arm designed for movement rehabilitation of a patient.

Keywords: Regulator, state space, controllability.

1 Introduction

Engineering has evolved, its participation in the life sciences has generated relatively new disciplines. Engineering in rehabilitation area is the biomedical area that produces more impact. The contribution of Biomedical Engineering to this problem is the design of useful devices to automate these therapies and give patients the autonomy necessary for a better performance [1]. Assistive devices and technologies such as wheelchairs, prostheses, mobility aides, hearing aids, visual aids, and specialized computer software and hardware increase mobility, hearing, vision and communication capacities. With the aid of these technologies, people with a loss in functioning are better able to live independently and participate in their societies. However, in many low-income and middle-income countries, only 5%-15% of people who require assistive devices and technologies have access to them [2].

Access to rehabilitation and habilitation can decrease the consequences of disease or injury, improve health and quality of life and decrease use of health services. While global data on the need for rehabilitation and habilitation, the type and quality of measures provided and estimates of unmet need do not exist, national-level data reveal large gaps in the provision of and access to such services in many low and middle-income countries [2].

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[1] Please note that the LNCS Editorial assumes that all authors have used the western naming convention, with given names preceding surnames. This determines the structure of the names in the running heads and the author index.

As engineers we apply mathematic and physic laws to solve problems, such as the ones mentioned. Through Modern Control Theory it is possible to deal any problematic situation using state equations. Some mathematical rules, as the superposition principle, present in recursive algorithms are excluded in this new approach. The consequent simplification of the developments will be reflected in new findings, giving value to new concepts such as Controllability and Stability [3].

This paper seeks to find new work items of the servomotor system by using a regulator, allowing a more controllable and stable performance. The servomotor presented here is designed to generate joint movements of a robotic arm designed for movement rehabilitation of a patient.

2 Materials and Methods

2.1 Pole placement method

The pole-placement method is to place the poles at desired locations in closed loop. It is assumed that all state variables are measurable and available for feedback, fig 1.

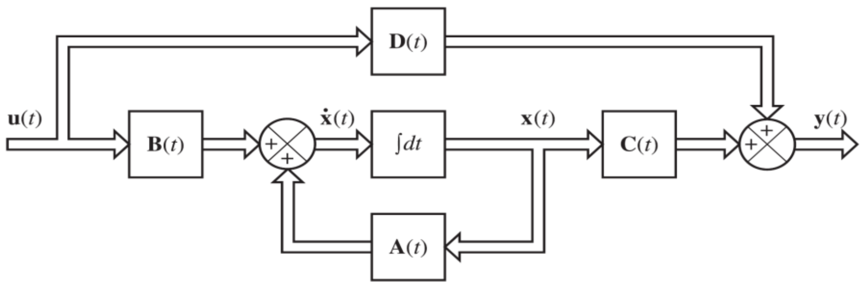

Fig. 1. Block diagram of the control system in the state space.

The design technique begins with the determination of the desired closed-loop poles from the transient response and/or specifications of the frequency response, as steady state requirements. Selecting an appropriate matrix for state feedback gains $\mathbf{K}$, it is possible that the system has the closed-loop poles at desired positions, only if the original system is completely controllable [4].

Let us consider the control system,

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y &= \mathbf{C}\mathbf{x} + Du \end{aligned} \qquad (1)$$

The control signal is selected as

$$u = -\mathbf{Kx} = \begin{bmatrix} K_1 & K_2 & \dots & K_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \tag{2}$$

This means that the control signal is determined by an instantaneous state.

Its goal is to keep the output to zero. As there may be disturbances, the output will deviate from zero. This output returns to the zero reference input because feedback scheme. A system of this kind is known as a system controller.

Replacing

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x} \tag{3}$$

The eigenvalues of the matrix $\mathbf{A} - \mathbf{BK}$ are called poles of the regulator.
Classic project procedures are based on the transfer function of the system, while the project by pole placement is based on the state model of the system.
It is also assumed that all system state variables can be measured and are available to be fed back. This technique based on the principle that if the system is completely controllable, it is possible to locate a set of poles of the closed-loop system in desired locations by state feedback from the system to meet certain specifications and transient dynamic response steady state. These specifications may be related to the characteristic parameters of the transient temporal response to step or impulse inputs.

The pole placement project is summarized in two steps:
(i) Specify the location of the desired roots of the characteristic equation of the closed loop system;
(ii) The calculation of earnings in order to place these roots in certain places in the previous section.

There are two ways of determining the components of matrix **K:**

1- Direct substitution method:

The characteristic equation of the system closed loop is given by:

$$\det[s\mathbf{I} - \mathbf{A} + \mathbf{BK}] = 0 \tag{4}$$

When this determinant is developed resulting in a polynomial of order n in s containing the matrix gains **K**. Now suppose that the desired pole locations are given by the roots $-\lambda_1,\ -\lambda_2,\ \ldots\ ,-\lambda_n$ then the desired characteristic equation is given by:

$$\alpha_c = (s+\lambda_1)(s+\lambda_2)\ldots(s+\lambda_n)$$

The project is completed by equating the coefficients of equal power in *s* of the determinant of the equations and the desired characteristic polynomial.

2 - Ackermann Formula:

Ackermann's formula is based on the similarity transformation which transforms a given model in the controllable canonical form state $(\mathbf{AB}) \rightarrow (\mathbf{A}_c\mathbf{B}_c)$, through a new state vector $\mathbf{x} = \mathbf{Tz}$, second the gains K_i are obtained, resulting in the control law $u = -\mathbf{K}_c\mathbf{z}$. To get the gains for the original state equation, thirdly the gain matrix is transformed back through the matrix $\mathbf{T}$, $\mathbf{K} = \mathbf{K}_c\mathbf{T}^{-1}$.
These three steps are grouped into Ackermann formula given by:

$$\mathbf{K} = [0\ 0\ 0\ .\ .\ .\ 1][\mathbf{B}\ \ \mathbf{AB}\ \ \mathbf{A}^2\mathbf{B}\ \ \ldots\ \ \mathbf{A}^{n-1}\mathbf{B}]^{n-1}\alpha_c(\mathbf{A}) \quad (5)$$

where $\alpha_c(\mathbf{A})$ is a polynomial matrices formed with the coefficients of the desired characteristic equation $\alpha_c(\mathbf{A}) = \mathbf{A}^n + \alpha_1\mathbf{A}^{n-1} + \alpha_2\mathbf{A}^{n-2} + \ldots + \alpha_n\mathbf{I}$ (6)

In the present work we used the sentence “acker” (Matlab) of the Ackermann formula to find the components of the gain matrix **K.**

3 Results

Parameters of the powerplant are as:

R_a= Housing resistence =1Ω
L_a = Housing inductance = 5mH
K_t= Constant torque =1N.m/A
K_b= Cosntant of the e.c.f.m =3V.s/rad
b= Bearings friction coefficient = 0,1N.m.s./rad
J= Inertia moment of the motor and load=0,2N.m./rad/s^2

$$\mathbf{A} = \begin{bmatrix} -5 & 50 \\ -600 & -200 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 200 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 \end{bmatrix}$$

Eigenvalues $\mathbf{A}$ (Original System):

$$\mathbf{J} = \begin{bmatrix} -102.50 + 143.16i & -102.50 - 143.16i \end{bmatrix}$$

After several experiments, and in an arbitrarily way, we opted for the three searches described below.

First search for new pole:

$$real(J) * 2$$

$$\mathbf{J1} = [[-205 + 143.16i \quad -205 - 143.16i]$$

$$\mathbf{K} = \begin{bmatrix} 3.0495 & 1.0250 \end{bmatrix}$$

New system 1:

$$\mathbf{AA} = 1.0e + 003 * \begin{bmatrix} -0.0050 & 0.0500 \\ -1.2099 & -0.4050 \end{bmatrix} \quad \mathbf{BB} = \begin{bmatrix} 0 \\ 609.8957 \end{bmatrix}$$

$$\mathbf{CC} = \mathbf{C} \qquad \mathbf{DD} = \mathbf{D}$$

Second search for a new pole

$$img(J)*2$$

$$\mathbf{J}2=\begin{bmatrix}-102.50+286.32i & -102.50-286.32i\end{bmatrix}$$

$$\mathbf{K}=\begin{bmatrix}6.1485 & 0\end{bmatrix}$$

New system 2

$$\mathbf{AA}=1.0e+003*\begin{bmatrix}-0.0050 & 0.0500\\ -1.8297 & -0.2000\end{bmatrix} \quad \mathbf{BB}=1.0e+003*\begin{bmatrix}0\\ 1.2297\end{bmatrix}$$

$$\mathbf{CC}=\mathbf{C} \qquad \mathbf{DD}=\mathbf{D}$$

Third search for a new pole

$$real(J)*2+img(J)*2$$

$$\mathbf{J}3=\left[[-205+286.32i \quad -205-286.32i\right]$$

$$\mathbf{K}=\begin{bmatrix}9.1979 & 1.0250\end{bmatrix}$$

New System 3:

$$\mathbf{AA}=1.0e+003*\begin{bmatrix}-0.0050 & 0.0500\\ -2.4396 & -0.4050\end{bmatrix} \quad \mathbf{BB}=1.0e+003*\begin{bmatrix}0\\ 1.8396\end{bmatrix}$$

$$\mathbf{CC}=\mathbf{C} \qquad \mathbf{DD}=\mathbf{D}$$

Poles search

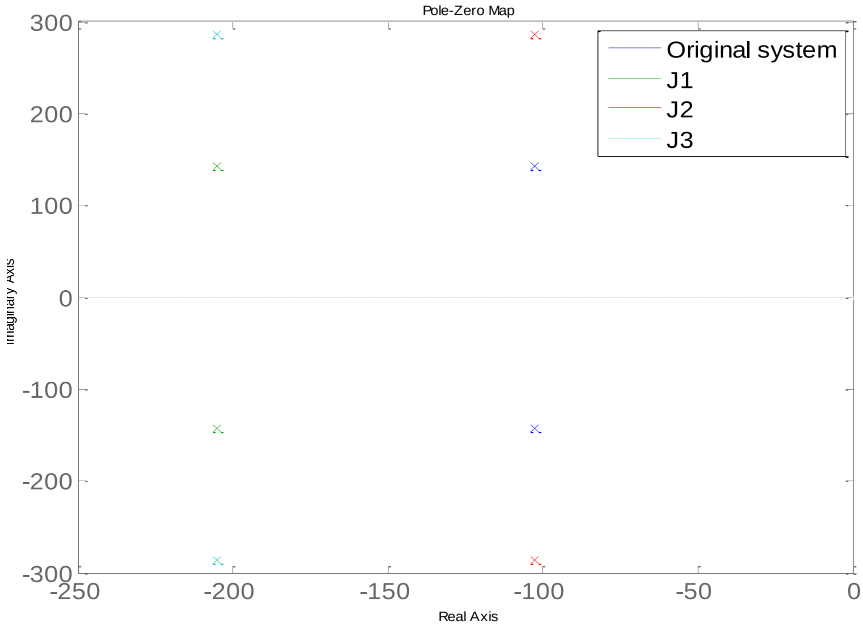

Fig 2. Poles of the diferents systems

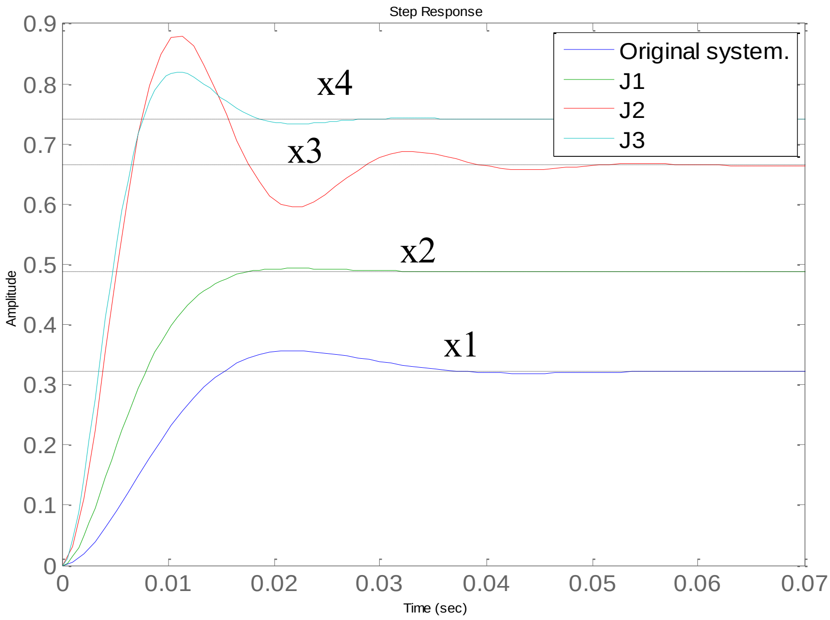

Fig. 3 Responses of different systems to new poles

The three lines x2, x3 and x4 show the adjustments made to the original system (blue), modifying and adjusting to **J1**, **J2** and **J3** respectively. It is possible to improve the system gain to the natural response, unit step, since the original system corresponds to the analysis of a Transfer Function in Open Loop, and settings, a feedback closed with the corresponding **K** gains loop to optimize system dynamics engine.

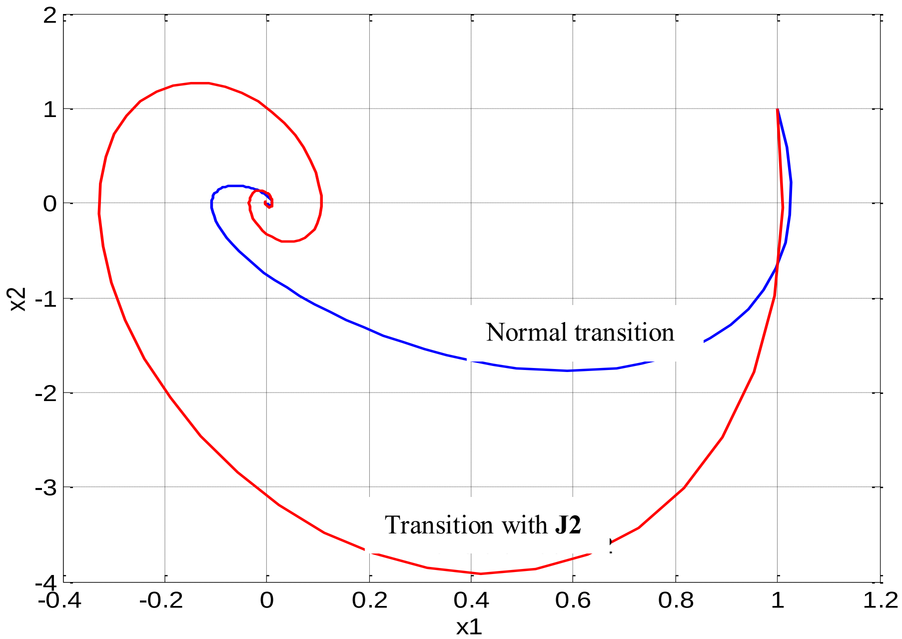

Fig. 4. Diagram in the state space. Search chosen. Improved gain with J2.

4 Conclusions

Note that, the **K** matrix is not unique for a given system, but depends on the desired positions of the closed-loop poles (which determine the speed and damping of the response). The selection of the desired closed loop poles, or of the property desired equation, is a compromise between speed of response and sensitivity to disturbances and measurement noise. That is, if the response speed is increased, generally the adverse effects of disturbances and measurement noise increases. If the system is second order, as in our case, its dynamics (response characteristics) correlates precisely with the position of the desired closed loop poles. Therefore determining the feedback gain matrix **K** for status of a particular system should be examined by a computer simulation in the response characteristics of the system for several different **K** matrices (based on some other desired characteristics equation) and choose the one that offers better overall system performance.

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