Well-formed defeat paths in abstract argumentation frameworks

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Abstract

Abstract argumentation systems are formalisms for argumentation where some components remains unspecified, usually the structure of arguments. In the dialectical process carried out to identify accepted arguments in the system, some controversial situations may be found, related to the reintroduction of arguments in this process, causing a circularity that must be treated in order to avoid an infinite analysis. Some systems apply a single restriction to argumentation lines: no previously considered argument is reintroduced in the process. In this work we show that a more specific restriction need to be applied, taking subarguments into account. We finally present a new definition of *acceptable argumentation lines*.

1 Introduction

Different formal systems of defeasible argumentation were defined inside Artificial Intelligence. The main idea in these systems is that any proposition will be accepted as true if there exists an argument that supports it, and this argument is acceptable according to an analysis between it and its counterarguments. Therefore, in the set of arguments of the system, some of them will be *acceptable* or *justified* arguments, while others not. In this manner, defeasible argumentation allows reasoning with incomplete and uncertain information and is suitable to handle inconsistency in knowledge-based systems.

Abstract argumentation systems [1, 4, 6, 8] are formalisms for argumentation, where some components remains unspecified, usually the structure of arguments. In this kind of systems, the emphasis is put on semantic notions, basically the task of finding the set of accepted arguments. Most of them are based on a single abstract notion called *attack relation*, and several argument extensions are defined as sets of possible accepted arguments. However, the task of comparing arguments to establish a preference is not always successful. Finding a preferred argument is essential to determine a defeat relation. In [6] an abstract framework for argumentation is presented, where two kind of argument defeat relations exists between arguments.

In the dialectical process carried out to identify accepted arguments in the system, some controversial situations may be found, as previously exposed in [2, 3]. These situations are related to the reintroduction of arguments in this process, causing a circularity that must be treated in order to avoid an infinite analysis.

Example 1.1. *Suppose* Φ *is an argumentative system and* $\mathcal{A}, \mathcal{B}$ *and* $\mathcal{C}$ *three arguments in* Φ *such that* $\mathcal{A}$ *is a defeater of argument* $\mathcal{B}$*,* $\mathcal{B}$ *is a defeater of* $\mathcal{C}$ *and* $\mathcal{C}$ *is defeating* $\mathcal{A}$*. In order to decide the acceptance of* $\mathcal{A}$*, the acceptance of its defeaters must be analyzed first, including* $\mathcal{A}$ *itself. As a consequence of this circular situation, none of the arguments are accepted.*

An *argumentation line* is a sequence of defeating arguments, such as $[\mathcal{A}, \mathcal{B}]$ or $[\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{A}]$ in the system of example 1.1. Whenever an argument $\mathcal{A}$ is encountered while analyzing arguments for and against $\mathcal{A}$, a circularity occurs. Some systems apply a single restriction to argumentation lines: no previously considered argument is reintroduced in the process. In [2, 7] the relation between circularity in argumentation and the comparison criteria used in the system is established. Arguments in such situations are called *fallacious arguments* and the circularity itself is called a *fallacy*. In somes sistems as [4, 5] these arguments are classified as *undecided* arguments: they are not accepted nor rejected.

In this work we show that a more specific restriction need to be applied, other than prohibit reintroduction of previous arguments in argumentation lines. In the next section we define the abstract framework in order to characterize *acceptable* argumentation lines.

2 Abstract framework

Our argumentation framework [6] is formed by three elements: a set of arguments, a binary conflict relation over this set, and some function used to evaluate the relative difference of conclusive force for any pair of arguments.

Definition 2.1. *An argumentation framework* Φ *is a triplet* $\langle Args, \mathbf{C}, \pi \rangle$*, where* $Args$ *is a finite set of arguments,* $\mathbf{C}$ *is a binary conflict relation between arguments,* $\mathbf{C} \subseteq Args \times Args$*, and* $\pi : Args \times Args \longrightarrow 2^{Args}$ *is a preference function for conflicting arguments.*

Arguments are abstract entities, as in [1], denoted by uppercase letters. No reference to the underlying logic is needed. It is sufficient to know that arguments support conclusions, which are denoted here by lowercase letters. If $\mathcal{A}$ is an argument, then $\mathcal{A}^-$ is a subargument of $\mathcal{A}$, and $\mathcal{A}^+$ is a superargument of $\mathcal{A}$. Subscript index denotes different subarguments or superarguments of A, when needed. Also, the symbol $\sqsubseteq$ denotes subargument relation: $\mathcal{A} \sqsubseteq \mathcal{B}$ means "$\mathcal{A}$ is a subargument of $\mathcal{B}$". Any argument $\mathcal{A}$ is considered a superargument and a subargument of itself. Any subargument $\mathcal{B} \sqsubseteq \mathcal{A}$ such that $\mathcal{B} \neq \mathcal{A}$ is said to be a non-trivial subargument. Non-trivial subargument relation is denoted by symbol $\sqsubset$.

Example 2.1. *The 4-tuple* $AF =< Args, \mathbf{C}, \pi, \sqsubseteq >$ *is an argumentation framework, where*

- $Args = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$
- $\mathbf{C} = \{(\mathcal{C}, \mathcal{B}), (\mathcal{C}, \mathcal{A}), (\mathcal{E}, \mathcal{D}), (\mathcal{E}, \mathcal{C})\}$
- $\pi(\mathcal{C}, \mathcal{B}) = \{\mathcal{C}\}, \pi(\mathcal{E}, \mathcal{D}) = \{\mathcal{E}\}, ...$
- $\mathcal{B} \sqsubseteq \mathcal{A}, \mathcal{D} \sqsubseteq \mathcal{C}$.

The conflict relation between two arguments $\mathcal{A}$ and $\mathcal{B}$ denotes the fact that these arguments can not be accepted simultaneously, usually because they contradict each other. For example, two arguments $\mathcal{A}$ and $\mathcal{B}$ that support complementary conclusions h and $\neg h$ can not be accepted together. The set of all conflict relations on AF is denoted by $\mathbf{C}$. For any argument $\mathcal{A}$, the set of all arguments in conflict with $\mathcal{A}$ is denoted by $Conf(\mathcal{A})$. Given a set of arguments T, an argument $\mathcal{A} \in S$ is said to be in conflict in S, if there is an argument $\mathcal{B} \in S$ such that $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$. Conflict relations are propagated to superarguments, as stated in the next lemma.

Lemma 2.1 (Conflict inheritance). *Let* $\mathcal{A}$ *and* $\mathcal{B}$ *be two arguments in* AF. *If* $\mathcal{A}$ *and* $\mathcal{B}$ *are in conflict, then the conflict is inherited by any superargument of* $\mathcal{A}$ *and* $\mathcal{B}$. *That is, if* $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$, *then*

- $(\mathcal{A}, \mathcal{B}^+) \in \mathbf{C}$,
- $(\mathcal{A}^+, \mathcal{B}) \in \mathbf{C}$,
- $(\mathcal{A}^+, \mathcal{B}^+) \in \mathbf{C}$,

for any superarguments $\mathcal{A}^+$ *and* $\mathcal{B}^+$.

The constraints imposed by the conflict relation leads to several sets of possible accepted arguments. For example, if $Args = \{\mathcal{A}, \mathcal{B}\}$ and $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$, then $\{\mathcal{A}\}$ is a set of possible accepted arguments, and so is $\{\mathcal{B}\}$. Therefore, a clear decision must be taken. In order to accomplish this task, function π is used to evaluate arguments, comparing them to establish a preference based on the conclusive force.

Definition 2.2. *An argument comparison criterion is a function* $\pi : S \times S \rightarrow 2^S$, *where* S *is the set of arguments in the framework and* $\pi(\mathcal{A}, \mathcal{B}) \subseteq \mathcal{P}(\{\mathcal{A}, \mathcal{B}\})$. *If* $\pi(\mathcal{A}, \mathcal{B}) = \{A\}$ *then* $\mathcal{A}$ *is preferred to* $\mathcal{B}$. *In the same way, if* $\pi(\mathcal{A}, \mathcal{B}) = \{B\}$ *then* $\mathcal{B}$ *is preferred to* $\mathcal{A}$. *If* $\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{A}, \mathcal{B}\}$ *then* $\mathcal{A}$ *and* $\mathcal{B}$ *are arguments with equal relative strength. If* $\pi(\mathcal{A}, \mathcal{B}) = \emptyset$ *then* $\mathcal{A}$ *and* $\mathcal{B}$ *are incomparable arguments.*

For two arguments $\mathcal{A}$ and $\mathcal{B}$, such that $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$ there are four possible outcomes:

- $\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{A}\}$. In this case a *defeat* relation is established. Because $\mathcal{A}$ is preferred to $\mathcal{B}$, in order to accept $\mathcal{B}$ it is necessary to analyze the acceptance of $\mathcal{A}$, but not the other way around. It is said that argument $\mathcal{A}$ *defeats* argument $\mathcal{B}$, and $\mathcal{A}$ is a *proper defeater* of $\mathcal{B}$.

- $\pi(\mathcal{A},\mathcal{B}) = \{\mathcal{B}\}$. In a similar way, argument $\mathcal{B}$ *defeats* argument $\mathcal{A}$, and therefore $\mathcal{B}$ is a *proper defeater* of $\mathcal{A}$.

- $\pi(\mathcal{A},\mathcal{B}) = \{\mathcal{A},\mathcal{B}\}$. Both arguments are equivalent, i.e, there is no relative difference of conclusive force, so $\mathcal{A}$ and $\mathcal{B}$ are said to be *indistinguishable*. No proper defeat relation can be established between these arguments.

- $\pi(\mathcal{A},\mathcal{B}) = \emptyset$. Both arguments are *incomparable* and no proper defeat relation is established.

In the first two cases, a concrete preference is made between two arguments, and therefore a defeat relation is established. The preferred arguments are called *proper defeaters*. In the last two cases, no preference is made, either because both arguments are indistinguishable to each other or they are incomparable. The conflict between these two arguments remains unsolved. Due to the fact that the conflict relation is a symmetric relation, an argument *blocks* the other one and it is said that both of them are *blocking defeaters*. An argument $\mathcal{B}$ is said to be a $defeater$ of an argument $\mathcal{A}$, if $\mathcal{B}$ is a blocking or a proper defeater of $\mathcal{A}$. Semantic functions used to characterize the set of accepted arguments in this framework can be found in [6].

Some authors leave the preference criteria unspecified, even when its one of the most important components in the system. However, in many cases it is sufficient to establish a set of properties that the criteria must exhibit. A very reasonable one states that an arguments is as strongest as it weakest subargument [8]. We formalize this idea in the next definition.

Definition 2.3 (Monotonic preference relation). *A preference relation π is said to be monotonic if, given $\pi(\mathcal{A},\mathcal{B}) = \{\mathcal{A}\}$, then $\pi(\mathcal{A},\mathcal{B}) = \pi(\mathcal{A},\mathcal{B}_i^+)$, for any arguments $\mathcal{A}$ and $\mathcal{B}$ in AF.* [1]

We will assume from now on that the criterion π included in Φ is monotonic. This is important because any argument $\mathcal{A}$ defeated by another argument $\mathcal{B}$ should also defeated by another argument $\mathcal{B}^+$.

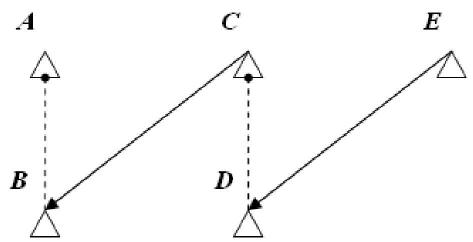

Figure 1: An abstract argumentation framework

In figure 1, a simple framework is depicted corresponding to example 2.1, where dotted lines denotes subargument relations and defeat relations are denoted by arrows. Here argument $\mathcal{C}$ defeats $\mathcal{B}$, but it should also be a defeater of $\mathcal{A}$, because $\mathcal{B}$ is its subargument. The same is true for arguments $\mathcal{E}, \mathcal{C}$ and $\mathcal{D}$.

[1]Note that $\pi(\mathcal{A},\mathcal{B}) = \pi(\mathcal{B},\mathcal{A})$.

3 Argumentation semantics

In [1] several semantic notions are defined. Other forms of clasifying arguments as *accepted* or *rejected* can be found in [4, 5]. However, these concepts are applied to abstract frameworks with single attack relation, as the one originally shown by Dung. It is widely accepted that defeat between arguments must be defined over two basic elements: contradiction and comparison. The first one states when two arguments are contradictory and therefore can not be accepted simultaneously. The second one determines which of these argument is preferred to the other, using a previously defined comparison method. Due to the possibility of lack of decision at comparison stage, the outcome of this process is not always equivalent to an attack relation as in [1]. According to this situation, our framework includes two kind of relations: proper defeat and blocking defeat. We will focus in this section on the task of defining the structure of a well-formed argumentation line, from an abstract point of view.

Definition 3.1 (Defeat path). *A* defeat path Δ *of an argumentation framework* $\langle Args, \mathbf{C}, \pi \rangle$*is a finite sequence of arguments* $[A_1, A_2, ..., A_n]$ *such that argument* A_{i+1} *is a defeater of argument* A_i *for any* $0 < i < n$. *The number of arguments in the path is denoted* $|\Delta|$.

A defeat path is a sequence of defeating arguments. The lenght of the defeat path is important for acceptance purposes, because an argument $\mathcal{A}$ defeated by an argument $\mathcal{B}$ may be reinstated by another argument $\mathcal{C}$. In this case, it is said that argument $\mathcal{C}$ *defends* $\mathcal{A}$ against $\mathcal{B}$. Note that three arguments are involved in a defense situation: the attacked, the attacker and the defender.

Definition 3.2 (Defeat paths for an argument). *Let* $AF =< Args, \mathbf{C}, \pi >$ *be an argumentation framework and* $\mathcal{A} \in Args$. *A defeat path for* $\mathcal{A}$ *is any defeat path* $[\mathcal{A}, \mathcal{D}_1, \mathcal{D}_2, ..., \mathcal{D}_n]$. *The set* $DP(\mathcal{A})$ *is the set of all defeat paths for* $\mathcal{A}$.

If the lenght of a defeat path for argument $\mathcal{A}$ is odd then the last argument in the sequence is playing a *defender* role. If the lenght is even, then the last argument is playing an *attacker* role [2, 3].

Definition 3.3 (Defending and interfering paths). *Let* Φ *be an argumentation framework,* $\mathcal{A}$ *an argument in* Φ *and* Δ *a defeat path for* $\mathcal{A}$. *If* $|\Delta|$ *is odd then* Δ *is said to be a* defending defeat path *for* $\mathcal{A}$. *If* $|\Delta|$ *is even, then* Δ *is said to be an* interfering defeat path *for* $\mathcal{A}$.

The notion of defeat path is very simple and only requires that any argument in the sequence must defeat the previous one. Under this unique constraint, which is the basis of argumentation processes, it is possible to obtain some controversial structures, as shown in the next examples.

Example 3.1. *Let* $\Phi = \langle Args, \mathbf{C}, \pi \rangle$ *an argumentation framework where*

$Args = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$
$\mathbf{C}$=$\{(\mathcal{A}, \mathcal{B}), (\mathcal{B}, \mathcal{C}), (\mathcal{A}, \mathcal{C})\}$ *and*
$\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{B}\}, \pi(\mathcal{B}, \mathcal{C}) = \{\mathcal{C}\}, \pi(\mathcal{A}, \mathcal{C}) = \{\}$

The sequence $\Delta = [\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{A}]$ is a defeat path in Φ, because $\mathcal{B}$ is a proper defeater of $\mathcal{A}$, $\mathcal{C}$ is a proper defeater of $\mathcal{B}$ and $\mathcal{A}$ and $\mathcal{C}$ are blocking defeaters of each other. The argument $\mathcal{A}$ appears twice in the sequence, as the first and last argument. Note that in order to analyze the acceptance of $\mathcal{A}$, it is necessary to analyze the acceptance of every argument in Δ, including $\mathcal{A}$. This is a circular defeat path for $\mathcal{A}$.

Example 3.2. *Let $\Phi = \langle Args, \mathbf{C}, \pi \rangle$ an argumentation framework where*

$Args = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\, \mathcal{A}_1{}^-\}$
$\mathbf{C}$=$\{(\mathcal{A}_1{}^-, \mathcal{B}), (\mathcal{B}, \mathcal{C}), (\mathcal{A}_1{}^-, \mathcal{C})...\}$ *and*
$\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{B}\}, \pi(\mathcal{B}, \mathcal{C}) = \{\mathcal{C}\}, \pi(\mathcal{A}_1{}^-, \mathcal{C}) = \{\}, \pi(\mathcal{A}, \mathcal{C}) = \{\}$

In this framework a subargument of $\mathcal{A}$ is included. Because $(\mathcal{A}_1{}^-, \mathcal{B}) \in \mathbf{C}$ then also $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$. The same is true for $(\mathcal{A}, \mathcal{C})$, due the inclusion of $(\mathcal{A}_1{}^-, \mathcal{C})$ in $\mathbf{C}$. According to this, the sequence $\Delta = [\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{A}_1{}^-]$ is a defeat path in Φ, because $\mathcal{B}$ is a proper defeater of $\mathcal{A}$, $\mathcal{C}$ is a proper defeater of $\mathcal{B}$ and $\mathcal{A}_1{}^-$ and $\mathcal{C}$ are blocking defeaters of each other. Note that even when no argument is repeated in the sequence, the subargument $\mathcal{A}_1{}^-$ was already taken into account in the argumentation line, as argument $\mathcal{B}$ is its defeater. This sequence may be considered another circular defeat path for $\mathcal{A}$.

Controversial situations are clear in examples 3.1 and 3.2. In the next example some piece of information is repeated in the sequence, but this is not a controversial situation.

Example 3.3. *Let $\Phi = \langle Args, \mathbf{C}, \pi \rangle$ an argumentation framework where*

$Args = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\, \mathcal{A}_1{}^-, \mathcal{A}_2{}^-\}$
$\mathbf{C}$=$\{(\mathcal{A}_1{}^-, \mathcal{B}), (\mathcal{B}, \mathcal{C}), (\mathcal{A}_2{}^-, \mathcal{C})...\}$ *and*
$\pi(\mathcal{A}, \mathcal{B}) = \{\mathcal{B}\}, \pi(\mathcal{B}, \mathcal{C}) = \{\mathcal{C}\}, \pi(\mathcal{A}_2{}^-, \mathcal{C}) = \{\}, \pi(\mathcal{A}, \mathcal{C}) = \{\}$

Again, because $(\mathcal{A}_1{}^-, \mathcal{B}) \in \mathbf{C}$ then $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$. Also $(\mathcal{A}, \mathcal{C}) \in \mathbf{C}$, because $(\mathcal{A}_1{}^-, \mathcal{B}) \in \mathbf{C}$. According to this, the sequence $\Delta = [\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{A}_2{}^-]$ is a defeat path in Φ, because $\mathcal{B}$ is a proper defeater of $\mathcal{A}$, $\mathcal{C}$ is a proper defeater of $\mathcal{B}$ and $\mathcal{A}_2{}^-$ and $\mathcal{C}$ are blocking defeaters of each other. In this case, a subargument $\mathcal{A}_2{}^-$ of $\mathcal{A}$ appears in the defeat path for $\mathcal{A}$. However, this is not a controversial situation, as $\mathcal{A}_2{}^-$ was not involved in any previous conflict in the sequence. Argument $\mathcal{B}$ is defeating $\mathcal{A}$ just because $(\mathcal{A}_1{}^-, \mathcal{B}) \in \mathbf{C}$, and is not related to $\mathcal{A}_2{}^-$. Defeat path Δ is correctly structured.

Note that $[\mathcal{A}, \mathcal{C}]$ is also a defeat path for $\mathcal{A}$. In this case, as stated in example 3.2, $\mathcal{A}_2{}^-$ should not appear in the sequence.

The initial idea of restricting the inclusion of arguments previously considered in the sequence is not enough. The examples 3.1, 3.2 and 3.3 show that the characterization of well-formed argumentation lines requires more restrictions.

4 Acceptable defeat paths

In this section we present the concept of acceptable defeat paths, as defined in [3], but in the context of an abstract argumentation framework. First, we formalize the consequences of removing an argument from a set of arguments. This is needed because is important to identify the set of arguments available for use in evolving defeat paths.

Suppose S is a set of available arguments used to construct a defeat path Δ. If an argument $\mathcal{A}$ in S is going to be discarded in that process (*i.e.*, its information content is not taken into account), then every argument that includes $\mathcal{A}$ as a subargument should be discarded too.

Definition 4.1 (Argument extraction). *Let S be a set of arguments and $\mathcal{A}$ an argument in S. The operator $\triangleq$ is defined as*

$$S \triangleq \mathcal{A} = S - Sp(\mathcal{A})$$

where $Sp(\mathcal{A})$ is the set of all superarguments of $\mathcal{A}$.

In figure 2 the extraction of arguments is depicted: $S \triangleq \mathcal{A}$ excludes $\mathcal{A}$ and all of its superarguments.

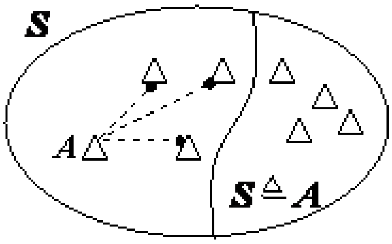

Figure 2: Argument extraction

Example 4.1. *Let $S = \{\mathcal{A}, \mathcal{A}^+, \mathcal{B}, \mathcal{B}^-, \mathcal{C}\}$ be a set of arguments. Then*

$S \triangleq \mathcal{A} \;=\; \{\mathcal{B}, \mathcal{B}^-, \mathcal{C}\}$ *and*
$S \triangleq \mathcal{B} \;=\; \{\mathcal{A}, \mathcal{A}^+, \mathcal{B}^-, \mathcal{C}\}$

As stated before in Lemma 2.1 , conflict relations are propagated through superarguments: if $\mathcal{A}$ and $\mathcal{B}$ are in conflict, then $\mathcal{A}^+$ and $\mathcal{B}$ are also conflictive arguments. On the other hand, whenever two arguments are in conflict, it is always possible to identify conflictive subarguments. This notion can be extended to defeat relations, as shown in the next lemma.

Lemma 4.1. *Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments such that $\mathcal{B}$ is a defeater of $\mathcal{A}$. Then there is a subargument $\mathcal{A}_i \sqsubseteq \mathcal{A}$ such that $\mathcal{B}$ is a defeater of $\mathcal{A}_i$.*

For any pair of conflictive arguments $(\mathcal{A}, \mathcal{B})$ there is another pair of conflictive arguments $(\mathcal{C}, \mathcal{D})$ where $\mathcal{C} \sqsubseteq \mathcal{A}$ and $\mathcal{D} \sqsubseteq \mathcal{B}$. Note that possibly $\mathcal{C}$ or $\mathcal{D}$ are trivial subarguments.

Definition 4.2 (Core conflict). *Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments such that $\mathcal{B}$ is a defeater of $\mathcal{A}$. A* core conflict *of $\mathcal{A}$ and $\mathcal{B}$ is a pair of arguments $(\mathcal{A}_i, \mathcal{B})$ where*

- *$\mathcal{A}_i \sqsubseteq \mathcal{A}$,*
- *$\mathcal{B}$ is a defeater of $\mathcal{A}_i$ and*

- *there is no other argument $\mathcal{A}_j \sqsubset \mathcal{A}_i$ such that $\mathcal{A}_j$ is defeated by $\mathcal{B}$.*

The core conflict is the underlying cause of a conflict relation between two arguments, due to the inheritance property. It is possible to identify the real disputed subargument, which is causing other arguments to fall in conflict.

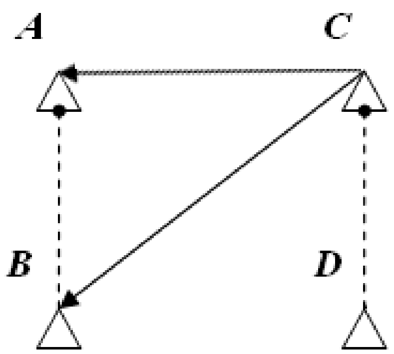

Figure 3: Argument $\mathcal{B}$ is a core conflict

In figure 3 argument $\mathcal{C}$ defeats $\mathcal{A}$ because it is defeating one of its subarguments $\mathcal{B}$. The core conflict of $\mathcal{A}$ and $\mathcal{C}$ is $\mathcal{B}$. In this case the defeat arc between the superarguments may not be drawn.

Definition 4.3 (Disputed subargument). *Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments such that $\mathcal{B}$ is a defeater of $\mathcal{A}$. A subargument $\mathcal{A}_i \sqsubseteq \mathcal{A}$ is said to be a* disputed subargument *of $\mathcal{A}$ with respect to $\mathcal{B}$ if $\mathcal{A}_i$ is a core conflict of $\mathcal{A}$ and $\mathcal{B}$.*

The notion of *disputed subargument* is very important in the construction of defeat paths in dialectical processes. Suppose argument $\mathcal{B}$ is a defeater of argument $\mathcal{A}$. It is possible to construct a defeat path $\Delta = [\mathcal{A}, \mathcal{B}]$. If there is a defeater of $\mathcal{B}$, say $\mathcal{C}$, then $[\mathcal{A}, \mathcal{B}, \mathcal{C}]$ is also a defeat path. However, $\mathcal{C}$ should not be a disputed argument of $\mathcal{A}$ with respect to $\mathcal{B}$, as circularity is introduced in the path. Even more, $\mathcal{C}$ should not be an argument that *includes* that disputed argument, because that path can always be extended by adding $\mathcal{B}$ again.

The set of arguments available to be used in the construction of a defeat path is formalized in the following definition.

Definition 4.4 (Defeat domain). *Let $AF =< Args, \mathbf{C}, \pi >$ be an argumentation framework and let $\Delta = [\mathcal{A}_1, \mathcal{A}_2, ..., \mathcal{A}_n]$ be a defeat path in AF. The function $D^i(\Delta)$ is defined as*

- $D^0(\Delta) = Args$
- $D^k(\Delta) = D^{k-1}(\Delta) \trianglelefteq \mathcal{B}_n$, *where $\mathcal{B}_n$ is the disputed subargument of $\mathcal{A}_{k-1}$ with respect to $\mathcal{A}_k$ in the sequence.*

The defeat domain discards controversial arguments. The function $D^k(\Delta)$ denotes the set of arguments that can be used to extend the defeat path Δ at stage k, *i.e.*, to defeat the argument $\mathcal{A}_k$. Choosing an argument from $D^k(\Delta)$ avoids the introduction of previous disputed arguments in the sequence. It is important to remark that if an argument including a previous disputed subargument is reintroduced in the defeat path, it is always possible to reintroduce its original defeater.

Therefore, in order to avoid controversial situations, any argument $\mathcal{A}_i$ of a defeat path Δ should be in $D^{i-1}(\Delta)$. Selecting an argument outside of this set implies the repetition of previously disputed information. The following definition characterizes well structured sequences of arguments, called *acceptable defeat paths*.

Definition 4.5 (Acceptable defeat path). *Let* $AF =< Args, \mathbf{C}, \pi >$ *be an argumentation framework. An acceptable defeat path is defined recursively in the following way:*

- $[\mathcal{A}]$ *is an acceptable defeat path, for any* $\mathcal{A} \in Args$.
- *If* $\Delta = [\mathcal{A}_1, \mathcal{A}_2, ..., \mathcal{A}_n]$, $n \geq 1$ *is an acceptable defeat path, then for any defeater* $\mathcal{B}$ *of* $\mathcal{A}_n$ *such that* $\mathcal{B} \in D^n(\Delta)$, $\Delta' = [\mathcal{A}_1, \mathcal{A}_2, ..., \mathcal{A}_n, \mathcal{B}]$ *is an acceptable defeat path.*

Defeat paths of examples 3.1 and 3.2 are not acceptable. Acceptable defeat paths are free of circular situations and guarantees progressive argumentation, as desired on every dialectical process. Note that it is possible to include a subargument of previous arguments in the sequence, as long as it is not a disputed subargument.

5 Conclusions

Abstract argumentation systems are formalisms for argumentation, where some components remains unspecified, usually the structure of arguments. In the dialectical process carried out to identify accepted arguments in the system, some controversial situations may be found, related to the reintroduction of arguments in this process, causing a circularity that must be treated in order to avoid an infinite analysis process. Some systems apply a single restriction to argumentation lines: no previously considered argument is reintroduced in the process. In this work we have shown that a more specific restriction need to be applied, taking subarguments into account. We finally presented a new definition of *acceptable argumentation lines*, based on the concept of *defeat domain*, where superarguments of previously disputed arguments are discarded.

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