

Current Applications of Ant Systems for Subset Problems

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1. Abstract

Early applications of Ant Colony Optimization (ACO) have been mainly concerned with solving ordering problems (e.g., the Traveling Salesman Problem). In this report we describe an Ant System algorithm, which would be appropriate for solving additional subset problems as was showed for solving the multiple knapsack problem in previous works. The experiments on progress show the potential power of the ACO approach for solving different subset problems.

1. Introduction

The Ant Colony Optimization (ACO) technique has emerged recently [8,9,10] as a new meta-heuristic for hard combinatorial optimization problems. ACO algorithms, that is, instances of the ACO meta-heuristics, are basically a multi-agent system where low level interactions between single agents (called artificial ants) result in a complex behavior of the whole system. ACO algorithms have been inspired by colonies of real ants [8], which deposit a chemical substance (called *pheromone*) on the ground. This substance influences the choices they make: the larger amount of pheromone is on a particular path, the larger probability is that an ant selects the path. Artificial ants, in ACO algorithms, behave in similar way.

ACO algorithms can be applied to discrete optimization problems that can be characterized as a graph $G=(C,L)$, where C is a finite set of *components* and $L \subseteq C \times C$ the set of *connections* between the components (see [8] for a complete description). The solutions to the optimization problem can be expressed in terms of feasible paths on the graph G . Thus, ACO algorithms can be used to find minimum cost paths (sequences) feasible with respect to the constraints Ω . For example, in the traveling salesman problem C is the set of cities, L is the set of arcs connecting cities, and Ω indicates that a solution ψ must be a Hamiltonian circuit.

In ACO algorithms a population (colony) of agents (or ants) collectively solve the optimization problem under consideration by using the above graph representation. Information collected by the ants during the search process is encoded in *pheromone trail* τ_{ij} associated to connection l_{ij} . Pheromone trails encode a long-term memory about the whole ant search process. Depending on the problem representation chosen, pheromone trails can be associated to all arcs of problem, or only to some of them. Arcs can also have an associated *heuristic value* η_{ij} representing *a priori* information about the problem instance definition or run-time information provided by a source different from the ants.

A subset problem can be seen in terms of the ACO graph representation where the connectivity is complete and, once a component is chosen, pheromone is deposited on all the connections among the component and all the other components. It is clear that a direct implementation of this would result inefficient. In the following we propose a straightforward implementation of the general concept of the ACO graph representation based on an Ant System, which takes into account the computational efficiency. Early experiments with ACO algorithms were connected with ordering problems such as the Traveling Salesman Problem and the Quadratic Assignment Problem [7]. More recently an ACO algorithm [14] was applied successfully to the MKP, an example of subset problem. This report briefly evaluates the feasibility of applying an ACO algorithm to other subset problems according to the general concept behind an ACO heuristic. The subset problems under consideration involve the set covering problem (SCP) and the maximum independent problem (MISP). These *NP-complete* problems are intractable by their nature, or sufficiently large to preclude the use of exact algorithms. In such cases, for the above subset problems as well as other combinatorial optimization problems, heuristics methods are

usually employed to find good, but not necessarily optimal solutions. The effectiveness of these methods depends upon their ability to adapt to a particular realization, avoid entrapment at local optima, and exploit the basic structure of the problem, such as a network or a natural ordering among its components. Various heuristic search techniques have been developed that have demonstrably improved our ability to obtain good solutions to difficult combinatorial optimization problems. Such techniques include simulated annealing, tabu search; greedy randomized adaptive search procedures (GRASP), genetic algorithms and more recently ant colony optimization.

Some studies include the application of evolutionary algorithms to the MKP. Khuri et al. [13] use the software package GENEsYs where a graded penalty term is applied to infeasible solutions. The test problems involved a set of instances from 15 to 50 components and from 5 to 10 constraints. More recently Beasley et al. [6] applied successfully genetic algorithms to a set of a larger instances of MKP by using specialized operators. Beasley [3,4] presented two heuristic algorithms applied to the SCP which include dual ascent, subgradient optimization, linear programming and Lagrange relaxation. Resende et al. [15,16] proposed an iterative randomized technique (GRASP) for solving a set of small but difficult set covering problems that arise when computing the 1-width of incidence matrices of Steiner triple systems. Larger instances of SCP were tested by using genetic algorithms [5] with specialized genetic operators. Grossman [12] conducted a comparative study of nine different approximation algorithms for the SCP, including several greedy variants, fractional relaxations, randomized algorithms and neural networks. Results for some small instances of MISP are reported in [1]. In Resende et al. [15], GRASP is applied to a set of larger and difficult instances of MISP.

2. Ant System for Subset Problems

The subset-based and permutation-based Ant Systems have many features in common. However, in the permutation-based Ant System the pheromone is laid on *paths* while for subset problems no path exists connecting the items. A subset-based Ant System takes advantage of one of the central ideas involved in the selection process of a permutation-based ant system: "the more amount of trail on a particular *path*, the more profitable is that *path*". This idea was adapted here in the following way: "the more pheromone trail on a particular *item*, the more profitable that *item* is". In other words, we move the pheromone from paths to items. At the same time, a local heuristic is also used in the new version, but now it considers *items* only instead of *connections* between them. Thus, the original Ant System must be modified accordingly. First of all, the pheromone trail is now laid on each element from set C, with the intended meaning that elements with a higher trail level are more profitable. For all subset problems under consideration, our version of the Ant System considers a special type of heuristic that takes into account both, problem knowledge and the partial solution being built by a particular ant.

The outline of the new version of the Ant System algorithm for subset problems is as follows:

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initialize
for t=1 to number of cycles do
  for k=1 to number of ants do
    repeat until solutionk is completed
      select item i to be incorporated with probability  $P(k, \tau_i, \eta_i)$ 
    end
    calculate the cost of solutionk
    save the best solution so far
  end
  update the trail levels  $\tau_i$  on all items
end
print the best solution found

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3. The Ant System applied to the Subset Problems

In order to apply the Ants System to SCP and MISP, it was only necessary to select appropriately the heuristic value η which influences the probability of item selection during the solution construction step. For each subset problems we defined a very simple heuristic based on earlier application of evolutionary algorithms for solving SCP [3,5] and MISP [1] respectively. In the experiments we considered a large number of well known instances of SCP [2] with sizes ranging from 200 to 1000 rows and from 1000 to 10000 columns. For all of them a reduction process was applied in order to decrease their sizes and hence increase the performance of the Ant System. On the other hand, several instances of MISP were generated by applying three different generation processes taken from literature [1,15]. The size of MISP instances considered ranged from 100 to 1000 nodes in the graph.

The Ant System was designed for running on a parallel platform. Its behavior was compared against an evolutionary algorithm called GENESyS [1,13] and some results reported in literature. The results obtained show that the performance of the Ant Systems on many test cases is comparable to the performance of "state-of-the-art" algorithms for solving the SCP and MISP.

Current applications of Ant Systems involve numerical optimization problems. Some preliminary results indicate the applicability of this approach to highly constrained continuous search spaces.

References

- [1] Bäck, T. and Khuri, S. (1994). "An Evolutionary Heuristic for the Maximum Independent Set Problem", Proceedings of the First International Conference on Evolutionary Computation. M. Michalewicz et al (eds) - (Piscataway, NJ; IEEE Press) pp 531-535.
- [2] Beasley, J., "OR-Library: Distributing Test Problems by Electronic Mail". e-mail: o.rlibrary@ic.ac.uk.
- [3] Beasley, J. (1987) "An Algorithm for Set Covering Problem". European Journal of Operational Research, vol 31, pp 85-93.
- [4] Beasley, J. (1990) "A Lagrangian Heuristics for Set Covering Problems". Naval Research Logistics, vol 37, pp. 151-164.
- [5] Chu, P. and Beasley, J (1996) "A Genetic Algorithm for the Set Covering Problem". European Journal of Operational Research, vol 58 , pp 392-404.
- [6] Chu, P. and Beasley, J. (1998). "A Genetic Algorithm for the Multi-constraint Knapsack Problem". Journal of Heuristics, Vol.4, pp.63-86.
- [7] Dorigo M., V. Maniezzo, and A. Colorni (1991) "The Ant System Applied to the Quadratic Assignment Problem". Technical Report IRIDIA 94/28, Université Libre de Bruxelles, Belgium.
- [8] Dorigo M. and G. Di Caro (1999). "The Ant Colony Optimization Meta-Heuristic". In D. Corne, M. Dorigo and F. Glover (eds), New Ideas in Optimization. McGraw-Hill, 1999. (Also available as: Tech. Rep. IRIDIA/99-1, Université Libre de Bruxelles, Belgium.)
- [9] Dorigo M., Di Caro G., Gambardella, L.M. (1999) "Ant Algorithms for Discrete Optimization". Artificial Life, 5(2):137-172 (Also available as Tech. Rep. IRIDIA/98-10, Université Libre de Bruxelles, Belgium)
- [10] Dorigo M. and L.M. Gambardella (1997). "Ant Colony System: A Cooperative Learning Approach to the Traveling Salesman Problem". IEEE Transactions on Evolutionary Computation, Vol.1, No.1, pp.53-66.
- [11] Dorigo M., V. Maniezzo, and A. Colorni (1991). "Positive feedback as a search strategy". Tech. Rep. No. 91-016, Politecnico di Milano, Italy.
- [12] Grossman, T. and Wool, A. (1997) "Computational Experience with Approximation Algorithms for the Set Covering Problem". European Journal of Operational Research, 101:81-92.
- [13] Khuri, S.; Bäck, T. and Heitkötter, J. (1994). "The Zero/One Multiple Knapsack Problem and Genetic Algorithms". Proceedings of the 1994 ACM Symposium on Applied Computing, E.Deaton and D.Oppenheim and J.Urban and H.Berghel (eds), pp.188-193. ACM Press, New York.
- [14] Leguizamón, G. And Michalewicz, Z. (1999) "A New Version of Ant Systems for Subset Problems". Proceedings of Congress on Evolutionary Computation, vol II, pp 1459-1468.
- [15] Redende, C.; Feo, T.; Mauricio, G. And Smith, S. (1994) "A Greedy Randomized Adaptive Search Procedure for Maximum Independent Problems". Operations Research , vol 42, pp 860-878.
- [16] Redende, C.; Feo, T. and Mauricio, G. (1994) "A Greedy Randomized Adaptive Search Problems". Journal of Global Optimization, vol 6, pp 109-133.