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Orbital-angular-momentum-multiplexed free-space optical communication link using transmitter lenses

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In this letter, we explore the potential benefits and limitations of using transmitter lenses in an orbital-angular-momentum (OAM)-multiplexed free-space optical (FSO) communication link. Both simulation and experimental results indicate that within certain transmission distances, using lenses at the transmitter to focus OAM beams could reduce power loss in OAM-based FSO links, and that this improvement might be more significant for higher-order OAM beams. Moreover, the use of transmitter lenses could enhance system tolerance to angular error between transmitter and receiver, but might degrade tolerance to lateral displacement.

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1. INTRODUCTION

Free-space optical (FSO) communication links can potentially benefit from the simultaneous transmission of multiple, spatially orthogonal beams through a single transmitter-receiver aperture pair [1-2]. In general, when transmitting multiple orthogonal beams—each carrying an independent data stream—through the same medium, the total capacity could be multiplied by the number of beams [3-5]. The use of orbital-angular-momentum (OAM) beams as an orthogonal modal basis set in FSO communication links has received some attention in the literature [6-7]. A beam carrying OAM presents a helical "twisting" phase-front as it propagates, described by $\exp(i\ell\varphi)$, where ℓ represents the OAM order and φ is the azimuthal angle. OAM beams with distinct ℓ are orthogonal to one another [8-9]. Orthogonality among these beams enables efficient multiplexing at the transmitter and demultiplexing at the receiver [10].

However, due to the beams' phase and amplitude structure, OAM multiplexed FSO communication links present unique challenges. An OAM beam has two important characteristics: (1) It has a "doughnut-shape" intensity profile, with little power at the center but more power at the annulus; (2) It has a rich phase-change at the center but less around the annulus [11-13]. To correctly recover an OAM beam, the rapid phase change that occurs in the center needs to be collected in order to ensure orthogonality among the OAM beams, while on the other hand, sufficient optical power should also be received to satisfy the system requirement for optical signal-to-noise-ratio (OSNR) [14-15]. Therefore, sufficient capture of an OAM beam is critical for an operational communication system and will thus limit the transmission distance and the number of modes that can be supported [16]. One approach to achieve sufficient capture is to increase the transmitter and/or receiver aperture sizes, although this might increase total cost and the weight of the FSO link, and degrade system flexibility. Another approach is to use lenses at the transmitter to focus OAM beams, thus achieving smaller beam sizes at the receiver [17]. For an OAM-multiplexed FSO link, it would be worth investigating the effect of using transmitter lenses, on both the performance and robustness of the system.

In this letter, we show the performance effects of using transmitter lenses in an OAM-multiplexed FSO communication link, through both simulation and experiment, exploring its potential benefits and limitations for system performance and robustness. Simulation results indicate that in 1 km and 10 km OAM-based FSO links, using transmitter lenses to focus OAM beams could reduce power loss; and that the use of transmitter lenses may reduce the power loss for higher-order OAM beams by more than 15 dB in both links. For system robustness, we investigate the effect of using transmitter lenses under conditions of link misalignment (i.e., angular error or displacement between the transmitter and receiver). Both simulation and experimental results show that transmitter lenses could also reduce angular-error-induced channel crosstalk by ~10 dB, while on the other hand channel crosstalk resulting from displacement might increase. A 1m FSO communication link multiplexing four different OAM beams each carrying 100 Gbit/s quadrature phase-shift keying (QPSK) signal was established in a laboratory environment. The results show that power loss could be reduced by >20 dB and that channel crosstalk could be reduced by 10~20 dB under angular errors, by using transmitter lenses.

2. CONCEPT AND SIMULATION

The concept and model simulating the use of transmitter lenses in an OAM-multiplexed FSO communication link are shown in Fig. 1. Independent data streams are carried by collimated Gaussian beams at the wavelength of 1550 nm. Each beam pass through a spiral phase plate (SPP) with a specific OAM order, to convert the data-carrying Gaussian beam into a corresponding data-carrying OAM beam. Multiple OAM beams are then multiplexed and passed through a pair of transmitter lenses (f_1 and f_2) before transmitting in free space. The equivalent focal length of the lens pair f_0 satisfies: $1/f_0 = 1/f_1 + 1/f_2 - d/f_1 f_2$, where the lens-spacing offset Δ is defined as $f_1 + f_2 - d$, with d representing the center-to-center spacing between these two lenses. Note that such a structure is widely used in traditional FSO systems as a telescope where the output beam is collimated (i.e., Δ=0) [18]. In our simulation, we use these transmitter lenses to focus OAM beams at the receiver by tuning Δ, as shown in Fig. 1. In the simulation, OAM beams with or without transmitter lenses are numerically propagated by using Kirchhoff–Fresnel diffraction to the receiver aperture placed at a certain transmission distance. At the receiver, for a specific incoming OAM beam, its power distribution in different OAM modes is analyzed via modal decomposition, which is based on the assumption that the power of a desired OAM beam could be perfectly collected by its own receiver, and that there is no power loss during the demultiplexing process [16].

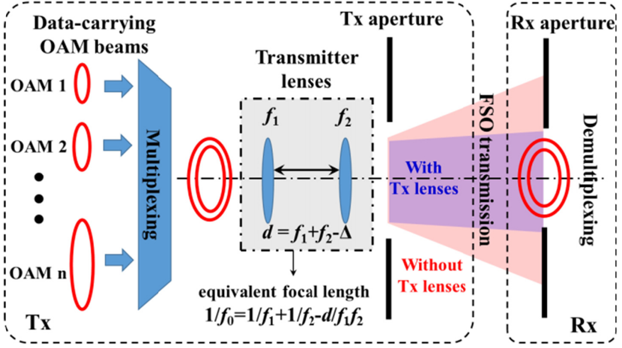

Fig. 1. Concept of an OAM-multiplexed free-space optical communication link using a pair of transmitter lenses. Tx: transmitter; Rx: receiver; f_0: equivalent focal length; d: center-to-center spacing between the two transmitter lenses; Δ: spacing offset between two transmitter lenses.

Since an OAM beam diverges when it propagates in free-space, its spot size might be larger than the hard-truncation receiver aperture. This results in a power loss that grows with the propagation distance and OAM order, as Fig. 2(a) and 2(b) shows. Transmitter lenses which focus the beams could be a potential solution up to a given propagation distance. Figure 2(b) shows that in an OAM-based FSO communication link, using transmitter lenses reduces power loss caused by beam divergence and limited size apertures. In this simulation, the transmitted beam diameter, which is defined as twice of the beam waist, is 10 cm; both transmitter and receiver aperture diameters are 10 cm; the equivalent focal length of the transmitter lenses, f_0, is 1 km. Figure 2(c) shows that in a 1 km FSO communication link with a transmitted beam diameter of 10 cm and both transmitter and receiver aperture diameters of 10 cm and $f_1 = f_2 = 0.5$ m, power loss decreases when the equivalent focal length f_0 of transmitter lenses is adjusted to be around the transmission distance. Moreover, both Fig. 2(b) and Fig. 2(c) show that higher-order OAM beams (e.g., OAM +7 in this simulation) would show greater benefit, which might result from their divergence characteristics. In Fig. 2(d), we also show the use of beam transmitter lenses to reduce power loss in a 10 km link that has a transmitted beam diameter of 30 cm and aperture diameters of 30 cm, and $f_1 = f_2 = 1$ m. Note that for FSO links of longer distances, more accurate adjustment of Δ would be required.

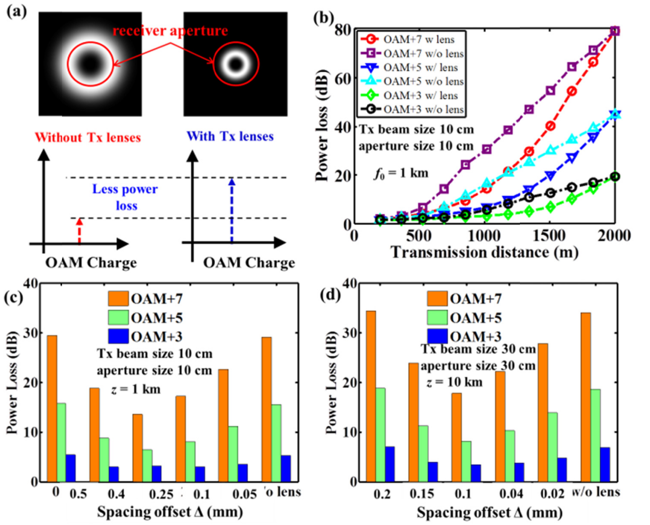

Fig. 2. (a) Intensity profiles of OAM beams with limited-size apertures, with and without transmitter lenses at the receiver. (b) Simulated power loss as a function of transmission distance of different orders of OAM beams; (c) and (d): Simulated power loss as a function of spacing offset between two transmitter lenses in 1 km and 10 km link, respectively. In (b), both transmitted beam size and aperture size are 10 cm, and the equivalent focal length of transmitter lenses is 1 km. In (c) and (d): both transmitted beam sizes and aperture sizes are 10 cm in (c), and 30 cm in (d); focal lengths of transmitter lenses are 0.5 m in (c) and 1 m in (d). Tx: transmitter; f_0: equivalent focal length of transmitter lenses; OAM ℓ: beam carrying OAM of ℓ; z: transmission distance.

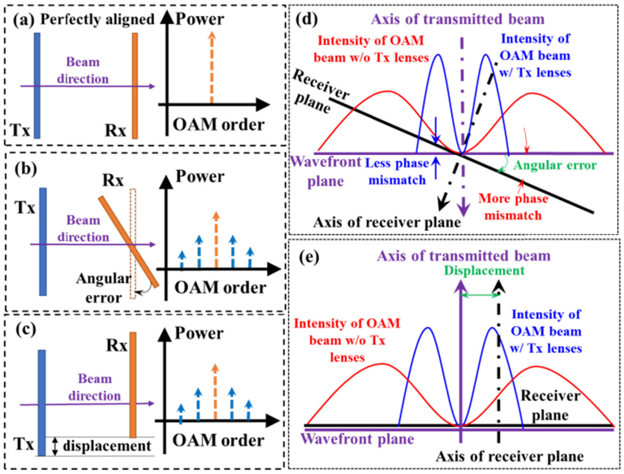

Fig. 3. Alignment between transmitter and receiver as well as received OAM spectrum in: (a) Perfectly aligned link; (b) Link with only angular error; (c) Link with only displacement (Tx: Transmitter; Rx: Receiver); (d) and (e): At the receiver, OAM beams with and without transmitter lenses under angular error and displacement, respectively.

We next analyze the effect of transmitter lenses on the robustness of FSO communication links. Ideally, transmitter and receiver are perfectly aligned, i.e., the centers of the receiver and the transmitted beam overlap, and both transmitter and receiver are perpendicular to the line connecting their centers (see Fig. 3(a)). When transmitter and receiver are perfectly aligned, all power of the transmitted OAM beam is received at the desired OAM mode, with no power leakage to neighboring modes. However, in a practical system, transmitter and receiver may have angular error; i.e., the center of the receiver still overlap that of the transmitter, but the receiver may have an angular shift (see Fig. 3(b)), and/or displacement; i.e., the receiver is still perpendicular to the transmission direction but its center has lateral shift (see Fig. 3(c)) [16]. Under displacement and/or angular error, some power of the transmitted OAM beam will spread into neighboring modes, thus causing channel crosstalk, as shown in Figs. 3(b) and 3(c). Here, channel crosstalk of a specific channel is defined as the power received on a desired channel when all other channels are transmitted and this desired channel is off (P_{others}), over the received power on this desired channel when only this specific channel is transmitted (P_{self}): $\eta_{xtalk}(\mathrm{dB}) = 10\log_{10}(P_{others}/P_{self})$.

Careful design of the transmitter lenses could help reduce power loss, and but it is also useful to know its effect on the robustness of an OAM-multiplexed FSO link under angular error or displacement. This is because angular error would introduce phase mismatch between the incoming OAM beam and the receiver plane due to different additional phase-shift increasing linearly along the radial direction (see Fig. 3(d)). Therefore, using transmitter lenses might reduce phase mismatch and, consequently, power leakage. Conversely, under the same displacement, beams with larger spot size would experience smaller mismatch, as Fig. 3(e) illustrates. Therefore, using transmitter lenses may increase channel crosstalk due to lateral displacement. Angular error and displacement could exist simultaneously in a practical FSO link, necessitating a trade-off in designing how the OAM beams are focused when using transmitter lenses at a certain level of angular error and displacement.

Simulated power distributions of OAM beam as a function of angular error with and without transmitter lenses are depicted in Fig. 4 (a) and 4(b), respectively. Power distributions as a function of displacement with and without transmitter lenses are shown in Fig. 4(c) and 4(d), respectively. In these simulations, OAM beam of $\ell = +3$ is transmitted to a distance of 1 km, with transmitted beam size and aperture sizes diameter 30 cm diameter, and focal length of transmitter lenses equivalent to 1 km. Figure 4(a) shows less power leakage to neighboring modes compared with Fig. 4(b) under the same angular error, indicating that the use of transmitter lenses could enhance system robustness under angular error, through possible decrease in channel crosstalk. Conversely, using transmitter lenses might reduce the link tolerance of displacement, as Figs. 4(c) and 4(d) indicate.

3. EXPERIMENTAL SETUP AND RESULTS

An experiment was conducted in a laboratory environment, over a transmission distance of ~1 m, in which spatial light modulators (SLMs) are functioned as SPP in simulation, (see Fig. 5). A narrow linewidth laser at 1550 nm is sent to a Mach–Zehnder modulator to produce a 100 Gbit/s QPSK signal. After amplification, this signal is split into two copies, one of which is delayed using a ~10 m length of single-mode-fiber (SMF) to decorrelate the data sequence. The two fiber branches are coupled to collimators, each of which emits a collimated Gaussian beam of diameter 2.2 mm. One of the Gaussian beams is converted to OAM+1 by SLM-1, while the other is converted to OAM+3 by SLM-2. After being combined via a beam splitter (BS-1), the multiplexed OAM beams are split into two identical copies by another beam splitter (BS-2). One of the copies is reflected by three mirrors to generate OAM -1 and OAM -3 beams, which are then multiplexed with the other copy (i.e., OAM +1 and +3) by a third beam splitter (BS-3).

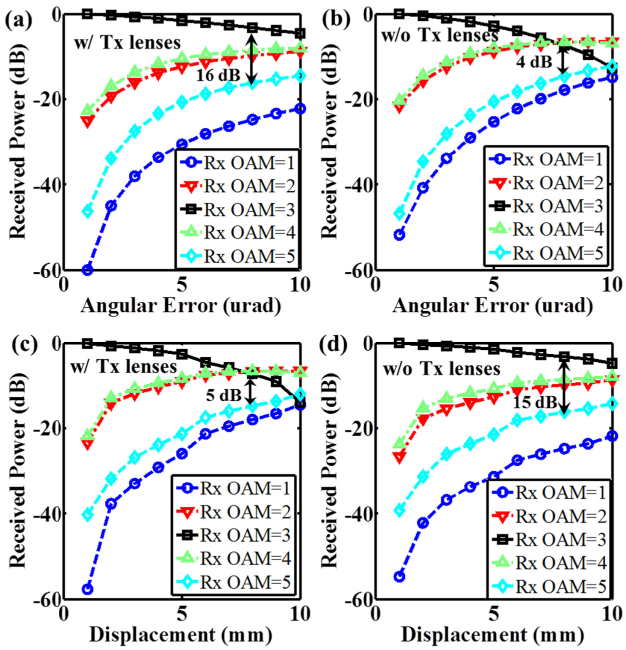

Fig. 4. Simulated OAM power distribution of 1 km free-space optical link: (a) With angular error and transmitter lenses; (b) With angular error but without transmitter lenses; (c) With displacement and transmitter lenses; and (d) With displacement but without transmitter lenses. Transmitted beam size is 10 cm, and only OAM+3 beam is transmitted. Rx OAM= ℓ: power coupled into the receiver for mode OAM= ℓ.

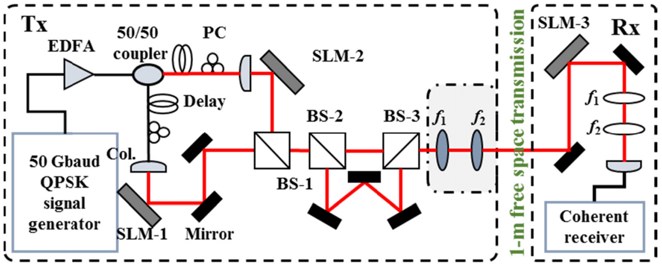

Fig. 5. Experimental setup for a 1 m OAM-multiplexed free-space optical link with transmitter lenses. Col: Collimator; SLM: spatial light modulator; BS: beam splitter; Tx: transmitter; Rx: receiver.

The four resulting multiplexed OAM beams pass through two transmitter lenses, both of focal length 10 cm. The spacing between these two lenses is adjustable such that the equivalent focal length is tunable. After free-space transmission of approximately 1 m, the beams are sent to SLM-3 loaded with an inverse-spiral phase pattern of the particular OAM channel to be detected. Such an OAM beam is converted to a Gaussian-like beam and then coupled into an SMF and sent for power measurement and/or coherent detection. First, we measured the power loss of the OAM beam when transmitted with/without transmitter lenses. Figure 6(a) and 6(b) shows simulated and experimental power losses of OAM +3 and OAM +7 as a function of receiver aperture size when only OAM +3 or OAM +7 respectively is transmitted with perfect alignment. Limited size receiver apertures are implemented by adding a truncated pattern onto SLM-3. The results show that an FSO link of ~1 m using transmitter lenses shows >20dB less power loss than that without transmitter lenses. Measured and simulated power losses show similar trends.

Next, we measured power distribution when OAM +3 is transmitted under angular errors with and without transmitter lenses (Fig. 6(c) and Fig. 6(d) respectively). The results indicate that when using transmitter lenses, the link is more tolerant of angular errors. For example, with an angular error of 200 μrad in the ~1 m FSO link, the power received by the designed mode (i.e., OAM +3) is still >20 dB more than the power leaked into channel OAM +1 and OAM +5; at the same angular error without using transmitter lenses, the power received by OAM +3 is almost the same as that leaked into OAM +1 and OAM +5. The power distributions when OAM +3 is transmitted under displacement with and without transmitter lenses are shown in Figs. 6(e) and 6(f), which indicate that using transmitter lenses might increase channel crosstalk under displacement in such an FSO link. Note that in Figs. 6(c)–(f), some power is still leaks into neighboring modes even when angular error or displacement is zero, which might be caused by imperfections in the alignment and free-space-to-fiber coupling.

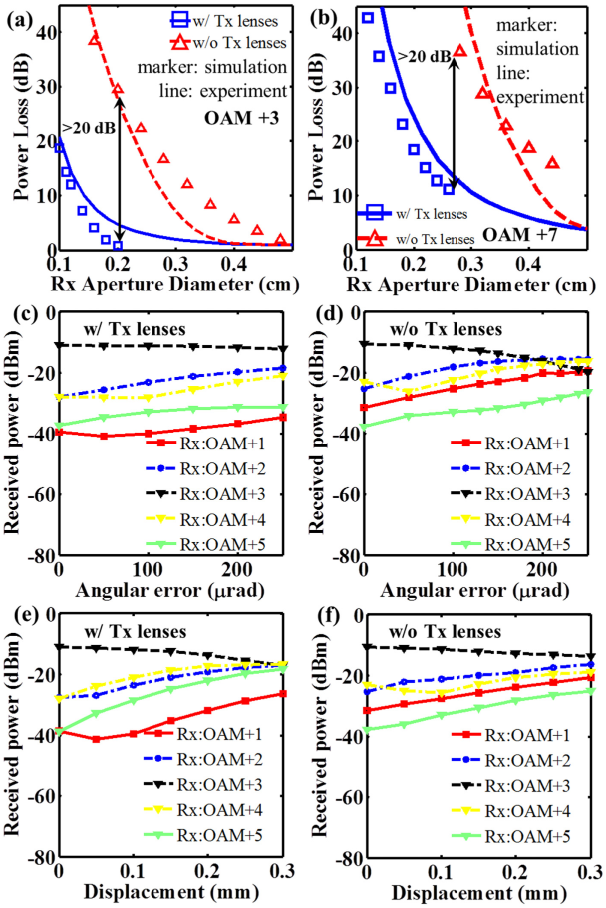

Fig. 6. (a) and (b) Simulated and experimental power loss as a function of receiver aperture size when only OAM +3 or only OAM +7 is transmitted with perfect alignment; (c) and (d) Experimental results of OAM power distribution when only OAM +3 is transmitted under angular errors with and without transmitter lenses, respectively; (e) and (f) Experimental results of OAM power distribution when only OAM +3 is transmitted under displacement with and without transmitter lenses, respectively.

Next, all four channels (i.e., OAM -3, OAM-1, OAM +1 and OAM+3) were transmitted simultaneously. Figures 7(a) and 7(b) show the crosstalk of OAM +3 channel under angular error and displacement, respectively. The results show that with transmitter lenses, crosstalk of channel OAM +3 from all three of the other channels decreases by 10~20 dB under angular errors ranging from 100 μrad to 500 μrad. On the other hand, with transmitter lenses, crosstalk of channel OAM +3 increases by 5~10 dB under displacement ranging from 0.1 mm to 0.5 mm. Measured crosstalk is in good agreement with the simulation results.

Figures 7(c) and 7 (d) show bit error rate (BER) curves of OAM +3 when all four channels are transmitted with or without transmitter lenses under angular errors and displacements, respectively (each beam carries a 50 Gbaud QPSK signal). Figure 7(c) shows that when the link has an angular error of 100 μrad, without transmitter lenses, this ~1 m FSO link could not achieve the 7% overhead forward error correlation (FEC) limit of 3.8×10^{-3}, whereas a link with transmitter lenses could. Moreover, with transmitter lenses, this FSO link could still achieve the FEC limit even under 300 μrad or 400 μrad angular error, with little power penalty compared with the 100 μrad case, demonstrating improved system robustness under angular errors. On the other hand, such a link would have higher power penalty under lateral displacement. Figure 7(d) shows that at 0.20 mm displacement, the use of transmitter lenses imposes a power penalty of ~8 dB compared with an FSO link without using transmitter lenses, and that higher power penalty might occur under larger displacement.

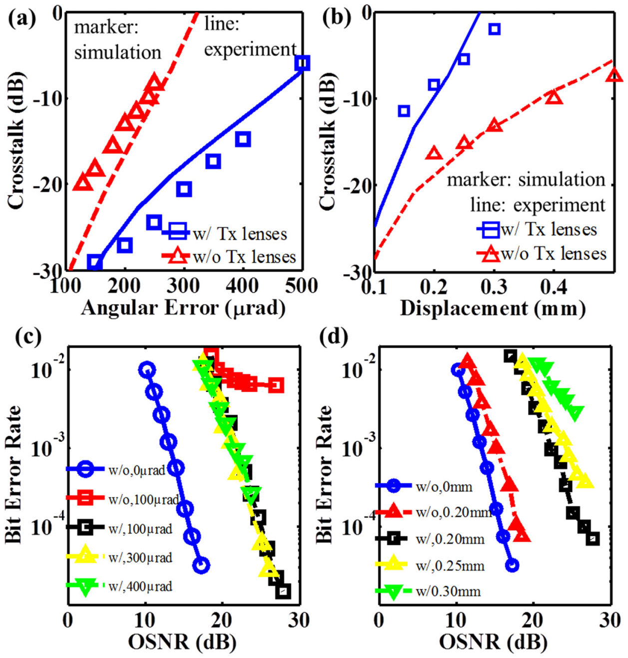

Fig. 7. (a) and (b): Experimental results of crosstalk for channel OAM +3 when OAM ±1 and ±3 are transmitted under angular error and displacement, respectively; (c) and (d): Experimental measurement of bit-error-rate as a function of optical signal-to-noise ratio (OSNR) for channel OAM +3 when OAM ±1 and ±3 are transmitted under angular error and displacement, respectively. w/: with transmitter lenses; w/o: without transmitter lenses; x μrad: with x μrad angular error; y mm: with y mm displacement.

4. CONCLUSION

In this work, the potential benefits and limitations of using transmitter lenses with FSO communication link multiplexing multiple OAM beams are investigated through simulation and experiments. We find that, within certain transmission distances, the use of transmitter lenses to focus OAM beams could reduce power losses in OAM-based FSO links with limited-size apertures; and could enhance system robustness under angular error but degrade tolerance of displacement. In practical cases, angular error and displacement are generally time-varying stochastic processes. Our approach may help analyze the upper and lower bounds of system robustness with or without transmitter lenses. This work only considers using continuous wave (CW) OAM beams. For OAM-carrying pulsed beams, the intensity profiles would be more complicated due to the broader bandwidth.

This may result in unique effects on OAM multiplexing in a lens system as well as different power loss and channel crosstalk compared with CW beams, which needs to be further studied [19-20]. Additionally, atmospheric turbulence might also result in distortion in OAM-based FSO links, the effects of which could be considered as angular error and displacement discussed above. However, severe signal fading might occur due to the intensity and phase fluctuation of the OAM beams. Such effects are not considered in this work and need further exploration.

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