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Indexed versus nominal government debt under inflation and price-level targeting

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Abstract

This paper presents a DSGE model in which long run inflation risk matters for social welfare. Optimal indexation of long-term government debt is studied under two monetary policy regimes: inflation targeting (IT) and price-level targeting (PT). Under IT, full indexation is optimal because long run inflation risk is substantial due to base-level drift, making indexed bonds a much better store of value than nominal bonds. Under PT, where long run inflation risk is largely eliminated, optimal indexation is substantially lower because nominal bonds become a better store of value relative to indexed bonds. These results are robust to the PT target horizon, imperfect credibility of PT and model calibration, but the assumption that indexation is lagged is crucial. From a policy perspective, a key finding is that accounting for optimal indexation has important welfare implications for comparisons of IT and PT.

Keywords: government debt, inflation risk, inflation targeting, price-level targeting.

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1. Introduction

Long-term contracts like government bonds and public pensions play an important role in many developed economies. Since contracts of this kind are often specified in nominal terms, unanticipated changes in inflation that are not reversed will lead to fluctuations in real wealth. These fluctuations are important for old generations because they rely on long-term contracts to fund their consumption in retirement. The magnitude of revaluations in long-term contracts due to unanticipated inflation depends crucially upon the amount of long run inflation risk in the economy. This observation motivates a comparison of the costs and benefits of inflation targeting (IT) and price-level targeting (PT) regimes. Under IT, unanticipated shocks to the price level are not reversed by policy, so there is base-level drift in the price level. Consequently, inflation risk rises with the forecast horizon.² By contrast, PT offsets unanticipated shocks to inflation in order to return the price level to a target path which is known *ex ante*. As a result, long run inflation risk is largely eliminated under a PT regime.

In this paper, optimal indexation of long-term government debt is studied under IT and PT regimes. Given that government debt accounts for a substantial fraction of net nominal wealth in developed economies (Dopeke and Schneider, 2006; Meh and Terajima, 2008), this analysis is important for comparing these two regimes. In recent years, both policymakers and academics have become interested in this comparison. Several papers have shown that PT offers short-term stabilisation benefits over IT when agents are forward-looking. Vestin (2006), for example, shows that in the standard New Keynesian model, PT reduces inflation variability for a given level of output gap variability if policy is discretionary. In the same model, the optimal commitment policy implies a stationary price level (Clarida, Gali and Gertler, 1999).³ In light of these results, the Bank of Canada recently conducted a detailed review of the costs and benefits of PT (see Bank of Canada, 2011).⁴ However, to the author's knowledge, no paper has assessed optimal indexation of long-term nominal debt contracts under IT and PT in a DSGE model where long run inflation risk matters for social welfare. The main contribution of this paper is to provide an initial assessment of this kind.

An overlapping generations (OG) model in the spirit of Diamond (1965) is calibrated to roughly match the UK economy. The model has three features that make it useful for investigating optimal indexation in an environment of long run inflation risk. First, long run inflation risk matters for social welfare since revaluations in the return on government debt due to unanticipated inflation are a source of consumption risk for old generations. In the model, long run inflation risk affects social welfare by two distinct channels: (i) variations in the real return on government debt lead to costly consumption variations for old generations; and (ii) a rise in long run inflation risk raises the risk premium the government pays on nominal debt and so requires higher taxes to maintain government spending. By contrast, only short-term inflation risk matters for social welfare in the standard New Keynesian model (Woodford, 2003). Second, the effects of optimal indexation on young and old generations can be assessed directly, hence providing useful information on the distributional effects of government debt policy under IT and PT regimes. Third, consistent with the standard OG

² That is, the price level follows a random walk. Inflation risk increases with the forecast horizon in this case because inflation between period t and $t+k$ depends on the ratio of the price level in $t+k$ to that in period t .

³ The issue of whether optimal policy in the New Keynesian model implies price stationarity is controversial. Negative results include Steinsson (2003), Levin *et al.* (2010) and Amano, Ambler and Shukayev (2012).

⁴ This work is surveyed in Ambler (2009), Crawford, Meh and Terajima (2009) and Bank of Canada (2011).

life-cycle model, each period in the model lasts 20 years. As a result, inflation risk and equilibrium asset prices can be modelled over a long horizon without introducing a large number of state variables. This feature of the model is crucial since a second-order approximation is needed to capture the full implications of inflation risk for social welfare,⁵ making a numerical solution computationally-intensive.

The main finding of the paper is that full indexation of government debt is optimal under IT, in stark contrast to PT where optimal indexation is substantially lower. Intuitively, despite the fact that the payoff on indexed bonds is subject to inflation risk due to a one-year indexation lag, return risk on nominal bonds is much higher since IT implies that cumulative inflation risk over a 20-year horizon is approximately 20 times that at a yearly horizon (due to base-level drift). Under a PT regime, by contrast, long run inflation risk does not increase with the forecast horizon and so is reduced to annual magnitudes. As a result, nominal bonds become a much better store of value relative to indexed bonds and optimal indexation is substantially lower. The indexation lag is crucial for explaining the sharp reduction in optimal indexation because the substantial reduction in long run inflation risk under a PT regime means that even a one-year indexation lag is sufficient to make return risk on indexed bonds comparable to that on nominal bonds. Consequently, there is no clear-cut benefit to indexation from the point of view of consumption stabilisation or government finances under a PT regime. It is important to note, however, that if the assumption that indexation is lagged is dropped, full indexation is optimal under both IT and PT.

In order to establish the main result, the analysis begins with a simple version of the model in which full indexation is optimal under IT and zero indexation is optimal under PT. Later sections then extend the model to more realistic settings and test sensitivity to calibration. Three different extensions are considered. First, if the price level is returned to its target path gradually over several years, optimal indexation remains somewhat lower under PT but rises to around 50 per cent at a 2-year target horizon, and around 75 per cent at a 4-year horizon. Second, optimal indexation is lower under a PT regime with imperfect credibility, but again the differential is narrowed somewhat: optimal indexation under PT rises to around 60 per cent under a regime with high credibility and 80 per cent under low credibility. The reason is that imperfect credibility raises the inflation risk premium on nominal government debt, because it reflects agents' belief that policy may revert to IT where long run inflation risk is much higher. Consequently, it is more costly for the government to issue nominal debt, implying higher taxes and (hence) lower average consumption for the young. Third, the baseline case assumes yearly money supply shocks are uncorrelated over time. Moderate correlation of money supply shocks raises optimal indexation under PT to around 50 per cent, while full indexation remains optimal under IT because indexing with a lag is less costly if inflation is persistent. In addition, sensitivity analysis suggests that the result that optimal indexation is substantially lower under PT is robust to model calibration.

An important finding from a policy perspective is that the potential long run welfare gains from PT are overstated substantially if indexation of government debt is held fixed across regimes at the current UK level. Indeed, the welfare gain from PT is reduced from around 0.20 per cent of aggregate consumption to almost zero if indexation is optimised under both

⁵ The reason is that in linear or log-linearised models there is 'certainty equivalence' – i.e. the coefficients of policy functions do not depend on risk (shock volatility). As pointed out by (Kim and Kim, 2003), failure to account for the effects of risk can lead to spurious welfare reversals. Since the model is solved using a second-order perturbation method, it captures the implications of risk for endogenous variables in the model.

IT and PT. These results highlight the importance of accounting for optimal indexation of government debt when comparing IT and PT regimes. More generally, the results suggest that analyses of IT and PT may produce misleading conclusions if they assume that nominal indexation is fixed across monetary regimes.

The analysis in this paper is directly related to two other strands of literature. The first is on the aggregate effects of unanticipated inflation. In a seminal paper, Doepke and Schneider (2006) document postwar nominal portfolios in the US and show that an unanticipated increase in inflation has substantial redistribution effects through revaluations of nominal assets and liabilities. Meh and Terajima (2008) later examined nominal portfolios in Canada. Building on these two papers, Meh, Rios-Rull and Terajima (2010) simulated aggregate and welfare effects from *one-off* episodes of unanticipated inflation in Canada under IT and PT in a quantitative OG model. They find that unanticipated inflation has greater redistribution effects under IT because the initial change in inflation is not reversed, so that long-term nominal contracts undergo substantial revaluations. Consequently, induced welfare losses are somewhat larger under IT. However, a limitation is that nominal portfolios remain fixed across monetary regimes in their analysis. As Meh *et al.* acknowledge, analysing how nominal portfolios change following implementation of PT may be important to reach more precise estimates of its aggregate and welfare effects. Since the current paper allows nominal portfolios to vary, it should provide additional insight into the aggregate and welfare effects of PT. It also provides a simple methodology for assessing optimal indexation that could potentially be extended to more realistic settings such as quantitative OG models.

The paper is also related to research on optimal indexation of wage contracts. In a seminal paper, Gray (1976) showed that optimal indexation increases with the nominal-to-real volatility ratio. More recently, Minford, Nowell and Webb (2003) build a model in which households cannot access financial markets and have an incentive to insure against real wage fluctuations. To do so, they optimise indexation of wage contracts. They find that optimal indexation is lower under a regime that aims at price rather than inflation stability, because nominal wage contracts become relatively better real wage stabilisers. Subsequently, Amano, Ambler and Ireland (2007) showed that the same conclusion holds in a model with staggered cohorts of labour-differentiated wage-setters who have unrestricted access to financial markets. An important difference in this paper is that indexation of government bonds has direct implications for government finances via the inflation risk premium on nominal government debt.⁶ Since the government must satisfy its budget constraint, inflation risk has knock-on effects on households and social welfare that are not present under optimal indexation of wage contracts. This problem therefore speaks to the need for a general equilibrium analysis that takes into account the main effects of inflation risk, including those for government finances. This paper provides such an analysis.

The remainder of the paper proceeds as follows. Sections 2 and 3 present the model and monetary policy in the baseline case. In Section 4, the optimal indexation problem and its solution are discussed. Then, in Section 5, the model is calibrated. Section 6 reports the optimal indexation results from the baseline model and is followed in Section 7 by extensions and sensitivity tests. Finally, Section 8 concludes and discusses implications for policy.

⁶ For a recent survey of the inflation risk premium, see Bekaert and Wang (2010). In the model that follows, indexed debt is risky and the inflation risk premium is defined as the expected difference between the real return on nominal bonds and the real return on indexed bonds.

2. Model

The model is a version of Diamond's (1965) model with capital and government bonds. It contains three sectors: a household sector, a government sector, and a sector devoted to production of a single output good. Each sector is described in detail below, starting with the household sector. This section also describes the aggregate resource constraint and explains how preferences of society are related to the preferences of individual generations.

2.1 Consumers

A simple overlapping generations (OG) model with generations that live for two periods is considered. Each generation is modelled as a representative consumer who inelastically supplies a unit of labour when young and retires when old, leaving no bequests. Let subscripts $\{Y, O\}$ denote, respectively, the young and the old. Each period in the model lasts 20 years. The number of generations born per period is constant and normalized to 1. The real wage income of each young generation is taxed by the government at a constant rate τ . Young agents' after-tax wage income is allocated to four assets: indexed government bonds, b^i ; nominal government bonds, b^n ; capital, k ; and fiat money, m .

Each young generation consumes and chooses an optimal portfolio of assets $z \equiv (k, b^i, b^n, m)$ which pays off in old age. Capital earns a real return r^k , which is taxed by the government at rate τ^k . Indexed bonds pay a risky real return r^i as a result of a one-year indexation lag, and nominal bonds pay a risky real return r^n . Nominal bonds are riskless but for unanticipated inflation over the holding horizon from youth to old age (i.e. 20 years). Consequently, the real return on nominal bonds is $r^n = R/(1+\pi)$, where π is inflation between youth and old age and R is the nominal interest rate.⁷ Since indexed bonds are subject to a one-year indexation lag, they pay a real return $r^i = r(1+\pi^{ind})/(1+\pi)$, where π^{ind} is the inflation rate that indexed bonds are linked to and r is the (*ex ante*) real interest rate. The interest rates R and r are endogenously determined and ensure that, for each bond, demand is equated to supply.

Money pays a real return $r^m = 1/(1+\pi)$. Positive money demand arises from the legal requirement that young agents hold real money balances of at least $\delta > 0$, so that $m_t \geq \delta$ as in Champ and Freeman (1990). The main advantage of this constraint is that it provides a role for money without requiring that it offer explicit transactions services, so that any differential in optimal indexation under IT and PT can be attributed to the implications of these regimes for long run inflation risk and not the impact of monetary policy on transactions costs or ease of exchange. The constraint binds with equality if $R_t > 1$ for all t , which is assumed to hold.⁸

Hence we have that

$$m_t = \delta, \quad \forall t \quad (1)$$

⁷ Bond returns are not taxed as this enables the model to better match the ratio of long-term government debt to GDP and the investment-GDP ratio. It is worth noting that UK government bonds with a maturity of 5 years or longer are exempt from tax if they are held in an ISA; see the Debt Management Office (DMO) website.

⁸ This condition is proven in the Appendix. It was comfortably satisfied in numerical simulations because steady-state inflation is positive and the steady-state real interest rate exceeds 1.

The budget constraints faced by the generation born in period t are given by

$$c_{t,Y} = (1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t \quad (2)$$

$$\begin{aligned} c_{t+1,O} &= (1-\tau^k)r_{t+1}^k k_{t+1} + r_{t+1}^i b_{t+1}^i + r_{t+1}^n b_{t+1}^n + r_{t+1}^m m_t \\ &= (1-\tau^k)r_{t+1}^k k_{t+1} + [vr_{t+1}^i + (1-v)r_{t+1}^n]b_t + r_{t+1}^m m_t \end{aligned} \quad (3)$$

where $0 \leq v \leq 1$ is the share of indexed bonds in total bonds portfolio, b .

Given the focus in this paper, it is important to use preferences that can match household attitudes to risk revealed in applied research. As is well known, standard CRRA preferences cannot match the risk-free rate and risk-premia as they imply that the elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion. Consequently, Epstein and Zin (1989) and Weil (1989) preferences are used here. With these preferences, the elasticity of intertemporal substitution and the coefficient of relative risk aversion can be calibrated separately. In a recent paper, Rudebusch and Swanson (2012) show that this feature enables an otherwise standard New Keynesian model to match the 10-year term premium on nominal bonds without compromising its ability to fit key macro variables.

Consumers solve a maximisation problem of the form

$$\max_{\{c_{t,Y}, z_t, c_{t+1,O}\}} u_t = \frac{1}{1-\gamma} \left[c_{t,Y}^\varepsilon + \beta \left[E_t c_{t+1,O}^{1-\gamma} \right]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma}{\varepsilon}} \quad \text{s.t. (1), (2) and (3)} \quad (4)$$

where $0 < \beta < 1$, γ is the coefficient of relative risk aversion, and $1/(1-\varepsilon)$ is the elasticity of intertemporal substitution.

The first-order conditions are summarized by the following Euler equations:

$$1 = E_t (sdf_{t+1} (1-\tau^k) r_{t+1}^k) \quad \text{for capital, } k \quad (5)$$

$$1 = E_t (sdf_{t+1} r_{t+1}^i) \quad \text{for indexed bonds, } b^i \quad (6)$$

$$1 = E_t (sdf_{t+1} r_{t+1}^n) \quad \text{for nominal bonds, } b^n \quad (7)$$

$$1 = E_t (sdf_{t+1} r_{t+1}^m) + \tilde{\mu}_t \quad \text{for money, } m \quad (8)$$

Here $\tilde{\mu}_t$ is the ratio of the Lagrange multiplier on the CIA constraint to that on the budget constraint of the young,⁹ and

$$sdf_{t+1} \equiv \beta \left(\frac{c_{t,Y}}{c_{t+1,O}} \right)^{1-\varepsilon} \left(\frac{c_{t+1,O}}{(E_t c_{t+1,O}^{1-\gamma})^{1/(1-\gamma)}} \right)^{1-\gamma-\varepsilon}; \quad r_{t+1}^n \equiv R_t / (1 + \pi_{t+1}); \quad r_{t+1}^i = r_t (1 + \pi_{t+1}^{ind}) / (1 + \pi_{t+1}).^{10}$$

⁹ For a full derivation of consumers' first-order conditions, see Section A of the Appendix.

¹⁰ The dating on R and r reflects the fact that these (*ex ante*) returns must clear the markets for nominal and indexed bonds at the time when bonds are purchased, that is, at the end of period t . Note that the inflation rate to which indexed bonds are linked is not equal to the previous period's inflation rate because the indexation lag is *one year*, whereas each period in the model lasts 20 years. For the determination of π and π^{ind} , see Section 3.

2.2 Firms

The production sector consists of a representative firm that produces output using a Cobb-Douglas production function. The share of capital in output is equal to α and the labour share is equal to $1-\alpha$. The firm hires capital and labour in competitive markets to maximise current period profits. Total factor productivity, A , is stochastic and follows an AR(1) in logs.¹¹

The real wage and the return on capital are given by

$$w_t = y_t - r_t^k k_t = (1-\alpha)A_t k_t^\alpha \quad (9)$$

$$r_t^k = \alpha y_t / k_t = \alpha A_t k_t^{\alpha-1} \quad (10)$$

2.3 Government

The government performs three functions. First, to meet government spending commitments, it taxes wage income of the young at a constant rate $\tau > 0$, and capital income of the old at a constant rate $\tau^k > 0$. Second, it conducts monetary policy by committing to a money supply rule. Third, the government sets the total supply of government bonds and chooses the share of indexed government bonds to maximise social welfare, subject to the monetary policy regime in place.

The government budget constraint is given by

$$\begin{aligned} g_t &= \tau w_t + \tau^k r_t^k k_t + b_{t+1}^i - r_t^i b_t^i + b_{t+1}^n - r_t^n b_t^n + m_t - r_t^m m_{t-1} \\ &= \tau w_t + \tau^k r_t^k k_t + b_{t+1} - [v r_t^i + (1-v)r_t^n] b_t + m_t - r_t^m m_{t-1} \end{aligned} \quad (11)$$

The total supply of government bonds is $b = b^i + b^n$, and the shares of indexed and nominal government bonds in the total bond portfolio are constant and equal to v and $1-v$, respectively. Since the tax rates on wage income and capital are constant, it follows that $\tau^k = a\tau$ for some constant $a > 0$, so that tax policy can be described by the single tax rate τ . The government sets the total supply of bonds to facilitate consumption smoothing between youth and old age. In particular, it chooses the total supply of government debt so that

$$E_t(sdf_{t+1}) = \beta \quad (12)$$

where E_t is the conditional expectations operator.

The bond supply rule in (12) implies a steady-state real interest rate of $1/\beta$ and hence perfect consumption smoothing in the deterministic steady-state. Consequently, there is a degree of social insurance without the burden of modelling a social security system. The government sets the nominal money supply according to an IT or PT rule. These policy rules are discussed in Section 3. The government sets the constant tax rate τ to ensure that it achieves a long run target ratio of government spending to output, or

$$E(g_t/y_t) = G^* > 0 \quad (13)$$

where E is the unconditional expectations operator.

¹¹ Long run productivity risk is included because the risk aversion coefficient is calibrated to match the 20-year Sharpe ratio on capital. Including long run real risk also ensures that the implications of indexation for consumption risk are not significantly overstated.

Although τ is constant over time, it will differ across IT and PT regimes because the level of long run inflation risk affects the average real return on money balances and the inflation risk premium on nominal government debt. It should thus be understood that the tax rate is regime-specific, though this dependence is suppressed in order to minimize notational burden. Taking into account the equilibrium conditions of the model and the requirement that τ be set so that $E(g_t/y_t) = G^*$, the government chooses the share of indexed government bonds to maximise social welfare. The full details of the government's optimal indexation problem are presented in Section 4.

2.4 Aggregate resource constraint

Capital depreciates fully within a period, an assumption which is empirically reasonable given that each period in the model lasts 20 years. It follows that investment in period t is $i_t = k_{t+1}$. The economy's aggregate resource constraint in period t is

$$y_t = c_{t,Y} + c_{t,O} + k_{t+1} + g_t \quad (14)$$

where the sum of the first two terms on the right hand side is aggregate consumption.

2.5 Social welfare

Welfare is given by the discounted sum of lifetime utilities across all generations:¹²

$$SW = (1 - \omega)E \left[\sum_{t=0}^{\infty} \omega^t u_t \right] = E(u_t) \quad (15)$$

where $0 < \omega < 1$ is the social discount factor, and E is the unconditional expectations operator.

It is clear from (15) that the social discount factor ω will not affect optimal indexation. Consequently, the social discount factor can be left unspecified.

3. Monetary Policy and inflation

The government conducts monetary policy using money supply rules set yearly with annual inflation in mind.¹³ The government can commit to these rules but cannot control the money supply perfectly and so has imperfect control over inflation. In order to obtain money supply rules consistent with the 20-year horizon of the model, the implications of these rules are traced out over a 20-year horizon. This section first derives expressions for equilibrium inflation under IT and PT, before turning to the one-year-lagged measure of inflation to which indexed bonds are linked. In the discussion that follows, M_n denotes the nominal

¹² This social welfare function ignores the utility of the initial old, but this does not affect the main results.

¹³ This assumption is more consistent with the policy horizon in practice and brings two additional advantages: (i) calibrating at a 20-year horizon is problematic given that IT has been in place for less than 20 years as part of an independent monetary policy regime, and (ii) the impact of a target horizon longer than one year is investigated as a robustness test, since proponents of PT argue that the short-term costs of undoing shocks to the price level could be diminished in this way (e.g. Gaspar, Smets and Vestin, 2007). This investigation is not possible with monetary policy rules that have a 20-year target horizon.

money stock at the end of year n , and ε_n is an IID-normal innovation in year n with mean zero and standard deviation σ .

3.1 Inflation targeting (IT)

Under IT, the yearly nominal money supply grows at the annual target inflation rate, π^* , plus any deviation due to an exogenous yearly money supply innovation ε :

$$M_n = M_{n-1}(1 + \pi^*)(1 + \varepsilon_n) \quad (16)$$

Substituting repeatedly for the previous year's money supply,

$$M_n = M_{n-20}(1 + \pi^*)^{20} \prod_{j=n-19}^n (1 + \varepsilon_j). \quad (17)$$

It is clear from this equation that the IT money supply rule aims at a constant inflation target and does not attempt to offset past money supply shocks – i.e. ‘bygones are bygones’. Given that each period lasts 20 years and the nominal money supply is the end-of-year stock, the implied money supply rule in period t is

$$M_t = M_{t-1}(1 + \pi^*)^{20} \prod_{j=1}^{20} (1 + \varepsilon_{j,t}) \quad (18)$$

where the money innovations are indexed by $j = 1, 2, \dots, 20$ and $M_t \equiv P_t m_t$ is the nominal money stock at the end of period t .

By money market equilibrium $M_t = P_t m_t$, where $m_t = \delta$ by the legal requirement on cash holdings. Hence $M_t/M_{t-1} = P_t/P_{t-1} = 1 + \pi_t$.

Inflation in period t is therefore given by

$$1 + \pi_t = (1 + \pi^*)^{20} \prod_{j=1}^{20} (1 + \varepsilon_{j,t}) \quad (19)$$

It is clear from (19) that there is base-level drift. As a result, inflation risk accumulates over a 20-year horizon. Note that this rule would stabilise inflation perfectly at the long-term inflation target $(1 + \pi^*)^{20}$ in the absence of money supply innovations, consistent with annual inflation of π^* every year. Finally, note that inflation expectations are anchored at target: $1 + E_{t-1}\pi_t = (1 + \pi^*)^{20}$.

Since indexed bonds are subject to a one-year indexation lag, the inflation rate to which indexed bond are linked is given by the one-year lagged value of (19):

$$1 + \pi_t^{ind} = (1 + \pi^*)^{20} (1 + \varepsilon_{20,t-1}) \prod_{j=1}^{19} (1 + \varepsilon_{j,t}) \quad (20)$$

Equation (20) shows that indexed inflation will covary strongly with actual inflation under IT: they have 19 of 20 shocks in common, with the difference accounted for by the one-year indexation lag. Consequently, indexed bonds will be excellent stabilisers of long run purchasing power under IT. This point is important for understanding the results that follow.

3.2 Price-level targeting (PT)

Under PT, policy aims to stabilize the price level around a target price path whose slope is consistent with an annual inflation target of π^* . The crucial difference relative to IT is that past deviations from the inflation target are offset to return the price level to target. The yearly money supply rule therefore includes a correction for the previous year's innovation:¹⁴

$$\begin{aligned} M_n &= M_{n-1}(1 + \pi^*) \frac{(1 + \varepsilon_n)}{(1 + \varepsilon_{n-1})} \\ &= M_{n-20}(1 + \pi^*)^{20} \frac{(1 + \varepsilon_n)}{(1 + \varepsilon_{n-20})} \end{aligned} \quad (21)$$

where the second equality follows from repeated substitution for the previous money supply.

Given that the nominal money supply is the end-of-year stock, this equation implies a period t money supply rule

$$M_t = M_{t-1}(1 + \pi^*)^{20} \frac{(1 + \varepsilon_t)}{(1 + \varepsilon_{t-20})} \quad (22)$$

Again, $M_t/M_{t-1} = P_t/P_{t-1} = 1 + \pi_t$, so inflation in period t is given by

$$1 + \pi_t = (1 + \pi^*)^{20} \frac{(1 + \varepsilon_{20,t})}{(1 + \varepsilon_{20,t-1})} \quad (23)$$

where $\varepsilon_{20,t}$ is the money supply innovation in year 20 of period t .

Notice that the PT money supply rule prevents base-level drift: money supply innovations have a temporary impact on the price level. As a result, long run inflation risk is somewhat lower than under IT.¹⁵ Intuitively, inflation in period t depends on the money supply innovation in year 20 of period t because policy offsets innovations after one year and so cannot offset the innovation in year 20 until the first year of the next period. Inflation in period t also depends on the money supply innovation in year 20 of period $t-1$, because this should be offset in year 1 of period t to correct for the past deviation from the target price path. Since rational agents expect past deviations from the target price path to be offset, inflation expectations vary with the past money supply innovation according to: $1 + E_{t-1}\pi_t = (1 + \pi^*)^{20} (1 + \varepsilon_{20,t-1})^{-1}$.

Since indexed bonds are subject to a one-year indexation lag, the inflation rate to which indexed bond are linked is given by

$$1 + \pi_t^{ind} = (1 + \pi^*)^{20} \frac{(1 + \varepsilon_{19,t})}{(1 + \varepsilon_{19,t-1})} \quad (24)$$

¹⁴ The intuition can be seen easily by taking logs: $M_n \approx M_{n-1} + \pi^* + \varepsilon_n - \varepsilon_{n-1}$. M_n and ε_n are defined as above.

¹⁵ It can be shown that the unconditional variance of inflation is approximately 10 times higher under an IT regime. (See the Appendix)

In this case, the inflation rate to which indexed bonds are linked will not covary with actual inflation *at all*, since yearly innovations to the money supply are uncorrelated.¹⁶ As a result, indexed bonds will be a poor stabiliser of purchasing power under a PT regime. The key to this result is that, under PT, cumulative inflation over a 20-year horizon depends only on two yearly innovations – the money supply innovation in year 20 of the current period, and the money supply innovation from year 20 of the previous period (i.e. the innovation from 20 years earlier) – because all innovations in intervening years have been offset by the end of period t in order to return the price level to its target path.

The key point is that since indexed bonds have a maturity of 20 years, the one-year indexation lag implies indexation to a measure of 20-year inflation whose start date and end date are one year earlier than those for actual inflation. This will tend to make indexed inflation a poor measure of actual inflation because *all* changes in actual inflation in period t come from the innovations that hit the economy at the start and end of each 20 year period. Of course, indexed inflation under IT also ‘misses’ innovations at the start and end of the period, but the crucial difference is that base-level drift implies that cumulative 20-year inflation under IT depends on *all* the other shocks that occur during period t , and not just those at the start and end of the period.

4. Optimal indexation

The government chooses the indexation share that maximises social welfare subject to generational budget constraints, the economy’s aggregate resource constraint, consumers’ and firms’ first-order conditions, fiscal policy (i.e. the total bond supply equation and the long run government spending target), and the monetary policy regime in place.

The government’s optimal indexation problem can be stated as follows:

$$\begin{aligned} \max_{v \in [0,1]} SW &= (1 - \omega) E \left[\sum_{t=0}^{\infty} \omega^t u_t \right] \quad \text{subject to: Eqs. (1) to (3) and (5) to (14),}^{17} \text{ and} \\ 1 + \pi_t &= \begin{cases} (1 + \pi^*)^{20} \prod_{j=1}^{20} (1 + \varepsilon_{j,t}) & \text{under IT} \\ (1 + \pi^*)^{20} \frac{(1 + \varepsilon_{20,t})}{(1 + \varepsilon_{20,t-1})} & \text{under PT} \end{cases} \\ 1 + \pi_t^{ind} &= \begin{cases} (1 + \pi^*)^{20} (1 + \varepsilon_{20,t-1}) \prod_{j=1}^{19} (1 + \varepsilon_{j,t}) & \text{under IT} \\ (1 + \pi^*)^{20} \frac{(1 + \varepsilon_{19,t})}{(1 + \varepsilon_{19,t-1})} & \text{under PT} \end{cases} \end{aligned} \tag{25}$$

¹⁶ This assumption is relaxed as a robustness test in Section 7. Allowing for correlation between yearly innovations does not overturn the main result unless innovations are strongly positively correlated.

¹⁷ Notice that we do not need to include the nominal money supply rules in this optimisation problem because, in conjunction with the binding constraint on cash holdings, they imply an equilibrium inflation rate under each regime (see Section 3 for the details).

The optimal indexation share that satisfies (25) is computed numerically. To do so, the model was solved using a second-order perturbation approximation in Dynare++. In particular, social welfare was computed for a discrete number of indexation shares in the interval $[0,1]$. This was achieved by looping over the parameter ν in discrete steps in Dynare++, with the aid of an algorithm available on Wouter Den Haan's webpage.¹⁸

The problem in (25) is more complicated than might appear at first sight since the researcher must solve for an indexation share that maximises social welfare, subject to the constraint that (13) holds, which pins down a unique tax rate τ^* . The optimal indexation share was therefore computed by simultaneously looping over ν and τ in discrete steps in order to find: (1) the tax rate $\tau^*(\nu_k)$ such that (13) holds for each discrete value of the indexation share $\nu_1, \nu_2, \dots, \nu_K$ in the interval $[0,1]$; and (2) the indexation share $\nu^*(\tau^*(\nu^*))$ in the set $\{\nu_1, \nu_2, \dots, \nu_K\}$ that maximises social welfare.

To understand the indexation results that follow, it is helpful to consider a second-order Taylor expansion of social welfare around the point $c_{t,Y} = E[c_{t,Y}]$ and $c_{t+1,O} = E[c_{t+1,O}]$:¹⁹

$$\begin{aligned} SW &\approx \tilde{u}_t + \frac{1}{2} (\tilde{u}_t^{c_Y c_Y} \text{var}[c_{t,Y}] + \tilde{u}_t^{c_O c_O} \text{var}[c_{t,O}] + 2\tilde{u}_t^{c_Y c_O} \text{cov}[c_{t,Y}, c_{t+1,O}]) \\ &= \frac{1}{1-\gamma} \left[(E[c_{t,Y}])^\varepsilon + \beta (E[c_{t,O}])^\varepsilon \right]^{\frac{1-\gamma}{\varepsilon}} \\ &\quad - \frac{1}{2} \left| \tilde{u}_t^{c_Y c_Y} \text{var}[c_{t,Y}] - \frac{1}{2} \tilde{u}_t^{c_O c_O} \text{var}[c_{t,O}] - \tilde{u}_t^{c_Y c_O} \text{cov}[c_{t,Y}, c_{t+1,O}] \right| \end{aligned} \quad (26)$$

where $\tilde{u}_t \equiv u_t(c_{t,Y} = E[c_{t,Y}], c_{t,O} = E[c_{t,O}])$; \tilde{u}_t^{xx} denotes the second derivative of \tilde{u}_t with respect to variable x , evaluated x at the point $E[x]$; and $|x| \equiv \text{abs}(x)$.

This expression shows that social welfare increases with mean consumption levels in youth and old age but falls, *ceteris paribus*, with the variance around these mean outcomes, due to risk-aversion. There is additionally a consumption covariance term which has a negative impact. However, since the correlation between consumption in youth and consumption in old age is fixed by the structure of consumer portfolios, we can say that this term will also depend upon the consumption variances.²⁰ Consequently, any difference in optimal indexation under IT and PT must be driven by the impact of policy on mean consumption levels or the variances of consumption. The numerical analysis that follows therefore explains optimal indexation with reference to these means and variances.

5. Calibration

The model was calibrated to roughly match the UK economy since 1997. Free parameters are calibrated to match standard values in the literature.

¹⁸ See <http://www.wouterdenhaan.com/numerical/dynareprograms.htm>

¹⁹ This expression makes use of the fact that, under stationarity, $E[c_{t+1,O}] = E[c_{t,O}]$ and $\text{var}[c_{t+1,O}] = \text{var}[c_{t,O}]$.

²⁰ This point follows since $\text{cov}[x,y] = \text{corr}[x,y] \cdot (\text{var}[x])^{1/2} (\text{var}[y])^{1/2}$.

5.1 Aggregate uncertainty

The model contains two aggregate shocks: a money supply innovation and a total factor productivity (TFP) shock. Calibrating the money supply rules requires a standard deviation for the annual money supply innovation. This standard deviation was set at $\sigma = 0.0105$ to match the standard deviation of annual CPI inflation from 1997 to 2011 in data from the Office for National Statistics (ONS). This calibration should give the model a good chance of matching the amount of long run inflation risk that would be observed with typical price level shocks in an IT regime.^{21, 22}

The AR(1) productivity shock is also calibrated for a generational horizon using annual data.²³ Productivity is thus given by

$$\ln A_t = (1 - \rho_A) \ln A_{mean} + \rho_A \ln A_{t-1} + e_t \quad (27)$$

where e_t is a zero-mean IID innovation to productivity and ρ_A and σ_e are calibrated based on the 20-year properties implied by an *annual* AR(1) productivity process.

In particular, if the AR(1) coefficient in the annual productivity process is ρ , the implied AR(1) coefficient at a 20-year horizon is $\rho_A \equiv \rho^{20}$ and the implied 20-year innovation standard deviation is the standard deviation of the annual innovation to productivity, multiplied by $[(1 - \rho^{40}) / (1 - \rho^2)]^{1/2}$. A standard calibration based on the business cycle literature is $\rho = 0.955$ and the annual innovation standard deviation was set at 0.018 based on the standard deviation of annual UK TFP growth from 1998-2010 in ONS data. Hence $\rho_A = 0.40$ and $\sigma_e = 0.0557$.

5.2 The indexation lag

As noted, indexed government bonds have a one-year indexation lag in the model. Up until September 2005, all index-linked gilts in the UK were indexed to the Retail Prices Index (RPI) with a lag of 8 months, but all index-linked gilts issued since this time have a 3-month lag. Consequently, both types of gilts are in existence today, with 3-month gilts accounting for 53 per cent of the index-linked gilts market as of March 2011 and 8-month gilts accounting for the remaining 47 per cent (DMO 2011, p. 10).²⁴ The proportion of 8-month gilts in the market will fall over time as debt issued before September 2005 reaches maturity; in fact, real-time data suggests that around two-thirds of the current market is in index-linked gilts with a 3-month indexation lag, and one-third in old-style gilts with an 8-month indexation lag.²⁵ Crucially, however, the main findings in the paper would not be overturned if the indexation lag was 3 months rather than 1 year, provided that it was assumed that shocks hit the economy at a quarterly frequency. The baseline assumption of an indexation

²¹ Under IT, annualised inflation, $(1 + \pi_t)^{1/20}$ has a standard deviation approximately equal to σ when $\pi^* \approx 0$.

²² The variance of actual inflation at a 20-year horizon cannot be used since IT was not adopted as part of an independent monetary policy regime until 1997.

²³ Lungu and Minford (2006) also calibrate a TFP shock at a generational horizon with annual data.

²⁴ These figures refer to the size of the market in uplifted nominal terms.

²⁵ See the UK Debt Management Office (DMO) website. The data was accessed on 10 May 2013.

lag of one year is made primarily to keep down the number of shocks in the model in the IT case, hence making a second-order approximation of the model computationally feasible.

5.3 Model parameters

Preference parameters

The parameter ε was set at -0.35 , which implies an elasticity of intertemporal substitution (EIS) of 0.74 . This calibration is consistent with micro studies that estimate an EIS less than 1 . The discount factor β was set at 0.70 , implying an annual discount factor of 0.982 and an annual risk-free real rate of 1.8 per cent per annum. The annual risk-free real rate was deliberately set below the average UK estimate of 2.9 per cent from 1965 to 2005 (Mills, 2008) since matching a real rate this high gives an investment-GDP ratio somewhat lower than in the data. Finally, the coefficient of relative risk aversion γ was calibrated to match the Sharpe ratio on capital, $E[r^k - r^f] / \text{std}(r^k - r^f)$.²⁶ The target value of 0.43 is based on Constantinides, Donaldson and Mehra (2002), who estimate the Sharpe ratio using 20-year holding period returns on equity and bonds in the US. Accordingly, γ was set at 15 .

Other model parameters

The parameter α was set at 0.263 , implying a share of capital income in GDP of 26.3 per cent. This value is on the low side of standard calibrations but helps the model to match a target ratio of long-term government bonds to GDP of around 10 per cent, which roughly matches the share of long-term government bonds in UK GDP over the past decade.²⁷ The tax rate on capital was set at 2.3 times the income tax rate, that is, $a = 2.3$. A substantially higher tax rate on capital is consistent with UK data over the period 1970 - 2005 : Angelopoulos, Malley and Phillippopoulos (2012) calculate that the average tax rate on capital was 0.44 , compared to an average tax rate on labour of 0.27 , implying that capital taxes should be roughly 1.6 times as high as labour taxes. The higher calibrated ratio of 2.3 enables the model to get close to a target ratio of government bonds to investment of $2/3$.²⁸

There are four additional parameters that need to be calibrated. First, an annual inflation target enters both the IT and PT money supply rules. This target was set at 0.02 , consistent with the 2 per cent UK inflation target for the Consumer Prices Index (CPI). Second, real money balances are equal to δ by the legal requirement on cash holdings. The calibration sets $\delta = 0.015$ so that money balances are around 3 per cent of steady-state GDP in the model, consistent with annual UK data on notes and coins (ONS 2011, Table 1). Third, the long run government spending to GDP target, G^* , was set at 0.11 since this implies a tax rate τ in the model solution such that government bonds and investment have plausible GDP shares.

5.4 Steady-state solution and key ratios

This section discusses the performance of the calibrated model against target ratios. In the model, investment equals the capital stock since there is full depreciation. The UK investment-GDP ratio has been close to 15 per cent over the past decade (ONS 2012, Table

²⁶ Returns are annualised here. The Sharpe ratio was computed using the after-tax return on capital.

²⁷ See ONS (2011) and historical data available on the Debt Management Office (DMO) website.

²⁸ This figure was reached by dividing the target ratio of government bonds to GDP of 0.10 by a target ratio of investment to GDP of 0.15 (see Section 5.4).

1.2) and over the same period the consumption share was around 65 per cent, implying target ratios of 0.15 and 0.65. Turning to government debt, the bonds-GDP ratio has fluctuated somewhat over the past decade but has averaged around one-third (ONS 2011, Table 1.1D). Together with a 2005 share of long-term government debt in total government debt of around 30 per cent,²⁹ this figure implies a target long-term government bonds to GDP ratio of around 0.1. Table 1 shows that the model does fairly well against target ratios.³⁰

Table 1 – Target versus model ratios

Ratio	Target	Model	Definition
b/y	0.10	0.11	Long-term bonds/GDP
$i/y (=k/y)$	0.15	0.14	Investment/GDP
b/i	0.67	0.79	Long-term bonds/Investment
$(c_Y + c_O)/y$	0.65	0.75	Aggregate consumption/GDP
$E[r^k - r^f] / \text{std}(r^k - r^f)$	0.43	0.40	Sharpe ratio
m/y	0.03	0.03	Notes and Coins/GDP

6. Results

The model was solved using a second-order perturbation approximation in Dynare++ (Julliard, 2001), with the optimal indexation share computed as described in Section 4. To determine optimal indexation, 100 simulations of length 1100 periods were run, with the first 100 periods of each disregarded to randomise initial conditions. Hence, a total of 100,000 simulated values were used to compute unconditional moments and social welfare for each indexation share. In this section, results are reported for the baseline model described above.

6.1 The baseline model

This section first investigates optimal indexation under inflation targeting (IT), before turning to the price-level targeting (PT) case. Optimal indexation is compared under IT and PT and the implications for welfare comparisons of these regimes are discussed.

²⁹ See historical data on the DMO website. The DMO classifies gilts as ‘long-term’ if maturity exceeds 15 years.

³⁰ The model overshoots the target ratio of long-term government bonds to GDP, but this can be justified by the presence of substantial unfunded public sector and state pension liabilities that play a similar role to long-term government bonds. The estimated annual cost of unfunded UK public sector pensions in 2007-8 was 1.5 per cent of GDP (Table 2.6, Public Sector Pensions Commission 2010) and Müller *et al.* (2009) report annual expenditure on social security pensions of 4.1 per cent of GDP in 2006. The Basic and Additional state pensions in the UK are indexed under a triple-lock system (highest of earnings, CPI inflation with an 8-month lag or 2.5 per cent), while public sector pensions are indexed to CPI inflation with an 8-month lag.

Inflation targeting (IT)

Figures 1 and 2 report the baseline results under IT:

Fig 1 – Indexation and social welfare under IT

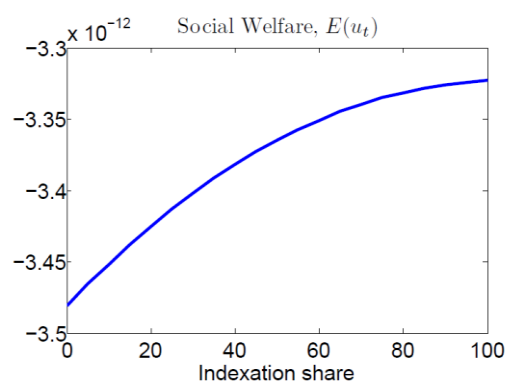
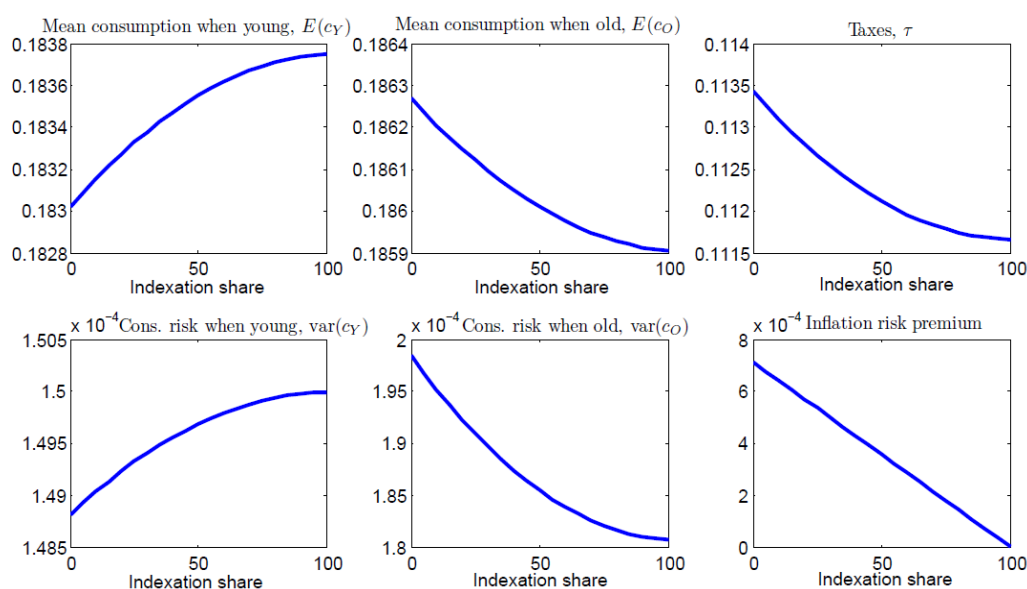


Fig 2 – Factors driving optimal indexation (IT)



As can be seen from Fig 1, social welfare is maximised when the government issues only indexed government debt – i.e. full indexation of 100 per cent is optimal. Fig. 2 sheds light on why this is the case. The key factor driving the full indexation result is old agents' consumption risk, which is minimised under full indexation. Old generations' consumption risk is crucial for two reasons: first, it has a direct impact on social welfare because consumers are risk-averse; second, higher consumption risk implies a higher inflation risk premium,³¹ so that higher taxes are necessary to meet the long run government spending target. In turn, a rise in taxes implies lower mean consumption by the young. Intuitively, old agents' consumption risk is minimised by full indexation of government debt because, as explained in Section 3.1, indexed government debt provides far better insurance against unanticipated inflation than nominal debt and so commands a much lower risk premium in equilibrium. Indeed, since cumulative inflation risk over 20 years is (approximately) 20 times

³¹ The inflation risk premium is defined as the differential between the expected real return on an indexed bond and the expected real return on a nominal bond – i.e. $E(r^i) - E(r^j)$. Graphs report the annualised risk premium.

that at a yearly horizon under IT due to base-level drift, the 1-year indexation lag on indexed debt exposes bondholders to only 1 year of inflation risk, as compared to exposure to the full 20 years in the case of nominal government debt.

Since consumption risk is minimised under full indexation, both the inflation risk premium and taxes are minimised with full indexation. The latter means that average consumption by the young is maximised. Consequently, the young gain from higher average consumption under full indexation, while the old gain directly from a substantial reduction in consumption risk (and indirectly from a lower capital tax rate). Indeed, although average consumption by the old falls as indexation is increased (because a higher inflation risk premium raises the average real return paid on nominal government debt) and consumption risk for the young rises marginally, the rise in average consumption by the young and the reduction in consumption risk for the old are sufficient to ensure social welfare rises under full indexation.

Price-level targeting (PT)

Figures 3 and 4 report the baseline results under PT:

Fig 3 – Indexation and social welfare under PT

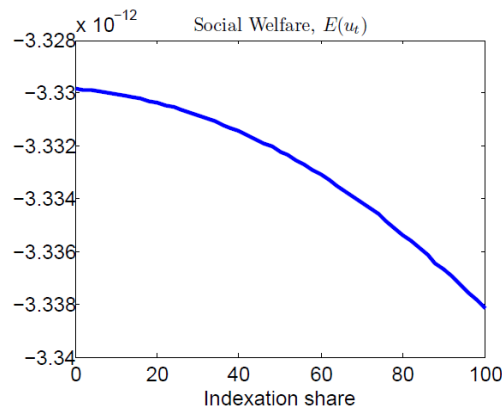
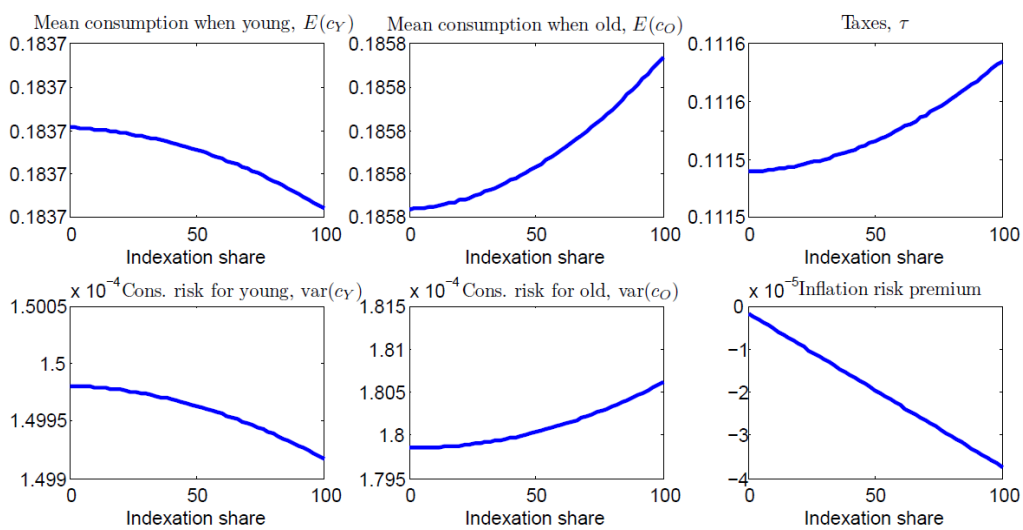


Fig 4 – Factors driving optimal indexation (PT)



In stark contrast to the IT case, zero indexation of government debt is optimal under PT. Figure 4 shows that this is because indexed bonds are more risky than nominal bonds, so that consumption risk for the old is minimised at close to zero indexation. The riskiness of indexed government debt shows itself clearly in a *negative* inflation risk premium on nominal bonds, in stark contrast to the IT case. As a result, the relationship between taxes and indexation is positive under PT, so that average consumption by the young is maximised under zero indexation. In short, the main factors driving optimal indexation work in exactly the opposite direction under PT, because nominal government debt is a better store of value than indexed debt and so commands a lower risk premium. The impact of the one-year indexation lag on indexed government debt is crucial for understanding this result. Indeed, the relatively poor performance of indexed debt under PT is driven solely by the indexation lag, since full indexation is optimal under both IT and PT in the absence of an indexation lag.

The one-year indexation lag is crucial because cumulative inflation risk over a 20-year horizon is reduced to yearly levels, due to the absence of base-level drift under PT. This makes nominal government bonds rather effective stabilisers of long run purchasing power, while indexed bonds ‘miss’ completely the yearly shocks that matter for 20-year inflation because they are indexed with a one-year lag. The results also highlight clearly the importance of allowing for changes in indexation of government debt when evaluating consumption risk and social welfare. In particular, changes in indexation are associated with much larger changes in consumption risk and social welfare under an IT regime – as can be seen clearly by comparing Figures 1 to 4. Consequently, if indexation is fixed at 20 per cent (which is close to the current share of index-linked gilts in the UK) there is an implied welfare gain from PT equivalent to 0.20 per cent of aggregate consumption.³² But when indexation is optimised under both regimes, this welfare gain is reduced to essentially zero.

7. Extensions and robustness

The baseline model makes a number of simplifying assumptions, including a PT target horizon of one year (i.e. all inflationary shocks are undone after one year);³³ perfect credibility of PT; and uncorrelated yearly money supply innovations. In this section, each of these assumptions is relaxed. Sensitivity to calibrated values is also discussed.

7.1 A flexible target horizon under PT

Gaspar, Smets and Vestin (2007) argue that the short-term stabilization costs of undoing price level shocks could be reduced by restoring the price level to its target path gradually following deviations from target. The analysis in this section therefore investigates optimal indexation under a PT regime that returns the price level to target over several years, in contrast to the maintained assumption of one year in the baseline case. To do so, the target horizon in the PT money supply rule is varied from 1 to 4 years.

For the general case where the price level is returned to target in uniform steps over H years, the yearly money supply rule is as follows:

³² The welfare gain (or loss) from PT was computed as the fractional increase in aggregate consumption, λ , necessary to equate social welfare under IT with that under PT, i.e. $SW^{IT}(1+\lambda)^{1-\gamma} = SW^{PT}$.

³³ Under IT, the target horizon is irrelevant because base-level drift means that shocks to inflation are not offset by policy.

$$M_n = M_{n-1}(1 + \pi^*) \frac{(1 + \varepsilon_n)}{\prod_{j=1}^H (1 + \varepsilon_{n-j})^{1/H}} \quad (28)$$

Note that innovations up to H years old enter in the denominator of this rule because each is offset only after H years in total, with a fraction $1/H$ offset each year.

By substitution, (28) implies that:

$$M_t = M_{t-1}(1 + \pi^*)^{20} \prod_{k=0}^{H-1} \left(\frac{1 + \varepsilon_{20-k,t}}{1 + \varepsilon_{20-k,t-1}} \right)^{(H-k)/H} \quad (29)$$

Inflation is therefore given by

$$1 + \pi_t = (1 + \pi^*)^{20} \prod_{k=0}^{H-1} \left(\frac{1 + \varepsilon_{20-k,t}}{1 + \varepsilon_{20-k,t-1}} \right)^{(H-k)/H} \quad (30)$$

Consider setting $H = 2$. In this case, only money supply shocks in years 19 and 20 of period t matter for inflation because shocks in years 1 to 18 will have been offset fully by the end of the period t (i.e. by year 20), since the target horizon is 2 years. Shocks in years 19 and 20 from the previous period enter in the denominator because these shocks will not have been offset before the end of period $t-1$ and so must be offset in period t to return the price level to its target path.

Since the inflation rate to which indexed bonds are linked is the one-year lagged value of actual inflation, indexed inflation for a target horizon of H years is now given by

$$1 + \pi_t^{ind} = (1 + \pi^*)^{20} \prod_{k=0}^{H-1} \left(\frac{1 + \varepsilon_{19-k,t}}{1 + \varepsilon_{19-k,t-1}} \right)^{(H-k)/H} \quad (31)$$

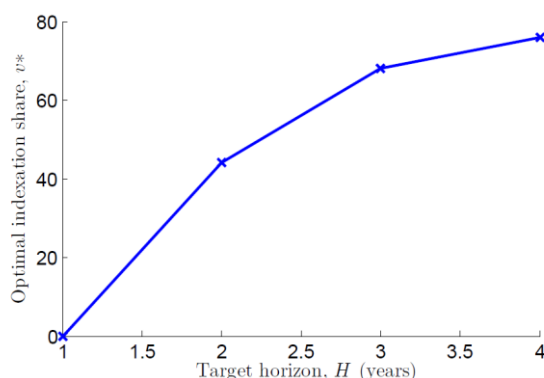
Equation (31) shows that a gradual return of the price level to target has two effects. First, returning the price level to target gradually raises long run inflation risk because the price level is allowed to deviate from its target path for longer. Second, a PT target horizon longer than one year implies that actual and indexed inflation are positively correlated, because past deviations from the target price path are ‘smoothed’ back to target over several years. Hence indexed bonds become better stabilisers purchasing power relative to indexed bonds.

The results for PT target horizons of 1 to 4 years are reported in Figure 5. Optimal indexation rises with the target horizon, from zero when $H = 1$ (the baseline case), to 44 per cent when $H = 2$, and up to 76 per cent when $H = 4$. The reason is that, as noted above, a target longer than one year implies that actual and indexed inflation are positively correlated, so that indexed bonds become better stabilisers of purchasing power relative to nominal bonds. This relative reduction in risk is reflected in a positive inflation risk premium on nominal bonds,³⁴ which implies that taxes can be reduced – and hence average consumption by the young increased – by issuing some indexed bonds. This rise in average consumption for the young is

³⁴ However, the inflation risk premium turns negative at relatively high indexation shares, which helps explain why full indexation is not optimal.

crucial for social welfare because it dominates the fall in average consumption for the old, and is not discounted in lifetime utility unlike the latter.

Fig 5 – Optimal indexation and the target horizon of policy (PT)



7.2 Imperfect credibility of PT

The argument that PT would be imperfectly credible is appealing since a regime of this kind has not been adopted in recent history. As such, imperfect credibility was an important consideration in the Bank of Canada’s deliberations about the relative merits of PT (Bank of Canada, 2011).

Imperfect credibility of PT has been studied by Gaspar, Smets and Vestin (2007) and Masson and Shukayev (2011). Gaspar et al. argue that PT would experience an initial period of imperfect credibility when agents would learn about the workings of the new regime. They use a New Keynesian model with learning and find that an initial period of imperfect credibility is sufficient to turn the net welfare gains from PT negative if agents are slow to learn, because expectations become backward-looking. Masson and Shukayev build a New Keynesian model where PT operates with an ‘escape clause’, such that sufficiently large shocks lead to rebasing of the target price path. They show that there are two stable equilibria: one with a low probability of rebasing, and one with a high probability. Consequently, both high and low credibility PT regimes are long run equilibrium outcomes. In contrast to these two papers, the analysis in this section concentrates on the impact of imperfect credibility through the long run inflation risk channel. The analysis also differs in that the model is non-linear, so that imperfect credibility influences aggregate outcomes through the inflation risk premium on nominal bonds, as well as by the inflation expectations channel.

In order to model imperfect credibility, it is assumed that young agents assign a constant probability p_{IT} to the event that that monetary policy will switch back to IT in the next period. Accordingly, agents assign a constant probability $1 - p_{IT}$ to the event that the current PT regime will remain in place next period. The probability p_{IT} can thus be taken as a measure of credibility, with $p_{IT} = 0$ corresponding to the perfect credibility case (the baseline case analysed above). It is important to note that although agents assign a positive probability to reversions to IT, no such reversions actually occur in equilibrium. Hence the analysis is one of imperfect credibility and not regime switching.

Given beliefs over regimes $s = \{IT, PT\}$ and period- t information Ω_t , lifetime utility is

$$u_t^C = \frac{1}{1-\gamma} \left[c_{t,Y}^\varepsilon + \beta \left[p_{IT} E[c_{t+1,O(IT)}^{1-\gamma} | \Omega_t] + (1-p_{IT}) E[c_{t+1,O(PT)}^{1-\gamma} | \Omega_t] \right]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma}{\varepsilon}} \quad (32)$$

The first-order conditions are given by the following Euler equations:³⁵

$$1 = R_t (p_{IT} E[SDF_{t+1(IT)} | \Omega_t] + (1-p_{IT}) E[SDF_{t+1(PT)} | \Omega_t]) \quad (33)$$

$$1 = r_t (p_{IT} E[SDF_{t+1(IT)} (1 + \pi_{t+1(IT)}^{ind}) | \Omega_t] + (1-p_{IT}) E[SDF_{t+1(PT)} (1 + \pi_{t+1(PT)}^{ind}) | \Omega_t]) \quad (34)$$

$$1 = \alpha k_{t+1}^{\alpha-1} (p_{IT} (1 - \tau_{IT}^k) E[sdf_{t+1(IT)} A_{t+1} | \Omega_t] + (1-p_{IT}) (1 - \tau_{PT}^k) E[sdf_{t+1(PT)} A_{t+1} | \Omega_t]) \quad (35)$$

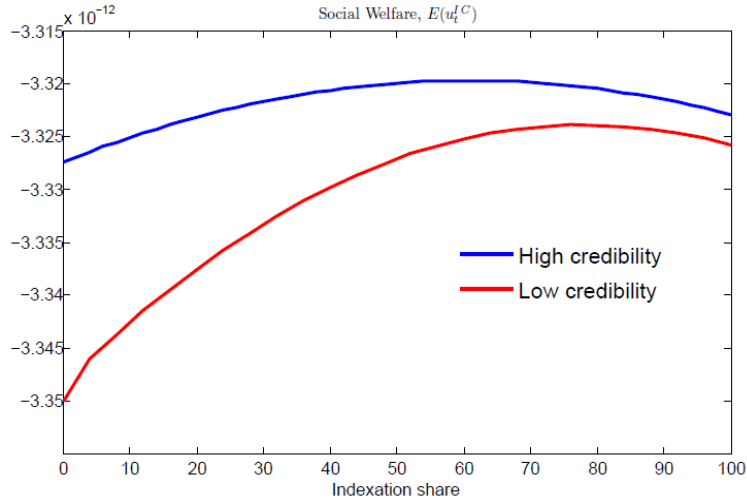
$$1 = (p_{IT} E[SDF_{t+1(IT)} | \Omega_t] + (1-p_{IT}) E[SDF_{t+1(PT)} | \Omega_t]) + \tilde{\mu}_t \quad (36)$$

where $SDF_{t+1(s)} \equiv sdf_{t+1(s)} / (1 + \pi_{t+1(s)})$, τ_s^k is the tax rate on capital in regime s , and

$$sdf_{t+1(s)} \equiv \beta \left(\frac{c_{t,Y}}{c_{t+1,O(s)}} \right)^{1-\varepsilon} \left[\frac{c_{t+1,O(s)}}{(p_{IT} E[c_{t+1,O(IT)}^{1-\gamma} | \Omega_t] + (1-p_{IT}) E[c_{t+1,O(PT)}^{1-\gamma} | \Omega_t])^{1/(1-\gamma)}} \right]^{1-\gamma-\varepsilon} \quad (37)$$

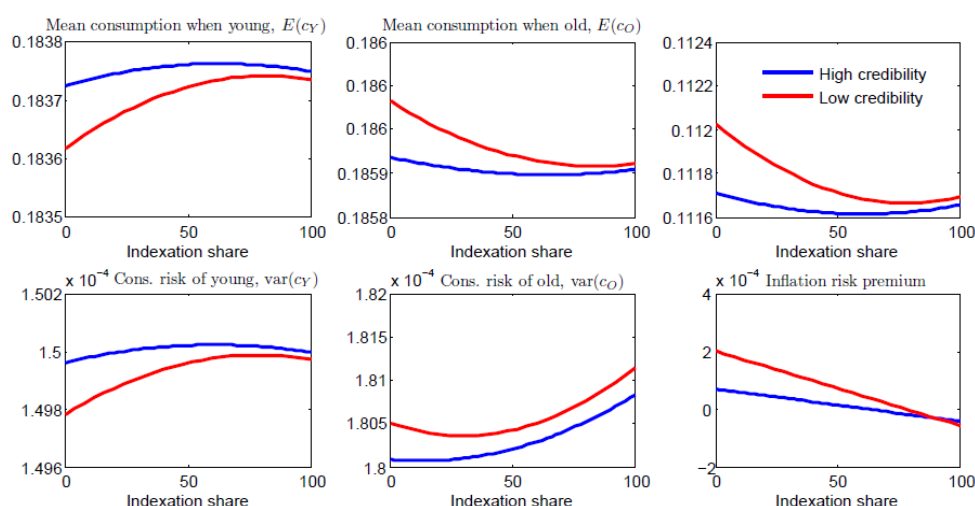
The model was simulated for two different values of p_{IT} : 0.1 and 0.3. These values represent fixed beliefs that policy will revert to IT next period with 10 and 30 per cent probability. The former is interpreted as a situation where PT has high credibility and the latter as a situation where it has low credibility. The results for these two cases are reported in Figures 6 and 7.

Fig 6 – Indexation and social welfare under imperfect credibility of PT



³⁵ The government is assumed to set the total bond supply so that the conditionally expected stochastic discount factor across regimes is equal to β (a natural extension of (12)), implying that $1 = \beta r$ in the deterministic steady-state. The first-order conditions are derived in the Appendix. Note that first-order condition for capital holdings reflects agents' belief that if there were a change in regime back to IT, the long run IT tax rate would apply.

Fig 7 – Factors driving optimal indexation (PT)



Optimal indexation is somewhat higher than in the baseline model, at around 60 per cent under high credibility and 80 per cent under low credibility. Optimal indexation rises because imperfect credibility increases the inflation risk premium on nominal government debt. The reason is that agents' expectations reflect their belief that policy may revert to the high inflation risk IT regime under which indexed debt is a far better store of value than nominal debt. In turn, this increase in the inflation risk premium makes it more costly (in real terms) to issue nominal rather than indexed government debt, implying higher taxes to meet the long run government spending target and lower average consumption for the young. An interior solution for optimal indexation balances the welfare loss from this increase in taxes against the welfare gain from the fact that nominal government debt stabilises old age consumption more effectively than indexed debt under PT.

Intuitively, the level of indexation is higher under low credibility because there is a larger rise in the inflation risk premium, so that the increase in taxes necessary to meet the long run government spending target is higher. Hence, while the welfare loss and welfare gain referred to above are of roughly equal magnitude under high credibility, a situation of low credibility shifts the balance in favour of indexed government debt because it raises the welfare loss from higher taxes whilst leaving the welfare gain side of the equation unaffected.³⁶

7.3 Correlated shocks to inflation

The baseline model assumes that yearly money supply innovations are uncorrelated. As discussed in Section 6, this assumption is likely to be important for optimal indexation because it implies that current and past shocks to inflation are uncorrelated, so that indexation to lagged inflation is more costly than it would under positive autocorrelation. In this section, the assumption of uncorrelated innovations is relaxed. In particular, it is assumed that the money supply innovation in any given year n of period t is positively correlated with the innovation in the previous year $n-1$. The correlation was set at 0.5 because empirical evidence suggests inflation persistence has fallen to moderate levels in the Great Moderation, in contrast to much of the postwar period (e.g. Benati, 2008; Minford et al. 2009).

³⁶ The latter is unchanged because it depends only on actual policy (i.e. PT) and not consumers' beliefs about which regime will be in place next period.

Since allowing for correlated innovations leaves optimal indexation unchanged under IT, the focus in this section is on the impact of correlated innovations under PT. More specifically, the question of interest is whether correlated innovations close substantially the optimal indexation differential between IT and PT. The results are reported in Figures 8 and 9.

Fig 8 – Indexation and social welfare (PT)

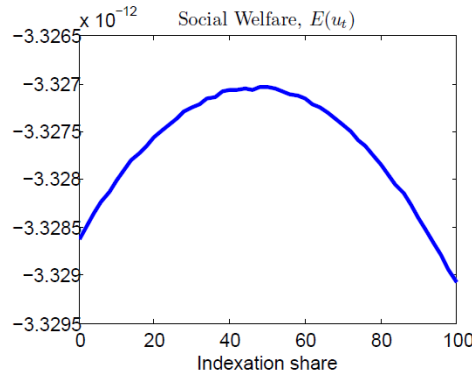
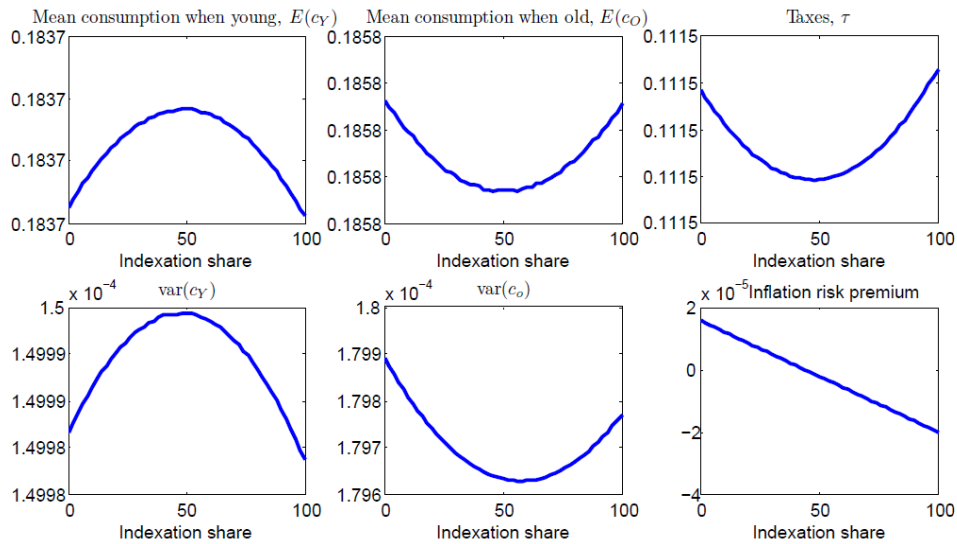


Fig 9 – Factors driving optimal indexation (PT)



Optimal indexation under PT is somewhat higher than in the baseline case at 48 per cent. Intuitively, correlated money supply innovations make indexed bonds a better store of value as compared to nominal bonds, so that consumption risk in old age is now minimised at around 50 per cent indexation, rather than at zero as in the baseline case. The fall in riskiness of indexed bonds is reflected directly in the inflation risk premium – it is now positive up to an indexation share of almost 50 per cent, so that the taxes necessary to meet the government spending target are minimised, and average consumption by the young maximised, at an indexation share of around 50 per cent. Since an indexation share of around 50 per cent maximises mean consumption by the young and minimises consumption risk for the old, it is intuitive that the optimal indexation share is close to 50 per cent.

7.4 Calibration sensitivity analysis

This final subsection is concerned with sensitivity of the baseline result to calibrated parameters. In particular, the baseline model was solved for the optimal indexation share under ‘high’ and ‘low’ calibrations of model parameters, including shock standard deviations. The main result was not overturned in any of these sensitivity tests.³⁷ Consequently, we can say that this finding is robust to (i) more realistic versions of the model, and (ii) alternative calibrations of the baseline model.

8. Conclusion

This paper has investigated optimal indexation of long-term government debt under inflation targeting (IT) and price-level targeting (PT). These two monetary regimes have very different long run implications. Under IT, inflation risk increases with the forecast horizon since there is base-level drift in the price level. By contrast, a PT regime rules out base-level drift. As a result, long run inflation risk is largely eliminated under PT, with the implication that the purchasing power of nominal assets is maintained over long horizons. Optimal indexation was studied in the context of a simple overlapping generations (OG) model that was roughly calibrated to match the UK economy. The model is well-suited for this task because each period lasts 20 years and long run inflation risk matters for social welfare. In order to capture base-level drift under IT – and its absence under PT – the standard OG model was augmented to include money supply shocks at a yearly horizon.

The main finding is that when indexed government debt is subject to a one-year indexation lag, optimal indexation is substantially lower under PT. In order to demonstrate this result, the analysis began with a simple version of the model in which full indexation is optimal under IT, and zero indexation is optimal under PT. Intuitively, return risk on long-term nominal bonds is somewhat higher than on indexed debt under IT, since cumulative inflation risk over a 20-year horizon is approximately 20 times that at a yearly horizon (due to base-level drift), while indexed bonds are subject to only one year’s worth of inflation risk. Under a PT regime, by contrast, 20-year inflation risk is lowered to annual magnitudes, and so is similar to the one year’s worth of inflation risk to which the return on indexed bonds is exposed. However, indexed bonds are worse stabilisers of purchasing power than nominal bonds under PT because actual and lagged inflation are entirely uncorrelated when money supply innovations are white noise. As a result, there is no clear-cut benefit to indexation from the point of view of consumption stabilisation or government finances under a PT regime.

The result that optimal indexation is substantially lower under PT holds in more realistic versions of the model in which the PT target horizon exceeds one year; where PT is imperfectly credible; and where yearly money innovations are serially correlated, so that indexing with a one-year lag is less costly than in the baseline case. In each case the differential between optimal indexation under IT and PT is closed by 50 per cent or more, indicating that moderate levels of indexation become optimal under PT. In the case of a low credibility PT regime there is quite a large impact: optimal indexation rises to 80 per cent, since the belief that policy may revert to IT next period (where long run inflation risk is much higher) raises the inflation risk premium on nominal government debt substantially.

³⁷ The results are available from the author on request and may be included in a separate web appendix.

Sensitivity analysis on model parameters does not overturn the main result, but it is important to note that relaxing the assumption that indexation is lagged makes full indexation of government debt optimal under both IT and PT.

An additional finding is that the long run welfare gains from PT are overstated non-trivially if indexation of government debt is held fixed under both regimes at the current UK level. Therefore, changes in the structure of the government bond portfolio have potentially important welfare implications for comparing IT and PT regimes. Policymakers and researchers should bear this in mind when assessing the case for a change in regime from IT to PT. More generally, the results in this paper suggest that it may be important to account for changes in the structure of long run nominal portfolios when comparing IT and PT regimes.

Technical appendix (Not for publication)

Section 1 – Derivations and proofs

A – Derivation of first-order conditions in the baseline case

Consumers solve a maximisation problem of the form

$$\max_{\{c_{t,Y}, z_t, c_{t+1,O}\}} u_t = \frac{1}{1-\gamma} \left[c_{t,Y}^\varepsilon + \beta [E_t c_{t+1,O}^{1-\gamma}]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma}{\varepsilon}} \quad \text{subject to} \quad (\text{A1})$$

$$c_{t,Y} = (1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t \quad (\text{Budget constraint of young})$$

$$c_{t+1,O} = (1-\tau^k)r_{t+1}^k k_{t+1} + r_{t+1}^i b_{t+1}^i + r_{t+1}^n b_{t+1}^n + r_{t+1}^m m_t \quad (\text{Budget constraint of old})$$

$$m_t = \delta \quad (\text{Cash constraint})$$

where $z_t \equiv (k_{t+1}, b_{t+1}^i, b_{t+1}^n, m_t)$ is the vector of assets chosen by households.

The Lagrangian for this problem is as follows:

$$L_t = E_t \left\{ u_t + \lambda_{t,Y} ((1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t - c_{t,Y}) + \lambda_{t+1,O} ((1-\tau^k)r_{t+1}^k k_{t+1} + r_{t+1}^i b_{t+1}^i + r_{t+1}^n b_{t+1}^n + r_{t+1}^m m_t - c_{t+1,O}) + \mu_t (m_t - \delta) \right\} \quad (\text{A2})$$

First-order conditions are as follows:

$$c_{t,Y} : \frac{\partial u_t}{\partial c_{t,Y}} = \lambda_{t,Y}, \quad c_{t+1,O} : \frac{\partial u_t}{\partial c_{t+1,O}} = \lambda_{t+1,O}, \quad k_{t+1} : \lambda_{t,Y} = E_t (\lambda_{t+1,O} (1-\tau^k) r_{t+1}^k)$$

$$b_{t+1}^i : \lambda_{t,Y} = E_t (\lambda_{t+1,O} r_{t+1}^i), \quad b_{t+1}^n : \lambda_{t,Y} = E_t (\lambda_{t+1,O} r_{t+1}^n), \quad m_t : \lambda_{t,Y} = E_t (\lambda_{t+1,O} r_{t+1}^m) + \mu_t$$

By substitution, this system can be reduced to four Euler equations:

$$\frac{\partial u_t}{\partial c_{t,Y}} = E_t \left(\frac{\partial u_t}{\partial c_{t+1,O}} (1-\tau^k) r_{t+1}^k \right), \quad \frac{\partial u_t}{\partial c_{t,Y}} = E_t \left(\frac{\partial u_t}{\partial c_{t+1,O}} r_{t+1}^n \right), \quad \frac{\partial u_t}{\partial c_{t,Y}} = E_t \left(\frac{\partial u_t}{\partial c_{t+1,O}} r_{t+1}^i \right),$$

$$\frac{\partial u_t}{\partial c_{t,Y}} = E_t \left(\frac{\partial u_t}{\partial c_{t+1,O}} r_{t+1}^m \right) + \mu_t$$

The partial derivatives of the utility function are as follows:

$$\frac{\partial u_t}{\partial c_{t,Y}} = \left[c_{t,Y}^\varepsilon + \beta [E_t c_{t+1,O}^{1-\gamma}]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}} c_{t,Y}^{-(1-\varepsilon)} \quad (\text{A3})$$

$$\frac{\partial u_t}{\partial c_{t+1,O}} = \left[c_{t,Y}^\varepsilon + \beta [E_t c_{t+1,O}^{1-\gamma}]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}} \beta [E_t (c_{t+1,O}^{1-\gamma})]^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} c_{t+1,O}^{-\gamma} \quad (\text{A4})$$

Dividing (A4) by (A3) gives

$$\frac{\partial u_t / \partial c_{t+1,O}}{\partial u_t / \partial c_{t,Y}} = \frac{\beta [E_t(c_{t+1,O}^{1-\gamma})]^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} c_{t+1,O}^{-\gamma}}{c_{t,Y}^{-(1-\varepsilon)}} = \beta \left(\frac{c_{t,Y}}{c_{t+1,O}} \right)^{1-\varepsilon} \left(\frac{c_{t+1,O}}{(E_t c_{t+1,O}^{1-\gamma})^{1/(1-\gamma)}} \right)^{1-\gamma-\varepsilon} \quad (\text{A5})$$

Defining $sdf_{t+1} \equiv \frac{\partial u_t / \partial c_{t+1,O}}{\partial u_t / \partial c_{t,Y}}$, the four Euler equations above can be written as follows:

$$1 = E_t(sdf_{t+1}(1 - \tau^k)r_{t+1}^k) \quad (\text{A6})$$

$$1 = E_t(sdf_{t+1}r_{t+1}^n) \quad (\text{A7})$$

$$1 = E_t(sdf_{t+1}r_{t+1}^i) \quad (\text{A8})$$

$$1 = E_t(sdf_{t+1}r_{t+1}^m) + \tilde{\mu}_t \quad (\text{A9})$$

where $\tilde{\mu}_t \equiv \mu_t / \lambda_{t,Y}$.

B – The binding legal constraint on money holdings

It is shown in this section that the constraint binds with strict equality if the gross money return on a nominal bond exceeds 1.

Proposition: The constraint binds with strict equality when $R_t > 1$

Proof.

By equations (A7) and (A9), the Lagrange multiplier on the cash constraint is given by

$$\mu_t = E_t[sdf_{t+1}(r_{t+1}^n - r_{t+1}^m)]\lambda_{t,Y} \quad (\text{B1})$$

Since the real return on nominal bonds is $r_{t+1}^n = R_t / (1 + \pi_{t+1}) = R_t r_{t+1}^m$, we can also write

$$\mu_t = E_t[sdf_{t+1}(R_t - 1)r_{t+1}^m]\lambda_{t,Y} = \lambda_{t,Y}(R_t - 1)E_t[sdf_{t+1}r_{t+1}^m] \quad (\text{B2})$$

since the nominal yield on nominal government bonds, R_t , is known at the end of period t .

The Kuhn-Tucker conditions associated with μ_t are as follows:

$$\{ \mu_t \geq 0 \quad \text{and} \quad \mu_t(m_t - \delta) = 0 \} \quad (\text{B3})$$

The second condition in (B3) is the complementary slackness condition. It implies that the cash constraint will be strictly binding iff $\mu_t > 0$ for all t .

Dividing (B2) by $1 = E_t[sdf_{t+1}r_{t+1}^n] = R_t E_t[sdf_{t+1}r_{t+1}^m]$, it follows that $\mu_t = \lambda_{t,Y}(R_t - 1) / R_t$.

Since $\lambda_{t,Y} > 0$ (as the budget constraint of the young will always hold with equality), it follows that $\mu_t > 0$ iff $R_t > 1$ for all t . **Q.E.D.**

C – Approximate analytical expressions for long run inflation risk under IT and PT

This appendix derives approximate expressions for the inflation variance under IT and PT.

Inflation Targeting (IT)

Under IT, inflation in period t is given by

$$1 + \pi_t = (1 + \pi^*)^{20} \prod_{j=1}^{20} (1 + \varepsilon_{j,t}) \quad (C1)$$

where $\varepsilon_{j,t}$ are IID-normal innovations with mean zero and variance σ^2 .

Since a general non-linear function $g(\varepsilon)$ (where ε is a vector of variables) can be approximated by $\text{var}(g(\varepsilon)) \approx \sum [g_j'(\mu)]^2 \text{var}(\varepsilon_j)$ using the ‘Delta method’ (where μ is the unconditional mean of the vector ε , and g_j' is the first derivative of $g(\varepsilon)$ with respect to variable ε_j), the inflation variance under IT can be approximated as follows:

$$\text{var}(\pi_t) \approx \sum_{j=1}^{20} (1 + \pi^*)^{40} \sigma^2 = (1 + \pi^*)^{40} 20 \sigma^2 \quad (C2)$$

Price-level targeting (PT)

Under PT, inflation in period t is given by

$$1 + \pi_t = (1 + \pi^*)^{20} \frac{(1 + \varepsilon_{30,t})}{(1 + \varepsilon_{30,t-1})} \quad (C3)$$

where $\varepsilon_{30,t}$ and $\varepsilon_{30,t-1}$ are IID-normal innovations with mean zero and variance σ^2 .

Using the same approximation method as above, the inflation variance under PT is given by

$$\text{var}(\pi_t) \approx [(1 + \pi^*)^{20}]^2 \sigma^2 + [-(1 + \pi^*)^{20}]^2 \sigma^2 = (1 + \pi^*)^{40} 2 \sigma^2 \quad (C4)$$

Hence the unconditional variance of inflation under IT is (approx.) 10 times that under PT.

D – First-order conditions under imperfect credibility

In this case, consumers solve the following problem where $s = \{IT, PT\}$:

$$\max_{\{c_{t,Y}, z_t, c_{t+1,O}\}} u_t^{IC} = \frac{1}{1 - \gamma} \left[c_{t,Y}^\varepsilon + \beta \left[p_{IT} E[c_{t+1,O(IT)}^{1-\gamma} | \Omega_t] + (1 - p_{IT}) E[c_{t+1,O(PT)}^{1-\gamma} | \Omega_t] \right]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma}{\varepsilon}} \quad (D1)$$

subject to

$$c_{t,Y} = (1 - \tau) w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t \quad (\text{Budget constraint of young})$$

$$c_{t+1,O(IT)} = (1 - \tau_{IT}^k) r_{t+1}^k k_{t+1} + r_{t+1(IT)}^i b_{t+1}^i + r_{t+1(IT)}^n b_{t+1}^n + r_{t+1(IT)}^m m_t \quad (\text{Budget constraint of old with IT})$$

$$c_{t+1,O(PT)} = (1 - \tau_{PT}^k) r_{t+1}^k k_{t+1} + r_{t+1(PT)}^i b_{t+1}^i + r_{t+1(PT)}^n b_{t+1}^n + r_{t+1(PT)}^m m_t \quad (\text{Budget constraint of old with PT})$$

$$m_t = \delta \quad (\text{Cash constraint})$$

where $E[X_{t+1(s)} | \Omega_t]$ is the expectation of X_{t+1} in regime s , conditional upon period- t information, Ω_t .

The Lagrangian for this problem is as follows:

$$L_t = E \left\{ \left(\begin{aligned} & u_t^{IC} + \lambda_{t,Y} ((1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t - c_{t,Y}) + \mu_t (m_t - \delta) \\ & + \lambda_{t+1,O(\Pi)} ((1-\tau_{\Pi}^k)r_{t+1}^k k_{t+1} + r_{t+1(\Pi)}^i b_{t+1}^i + r_{t+1(\Pi)}^n b_{t+1}^n + r_{t+1(\Pi)}^m m_t - c_{t+1,O(\Pi)}) \\ & + \lambda_{t+1,O(PT)} ((1-\tau_{PT}^k)r_{t+1}^k k_{t+1} + r_{t+1(PT)}^i b_{t+1}^i + r_{t+1(PT)}^n b_{t+1}^n + r_{t+1(PT)}^m m_t - c_{t+1,O(PT)}) \end{aligned} \right) \Bigg| \Omega_t \right\} \quad (D2)$$

First-order conditions are as follows:

$$c_{t,Y} : \frac{\partial u_t}{\partial c_{t,Y}} = \lambda_{t,Y}, \quad c_{t+1,O(\Pi)} : \frac{\partial u_t}{\partial c_{t+1,O(\Pi)}} = \lambda_{t+1,O(\Pi)}, \quad c_{t+1,O(PT)} : \frac{\partial u_t}{\partial c_{t+1,O(PT)}} = \lambda_{t+1,O(PT)}$$

$$k_{t+1} : \lambda_{t,Y} = E([\lambda_{t+1,O(\Pi)} (1-\tau_{\Pi}^k)r_{t+1}^k + \lambda_{t+1,O(PT)} (1-\tau_{PT}^k)r_{t+1}^k] | \Omega_t)$$

$$b_{t+1}^i : \lambda_{t,Y} = E([\lambda_{t+1,O(\Pi)} r_{t+1(\Pi)}^i + \lambda_{t+1,O(PT)} r_{t+1(PT)}^i] | \Omega_t)$$

$$b_{t+1}^n : \lambda_{t,Y} = E([\lambda_{t+1,O(\Pi)} r_{t+1(\Pi)}^n + \lambda_{t+1,O(PT)} r_{t+1(PT)}^n] | \Omega_t)$$

$$m_t : \lambda_{t,Y} = E([\lambda_{t+1,O(\Pi)} r_{t+1(\Pi)}^m + \lambda_{t+1,O(PT)} r_{t+1(PT)}^m] | \Omega_t) + \mu_t$$

By substitution, this system can be reduced to four Euler equations:

$$\frac{\partial u_t^{IC}}{\partial c_{t,Y}} = E \left(\left[\frac{\partial u_t}{\partial c_{t+1,O(\Pi)}} (1-\tau_{\Pi}^k)r_{t+1}^k + \frac{\partial u_t}{\partial c_{t+1,O(PT)}} (1-\tau_{PT}^k)r_{t+1}^k \right] \Bigg| \Omega_t \right) \quad (D3)$$

$$\frac{\partial u_t^{IC}}{\partial c_{t,Y}} = E \left(\left[\frac{\partial u_t}{\partial c_{t+1,O(\Pi)}} r_{t+1(\Pi)}^n + \frac{\partial u_t}{\partial c_{t+1,O(PT)}} r_{t+1(PT)}^n \right] \Bigg| \Omega_t \right) \quad (D4)$$

$$\frac{\partial u_t^{IC}}{c_{t,Y}} = E \left(\left[\frac{\partial u_t}{\partial c_{t+1,O(\Pi)}} r_{t+1(\Pi)}^i + \frac{\partial u_t}{\partial c_{t+1,O(PT)}} r_{t+1(PT)}^i \right] \Bigg| \Omega_t \right) \quad (D5)$$

$$\frac{\partial u_t^{IC}}{\partial c_{t,Y}} = E \left(\left[\frac{\partial u_t}{\partial c_{t+1,O(\Pi)}} r_{t+1(\Pi)}^m + \frac{\partial u_t}{\partial c_{t+1,O(PT)}} r_{t+1(PT)}^m \right] \Bigg| \Omega_t \right) + \mu_t \quad (D6)$$

The partial derivatives of the utility function are as follows:

$$\frac{\partial u_t^{IC}}{\partial c_{t,Y}} = \Phi c_{t,Y}^{-(1-\varepsilon)} \quad (D7)$$

$$\frac{\partial u_t^{IC}}{\partial c_{t+1,O(s)}} = \beta \Phi p_s (p_{\Pi} E[c_{t+1,O(\Pi)}^{1-\gamma} | \Omega_t] + (1-p_{\Pi}) E[c_{t+1,O(PT)}^{1-\gamma} | \Omega_t])^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} c_{t+1,O(s)}^{-\gamma} \quad (D8)$$

where $\Phi \equiv \left[c_{t,Y}^\varepsilon + \beta \left[p_{II} E[c_{t+1,O(II)}^{1-\gamma} | \Omega_t] + (1 - p_{II}) E[c_{t+1,O(PT)}^{1-\gamma} | \Omega_t] \right]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}}$ and $p_{PT} \equiv 1 - p_{II}$.

Dividing (D8) by (D7) gives

$$\frac{\partial u_t^{IC} / \partial c_{t+1,O(s)}}{\partial u_t^{IC} / \partial c_{t,Y}} = \beta p_s \left(\frac{c_{t,Y}}{c_{t+1,O(s)}} \right)^{1-\varepsilon} \left[\frac{c_{t+1,O(s)}}{(p_{II} E[c_{t+1,O(II)}^{1-\gamma} | \Omega_t] + (1 - p_{II}) E[c_{t+1,O(PT)}^{1-\gamma} | \Omega_t])^{1/(1-\gamma)}} \right]^{1-\gamma-\varepsilon} \quad (D9)$$

Defining $sdf_{t+1(s)} \equiv \frac{1}{p_s} \frac{\partial u_t^{IC} / \partial c_{t+1,O(s)}}{\partial u_t^{IC} / \partial c_{t,Y}}$, the four Euler equations above can be written as

$$1 = R_t (p_{II} E[SDF_{t+1(II)} | \Omega_t] + (1 - p_{II}) E[SDF_{t+1(PT)} | \Omega_t]) \quad (D10)$$

$$1 = r_t (p_{II} E[SDF_{t+1(II)} (1 + \pi_{t+1(II)}^{ind}) | \Omega_t] + (1 - p_{II}) E[SDF_{t+1(PT)} (1 + \pi_{t+1(PT)}^{ind}) | \Omega_t]) \quad (D11)$$

$$1 = \alpha k_{t+1}^{\alpha-1} ([p_{II} (1 - \tau_{II}^k) E[sdf_{t+1(II)} A_{t+1} | \Omega_t] + (1 - p_{II}) (1 - \tau_{PT}^k) E[sdf_{t+1(PT)} A_{t+1} | \Omega_t]) \quad (D12)$$

$$1 = (p_{II} E[SDF_{t+1(II)} | \Omega_t] + (1 - p_{II}) E[SDF_{t+1(PT)} | \Omega_t]) + \tilde{\mu}_t \quad (D13)$$

where $SDF_{t+1(s)} \equiv sdf_{t+1(s)} / (1 + \pi_{t+1(s)})$ and $\tilde{\mu}_t \equiv \mu_t / \lambda_{t,Y}$.

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