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Accounting for palaeoclimate and topography: a rigorous approach to correction of the British geothermal dataset

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Abstract

Raw heat flow measurements typically require correction for both palaeoclimate and topography if temperatures are to be reliably extrapolated to depths greater than those where temperature is measured. Such corrections are thus an essential step in quantifying geothermal energy resources. However, although both types of correction were pioneered decades ago by British workers, they have been omitted or underplayed in subsequent assessments of the UK geothermal dataset. Furthermore, as most UK heat flow measurements to date have utilised shallow boreholes, the magnitudes of the required corrections (for both effects) are exacerbated. In addition, the location of Britain, at a range of latitude with a temperate climate at present but where arctic conditions prevailed during much of the Pleistocene, means that the palaeoclimate correction, for a borehole of a given depth, is particularly large. Outside regions of high relief relative to borehole depth, palaeoclimate corrections at sites in Britain are indeed larger in magnitude than topographic corrections, and for almost all boreholes are additive. The magnitude of the palaeoclimate correction depends on assumptions about palaeotemperature anomalies and their durations, but from the available evidence it can be assessed, for a very shallow borehole in an unglaciated part of southern Britain with rocks of thermal conductivity 3 W m^{-1} °C^{-1}, as 27 mW m^{-2}. Air temperatures during Pleistocene cold stages decreased northward, but in much of northern Britain the Earth's surface was not exposed to these low temperatures for part of the Late Pleistocene due to the insulating effect of cover by ice sheets; the detailed correction for each locality thus depends on the local histories of air temperature anomalies and of ice cover, and may therefore potentially be greater or less than is typical for southern England. The past failure to recognise the magnitude of palaeoclimate corrections at sites in Britain, and to incorporate them into studies of geothermics, has led to systematic underestimation of temperatures at depth and, thus, of the overall geothermal energy resource.

Key words:
Britain; heat flow; palaeoclimate correction; topographic correction

Highlights

- UK heat flow data have typically not been corrected for palaeoclimate or topography
- The required corrections are exacerbated by the national shallow-borehole strategy
- The palaeoclimate correction is also exacerbated by Gulf-Stream-related climate fluctations
- Predicted biostratigraphic palaeoclimate corrections are large, up to ~30 mW m^{-2}
- Past omission of corrections has led to underestimation of the geothermal resource

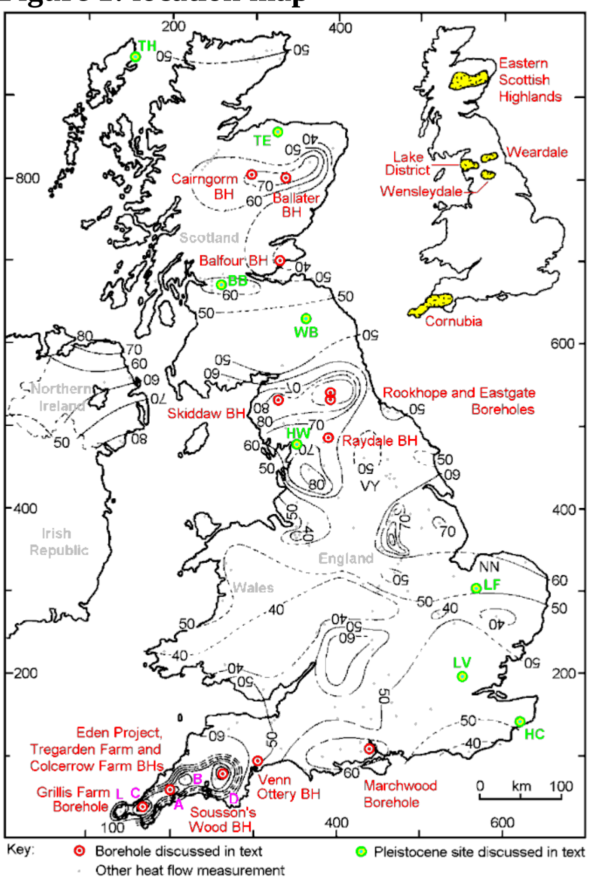
1. Introduction

Increased use of renewable energy is important for mitigation of the effects of anthropogenic carbon dioxide emissions; geothermal energy is of evident significance in any national mix of renewable energy resources. The recognition that geothermal energy has a very low 'carbon footprint' and does not suffer from the intermittency problems of other renewable energy sources such as wind and solar, and that economies of scale are resulting in increasingly competitive production costs, has led to a resurgence of interest in this technology in recent years (Younger et al., 2012). Heat flow can be measured in boreholes using standard techniques, as part of the assessment of geothermal energy resources. However, previous work has established that raw heat flow measurements require correction for effects of palaeoclimate and topography before they can give accurate predictions of temperatures at greater depths than those at which the measurements were obtained. The aim of this study is to assess the significance of such corrections for the British geothermal dataset.

To anticipate our conclusions, corrections of the types mentioned above are of particular importance for the British geothermal dataset for a number of reasons. First, although the effect of palaeoclimate on heat flow was first recognised decades ago (e.g., Benfield, 1939; Anderson, 1940), this effect has not been acknowledged, or corrected for, in many more recent studies. Given that the present-day climate of Britain is warmer than the time-averaged conditions during the Pleistocene, failure to correct for climate change will result in underestimation of heat flow and of temperatures at depth. Second, the location of Britain (between latitudes 50°N and 59°N), south of the arctic climatic front at present but north of it during Pleistocene cold stages, means that temperature fluctuations have been particularly large, necessitating correspondingly large corrections. Third, corrections have previously been applied for the effect of topography on focusing heat flow; in general, heat flow will thus increase beneath valleys and decrease beneath intervening hills. However, in Britain, such corrections have hitherto only been applied crudely. Fourth, the fact that prior evaluations of heat flow in the UK (e.g., Downing and Gray 1986) have relied largely on shallow boreholes (often <300 m deep and sometimes only ~100 m deep) implies that particularly large corrections might be needed to account for palaeoclimate and topography. In the absence of such corrections, reported heat flows may only be loose approximations of true conditions at depth. Indeed, the most recent update of UK geothermal data (Busby et al., 2011), published as an aid to identifying geothermal energy resources in Britain, still includes data that have not been corrected for palaeoclimate or topography, and is therefore unlikely to prove a reliable guide to temperatures when extrapolated beyond the depths at which temperature was measured.

In the account which follows, corrections to heat flow for palaeoclimate will be discussed first, followed by corrections for topography, then the joint application of both corrections will be considered. The corrections will be exemplified using data from a range of case study localities in Britain (Fig. 1). It should be noted that, in general, heat flow measurements may also be perturbed by a range of other effects besides palaeoclimate and topography, for example, as a result of hydrothermal circulation (e.g., Holliday, 1986; Younger et al., 2012), lateral refraction of heat flow between rocks of different composition, such as granite intrusions and the surrounding country rock (e.g., Lee, 1986), or advection of heat due to erosion (e.g., Stüwe et al., 1994; Westaway, 2002). Discussion of these effects is, however, beyond the scope of the present study.

Figure 1: location map



2. Correction for palaeoclimate

Past climate change has affected the temperature at the Earth's surface and thus the temperature distribution at depth, and has, therefore, also affected the geothermal gradient and heat flow within the Earth's crust. As this section of text will show, the importance of this effect on heat flow measurements in Britain was first recognised many decades ago, but it has subsequently been downplayed or overlooked (Younger et al. 2012). This is alarming, as palaeoclimate corrections to heat flow within Britain can be expected to be large, on the order of ~20 mW m^{-2}, and will typically be positive, due primarily to the effect of postglacial temperature rise, such that failure to correct for this effect, other factors being equal, will result in the underestimation of both heat flow and the magnitude of the UK's geothermal resource base. It is now apparent that this effect is particularly acute in Britain because of the large magnitude of temperature changes that result from the location adjoining the Gulf Stream. During times of temperate climate, as at present, the Gulf Stream causes temperatures in Britain to be significantly higher than they would otherwise be, whereas during cold-climate stages, this thermohaline circulation in the North Atlantic Ocean ceases (e.g., Broecker, 1981) and the climate cools dramatically.

This section will, first, recap on established theory for correcting heat flow measurements for changes in surface temperature, discussing how this theory has been incorporated into a computer program for calculation of this effect. Previous work on palaeoclimate correction of UK heat flow data will then be summarised, after which data pertaining to the estimation of surface palaeotemperature will be reviewed. Finally, quantitative calculations of corrections to heat flow measurements for conditions representing different parts of Britain will be presented.

Theory for palaeoclimate correction of heat flow data has previously been presented both in scholarly articles (e.g., Birch, 1948; Beck, 1977) and in textbooks (e.g., Turcotte and Schubert, 1982; Beardsmore and Cull, 2001). Past variations in surface temperature ΔT_o relative to its present value can be approximated as a series of step changes, each starting at a particular time t' before the present day. The contributions from each of these step changes are then summed to obtain the overall perturbation to the geotherm, $\delta T(z)$, at each depth z, at the present day. This theory is summarized in Appendix 1, with an analytic formula for $\delta T(z)$ as a result of n past step changes ΔT_o in surface temperature, given in equation (1.09). The perturbation to the temperature gradient, $\partial\delta T/\partial z$, can thus be obtained analytically by term-by-term differentiation of equation (1.09), and is given as equation (1.10). The associated perturbation to the heat flow at depth z, $\delta Q(z)$, can thus be determined as $k\times\partial\delta T/\partial z$, where k is the thermal conductivity of the bedrock. The assumed history of surface temperature variation thus determines the present-day perturbation to the geothermal gradient; the resulting heat flow perturbation scales in proportion to k, and so can be readily calculated for different vaues of k to those adopted, in proportion. A computer program was developed to explore this effect. This evaluates equation (1.10) to determine $\partial\delta T/\partial z$ and Q as functions of depth z. It then numerically integrates this solution for $\partial\delta T/\partial z$ (using Simpson's rule, to ensure no significant loss of accuracy) to recover the corresponding perturbation to temperature, $\delta T(z)$. Furthermore, to facilitate direct comparison with existing borehole heat flow datasets, the perturbation to the mean heat flow between depths z_1 and z_2 can also be evaluated as $\delta Q_m(z)$ where

$$\delta Q_m(z_1, z_2) \;=\; \frac{k}{2}\left[\frac{\partial\delta T}{\partial z}(z_1) \;+\; \frac{\partial\delta T}{\partial z}(z_2)\right]. \qquad (1)$$

Input to this program consists of specifying the crustal parameters k and the associated thermal diffusivity κ, the depth interval Δz over which calculations are performed, and a time-series of variations in surface temperature. The program was tested against numerical results from Birch (1948) and Turcotte and Schubert (1982); it reproduced these results to the precision at which they were presented.

2.1 Previous work on palaeoclimate corrections to heat flow in Britain

The importance of correction to heat flow data for Britain for palaeoclimate will be emphasised below (section 2.3). The significance of this effect was first recognised many decades ago, for example by Benfield (1939) and Anderson (1940). However, in subsequent investigations its importance has been rather downplayed. For example, Richardson and Oxburgh (1978) commented that the post-glacial rise in surface temperature will have reduced the present-day heat flow at shallow depths by ~0.2 hfu (i.e., by ~8.4 mW m^{-2}), although without indicating how this numerical value was determined. They also argued that the uncertainty in such corrections makes it preferable to avoid them, and so discussed uncorrected data even when previous authors had applied palaeoclimate corrections. Bloomer et al. (1979) subsequently compiled palaeoclimate corrections for UK geothermal data where these had been previously calculated by others, but did not make any further corrections themselves, despite noting that the reported corrections were sometimes rather large (up to 26 mW m^{-2}); nevertheless, they recommended neglecting corrections to facilitate comparison of the overall dataset. The detailed compilation of heat flow data by Burley et al. (1984) only included palaeoclimate corrections for some boreholes shallower than ~300 m in SW England (i.e., it omitted these corrections for deeper boreholes throughout Britain, for boreholes shallower than ~300 m outside SW England, and for some boreholes shallower than ~300 m within SW England) and these corrections vary significantly between adjacent boreholes of equivalent depth, for which one would expect the palaeoclimate corrections to be very similar. The otherwise detailed analysis by Wheildon and Rollin (1986) provided only a cursory examination of the effect on heat flow of palaeoclimate. This included the statements that ‘most authors have ... been inclined to neglect the effect ...’ and that ‘for boreholes deeper than 300 m the effect is reasonably uniform so that for comparative regional studies this neglect is of no great significance. Most of the UK data are presented without climatic correction.’ However, Wheildon and Rollin (1986) went on to explain that many geothermal boreholes have been drilled no deeper than 100 m, that such shallow boreholes are sensitive to surface temperature variations over the past 500 years, and that the data have therefore been corrected to bring them into line with measurements from deeper boreholes. Downing and Gray (1986) listed the palaeoclimate-corrected heat flow data for these boreholes (where available) but not the raw data, so the magnitude of the corrections was unclear from their report; it would appear that they adopted the corrections applied by Burley et al. (1984). Wheildon and Rollin (1986) also commented that it is not necessary to have accurate corrections to heat flow data if one is only interested in relative comparisons, but this does not apply when trying to make a quantitative assessment of geothermal resources. Furthermore, as will become clear below, the climate history of Britain requires significant corrections to recover the ‘true’ temperature variations at depth from geothermal measurements, the magnitudes of the corrections being particularly large for shallow boreholes (cf. Rollin, 1995). The magnitudes of these corrections also depend on location; for example, they will be less in regions that have been glaciated, as the presence of ice sheets will have insulated the Earth’s surface from the arctic conditions otherwise pertaining, than in unglaciated regions. Rollin (1995) noted that the potential magnitude of any palaeoclimate correction will vary inversely with the depth of each borehole in which heat flow has been measured. He inferred that the effect of palaeoclimate would be to perturb heat flow by no more than ~10% for a 320 m deep borehole and that any effect would be negligible for a ≥1000 m deep borehole; however, it is evident that he was thinking in terms of the effects of 2 °C changes to surface temperature, whereas it is now clear that the surface temperature changes in Britain have been much larger than this (see below; section 2.2). The most recent inventory of geothermal data, by Busby et al. (2011), has been processed in a similar manner to that of Rollin (1995), which means the inclusion of many data for which palaeoclimate corrections have still not been applied.

The first estimate of a palaeoclimate correction for a borehole heat flow measurement in the UK was made by Benfield (1939), for the 1216 m deep Balfour borehole in Fife, Scotland (at National Grid reference NO 323 003). He estimated that the local surface temperature rose from 0 °C to 10.83 °C at 11 ka, that the unperturbed geothermal gradient would be 26.7 °C km^{-1}, and that the measured heat flow of 28.5 mW m^{-2} indicates an unperturbed heat flow of 48.5 mW m^{-2}. Anderson (1940) noted difficulties with Benfield’s (1939) analysis, including its over-prediction of the present-day surface temperature and its omission of any effect of the Early Holocene climatic optimum, when the surface temperature was significantly higher than at present. Anderson (1940) thus inferred that the surface temperature rose from 0 °C to 11.5 °C at 9.5 ka, before falling to the present 9.5 °C at 3.5 ka (or ~1500 B.C.), and that the observed and unperturbed surface heat flow in this borehole are 34.7 and 50.0 mW m^{-2}, both higher than Benfield’s (1939) estimates. The palaeoclimate correction to surface heat flow for this particular borehole was thus ~20 mW m^{-2} according to Benfield (1939) or ~15 mW m^{-2} according to Anderson (1940).

The only other instances where palaeoclimate corrections to heat flow of ~20 mW m^{-2} have previously been reported in Britain are from the analyses by Wheildon et al. (1977), as cited by Bloomer et al. (1979), and by Burley et al. (1984). Wheildon et al. (1977) reported palaeoclimate corrections for several ~100 m deep boreholes in the granite of SW England of up to 26 mW m^{-2}, the highest corrected heat flow estimate thus obtained (at Grillis Farm, near Redruth, Cornwall; SW 680 385) being 129 mW m^{-2}. However, when the same dataset was summarised by Downing and Gray (1986), the numerical values for heat flow (inclusive of climate correction) were significantly smaller (including 113 mW m^{-2} at Grillis Farm), suggesting that these latter authors either applied a more conservative palaeoclimate correction, although without explanation of how this had been calculated. Burley et al. (1984) indeed reported that this 113 mW m^{-2} value included a 21 mW m^{-2} palaeoclimate correction, indicating that the raw heat flow measurement was 92 mW m^{-2}. The analysis in section 2.2, below, suggests that an appropriate palaeoclimate correction for such a shallow borehole in this part of Britain is ~27 mW m^{-2} if the thermal conductivity k of the rock is 3 W m^{-1} °C^{-1}. Thus, if k=3.3 W m^{-1} °C^{-1} in the Carnmenellis Granite at Grillis Farm (Lee, 1986), the correction would adjust to ~30 mW m^{-2} making the corrected heat flow ~122 mW m^{-2}. As already noted, Burley et al. (1984) included palaeoclimate corrections for this and some other shallow boreholes (depth ≤300 m) but not others, so the dataset was not reduced in a consistent way; although this is not made clear in the Downing and Gray (1986) publication, the set of data they reported likewise includes some values for which palaeoclimate corrections have been applied but others where they have not been.

2.2 Palaeoclimatic conditions in Britain

It is apparent from the foregoing, especially from published discussions of palaeoclimate corrections to heat flow such as those by Birch (1948) and Beck (1977), that this field of investigation has been hampered by the uncertainty that existed decades ago regarding Quaternary climate history. Indeed, some workers (e.g., Hotchkiss and Ingersoll, 1934; Benfield, 1939) used the fitting of palaeoclimate corrections to borehole heat flow data to try to estimate the timing of the most recent deglaciation, which was subject to great uncertainty at the time. In contrast, the abundance of information now available regarding Quaternary palaeoclimate enables corrections for this effect to be calculated with significantly greater confidence than was possible many decades ago.

The limits of glaciation in Britain around the Last Glacial Maximum (LGM) are now well known; however, the timing of this glaciation remains subject to considerable uncertainty, as is reflected by recent investigations (e.g., Bradwell et al., 2008; Sejrup et al., 2009; Clark et al., 2012). The most recent British-Irish Ice Sheet is thought from these studies to have reached its maximum volume around 27-25 ka, although it did not necessarily attain its maximum extent at this time, as ice surges subsequently reached more southerly glacial limits in localities such as the Vale of York and north Norfolk during general deglaciation. Furthermore, although the presence of an ice sheet clearly indicates very cold conditions, depending on its thickness, ice cover will insulate the underlying land surface from the prevailing arctic climatic regime. Counter-intuitively, perhaps, localities that have been covered by ice for part of the most recent climate cycle will therefore require smaller palaeoclimate corrections to heat flow than will localities that have experienced no glaciation (cf. Jessop, 1971).

The arctic climatic conditions prevalent in Britain during much of the last climate cycle can be inferred from many kinds of data, such as marine and terrestrial sedimentary records and ice cores, as well as from numerical simulations of climate. For example, Renssen and Bogaart (2003) simulated the palaeoclimate for a region adjoining latitude 52°N, longitude 6°E, near Arnhem in the Netherlands for the early part of the Bölling-Allerød interstadial (~14.5 ka; equivalent to the early part of the Lateglacial or Windermere interstadial of Britain) and the latter part of the LGM (~15 ka); the latter simulation indicates winter temperatures that fluctuated between circa -5 and -45 °C. At more northerly sites, for example in much of Britain, palaeotemperatures can be expected to have remained around the lower of these limits for longer. Kageyama et al. (2001) indeed presented an ensemble of predictions of annual mean palaeotemperature at the LGM, spatially averaged across longitudes 10°W-15°E. This ensemble ranges roughly linearly from circa -10±5 °C at 50°N to circa -21±7 °C at 60°N. During the early part of the Bölling-Allerød interstadial, the Renssen and Bogaart (2003) simulation indicates that winter temperatures fluctuated between circa 0 and -20 °C with summer temperatures of ~15 °C. The first estimates of Late Pleistocene palaeotemperatures for Britain, using the fossil occurrence of temperature-sensitive taxa such as beetles, were made by Coope et al. (1971) and Coope (1975). However, these estimates only covered summer palaeotemperatures, and so provide no direct indication of the annual mean values that are required for the present analysis. Nonetheless, Coope et al. (1971) estimated the mean July tempeature in southeast England at the LGM as ~8 °C and inferred (by analogy with the present-day climate of arctic Russia) a ~30 °C difference between mean January and mean July temperatures, tentitatively estimating that the annual mean temperature was circa -7 °C or some 18 °C below its present-day value. Table 1 illustrates a more recent example of the use of biostratigraphy to infer palaeoclimate, for biostratigraphic data from Late Pleistocene / Holocene sediments at Holywell Coombe near Folkestone, Kent, in SE England (TR 171 394). Certain taxa, notably snails and beetles used here, are sensitive environmental indicators. The present-day climate at Holywell Coombe has thus been characterised by mean February temperatures of 4.5 °C and mean August temperatures of 16 °C (Rousseau et al., 1998), giving an annual mean temperature of ~10.25 °C. Table 1 indicates estimates of the annual mean palaeotemperature, likewise calculated as the mean of the February and August palaeotemperatures. Again using the beetle fauna, Atkinson et al. (1987) estimated that during the LGM the mean temperatures were 6-13 °C during the warmest month and -11 to -34 °C during the coldest month at a site in the Lea Valley, northeast London (TQ 357 936). The mid-ranges of these values are 9.5 and -22.5 °C, indicating a mean annual temperature of -13 °C. The present-day climate of London is characterised by mean temperatures in July and January of ~18 °C and ~5 °C, with an annual mean of ~11 °C. These data thus indicate that the mean temperature in the London area was ~24 °C less than at present, albeit with a considerable margin of uncertainty. For comparison, Glasser (1995) estimated using biostratigraphic and other data that the mean annual temperature at the LGM was -9 °C, slightly higher than our estimate of -13 °C.

In detail, the agreement between Table 1 and the Renssen and Bogaart (2003) simulation is significantly better for summer than for winter palaeotemperatures. This is true for palaeoclimate estimates from biostratigraphy in general, and is thought to occur because winter survival of, say, beetles under arctic conditions is determined by extrema of temperature rather than seasonal mean values, and because these insects can adopt survival strategies such as burrowing underground to escape these temperature minima (e.g., Bray et al., 2006; Alfimov and Berman, 2009). Other insect taxa, such as midge (chironomid) larvae, likewise provide more reliable proxies for summer rather than winter palaeotemperatures (e.g., Eggermont and Heiri, 2012). For example, these taxa have been used to determine mean July temperatures of 11-12 °C during the Lateglacial Interstadial and 7.5-9 °C during the Younger Dryas cold stage at Whitrig Bog (NT 621 348) in SE Scotland (Brooks and Birks, 2000). The present-day July and January mean temperatures in this locality are ~14 °C and ~3 °C making the annual mean temperature ~8 °C. For comparison, using variations in oxygen isotope ratios in lacustrine sediments, Marshall et al. (2002) estimated summer palaeotemperatures of ~11 °C and ~6-7 °C during the Lateglacial Interstadial and Younger Dryas cold stage at Hawes Water, a small lake in northern Lancashire, NW England (SD 478 766), where the present-day July, January and annual mean temperatures are ~15, ~3 and ~9 °C, respectively. Although no winter palaeotemperature estimates are available from these sites, the differences in summer temperature relative to the present day exceed those at Holywell Coombe (Table 1), suggesting that the overall palaeotemperature anomalies for northerly parts of Britain are indeed larger than at the latter site (as climate simularions such as Kageyama et al., 2001, suggest), so palaeoclimate corrections to heat flow estimated on the latter basis may well be conservative.

Another site, which provides an indication of the palaeoclimate earlier in the Late Pleistocene, is at Lynford in west Norfolk (TL 825 948), recently documented in detail by Boismier et al. (2012). This site is thought to date from early in Marine oxygen Isotope Stage (MIS) 3 and may mark the first recolonization by Neanderthals after their prolonged absence from Britain during late MIS 6 to MIS 4. Optically stimulated luminescence dating indicates ages for the sediments at this site of 65-57 ka, with a best estimate of ~59 ka, suggesting a correlation with a brief interval of relative warmth previously identified in Germany and known as the Oerel Interstadial. The Lynford beetle fauna indicates mean temperatures of 12-14 °C in summer and circa -15 °C in winter, suggesting an annual mean temperature of circa -1 °C, although other biostratigraphic proxies suggest somewhat different values. The present-day mean temperatures in this part of Britain are ~17 °C in summer and ~4 °C in winter, indicating an annual mean of ~10 °C; the estimated annual mean palaeotempeature was thus some 11 °C below its present-day value. The brief phase of relative warmth represented at Lynford is one of many such events now recognised during the Late Pleistocene, in Greenland ice cores and other high-resolution records, as Greenland interstadials (e.g., Wang et al., 2001; Svensson et al., 2008; Fleitmann et al., 2009). The warm phase represented at Lynford is indeed thought to probably represent Greenland Interstadial 17 (Boismier et al., 2012). The Greenland interstadials only represent a small proportion of the Late Pleistocene; outside these brief spans of time the climate of NW Europe was significantly colder (e.g., Voelker, 2002). The data from Lynford thus suggest that for much of the Late Pleistocene the mean annual temperature was below that at present by a margin significantly in excess of 10 °C.

Regarding the more recent part of the record, the biostratigraphic data (summarised above) indicate that the climate of Britain was significantly colder than at present during the Lateglacial Interstadial (in reasonable agreement with the Renssen and Bogaart, 2003, climate simulation), even colder during the Younger Dryas stadial, but generally warmer than at present during the Early-Mid Holocene ‘climatic optimum’. A notable exception is the relatively cold ‘8.2 ka event’ (Table 1). This is thought to have been triggered by a major outburst flood from a proglacial lake in North America, Lake Agassiz (located to the south of the melting Laurentian ice sheet), which released ~80,000 km^3 of water into the Arctic Ocean in six months, at an estimated flow rate of $\sim 5\times10^6$ m^3 s^{-1} (Clarke et al., 2004). This release of water is indeed thought to have reduced the salinity of the surface layer of the North Atlantic Ocean sufficiently to shut down the thermohaline circulation in the latter (cf. Broecker, 1981) and to thus have affected global climate (e.g., Clarke et al., 2004). Palaeotemperature evidence such as that in Table 1 can thus guide the calculation of palaeoclimate corrections for heat flow data (see section 2.3).

The biostratigraphic and other data discussed above cannot, of course, constrain palaeotemperatures at the bedrock surface beneath ice sheets; other input is therefore needed. It is now well established that the British Ice Sheet was highly unstable; this is demonstrated by abundant evidence, including surges and retreats of the ice margin and changes in ice flow directions and in the locations of ice divides in the interior of the ice sheet (e.g., Evans et al., 2009; Hubbard et al., 2009; Bridgland et al., 2011; Clark et al., 2012; Faulkner, 2012). Such instability is attributed to the presence of meltwater at the base of an ice sheet (e.g., Arnold and Sharp, 2002), which facilitates ice movement by lubricating the ice-bedrock contact. Conversely, ice movement relative to bedrock can result in frictional heating, which facilitates meltwater production (e.g., Glasser and Siegert, 2002; Hall and Glasser, 2003). As a result, complex feedbacks between ice movement and meltwater are envisaged to have occurred beneath the British Ice Sheet; modelling studies (e.g., Glasser and Siegert, 2002; Hall and Glasser, 2003) indicate that these feedbacks maintained the temperature at the melting point of ice at the base of much of the ice sheet. Exceptions are near ice divides, where slow ice movement was conducive to basal temperatures below the melting threshold, especially where the ice was relatively thin (e.g., Glasser and Siegert, 2002; Hall and Glasser, 2003). Modern reconstructions (e.g., Evans et al., 2009) indicate that the maximum height of the surface of the British Ice Sheet was ~1500 m above sea-level, such that after allowance for glacio-isostasy, the maximum thickness of ice was ~2 km. However, when the presence of topography is taken into account, the thickness of ice in much of the formerly glaciated region was less than 1 km and in many regions no more than hundreds of metres. Such conclusions are supported by analyses of subglacial landforms (e.g., drumlins) in formerly glaciated upland parts of Britain, which indicate that these regions were completely submerged by ice that was thick enough to form an overall ice sheet whose shape was independent of topography but thin enough for the basal ice flow to be influenced by the subglacial topography (e.g., Mitchell and Hughes, 2012); this means ice thicknesses over these uplands of no more than hundreds of metres. From standard data books, the melting point of ice is depressed to -1 °C by a pressure of 13.35 MPa. Taking the density of ice as 920 kg m^{-3}, this means that melting occurs at -0.5 °C beneath ice of thickness ~730 m and at -1.5 °C beneath ice of thickness ~2200 m. Given the evident thinness of much of the British Ice Sheet, from the above observations, the basal temperature can be estimated as 0 °C to the nearest °C; beneath the thickest parts of this former ice sheet the basal temperature could be estimated as -1 °C to the nearest °C.

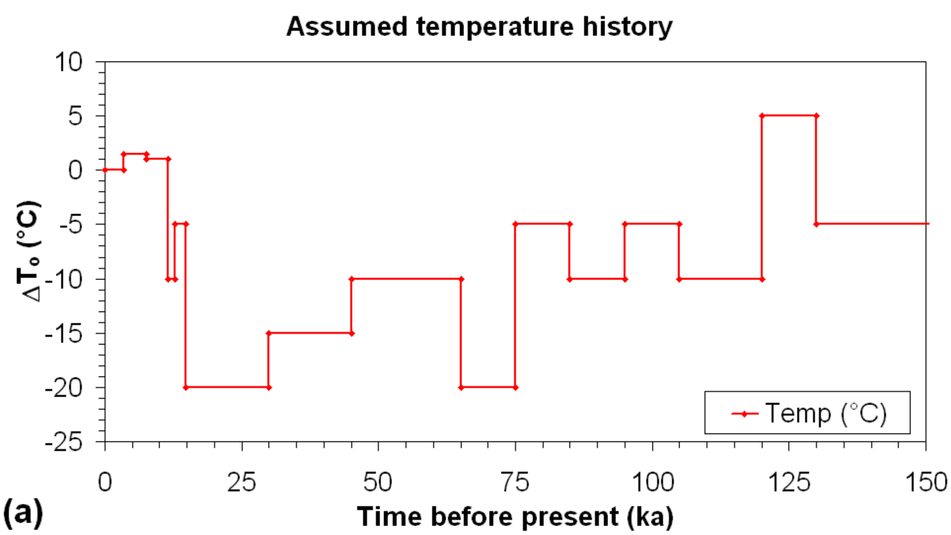
2.3 Calculation of palaeoclimate corrections to heat flow data for Britain

Figure 2(a) shows a plausible model palaeotemperature history (tabulated in Table 2) representing a part of southern England that remained unglaciated during the Late Pleistocene. Figure 2(b) shows the associated palaeoclimate correction to heat flow, calculated as summarised above. The resulting correction is rather large, amounting to the addition of 27.2 mW m^{-2} to convert measured heat flow to 'true' heat flow in the limit of a very shallow borehole. However, for a 1 km deep borehole, the correction would reduce to 19.3 mW m^{-2} if heat flow has been calculated as the average over the whole depth range of the borehole, or to 11.4 mW m^{-2} if it has been calculated from temperature measurements over a limited range of depth at the bottom of the borehole. For a 1.5 km deep borehole the corresponding corrections would reduce to 13.3 mW m^{-2} and -0.7 mW m^{-2}, the latter correction involving subtraction, rather than addition, to the observed heat flow. At relatively shallow depths the principal cause of the corrections is the low temperature regime assumed around the LGM, whereas the negative correction at greater depths indicates that this contribution is locally outweighed by that of the relatively high temperatures (5 °C above present; Table 2) inferred to have existed during the last interglacial (the Ipswichian interglacial; MIS 5e).

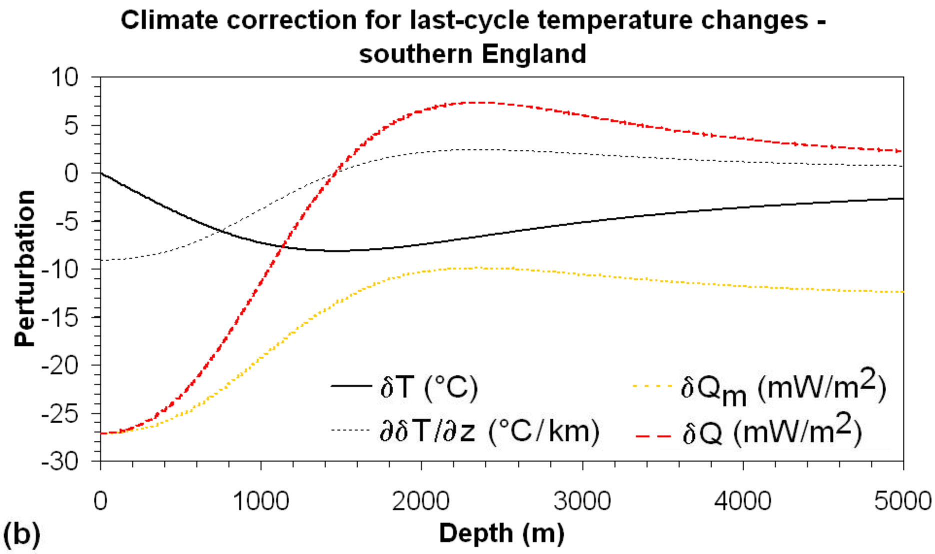
Table 2: Model temperature history for Fig. 2

t_2 (ka)	t_1 (ka)	ΔT_o (°C)	Name
0	3.5	0	Present conditions
3.5	7.5	+1.5	Mid Holocene climatic optimum
7.5	11.5	+1	Early Holocene
11.5	12.8	-10	Younger Dryas Stadial
12.8	14.7	-5	Lateglacial Interstadial
14.7	30	-20	Last Glacial Maximum
30	45	-15	earlier MIS 2
45	65	-10	MIS 3
65	75	-20	MIS 4
75	85	-5	MIS 5a
85	95	-10	MIS 5b
95	105	-5	MIS 5c
105	120	-10	MIS 5d
120	130	+5	Ipswichian (MIS 5e)
130	3000	-5	Earlier Pleistocene
3000	65000	+5	Pre-Quaternary

Times t_1 and t_2 represent the start and end of each phase, for which the surface temperature (relative to the present-day value) is assumed to have been ΔT_o. Individual temperature phases are named in the final column of the Table, several being associated with particular Marine oxygen Isotope Stage (MIS) numbers.



(a)



(b)

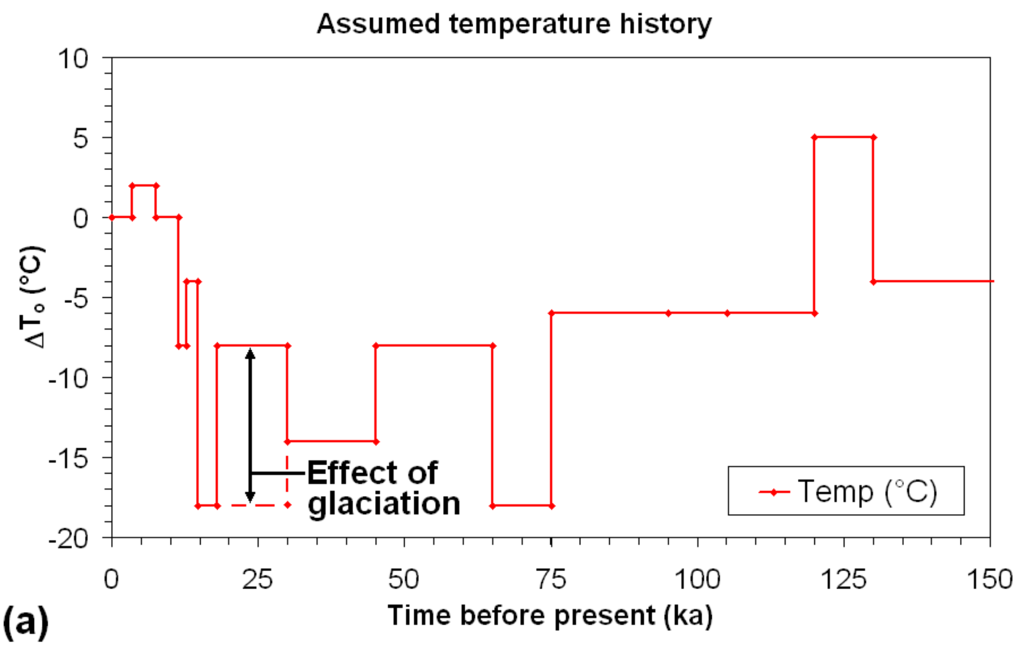
Although one may debate individual details of the assumed temperature history (see section 2.2; see, also, section 5), it is apparent that for any plausible temperature history the palaeoclimate correction for southern England is large; moreover, its magnitude varies with both the depth range over which a borehole has been drilled and the manner in which heat flow has been calculated (whether from the temperature difference throughout the borehole or from measurements over a narrow depth range). These deductions contrast with the views expressed in many publications regarding the insignificance of palaeoclimate corrections for UK heat flow data.

Figure 2 indeed indicates that climate corrections to heat flow are particularly important for shallow boreholes and, depending upon how heat flow has been measured, they can also be significant for deeper boreholes. For example, logging of the Marchwood borehole in Hampshire (SU 399 112) indicated a temperature of ~75 °C in the Triassic Sherwood Sandstone at a depth of ~1.7 km, from which a heat flow measurement of 61 mW m^{-2} has been determined (Lee, 1986; Smith, 1986; Downing and Gray, 1986). It is apparent, from Downing and Gray (1986), that this heat flow measurement has been based on the temperature difference over this ~1.7 km depth range. Thus, from Fig. 2, the measurement should be revised upwards by ~12 mW m^{-2} to yield the steady-state heat flow in this locality, taking account of past climate change. However, if the heat flow had been measured over a narrow depth interval at ~1.7 km depth, Fig. 2 also indicates that any climate correction to it would be minimal. Strictly speaking, however, the correction to heat flow for this borehole is more complex than this; it has been determined for k=3 W m^{-1} $°C^{-1}$, but should take account of the variation in thermal conductivities within the succession of Mesozoic and Cenozoic sediments penetrated by the borehole (cf. Richardson and Oxburgh, 1978).

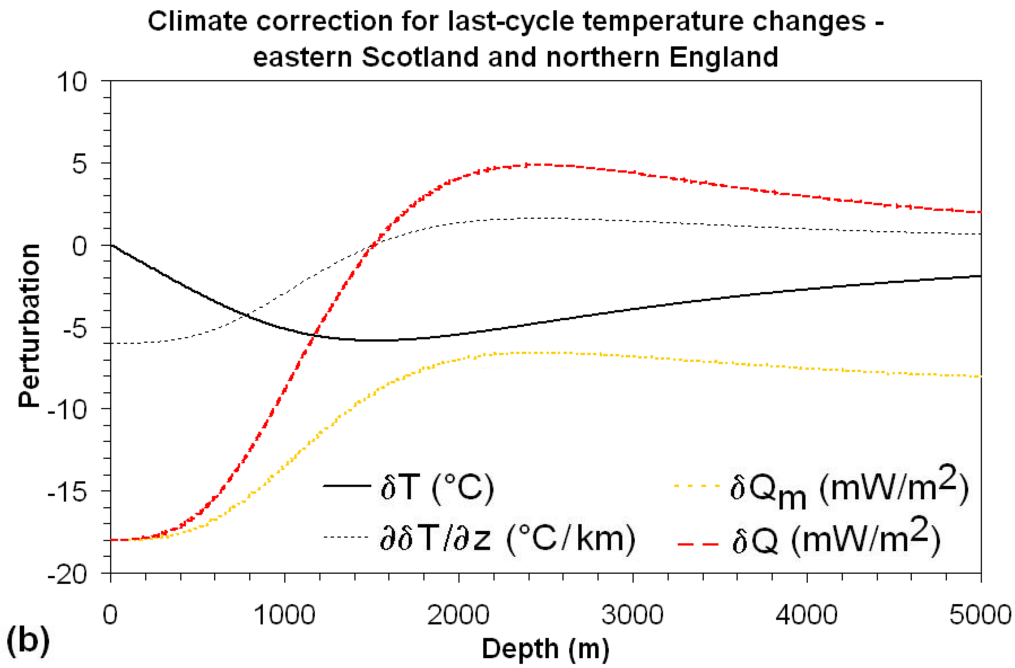
Table 3: Model temperature history for Fig. 3

t_2 (ka)	t_1 (ka)	ΔT_o (°C)	Name
0	3.5	0	Present conditions
3.5	7.5	+2	Mid Holocene climatic optimum
7.5	11.5	+0	Early Holocene
11.5	12.8	-8	Younger Dryas Stadial
12.8	14.7	-4	Lateglacial Interstadial
14.7	18	-18	Last Glacial Maximum - after deglaciation
18	30	-8	Last Glacial Maximum - during glaciation
30	45	-14	earlier MIS 2
45	65	-8	MIS 3
65	75	-18	MIS 4
75	120	-6	MIS 5d-5a
120	130	+5	Ipswichian (MIS 5e)
130	3000	-4	Earlier Pleistocene
3000	65000	+5	Pre-Quaternary

Display format is the same as for Table 2.



(a)



For comparison, Fig. 3 shows perturbations to the same geothermal parameters as in Fig. 2(b), calculated for an assumed surface temperature history intended to represent regions of northern England and eastern Scotland that were glaciated around the LGM (Table 3). It has thus been assumed that the model region was glaciated between 30 ka and 18 ka, and that the thickness of the overlying ice sheet raised the temperature at the bedrock surface by 10 °C relative to what it would have been with no ice sheet present, thus effectively assuming a surface temperature at this time of circa 0 °C, consistent with the presence of a melting layer at the base of the ice sheet. The onset of glaciation in Scotland is constrained by the presence of preglacial sediments (i.e., sediments overlain by Late Pleistocene glacial deposits), dated to the span of time ~38-32 ka (or Greenland interstadials 8 to 5), an interval known in the Netherlands as the 'Denekamp Interstadial', at North Tolsta in the extreme northwest of Scotland (NB 557 468; Whittington and Hall, 2002). At the more centrally-located (i.e., closer to any potential site of initial ice accumulation in the western Scottish Highlands) site of Bishopbriggs, northeast of Glasgow (NS 601 722 to NS 625 732), preglacial sediments have been dated to 35 ka (Jacobi et al., 2009); these deposits contain fossil material indicating the youngest known occurrence in Britain of the woolly rhinoceros *Coelodonta antiquitatis*, the age having been determined by radiocarbon dating of rhinoceros bone. Conversely, evidence that a substantial ice sheet existed in Scotland by 29 ka is provided by the significant increase in ice-rafted sediment reaching the Atlantic Ocean to the north of Ireland at this time (Scourse et al., 2009). The onset of ice accumulation is thus envisaged circa 30 ka, as noted above, the ice sheet having expanded rapidly to its maximum extent by ~27 ka (Clark et al., 2012). The corrections to the calculated geothermal parameters for a 30 ka onset of ice accumulation remain rather large, amounting to the addition of 18.0 mW m^{-2} to convert measured heat flow to 'true' heat flow in the limit of a very shallow borehole, in rocks with k=3 W m^{-1} $°C^{-1}$. For a 1 km deep borehole, the correction would reduce to

13.5 mW m^{-2} if heat flow had been calculated as the average over the whole depth range of the borehole, or to 8.8 mW m^{-2} if it had been calculated from temperature measurements over a limited range of depth at the bottom of the borehole. For a 1.5 km deep borehole the corresponding corrections would reduce further, to 9.1 mW m^{-2} and 0.2 mW m^{-2}.

For the same temperature history, but with the insulating effect of glaciation reduced to 4 °C, the correction becomes 21.1 mW m^{-2} to convert measured heat flow to ‘true’ heat flow in the limit of a very shallow borehole. For a 1 km deep borehole, it would reduce to 15.3 mW m^{-2} if heat flow has been calculated as the average over the whole depth range of the borehole, or to 9.5 mW m^{-2} if it has been calculated from temperature measurements over a limited depth range at the bottom of the borehole. For a 1.5 km deep borehole the corresponding corrections would again reduce further, to 10.3 mW m^{-2} and -0.6 mW m^{-2}. Likewise, with the same temperature history, but with no insulating effect of glaciation assumed, the correction becomes 23.1 mW m^{-2} in the limit of a very shallow borehole. For a 1 km deep borehole, it would reduce to 16.6 mW m^{-2} if heat flow had been calculated as the average over the whole depth range of the borehole, or to 9.9 mW m^{-2} if it had been calculated from temperature measurements over a limited range of depth at the bottom of the borehole. For a 1.5 km deep borehole the corresponding corrections would again reduce further, to 11.0 mW m^{-2} and -1.1 mW m^{-2}.

The above suggestion, that corrections for palaeoclimate decrease as one moves into regions that were formerly glaciated, at first sight seems counter-intuitive, but arises because of the insulating effect of the former ice sheet, which ensures that the Earth’s surface was not directly exposed to the arctic air temperatures that would otherwise prevail. The same effect, albeit in a more extreme form, was recognised decades ago in Canada by Jessop (1971). He established that palaeoclimate corrections to heat flow typically increase southward across Canada, reflecting the southward thinning of the former ice sheet, and are indeed negative for sites in the Canadian high arctic, which were formerly insulated by a thick ice sheet but are now exposed to the arctic climate, so these sites are colder now than they were around the LGM.

2.4 Comparison with previous work outside the UK

Kukkonen et al. (1998) investigated the heat flow in a number of shallow (<~750 m deep) boreholes in eastern Karelia, Russia (latitude 63.2-63.3° N; longitude ~36.1-36.2°E; south of the White Sea) which yielded very low heat flow values (≤12 mW m^{-2}) over depth ranges of up to ~300 m; they concluded that the main cause of this was palaeoclimate. The present-day surface temperature in this region is ~5 °C; by assuming a palaeotemperature history with the surface temperature -15 °C between 60 and 11 ka, Kukkonen et al. (1998) determined palaeoclimate corrections which raised the corrected heat flow to ≤40 mW m^{-2}, values more in keeping with an Archaean crustal province with low radioactive heat production in the crust. This region lay inside the eastern margin of the Scandinavian ice sheet at its maximum Late Pleistocene extent, but is not thought to have been ice-covered for more than a few thousand years, hence the palaeoclimate correction procedure that omitted ice cover. The magnitude of this correction,

~28 mW m^{-2}, may be compared with the 27 mW m^{-2} estimated for parts of Britain that were not glaciated during the Late Pleistocene (Fig. 2(b)).

Slagstad et al. (2009) determined palaeoclimate corrections to heat flow data from Norway. However, their corrections for palaeoclimate are quite small, the largest being 14 mW m^{-2} for one particularly shallow borehole (where heat flow was measured between 180 and 330 m depths). These corrections took account of the effect of the Scandinavian Ice Sheet shielding the bedrock surface from the arctic air temperatures that prevailed during much of the Late Pleistocene, which is reasonable. It also assumed that at times when no ice sheet was present the bedrock surface was no more than 5 °C below its present temperature; however, this 5 °C temperature difference was an assumption, not supported by any evidence. By analogy with the data from Britain (discussed above) it is probably a significant underestimate; as a result, it is likely that Slagstad et al. (2009) have under-corrected their heat flow data and the true heat flow values in Norway therefore significantly exceed the values they have suggested.

Majorowicz and Wybraniec (2011) recently investigated the effect of palaeoclimate on heat flow data across Europe. After presenting a brief synthesis of palaeoclimate evidence, they concluded that representative values for the temperature difference at the LGM are 14, 10 and 7 °C, respectively, for NW, central and southern Europe. They thus calculated a set of palaeoclimate corrections to heat flow, which were presented graphically in a manner similar to Figs. 2(b) and 3(b). For example, they inferred that the correction at zero depth in NW Europe is 19 mW m^{-2}, which compares with 27 mW m^{-2} in Fig. 2(b) and 18 mW m^{-2} in Fig. 3(b). Although the corrections to heat flow for palaeoclimate advocated by Majorowicz and Wybraniec (2011) and in the present study are thus in broad agreement, it is apparent that theirs are somewhat smaller in magnitude and, therefore, more conservative. This is for two main reasons. First, they envisaged a shorter duration of the LGM, between 25 and 15 ka, whereas for the present analysis it has been assigned an age span of ~30-15 ka, consistent with the modern literature (cited in section 2.2). Second, they envisaged a smaller temperature difference between the LGM and the present day. Moreover, it is clear from the wording of their paper that their adopted 14 °C temperature difference was applicable to a region that was glaciated throughout the LGM (hence the better agreement between their predictions and Fig. 3(b) rather than Fig. 2(b)); they thus did not consider the effect of surface exposure to arctic temperatures, with no overlying insulating layer of ice, for all or part of the LGM.

3. Topographic correction to heat flow

It has long been recognised that topography will affect heat flow; for example, a valley will focus heat flow towards its axis, causing a localised increase in the heat flow, whereas a hill will have the opposite effect. This section will review previous work on topographic correction of heat flow data, contrasting the limited nature of such work in the UK with the more sophisticated analyses undertaken elsewhere. It will then investigate the use of one type of analytic correction for heat flow in the presence of topography, that for a ‘Lees Valley’ (section 3.3). Example calculations of this type will then be undertaken for a representative subset of borehole heat flow measurements in Britain.

3.1 Previous work

Lees (1910) was the first worker to publish any quantitative solution for the perturbation to terrestrial heat flow caused by topography. He showed that there exists a particular two-dimensional surface profile, nowadays known as a 'Lees Hill', for which the associated perturbation to heat flow can be calculated analytically (see Appendix 2). Jeffreys (1938) subsequently developed equations for the approximate calculation of a correction to heat flow for more general topography. Many subsequent studies, particularly of UK heat flow (e.g., Bott et al., 1972; Richardson and Oxburgh, 1978; and Bloomer et al., 1979), have stated that topographic corrections have been calculated 'after Jeffreys (1938)', but without specifying what calculations were carried out. Richardson and Oxburgh (1978) explained that the correction method 'generally involved digitisation of the topography on a 1 km grid for a maximum of 20 km from the site; in some cases it was necessary to digitise on a 0.5 km grid in the neighbourhood of a hole.' This (and the illustration of the method by Bodmer et al., 1979, for a site outside Britain) gives the impression of a rather crude correction procedure, which is likely to significantly smooth the actual variations in topography in any given region, and which may well thus underestimate its true effect. Richardson and Oxburgh (1978) also commented that 'the effects of irregular topography on heat flow are slight at depths more than 100 m in most of our boreholes as they are situated in areas of subdued topography.' However, in the compilation of heat flow data by Bloomer et al. (1979), the largest topographic corrections were for relatively deep boreholes situated in valley-floor localities with considerable topographic relief: Raydale, North Yorkshire (depth 593 m; correction 6.3 mW m^{-2}); and Rookhope, County Durham (depth 799 m; correction 4.2 mW m^{-2}). The impression created by these publications is that topographic corrections are of minor importance and can typically be neglected in any inventory of UK geothermal data. Downing and Gray (1986) presented a table of geothermal data for the UK (their pp. 172-175) but this simply reported their preferred heat flow values, with no indication of whether a topographic correction had been made, let alone how it has been determined. Subsequent works on UK geothermics, such as Rollin (1995), and Busby et al. (2011) have indeed omitted discussion of this topic, notwithstanding the thorough analyses that have been made for non-UK datasets (e.g., Blackwell et al., 1980; Henry and Pollack, 1985).

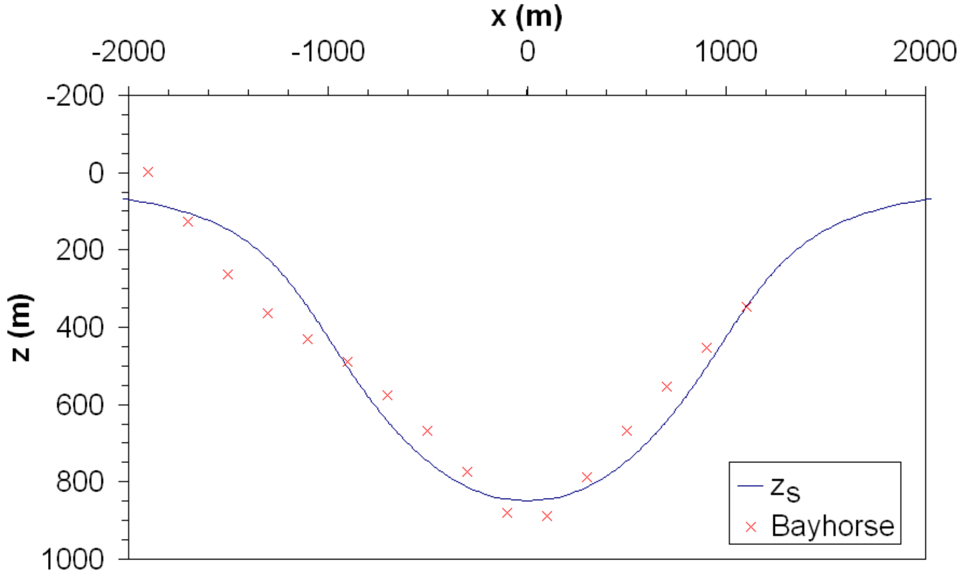
The above discussion indicates that the very small topographic corrections that have hitherto characterised the UK geothermal dataset (e.g., Bott et al., 1972; Richardson and Oxburgh, 1978; Bloomer et al., 1979) may well be invalid, as they have been based on an approximate algorithm that is only a good approximation where the depth of measurement is greater than the magnitude of the topography (cf. Birch, 1950). In addition, as the summary from Richardson and Oxburgh (1978) indicates, the application of this method has hitherto only provided a crude approximation to the local topography in the vicinity of any geothermal borehole.

3.2 Lees Valley topographic correction procedure

Rather than attempting to calculate topographic corrections to heat flow in the presence of general topography (cf. Blackwell et al., 1980; Henry and Pollack, 1985), the present study shall focus on the use of analytic corrections. Since many UK geothermal boreholes are located in valleys, the adopted analytic correction shall be that for a Lees Valley (cf. Lees, 1910; Appendix 2). Such a correction would be exact if the topography matched the form assumed for calculation purposes; it will therefore approximate the correction that is appropriate for topography similar to that for the assumed shape.

A Lees Valley is an analytic two-dimensional profile of topography, with the dependence of surface topography z_s against horizontal position x defined by equation (2.08), which approximates the cross-sections of many fluvial or glacial valleys. The associated subsurface temperature distribution T(x,z) for $z \geq z_s$ is given by equation (2.05) and that for the vertical component of the geothermal gradient by equation (2.09). Perturbations, relative to the case with no topography, can be readily calculated for temperature, geothermal gradient and heat flow, and the mean heat flow between two vertically-separated points, representing two depths within a given borehole, can also be calculated analogously to equation (1). By analogy with the palaeoclimate correction procedure discussed in section 2, the assumed topographic profile thus determines the present-day perturbation to the geothermal gradient; the resulting heat flow perturbation scales in proportion to k, and so can be readily calculated for different vaues of k to those adopted, in proportion. Lee (1991) established that it is algebraically invalid to superpose multiple analytic solutions of this general type to represent more complex topography and also noted that use of a two-dimensional correction (such as this) in three-dimensional terrain may over- or under-estimate the true heat-flow correction, depending on the distance of a borehole from the dominant topographic feature in its vicinity. In each individual case, therefore, it is unclear whether this correction procedure will over- or under-estimate the true heat flow.

A computer program was developed to calculate corrections to the above-mentioned geothermal parameters subject to the assumption of Lees Valley fits to observed topography. The Lees Valley fit is specified by three parameters: H is the depth of the valley floor at x=0, measured below the reference level z_o (itself measured relative to sea-level) that specifies the height of the flanking interfluves; and B is a measure of the half-width of the valley, the depth below z_o of the model valley floor decreasing to H/2 at x=±B. In addition, the correction to temperature depends on the unperturbed near-surface geothermal gradient u (i.e., the geothermal gradient at z=0), the vertical temperature gradient at the bedrock surface u', the latter being equal, for a subaerial bedrock surface, to the atmospheric lapse rate, and the radioactive heat production Y in the bedrock within which the valey is entrenched. Finally, as noted above, the perturbation to vertical heat flow scales in proportion to the thermal conductivity of the bedrock, k. This program was tested against the data presented in Fig. 8 of Blackwell et al. (1980), for which a heat flow of 87 mW m^{-2} can be estimated across a notional D=250 m deep borehole in the valley floor, in a locality where the regional geothermal gradient is 20 °C km^{-1}, subject to the assumption that u'=5 °C km^{-1} and, as before, k=3 W m^{-1} $°C^{-1}$. The value of Y was set to zero as Blackwell et al. (1980) did not specify a value for this parameter in this locality. The resulting corrections to the geothermal gradient are -11.7 °C km^{-1} at ground level (i.e., at x=0, $z=z_s=H$) and -7.8 °C km^{-1} at the base of this notional borehole (i.e., at x=0, z=H+D). The mean geothermal gradient (at x=0, between $z=z_s=H$ and z=H+D) thus requires correction by -9.5 °C km^{-1} and the associated heat flow by -9.5 °C km^{-1} × 3 W m^{-1} $°C^{-1}$, or -28.6 mW m^{-2}. The resulting estimate for the corrected regional heat flow is thus 87 - 28.6 mW m^{-2} or 58.4 mW m^{-2}, close to the 60 mW m^{-2} value estimated by Blackwell et al. (1980) using a different method. It is thus evident that, even though the fit of the Lees Valley to the actual topography of this study locality is not particularly good (Fig. 4), this method gives a reasonable indication of the topographic correction to heat flow.



3.3 Lees Valley topographic corrections for sites in Britain

We shall now determine the corrections to heat flow applicable for several geothermal boreholes in Britain. The sites to be discussed comprise Rookhope, Raydale, Eastgate, Skiddaw, Tomnakeist (Ballater), and Soussons Wood (Dartmoor), each of which is located in a valley. In each case a topographic profile subperpendicular to the valley in which the borehole is located has been digitised from local 1:25,000 or 1:50,000 scale map coverage. The resulting profile has then been modelled as a Lees Valley (section 3.3) to determine the correction. The topographic corrections thus obtained are listed in Table 4 and are compared with previous estimates in Table 5.

As section 3.2 notes, modelling of this type ideally requires knowledge of the heat flow in the limit of z=0, after taking account of the upward increase in heat flow caused by the radioactive heat production in the bedrock. However, in each of the cases to be modelled, heat flow has been calculated across a range of depth within each borehole, rather than at the notional z=0 level that is ideally required. In principle, these observed heat flow values could be used to calculate the heat flow at z=0, then the analysis could proceed, for example by applying equation (2.08) to determine the topography of the Lees Valley, using the heat flow at z=0 and the appropriate value for the radioactive heat production in the bedrock. However, tests have shown that, given the limited extent of relief in each of the modelled localities, such elaborations make no significant difference to any of the solutions, compared with the simpler procedure of approximating the heat flow at the depth of measurement as the heat flow at z=0 and neglecting any vertical variations due to radioactive heat production over the intervening range of depth. The latter, simpler, procedure has therefore been adopted for each of the analyses presented in this section.

3.3.1 Rookhope The Rookhope borehole (at NY 938 428) was the first to penetrate the Palaeozoic Weardale Granite that underlies most of County Durham in northeast England; the heat flow in this borehole has been extensively studied (e.g., Bott et al., 1972; England et al., 1980). Hitherto, the definitive analysis of the geothermics of this borehole is arguably that by England et al. (1980). These authors argued that the best estimate of the heat flow came from the deeper part of the borehole, between 390 and 789 m depths within the Weardale Granite. They measured a topographically corrected temperature change across this depth range of 12.28 °C giving a heat flow of 95.4 mW m^{-2}, but did not state the raw data values. However, it can reasonably be assumed that their topographic correction was 4.2 mW m^{-2}, the same as for Bloomer et al. (1979), making their raw heat flow estimate 99.6 mW m^{-2}.

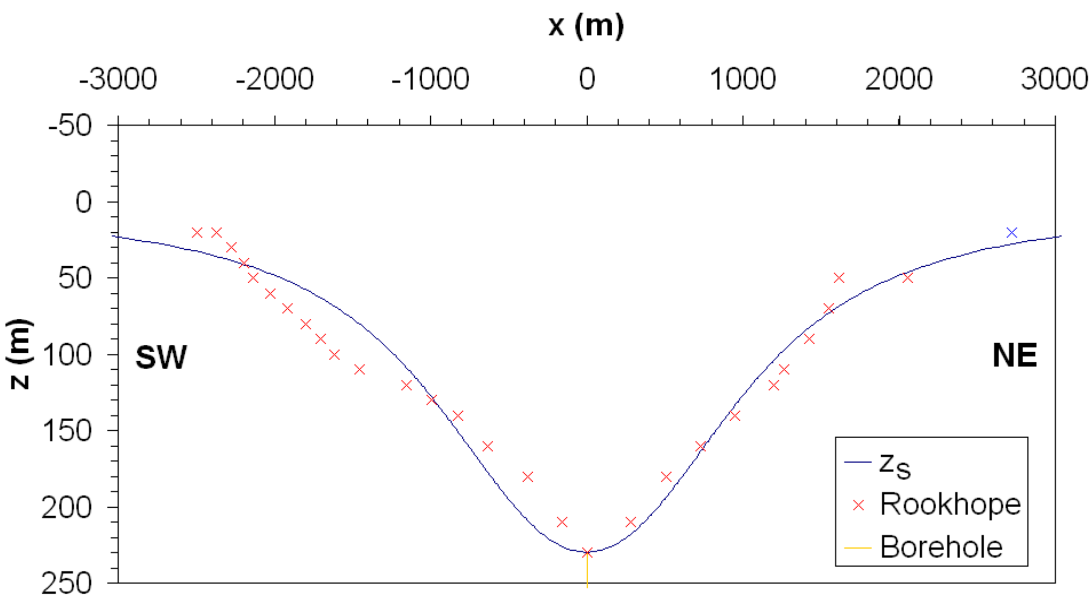
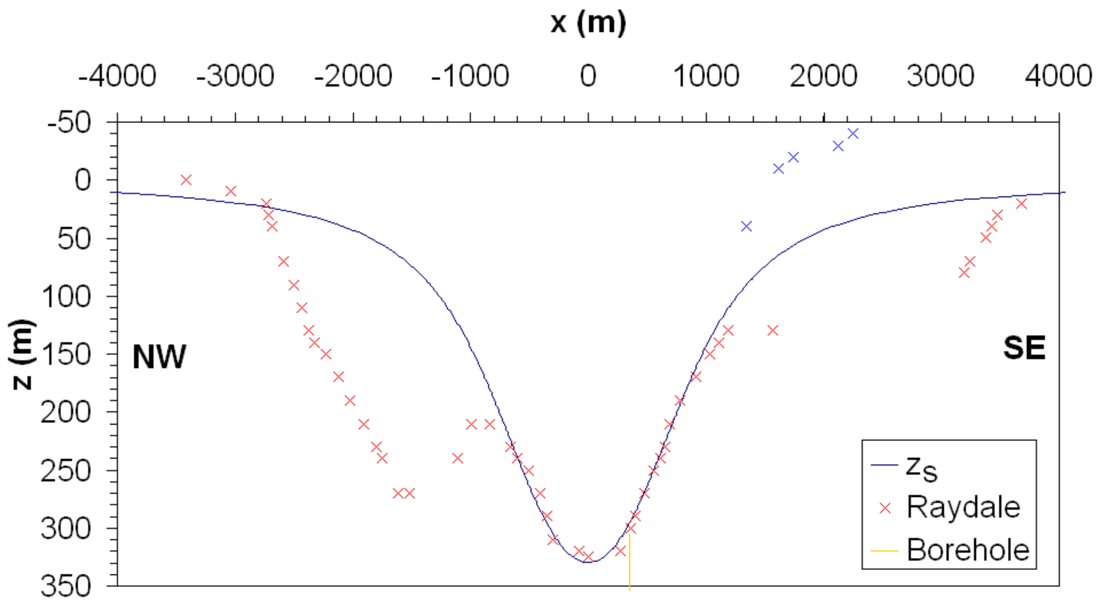


Figure 5 illustrates a Lees Valley fit to the adjoining topography; Table 4 quantifies the associated perturbation to heat flow. This fit predicts (using equation (2.13)) temperature perturbations due to the topography of 4.23 and 3.34 °C at 390 and 789 m depths. The geothermal gradient and heat flow are thus perturbed over this depth range by 2.2 °C km^{-1} and 6.9 mW m^{-2}, making the preferred estimate for the topographically corrected heat flow in this borehole 92.7 mW m^{-2} (Table 5).

3.3.2 Raydale The Raydale borehole (at SD 903 847) was the first to penetrate the Palaeozoic Wensleydale Granite that underlies the NW part of North Yorkshire in northern England. Hitherto, the definitive analysis of the geothermics of this borehole is again arguably that by England et al. (1980). They determined that the best estimate of the heat flow came from the deeper part of the borehole within the Wensleydale granite, but did not specify the depth range of measurement. However, it can be seen from their

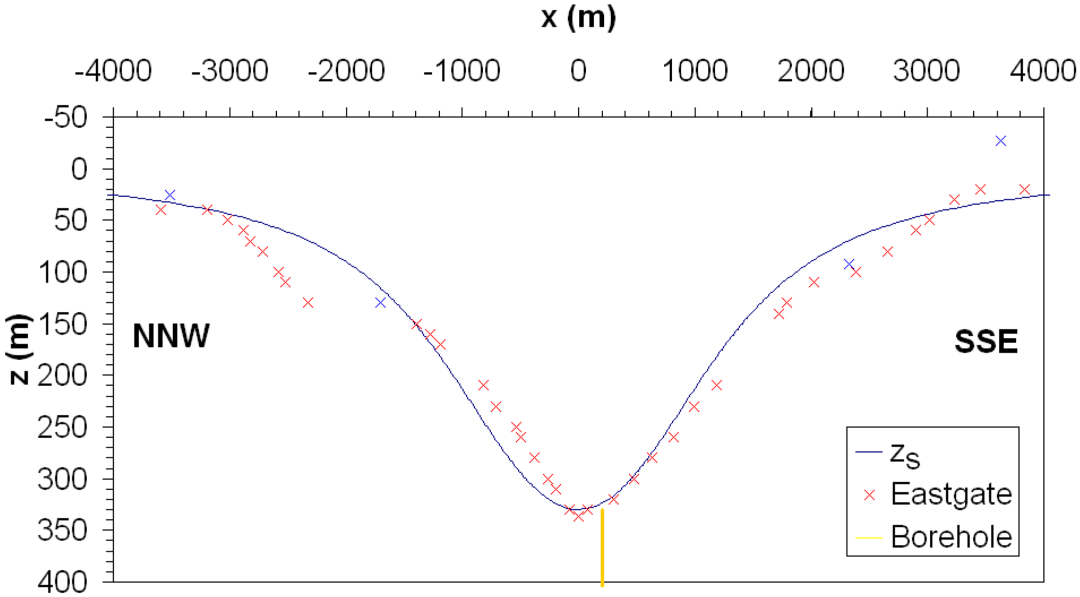
Fig. 4 that the relevant depth range is ~525 to ~590 m. There are minor discrepancies between the text and Figure 4 of England et al. (1980), as well as between their publication and Bloomer et al. (1979), regarding heat flow measurements, but for calculation purposes their raw and topographically corrected heat flow are taken as 71.1 and 64.9 mW m^{-2}.



In Fig. 6 a Lees Valley has been fitted through the cross-sectional profile of the main Raydale Valley and the adjoining moorland summit flats. Using equation (2.05), with the borehole at x=350 m (i.e., offset by 350 m from the valley axis) and its top at 285 m O.D. modelled at a depth of z=H-25 m, this fit predicts temperature perturbations due to the topography of 2.59 and 2.49 °C at 525 and 590 m depths. The geothermal gradient and heat flow are thus perturbed over this depth range by 1.6 °C km^{-1} and 5.7 mW m^{-2}, making the preferred estimate for the topographically corrected heat flow in this borehole 65.4 mW m^{-2} (Table 5). In this case, the off-axis location of the borehole does not affect the topographic correction significantly. Keeping other parameters constant, but moving the borehole to x=0 with its top at z=H would increase the topographic correction to 6.6 mW m^{-2} and thus reduce the estimate of the corrected heat flow to 64.5 mW m^{-2}.

3.3.3 Eastgate The Eastgate borehole (Manning et al., 2007; Younger et al., 2012) was drilled in 2005 into the valley floor of Weardale (at NY 93890 38200), some 4 km south of Rookhope; with its wellhead at 251 m O.D., it penetrated 995 m, mostly into the Weardale Granite (like at Rookhope). This project was designed to intersect the Slitt Vein, to ascertain the geothermal potential of the hot groundwater flowing upward through this mineral vein within the granite. However, due to the resulting large magnitude of groundwater circulation detected by the borehole (Younger and Manning 2010), only limited analysis of conductive heat flow has been carried out; in particular, no topographic correction has hitherto been determined. Manning et al. (2007) noted the difference between the bottom hole temperature of 46 °C and the annual mean surface temperature of ~8 °C; dividing this difference by the depth of the borehole and multiplying by the estimated mean thermal conductivity, of 2.99 W m^{-1} °C^{-1}, they determined the heat flow as 115 mW m^{-2}, a markedly higher value than for Rookhope (section 3.3.1).

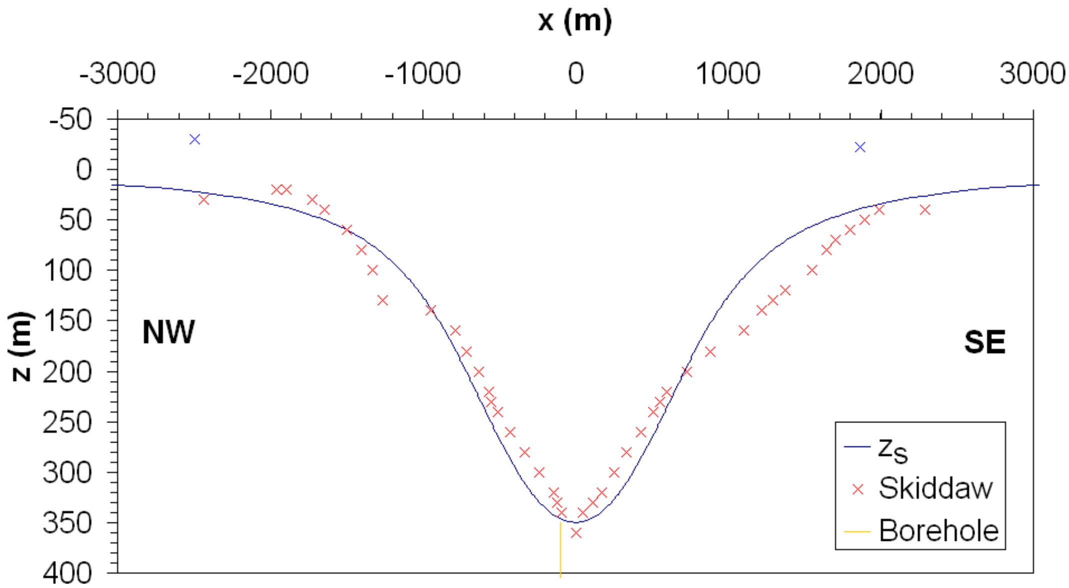
Figure 7.



The Lees Valley fit to Weardale near Eastgate in Fig. 7 indicates that the local topography has perturbed the thermal state of the crust as indicated in Table 4. A much larger topographic correction is therefore appropriate at Eastgate in comparison with Rookhope, a conclusion that remains virtually unaffected irrespective of whether the borehole is assumed for calculation purposes to be on the valley axis or projected to the section line at x=200 m, consistent with Fig. 7. This is partly because Weardale is more deeply entrenched than the valley of Rookhope Burn (~350 m deep, compared with ~200 m), but also partly due to the different manner in which the heat flow in the Eastgate borehole has been estimated. Notably, because the heat flow was estimated across the entire vertical extent of the Eastgate borehole it is affected at shallow depths where the temperature is most severely perturbed. Taking x=200 m, application of equation (2.05) indicates temperature perturbations at depths 0 and 995 m of 9.48 and 5.47 °C. The geothermal gradient over this interval is thus perturbed by 4.0 °C km^{-1} and the associated heat flow by 12.0 mW m^{-2}, making the corrected heat flow 103.0 mW m^{-2}. Application of this correction indeed brings the heat flow measurements for the Eastgate and Rookhope boreholes into closer agreement (103.0 versus 92.7 mW m^{-2}) than is apparent from the raw data. However, this correction procedure presupposes that the temperaure differences within the Eastgate borehole are due to conductive heat flow; if part of the effect is due to groundwater convection then no such correction would be applicable. In principle, a revised measurement of the conductive heat flow in the Eastgate borehole could be made over a limited range of depth towards the bottom of the borehole, within the Weardale granite and below the level at which the Slitt Vein was intersected; it may well provide a better indication of the ‘true’ heat flow at this site. This measurement would still require topographic correction, but the magnitude of the correction would be much less than that required given the way Manning et al. (2007) measured the heat flow.

3.3.4 Skiddaw The Skiddaw (Burdell Gill) borehole (at NY 314 314) was one of many drilled for geothermal prospecting in the early 1980s; in accordance with the research strategy followed, it penetrated only 281 m. The objective was to measure the heat flow through the small Skiddaw pluton of Palaeozoic granite in the northern part of the Lake District of Cumbria, in northwest England. The borehole is documented by Lee (1986), some details being contained elsewhere in the Downing and Gray (1986) report. Downing and Gray (1986) indeed reported that heat flow as 118.5 mW m^{-2}, measured between the depths of 100 and 281 m. Notably, Lee (1986) estimated a very large topographic correction to this heat flow measurement, of 17.6 mW m^{-2} (Table 5).

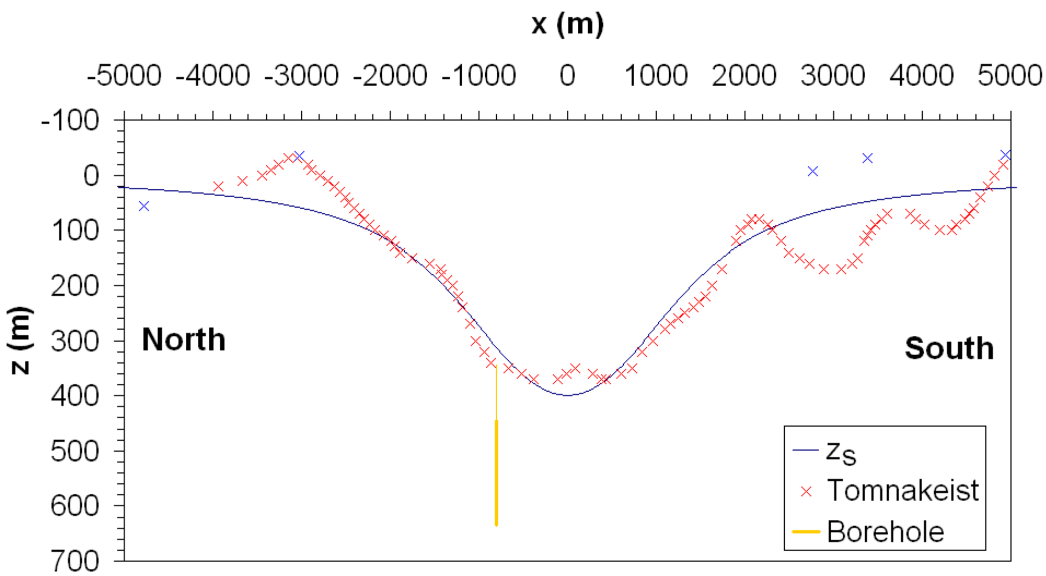
Figure 8.



The Lees Valley fit to the Burdell Gill valley in Fig. 8 indicates that the local topography has perturbed the thermal state of the crust as indicated in Table 4. This fit predicts (using equation (2.05), for x=-100 m, given the offset between the borehole and the valley axis indicated in Fig. 8) temperature perturbations due to the topography of 6.97 and 5.84 °C at 100 and 281 m depths. The geothermal gradient and heat flow are thus perturbed over this depth range by 6.2 °C km^{-1} and 21.8 mW m^{-2}, making the preferred estimate for the topographically corrected heat flow 96.7 mW m^{-2} (Table 5). It can thus be seen that a large topographic correction is indeed appropriate for this borehole, due to the combination of rugged terrain and shallow depth of penetration, the 21.8 mW m^{-2} value deduced here being somewhat larger than Lee’s (1986) estimate.

3.3.5 Ballater (Tomnakeist) Tomnakeist in the valley of the River Dee near Ballater (at NO 401 986) is one of a number of shallow boreholes drilled in the early 1980s for geothermal prospecting in the Palaeozoic Caledonian granites of northeast Scotland (the Eastern Scottish Highlands; Fig. 1); this particular borehole reached 290 m. Geothermal parameters for this borehole were documented by Lee (1986) and elsewhere in the Downing and Gray (1986) report. These authors reported that a heat flow of 75.6 mW m^{-2} was estimated between depths of 100 and 290 m. The thermal conductivity of the granite penetrated by this borehole was not reported, but Lee (1986) mentioned that values for this suite of granites fall within the range 3.1-3.5 W m^{-1} °C^{-1}, so a representative value of 3.3 W m^{-1} °C^{-1} can be adopted.

Figure 9.



A Lees Valley fit to the local topography is illustrated in Fig. 9, with the resulting perturbation to the thermal state of the crust indicated in Table 4. The wellhead, at ~225 m O.D., corresponds in terms of Fig. 9 to x=-800 m, the depth range of heat flow measurement being equivalent to z ~445 to ~635 m. This fit predicts (using equation (2.05)) temperature perturbations due to the topography of 4.83 and 4.50 °C at these points. The geothermal gradient and heat flow are thus perturbed over this depth range by 1.7 °C km^{-1} and 5.7 mW m^{-2}, making the preferred estimate for the topographically corrected heat flow in this borehole 69.9 mW m^{-2}, in reasonable agreement with Lee's (1986) calculations (Table 5). For this particular topographic profile, the correction to heat flow is therefore relatively small, despite the high local relief and the shallowness of the borehole. This is largely due to the borehole location significantly 'off axis' from the modelled Lees Valley; for a borehole of the same dimensions, with its wellhead at x=0, $z=z_s=H$, but with all other parameters as before, the temperature perturbations at 100 and 290 m depths would be 6.20 and 5.51 °C. The geothermal gradient between these depths would thus be perturbed by 3.6 °C km^{-1}, making the associated heat flow perturbation 12.0 mW m^{-2}, so the topographically corrected heat flow in this borehole would be 63.6 mW m^{-2}.

3.3.6 Soussons Wood, Dartmoor Soussons Wood (at SX 6733 7971) is one of several very shallow (≤100 m deep) boreholes drilled in the early 1980s for geothermal prospecting within the Variscan Dartmoor granite in Devon, southwest England (Lee, 1986). Although the local landscape has little relief compared with the other examples investigated here, a Lees Valley fit has nonetheless been undertaken to quantify the resulting correction, given the shallowness of the borehole (Fig. 10). The wellhead is 200 m ESE of and 10 m higher than the valley axis; thus, with the specified parameters, it corresponds to x=200 m and z=75 m.

A complicating factor affecting the analysis of this particular borehole dataset is that the heat flow of 132.2 mW m^{-2}, quoted by Downing and Gray (1986) and Lee (1986), includes (according to Burley et al., 1984) a 9 mW m^{-2} correction for palaeoclimate. The calculation of the topographic correction should utilise the raw heat flow measurement, so this palaeoclimate correction must first be removed, yielding 123.2 mW m^{-2}. Using equation (2.05), temperature perturbations due to the topography of 2.16 and 1.86 °C are determined at the wellhead and at the 100 m deep bottom of this borehole. The geothermal gradient and heat flow are thus resulting perturbed over this depth range by 2.9 °C km^{-1} and 9.1 mW m^{-2}, making the preferred estimate for the topographically corrected heat flow in this borehole 114.1 mW m^{-2} (Table 5). Despite the subdued topography, the correction to heat flow is indeed substantial, due to the shallowness of the borehole. If the borehole were located beneath the valley axis, with the wellhead at x=0, z=z_s=H but with other parameters unchanged, the heat flow perturbation would be even larger, 14.4 mW m^{-2}, making the topographically-corrected heat flow 108.8 mW m^{-2}. The reinstatement of the palaeoclimate correction for this particular borehole will be discussed below (section 4; Table 6).

Figure 10.

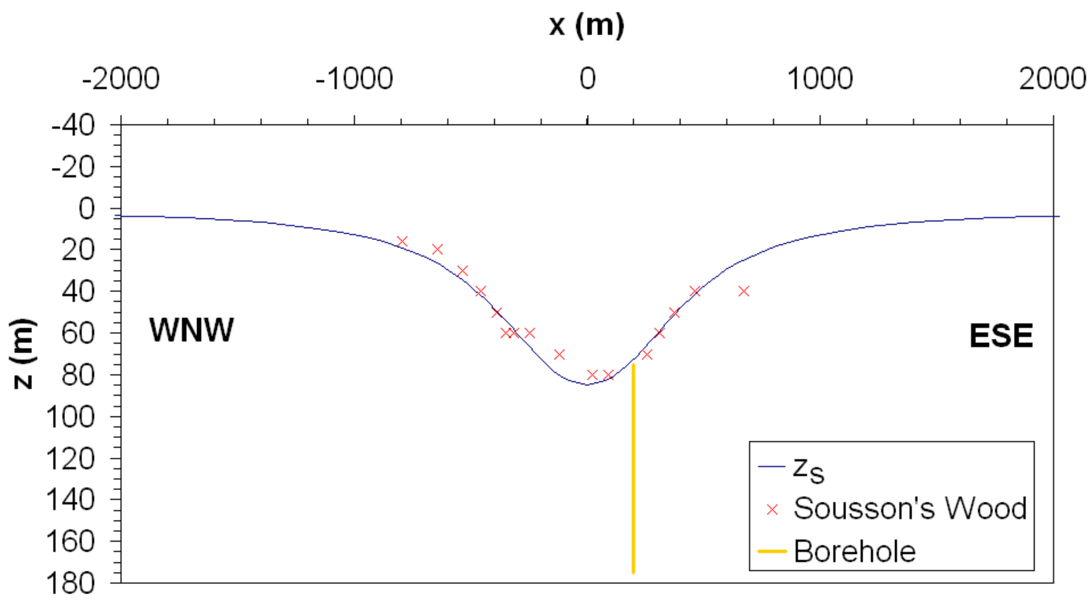


Table 6: Combined topographic and palaeoclimate corrections

Borehole	Q_o (mW m^{-2})	z_1 (m)	z_2 (m)	k (W m^{-1} °C^{-1})	ΔQ_T (mW m^{-2})	ΔQ_p (mW m^{-2})	Q_c (mW m^{-2})
Rookhope	99.6	390	789	3.1	-6.9	15.5	108.2
Raydale	71.1	525	590	3.65	-5.7	19.3	84.7
Eastgate	115.0	0	995	2.99	-12.0	13.4	116.4
Skiddaw	118.5	100	281	3.5	-21.8	20.8	117.5
Ballater	75.6	100	290	3.3	-5.7	19.6	89.5
Soussons Wood	123.2	0	100	3.12	-9.1	28.2	142.3

Q_o is raw heat flow, measured between depths z_1 and z_2, in rock of thermal conductivity k. ΔQ_T and ΔQ_p are the corrections to heat flow for topography and palaeoclimate, $Q_c = Q_o + \Delta Q_T + \Delta Q_p$ being the corrected heat flow. Values of Q_o, z_1, z_2, k, and ΔQ_T are the same as in Table 5. Values of ΔQ_p are calculated from the climatically-perturbed geotherms predicted in section 2 (based on Fig. 2 for the Soussons Wood borehole and Fig. 3 for the others) using equation (1) with the appropriate values of z_1, z_2 and k. As is discussed in the main text (section 4) these corrections may well be conservative.

3.5 Discussion

The results of this summary investigation of topographic corrections to heat flow data are summarised in Table 5. Notwithstanding the fact that the method used hitherto to determine such corrections for UK heat flow data has not been transparent, and is known to be approximate, it is evident that there is reasonable agreement between the set of corrections determined in the present study and the extant set of values. In general, the new corrections exceed those determined previously, as might well be expected from the smoothing of topography evident with the previous method. The largest mismatches in Table 5 are 4.2 and 3.2 mW m^{-2}, respectively for the Skiddaw and Ballater boreholes, which are in the localities with the greatest relief.

The use of a Lees Valley fit, suggested here as a method for topographic correction, will not work for all boreholes, as in many cases the local topography will be quite unlike that of a Lees Valley. For example, the Cairngorm geothermal borehole in northeast Scotland was located at NH 989 062 (from Lee, 1986), the wellhead being at ~630 m O.D., more than halfway up the 1245 m high Cairngorm mountain range. Lee (1986) reported a heat flow measurement in this borehole of 72.2 mW m^{-2}, then subtracted a topographic correction of 2.7 mW m^{-2} to obtain a corrected heat flow value of 69.5 mW m^{-2}. However, given the location of the borehole, it is not at all obvious why the topographic correction should be negative; from the nature of the terrain, one would instead expect a zero or small positive correction. It thus seems probable that the 69.5 mW m^{-2} value underestimates the heat flow in this locality and that it is, therefore, arguably better to use the raw data value of 72.2 mW m^{-2} as the best estimate of local heat flow. However, at this stage it is impossible to be more specific as to the reasons why this particular correction has been reported for this site, and likewise for other sites for which the adjacent terrain does not approximate a Lees Valley; these tasks will require a more general method for determining topographic corrections to heat flow, beyond the scope of the present study.

The need for topographic corrections is particularly evident for shallow boreholes; for example, the correction calculated for the 100 m deep Soussons Wood borehole, located in a subdued valley, is almost as large as for the 995 m deep Eastgate borehole, located in a deeply entrenched valley in quite rugged terrain (Table 5). The importance of corrections of this type for shallow boreholes does not seem to have previously been recognised, notwithstanding the national strategy of drilling shallow boreholes for geothermal prospecting. With a reported heat flow of 132 mW m^{-2}, the Soussons Wood borehole has stood as a clear outlier in relation to others in Dartmoor, which have yielded heat flow values of 106, 107, 108, 111 and 114 mW m^{-2} (Lee, 1986); this remains so when the palaeoclimate corrections for these boreholes, applied by Burley et al. (1984) are removed, yielding a heat flow of 123 mW m^{-2} at Soussons Wood and 86, 89, 89, 90, and 93 mW m^{-2} for the others. The topographic correction now determined for this borehole brings it closer into line with the others; moreover, some of these other Dartmoor boreholes are located on hilltops, rather than in a valley, so they probably require positive topographic corrections, which may well bring them all into even closer agreement.

4. Interaction between topographic and palaeoclimate corrections

Corrections to heat flow for palaeoclimate and for topography have been discussed separately in sections 2 and 3. The logic of these correction procedures requires that the topographic correction be applied first, to convert raw data to measurements of present-day heat flow in the absence of topography. The palaeoclimate correction should be subsequently applied, to obtain a best estimate of what the present-day heat flow would be in the absence of past climate change. Fortunately, the palaeoclimate correction is simply an additive correction (see section 2); it does not depend on the present-day heat flow, just on the depth of a borehole (or the depth range of heat flow measurement) and on the past climate history. As a result, applying this topographic correction followed by the palaeoclimate correction is equivalent to working out both corrections separately and then simply adding them together. An exception to this rule, illustrated in section 3.3.6 for the Soussons Wood borehole, arises where the reported heat flow data have already been corrected for palaeoclimate (by Burley et al., 1984). This prior correction must first be removed, then the data corrected for topography and subsequently again for palaeoclimate.

Figure 11.

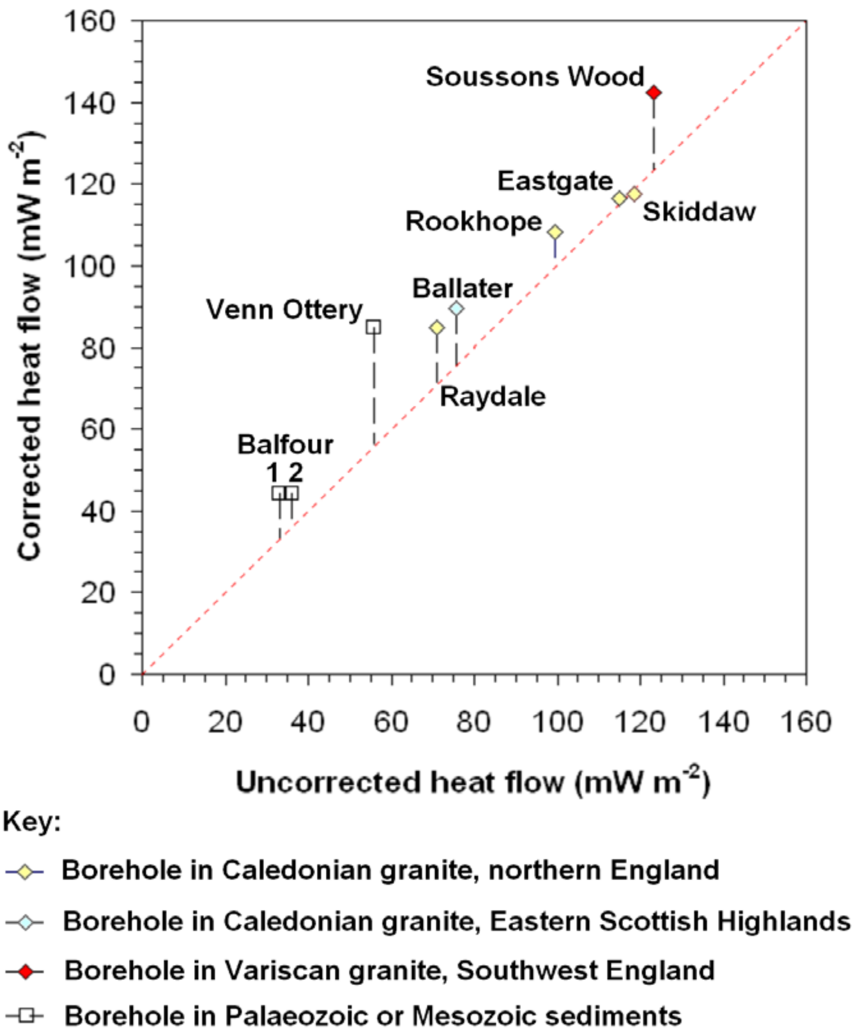


Table 6 lists the results of applying corrections for topography followed by corrections for palaeoclimate for the set of boreholes discussed in section 3. Except for the Skiddaw borehole, the latter corrections outweigh the former, such that the corrected heat flow exceeds the raw measurement. Nonetheless, for the Skiddaw and Eastgate boreholes, the two corrections are opposite in sign but almost equal in magnitude, so the raw heat flow measurements provide a good approximation to the corrected values; in all other cases, the corrected values are significantly larger than the raw measurements. For the sites listed, the greatest absolute difference between corrected and raw heat flow measurements is 19 mW m^{-2} for the Soussons Wood borehole; the greatest percentage differences are ~18-19% for Raydale and Ballater boreholes (Fig. 11). However, greater percentage differences can be expected for boreholes in areas of relatively low heat flow.
For example, for the Balfour borehole Downing and Gray (1986) reported the heat flow as 36 mW m^{-2} between depths of 543 and 1205 m, between which the temperature increases by 14.9 °C (Benfield, 1939). These values are consistent with a geothermal

gradient of 22.5 °C km^{-1} and a mean thermal conductivity over this depth range of 2.4 W m^{-1} $°C^{-1}$. Using equation (1), the resulting correction to the heat flow over this depth range can be estimated as 8.4 mW m^{-2}, making the corrected value 44.4 mW m^{-2}, a correction by 23%, and indicating a corrected geothermal gradient of 18.5 °C km^{-1}. On the other hand, for the same borehole, the temperature difference between 0 and 543 m depths is 9 °C (Burley et al., 1984), indicating a geothermal gradient of 16.6 °C km^{-1}. From equation (1), the palaeoclimate correction to the geothermal gradient over this depth range is estimated as 5.7 °C km^{-1}, making the corrected value 22.3 °C km^{-1}, a correction by 34%. If the heat flow at greater depths has been determined correctly, the thermal conductivity in the uppermost part of this borehole is only 2.0 W m^{-1} $°C^{-1}$, such that the uncorrected heat flow in this uppermost part of the borehole would be 33.2 mW m^{-2}. The palaeoclimate corrections for this particular borehole, of some historical interest (section 2.1), are indeed significant, albeit smaller than the estimates by Benfield (1939) and Anderson (1940). As another example, one may consider the Venn Ottery borehole, located in east Devon (SY 066 911), east of the Cornubian Batholith (Fig. 1). The reported heat flow in this borehole is 56 mW m^{-2}, measured between depths of 100 and 308 m and uncorrected for palaeoclimate (e.g., Burley et al., 1984; Downing and Gray, 1986). The thermal conductivity of the rocks penetrated by this borehole was not noted by these authors, but Richardson and Oxburgh (1978) reported k for 'New Red Sandstone' (the local lithology in outcrop and in the shallow subsurface; Edwards and Scrivener, 1999) as 7.9 mcal s^{-1} cm^{-1} $°C^{-1}$, equivalent to ~3.3 W m^{-1} $°C^{-1}$. Using equation (1), for the temperature history in Fig. 2(a), the palaeoclimate correction to heat flow can be determined for this borehole as 29 mW m^{-2}. The location of the borehole on a hillside prevents the application of any straightforward correction to heat flow for topography, but given the limited local relief any resulting correction (whether positive or negative) is unlikely to be large. The best available estimate for the corrected heat flow in this borehole is thus 85 mW m^{-2}, ~52% higher than the raw measurement. For other boreholes to the east and north of the Cornubian Batholith, palaeoclimate corrections have likewise not previously been determined or have previously been estimated as very small even though the boreholes are shallow, whereas palaeoclimate corrections have previously been determined for many of the shallow boreholes within the granite of this batholith (Burley et al., 1984). As a result, existing depictions such as Fig. 1 and Fig. 6 of Busby et al. (2011) exaggerate the heat flow contrast between the Cornubian Batholith and its surroundings.

As Younger et al. (2012) have discussed, several projects are currently under way to exploit the geothermal potential of the Cornubian Batholith. One of these involves drilling into the St Austell granite to provide an energy source for the Eden Project ecological theme park (at SX 005 555); it has been estimated that temperatures of ~190 °C will be encountered by depths of ~5000 m, apparently on the basis of previous calculations by Lee (1986) for the adjacent Carnmenellis granite. Geothermal data from the St Austell granite in the vicinity of the Eden Project are available from two adjacent 100 m deep boreholes: Tregarden (SX 055 595) with heat flow 125.8 mW m^{-2}; and Colcerrow (SX 068 576) with heat flow 126.5 mW m^{-2} (Lee, 1986). However, these values incorporate palaeoclimate corrections of 20 and 24 mW m^{-2}, respectively (Burley et al., 1984), so the raw heat flow measurements are 105.8 and 102.8 mW m^{-2}. The

Tregarden borehole was drilled near the summit of a knoll rising ~25 m above its surroundings, so the topographic correction to heat flow, calculated for a Lees Hill (cf. Appendix 2; but not illustrated here) would be ~7 mW m^{-2}. Likewise, the Colcerrow borehole was drilled on the flank of a larger knoll, rising by ~60 m, so the topographic correction would be ~9 mW m^{-2}. Using equation (1) between depths of 0 and 100 m, for the temperature history in Fig. 2(a), and with k=3.14 W m^{-1} °C^{-1} at Tregarden and 3.38 W m^{-1} °C^{-1} at Colcerrow (Lee, 1986), palaeoclimate corrections at these two localities can be estimated as 28 and 30 mW m^{-2}, so the corrected heat flow values are ~141 and ~142 mW m^{-2}, respectively. These corrected values are indeed similar to that at Soussons Wood in the Dartmoor granite (Table 6), suggesting that ~140 mW m^{-2} is a reasonable upper bound to the surface heat flow over any part of the Cornubian Batholith. To estimate the temperature at depth it can be noted that the radioactive heat production was measured as 3.5±0.6 μW m^{-3} in the Tregarden borehole and 4.8±0.7 μW m^{-3} in the Colcerrow borehole (Lee, 1986), suggesting a typical value for the St Austell granite of ~4.0-4.5 μW m^{-3}. From standard theory (e.g., Lachenbruch, 1970; Westaway, 2009), in granite of uniform radioactive heat production Y and vertical extent D, the surface heat flow Q_s will equal the basal heat flow Q_o plus $Y \times D$. One may thus tentatively estimate for the St Austell granite that Q_o is ~50 mW m^{-2} and the radioactive contribution is a consequence of Y ~4.5 μW m^{-3} across D ~20 km. Subject to these assumptions, with a surface temperature of ~10 °C, one may calculate using this standard theory that a temperature of 190 °C is to be expected at a depth of ~4.5 km, a shallower depth than is expected for the previous calculations. The difference is due primarily to the use of less conservative palaeoclimate corrections than were favoured by Burley et al. (1984) and the inclusion of modest topographic corrections, but relative to the previous calculations by Lee (1986) the effect of these changes has been partly offset by the adoption of a greater contribution from radioactive heating, which means a faster rate of decrease of heat flow with depth. Nonetheless, the saving of the cost of an estimated ~500 m of drilling to reach the target temperature would significantly benefit the economics of this geothermal energy project.

Taken together, the analyses in the present study (sections 2 and 3) confirm the impression previously apparent from Bloomer et al. (1979), that palaeoclimate corrections are typically much larger than topographic corrections for UK heat flow data. As discussed in section 2, palaeoclimate correction will typically cause the ‘true’ heat flow to significantly exceed the measured present-day value; this is usually the predominant effect, although in occasional cases, such as the Skiddaw borehole (section 3 and Tables 5 and 6), the topographic correction can be of comparable magnitude to, and thus largely cancel, the palaeoclimate correction. Because workers have often neglected or underestimated the importance of palaeoclimate correction (section 2), in general measured UK heat flow data underestimate temperatures at depth and thus underestimate geothermal resources.

The relative magnitudes of these two corrections, and the greater relative importance of the palaeoclimate correction, therefore has significant implications for assessment of the thermal state of the crust. One region where this effect is of some importance is northeast Scotland, where relatively low heat flow measurements have been obtained from highly radioactive Palaeozoic granites that extend to considerable depths within the crust (e.g., Lee, 1986; Lee et al., 1987; Webb et al., 1987; Younger et al., 2012). To anticipate the conclusions of the analysis below, it is suggested that the resolution of this apparent paradox is that the 'true' heat flow in this region is significantly greater than the measured value, because the measurements have not hitherto been corrected for palaeoclimate.

Analysis of the distribution of heat flow and heat production data suggests that the basal component of heat flow, due to conduction from the Earth's mantle, is 36 mW m^{-2} in northeast Scotland (Lee, 1986). The granites of northeast Scotland are associated with a negative gravity anomaly of circa -50 milligals (e.g., Lee, 1986), which has been modelled in terms of the base of the granite no shallower than 13 km depth (see Lee et al., 1987, and references cited therein). The radioactive heat production within the different granites is somewhat variable, reflecting their different compositions (e.g., Gould, 2001; Smith et al., 2002), but it was reported by Lee (1986) as 5.0 μW m^{-3} in the Cairngorm granite and 5.7 μW m^{-3} in the Ballater granite (although it is rather less in some other granites); the corresponding values quoted by Lee et al. (1987) were 7.3 and 6.8 μW m^{-3}, respectively. These measurements are based on samples recovered from the various geothermal boreholes, which penetrate no more than ~300 m into these granites. As already discussed (section 3), after topographic correction the surface heat flow is ~70 mW m^{-2} in the Ballater granite and probably ~72 mW m^{-2} in the Cairngorm granite.

Once again making use of standard theory (e.g., Lachenbruch, 1970; Westaway, 2009), if D is 13 km and Y 5.0 μW m^{-3}, Q_s will exceed Q_o by 65 mW m^{-2}, whereas if Y is 5.7 μW m^{-3} the difference will be 74 mW m^{-2}. From the measured values the differences between Q_s and the expected value of Q_o are only ~35 mW m^{-2}. However, following upward revision to Q_s as a result of a correction for palaeoclimate, the values would be in better agreement. Section 2 indicates that for shallow boreholes (<300 m deep), as at Ballater and Cairngorm, the palaeoclimate correction to heat flow will be ~20 mW m^{-2}, so the best estimate of the unperturbed heat flow in these boreholes will increase from ~70 to ~90 mW m^{-2}, with some ~55 mW m^{-2} of the latter value ascribed to radioactive heat production in the granite. One may thus tentatively infer that this granite has uniform heat production (at ~5.5 μW m^{-3}) to a depth of ~10 km. If so, then assuming a surface temperature of 7 °C and standard theory for conductive heat flow with a representative thermal conductivity of 3.3 W m^{-1} °C^{-1} and a basal heat flow of 35 mW m^{-2}, the temperature at the 10 km deep base of the granite would be 196 °C, decreasing to 157 °C at 7 km and 123 °C at 5 km depths.

For a proposal to drill a geothermal borehole to a given depth, one may obtain an approximate correction for the temperature difference expected by extrapolation of extant shallow borehole data and the true temperature after palaeoclimate correction, by linear extrapolation of the trend at shallow depth of a graph of δT (as in Fig. 2 or 3) to the required depth and comparison of with the part of the graph of δT at that depth. For example, suppose one wishes to drill in a region where the surface temperature is 10 °C and on the basis of shallow borehole date the near-surface heat flow and geothermal gradient are 70 mW m^{-2} and 20 °C km^{-1}, such that k=3.5 W m^{-1} °C^{-1} (parameter values roughly representative of the Ballater and Cairngorm granites; see above). Simple extrapolation, neglecting any effect of radioactive heat production, would thus predict a temperature of 110 °C at a depth of 5 km. Extrapolation of the near-surface trend of the δT graph in Fig. 3 would place it some 30 °C below the predicted value of δT at 5 km depth. However, the prediction in Fig. 3 is for k=3 W m^{-1} $°C^{-1}$ and so this mismatch would adjust to 35 °C for 3.5 W m^{-1} $°C^{-1}$. The predicted temperature at 5 km depth would thus adjust after palaeoclimate correction to ~145 °C, rather more than the target temperature of 110 °C. It can indeed be seen from Fig. 3 that if this hypothetical borehole were to be drilled to 3.85 km depth, the predicted bottom temperature would be 87 °C (10 °C + 3.85 km × 20 °C km^{-1}) in the absence of a palaeoclimate correction, but would adjust upwards by ~23 °C to the target of ~110 °C when such a correction is applied. Once again, the saving of the cost of ~1150 m of drilling to reach this target temperature would significantly benefit the economics of any such project. The proposed correction is greater here than for the earlier calculations for the St Austell granite, essentially because in the former case the local geothermal data had been subjected to palaeoclimate corrections that were too conservative, whereas the corresponding data for the Eastern Highlands of Scotland have not hitherto been corrected for palaeoclimate at all.

For comparison, Lee (1986), Lee et al. (1987) and Webb et al. (1987) tried to explain the apparent paradox in the Eastern Highlands of Scotland of high radioactive heat production and great vertical extent of the granites but low surface heat flow without recourse to climate correction, by postulating that the radioactive heat production decreases exponentially with depth. However, the concentration of radioactive elements (such as potassium) in granite that gives rise to the heat production also gives rise to the low density, so with the assumption of lower radioactive heat production at depth one would expect less density contrast relative to the surrounding country rock; one would therefore be unable to account for the observed gravity anomalies. Notwithstanding the compositional variability between the different granite intrusions, each intrusion shows only a limited range of composition (e.g., Gould, 2001; Smith et al., 2002), thus providing no basis for any assumption that radioactive heat production decreases with depth, although the ranges of depth within these intrusions that have been sampled are limited, due to the absence of deep boreholes. Nonetheless, assuming an exponential decrease in radioactive heat production with depth, Lee (1986) estimated the temperature in the Ballater granite as 98 °C at 5 km depth and 126 °C at 7 km depth, rather lower estimates than for the revised geotherm proposed above (which is, itself, arguably conservative). A test, which would enable this difference in interpretation to be resolved, would be to drill a borehole into one of these granites to a depth of ~1-2 km. At such a depth the topographic and palaeoclimate corrections would be minimal; analysis of the geochemitry of the recovered granite core would also establish the radioactive heat production at each depth.

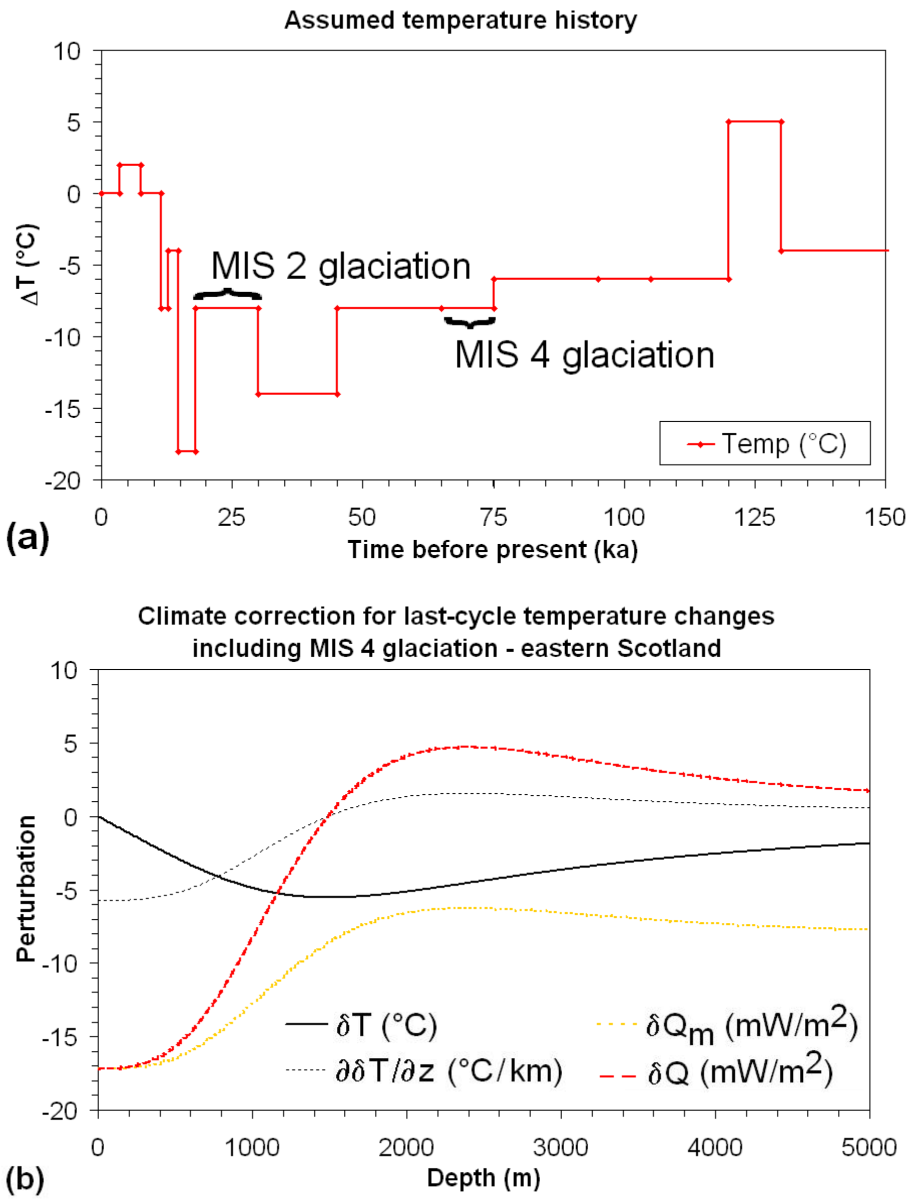
5. Discussion

As noted above, the corrections for topography, proposed here, are only accurate to the extent that the landscape in each locality approximates the form of a Lees Valley. The general agreement between these corrections and previous estimates, calculated using a different method (e.g., Table 4), suggests that the proposed corrections are unlikely to be significantly in error. Nonetheless, it is evident that topographic corrections should ideally be calculated using a more general, but transparent, method that takes account of the three-dimensional character of the topography in each locality; however, such a refinement is beyond the scope of this study. Another potential refinement concerns the calculation of topographic corrections in formerly glaciated regions. Throughout Britain, the unperturbed geothermal gradient u exceeds the atmospheric lapse rate u' such that, given the algebraic form of the topographic correction (e.g., in equation 2.15), under present-day conditions the topographic perturbation to heat flow for a borehole in a valley floor will always be positive, so that (other factors being equal) the corrected heat flow will be less than the raw value. However, as a result of the aforementioned frictional heating effects, the variations in temperature with height at the base of a moving ice sheet can create temperature gradients that dramatically exceed both the atmospheric lapse rate and the steady-state geothermal gradient. For example, on the eastern flank of the Cairngorm Mountains (circa NJ 040 000, in the SE corner of the area depicted in Fig. 9 of Hall and Glasser, 2003), around the LGM the subglacial temperature at the bedrock surface is predicted by Hall and Glasser (2003) to have decreased upward by ~11 °C across a height range of ~250 m between the floor and flanks of a glaciated valley. This corresponds to a vertical temperature gradient of >40 °C km^{-1}, which exceeds the steady-state geothermal gradient in this region. Thus, to calculate the topographic correction to heat flow in this locality at the LGM, one should use this ~40 °C km^{-1} value in lieu of the atmospheric lapse rate for u' in the correction formula (e.g., in equation 2.15). The topographic perturbation to heat flow for a hypothetical borehole in the floor of this glaciated valley around the LGM would thus be negative, so the corrected heat flow will exceed the raw value. Given the thermal inertia of the bedrock in such a locality, the present-day topographic correction to heat flow will therefore be intermediate between the value calculated for the present-day conditions and that calculated for the conditions at the LGM. This correction will, therefore be smaller in magnitude than that calculated for the present-day contions, but may conceivably be positive, rather than the negative value expected for the present-day conditions. Such complexities (involving interactions between topographic and palaeoclimate corrections that lie outside the scope of section 4) may thus contribute to explaining the low raw heat flow values measured in NE Scotland; however, further investigation of such interactions between corrections is beyond the scope of this study.

The corrections for palaeoclimate could likewise be refined further in future, to take account of lateral variations in surface temperature and in durations of ice cover; the latter would be significant when investigating regions near the margins of the MIS 2 ice sheet, which have not been considered in the present study. Additional types of data could also be factored in, such as sedimentary textures indicative of periglacial conditions (e.g., Williams, 1975). Furthermore, the magnitudes of the temperature anomalies listed in Tables 2 and 3 for much of the Late Pleistocene roughly reflect differences between present-day temperatures and those during Greenland interstadials (see section 2.2; discussion of the data from Lynford) rather than the differences relative to the time-averaged conditions that existed throughout this span of time, treating the Greenland interstadials and stadials separately. It could indeed be argued that the corrections for palaeoclimate proposed in Table 3 as applicable to Scotland are conservative, because they may underestimate the duration of extreme cold (cf. Kukkonen et al., 1998) and (at such a high latitude as this; ~57°N) may also underestimate the magnitude of the cooling effect around the LGM at times when the region was unglaciated (assumed to be -20 °C in Table 3) or overlain by ice that was well below its melting point (cf. Hall and Glasser, 2003) (not to mention the perturbation to the near-surface temperature gradient caused by the heating effect of flow of ice, discussed above). For example, the magnitudes of the temperature anomalies listed in Tables 2 and 3 for much of the Late Pleistocene roughly reflect differences between present-day temperatures and those during Greenland interstadials (see section 2.2; discussion of the data from Lynford) rather than the differences relative to the time-averaged conditions that existed throughout this span of time. Given that the present-day mean surface temperature of ~7 °C in northeast Scotland, the mid-range of the ensemble of climate predictions from Kageyama et al. (2001) of -18 °C at this latitude indicates a temperature anomaly of circa -25 °C, whereas the coldest of these predictions, -25 °C, indicates a temperature anomaly of -32 °C. The nominal ~12,000 year duration of ice cover may also be an overestimate for some parts of Scotland (cf. Clark et al., 2012); in combination, these factors may potentially result in a larger-magnitude palaeoclimate correction, possibly in excess of 30 mW m^{-2}.

One potential complicating factor is the possibility of glaciation during MIS 4. This can be inferred from glacial sediments that have been dated to this span of time in onshore localities (e.g., at Teindland in NE Scotland, NJ 297 570; Hall et al., 1995) and from other sediments in offshore areas both east and west of Scotland (e.g., Stoker et al., 1993; Stoker and Bradwell, 2005; Stewart and Lonergan, 2011). Clapperton (1997) indeed estimated that Scotland was covered by a substantial ice sheet between 70 and 57 ka. Palaeoclimate simulations indicate that any ice sheet that formed at this time melted subsequently, with little or no ice volume in Britain until the Late Devensian ice sheet began to form circa 30 ka (e.g., Evans et al., 2009). To investigate the palaeoclimate effect of a MIS 4 ice sheet on heat flow, Figure 12 has been constructed, assuming the same temperature history as for Fig. 3 except between 75 and 65 ka the bedrock surface temperature anomaly has been set to -8 °C rather than the previous -18 °C. Comparison of Fig. 12 with Fig. 3 indicates that the inclusion of a MIS 3 ice sheet does not change the predicted palaeoclimate correction much; its variation is found to be only ~4%, for a nominal thermal conductivity of 3 W m^{-1} $°C^{-1}$ its value at the bedrock surface being reduced from 18.0 to 17.2 mW m^{-2}.

Figure 12. Palaeoclimate corrections including a MIS 4 ice sheet.



6. Conclusions

Raw heat flow measurements typically require correction for both palaeoclimate and topography if temperatures are to be reliably extrapolated to depths greater than those where temperature is measured. However, although both types of correction were pioneered decades ago by British workers (e.g., Lees, 1910; Jeffreys, 1938; Benfield, 1939; Anderson, 1940) they have been omitted or underplayed in recent assessments of the UK geothermal dataset. Furthermore, the former strategy of measuring heat flow utilising shallow boreholes (e.g. Downing and Gray 1986) exacerbates the magnitude of both types of correction. In addition, the location of Britain at a range of latitude with a temperate climate at present but where arctic conditions prevailed during much of the Pleistocene means that the palaeoclimate correction, for a borehole of a given depth, is particularly large. Outside regions of high relief relative to borehole depth, palaeoclimate corrections at sites in Britain are indeed larger than topographic corrections, and for almost all boreholes are additive. The magnitude of the palaeoclimate correction depends on assumptions about palaeotemperature anomalies and their durations, but from the available evidence it can be assessed, for a very shallow borehole in an unglaciated part of southern Britain where the bedrock has thermal conductivity 3 W m^{-1} $°C^{-1}$, as 27 mW m^{-2}. Air temperatures during Pleistocene cold stages decreased northward, but in much of northern Britain the Earth's surface was not exposed to these low temperatures for part of the Late Pleistocene due to the insulating effect of cover by ice sheets; the detailed correction for each locality thus depends on the local histories of air temperature anomalies and of ice cover, and may therefore potentially be greater or less than is typical for southern England. The past failure to recognise the magnitude of palaeoclimate corrections at sites in Britain, and to incorporate them into studies of geothermics, has led to systematic underestimation of temperatures at depth and, thus, of the geothermal energy resource.

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Appendix 1: Theory for palaeoclimate correction

The following analysis is based on previous treatments, notably those by Birch (1948) and Turcotte and Schubert (1982). The aim is to find the temperature T as a function of depth z and time t in crust of thermal conductivity k and thermal diffusivity κ, for which the surface temperature Φ(t) has varied in a specified manner. The crust is represented as a halfspace with its upper surface at z=0; it is inferred to have been in a steady thermal state with a uniform geothermal gradient before the surface temperature variations started, at $t=t_o$. From first principles, the perturbation δT relative to this assumed initial state is given by

$$\delta T\ (z,t) = \int_{\tau=0}^{t} \Phi(\tau) \exp\left(-z^2 /(4\kappa\ (t-\tau))\right) (t - \tau)^{-3/2}\, d\tau \qquad (1.01)$$

where τ is a dummy variable for time, which varies from 0 at the start of the temperature perturbations to t at the present day.

Equation (1.01) can be solved by assuming that Φ(t) varies in a series of n step changes, such that

$$\begin{aligned}
\Phi(t) &= \Phi_1;\ 0 < \tau < \tau_1 \\
&= \Phi_2;\ \tau_1 < \tau < \tau_2 \\
&\ \ \ldots \\
&= \Phi_{n-1};\ \tau_{n-2} < \tau < \tau_{n-1} \\
&= \Phi_n;\ \tau_{n-1} < \tau < t\,.
\end{aligned} \tag{1.02}$$

With Φ thus taken as constant during each time step, equation (1.01) can be solved by introducing a change of variable $\alpha = z / \sqrt{(4\ \kappa\ (t - \tau))}$. After making this substitution and several algebraic steps of simplification, it can be evaluated as a sequence of n integrals

$$\begin{aligned}
\delta T\ (z,t) = (2/\sqrt{\pi}) \times [&\int_{\alpha=0}^{z/\sqrt{(4\kappa\,(t-\tau_1))}} \Phi_1 \exp(-\alpha^2)\, d\alpha + \\
&\int_{\alpha=z/\sqrt{(4\kappa\,(t-\tau_1))}}^{z/\sqrt{(4\kappa\,(t-\tau_2))}} \Phi_2 \exp(-\alpha^2)\, d\alpha + \\
&\ldots + \\
&\int_{\alpha=z/\sqrt{(4\kappa\,(t-\tau_{n-2}))}}^{z/\sqrt{(4\kappa\,(t-\tau_{n-1}))}} \Phi_{n-1} \exp(-\alpha^2)\, d\alpha + \\
&\int_{\alpha=z/\sqrt{(4\kappa\,(t-\tau_{n-1}))}}^{\infty} \Phi_n \exp(-\alpha^2)\, d\alpha\ ]\,.
\end{aligned} \tag{1.03}$$

The Gaussian error function erf(x) is defined as

$$\mathrm{erf}(x) \equiv (2/\sqrt{\pi}) \times \int_0^x \exp(-\alpha^2)\, d\alpha\,, \tag{1.04}$$

such that erf(0)=0 and erf(∞)=1. One may thus write

$$\begin{aligned}
\delta T\ (z,t) = \ &\Phi_1\, \mathrm{erf}(z/\sqrt{(4\kappa\,(t-\tau_1))}) \\
&- \Phi_2\, \mathrm{erf}(z/\sqrt{(4\kappa\,(t-\tau_1))}) + \Phi_2\, \mathrm{erf}(z/\sqrt{(4\kappa\,(t-\tau_2))}) \\
&- \ldots \\
&- \Phi_{n-1}\, \mathrm{erf}(z/\sqrt{(4\kappa\,(t-\tau_{n-2}))}) + \Phi_{n-1}\, \mathrm{erf}(z/\sqrt{(4\kappa\,(t-\tau_{n-1}))}) \\
&- \Phi_n\, \mathrm{erf}(z/\sqrt{(4\kappa\,(t-\tau_{n-1}))}) + \Phi_n\,.
\end{aligned} \tag{1.05}$$

Subject to the assumption that no other processes affect the geothermal gradient and that at large depth this gradient attains a uniform value u, one may thus write

$$T(z,t) = T_o + u\,z + \delta T\ (z,t)\,. \tag{1.06}$$

One may thus determine the geothermal gradient by partial differentiation of equation (1.06), given that if

$$K(z,t) = L\ \mathrm{erf}((z/\sqrt{(4\ \kappa\ (t - c))}) \tag{1.07}$$

with L and c constant, then

$$\frac{\partial K}{\partial z}(z,t) = \frac{2L}{\sqrt{\pi}} \times \frac{1}{\sqrt{(4\,\kappa\,(t-c))}} \exp\left(-z^2/(4\,\kappa\,(t-c))\right) . \qquad (1.08)$$

Having obtained this solution, one may differentiate equation (1.05) term by term to obtain the geothermal gradient at any time t at any depth z within the model crust.

Although algebraically valid, the above solutions would be inconvenient to use; it is preferable to express the solutions in terms of time before the present day rather than measuring time forwards from the start of any succession of temperature perturbations. One may thus redefine the n step-changes in temperature in terms of a new symbol ΔT, such that for each of these, $\Delta T_i = \Phi_{n+1-i}$ (i.e., $\Delta T_1 = \Phi_n$; $\Delta T_2 = \Phi_{n-1}$; ... ; $\Delta T_{n-1} = \Phi_2$; $\Delta T_n = \Phi_1$), and make a corresponding re-definition of the ages of the starts and ends of the time steps, such that $t'_i = t - \tau_{n-i}$ (i.e., $t'_1 = t - \tau_{n-1}$; $t'_2 = t - \tau_{n-2}$; ... ; $t'_1 = t - \tau_1$; $t'_n = t$). Equation (1.05) thus adjusts to

$$\begin{aligned}\delta T\,(z, t') = \;& \Delta T_1\,(1 - \mathrm{erf}(z/\sqrt{(4\kappa\, t'_1)}))\\ &+ \Delta T_2\,(\mathrm{erf}(z/\sqrt{(4\kappa\, t'_1)}) - \mathrm{erf}(z/\sqrt{(4\kappa\, t'_2)}))\\ &+ \ldots\\ &+ \Delta T_{n-1}\,(\mathrm{erf}(z/\sqrt{(4\kappa\, t'_{n-2})}) - \mathrm{erf}(z/\sqrt{(4\kappa\, t'_{n-1})}))\\ &+ \Delta T_n\,(\mathrm{erf}(z/\sqrt{(4\kappa\, t'_{n-1})}) - \mathrm{erf}(z/\sqrt{(4\kappa\, t'_n)}))\,, \qquad (1.09)\end{aligned}$$

with the corresponding perturbation to the geothermal gradient given by:

$$\begin{aligned}\frac{\partial \delta T}{\partial z}(z, t') = \;& \frac{1}{\sqrt{\pi}} \times \Big[\Delta T_1 \frac{1}{\sqrt{(\kappa\, t'_1)}} \left(-\exp(-z^2/(4\kappa\, t'_1))\right)\\ &+ \Delta T_2 \left(\frac{1}{\sqrt{(\kappa\, t'_1)}} \exp(-z^2/(4\kappa\, t'_1)) - \frac{1}{\sqrt{(\kappa\, t'_2)}} \exp(-z^2/(4\kappa\, t'_2))\right)\\ &+ \ldots\\ &+ \Delta T_{n-1} \left(\frac{1}{\sqrt{(\kappa\, t'_{n-2})}} \exp(-z^2/(4\kappa\, t'_{n-2})) - \frac{1}{\sqrt{(\kappa\, t'_{n-1})}} \exp(-z^2/(4\kappa\, t'_{n-1}))\right)\\ &+ \Delta T_n \left(\frac{1}{\sqrt{(\kappa\, t'_{n-1})}} \exp(-z^2/(4\kappa\, t'_{n-1})) - \frac{1}{\sqrt{(\kappa\, t'_n)}} \exp(-z^2/(4\kappa\, t'_n))\right)\Big] . \qquad (1.10)\end{aligned}$$

Appendix 2: Theory for topographic correction using Lees hills and valleys

As was noted in the main text, Lees (1910) established a geologically-useful characteristic shape of topography for which the temperature perturbation can be determined analytically; this solution has subsequently become known as a 'Lees Hill'. This is a two-dimensional solution, applicable to a topographic profile in one horizontal direction, for which the topography and other parameters do not vary in the perpendicular horizontal direction. The horizontal co-ordinate x is measured from the axis of the hill, the vertical co-ordinate z being positive downwards. Lees (1910) showed that if the surface topography has the form $z_s(x)$ that satisfies

$$(u - u') z_s = \frac{Y z_s^2}{2 k} - \frac{A (z_s + a)}{x^2 + (z_s + a)^2} , \qquad (2.01)$$

where u is the near-surface geothermal gradient unperturbed by the terrain, u' is the rate at which the surface temperature decreases with height (i.e., the atmospheric lapse rate), Y is the radioactive heat production in the crust, including the volume of crust above z=0 (i.e., for z<0, z being positive downwards), and A and a are parameters that specify the height and shape of the terrain, then the associated temperature perturbation will satisfy

$$T(x, z) = T_o + u z - \frac{Y z^2}{2 k} + \frac{A (z + a)}{x^2 + (z + a)^2} \quad (z \geq z_s; z_s \leq 0), \qquad (2.02)$$

where T is the perturbed temperature at co-ordinates (x, z) and T_o is the surface temperature at z=0. This solution is consistent with the physics of heat generation and transport in the modelled rock mass and with the boundary conditions that the surface temperature decreases upward as $T = T_o + u' z$, due to the upward cooling of the atmosphere, and that at large (positive) z or large (positive or negative) x the temperature tends to the unperturbed value $T = T_o + u z - Y z^2 / (2 k)$.

Lees (1910) showed that the parameters specifying the height and width of the model topography can be interrelated by considering the half width of the topography b at which the height decreases to z_s=h=H/2 (i.e., x=±b when z_s=h), as follows:

$$a = H + \sqrt{(H - h)^2 + b^2} \text{ and}$$
$$A = (u - u') H \sqrt{(H - h)^2 + b^2} (1 + \beta H) \text{ where}$$
$$\beta = Y / (2 k (u - u')) \quad . \qquad (2.03)$$

Lee (1991) proposed the following parameterization for the case where Y=0 (i.e., no radioactivity in the crust): where (again) H is the height of the topography at x=0 and 2 B is the half-width of the hill (i.e., the topography decreases to H/2 at x=±B):

$$A = H^2 \gamma (u - u')$$
$$a = H (1 + \gamma)$$
$$\gamma = \sqrt{1/4 + \beta^2} \text{ and}$$
$$\beta = B / H \quad . \qquad (2.04)$$

In both cases there are limiting conditions on the validity of the solutions, as discussed in the respective publications.

For a long time it was accepted that one could generate the temperature perturbation caused by a valley very simply from Lees's (1910) solution for a hill, by substitution of a negative value of a (e.g., Birch, 1950; Jaeger and Sass, 1963). This turns out not to be so; Lee (1991) was the first to work through the derivation of the solution for a valley from first principles, and to thus demonstrate otherwise.

Lee's (1991) solution excluded effects of radioactive heat production, but can be modified to include this contribution to heat flow to give:

$$T(x, z) \quad = \quad T_o \quad + \quad u\,z \; - \frac{Y\,z^2}{2\,k} + \frac{A\,(z - a)}{x^2 + (z - a)^2} \qquad (z \geq z_s \geq 0)\,. \qquad (2.05)$$

In the absence of radioactive heat production, Lee (1991) established that

$$A \quad = \quad -H^2\,\gamma\,(u - u')$$
$$a \quad = \quad H\,(1 - \gamma)$$
$$\gamma \quad = \quad \sqrt{(1/4 + \beta^2)} \text{ and}$$
$$\beta \quad = \quad B / H \quad , \qquad (2.06)$$

It is thus evident that the required changes to switch from a hill to a valley are more complex than had hitherto been assumed; even if B and H, and thus β and γ, remain constant, the value of a changes, in addition to the terms in a changing sign; the value of A also becomes negative. As a result of the different values of a, a hill with given values of H and B and a valley with the same values of H and B do not have equivalent profiles; the resulting perturbations to the heat flow and geothermal gradient also differ somewhat. The resulting topography is given by (cf. equation (2.01))

$$(u - u')\,z_s \quad = \quad \frac{Y\,z_s^2}{2\,k} \quad - \quad \frac{A\,(z_s - a)}{x^2 + (z_s - a)^2}\,, \qquad (2.07)$$

which can be expressed in explicit form as

$$x \quad = \quad \pm \left[\frac{2\,k\,A\,(z_s - a)}{Y\,z_s^2 - 2\,k\,z_s\,(u - u')} \quad - \; (z_s - a)^2 \right]. \qquad (2.08)$$

Differentiating equation (2.05) gives the vertical geothermal gradient as:

$$\frac{\partial T}{\partial z} \quad = \quad u \; - \; \frac{Y\,z}{k} \; + \; \frac{A\,(x^2 - (z - a)^2)}{(x^2 + (z - a)^2)^2}\,. \qquad (2.09)$$

Substituting equation (2.09) in terms of equation (2.06) gives

$$\frac{\partial T}{\partial z} \quad = \quad u \; - \; \frac{Y\,z}{k} \; + \; \frac{H^2\,\gamma\,(u\text{-}u')\,(x^2 - (z - H(1\text{-}\gamma))^2)}{(x^2 + (z - H(1\text{-}\gamma))^2)^2}\,. \qquad (2.10)$$

The perturbation to $\partial T/\partial z$, due to the terrain, $\Delta\partial T/\partial z$, can thus be inferred (after several algebraic steps) to be

$$\frac{\Delta\partial T}{\partial z} \quad = \quad \frac{u\text{-}u'}{\gamma} \quad = \quad \frac{u\text{-}u'}{\sqrt{(B^2 / H^2 + 1/4)}} \quad \approx \quad \frac{(u\text{-}u')\,H}{B} \qquad (2.11)$$

at $x=0$, $z=z_s=H$, i.e., at the Earth's surface along the axis of the valley floor. The approximate form of the expression for $\Delta\partial T/\partial z$ is justified since $B/H >> 1/2$ for the relatively low-relief topography that characterises most of Britain. Since u>>u', this perturbation will be positive; i.e., the terrain will cause the surface heat flow to be greater along the axis of any valley than in surrounding areas, as is expected. Counter-intuitively, perhaps, this perturbation depends only on the aspect ratio of the model valley (i.e., the ratio of depth to half-width, H/B) and on the difference between the (unperturbed) near-surface geothermal gradient and the atmospheric lapse rate.

From equations (2.05) and (2.06), the temperature perturbation at the Earth's surface along the axis of the valley, i.e., at x=0, z= z_s=H, is

$$\Delta T(x=0, z=H) = - H (u - u') \qquad (2.12)$$

and the temperature perturbation at a greater depth beneath the same point, at x=0, z=H+D, is

$$\Delta T(x=0, z=H+D) = \frac{- H^2 \gamma (u - u')}{D + \gamma H} \qquad (2.13)$$

The difference between these temperature perturbations, δT, can thus be determined by subtraction; after several algebraic steps it can be shown that

$$\delta T = \frac{H D (u - u')}{D + \gamma H} \approx \frac{H D (u-u')}{D + B} . \qquad (2.14)$$

The perturbation to the geothermal gradient, spatially averaged between z=H and z=H+D, is thus H (u - u') / (D + B), and the associated perturbation to the heat flow, likewise spatially averaged, is δQ, where

$$\delta Q = \frac{H k (u - u')}{D + \gamma H} \approx \frac{H k (u - u')}{D + B} \qquad (2.15)$$

This latter expression can be used to apply a topographic correction to raw heat flow data obtained from a borehole extending to a depth D beneath the axis of a valley, where the raw heat flow measurement is itself vertically averaged across the depth range of the borehole. Corrections to heat flow can also be calculated where a borehole is to the side of the valley axis and/or the depth range of measurement is less than the overall depth of the borehole, as is the case for most of the examples discussed in the main text.

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Tables

Table 1: Palaeotemperature estimates for Holywell Coombe

Timespan (ka)	Name	T_f (°C)	T_a (°C)	T_m (°C)
0	Present day	4.5	16	10.25
~8 - ~3	Mid Holocene	3	19.5	11.25
~8.2	'8.2 ka event'	3	18	10.5
~10.5 - ~8.2	Early Holocene	3.5	18.5	11
~13 - ~11	Younger Dryas Stadial	-4	16	6
~14.5 - ~13	Lateglacial Interstadial	-1	17	8

Based on the biostratigraphic data in Preece and Bridgland (1998) and in Fig. 3 of Rousseau et al. (1998), with the chronology expressed in terms of calibrated radiocarbon years. T_f and T_a are the February and August mean palaeotemperatures, estimated from Fig. 3 of Rousseau et al. (1998). T_m is the estimated annual mean palaeotemperature, calculated as explained in the text.

Table 2: Model temperature history for Fig. 2

t_2 (ka)	t_1 (ka)	ΔT_o (°C)	Name
0	3.5	0	Present conditions
3.5	7.5	+1.5	Mid Holocene climatic optimum
7.5	11.5	+1	Early Holocene
11.5	12.8	-10	Younger Dryas Stadial
12.8	14.7	-5	Lateglacial Interstadial
14.7	30	-20	Last Glacial Maximum
30	45	-15	earlier MIS 2
45	65	-10	MIS 3
65	75	-20	MIS 4
75	85	-5	MIS 5a
85	95	-10	MIS 5b
95	105	-5	MIS 5c
105	120	-10	MIS 5d
120	130	+5	Ipswichian (MIS 5e)
130	3000	-5	Earlier Pleistocene
3000	65000	+5	Pre-Quaternary

Times t_1 and t_2 represent the start and end of each phase, for which the surface temperature (relative to the present-day value) is assumed to have been ΔT_o. Individual temperature phases are named in the final column of the Table, several being associated with particular Marine oxygen Isotope Stage (MIS) numbers.

Table 3: Model temperature history for Fig. 3

t_2 (ka)	t_1 (ka)	ΔT_o (°C)	Name
0	3.5	0	Present conditions
3.5	7.5	+2	Mid Holocene climatic optimum
7.5	11.5	+0	Early Holocene
11.5	12.8	-8	Younger Dryas Stadial
12.8	14.7	-4	Lateglacial Interstadial
14.7	18	-18	Last Glacial Maximum - after deglaciation
18	30	-8	Last Glacial Maximum - during glaciation
30	45	-14	earlier MIS 2
45	65	-8	MIS 3
65	75	-18	MIS 4
75	120	-6	MIS 5d-5a
120	130	+5	Ipswichian (MIS 5e)
130	3000	-4	Earlier Pleistocene
3000	65000	+5	Pre-Quaternary

Display format is the same as for Table 2.

Table 4: Topographic corrections assuming Lees Valley analytic solutions

Locality	Q_o	u_o	k	D	u_t	Δu_s	Δu_b	Δu_m	ΔQ_m	Q_f
Rookhope	99.6	32.1	3.1	799	29.9	-5.2	-1.7	-3.0	-9.3	90.3
Raydale	71.1	19.5	3.65	601	17.9	-4.7	-1.7	-2.8	-10.2	60.9
Eastgate	115	38.5	2.99	995	34.4	-7.4	-2.4	-4.2	-12.6	102.4
Skiddaw	118.5	33.9	3.5	281	27.6	-9.7	-5.4	-7.2	-25.2	93.3
Ballater	75.6	22.9	3.3	290	21.2	-4.6	-3.2	-3.8	-12.5	63.1
Soussons Wood	123.2	3.95	3.12	100	34.89	-5.6	-3.8	-4.6	-14.4	108.8

Q_o (in mW m^{-2}) is the measured heat flow, u_o (in °C km^{-1}) the corresponding geothermal gradient and k (in W m^{-1} °C^{-1}) the corresponding thermal conductivity (with $Q_o = k \times u_o$). D is the depth of the borehole (in m). u_t (°C km^{-1}) is the 'target' geothermal gradient, the estimate of the regional geothermal gradient relative to which the topographic correction is derived. Δu_s is the topographic correction to the geothermal gradient at the Earth's surface on the axis of the Lees Valley (i.e., at x=0, $z=z_s=H$), derived from equation (2.10). Δu_b is the topographic correction to the geothermal gradient at the bottom of the borehole on the axis of the Lees Valley (i.e., at x=0, z=H+D), also derived from equation (2.10). Δu_m and ΔQ_m are the perturbations to the mean geothermal gradient and mean heat flow, beneath the axis of the Lees Valley, spatially averaged between $z=z_s=H$ and z=H+D, calculated using equation (2.15). Q_f is the corrected heat flow, calculated as $Q_o + \Delta Q_m$. Sources of data are cited in the text, although it should be noted that different references often quote slightly different values for a given measurement from a given borehole. Parameters for the Soussons Wood borehole are listed after removal of the 9 mW m^{-2} palaeoclimate correction that was applied by Burley et al. (1984). All solutions assume a value of 5 °C km^{-1} for u', the atmospheric lapse rate.

Table 5: Comparison of topographic corrections with previous work

	Previous work				This study		
Locality	Q_o	ΔQ_T	Q_f	Ref.	Q_o	ΔQ_T	Q_f
Rookhope	99.5	-4.2	95.3	[1]	99.6	-6.9	92.7
Raydale	71.1	-6.3	64.9	[1]	71.1	-5.7	65.4
Eastgate	-	-	-	-	115.0	-12.0	103.0
Skiddaw	118.5	-17.6	100.9	[2]	118.5	-21.8	96.7
Ballater	75.6	-4.2	71.4	[2]	75.6	-5.7	69.9
Soussons Wood	-	-	-	-	123.2	-9.1	114.1

Localities, notation and units are as for Table 4. References for previous work are: [1], Bloomer et al. (1979); and [2], Lee (1986). Parameters for the Soussons Wood borehole are listed after removal of the 9 mW m^{-2} palaeoclimate correction that was applied by Burley et al. (1984). Note that in the present study the preferred corrections to heat flow listed here correspond to the depth ranges over which heat flow has previously been measured and the horizontal position of each borehole, and so they may differ from the values listed in Table 4 that relate to ground level and the bottom of each borehole and were calculated as if the borehole were located on the axis of the fitted Lees Valley.

Table 6: Combined topographic and palaeoclimate corrections

Borehole	Q_o (mW m^{-2})	z_1 (m)	z_2 (m)	k (W m^{-1} °C^{-1})	ΔQ_T (mW m^{-2})	ΔQ_p (mW m^{-2})	Q_c (mW m^{-2})
Rookhope	99.6	390	789	3.1	-6.9	15.5	108.2
Raydale	71.1	525	590	3.65	-5.7	19.3	84.7
Eastgate	115.0	0	995	2.99	-12.0	13.4	116.4
Skiddaw	118.5	100	281	3.5	-21.8	20.8	117.5
Ballater	75.6	100	290	3.3	-5.7	19.6	89.5
Soussons Wood	123.2	0	100	3.12	-9.1	28.2	142.3

Q_o is raw heat flow, measured between depths z_1 and z_2, in rock of thermal conductivity k. ΔQ_T and ΔQ_p are the corrections to heat flow for topography and palaeoclimate, $Q_c = Q_o + \Delta Q_T + \Delta Q_p$ being the corrected heat flow. Values of Q_o, z_1, z_2, k, and ΔQ_T are the same as in Table 5. Values of ΔQ_p are calculated from the climatically-perturbed geotherms predicted in section 2 (based on Fig. 2 for the Soussons Wood borehole and Fig. 3 for the others) using equation (1) with the appropriate values of z_1, z_2 and k. As is discussed in the main text (section 4) these corrections may well be conservative.

Figure Captions

Figure 1. Location map showing heat flow measurement sites and Pleistocene sites discussed in the text, modified after Fig. 1 of Lee et al. (1987). The Pleistocene sites are identified thus: BB, Bishopbriggs; HC, Holywell Coombe; HW, Hawes Water; LF, Lynford; LV, Lea Valley; TE, Teindland; TH, Tolsta Head; and WB, Whitrig Bog. VY and NN denote the Vale of York and North Norfolk, other localities mentioned in the text. Interpreted heat flow contours in mW m^{-2}, based mostly on raw measurements, are from Lee et al. (1987); see Fig. 6 of Busby et al. (2011) for a colour-shaded map of the same heat flow variations. The grid indicates boundaries of 100 km $\times$ 100 km quadrangles of the British National Grid, whose letter designations are used for location purposes in the text. The inset shows the principal Palaeozoic granite batholiths in Britain, within which the highest heat flow measurements are obtained. Letters adjoining SW England on the main map indicate the principal onshore plutons of the Cornubian Batholith; from west to east these are: L, the Land's End granite; C, the Carnmenellis granite; A, the St Austell granite; B, the Bodmin Moor granite (all in Cornwall); and D, the Dartmoor granite (in Devon).

Figure 2. (a) Assumed temperature history (also tabulated in Table 2), to represent conditions in southern England (present-day surface temperature ~10 °C) during the Late Pleistocene. (b) Output of the resulting perturbations to the present-day geotherm (δT), the geothermal gradient ($\partial \delta T/\partial z$), and the heat flow (expressed both as $\delta Q \equiv k\ \partial \delta T/\partial z$, the heat flow perturbation at depth z, and as $\delta Q_m(z)$, the mean perturbation to the heat flow between z=0 and depth z, calculated using equation (1)). Calculations assume k=3 W m^{-1} °C^{-1} and κ=1 mm^2 s^{-1}, and have been carried out over a succession of depth intervals spaced by Δz=10 m.

Figure 3. (a) Assumed temperature history (also tabulated in Table 3), representative of conditions in northern England and eastern Scotland (present-day surface temperature ~8 °C) during the Late Pleistocene. (b) Output of the resulting perturbations to the present-day geotherm, geothermal gradient, and heat flow. With minor exceptions, values of ΔT_o have been determined for the same surface temperatures as for Fig. 2, except that during 30-18 ka it has been assumed that the Earth's surface was covered by a thick enough ice sheet that the insulation effect resulted in a temperature at the bedrock surface 10 °C higher. Apart from the different temperature history, calculations represent the same parameters and are based on the same input data as for Fig. 2.

Figure 4. Valley cross-section at Bayhorse, Idaho, showing the idealised cross-sectional profile of a Lees Valley with H=850 m, z_o=2850 m a.s.l., and B=1000 m (solid line) fitted to the topographic dataset in Fig. 8 of Blackwell et al. (1980) ($\times$ symbols), with the horizontal co-ordinate x measured from zero at a point 1.9 km across the original profile. Blackwell et al. (1980) predicted that the geothermal gradient in the absence of topography would be 20 °C km^{-1} but is 1.6 times this value (or 32 °C km^{-1}) at the Earth's surface in the valley floor and 1.3 times this value (or 26 °C km^{-1}) at a point 250 m deeper. Assuming that the thermal conductivity of the bedrock is 3 W m^{-1} °C^{-1}, a 250 m deep borehole in this locality would indicate a mean geothermal gradient of ~1.45 $\times$ 20 °C km^{-1} or 29 °C km^{-1} and thus a heat flow of 87 mW m^{-2}, whereas the 'true' regional heat flow in this area is expected to be 20°C km^{-1} $\times$ 3 W m^{-1} °C^{-1}, or 60 mW m^{-2}. The local topographic correction to heat flow, to recover the regional value from measurements in such a hypothetical borehole, would thus be 27 mW m^{-2}.

Figure 5. Fit of a Lees Valley to a SW-NE topographic profile across the Rookhope Burn valley in County Durham. This profile extends between National Grid references NY 920 410 and NY 960 450, passing through the borehole at NY 938 428 (at the axis of the valley, at x=0), ~2.5 km from the SW end of the topographic profile. This Lees Valley fit has H=230 m, z_o=560 m O.D., and B=1100 m. Display format is the same as for Fig. 4; labelling of points that lie on the specified topographic profile uses more prominent ornament and that of points adjoining it (e.g., at summits) uses fainter ornament. Note that the Rookhope Burn valley has a ~90° bend, from west-east to north-south, in the vicinity of this profile, so the topography is not two-dimensional; the valley sides nonetheless provide a reasonable approximation to a Lees Valley.

Figure 6. Fit of a Lees Valley to a NW-SE topographic profile across the Raydale valley in North Yorkshire. This profile extends between SD 8765 8735 and SD 9265 8235, passing through the valley axis at SD 9005 8495 (x=0) and the Raydale borehole at SD 903 847 (x≈0.35 km), ~3.75 km from the NW end of the topographic profile. This Lees Valley fit has H=330 m, z_o=590 m O.D., and B=900 m, and is plotted using the same display format as for Fig. 5 but with the off-axis location of the borehole also shown. Irregularities in the observed topographic profile are caused by tributary valleys, and are smoothed out by the Lees Valley fit.

Figure 7. Fit of a Lees Valley to a NNW-SSE topographic profile across Weardale in County Durham. This profile extends between NY 918 414 and NY 951 348, passing through the valley axis at NY 934 402 (x=0) and making its closest approach to the Eastgate borehole (which is at NY 93890 38200) at NW 9350 3795 (x≈0.2 km), ~3.85 km from the NNW end of the topographic profile. The Lees Valley fit has H=330 m, z_o=580 m O.D., and B=1300 m, and is plotted using the same display format as for Fig. 6.

Figure 8. Fit of a Lees Valley to a NW-SE topographic profile across the Burdell Gill valley in Cumbria. This profile extends between NY 2980 3320 and NY 3315 2985, passing through the valley axis at NY 3155 3145 (x=0) and making its closest approach to the Burdell Gill (Skiddaw) borehole (which is at NY 314 314) at NY 3150 3150 (x≈-0.1 km), ~2.4 km from the NW end of the topographic profile. The Lees Valley fit has H=350 m, z_s=680 m O.D., and B=800 m, and is plotted using the same display format as for Fig. 6.

Figure 9. Fit of a Lees Valley to a north-south topographic profile across the Dee valley near Ballater in Aberdeenshire, Scotland. This profile extends between NJ 400 020 and NO 400 925, passing through the valley axis at NO 400 978 (x=0) and making its closest approach to the Ballater (Tomnakeist) borehole (which is at NO 401 986) at NO 400 986 (x≈-0.8 km), ~3.4 km from the northern end of the topographic profile. The Lees Valley fit has H=400 m, z_s=570 m O.D., and B=1400 m, and is plotted using the same display format as for Fig. 6. The borehole is plotted at x=-800 m with its wellhead at ~225 m O.D. corresponding to z_s ~345 m, such that the 100-290 m depth range across which the heat flow has been measured (depicted with a thicker line) corresponds to z ~445-635 m.

Figure 10. Fit of a Lees Valley to a WNW-ESE topographic profile through the Soussons Wood area of Dartmoor, Devon. This profile extends between SX 6640 7995 and SX 6780 7960, passing through the valley axis at SX 6715 7980 (x=0) and the Soussons Wood borehole at SX 6733 7971 ($x \approx 0.2$ km), ~1 km from the WNW end of the topographic profile. The Lees Valley fit has H=85 m, z_o=440 m O.D., and B=450 m, and is plotted using the same display format as for Fig. 6.

Figure 11. Comparison of corrected and uncorrected heat flow data for sites discussed in the text (plotting data listed in Table 6 and in section 4 of the text). For each site, dashed lines indicate the difference between corrected and uncorrected values; the diagonal dotted line corresponds to both values being equal. The two points for the Balfour borehole are derived from the two depth ranges discussed in the text.

Figure 12. (a) Assumed temperature history, representative of conditions in eastern Scotland (present-day surface temperature ~8 °C) during the Late Pleistocene, subject to the assumption that this region was glaciated during MIS 4. This assumed temperature history the same as in Fig. 3 and Table 3, except a surface temperature anomaly of -8 °C has been assumed during 75-65 ka to represent the MIS 4 glaciation. (b) Output of the resulting perturbations to the present-day geotherm, geothermal gradient, and heat flow. Apart from the different temperature history, calculations represent the same parameters and are based on the same input data as for Fig. 2.