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Characterization and ply mixing rules for non-symmetric forms of fully orthotropic laminates.

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Stacking sequence listings are presented for fully orthotropic angle-ply laminates, with up to 21 plies, together with rules for mixing these sequences to form laminates containing any number of plies. The mixing rules are demonstrated through an abridged set of sequences, which are characterized in terms of angle- and cross-ply sub-sequence symmetries. The abridged set of sequences is derived from a new definitive list that supersedes previously published listings. Stacking sequences are presented together with dimensionless parameters from which the bending stiffness terms are readily calculated and an assessment of the bending stiffness efficiency made for angle- and cross-ply sub-sequences. Expressions relating the dimensionless parameters to the well-known lamination parameters are also given, together with graphical representations of feasible domains for all sub-sequence symmetries contained in the definitive list. Feasible domains for extensionally isotropic and fully isotropic laminates are also presented as important sub-sets of fully orthotropic laminates. Finally, examples are given for tapered laminates with fully orthotropic properties, derived from compatible sequences in the definite list.

Nomenclature

$\mathbf{A}$,A_{ij}	=	extensional (membrane) stiffness matrix and its elements (i,j = 1, 2, 6)
$\mathbf{B}$,B_{ij}	=	bending-extension-coupling stiffness matrix and its elements (i,j = 1, 2, 6)
$\mathbf{D}$,D_{ij}	=	bending (flexural) stiffness matrix and its elements (i,j = 1, 2, 6)
H	=	laminate thickness ($= n \times t$)
n	=	number of plies in laminate stacking sequence
Q_{ij}	=	reduced stiffness (i,j = 1, 2, 6)
Q'_{ij}	=	transformed reduced stiffness (i,j = 1, 2, 6)
t	=	ply thickness
ξ_1^A,ξ_2^A	=	lamination parameters for extensional stiffness
ξ_1^D,ξ_2^D	=	lamination parameters for bending stiffness ($\xi_1^D = \xi_9$, $\xi_2^D = \xi_{10}$)
ζ,$\zeta_\pm$,$\zeta_\circ$,$\zeta_\bullet$	=	bending stiffness parameter for laminate, and angle-ply and cross-ply sub-sequences
+,−,±	=	angle plies, used in stacking sequence definition
○,●	=	cross-plies, used in stacking sequence definition

Matrix sub-scripts

0	=	All elements zero
F	=	All elements Finite
I	=	Isotropic form, see Eqs. (3) – (5)
S	=	Specially orthotropic form, see Eqs. (1) - (2)

Keywords

Stacking Sequences; Fully (or Specially) Orthotropic Laminates; Extensionally (or Membrane or Quasi) Isotropic Laminates; Fully Isotropic Laminates; Bending Stiffness Parameters; Lamination Parameters; Feasible Domains; Laminate Taper.

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I. Introduction

Composite laminate materials are typically characterized in terms of their response to mechanical or thermal loading, which is generally associated with a description of the coupling behavior, unique to this type of material, i.e. coupling between in-plane (extension or membrane) and out-of-plane (bending or flexure) actions, coupling between in-plane shear and extension, and coupling between out-of-plane bending and twisting. One such classification system is offered by the Engineering Sciences Data Unit or ESDU[1], in which the extensional (**A**), coupling (**B**) and bending (**D**) stiffness matrices are used together with an extended subscript notion to describe the form of the elements in each matrix. For instance, balanced and symmetric stacking sequences, which generally possess bending anisotropy, are referred to by the designation $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F$, signifying that the elements of the extensional stiffness matrix (**A**) are specially orthotropic in nature, i.e.

$$A_{16} = A_{26} = 0, \tag{1}$$

the bending-extension coupling matrix (**B**) is null, whilst all elements of the bending stiffness matrix (**D**) are finite.

Laminates possessing extensional anisotropy give rise to coupling between in-plane shear and extension only and, by the same rationale, are referred to by the designation $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$, signifying that all elements of the extensional stiffness matrix (**A**) are finite, the bending-extension coupling matrix (**B**) is null, whilst the elements of the bending stiffness matrix (**D**) are specially orthotropic in nature, i.e.

$$D_{16} = D_{26} = 0 \tag{2}$$

A designation for extensional anisotropy is not listed as part of the ten laminate classifications described in the ESDU data item[1], but is however the subject of a recent article[2], identifying the definite list of $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$ stacking sequences with up to 21 plies, thus complementing a new definitive list[3] of Fully Orthotropic Laminates or **FOL**s. Note that the term **FOL** is synonymous with specially orthotropic laminates, which possess none of the coupling characteristics described above and are represented by the designation $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$.

A related article[4] presents the characterization of Extensionally Isotropic Laminates or **EIL**s, with the designation $\mathbf{A}_I\mathbf{B}_0\mathbf{D}_S$ and Fully Isotropic laminates or **FIL**s, with the designation $\mathbf{A}_I\mathbf{B}_0\mathbf{D}_I$. These laminates represent a sub-set of **FOL**s and are contained within the definitive list[3], since in addition to the specially orthotropic form of each matrix, see Eqs. (1) and (2), elements simply further in **EIL**s and the designation $\mathbf{A}_S$ is replaced with $\mathbf{A}_I$ to indicate that:

$$A_{11} = A_{22} \tag{3}$$

and

$$A_{66} = (A_{11} – A_{12})/2 \tag{4}$$

and further still in **FIL**s, in which the designation $\mathbf{D}_S$ is replaced with $\mathbf{D}_I$ to indicate that:

$$D_{ij} = A_{ij}\, H^2/12, \tag{5}$$

where H is the laminate thickness, corresponding to the total number of plies (n) of thickness (t).

Fully orthotropic laminates minimize distortion during manufacturing and maximize compression buckling strength[5] in comparison to balanced and symmetric laminates, which generally possess flexural anisotropy, and are therefore of continuing interest to industry and the academic community alike. For instance, Valot and Vannucci provide examples of **FOL**s with anti-symmetric sequences[6], following a previous article[7] on **FIL**s. Indeed these two articles are part of, and provide reference to, a growing number of related articles by a community of co-workers addressing the development of laminate stacking sequences for a range of behavioral characteristics, which can be traced back to an original article by Caprino and Crivelli-Visconti[8], identifying the specially orthotropic angle-ply laminate with eight plies. Much of this related work is focused on establishing stacking sequences for **EIL**s, **FIL**s and laminates with material homogeneity. However, with the exception of **FIL**s, these material characterizations generally provide little distinction between laminates with different coupling behavior. For instance, the quasi-homogenous laminate, which is defined as possessing the same elastic propertied in both flexural and membrane actions, satisfying Eq. (5), but with no coupling between these two actions, may be designated as either $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$ or

$\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$, and **EIL**s may be designated as either $\mathbf{A}_I\mathbf{B}_0\mathbf{D}_F$ or $\mathbf{A}_I\mathbf{B}_0\mathbf{D}_S$. Such sequences have been obtained through an inverse polar (or Mohr's circle) representation method developed by Verchery and co-workers; acknowledged[9] to be similar to the method presented previously by Tsai and Pagano[10].

In contrast, original work by Bartholomew[11,12], which forms the basis of the Engineering Sciences Data Unit (ESDU) publication[13] for the so called definitive list of fully orthotropic angle-ply laminates, precedes the findings of Caprino and Crivelli-Visconti[8], but appears to have been completely overlooked in the literature described above. Reference 13 contains 75 symmetric sequences, for laminates with up to 21 plies, and 653 anti-symmetric sequences, for laminates with up to 20 plies, together with 49 additional non-symmetric (asymmetric) sequences, which were derived by combining the symmetric and anti-symmetric sequences. Further inspection reveals that there are no angle-ply laminates possessing specially orthotropic properties with fewer than 7 plies. Indeed, there is but a single generic 7-layer angle-ply anti-symmetric stacking sequence. This number increases to 233 generic anti-symmetric sequences with 20 ply layers. There are no symmetric stacking sequences with less than 12 layers, and only 25 generic combinations with 20 layers. These twenty-five generic stacking sequences possess balanced and symmetric combinations of angle plies, together with cross plies, which may be 0 and/or 90°, symmetrically disposed about the laminate mid-plane; all possess angle-ply layers on the outer surfaces of the laminate. The term 'generic' is used here to describe the form of the stacking sequences adopted, defined by three parameters: +, − and ∗, relating to positive and negative angle plies with general orientation, θ, and cross ply, respectively.

The derivation[11,12] adopted in the ESDU data item[13], made the explicit assumption that cross plies, as well as angle plies, are symmetrically disposed about the laminate mid-plane, i.e. the mixing of 0 and 90° plies is permitted only in one half of the laminate, which is then reflected symmetrically about the laminate mid-plane. This rule applies to both symmetric and anti-symmetric angle-ply stacking sequences. For this reason, cross plies are legitimately represented by the single parameter ∗.

The relatively small number of fully orthotropic sequences for thin laminates clearly leaves limited scope for composite tailoring, particularly where ply terminations are necessary and fully orthotropic characteristics are a design requirement. This was the key motivation leading to the redevelopment of a definitive list[3] for specially orthotropic angle-ply laminates with up to 21 plies, presented in the current article in abridged form.

In the derivation of this list for (but not restricted to) standard angle-ply configurations, i.e. ±45, 0 and 90°, the general rule of symmetry is relaxed. Cross plies, as well as angle plies, are therefore no longer constrained to be symmetric about the laminate mid-plane, leading to an increase in the number of possible solutions. For 16 ply laminates, there are approximately one million possible stacking sequence combinations, of which 368 comply with the requirements of special orthotropy, increasing to approximately 1 billion combinations for 21 plies, giving rise to a hundred-fold increase in the number of fully orthotropic laminates. The scope for composite tailoring is now vastly increased, particularly in the context of fully orthotropic tapered laminates. The numbers of sequences for each ply number grouping are summarized in Table 1, which also provides cross-referencing to the tables of abridged listings that follow. New laminate mixing rules are also presented, which will help the designer develop **FOL**s with any number of plies by combing sequences from the definitive list.

Because of the substantial number of sequences identified in the definitive list, it is beneficial, for design purposes, to express the stiffness properties in terms of lamination parameters, which can be conveniently presented in graphical form, as originally conceived by Fukunaga and Vanderplaats[14] for the purposes of optimum design: the membrane or flexural stiffness terms are now defined by two linear design variables. Optimized lamination parameters may then be matched against a corresponding set of laminate stacking sequences. Graphical representations of feasible domains of lamination parameters are readily extended to other laminate classifications: **EIL**s are reduced to a single line representation, whilst **FIL**s are represented by a single point.

II. Stacking sequence derivation

In the derivation of the stacking sequences that follow, the general rule of symmetry is relaxed. Cross plies, as well as angle plies, are therefore no longer constrained to be symmetric about the laminate mid-plane. Consequently, the mixing of 0 and 90° plies needs special attention to avoid violation of the rules for special orthotropy, see Eqs. (1) - (2). Examples of symmetric angle-ply laminates with non-symmetric cross-ply sub-sequences have been presented previously[15], in the context of composite plate instability, and represent one of the many non-symmetric forms contained in the definitive list of **FOL**s possessing standard angle-ply configurations, e.g. ±45, 0 and 90°, many of which appear both extra-ordinary in appearance and infeasible in terms of the uncoupled behavior that the laminates possess. The great majority of **FOL**s are non-symmetric and many of these are without any sub-sequence patterns, e.g. symmetries or repeating groups, which is contrary to the assumptions on which many previous studies have been based.

Arrangement and form of stacking sequence data

For compatibility with the previously published data, similar symbols have been adopted for defining all stacking sequences that follow. Additional symbols and parameters are necessarily included to differentiate between cross plies (0° and 90°), given that symmetry about the laminate mid-plane is no longer assumed.

The resulting sequences are characterized by sub-sequence symmetries using a double prefix notation, the first character of which relates to the form of the angle-ply sub-sequence and the second character to the cross-ply sub-sequence. The double prefix contains combinations of the following characters: *A* to indicate Anti-symmetric form; *N* for Non-symmetric; and *S* for Symmetric. Additionally, for cross-ply sub-sequence only, *C* is used to indicate Cross-symmetric form.

To avoid the trivial solution of a stacking sequences with cross plies only, all sequences have an angle-ply (+) on one outer surface of the laminate. As a result, the other outer surface may have an angle-ply of equal (+) or opposite (–) orientation or a cross ply (○), which may be either 0 or 90°. A subscript notation, using these three symbols, is employed to deferential between similar forms of sequence.

The form (and number) of the stacking sequences contained in the definitive list[3] for up to 21 plies can therefore be summarized as: *AC* (210), *AN* (14,532), *AS* (21,906), *SC* (12), *SN* (192), *SS* (1,029), $_{+}NS_{+}$ (220), $_{+}NS_{-}$ (296), $_{+}NN_{+}$ (5,498), $_{+}NN_{-}$ (15,188) and $_{+}NN_{\circ} = {}_{+}NN_{\bullet}$ (10,041).

Development of parameters

As adopted in the published ESDU listings[13], the new sequences are ordered in terms of ascending numbers of plies, n, or ζ ($= n^3$), which are in turn ordered by ascending value of the bending stiffness parameter for the angle plies ($\zeta_{\pm}$) and finally by one of the two cross-ply sub-sequences ($\zeta_{\circ}$) within the laminate. Hence, the numbering of sequences for each sub-symmetric form, described in the previous section, may be readily extended.

The calculation of the bending stiffness parameter, $\zeta_{\pm}$, is readily demonstrated for the 7 ply laminate, designated *A381* in the ESDU listings[13] and *AS 2* in the definitive list[3], with stacking sequence $[\pm/-/\circ/+/\pm]_T$, where the bending stiffness terms,

$$D_{ij} = \sum_{k=1}^{n} Q'_{ij}(z_k^3 - z_{k-1}^3)/3 \tag{6}$$

may be written in sequence for the 7 individual plies, where z, representing the distance from the laminate mid-plane, is expressed here in terms of the uniform ply thickness t:

$$D_{ij} = \{Q'_{ij+}((-5t/2)^3 - (-7t/2)^3) + Q'_{ij-}((-3t/2)^3 - (-5t/2)^3) + Q'_{ij-}((-t/2)^3 - (-3t/2)^3) + Q'_{ij\circ}((t/2)^3 - (-t/2)^3) + Q'_{ij+}((3t/2)^3 - (t/2)^3) + Q'_{ij+}((5t/2)^3 - (3t/2)^3) + Q'_{ij-}((7t/2)^3 - (5t/2)^3)\}/3 \tag{7}$$

where subscripts i, j = 1, 2, 6.

The bending stiffness contribution from the angle plies is therefore:

$$D_{ij\pm} = 85.5t^3/3 \times Q'_{ij\pm} \tag{8}$$

and for orthotropic plies:

$$D_{ij\circ} = 0.25t^3/3 \times Q'_{ij\circ} \tag{9}$$

Note that the decoupling of in- and out-of-plane actions for the laminate may be verified in a similar manner from:

$$B_{ij} = \sum_{k=1}^{n} Q'_{ij}(z_k^2 - z_{k-1}^2)/2 = 0 \qquad (10)$$

Bending stiffness terms are written in alternative form in Ref. 13 as:

$$D_{ij\pm} = \zeta_{\pm} t^3/12 \times Q'_{ij\pm} \qquad (11)$$

and

$$D_{ij\circ} = \zeta_{\circ} t^3/12 \times Q'_{ij\circ} \qquad (12)$$

respectively, since the number of plies (n = 7) are now related directly to the bending stiffness terms by the expression:

$$\zeta_{\pm} + \zeta_{\circ} = \zeta = n^3 \qquad (13)$$

where $\zeta_{\pm}$ = 342 and $\zeta = n^3$ = 343.

The stiffness parameters are hereby extended to include both cross plies ($\zeta_{\circ}$ and $\zeta_{\bullet}$), including percentage values to indicate the relative proportion ($n_{\pm}/n$, $n_{\circ}/n$ and $n_{\bullet}/n$) and relative contribution to bending stiffness ($\zeta_{\pm}/\zeta$, $\zeta_{\circ}/\zeta$ and $\zeta_{\bullet}/\zeta$) of each ply sub-sequences within the laminate, i.e. a sub-sequence containing either ±, ○ or ● plies.

Comparison of the relative proportion and the contribution to bending stiffness provides a measure of efficiency of the sub-laminate for each ply orientation, in the same sense that the radius of gyration, relating cross-sectional area and second moment of area, provides as assessment of the geometric efficiency of a beam to resist bending.

Validation

Table 1 provides a summary of the number of fully orthotropic stacking sequences in the definitive list for up to 21 plies together with cross-references to Tables 2-12 of abridged sequences for upper- and lower-bound $\zeta_{\pm}$ values; these bounds have been demonstrated[15] to give upper- and lower-bound closed form buckling solutions respectively in compression loaded rectangular plates. Care must however be exercised when applying this rule to other planform geometries and load combinations.

The new stacking sequences have been validated against the published ESDU listing[13], and reference numbers with prefix designations *A* and *S* are included in Tables 4 and 7, respectively, for cross-referencing purposes. Note that sequences containing only angle plies may share the same value of $\zeta_{\pm}$ and therefore the order of such sequence may be differ from the ESDU listing. For such laminates the second prefix has no significance, but has been given *AS* or *SS* designations, which is justified in the sense that *AS* sequences are of the general form [+/–/–/+/∗/–/+/+/–], and the single *SS* sequence is of the general form $[+/-/-/+/-/+/+/-/*]_S$, where ∗ represents any symmetric cross-ply sequence.

Furthermore, the ESDU data item listed even and odd ply sequences separately, which explains the apparent inconsistency between the two numbering systems. Note also that Table 12 begins with laminate designation *NN 12*, which is explained by the fact that ${}_{+}NN_{\bullet}$ sequences are not shown, since they are identical to the ${}_{+}NN_{\circ}$ listing with ● and ○ cross plies interchanged. The first sequence, i.e. that with lowest $\zeta_{\pm}$ and $\zeta_{\circ}$, is given by sequence *NN 1*: $[+/-/\bullet/\bullet/\bullet/\bullet/-/+/-/\bullet/+/+/\bullet/-/\bullet]_T$, which is of the form ${}_{+}NN_{\bullet}$.

For each unique stacking sequence with the prefix *SN*, denoting <u>S</u>ymmetric angle plies and <u>N</u>on-symmetric cross plies, alternative sequences also exist, since each cross ply in the sub-sequence is interchangeable, ○ with ●, and the order of these alternative non-symmetric sub-sequences are themselves reversible. For each laminate with either the prefix *SC* or *SS*, denoting <u>S</u>ymmetric angle plies with <u>C</u>ross-symmetric or <u>S</u>ymmetric cross plies respectively, an alternative arises again from the interchangeability of each cross ply (○ with ●) in the sub-sequence.

Table 1 - Number of **FOL**s for 7 through 21 ply laminates, corresponding to prefix designations for Anti-symmetric (*A*), Cross-symmetric (*C*), Non-symmetric (*N*) and Symmetric (*S*) ply sub-sequences, listed in abridged form in Tables 2 - 12. Subscripts arranged before and after the prefix designations denote angle (+, –) and cross plies (○ = 0 or 90°), and correspond to the orientations of the top and bottom plies of the laminate respectively. Numbers given in brackets represent those listed in Ref. 13.

Prefix:	Number of plies, *n*															Table:
	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
AC	-	-	-	-	-	-	-	-	24	12	84	-	-	90	-	2
AN	-	-	-	-	-	-	-	-	24	-	232	132	1984	920	11240	3
AS	2	1	6	6	24	21	84	76	288	268	1002	934	3512	3290	12392	4
(*A*)	(1)	(1)	(2)	(3)	(4)	(6)	(10)	(14)	(26)	(34)	(65)	(89)	(165)	(233)	(–)	
SC	-	-	-	-	-	-	-	-	-	2	4	-	-	6	-	5
SN	-	-	-	-	-	-	-	-	-	-	8	8	64	88	40	6
SS	-	-	-	-	-	4	-	12	4	33	50	110	120	352	344	7
(*S*)						(1)		(3)	(1)	(6)	(4)	(13)	(6)	(25)	(16)	
$_{+}NS_{+}$	-	-	-	-	-	-	-	-	-	2	12	8	64	94	40	8
$_{+}NS_{-}$	-	-	-	-	-	-	-	-	-	-	-	16	24	88	168	9
$_{+}NN_{+}$	-	-	-	-	-	-	-	-	4	4	136	120	476	788	3970	10
$_{+}NN_{-}$	-	-	-	-	-	-	-	-	-	24	116	124	1444	1372	12108	11
$_{+}NN_{\bigcirc}$	-	-	-	-	-	-	-	-	16	15	188	151	878	838	7955	12

By contrast, the sequences presented in Refs 6, 7, 9 and 16 have been algorithmically filtered to provide mathematically and mechanically unique sequences. Applying this filtering to the 18 **FIL**s, with π/3 isotropy, identified as a sub-set of **FOL**s, which are of the form (and number) $_{+}NN_{+}$ (2), $_{+}NN_{-}$ (8) and $_{+}NN_{\bigcirc}$ (8), and correspond to sequences numbers *NN* 1071 – *NN* 1088 in the definitive list[3], reveals: only 1 of the 2 $_{+}NN_{+}$ sequences is unique when the order is reversed; only 4 from the 8 $_{+}NN_{-}$ sequences are unique when the order is reversed and + and – plies are inter-changed; and $_{+}NN_{\bigcirc}$ sequences are identical to $_{+}NN_{-}$ sequences with – and ○ plies inter-changed, leaving the 5 mathematically and mechanically unique sequences identified in Ref. 7, i.e.:

$$[\pm/-/\bigcirc_3/+_2/\bigcirc/\mp/\pm/-_2/\bigcirc_2/+]_T \qquad (NN\ 1071)$$

$$[+/\bigcirc/-_2/\bigcirc/+/\bigcirc/\mp/\pm/\bigcirc/\mp/\bigcirc_2/\pm]_T \qquad (NN\ 1073)$$

$$[+/\bigcirc/-/\bigcirc/\mp_2/\bigcirc/-/\bigcirc/+/\pm/\bigcirc_2/\pm]_T \qquad (NN\ 1074)$$

$$[+/\bigcirc_2/-_2/\pm/\mp/+/\bigcirc_3/\pm/\bigcirc/\pm]_T \qquad (NN\ 1075)$$

$$[+/\bigcirc/-/\bigcirc/\mp_2/\bigcirc/\mp/\bigcirc_2/\mp/+/\bigcirc/-]_T \qquad (NN\ 1077)$$

in which +/–/○ represent 60/-60/0°, respectively, or indeed any angle combination with π/3 separation.

Sequence *AC* 127, given in Table 2, also has special significance: It possesses quasi-homogeneous properties and verifies the only sequence containing all four (+/ –/○/●) ply orientations, presented in Ref 16 for a 20 ply laminate satisfying Eqs. (3) and (5), but not Eq. (4). Note that *AC* 127 and *AC* 128 are identical when the cross ply is interchanged, ○ with ●.

Clearly, filtering for mathematically unique sequences is inappropriate in the context of structural design since interchanging – and ○ plies has a significant effect of the structural response of the laminate and any database of laminate stacking sequences therefore needs to contain both sets of sequences. The same argument can be applied to mechanical uniqueness, in the sense that the through thickness stress distribution, resulting from a flexural action, is very different if the upper surface of a non-symmetric laminate is confused with the lower surface.

III. Calculation of membrane and bending stiffness terms

The calculation procedure for the elements (A_{ij} and D_{ij}) of the extensional (**A**) and bending (**D**) stiffness matrices, using the dimensionless parameters provided in Tables 2-12, are as follows:

$$A_{ij} = \{n_{\pm}/2 \times Q'_{ij+} + n_{\pm}/2 \times Q'_{ij-} + n_{\circ}Q'_{ij\circ} + n_{\bullet}Q'_{ij\bullet}\} \times t \tag{14}$$

$$D_{ij} = \{\zeta_{\pm}/2 \times Q'_{ij+} + \zeta_{\pm}/2 \times Q'_{ij-} + \zeta_{\circ}Q'_{ij\circ} + \zeta_{\bullet}Q'_{ij\bullet}\} \times t^3/12 \tag{15}$$

The form of Eqs. (14) and (15) was chosen because they are then readily modified to account for laminates with extensional and bending anisotropy by replacing $n_{\pm}/2 \times Q'_{ij}+$ with $n_{\pm}(n_+/n_{\pm})Q'_{ij+}$ and $n_{\pm}/2 \times Q'_{ij-}$ with $n_{\pm}(1 - n_+/n_{\pm})Q'_{ij-}$, and $\zeta_{\pm}/2 \times Q'_{ij+}$ with $\zeta_{\pm}(\zeta_+/\zeta_{\pm}) \times Q'_{ij+}$ and $\zeta_{\pm}/2 \times Q'_{ij-}$ with $\zeta_{\pm}(1 - \zeta_+/\zeta_{\pm}) \times Q'_{ij-}$. The use of these modified equation requires the calculation of an additional stiffness parameter, n_+ and ζ_+, relating to the extensional and bending stiffness contribution of positive (θ) angle plies, respectively.

The transformed reduced stiffness terms in Eqs. (14) and (15) are given by:

$$Q'_{11} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\cos^2\theta\sin^2\theta + Q_{22}\sin^4\theta$$

$$Q'_{12} = Q'_{21} = (Q_{11} + Q_{22} - 4Q_{66})\cos^2\theta\sin^2\theta + Q_{12}(\cos^4\theta + \sin^4\theta)$$

$$Q'_{16} = Q'_{61} = \{(Q_{11} - Q_{12} - 2Q_{66})\cos^2\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^2\theta\}\cos\theta\sin\theta$$

$$Q'_{22} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\cos^2\theta\sin^2\theta + Q_{22}\cos^4\theta$$

$$Q'_{26} = Q'_{62} = \{(Q_{11} - Q_{12} - 2Q_{66})\sin^2\theta + (Q_{12} - Q_{22} + 2Q_{66})\cos^2\theta\}\cos\theta\sin\theta$$

$$Q'_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\cos^2\theta\sin^2\theta + Q_{66}(\cos^4\theta + \sin^4\theta) \tag{16}$$

and the reduced stiffness terms by:

$$Q_{11} = E_1/(1 - \nu_{12}\nu_{21})$$

$$Q_{12} = \nu_{12}E_2/(1 - \nu_{12}\nu_{21}) = \nu_{21}E_1/(1 - \nu_{12}\nu_{21})$$

$$Q_{22} = E_2/(1 - \nu_{12}\nu_{21})$$

$$Q_{66} = G_{12} \tag{17}$$

For optimum design of angle-ply laminates, lamination parameters are often preferred, since these allow the stiffness terms to be expressed as linear variables. The optimized lamination parameters may then be matched against a corresponding set of laminate stacking sequences. In the context of the parameters presented in the current article, the four lamination parameters are related through the following expressions:

$$\xi_1^A = \xi_1 = \{n_{\pm}(1 - n_+/n_{\pm})\mathrm{Cos}(2\theta_+) + n_{\pm}(n_+/n_{\pm})\mathrm{Cos}(2\theta_-) + n_{\circ}\mathrm{Cos}(2\theta_{\circ}) + n_{\bullet}\mathrm{Cos}(2\theta_{\bullet})\}/n$$

$$\xi_2^A = \xi_2 = \{n_{\pm}(1 - n_+/n_{\pm})\mathrm{Cos}(4\theta_+) + n_{\pm}(n_+/n_{\pm})\mathrm{Cos}(4\theta_-) + n_{\circ}\mathrm{Cos}(4\theta_{\circ}) + n_{\bullet}\mathrm{Cos}(4\theta_{\bullet})\}/n \tag{18}$$

and

$$\xi_1^D = \xi_9 = \{\zeta_{\pm}(1 - \zeta_+/\zeta_{\pm})\mathrm{Cos}(2\theta_+) + \zeta_{\pm}(\zeta_+/\zeta_{\pm})\mathrm{Cos}(2\theta_-) + \zeta_{\circ}\mathrm{Cos}(2\theta_{\circ}) + \zeta_{\bullet}\mathrm{Cos}(2\theta_{\bullet})\}/\zeta$$

$$\xi_2^D = \xi_{10} = \{\zeta_{\pm}(1 - \zeta_+/\zeta_{\pm})\mathrm{Cos}(4\theta_+) + \zeta_{\pm}(\zeta_+/\zeta_{\pm})\mathrm{Cos}(4\theta_-) + \zeta_{\circ}\mathrm{Cos}(4\theta_{\circ}) + \zeta_{\bullet}\mathrm{Cos}(4\theta_{\bullet})\}/\zeta \tag{19}$$

where the bending stiffness parameter $\zeta_{+} = \zeta_{\pm}/2$ for **FOL**s, **EIL**s and **FIL**s, hence Eqs. (19) reduce to:

$$\xi_1^D = \xi_9 = \{\zeta_{\pm}\text{Cos}(2\theta_{\pm}) + \zeta_{\circ}\text{Cos}(2\theta_{\circ}) + \zeta_{\bullet}\text{Cos}(2\theta_{\bullet})\}/\zeta$$

$$\xi_2^D = \xi_{10} = \{\zeta_{\pm}\text{Cos}(4\theta_{\pm}) + \zeta_{\circ}\text{Cos}(4\theta_{\circ}) + \zeta_{\bullet}\text{Cos}(4\theta_{\bullet})\}/\zeta \qquad (20)$$

Elements of the extensional and bending stiffness matrices are related to the lamination parameters, respectively, by:

$$A_{11} = \{U_1 + \xi_1 U_2 + \xi_2 U_3\} \times H$$

$$A_{12} = A_{21} = \{-\xi_2 U_3 + U_4\} \times H$$

$$A_{22} = \{U_1 - \xi_1 U_2 + \xi_2 U_3\} \times H$$

$$A_{66} = \{-\xi_2 U_3 + U_5\} \times H \qquad (21)$$

and

$$D_{11} = \{U_1 + \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12$$

$$D_{12} = \{U_4 - \xi_{10} U_3\} \times H^3/12$$

$$D_{22} = \{U_1 - \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12$$

$$D_{66} = \{-\xi_{10} U_3 + U_5\} \times H^3/12 \qquad (22)$$

where the laminate invariants are given in terms of the reduced stiffnesses of Eq. (17) by:

$$U_1 = \{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}\}/8$$

$$U_2 = \{Q_{11} - Q_{22}\}/2$$

$$U_3 = \{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}\}/8$$

$$U_4 = \{Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}\}/8$$

$$U_5 = \{Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}\}/8 \qquad (23)$$

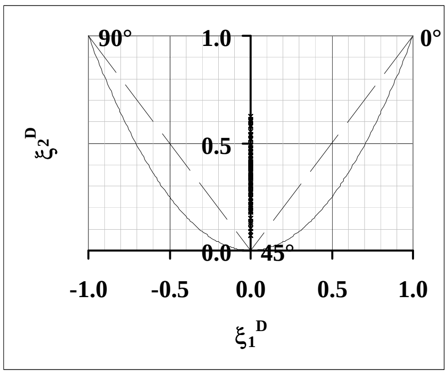
In Ref. 14, a modified set of lamination parameters were adopted for the purposes of optimum design, since bounds are then defined by the parabola $\xi_{10} = \xi_9^2$ with limits $-1 \le \xi_9 \le 1$ and $0 \le \xi_9^2$, hence Eqs. (20) may be re-written as:

$$\xi_1^D = \xi_9 = \{\zeta_{\pm}\text{Cos}(2\theta_{\pm}) + \zeta_{\circ}\text{Cos}(2\theta_{\circ}) + \zeta_{\bullet}\text{Cos}(2\theta_{\bullet})\}/\zeta$$

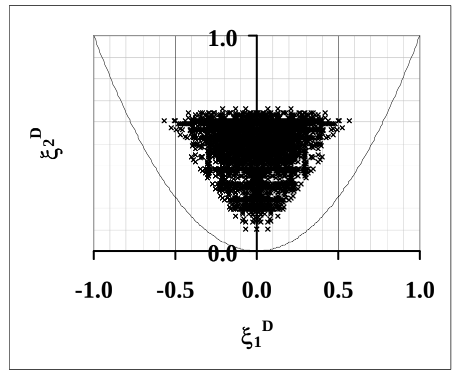
$$\xi_2^D\ (= \xi_{10}) = \{\zeta_{\pm}\text{Cos}^2(2\theta_{\pm}) + \zeta_{\circ}\text{Cos}^2(2\theta_{\circ}) + \zeta_{\bullet}\text{Cos}^2(2\theta_{\bullet})\}/\zeta \qquad (24)$$

Figure 1(a) – (k) illustrates the feasible domains of lamination parameters, in the form of Eqs. (24), for all **FOL**s from the definitive list, in which the +/–/○/● of Tables 2 - 12 correspond to 45/-45/0/90°, respectively. Additionally Fig. 1(l) provides the feasible domain for the corresponding sub-set of **EIL**s with π/3 isotropy in which +/–/○ now represent 60/-60/0°, respectively. Stacking sequences lying along the broken line drawn between (ξ_1^D, ξ_2^D) = (0, 0) and (1, 1) on Fig. 1(a), consist of 0 and ±45° plies only, whereas those along the line between (0, 0) and (-1, 1) consist of ±45 and 90° plies only. Similarly any stacking sequences corresponding to the points (-1, 1), (0, 0) or (1, 1), would respectively contain 90° plies, ±45° plies or 0° plies only. In Fig. 1(l), any sequence corresponding to point (-0.5, 0.25) would contain ±60° plies only. Finally, the corresponding sub-set of **FIL**s, with π/3 isotropy, identified in the previous section, correspond to the point (0, 0.5).

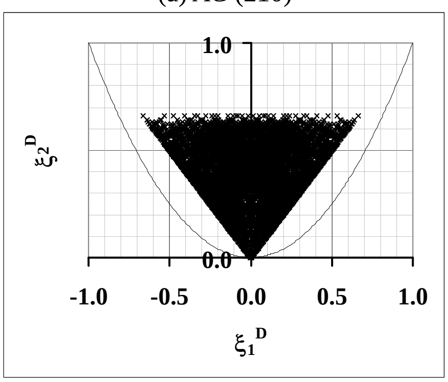
All feasible domains of **FOL**s have symmetry about the vertical (ξ_2^D) axis, with the exception of Fig. 1(k), in which there is a bias towards the right-hand region of the graph. This is explained by the fact that $_{+}NN_{\circ}$ laminates possess an angle ply on one surface and a cross-ply the other, hence cross-plies have a significant effect on the overall bending stiffness, causing the bias towards the top right-hand corner of the feasible domain, representing a laminate with 0° plies only. The feasible domain for $_{+}NN_{\bullet}$ laminates is the mirror image, with a bias towards the left-hand region of the graph.



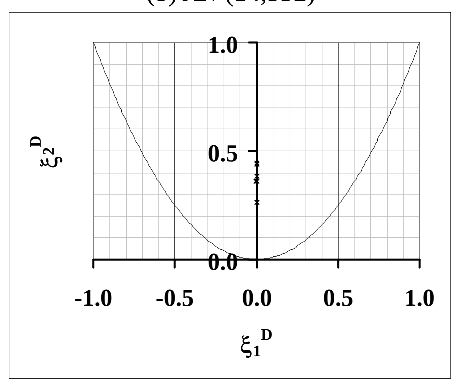
(a) *AC* (210)



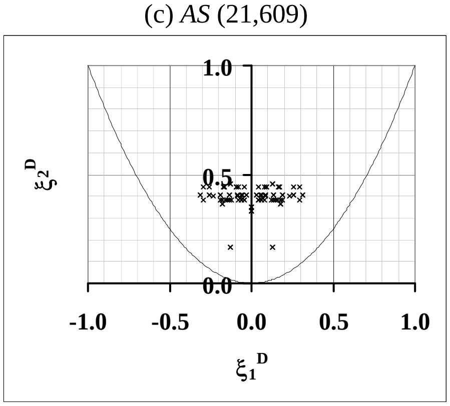
(b) *AN* (14,532)



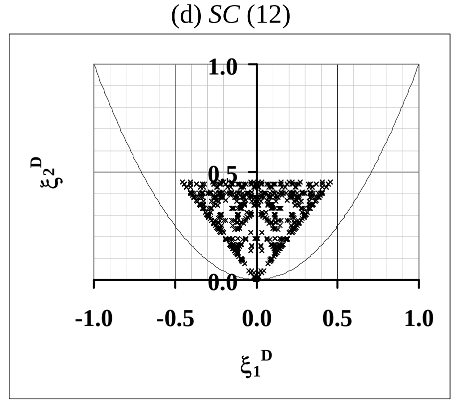
(c) *AS* (21,609)



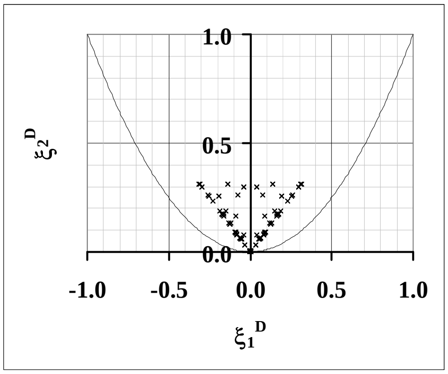
(d) *SC* (12)



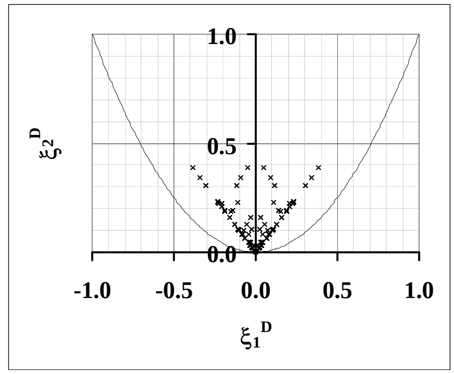
(e) *SN* (192)



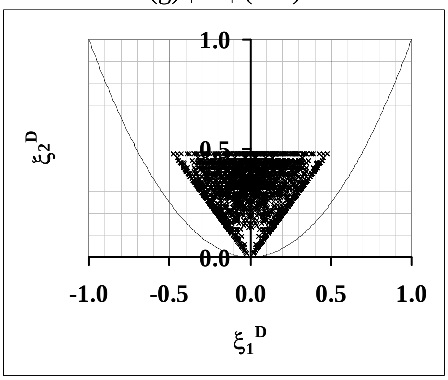
(f) *SS* (1,029)



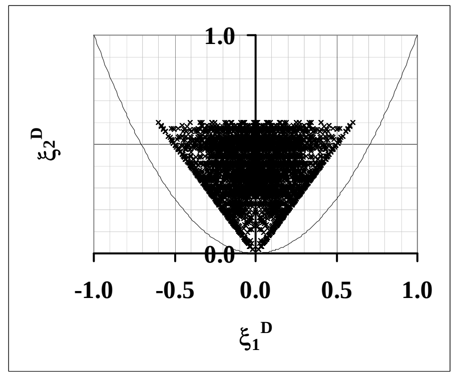
(g) $_+NS_+$ (220)



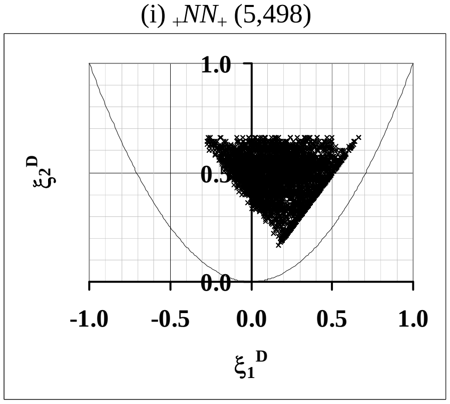
(h) $_+NS_-$ (296)



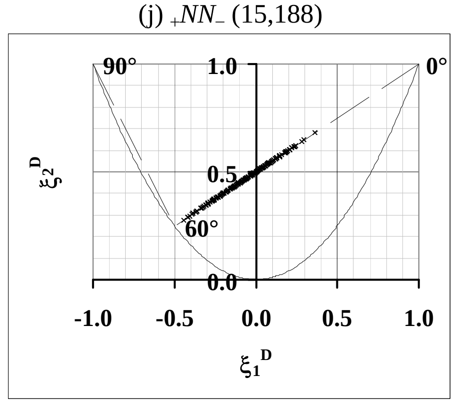
(i) $_+NN_+$ (5,498)



(j) $_+NN_-$ (15,188)



(k) $_+NN_\circ$ (10,041)



(l) **EIL**s

Figure 1 – Feasible domains of lamination parameters, in the form of Eqs. (24), for: (a) – (k) **FOL**s, including form and number of sequences represented and; (l) the sub-set of $\pi/3$ **EIL**s from the definitive list of **FOL**s.

IV. Mixing rules for combining stacking sequences

Simple mixing rules are presented in this section by which **FOL**s from the definitive list may be combined to provide new **FOL**s with any number of plies.

Caprino and Crivelli-Visconti[8] demonstrated that a fully orthotropic angle-ply laminate could be obtained by stacking two balanced and symmetric semi-laminates, i.e. $[\bigcirc/+/-]_S$ and $[\bigcirc/-/+]_S$, to give $[\bigcirc/+/-/-/+/\bigcirc]_A$, the simplest form of which is $[(\pm)_S/(\mp)_S]$, and corresponds to the 8-ply laminate *AS 3* given in Table 4.

Bartholomew[12] identified 7 **FOL**s with asymmetric form by combining symmetric and anti-symmetric sequences using a method referred to as interlocking, in which the cross-plies of one compatible sequence are substituted by the corresponding angle-plies from another, e.g. combining *AS 43* and *SS 36*, from Tables 4 and 7 respectively, produces laminate *NS 2* from Table 8:

AS43   + ○ – ○ – – + + ○ + ○ –

SS36   + – ○ – ○ + ○ ○ ○ ○ + ○ – ○ – +

NS2   + – + – – + – – + + + + – – – +

This technique is also applied in the ESDU data item[13], in which listings of interlocking sequences are presented. The listings reveal that there are no asymmetric sequences with less than 16 plies; the single asymmetric sequence corresponds to *NS 2*, derived above. Additionally, the Tables reveal that there are 6 and 28 sequences with 18 and 20 ply asymmetric sequences, respectively; errors in the referencing of interlocking sequences with odd numbers of layers make the exact number of asymmetric sequences, amounting to 2, 7 and 5 sequences with 17, 19 and 21 plies respectively, difficult to verify independently[†].

Interlacing of laminate stacking sequences

A more intuitive method for combining laminates is now demonstrated, involving the interlacing of individual plies of one laminate with those of another. In this way two laminates may be combined: one with n plies, the other with $n + 1$ plies, e.g. sequences *AS 1* and *AS 3* give rise to sequence *AS 504* from Table 4:

AS1   + – – ● + + –

AS504   + + – – – – + ● – + + + + – –

AS3   + – – + – + + –

Interlacing can also be applied to sequences with equal numbers of plies, but only for *AS* and *SS* sequences or combinations thereof; the new sequence being of the same form or non-symmetric in the latter, i.e. *AS 40* and *SS 1* of Tables 4 and 7 respectively, give rise to a 24-ply sequence of the form ${}_{+}NS_{-}$ (cf. Table 9):

SS1   + – – ● + ● ● + ● – – +

  + + – ● – – ● ● + – ● – ● + + + ● ● – + – ● + –

AS40   + ● – ● – – + + ● + ● –

The method is generally not applicable for combining Non-symmetric or Cross-symmetric sequences with equal numbers of plies, although exceptions to this rule have been identified.

Clearly interlacing gives rise almost exclusively to sequences with odd numbers of plies, given that the technique is only universally applicable to **FOL**s with odd- and even-ply combinations. This leads to laminates with ply number groupings greater than or equal to 15, as derived above from *AS 1* and *AS 3* with 7 and 8 plies respectively. Hence, neither interlacing nor interlocking can account for the non-symmetric 15-ply laminates listed in Tables 10 and 12.

[†] The error in Ref. 13 relates specifically to the first 14 lines of Table 9.7, where *S382*, *S386*, *S392*, *S395*, *S396*, *S398* and *S403* should read *S50*, *S54*, *S60*, *S63*, *S64*, *S66* and *S71*, respectively.

The final example demonstrates the interlacing of non-symmetric laminates of the form ${}_{+}NN_{\bigcirc}$ with n plies and ${}_{+}NN_{-}$ with (n + 1) plies, i.e. the 15-ply sequence *NN 12* and 16-ply sequence *NN 67* from Tables 12 and 11, respectively. Together they form a 31-ply **FOL** of the form ${}_{+}NN_{-}$:

NN12: + ● – ○ – ○ ○ + ● – + + ● – ○

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

\+ + – ● ● – ● ○ – – + ○ + ○ – + – ● + – – + ● + + ● + – ● ○ –

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

NN67: + – ● ● – + + – – + – ● + + ● –

Addition and mirroring of laminate stacking sequences

The addition of laminate stacking sequences is another intuitive methods of combining **FOL**s with equal ply numbers, in series, to provide provides the even-number sequences that interlacing cannot. However, the fully orthotropic nature of combined laminate is generally lost when laminates with arbitrary non-symmetric form are used and is the subject of on-going research.

Even-ply sequences can also be developed through the addition of sequences in series or by splitting one even-ply sequence about the laminate mid-plane and inserting another sequence with even- or odd-plies to produce a sandwiched sequence. This includes the sandwiching of cross-ply sequences or isotropic material at the laminate mid-plane, as discussed in the following section.

Finally, mirroring offers the final intuitive method of combining **FOL**s discussed in this section. The technique applies to every sequence defined in the definitive list with up to 21 plies, such that the list can now be regarded as containing the anti-symmetric or symmetric halves for even-ply laminates with up to 42 plies.

V. Fully orthotropic tapered laminates

The following examples provide some insight into the scope for composite tailoring that the new definitive list provides. Two examples of tapered laminates are illustrated, both of which possess non-symmetric stacking sequences. The first example involves single-ply terminations at the laminate mid-plane, whilst the second involves two-ply terminations on either side of the laminate mid-plane. Note that the two examples are similar if the sequences from the first are taken to represent the symmetric half of the full laminate.

The following series provides the laminate number, together with the details of the particular ply to be dropped, i.e. ply orientation (+/–/●/○) and corresponding ply number:

NN 3979(●$_{11}$) ➡ *NN 1796*(●$_{10}$) ➡ *NN 560*(○$_{11}$) ➡ *NN 153*(○$_{10}$) ➡ *NN 21*(○$_{9}$) ➡ *NN 2*

Hence terminating the 11th ply of sequence *NN 3979*, corresponding to a ● ply, gives sequence *NN 1796*. Terminating the 10th ply of sequence *NN 1796*, which is also a ● ply, gives sequence *NN 560*, and so on. The laminate sequences, corresponding to this tapering series can be illustrated in full as follows:

NN 3979: +/–/●/○/○/●/–/+/○/●/●/○/○/–/●/+/+/●/–/○

NN 1796: +/–/●/○/○/●/–/+/○/●/○/○/–/●/+/+/●/–/○

NN 560: +/–/●/○/○/●/–/+/○/○/○/–/●/+/+/●/–/○

NN 153: +/–/●/○/○/●/–/+/○/○/–/●/+/+/●/–/○

NN 21: +/–/●/○/○/●/–/+/○/–/●/+/+/●/–/○

NN 2: +/–/●/○/○/●/–/+/–/●/+/+/●/–/○

In this second example the series provides the laminate number, together with the details of each of the two plies to be dropped:

NN 4104(○$_{6,16}$) ➡ *NN 600*(●$_{4,15}$) ➡ *NN 33*

Hence terminating the 6th and 16th ply of sequence *NN 4104*, both of which are ○ plies, gives sequence *NN 600*. Terminating the 4th and 15th ply of sequence *NN* 600, both of which are ● plies, gives sequence *NN* 33. The laminate sequences, corresponding to this tapering series can be illustrated in full as follows:

NN 4104: +/–/○/●/○/○/–/+/–/+/+/–/○/–/+/○/●/+/–/○

NN 600: +/–/○/●/○/–/+/–/+/+/–/○/–/+/●/+/–/○

NN 33: +/–/○/○/–/+/–/+/+/–/○/–/+/+/–/○

A very different form taper was demonstrated in Ref 17, but which is of relevance to the current article. Here, a lightweight Syntactic film core was used as a partial replacement for carbon fiber material, whilst maintaining the fully orthotropic laminate properties. This was achieved by combining both symmetric and anti-symmetric fully orthotropic angle-ply stacking sequences, which produced a **FOL** insensitive to the replacement of carbon fiber plies with equivalent thickness isotropic Syntactic film core at the laminate mid-plane. This would not have been possible with non-symmetric angle-ply sub-sequences, since the replacement of the angle plies by isotropic material, symmetrically displaced about the laminate mid-plane, would have caused an imbalance in the angle-plies that remained, thus destroying the fully orthotropic nature of the laminate.

VI. Conclusions

A new definitive list of Fully Orthotropic Laminates, or **FOL**s, with up to 21 plies has been presented, in abridged form. The great majority of sequences have been shown to be non-symmetric in form and many of these are without any sub-sequence patterns, e.g. symmetry or repeating groups, which is contrary to the assumptions on which many previous studies have been based. Many sequences appear both extra-ordinary in appearance and infeasible in terms of the uncoupled behavior that the laminates possess.

The definitive list has also been shown to contain all Quasi-Isotropic or Extensionally Isotropic Laminates (**EIL**s) and Fully Isotropic Laminates (**FOL**s), with up to 21 plies, which are sub-sets of Fully Orthotropic Laminates (**FOL**s).

Simple mixing rules have been presented by which **FOL**s from the new definitive list may be combined in order to develop new **FOL**s with any number of plies; some of the mixing rules represent work in progress.

The scope for composite tailoring using **FOL**s has also been demonstrated in the context of fully orthotropic tapered laminates, which hitherto fore were not possible. Continuing work in this area aims to provide a definitive list of tapered **FOL**s.

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Table 2 - Number of **FOL**s for 7 through 21 ply laminates corresponding to prefix designations *AC* for Anti-symmetric (*A*) angle-plies and Cross-symmetric (*C*) cross-plies.

Ref.	Sequence	n	$n_\pm$	$n_\circ$	$n_\bullet$	ζ	$\zeta_\pm$	$\zeta_\circ$	$\zeta_\bullet$	$n_\pm/n$ (%)	$n_\circ/n$ (%)	$n_\bullet/n$ (%)	$\zeta_\pm/\zeta$ (%)	$\zeta_\circ/\zeta$ (%)	$\zeta_\bullet/\zeta$ (%)
AC 1	+ ○ ● – – ● ○ ● ● ○ + + ○ ● –	15	6	4	5	3375	1782	796	797	40.0	26.7	33.3	52.8	23.6	23.6
:		:													
AC 24	+ – – + ● ○ ○ ○ ● ● ○ – + + –	15	8	4	3	3375	3032	172	171	53.3	26.7	20.0	89.8	5.1	5.1
AC 25	+ ○ ● – – ● ○ + – ● ○ + + ○ ● –	16	8	4	4	4096	2144	976	976	50.0	25.0	25.0	52.3	23.8	23.8
:		:													
AC 36	+ – – + ● ○ ○ ● ○ ● ● ○ – + + –	16	8	4	4	4096	3584	256	256	50.0	25.0	25.0	87.5	6.3	6.3
AC 37	+ ○ ● – ● ○ – – ● + + ● ○ + ○ ● –	17	8	4	5	4913	2264	1324	1325	47.1	23.5	29.4	46.1	26.9	27.0
:		:													
AC 120	+ + – – – ● ○ ○ ○ ● ● ○ + + + – –	17	10	4	3	4913	4570	172	171	58.8	23.5	17.6	93.0	3.5	3.5
AC 121	+ ○ ● ○ ● – – ● – ○ ● + ○ + + ○ ● ○ ● –	20	8	6	6	8000	3008	2496	2496	40.0	30.0	30.0	37.6	31.2	31.2
:		:													
AC 127	+ ○ ● ● – ○ – ○ ● – + ○ ● + ● + ○ ○ ● –	20	8	6	6	8000	3200	2400	2400	40.0	30.0	30.0	40.0	30.0	30.0
:		:													
AC 210	+ – + – – ● + ○ ○ ○ ● ● ● – ○ + + – + –	20	12	4	4	8000	7296	352	352	60.0	20.0	20.0	91.2	4.4	4.4

Table 3 - Number of **FOL**s for 7 through 21 ply laminates corresponding to prefix designations *AN* for Anti-symmetric (*A*) angle-plies and Non-symmetric (*N*) cross-plies.

Ref.	Sequence	n	$n_\pm$	$n_\circ$	$n_\bullet$	ζ	$\zeta_\pm$	$\zeta_\circ$	$\zeta_\bullet$	$n_\pm/n$ (%)	$n_\circ/n$ (%)	$n_\bullet/n$ (%)	$\zeta_\pm/\zeta$ (%)	$\zeta_\circ/\zeta$ (%)	$\zeta_\bullet/\zeta$ (%)
AN 1	+ ○ ● – – ● ● ● ○ ● + + ○ ● –	15	6	3	6	3375	1782	747	846	40.0	20.0	40.0	52.8	22.1	25.1
:		:													
AN 24	+ – ● ○ ○ ○ – ○ + ● ● ○ ○ + –	15	6	6	3	3375	2070	846	459	40.0	40.0	20.0	61.3	25.1	13.6
AN 25	+ ○ ● ● – – ● – ● + ○ + + ○ ● ● –	17	8	3	6	4913	2168	939	1806	47.1	17.6	35.3	44.1	19.1	36.8
:		:													
AN 256	+ – – + ○ ● ○ ○ ○ ● ● ○ ○ – + + –	17	8	6	3	4913	4184	558	171	47.1	35.3	17.6	85.2	11.4	3.5
AN 257	+ ● ○ ● – – ● ○ – + ● ○ + + ○ ● ● –	18	8	4	6	5832	2528	976	2328	44.4	22.2	33.3	43.3	16.7	39.9
:		:													
AN 388	+ – – + ○ ● ○ ○ ● ○ ● ● ○ ○ – + + –	18	8	6	4	5832	4832	744	256	44.4	33.3	22.2	82.9	12.8	4.4
AN 389	+ ● ● ○ ● – – – ● ● ○ + + + ○ ● ● ● –	19	8	3	8	6859	2648	747	3464	42.1	15.8	42.1	38.6	10.9	50.5
:		:													
AN 2372	+ + – – ○ – ● ○ ○ ○ ● ● ○ + ○ + + – –	19	10	6	3	6859	5914	774	171	52.6	31.6	15.8	86.2	11.3	2.5
AN 2373	+ ● ○ ● ● – – ● – ○ ● + ○ + + ○ ● ● ● –	20	8	4	8	8000	3008	1120	3872	40.0	20.0	40.0	37.6	14.0	48.4
:		:													
AN 3292	+ – – + ○ ○ ● ○ ○ ● ○ ● ● ○ ○ ○ – + + –	20	8	8	4	8000	6272	1472	256	40.0	40.0	20.0	78.4	18.4	3.2
AN 3293	+ ● ○ ● ● ○ – – – – ● + + + + ● ○ ○ ● ● –	21	10	4	7	9261	3130	2092	4039	47.6	19.0	33.3	33.8	22.6	43.6
:		:													
AN 14532	+ – + – – ○ + ● ○ ○ ○ ● ● ○ – ○ + + – + –	21	12	6	3	9261	8316	774	171	57.1	28.6	14.3	89.8	8.4	1.8

Table 4 - Number of **FOL**s for 7 through 21 ply laminates corresponding to prefix designations *AS* for Anti-symmetric (*A*) angle-plies and Symmetric (*S*) cross-plies.

Ref.	Sequence	n	$n_{\pm}$	$n_{\circ}$	$n_{\bullet}$	ζ	$\zeta_{\pm}$	$\zeta_{\circ}$	$\zeta_{\bullet}$	$n_{\pm}/n$ (%)	$n_{\circ}/n$ (%)	$n_{\bullet}/n$ (%)	$\zeta_{\pm}/\zeta$ (%)	$\zeta_{\circ}/\zeta$ (%)	$\zeta_{\bullet}/\zeta$ (%)	ESDU 83012
AS 1	+ − − ● + + −	7	6	0	1	343	342	0	1	85.7	0.0	14.3	99.7	0.0	0.3	
AS 2	+ − − ○ + + −	7	6	1	0	343	342	1	0	85.7	14.3	0.0	99.7	0.3	0.0	*A 381*
AS 3	+ − − + − + + −	8	8	0	0	512	512	0	0	100.0	0.0	0.0	100.0	0.0	0.0	*A 1*
AS 4	+ − ● − ● + ● + −	9	6	0	3	729	630	0	99	66.7	0.0	33.3	86.4	0.0	13.6	
:		:														
AS 9	+ − − + ○ − + + −	9	8	1	0	729	728	1	0	66.7	33.3	0.0	86.4	13.6	0.0	*A 383*
AS 10	+ ● − − − + + + ● −	10	8	0	2	1000	704	0	296	80.0	0.0	20.0	70.4	0.0	29.6	
:		:														
AS 15	+ − − + ○ ○ − + + −	10	8	2	0	1000	992	8	0	80.0	20.0	0.0	70.4	29.6	0.0	*A 4*
AS 16	+ ● − − ● ● ● + + ● −	11	6	0	5	1331	918	0	413	54.5	0.0	45.5	69.0	0.0	31.0	
:		:														
AS 39	+ − − + ○ ○ ○ − + + −	11	8	3	0	1331	1304	27	0	72.7	27.3	0.0	98.0	2.0	0.0	*A 387*
AS 40	+ ● − ● − − + + ● + ● −	12	8	0	4	1728	1088	0	640	66.7	0.0	33.3	63.0	0.0	37.0	
:		:														
AS 60	+ − + − − − + + + − + −	12	12	0	0	1728	1728	0	0	100.0	0.0	0.0	100.0	0.0	0.0	*A 10*
AS 61	+ ● ● − − − ● + + + ● ● −	13	8	0	5	2197	1208	0	989	61.5	0.0	38.5	55.0	0.0	45.0	
:		:														
AS 144	+ − + − − ○ ○ ○ + + − + −	13	10	3	0	2197	2170	27	0	76.9	23.1	0.0	98.8	1.2	0.0	*A 397*
AS 145	+ ● ● − − ● − + ● + + ● ● −	14	8	0	6	2744	1472	0	1272	57.1	0.0	42.9	53.6	0.0	46.4	
:		:														
AS 220	+ + − − − − ○ ○ + + + + − −	14	12	2	0	2744	2736	8	0	85.7	14.3	0.0	99.7	0.3	0.0	*A 24*
AS 221	+ ● ● − ● − − ● + + ● + ● ● −	15	8	0	7	3375	1688	0	1687	53.3	0.0	46.7	50.0	0.0	50.0	
:		:														
AS 508	+ + − − − − + ○ − + + + + − −	15	14	1	0	3375	3374	1	0	93.3	6.7	0.0	100.0	0.0	0.0	*A 422*
AS 509	+ ● ● ● − − − ● ● + + + ● ● ● −	16	8	0	8	4096	1856	0	2240	50.0	0.0	50.0	45.3	0.0	54.7	
:		:														
AS 776	+ + − − − − + + − − + + + + − −	16	16	0	0	4096	4096	0	0	100.0	0.0	0.0	100.0	0.0	0.0	*A 56*
AS 777	+ ● ● ● − − ● − ● + ● + + ● ● ● −	17	8	0	9	4913	2168	0	2745	47.1	0.0	52.9	44.1	0.0	55.9	
:		:														
AS 1778	+ + − − − + − − ○ + + − + + + − −	17	16	1	0	4913	4912	1	0	94.1	5.9	0.0	100.0	0.0	0.0	*A 484*
AS 1779	+ ● ● ● − ● − − ● ● + + ● + ● ● ● −	18	8	0	10	5832	2432	0	3400	44.4	0.0	55.6	41.7	0.0	58.3	
:		:														
AS 2712	+ + − − − − + + ○ ○ − − + + + + − −	18	16	2	0	5832	5824	8	0	88.9	11.1	0.0	99.9	0.1	0.0	*A 144*
AS 2713	+ ● ● ● ● − − − ● ● ● + + + ● ● ● ● −	19	8	0	11	6859	2648	0	4211	42.1	0.0	57.9	38.6	0.0	61.4	
:		:														
AS 6224	+ + − − + − − − ○ ○ ○ + + + − + + − −	19	16	3	0	6859	6832	27	0	84.2	15.8	0.0	99.6	0.4	0.0	*A 652*
AS 6225	+ ● ● ● ● − − ● − ● ● + ● + + ● ● ● ● −	20	8	0	12	8000	3008	0	4992	40.0	0.0	60.0	37.6	0.0	62.4	
:		:														
AS 9514	+ + − + − − − − − + − + + + + + − + − −	20	20	0	0	8000	8000	0	0	100.0	0.0	0.0	100.0	0.0	0.0	*A 373*
AS 9515	+ ● ● ● ● ● − − − − ● + + + + ● ● ● ● ● −	21	10	0	11	9261	3130	0	6131	47.6	0.0	52.4	33.8	0.0	66.2	
:																
AS 21906	+ + + − − − − − − ○ ○ ○ + + + + + + − − −	21	18	3	0	9261	9234	27	0	85.7	14.3	0.0	99.7	0.3	0.0	

Table 5 - Number of **FOL**s for 7 through 21 ply laminates corresponding to prefix designations *SC* for Symmetric (*S*) angle-plies and Cross-symmetric (*C*) cross-plies.

Ref.	Sequence	n	$n_{\pm}$	$n_{\circ}$	$n_{\bullet}$	ζ	$\zeta_{\pm}$	$\zeta_{\circ}$	$\zeta_{\bullet}$	$n_{\pm}/n$ (%)	$n_{\circ}/n$ (%)	$n_{\bullet}/n$ (%)	$\zeta_{\pm}/\zeta$ (%)	$\zeta_{\circ}/\zeta$ (%)	$\zeta_{\bullet}/\zeta$ (%)
SC 1	+ − ○ − ● + ● ● ○ ○ + ○ − ● − +	16	8	4	4	4096	3008	544	544	50.0	25.0	25.0	73.4	13.3	13.3
SC 2	+ − ● − ○ + ○ ○ ● ● + ● − ○ − +	16	8	4	4	4096	3008	544	544	50.0	25.0	25.0	73.4	13.3	13.3
SC 3	+ − ○ ● − ● ○ + ● + ● ○ − ○ ● − +	17	8	4	5	4913	3128	892	893	47.1	23.5	29.4	63.7	18.2	18.2
SC 4	+ − ● ○ − ○ ● + ● + ○ ● − ● ○ − +	17	8	4	5	4913	3128	892	893	47.1	23.5	29.4	63.7	18.2	18.2
SC 5	+ − ○ ● − ● ○ + ○ + ● ○ − ○ ● − +	17	8	5	4	4913	3128	893	892	47.1	29.4	23.5	63.7	18.2	18.2
SC 6	+ − ● ○ − ○ ● + ○ + ○ ● − ● ○ − +	17	8	5	4	4913	3128	893	892	47.1	29.4	23.5	63.7	18.2	18.2
SC 7	+ − ○ ● ○ − ● ● + ● ○ + ○ ○ − ● ○ ● − +	20	8	6	6	8000	4448	1776	1776	40.0	30.0	30.0	55.6	22.2	22.2
SC 8	+ − ● ○ ● − ○ ○ + ○ ● + ● ● − ○ ● ○ − +	20	8	6	6	8000	4448	1776	1776	40.0	30.0	30.0	55.6	22.2	22.2
SC 9	+ ● ○ − − − ○ ● + + + + ○ ● − − − ● ○ +	20	12	4	4	8000	4464	1768	1768	60.0	20.0	20.0	55.8	22.1	22.1
SC 10	+ ○ ● − − − ● ○ + + + + ● ○ − − − ○ ● +	20	12	4	4	8000	4464	1768	1768	60.0	20.0	20.0	55.8	22.1	22.1
SC 11	+ − ○ ● − ● + ○ ○ ● ○ ● ● + ○ − ○ ● − +	20	8	6	6	8000	4928	1536	1536	40.0	30.0	30.0	61.6	19.2	19.2
SC 12	+ − ● ○ − ○ + ● ● ○ ● ○ ○ + ● − ● ○ − +	20	8	6	6	8000	4928	1536	1536	40.0	30.0	30.0	61.6	19.2	19.2

Table 6 - Number of **FOL**s for 7 through 21 ply laminates corresponding to prefix designations *SN* for Symmetric (*S*) angle-plies and Non-symmetric (*N*) cross-plies.

Ref.	Sequence	n	$n_{\pm}$	$n_{\circ}$	$n_{\bullet}$	ζ	$\zeta_{\pm}$	$\zeta_{\circ}$	$\zeta_{\bullet}$	$n_{\pm}/n$ (%)	$n_{\circ}/n$ (%)	$n_{\bullet}/n$ (%)	$\zeta_{\pm}/\zeta$ (%)	$\zeta_{\circ}/\zeta$ (%)	$\zeta_{\bullet}/\zeta$ (%)
SN 1	+ – ● ○ – ● ● + ● + ○ ○ – ● ● – +	17	8	3	6	4913	3128	459	1326	47.1	17.6	35.3	63.7	9.3	27.0
SN 2	+ – ● ● – ○ ○ + ● + ● ● – ○ ● – +	17	8	3	6	4913	3128	459	1326	47.1	17.6	35.3	63.7	9.3	27.0
SN 3	+ – ● ○ – ● ● + ○ + ○ ○ – ● ● – +	17	8	4	5	4913	3128	460	1325	47.1	23.5	29.4	63.7	9.4	27.0
SN 4	+ – ● ● – ○ ○ + ○ + ● ● – ○ ● – +	17	8	4	5	4913	3128	460	1325	47.1	23.5	29.4	63.7	9.4	27.0
SN 5	+ – ○ ○ – ● ● + ● + ○ ○ – ● ○ – +	17	8	5	4	4913	3128	1325	460	47.1	29.4	23.5	63.7	27.0	9.4
SN 6	+ – ○ ● – ○ ○ + ● + ● ● – ○ ○ – +	17	8	5	4	4913	3128	1325	460	47.1	29.4	23.5	63.7	27.0	9.4
SN 7	+ – ○ ○ – ● ● + ○ + ○ ○ – ● ○ – +	17	8	6	3	4913	3128	1326	459	47.1	35.3	17.6	63.7	27.0	9.3
SN 8	+ – ○ ● – ○ ○ + ○ + ● ● – ○ ○ – +	17	8	6	3	4913	3128	1326	459	47.1	35.3	17.6	63.7	27.0	9.3
SN 9	+ ● – – ○ ● ● ○ + + ● ○ ○ ● – – ● +	18	8	4	6	5832	3488	496	1848	44.4	22.2	33.3	59.8	8.5	31.7
SN 10	+ ● – – ● ○ ○ ● + + ○ ● ● ○ – – ● +	18	8	4	6	5832	3488	496	1848	44.4	22.2	33.3	59.8	8.5	31.7
SN 11	+ ● – – ● ○ ○ ○ + + ● ● ● ● – – ○ +	18	8	4	6	5832	3488	928	1416	44.4	22.2	33.3	59.8	15.9	24.3
SN 12	+ ○ – – ● ● ● ● + + ○ ○ ○ ● – – ● +	18	8	4	6	5832	3488	928	1416	44.4	22.2	33.3	59.8	15.9	24.3
SN 13	+ ● – – ○ ○ ○ ○ + + ● ● ● ○ – – ○ +	18	8	6	4	5832	3488	1416	928	44.4	33.3	22.2	59.8	24.3	15.9
SN 14	+ ○ – – ○ ● ● ● + + ○ ○ ○ ○ – – ● +	18	8	6	4	5832	3488	1416	928	44.4	33.3	22.2	59.8	24.3	15.9
SN 15	+ ○ – – ○ ● ● ○ + + ● ○ ○ ● – – ○ +	18	8	6	4	5832	3488	1848	496	44.4	33.3	22.2	59.8	31.7	8.5
SN 16	+ ○ – – ● ○ ○ ● + + ○ ● ● ○ – – ○ +	18	8	6	4	5832	3488	1848	496	44.4	33.3	22.2	59.8	31.7	8.5
SN 17	+ ● – – ● ○ ● + ● ● ○ + ○ ● ● – – ● +	19	8	3	8	6859	4088	315	2456	42.1	15.8	42.1	59.6	4.6	35.8
⋮		⋮													
SN 80	+ ○ – – ○ ● ○ + ○ ○ ● + ● ○ ○ – – ○ +	19	8	8	3	6859	4088	2456	315	42.1	42.1	15.8	59.6	35.8	4.6
SN 81	+ – ● ● ○ – ● ● + ○ ● + ○ ○ – ● ● ● – +	20	8	4	8	8000	4448	592	2960	40.0	20.0	40.0	55.6	7.4	37.0
⋮		⋮													
SN 168	+ – ○ ○ – ● + ○ ○ ○ ● ● ● + ○ – ○ ○ – +	20	8	8	4	8000	4928	2720	352	40.0	40.0	20.0	61.6	34.0	4.4
SN 169	+ ● – ● ○ – – ○ + + ● + + ● – – ● ● – ○ +	21	12	3	6	9261	5052	1515	2694	57.1	14.3	28.6	54.6	16.4	29.1
⋮		⋮													
SN 192	+ – – ○ + + – ● ○ ○ ○ ● ● ○ – + + ○ – – +	21	12	6	3	9261	7740	1350	171	57.1	28.6	14.3	83.6	14.6	1.8

Table 7 - Number of **FOL**s for 7 through 21 ply laminates corresponding to prefix designations *SS* for Symmetric (*S*) angle-plies and Symmetric (*S*) cross-plies.

Ref.	Sequence	n	$n_{\pm}$	$n_{\circ}$	$n_{\bullet}$	ζ	$\zeta_{\pm}$	$\zeta_{\circ}$	$\zeta_{\bullet}$	$n_{\pm}/n$ (%)	$n_{\circ}/n$ (%)	$n_{\bullet}/n$ (%)	$\zeta_{\pm}/\zeta$ (%)	$\zeta_{\circ}/\zeta$ (%)	$\zeta_{\bullet}/\zeta$ (%)	ESDU 83012
SS 1	+ − − ● + ● ● + ● − − +	12	8	0	4	1728	1568	0	160	66.7	0.0	33.3	90.7	0.0	9.3	
SS 2	+ − − ● + ○ ○ + ● − − +	12	8	2	2	1728	1568	8	152	66.7	16.7	16.7	90.7	0.5	8.8	
SS 3	+ − − ○ + ● ● + ○ − − +	12	8	2	2	1728	1568	152	8	66.7	16.7	16.7	90.7	8.8	0.5	
SS 4	+ − − ○ + ○ ○ + ○ − − +	12	8	4	0	1728	1568	160	0	66.7	33.3	0.0	90.7	9.3	0.0	*S 1*
SS 5	+ − ● − ● ● + + ● ● − ● − +	14	8	0	6	2744	2048	0	696	57.1	0.0	42.9	74.6	0.0	25.4	
SS 6	+ − ● − ● ○ + + ○ ● − ● − +	14	8	2	4	2744	2048	56	640	57.1	14.3	28.6	74.6	2.0	23.3	
SS 7	+ − ● − ○ ● + + ● ○ − ● − +	14	8	2	4	2744	2048	152	544	57.1	14.3	28.6	74.6	5.5	19.8	
SS 8	+ − ● − ○ ○ + + ○ ○ − ● − +	14	8	4	2	2744	2048	208	488	57.1	28.6	14.3	74.6	7.6	17.8	
SS 9	+ − ○ − ● ● + + ● ● − ○ − +	14	8	2	4	2744	2048	488	208	57.1	14.3	28.6	74.6	17.8	7.6	
SS 10	+ − ○ − ● ○ + + ○ ● − ○ − +	14	8	4	2	2744	2048	544	152	57.1	28.6	14.3	74.6	19.8	5.5	
SS 11	+ − ○ − ○ ● + + ● ○ − ○ − +	14	8	4	2	2744	2048	640	56	57.1	28.6	14.3	74.6	23.3	2.0	
SS 12	+ − ○ − ○ ○ + + ○ ○ − ○ − +	14	8	6	0	2744	2048	696	0	57.1	42.9	0.0	74.6	25.4	0.0	*S 2*
SS 13	+ − − ● + + − − + + ● − − +	14	12	0	2	2744	2448	0	296	85.7	0.0	14.3	89.2	0.0	10.8	
SS 14	+ − − ○ + + − − + + ○ − − +	14	12	2	0	2744	2448	296	0	85.7	14.3	0.0	89.2	10.8	0.0	*S 3*
SS 15	+ − − + − + ● ● + − + − − +	14	12	0	2	2744	2736	0	8	85.7	0.0	14.3	99.7	0.0	0.3	
SS 16	+ − − + − + ○ ○ + − + − − +	14	12	2	0	2744	2736	8	0	85.7	14.3	0.0	99.7	0.3	0.0	*S 4*
SS 17	+ − − ● + + − ● − + + ● − − +	15	12	0	3	3375	2988	0	387	80.0	0.0	20.0	88.5	0.0	11.5	
:		:														
SS 20	+ − − ○ + + − ○ − + + ○ − − +	15	12	3	0	3375	2988	387	0	80.0	20.0	0.0	88.5	11.5	0.0	*S 49*
SS 21	+ − ● − ● + ● ● ● ● + ● − ● − +	16	8	0	8	4096	3008	0	1088	50.0	0.0	50.0	73.4	0.0	26.6	
:		:														
SS 53	+ − − + − + + − − + + − + − − +	16	16	0	0	4096	4096	0	0	100.0	0.0	0.0	100.0	0.0	0.0	*S 10*
SS 54	+ − ● ● − ● ● + ● + ● ● − ● ● − +	17	8	0	9	4913	3128	0	1785	47.1	0.0	52.9	63.7	0.0	36.3	
:		:														
SS 103	+ − − + − + + − ○ − + + − + − − +	17	16	1	0	4913	4912	1	0	94.1	5.9	0.0	100.0	0.0	0.0	*S 53*
SS 104	+ ● − − ● ● ● ● + + ● ● ● ● − − ● +	18	8	0	10	5832	3488	0	2344	44.4	0.0	55.6	59.8	0.0	40.2	
:		:														
SS 213	+ − − + − + + − ○ ○ − + + − + − − +	18	16	2	0	5832	5824	8	0	88.9	11.1	0.0	99.9	0.1	0.0	*S 23*
SS 214	+ ● − − ● ● ● + ● ● ● + ● ● ● − − ● +	19	8	0	11	6859	4088	0	2771	42.1	0.0	57.9	59.6	0.0	40.4	
:		:														
SS 333	+ − − + − + + − ○ ○ ○ − + + − + − − +	19	16	3	0	6859	6832	27	0	84.2	15.8	0.0	99.6	0.4	0.0	*S 59*
SS 334	+ − ● ● ● − ● ● + ● ● + ● ● − ● ● ● − +	20	8	0	12	8000	4448	0	3552	40.0	0.0	60.0	55.6	0.0	44.4	
:		:														
SS 685	+ − − + − + + − ○ ○ ○ ○ − + + − + − − +	20	16	4	0	8000	7936	64	0	80.0	20.0	0.0	99.2	0.8	0.0	*S 48*
SS 686	+ ● − ● ● − − ● + + ● + + ● − − ● ● − ● +	21	12	0	9	9261	5052	0	4209	57.1	0	42.9	54.6	0	45.4	
:		:														
SS 1029	+ − − + − + + − ○ ○ ○ ○ ○ − + + − + − − +	21	16	5	0	9261	9136	125	0	76.2	23.8	0	98.7	1.3	0	*S 75*

Table 8 - Number of **FOL**s for 7 through 21 ply laminates corresponding to prefix designations $_{+}NS_{+}$ for <u>N</u>on-symmetric (*N*) angle-plies and <u>S</u>ymmetric (*S*) cross-plies.

Ref.	Sequence	n	$n_{\pm}$	$n_{\circ}$	$n_{\bullet}$	ζ	$\zeta_{\pm}$	$\zeta_{\circ}$	$\zeta_{\bullet}$	$n_{\pm}/n$ (%)	$n_{\circ}/n$ (%)	$n_{\bullet}/n$ (%)	$\zeta_{\pm}/\zeta$ (%)	$\zeta_{\circ}/\zeta$ (%)	$\zeta_{\bullet}/\zeta$ (%)
NS 1	+ − − − + + + + − − + − − + − +	16	16	0	0	4096	4096	0	0	100.0	0.0	0.0	100.0	0.0	0.0
NS 2	+ − + − − + − − + + + + − − − +	16	16	0	0	4096	4096	0	0	100.0	0.0	0.0	100.0	0.0	0.0
NS 3	+ − ● − − + + + ● + − − − + ● − +	17	14	0	3	4913	4046	0	867	82.4	0.0	17.6	82.4	0.0	17.6
NS 4	+ − ● + − − − + ● + + + − − ● − +	17	14	0	3	4913	4046	0	867	82.4	0.0	17.6	82.4	0.0	17.6
NS 5	+ − ● − − + + + ○ + − − − + ● − +	17	14	1	2	4913	4046	1	866	82.4	5.9	11.8	82.4	0.0	17.6
NS 6	+ − ● + − − − + ○ + + + − − ● − +	17	14	1	2	4913	4046	1	866	82.4	5.9	11.8	82.4	0.0	17.6
NS 7	+ − ○ − − + + + ● + − − − + ○ − +	17	14	2	1	4913	4046	866	1	82.4	11.8	5.9	82.4	17.6	0.0
NS 8	+ − ○ + − − − + ● + + + − − ○ − +	17	14	2	1	4913	4046	866	1	82.4	11.8	5.9	82.4	17.6	0.0
NS 9	+ − ○ − − + + + ○ + − − − + ○ − +	17	14	3	0	4913	4046	867	0	82.4	17.6	0.0	82.4	17.6	0.0
NS 10	+ − ○ + − − − + ○ + + + − − ○ − +	17	14	3	0	4913	4046	867	0	82.4	17.6	0.0	82.4	17.6	0.0
NS 11	+ − − + − + − + ● + + − − − + − +	17	16	0	1	4913	4912	0	1	94.1	0.0	5.9	100.0	0.0	0.0
NS 12	+ − + − − − + + ● + − + − + − − +	17	16	0	1	4913	4912	0	1	94.1	0.0	5.9	100.0	0.0	0.0
NS 13	+ − − + − + − + ○ + + − − − + − +	17	16	1	0	4913	4912	1	0	94.1	5.9	0.0	100.0	0.0	0.0
NS 14	+ − + − − − + + ○ + − + − + − − +	17	16	1	0	4913	4912	1	0	94.1	5.9	0.0	100.0	0.0	0.0
NS 15	+ ● − − − + + − + + + − − + − − ● +	18	16	0	2	5832	4480	0	1352	88.9	0.0	11.1	76.8	0.0	23.2
⋮		⋮													
NS 32	+ + − − ○ − − − + + + + + ○ − − − +	18	16	2	0	5832	5344	488	0	88.9	11.1	0.0	91.6	8.4	0.0
NS 39	+ ● − − ● − + + + ● − + − + ● − − ● +	19	14	0	5	6859	4718	0	2141	73.7	0.0	26.3	68.8	0.0	31.2
⋮		⋮													
NS 124	+ + − − − − ○ + + ○ − + ○ + + − − − +	19	16	3	0	6859	6640	219	0	84.2	15.8	0.0	96.8	3.2	0.0
NS 143	+ − ● ● − − + + + − + + − − − + ● ● − +	20	16	0	4	8000	5632	0	2368	80.0	0.0	20.0	70.4	0.0	29.6
⋮		⋮													
NS 300	+ − + − + − − − + − + + + + − − + − − +	20	20	0	0	8000	8000	0	0	100.0	0.0	0.0	100.0	0.0	0.0
NS 365	+ − − ● + − − + + + ● + + − − − − ● − + +	21	18	0	3	9261	8082	0	1179	85.7	0.0	14.3	87.3	0.0	12.7
⋮		⋮													
NS 514	+ + − − − + − − + ○ ○ ○ + + − + + − − − +	21	18	3	0	9261	9234	27	0	85.7	14.3	0.0	99.7	0.3	0.0

Table 9 - Number of **FOL**s for 7 through 21 ply laminates corresponding to prefix designations $_{+}NS_{-}$ for Non-symmetric (*N*) angle-plies and Symmetric (*S*) cross-plies.

Ref.	Sequence	n	$n_{\pm}$	$n_{\circ}$	$n_{\bullet}$	ζ	$\zeta_{\pm}$	$\zeta_{\circ}$	$\zeta_{\bullet}$	$n_{\pm}/n$ (%)	$n_{\circ}/n$ (%)	$n_{\bullet}/n$ (%)	$\zeta_{\pm}/\zeta$ (%)	$\zeta_{\circ}/\zeta$ (%)	$\zeta_{\bullet}/\zeta$ (%)
NS 17	+ ● − + − − − + + − + + − − + + ● −	18	16	0	2	5832	4480	0	1352	88.9	0.0	11.1	76.8	0.0	23.2
NS 18	+ ● − − + + − − + − − + + + − + ● −	18	16	0	2	5832	4480	0	1352	88.9	0.0	11.1	76.8	0.0	23.2
NS 19	+ ● + − − − − − + + + + − + − + ● −	18	16	0	2	5832	4480	0	1352	88.9	0.0	11.1	76.8	0.0	23.2
NS 20	+ ● − + − + − − − − + + + + + − ● −	18	16	0	2	5832	4480	0	1352	88.9	0.0	11.1	76.8	0.0	23.2
NS 23	+ ○ − + − − − + + − + + − − + + ○ −	18	16	2	0	5832	4480	1352	0	88.9	11.1	0.0	76.8	23.2	0.0
NS 24	+ ○ − − + + − − + − − + + + − + ○ −	18	16	2	0	5832	4480	1352	0	88.9	11.1	0.0	76.8	23.2	0.0
NS 25	+ ○ + − − − − − + + + + − + − + ○ −	18	16	2	0	5832	4480	1352	0	88.9	11.1	0.0	76.8	23.2	0.0
NS 26	+ ○ − + − + − − − − + + + + + − ○ −	18	16	2	0	5832	4480	1352	0	88.9	11.1	0.0	76.8	23.2	0.0
NS 29	+ − + − ● − − + + + − + − ● − + + −	18	16	0	2	5832	5344	0	488	88.9	0.0	11.1	91.6	0.0	8.4
NS 30	+ − − + ● + − + − − − + + ● + − + −	18	16	0	2	5832	5344	0	488	88.9	0.0	11.1	91.6	0.0	8.4
NS 33	+ − + − ○ − − + + + − + − ○ − + + −	18	16	2	0	5832	5344	488	0	88.9	11.1	0.0	91.6	8.4	0.0
NS 34	+ − − + ○ + − + − − − + + ○ + − + −	18	16	2	0	5832	5344	488	0	88.9	11.1	0.0	91.6	8.4	0.0
NS 35	+ + − − − − + ● + − ● + + − + − + −	18	16	0	2	5832	5776	0	56	88.9	0.0	11.1	99.0	0.0	1.0
NS 36	+ − + − + − − ● + − ● − + + + + − −	18	16	0	2	5832	5776	0	56	88.9	0.0	11.1	99.0	0.0	1.0
NS 37	+ + − − − − + ○ + − ○ + + − + − + −	18	16	2	0	5832	5776	56	0	88.9	11.1	0.0	99.0	1.0	0.0
NS 38	+ − + − + − − ○ + − ○ − + + + + − −	18	16	2	0	5832	5776	56	0	88.9	11.1	0.0	99.0	1.0	0.0
NS 71	+ + − − ● − − ● + ● + ● + − ● + − + −	19	14	0	5	6859	6158	0	701	73.7	0.0	26.3	89.8	0.0	10.2
:		:													
NS 126	+ − + − − + ○ + − ○ − − ○ + + + + − −	19	16	3	0	6859	6640	219	0	84.2	15.8	0.0	96.8	3.2	0.0
NS 127	+ ● ● − + − − − + − + + + − − + + ● ● −	20	16	0	4	8000	4912	0	3088	80.0	0.0	20.0	61.4	0.0	38.6
:		:													
NS 308	+ − − + + − + − + − − − + − + + + + − −	20	20	0	0	8000	8000	0	0	100.0	0.0	0.0	100.0	0.0	0.0
NS 309	+ ● + − ● − − − − + ● + + + − + ● − + ● −	21	16	0	5	9261	6448	0	2813	76.2	0.0	23.8	69.6	0.0	30.4
:		:													
NS 515	+ − − + + + − − − ○ ○ ○ − + + − + + + − −	21	18	3	0	9261	9234	27	0	85.7	14.3	0.0	99.7	0.3	0.0

Table 10 - Number of **FOL**s for 7 through 21 ply laminates corresponding to prefix designations ${}_{+}NN_{+}$ for Non-symmetric (N) angle-plies and Non-symmetric (N) cross-plies.

Ref.	Sequence	n	$n_{\pm}$	$n_{\circ}$	$n_{\bullet}$	ζ	$\zeta_{\pm}$	$\zeta_{\circ}$	$\zeta_{\bullet}$	$n_{\pm}/n$ (%)	$n_{\circ}/n$ (%)	$n_{\bullet}/n$ (%)	$\zeta_{\pm}/\zeta$ (%)	$\zeta_{\circ}/\zeta$ (%)	$\zeta_{\bullet}/\zeta$ (%)
NN 33	+ ● − − − + + + − ● + ● − − +	15	12	0	3	3375	2700	0	675	80.0	0.0	20.0	80.0	0.0	20.0
NN 34	+ − − ● + ● − + + + − − − ● +	15	12	0	3	3375	2700	0	675	80.0	0.0	20.0	80.0	0.0	20.0
NN 35	+ − − ○ + ○ − + + + − − − ○ +	15	12	3	0	3375	2700	675	0	80.0	20.0	0.0	80.0	20.0	0.0
NN 36	+ ○ − − − + + + − ○ + ○ − − +	15	12	3	0	3375	2700	675	0	80.0	20.0	0.0	80.0	20.0	0.0
NN 65	+ ● − − ● − + + + + − − ● ● − +	16	12	0	4	4096	2832	0	1264	75.0	0.0	25.0	69.1	0.0	30.9
NN 66	+ − ● ● − − + + + + − ● − − ● +	16	12	0	4	4096	2832	0	1264	75.0	0.0	25.0	69.1	0.0	30.9
NN 69	+ − ○ ○ − − + + + + − ○ − − ○ +	16	12	4	0	4096	2832	1264	0	75.0	25.0	0.0	69.1	30.9	0.0
NN 70	+ ○ − − ○ − + + + + − − ○ ○ − +	16	12	4	0	4096	2832	1264	0	75.0	25.0	0.0	69.1	30.9	0.0
NN 457	+ ● − ● − ● − + + + ● ● ● − ● − +	17	10	0	7	4913	3130	0	1783	58.8	0.0	41.2	63.7	0.0	36.3
⋮		⋮													
NN 722	+ − + − − ○ ○ ○ + ○ + − + ○ − − +	17	12	5	0	4913	4428	485	0	70.6	29.4	0.0	90.1	9.9	0.0
NN 935	+ ● − − ● ● ● + ● ● + ● ● − ● ● − +	18	8	0	10	5832	3584	0	2248	44.4	0.0	55.6	61.5	0.0	38.5
⋮		⋮													
NN 1258	+ − − + ○ ○ − + ○ ○ ○ + − + − ○ − +	18	12	6	0	5832	4896	936	0	66.7	33.3	0.0	84.0	16.0	0.0
NN 3291	+ − ● ● − ● ● + ● + − + ● − ● ● ● − +	19	10	0	9	6859	4090	0	2769	52.6	0.0	47.4	59.6	0.0	40.4
⋮		⋮													
NN 4944	+ − + − − + ○ − + − ○ ○ + + + − − − +	19	16	3	0	6859	6688	171	0	84.2	15.8	0.0	97.5	2.5	0.0
NN 6041	+ ● − ● ● − − + ● + + − + ● ● − ● ● − +	20	12	0	8	8000	4656	0	3344	60.0	0.0	40.0	58.2	0.0	41.8
⋮		⋮													
NN 8778	+ + − − − − + ○ ○ + ○ − + ○ + + − − − +	20	16	4	0	8000	7744	256	0	80.0	20.0	0.0	96.8	3.2	0.0
NN 21309	+ ● − − ● ● ● ● ● + + + − ● ● ● − − ● ● +	21	10	0	11	9261	4858	0	4403	47.6	0.0	52.4	52.5	0.0	47.5
⋮		⋮													
NN 40762	+ + − − − − + ○ + − + ○ ○ − + + + − − − +	21	18	3	0	9261	9090	171	0	85.7	14.3	0.0	98.2	1.8	0.0

Table 11 - Number of **FOL**s for 7 through 21 ply laminates corresponding to prefix designations $_{+}NN_{-}$ for Non-symmetric (*N*) angle-plies and Non-symmetric (*N*) cross-plies.

Ref.	Sequence	n	$n_{\pm}$	$n_{\circ}$	$n_{\bullet}$	ζ	$\zeta_{\pm}$	$\zeta_{\circ}$	$\zeta_{\bullet}$	$n_{\pm}/n$ (%)	$n_{\circ}/n$ (%)	$n_{\bullet}/n$ (%)	$\zeta_{\pm}/\zeta$ (%)	$\zeta_{\circ}/\zeta$ (%)	$\zeta_{\bullet}/\zeta$ (%)
NN 67	+ − ● ● − + + − − + − ● + + ● −	16	12	0	4	4096	2832	0	1264	75.0	0.0	25.0	69.1	0.0	30.9
NN 68	+ ● − − ● + − + + − − + ● ● + −	16	12	0	4	4096	2832	0	1264	75.0	0.0	25.0	69.1	0.0	30.9
NN 71	+ ○ − − ○ + − + + − − + ○ ○ + −	16	12	4	0	4096	2832	1264	0	75.0	25.0	0.0	69.1	30.9	0.0
NN 72	+ − ○ ○ − + + − − + − ○ + + ○ −	16	12	4	0	4096	2832	1264	0	75.0	25.0	0.0	69.1	30.9	0.0
NN 75	+ − − ● + + ● ● − − − + + + ● −	16	12	0	4	4096	3312	0	784	75.0	0.0	25.0	80.9	0.0	19.1
NN 76	+ − + ● − − ● ● − + + + − + ● −	16	12	0	4	4096	3312	0	784	75.0	0.0	25.0	80.9	0.0	19.1
NN 77	+ ● − − − + + + ● ● − − ● + + −	16	12	0	4	4096	3312	0	784	75.0	0.0	25.0	80.9	0.0	19.1
NN 78	+ ● − + − − − + ● ● + + ● − + −	16	12	0	4	4096	3312	0	784	75.0	0.0	25.0	80.9	0.0	19.1
NN 79	+ ○ − − − + + + ○ ○ − − ○ + + −	16	12	4	0	4096	3312	784	0	75.0	25.0	0.0	80.9	19.1	0.0
NN 80	+ ○ − + − − − + ○ ○ + + ○ − + −	16	12	4	0	4096	3312	784	0	75.0	25.0	0.0	80.9	19.1	0.0
NN 81	+ − − ○ + + ○ ○ − − − + + + ○ −	16	12	4	0	4096	3312	784	0	75.0	25.0	0.0	80.9	19.1	0.0
NN 82	+ − + ○ − − ○ ○ − + + + − + ○ −	16	12	4	0	4096	3312	784	0	75.0	25.0	0.0	80.9	19.1	0.0
NN 83	+ − − ● + + ● − ● − + − + ● + −	16	12	0	4	4096	3456	0	640	75.0	0.0	25.0	84.4	0.0	15.6
NN 84	+ − ● − + − + ● + ● − − ● + + −	16	12	0	4	4096	3456	0	640	75.0	0.0	25.0	84.4	0.0	15.6
NN 85	+ − ○ − + − + ○ + ○ − − ○ + + −	16	12	4	0	4096	3456	640	0	75.0	25.0	0.0	84.4	15.6	0.0
NN 86	+ − − ○ + + ○ − ○ − + − + ○ + −	16	12	4	0	4096	3456	640	0	75.0	25.0	0.0	84.4	15.6	0.0
NN 87	+ − + − ● − ● ● + − + + − ● + −	16	12	0	4	4096	3552	0	544	75.0	0.0	25.0	86.7	0.0	13.3
NN 88	+ − ● + − − + − ● ● + ● + − + −	16	12	0	4	4096	3552	0	544	75.0	0.0	25.0	86.7	0.0	13.3
NN 89	+ − + − ○ − ○ ○ + − + + − ○ + −	16	12	4	0	4096	3552	544	0	75.0	25.0	0.0	86.7	13.3	0.0
NN 90	+ − ○ + − − + − ○ ○ + ○ + − + −	16	12	4	0	4096	3552	544	0	75.0	25.0	0.0	86.7	13.3	0.0
NN 91	+ − − + ● ● − + + ● − − ● + + −	16	12	0	4	4096	3600	0	496	75.0	0.0	25.0	87.9	0.0	12.1
NN 92	+ − − ● + + ● − − + ● ● − + + −	16	12	0	4	4096	3600	0	496	75.0	0.0	25.0	87.9	0.0	12.1
NN 93	+ − − + ○ ○ − + + ○ − − ○ + + −	16	12	4	0	4096	3600	496	0	75.0	25.0	0.0	87.9	12.1	0.0
NN 94	+ − − ○ + + ○ − − + ○ ○ − + + −	16	12	4	0	4096	3600	496	0	75.0	25.0	0.0	87.9	12.1	0.0
NN 299	+ ● ● − − ● + ● − − + + ● + ● ● −	17	10	0	7	4913	2554	0	2359	58.8	0.0	41.2	52.0	0.0	48.0
:		:													
NN 714	+ − ○ + − − + − + − ○ + ○ + − + −	17	14	3	0	4913	4238	675	0	82.4	17.6	0.0	86.3	13.7	0.0
NN 991	+ − ● ● + ● − − − + ● + − + + ● ● −	18	12	0	6	5832	3600	0	2232	66.7	0.0	33.3	61.7	0.0	38.3
:		:													
NN 1268	+ − − + ○ + − ○ ○ ○ − ○ ○ + + − + −	18	12	6	0	5832	5328	504	0	66.7	33.3	0.0	91.4	8.6	0.0
NN 1403	+ ● ● − ● ● ● − − + ● ● ● + + ● ● ● −	19	8	0	11	6859	2936	0	3923	42.1	0.0	57.9	42.8	0.0	57.2
:		:													
NN 4940	+ − − + + ○ − + − − ○ − ○ + + + + − −	19	16	3	0	6859	6544	315	0	84.2	15.8	0.0	95.4	4.6	0.0
NN 5377	+ ● − ● ● ● − + + − − − ● + + + ● ● ● −	20	12	0	8	8000	3888	0	4112	60.0	0.0	40.0	48.6	0.0	51.4
:		:													
NN 8780	+ − + − − + ○ + − ○ − ○ ○ − + + + + − −	20	16	4	0	8000	7744	256	0	80.0	20.0	0.0	96.8	3.2	0.0
NN 10839	+ ● ● ● − ● − ● + − ● − ● + + + ● ● ● ● −	21	10	0	11	9261	3706	0	5555	47.6	0.0	52.4	40.0	0.0	60.0
:		:													
NN 40768	+ − + − + − − + ○ ○ − − + ○ + − + + + − −	21	18	3	0	9261	9090	171	0	85.7	14.3	0.0	98.2	1.8	0.0

Table 12 - Number of **FOL**s for 7 through 21 ply laminates corresponding to prefix designations ${}_{+}NN_{\circ}$ for Non-symmetric (N) angle-plies and Non-symmetric (N) cross-plies.

Ref.	Sequence	n	$n_{\pm}$	$n_{\circ}$	$n_{\bullet}$	ζ	$\zeta_{\pm}$	$\zeta_{\circ}$	$\zeta_{\bullet}$	$n_{\pm}/n$ (%)	$n_{\circ}/n$ (%)	$n_{\bullet}/n$ (%)	$\zeta_{\pm}/\zeta$ (%)	$\zeta_{\circ}/\zeta$ (%)	$\zeta_{\bullet}/\zeta$ (%)
NN 12	+●–○–○○ + ●–++●–○	15	8	4	3	3375	1784	844	747	53.3	26.7	20.0	52.9	25.0	22.1
NN 13	+–●○○●– + –●++●–○	15	8	3	4	3375	1784	891	700	53.3	20.0	26.7	52.9	26.4	20.7
NN 14	+–○●●○– + –●++●–○	15	8	3	4	3375	1784	939	652	53.3	20.0	26.7	52.9	27.8	19.3
NN 15	+–●○○○– + –○++●–○	15	8	5	2	3375	1784	989	602	53.3	33.3	13.3	52.9	29.3	17.8
NN 16	+○–●–●○ + ●–++●–○	15	8	3	4	3375	1784	1035	556	53.3	20.0	26.7	52.9	30.7	16.5
NN 17	+○–●–○● + ○–++●–○	15	8	4	3	3375	1784	1084	507	53.3	26.7	20.0	52.9	32.1	15.0
NN 18	+–○○●●– + –○++●–○	15	8	4	3	3375	1784	1132	459	53.3	26.7	20.0	52.9	33.5	13.6
NN 19	+–○○○●– + –●++○–○	15	8	5	2	3375	1784	1493	98	53.3	33.3	13.3	52.9	44.2	2.9
NN 20	+○–○–○● + ●–++○–○	15	8	5	2	3375	1784	1565	26	53.3	33.3	13.3	52.9	46.4	0.8
NN 21	+–○○○○– + –○++○–○	15	8	7	0	3375	1784	1591	0	53.3	46.7	0.0	52.9	47.1	0.0
NN 22	+○–○–○○ + ○–++○–○	15	8	7	0	3375	1784	1591	0	53.3	46.7	0.0	52.9	47.1	0.0
NN 28	+–●○–○○ + +●●––+○	15	8	4	3	3375	2072	844	459	53.3	26.7	20.0	61.4	25.0	13.6
NN 29	+–○●–○● + +●●––+○	15	8	3	4	3375	2072	939	364	53.3	20.0	26.7	61.4	27.8	10.8
NN 30	+–○○–●● + +○●––+○	15	8	4	3	3375	2072	1132	171	53.3	26.7	20.0	61.4	33.5	5.1
NN 31	+–○○–●○ + +●○––+○	15	8	5	2	3375	2072	1205	98	53.3	33.3	13.3	61.4	35.7	2.9
NN 32	+–○○–○○ + +○○––+○	15	8	7	0	3375	2072	1303	0	53.3	46.7	0.0	61.4	38.6	0.0
NN 43	+–●○○●–+ ○–●++●–○	16	8	4	4	4096	2144	1072	880	50.0	25.0	25.0	52.3	26.2	21.5
⋮		⋮													
NN 74	+–○+––○○ +–++–+–○	16	12	4	0	4096	3024	1072	0	75.0	25.0	0.0	73.8	26.2	0.0
NN 105	+–●○●○●– ● +–++●–●○	17	8	3	6	4913	2168	1179	1566	47.1	17.6	35.3	44.1	24.0	31.9
⋮		⋮													
NN 650	+–+○–○–○ – ○++++––○	17	12	5	0	4913	3708	1205	0	70.6	29.4	0.0	75.5	24.5	0.0
NN 737	+–●●○○●–○ ●++–+–●●○	18	8	4	6	5832	2432	1264	2136	44.4	22.2	33.3	41.7	21.7	36.6
⋮		⋮													
NN 1236	+–+–○–○○○ ○+++–––+○	18	12	6	0	5832	4608	1224	0	66.7	33.3	0.0	79.0	21.0	0.0
NN 1289	+●●–○○–●● ● –+++●●●–○	19	8	3	8	6859	2648	1467	2744	42.1	15.8	42.1	38.6	21.4	40.0
⋮		⋮													
NN 4544	+–+––○○○+ ○ +–+–+––+○	19	14	5	0	6859	5534	1325	0	73.7	26.3	0.0	80.7	19.3	0.0
NN 4958	+●–●●○○–○● ●+–++●●–●○	20	8	4	8	8000	3008	1504	3488	40.0	20.0	40.0	37.6	18.8	43.6
⋮		⋮													
NN 8380	++–––○○+○– –++–+++––○	20	16	4	0	8000	6496	1504	0	80.0	20.0	0.0	81.2	18.8	0.0
NN 8807	+●–●○●○●–– ● ++++––●●●○	21	10	3	8	9261	3130	1827	4304	47.6	14.3	38.1	33.8	19.7	46.5
⋮		⋮													
NN 39424	+––++–○○○○ – +–+–++–+–○	21	16	5	0	9261	7696	1565	0	76.2	23.8	0	83.1	16.9	0