A Formal Solution to Reichenbach's Reference Class Problem

Paul D. THORN[†]

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ABSTRACT

Following Reichenbach, it is widely held that in making a direct inference, one should base one's conclusion on a relevant frequency statement concerning the most specific reference class for which one is able to make a warranted and relatively precise-valued frequency judgment. In cases where one has accurate and precise-valued frequency information for two relevant reference classes, R_1 and R_2, and one lacks accurate and precise-valued frequency information concerning their intersection, $R_1 \cap R_2$, it is widely held, following Reichenbach, that no inference may be drawn. In contradiction to Reichenbach and the common wisdom, I argue for the view that it is often possible to draw a reasonable informative conclusion, in such circumstances. As a basis for drawing such a conclusion, I show that one is generally in a position to formulate a reasonable direct inference for a reference class that is more specific than either of R_1 and R_2.

1. Introduction

Typical instances of direct inference proceed from a premise stating that the frequency with which members of a given reference class, R, are members of a respective target class, T, is *r*, and a premise stating that an object, c, is an element of R, and yields the conclusion that the probability that c is a member of T is *r*. In order to abbreviate the description of such inferences, I use "PROB" to refer to a probability function that takes propositions as arguments, and is understood as designating the personal probabilities that are rational for a respective agent, given the evidence that the agent has. I use "freq" to denote a function that takes a pair of sets as an argument, and returns the relative frequency of the first set among the second (i.e., $\text{freq}(T|R) = |T \cap R|/|R|$). So "$\text{freq}(T|R) = 0.75$" expresses that the relative frequency of Rs (elements of R) that are Ts (elements of T) is 0.75. Given this notation, typical direct inferences conform to the following principle:

Direct Inference [DI]: $c \in R$ and $\text{freq}(T|R) = r$ is a reason for concluding that $\text{PROB}(c \in T) = r$.[1]

Following Reichenbach, it is widely held that in making a direct inference, we should base our conclusion on a relevant frequency statement concerning the most specific reference class for which we are able to make a warranted and (relatively) precise-valued frequency judgment (Reichenbach 1949, 374; cf. Venn 1866; Pollock 1990; Bacchus 1990; Kyburg & Teng 2001; Thorn 2012; Thorn 2017). In cases where we have accurate and precise-valued frequency information for two relevant reference classes, R_1 and R_2, and we lack accurate and precise-valued frequency information concerning their intersection, it is widely held,

[†] Philosophisches Institut, Heinrich-Heine-Universität Düsseldorf; Email: thorn@phil.hhu.de

[1] While many, including Venn (1866), Reichenbach (1949), Kyburg (1974), and Kyburg & Teng (2001), have assumed that the proper major premises for direct inference are statements of frequency or limiting frequency, other proposals have been made, by Pollock (1990), Bacchus (1990), and Thorn (2012, 2017). The modest assumption that point-valued frequency statements *may* serve as major premises for direct inference will be sufficient for the present paper.

following Reichenbach, that no inference may be drawn. In such cases, Reichenbach holds that there is no formal solution to the problem, and that our only course, if we hope to make an inference, is to gather more information concerning the intersection of the two reference classes (Reichenbach 1949, 375).[2] In contradiction to Reichenbach and the common wisdom, I follow in the footsteps of Kyburg and Teng (2001) and Pollock (2011), and argue that it is often possible to draw a reasonable informative conclusion, in such circumstances. As a basis for drawing such a conclusion, I show that one is generally in a position to formulate a reasonable direct inference for a reference class that is more specific than either of R_1 and R_2. After presenting my approach in Section 3, I compare it to the approaches of Kyburg and Teng (2001) and Pollock (2011), in Section 4.

2. Preliminary Discussion

Before proceeding, it will be helpful to get a bit clearer about the sorts of conditions under which direct inferences are defeated. There are two plausible principles that provide a partial specification of the conditions under which instances of [DI] are defeated:[3]

Specificity Defeat [SD]: If D_1 and D_2 are direct inferences leading to mutually inconsistent conclusions, and the reference class for D_1 is more specific than the reference class for D_2, then D_2 is subject to *specificity defeat*.

Rebutting Defeat [RD]: If D_1 and D_2 are direct inferences leading to mutually inconsistent conclusions, and neither inference is defeated via an application of [SD], then both D_1 and D_2 are subject to rebutting defeat.

The preceding principles encapsulate the two core ‘mechanisms’ by which instances of direct inference may interact. In particular, they express the idea that (1) direct inferences based on more specific reference classes have priority, and (2) absent the possibility of prioritizing by appeal to specificity, conflicting direct inferences are subject to mutual defeat.

[SD] and [RD] do not provide a complete specification of the conditions under which instances of direct inference are defeated. For one, the principles do not address the treatment of direct inferences based on gerrymandered reference and target classes. I will not attempt to provide formal criteria for identifying such direct inferences here, but see (Pollock 1990, 84), for examples. Rather I will simply assume that direct inferences based on gerrymandered reference and target classes are defeated, and cannot serve as defeaters for other direct inferences. There is another possible shortcoming of [SD] and [RD], namely: It is plausible that direct inferences leading to mutually *consistent* conclusions can interact, thereby resulting in situations where one or both of the direct inferences are defeated (cf. Stone 1987). For

[2] See, for example, Pust (2011, 292) who appeals to the common wisdom in attempting to refute the arguments made in (Seminar 2008), or even Venn (1866, 222-3), who appears to have held the Reichenbachian view before Reichenbach. Throughout his early work on direct inference (e.g., Pollock 1990), Pollock was committed to the mutual defeat of conflicting direct inferences in cases where neither has a more specific reference class, provisionally accepting Reichenbach’s view without excluding the possibility that some acceptable formal solution to the problem could be formulated.

[3] Many accounts of direct inference, including those of Venn (1866), Reichenbach (1949), Kyburg (1974), Pollock (1990), Kyburg & Teng (2001), and Thorn (2012, 2017), have recognized something like [SD] and [RD], with some variation concerning the treatment of imprecise-valued frequency data. The account of Bacchus (1990) recognizes something like [SD], but takes a ‘credulous’ approach to conflicting direct inferences, in cases where considerations of specificity are inapplicable.

example, suppose we are in a position to make direct inferences in accord with the following (generalized) instances of [DI], in the case where R′ is a proper subset of R:[4]

c∈R and freq(T|R) ∈ [0, 0.6] is a reason for concluding that PROB(c∈T) ∈ [0, 0.6].

c∈R′ and freq(T|R′) ∈ [0.6, 1] is a reason for concluding that PROB(c∈T) ∈ [0.6, 1].

In this case, the conclusions of the two direct inferences are consistent. But if neither of the two direct inferences were defeated, it would be correct to infer that PROB(c∈T) = 0.6, which is not plausible. So it appears that a direct inference based on a more specific reference class can trigger the defeat of a direct inference based on a less specific reference class, even if the conclusions of the two direct inferences are mutually consistent. On the other hand, the possibility of formulating a direct inference based on imprecise-valued frequency information for a narrower reference class does not always result in the defeat of a direct inference for a broader reference class. If this was not the case, then all interesting instances of [DI] would be defeated, due to (generalized) instances of [DI] of the following form:

c∈{c} and freq(T|{c})∈{0,1} is a reason for concluding that PROB(c∈T)∈{0,1}.[5]

In the present paper, I will not address the difficult problem of precisely characterizing when imprecise-valued frequency information for a more specific reference class yields the defeat of a direct inference based on a broad reference class. Rather than do this, I will focus on two types of clear case where this problem does not arise, namely: cases where the two direct inferences yield mutually inconsistent point-valued conclusions (so direct inference based on the broader reference class is defeated by appeal to [SD]), and cases where we have *no* non-trivial frequency information about the value of freq(T|R′) for the narrower reference class R′ (so direct inference based on the broad reference class is *not* defeated by appeal to [SD]).

So far I have proceeded under the assumption that a reference class R′ is more specific than a reference class R, if R′ is a proper subset of R. For the sake of addressing Reichenbach's reference class problem, I will need to apply specificity criteria that permit arbitration between reference classes whose elements are tuples of different arity (e.g., reference classes whose elements are individual objects versus reference classes whose elements are ordered pairs of objects). My proposal will involve comparing open first order formulae that correspond to respective reference classes. To this end, I will say that an open first order formula $R(x_1,\ldots,x_n)$ *encapsulates* a set R if and only if $R = \{\langle c_1,\ldots,c_n\rangle : \langle c_1,\ldots,c_n\rangle$ satisfies $R(x_1,\ldots,x_n)\}$, or $R = \{c_1 : c_1$ satisfies $R(x_1)\}$, in the case where $n = 1$. The specificity condition that is used in determining the applicability of [SD] is then characterized as follows (where "⊃" is used to express material implication).

Specificity: R′ is *more specific* than R *if and only if*

[4] The following are described as generalized instances of [DI], since they involve inference from imprecise-valued frequency statements to imprecise-valued probability statements.

[5] In explaining why direct inferences of the present form are not admissible, and do not result in the defeat of direct inferences based on non-trivial frequency information, I favor the proposal that it is statements of *expected* frequency (and not of frequency) that are the proper major premises for direct inference (Bacchus 1990, Thorn 2012). This proposal delivers the correct results, since it permits the use of known frequencies as premises for direct inference (since if PROB(freq(T|R) = r) = 1, then the expectation of freq(T|R) is r), and explains why imprecise-valued frequency information for more specific reference classes does not yield the defeat of direct inferences based on precise-valued frequency information for less specific reference classes (since PROB(freq(T|R)∈S) = 1 does not imply that the expectation of freq(T|R) is in S) (Thorn 2012, 311-13).

(1) there are encapsulations $R(x_1,\ldots,x_n)$ and $R'(x_1,\ldots,x_m)$ of R and R′, such that:
$\forall x_1,\ldots,x_k$: $R'(x_1,\ldots,x_m) \supset R(x_1,\ldots,x_n)$ (where $k = \max\{m, n\}$), and
(2) there are no encapsulations $R(x_1,\ldots,x_n)$ and $R'(x_1,\ldots,x_m)$ of R and R′, such that:
$\forall x_1,\ldots,x_k$: $R(x_1,\ldots,x_n) \supset R'(x_1,\ldots,x_m)$ (where $k = \max\{m, n\}$).

The idea behind the preceding proposal (which is similar to the proposals of Pollock 1990, Kyburg & Teng 2001, and Thorn 2011, cf. Pust 2011) is that a reference class R′ is more specific than a reference class R just in case (for some first order specification of R′ and R) any tuple/object that satisfies the first order formula for R′ also satisfies the first order formula for R, but not vice versa. Despite capturing the preceding idea, it must be acknowledged that the proposed specificity criteria fail to recognize some intuitive cases of specificity. For example, the proposed criteria fail to recognize that $\{x : x \text{ is taller than Bob}\}$ is more specific than $\{\langle x,y\rangle : x \text{ is taller than } y\}$. Note, however, that the present omission is harmless, since in any situation where one is in a position to make a direct inference using the reference class $\{x : x \text{ is taller than Bob}\}$, one is also in a position to make a direct inference, to the very same conclusion, using the reference class $\{\langle x,y\rangle : x \text{ is taller than } y \wedge y = \text{Bob}\}$. The preceding fact effectively 'disarms' the counterexample to the proposed specificity criteria, since those criteria classify $\{\langle x,y\rangle : x \text{ is taller than } y \wedge y = \text{Bob}\}$ as more specific than $\{\langle x,y\rangle : x \text{ is taller than } y\}$, which is sufficient for delivering the correct conclusion about which direct inferences are subject to specificity defeat via [SD].

3. Reichenbach's Problem and Its Solution

I here consider the problem of determining what to conclude when one is in a position to make two competing direct inferences, of the following form, in cases where (i) neither R_1 is more specific than R_2 nor R_2 is more specific than R_1, and (ii) for all S: if one has non-trivial information about the value of freq(T|S), then c is not in S, or R_1 is more specific than S, or R_2 is more specific than S:

(DI_{R_1}): $c \in R_1$ and $\text{freq}(T|R_1) = r_1$ is a reason for concluding that $\text{PROB}(c \in T) = r_1$.

(DI_{R_2}): $c \in R_2$ and $\text{freq}(T|R_2) = r_2$ is a reason for concluding that $\text{PROB}(c \in T) = r_2$.

I call the problem of determining what to conclude in such cases "Reichenbach's reference class problem".[6] Notice that (i) excludes cases where (DI_{R_1}) is subject to specificity defeat via (DI_{R_2}), and cases where (DI_{R_2}) is subject to specificity defeat via (DI_{R_1}). In interpreting (ii), "non-trivial" information should be understood as information that permits one to validly infer a conclusion about the value of freq(T|S) that is not implied by one's frequency information for R_1 and R_2 (i.e., the information that $\text{freq}(T|R_1) = r_1$ and $\text{freq}(T|R_2) = r_2$). So (ii) excludes cases where (DI_{R_1}) and (DI_{R_2}) are subject to specificity defeat based on information concerning the value of $\text{freq}(T|R_1 \cap R_2)$, since $\text{freq}(T|R_1)$ and $\text{freq}(T|R_2)$ place no constraints on the value of $\text{freq}(T|R_1 \cap R_2)$ (assuming $\{r_1, r_2\} \cap \{0, 1\} = \varnothing$). Beyond this, (ii) probably excludes more cases than necessary. But rather than attempt to delineate better fitting criteria for when the approach to be proposed is applicable, I play it safe, and stick with a core set of cases where I am confident that the approach is appropriate. In the end,

[6] I introduce the expression "Reichenbach's reference class problem," in order to distinguish the problem considered here from other worries concerning the selection of appropriate reference classes (cf. Hájek 2007; Thorn 2012).

conditions (i) and (ii) will not be very important, since it is intended that the range of cases to which the proposed approach is applicable is ultimately determined by [SD]. I will return to this point a little later.

My approach to Reichenbach's reference class problem proceeds by applying our frequency information for the two competing reference classes, R_1 and R_2, in order to draw a reasonable conclusion about the likelihood that the given object, c, is an element of the given target class T. The basic idea is to: (1) form a reference class that is more specific than R_1 and R_2 that includes an object that corresponds to c, (2) determine the frequency of membership in T among that reference class, and (3) use the resulting frequency statement to formulate a new direct inference, in order to draw a conclusion about the probability that c is in T. As a basis for the proposed direct inference, consider the set of pairs whose first element is in R_1 and whose second element is in R_2, such that both elements are in T or both are not, namely: $\{\langle x,y\rangle : x\in R_1 \wedge y\in R_2 \wedge ((x\in T \wedge y\in T) \vee (x\notin T \wedge y\notin T))\}$. For ease of reference, call this set R*. Note that while c is not a member of R*, $\langle c,c\rangle$ is. The next thing to notice about R* is that it is more specific than R_1, and more specific than R_2. In particular, $\forall x,y$: $(x\in R_1 \wedge y\in R_2 \wedge ((x\in T \wedge y\in T) \vee (x\notin T \wedge y\notin T))) \supset x\in R_1$, but not $\forall x,y$: $x\in R_1 \supset (x\in R_1 \wedge y\in R_2 \wedge ((x\in T \wedge y\in T) \vee (x\notin T \wedge y\notin T)))$, and similarly for R_2. Given [SD], the preceding implies that direct inference using frequency information for R* takes priority over direct inference using frequency information for either R_1 or R_2, in cases where one has reliable precise-valued frequency information for R*. Now notice that in cases where Reichenbach's reference class problem arises, we are *always* in a position to deduce the precise value of $\text{freq}(\{\langle x,y\rangle : x\in T\}|R^*)$, via the following theorem:[7]

Theorem 1: $\forall T,R_1,R_2$: $R^* \neq \varnothing \Rightarrow \text{freq}(\{\langle x,y\rangle : x\in T\}|R^*) = (\text{freq}(T|R_1)\times\text{freq}(T|R_2)) / (\text{freq}(T|R_1)\times\text{freq}(T|R_2) + \text{freq}(T^C|R_1)\times\text{freq}(T^C|R_2))$.

Proof:

Recall that $R^* = \{\langle x,y\rangle : x\in R_1 \wedge y\in R_2 \wedge ((x\in T \wedge y\in T) \vee (x\notin T \wedge y\notin T))\}$.

Let $R^*_T = \{\langle x,y\rangle : x\in R_1 \wedge y\in R_2 \wedge x\in T \wedge y\in T\}$, and
$R^*_{T^c} = \{\langle x,y\rangle : x\in R_1 \wedge y\in R_2 \wedge x\notin T \wedge y\notin T\}$.

Then we have the following:

$$R^* = R^*_T \cup R^*_{T^c},$$
$$R^*_T \cap R^*_{T^c} = \varnothing,$$
$$|R^*| = |R^*_T| + |R^*_{T^c}|,$$
$$|R^*_T| = |T\cap R_1|\times|T\cap R_2| = \text{freq}(T|R_1)\times|R_1| \times \text{freq}(T|R_2)\times|R_2|, \text{ and}$$
$$|R^*_{T^c}| = |T^C\cap R_1|\times|T^C\cap R_2| = \text{freq}(T^C|R_1)\times|R_1| \times \text{freq}(T^C|R_2)\times|R_2|.$$

So, assuming $R^* \neq \varnothing$, we have:

$$\text{freq}(\{\langle x,y\rangle : x\in T\}|R^*) = |\{\langle x,y\rangle : x\in T\}\cap R^*|/|R^*| = |R^*_T|/(|R^*_T|+|R^*_{T^c}|)$$
$$= (\text{freq}(T|R_1)\times\text{freq}(T|R_2)) / (\text{freq}(T|R_1)\times\text{freq}(T|R_2) + \text{freq}(T^C|R_1)\times\text{freq}(T^C|R_2)).\square$$

By appeal to the preceding theorem, we may infer the value of $\text{freq}(\{\langle x,y\rangle : x\in T\}|R^*)$, and then draw a conclusion about the value of $\text{PROB}(c\in T)$, via the following instance of [DI]:

[7] Note that if $R^* = \varnothing$, then the respective case is not one where Reichenbach's reference class problem arises, since $R^* = \varnothing$ implies that $R_1\cap R_2 = \varnothing$, and thus that $c\notin R_1$ or $c\notin R_2$.

(DI_{R*}): $\langle c,c\rangle \in R^*$ and freq($\{\langle x,y\rangle : x\in T\}|R^*$) = r is a reason for concluding that PROB($\langle c,c\rangle \in \{\langle x,y\rangle : x\in T\}$) = r (i.e., that PROB($c\in T$) = r).

For example, suppose we want to form a judgment about the likelihood that Robert, one of the residents of a given apartment building, owns a car, and we are in a situation that is an instance of Reichenbach's reference class problem: We know that Robert lives in a south facing apartment (i.e., is in R_1) and that the frequency of car ownership among the members of R_1 is 0.6, and we know that Robert lives on the third floor (i.e., is in R_2) and that the frequency of car ownership among the members of R_2 is 0.4, but we have no information concerning the frequency of car ownership among residents of third floor west facing apartments (i.e., among $R_1 \cap R_2$). At this point, we notice that we can form a reference class formed of pairs drawn from R_1 and R_2, as specified by R* (i.e., the set of pairs whose first element is a resident of a south facing apartment, and whose second element is a resident of a third floor apartment, where both elements are car owners or neither is a car owner). We also know that ⟨Robert, Robert⟩ is an element of R*, and, in the present situation, we can calculate that the frequency of pairs whose elements are car owners among R* is 0.5 (by an application of Theorem 1). So we are in a position to make a direct inference, via an instance of (DI_{R*}), to the conclusion that PROB(⟨Robert, Robert⟩$\in \{\langle x,y\rangle : x\in T\}$) = 0.5, and thus to the conclusion that the probability that Robert owns a car is 0.5.

In cases where Reichenbach's reference class problem arises, it is intended that the justification for reasoning by appeal to instances of (DI_{R*}) derives from [SD]. Indeed, R* is more specific than R_1, and more specific than R_2. So [SD] tells us that direct inference based on R* takes priority over direct inference by R_1 or R_2. On the other hand, $\{\langle x,y\rangle : x\in R_1 \wedge y\in R_2 \wedge x = y\}$ is more specific than R*, and in any situation where one is in a position to make a direct inference using $\{\langle x,y\rangle : x\in R_1 \wedge y\in R_2 \wedge x = y\}$ as one's reference class, one is in a position to make a direct inference to the very same conclusion using $R_1 \cap R_2$ as one's reference class. This means that direct inference using informative frequency information for $R_1 \cap R_2$ takes priority to direct inference using frequency information for R*. Note, however, that while making a direct inference based on $R_1 \cap R_2$ would be preferable to making a direct inference based on R*, we lack informative frequency information for $R_1 \cap R_2$ in cases where Reichenbach's reference class problem arises. For this reason, direct inference based on R* may be reasonable, in such cases. I here say that direct inference based on R* "may be reasonable" in such cases, since in addition to $R_1 \cap R_2$ (and $\{\langle x,y\rangle : x\in R_1 \wedge y\in R_2 \wedge x = y\}$), there are many reference classes that are more specific than R*, and in cases where we have reliable precise-valued frequency information for a reference class that is more specific than R*, direct inference based on R* will be defeated via [SD]. For example, in a case where one is justified in accepting the premises for (DI_{R_1}) and (DI_{R_2}) and one has no non-trivial frequency information about the value of freq($T|R_1 \cap R_2$), one might yet know that c is a member of T (or one might know the frequency of T among some other subset of $R_1 \cap R_2$ that is known to contain c). In such cases, it is clear that one should not form one's judgment concerning the probability that c is a member of T using (DI_{R*}).

As I have already stated, the justification for forming one's conclusion about the value of PROB($c\in T$) via an instance of (DI_{R*}) (in cases where Reichenbach's reference class problem arises) derives from [SD]. Beyond this justification, the following four propositions suggest that direct inference via (DI_{R*}) yields reasonable conclusions about the value of PROB($c\in T$), in the range of cases under consideration:

Symmetry: $\forall T,R_1,R_2$: freq($T|R_1$) = freq($T^C|R_2$) $\Rightarrow$ freq($\{\langle x,y\rangle : x\in T\}|R^*$) = 0.5.

Asymmetry: $\forall$T,R_1,R_2: freq(T|R_1) > freq(T^C|R_2) $\Rightarrow$ freq($\{\langle x,y\rangle : x\in T\}$|R*) > 0.5.

Balance: $\forall$T,R_1,R_2,*r*: freq(T|R_1) = 0.5 $\wedge$ freq(T|R_2) = *r* $\Rightarrow$ freq($\{\langle x,y\rangle : x\in T\}$|R*) = *r*.

Synergy: $\forall$T,R_1,R_2,*r*: 1 > freq(T|R_1) = *r* > 0.5 $\wedge$ freq(T|R_2) > 0.5 $\Rightarrow$ freq($\{\langle x,y\rangle : x\in T\}$|R*) > *r*.[8]

In the range of cases under consideration, I regard the prescriptions deriving from the first of the above propositions, *Symmetry*, as more or less indubitable: If the frequency of Ts in R_1 is identical to the frequency of non-Ts in R_2, then the information bearing on the likelihood of T vs. non-T, for an object that is a member of both R_1 and R_2, is *symmetric*, suggesting that one ought to conclude that the object is as likely to be a member of T as not. On the other hand, if the frequency of Ts in R_1 is greater than the frequency of non-Ts in R_2, then information bearing on the likelihood of being in T vs. non-T is *asymmetric*, with the balance tilted toward T, suggesting that one ought to conclude that the object is more likely to be a member of T than not. *Balance* yields the result that the likelihood that an object is in T be identical to the frequency of T in R_2, in cases where a member of R_1 is as likely to be a member of T as not. Although this principle is probably dubitable, it is intuitively plausible: If membership in R_1 is neither positively nor negatively probative for membership in T, then one should base one's judgment concerning the likelihood of an object's membership in T on the fact that the object is a member of R_2. *Synergy* yields the result that the likelihood that an object is in T is greater than the frequency of T in R_1, in the case where the object is a member of both R_1 and R_2, and the frequency of T is greater than 0.5 in both R_1 and R_2. This principle is also intuitively plausible: If having either of two distinct characteristics (R_1 and R_2) makes it likely that an object has some third characteristic (T), then, in the absence of countervailing evidence, having both characteristics (R_1 and R_2) makes it even more likely that the object has the third characteristic.

Note that the consequent of the preceding conditional does not rely on a commitment to the claim that freq(T|$R_1\cap R_2$) is greater than both freq(T|R_1) and freq(T|R_2), though it does presumably rely on a commitment to the claim that the *expectation* of freq(T|$R_1\cap R_2$) is greater than both freq(T|R_1) and freq(T|R_2) (i.e., that the probability weighted average of the possible values of freq(T|$R_1\cap R_2$) is greater than both freq(T|R_1) and freq(T|R_2)). More generally, it must be emphasized that the proposed approach to Reichenbach's reference class problem does not involve a commitment to the claim that freq(T|$R_1\cap R_2$) = freq($\{\langle x,y\rangle : x\in T\}$|R*), in cases where the approach is applied. Rather, when reasoning in accordance with (DI_{R*}), ignorance of the value of freq(T|$R_1\cap R_2$) is on par with ignorance of the value of freq(T|{c}), and presents no defeater for instances of (DI_{R*}). The present point is important, since the values of freq($\{\langle x,y\rangle : x\in T\}$|R*) and freq(T|$R_1\cap R_2$) may differ dramatically, as implied by the fact that the values of freq(T|R_1) and freq(T|R_2) place no constraints on the value of freq(T|$R_1\cap R_2$), assuming $\{r_1, r_2\}\cap\{0, 1\} = \varnothing$.

A significant worry regarding (DI_{R*}) was pointed out by a referee for this paper. The worry is that there are many reference classes narrower than R* whose relative frequencies

[8] All of the principles follow straightforwardly from Theorem 1. For example, asymmetry is straightforward consequence of the fact that freq(T|R_1) > freq(T^C|R_2) implies freq(T|R_2) > freq(T^C|R_1), for all T, R_1, and R_2. For synergy, notice that for all T, R_1, R_2: if ε = freq(T|R_2) – 1/2, then (freq(T|R_1)×freq(T|R_2)) / (freq(T|R_1)×freq(T|R_2) + freq(T^C|R_1)×freq(T^C|R_2)) = (freq(T|R_1)/2 + freq(T|R_1)×ε) / (1/2 + (2×freq(T|R_1) – 1)×ε). So, for reductio, one may assume that (freq(T|R_1)/2 + freq(T|R_1)×ε) / (1/2 + (2×freq(T|R_1) – 1)×ε) ≤ freq(T|R_1) (for some T, R_1, R_2), which implies that (freq(T|R_1)/2 + freq(T|R_1)×ε) / (1/2 + (2×freq(T|R_1) – 1)×ε) ≤ (freq(T|R_1)/2 + freq(T|R_1)×(2×freq(T|R_1) – 1)×ε)) / (1/2 + (2×freq(T|R_1) – 1)×ε), and thus that freq(T|R_1)/2 + freq(T|R_1)×ε ≤ freq(T|R_1)/2 + freq(T|R_1)×(2×freq(T|R_1) – 1)×ε, which is absurd, since 2×freq(T|R_1) – 1 < 1.

can be determined in a manner similar to the value of freq($\{\langle x,y\rangle : x\in\mathrm{T}\}|\mathrm{R}^*$). Consider, for example, the following reference class:

$$\mathrm{R}^\dagger = \{\langle x,y,z\rangle : x\in \mathrm{R}_1 \wedge y\in \mathrm{R}_2 \wedge z\in \mathrm{R}_2 \wedge ((x\in \mathrm{T} \wedge y\in \mathrm{T} \wedge z\in \mathrm{T}) \vee (x\notin \mathrm{T} \wedge y\notin \mathrm{T} \wedge z\notin \mathrm{T}))\}.$$

In a manner similar to freq($\{\langle x,y\rangle : x\in\mathrm{T}\}|\mathrm{R}^*$), we can compute the value of freq($\{\langle x,y,z\rangle : x\in\mathrm{T}\}|\mathrm{R}^\dagger$), namely:

$$\mathrm{freq}(\{\langle x,y,z\rangle : x\in\mathrm{T}\}|\mathrm{R}^\dagger) = (\mathrm{freq}(\mathrm{T}|\mathrm{R}_1)\times\mathrm{freq}(\mathrm{T}|\mathrm{R}_2)^2) \ / \ (\mathrm{freq}(\mathrm{T}|\mathrm{R}_1)\times\mathrm{freq}(\mathrm{T}|\mathrm{R}_2)^2 + \mathrm{freq}(\mathrm{T}^\mathrm{C}|\mathrm{R}_1)\times\mathrm{freq}(\mathrm{T}^\mathrm{C}|\mathrm{R}_2)^2).$$

This leaves the possibility of formulating the following instance of [DI]:

($\mathrm{DI}_{\mathrm{R}^\dagger}$): $\langle c,c,c\rangle\in\mathrm{R}^\dagger$ and freq($\{\langle x,y,z\rangle : x\in\mathrm{T}\}|\mathrm{R}^\dagger$) = $r_\dagger$ is a reason for concluding that PROB($\langle c,c,c\rangle\in\{\langle x,y,z\rangle : x\in\mathrm{T}\}$) = $r_\dagger$ (i.e., that PROB($c\in\mathrm{T}$) = $r_\dagger$).

Notice ($\mathrm{DI}_{\mathrm{R}^*}$) and ($\mathrm{DI}_{\mathrm{R}^\dagger}$) will typically yield conflicting conclusions. For example, if freq($\mathrm{T}|\mathrm{R}_1$) = 0.75 and freq($\mathrm{T}|\mathrm{R}_2$) = 0.25, then freq($\{\langle x,y\rangle : x\in\mathrm{T}\}|\mathrm{R}^*$) = 0.5 and freq($\{\langle x,y,z\rangle : x\in\mathrm{T}\}|\mathrm{R}^\dagger$) = 0.25. So ($\mathrm{DI}_{\mathrm{R}^*}$) tells us to infer that PROB($c\in\mathrm{T}$) = 0.5, and ($\mathrm{DI}_{\mathrm{R}^\dagger}$) tells us to infer that PROB($c\in\mathrm{T}$) = 0.25. Further, note that $\mathrm{R}^\dagger$ is more specific than R*, and this yields the defeat of ($\mathrm{DI}_{\mathrm{R}^*}$), via [SD].

The key to addressing the present problem is to notice that for any direct inference such as ($\mathrm{DI}_{\mathrm{R}^\dagger}$), it is possible to formulate a direct inference with a reference class that is more specific than $\mathrm{R}^\dagger$ that yields the same conclusion as ($\mathrm{DI}_{\mathrm{R}^*}$). For example, in the face of $\mathrm{R}^\dagger$ and ($\mathrm{DI}_{\mathrm{R}^\dagger}$), we can formulate the following reference class, and a corresponding instance of [DI]:

$$\mathrm{R}^{\dagger *} = \{\langle x,y,z\rangle : x\in \mathrm{R}_1 \wedge y\in \mathrm{R}_2 \wedge y = z \wedge ((x\in \mathrm{T} \wedge y\in \mathrm{T}) \vee (x\notin \mathrm{T} \wedge y\notin \mathrm{T}))\}.$$

($\mathrm{DI}_{\mathrm{R}^{\dagger *}}$): $\langle c,c,c\rangle\in\mathrm{R}^{\dagger *}$ and freq($\{\langle x,y,z\rangle : x\in\mathrm{T}\}|\mathrm{R}^{\dagger *}$) = r is a reason for concluding that PROB($\langle c,c,c\rangle\in\{\langle x,y,z\rangle : x\in\mathrm{T}\}$) = r (i.e., that PROB($c\in\mathrm{T}$) = r).

Notice that freq($\{\langle x,y,z\rangle : x\in\mathrm{T}\}|\mathrm{R}^{\dagger *}$) = freq($\{\langle x,y\rangle : x\in\mathrm{T}\}|\mathrm{R}^*$). So ($\mathrm{DI}_{\mathrm{R}^{\dagger *}}$) permits us to reason to the same conclusion as ($\mathrm{DI}_{\mathrm{R}^*}$). Next, notice that $\mathrm{R}^{\dagger *}$ is more specific that $\mathrm{R}^\dagger$ (which is evident, since $\mathrm{R}^{\dagger *}$ is a proper subset of $\mathrm{R}^\dagger$). So ($\mathrm{DI}_{\mathrm{R}^\dagger}$) is defeated due to our frequency information for $\mathrm{R}^{\dagger *}$. If we like, we can adopt the terminology of Pollock (1990, 87), and say that ($\mathrm{DI}_{\mathrm{R}^*}$) is "reinstated", due to the defeat of ($\mathrm{DI}_{\mathrm{R}^\dagger}$). Regardless, if ($\mathrm{DI}_{\mathrm{R}^{\dagger *}}$) is undefeated, we can use it to reason to the very same conclusion as the one that we would have reached using ($\mathrm{DI}_{\mathrm{R}^*}$). There is, however, a residual problem: It is possible to formulate variants of $\mathrm{R}^\dagger$ that yield the defeat of direct inferences such as ($\mathrm{DI}_{\mathrm{R}^{\dagger *}}$). Consider, for example, the reference class: $\{\langle x,y,z,w\rangle : x\in \mathrm{R}_1 \wedge y\in \mathrm{R}_2 \wedge y = z \wedge w\in \mathrm{R}_2 \wedge ((x\in \mathrm{T} \wedge y\in \mathrm{T} \wedge w\in \mathrm{T}) \vee (x\notin \mathrm{T} \wedge y\notin \mathrm{T} \wedge w\notin \mathrm{T}))\}$. Of course, we can also formulate further variants of $\mathrm{R}^{\dagger *}$ (e.g., $\{\langle x,y,z,w\rangle : x\in \mathrm{R}_1 \wedge y\in \mathrm{R}_2 \wedge y = z = w \wedge ((x\in \mathrm{T} \wedge y\in \mathrm{T}) \vee (x\notin \mathrm{T} \wedge y\notin \mathrm{T}))\}$) that can be used to formulate defeaters for direct inferences based on the variants of $\mathrm{R}^\dagger$, and so on. So it looks like it is possible to formulate an infinitely extending 'regress' of direct inferences, with each direct inference yielding the defeat of the preceding element of the regress. A reasonable means to halt such a regress (and reach the conclusion recommended by ($\mathrm{DI}_{\mathrm{R}^*}$)) is to demand a finite upper bound on the size of the tuples that may form the elements of a permissible reference class. Such a measure will stop the regress, since, for all n, the n-tuple variant of $\mathrm{R}^{\dagger *}$ is more specific than the n-tuple variant of $\mathrm{R}^\dagger$, which means that an upper bound on the arity of tuples for permissible reference classes, will result in the defeat of all variants of $R^{\dagger}$, but not of all variants of $R^{\dagger}*$. In fact, we need only demand that for every context, there is some (as large as one likes) finite upper bound that applies in that context. Once again, we can adopt the terminology of Pollock (1990, 87), and say that (DI_{R*}) is "reinstated", due to the defeat of all variants of ($DI_{R^{\dagger}}$). Regardless, demanding an upper bound on the arity of tuples for permissible reference classes, will leave an undefeated variant of ($DI_{R^{\dagger *}}$), and the conclusion of this direct inference will be equivalent to the conclusion of (DI_{R^*}).

It strikes me as plausible that, for each context, there is some upper bound on the size of the tuples that may form the elements of a suitable reference class (e.g., the bound n^n, in the case where n is the number of elements in the domain about which one is reasoning). That said, I am not aware of a *direct* and compelling argument in favor of such bounds. There is, however, a relatively compelling *indirect* argument in favor of such bounds.[9] The argument (by reductio) turns on the assumption that principles in the vicinity of the principles expressed in the preceding sections (namely, [DI], [SD], and [RD]) are correct, and the assumption that *some* direct inferences are undefeated. In that case, notice that absent an upper bound on the size of the tuples that may form the elements of a suitable reference class (or absent an equivalent mechanism, e.g., as suggested in footnote 7), the application of [SD] (assuming something in the vicinity of the specificity condition introduced in the preceding section) would yield the defeat of *all* direct inferences, via infinitely extending 'regresses' of direct inferences. As an illustration, suppose we wish to make a respectable direct inference based on the frequency statement freq(T|R) = 0.6. Notice that we may formulate a defeater for the proposed direct inference based on the following frequency statement: $\text{freq}(\{\langle x,y\rangle : x\in T\}|\{\langle x,y\rangle : x\in R \wedge y\in R \wedge ((x\in T \wedge y\in T) \vee (x\notin T \wedge y\notin T))\}) = 9/13$. Yet we can also formulate a defeater for the latter direct inference based on the following frequency statement: $\text{freq}(\{\langle x,y\rangle : x\in T\}|\{\langle x,y\rangle : x\in R \wedge x = y\}) = 0.6$. And so on. So if we think that some direct inferences are uniquely reasonable, then we have reason to adopt a measure of the sort that is needed to rescue (DI_{R*}) from ($DI_{R^{\dagger}}$) and its variants.

Before proceeding, I should mention that the proposed approach to Reichenbach's reference class problem permits of generalization along at least *four* dimensions, in order to apply in situations where: (i) the number of relevant intersecting reference classes for which we have frequency information (here R_1 and R_2) exceeds two, (ii) we have frequency information for finer-grained target classes (e.g., we know the values of $\text{freq}(T\cap S|R_1)$, $\text{freq}(T\cap S^C|R_1)$, $\text{freq}(T^C\cap S|R_1)$, and $\text{freq}(T^C\cap S^C|R_1)$), (iii) our frequency information for the intersecting reference classes (here R_1 and R_2) is imprecise-valued, and (iv) we have non-trivial frequency information for the intersection of the relevant reference classes (here $R_1\cap R_2$). Discussion of situations where any of (i) through (iv) apply is left to another occasion. That said, it is important to acknowledge that (along with situations of type (iv)) situations of type (i) and (ii) can generate defeaters for instances of (DI_{R*}).

4. Other Proposals

The treatment of Reichenbach's reference class problem proposed here has some competitors. Before concluding the present paper, I briefly compare my proposal with the two most plausible competitors, namely: the proposal of Kyburg and Teng (2001), and the proposal of Pollock (2011).

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[9] Alternatively, one could achieve similar ends by more or less equivalent means. For example, one could maintain that inference to a particular probability statement is 'ultimately' undefeated (cf. Pollock 1990, 89-90), if, for all n, there is an undefeated direct inference to that probability statement, so long as we only consider direct inferences whose reference classes consist of tuples of length n or less.

4.1. Comparison with Kyburg and Teng

In adjudicating between direct inferences such as (DI_{R_1}) and (DI_{R_2}) (where neither R_1 is a proper subset of R_2 nor R_2 is a proper subset of R_1), Kyburg and Teng (2001) propose that we conclude that PROB($c \in T$) lies within the smallest interval that includes r_1 and r_2. For example, if freq($T|R_1$) = 0.4 and freq($T|R_2$) = 0.6, then the account of Kyburg and Teng recommends the conclusion that PROB($c \in T$) $\in$ [0.4, 0.6].

The proposal of Kyburg and Teng frequently leads to conclusions that are more conservative than the ones proposed in the present paper. Indeed, while the account of Kyburg and Teng recommends the conclusion that PROB($c \in T$) $\in$ [0.4, 0.6], in the case where freq($T|R_1$) = 0.4 and freq($T|R_2$) = 0.6, my proposal recommends the conclusion that PROB($c \in T$) = 0.5, in accordance with *symmetry* (as described in the preceding section). If freq($T|R_1$) = 0.4 and freq($T|R_2$) = 0.7, the account of Kyburg and Teng recommends the conclusion that PROB($c \in T$) $\in$ [0.4, 0.7], whereas my proposal recommends the conclusion that PROB($c \in T$) = 14/23 $\approx$ 0.61, in accordance with *asymmetry*. If freq($T|R_1$) = 0.5 and freq($T|R_2$) = 0.7, then the account of Kyburg and Teng recommends the conclusion that PROB($c \in T$) $\in$ [0.5, 0.7], while mine recommends the conclusion that PROB($c \in T$) = 0.7, in accordance with *balance*. In other cases, the proposal of Kyburg and Teng recommends conclusions that are *inconsistent* the ones proposed within the present paper. For example, if freq($T|R_1$) = 0.6 and freq($T|R_2$) = 0.7, then the account of Kyburg and Teng recommends the conclusion that PROB($c \in T$) $\in$ [0.6, 0.7], whereas the approach of the present paper recommends the conclusion that PROB($c \in T$) = 7/9 $\approx$ 0.78, in accordance with *synergy*.

In the face of direct inferences that support contradictory conclusions, the basic idea of Kyburg and Teng is to adopt a conclusion that is: (1) consistent with the conclusions of the two direct inferences, and (2) relatively informative. While their approach is not without some prima facie plausibility, it is also, arguably, ad hoc. Regardless, it appears that advocates of Kyburg and Teng's approach should be open (if not compelled) to accept the approach to Reichenbach's reference class problem proposed in this paper, since the approach of Kyburg and Teng is committed to the preference for direct inferences based on frequency information for more specific reference classes. In particular, Kyburg and Teng are committed to prefer direct inference via R^* over either of R_1 and R_2, according to their criteria for sharpening by richness (Kyburg & Teng 2001, 215-7). Furthermore, since $\text{freq}(\{\langle x,y\rangle : x \in T\}|R^*) = (\text{freq}(T|R_1)\times\text{freq}(T|R_2)) / (\text{freq}(T|R_1)\times\text{freq}(T|R_2) + \text{freq}(T^C|R_1)\times\text{freq}(T^C|R_2))$ is a mathematical truth (given $R^* \neq \varnothing$), an appropriate direct inference based on frequency information for R^* is available in all cases where Reichenbach's reference class problem arises.

4.2. Comparison with Pollock (2011)

Pollock's approach to direct inference differs from the approach taken here, due to the former's formulation within Pollock's general framework of 'nomic probabilities', where statements of nomic probability, rather than frequency, are taken as the proper major premises for direct inference (Pollock 1990). The framework of Pollock's approach is thus idiosyncratic and highly non-standard. However, it turns out that most of Pollock's insights can be re-expressed within a framework where statements of frequency are taken as the proper major premises for direct inference.[10] I think much clarity is gained when Pollock's insights are so expressed, and so I will present Pollock's ideas concerning the reference class problem within such a framework. In that case, the account of Pollock has several affinities to

[10] Pollock's system also permits direct inference using frequency statements, by appeal to a principle that Pollock calls "PFREQ" (Pollock 1990, 70).

the approach to Reichenbach's reference class problem proposed within the present paper. Indeed, in the face of competing direct inferences based on a pair of reference classes R_1 and R_2, Pollock proposes to (1) form a judgment about the frequency of T among a reference class that contains c and is more specific than either R_1 or R_2, and (2) use the resulting frequency judgment as the major premise for a new direct inference. Furthermore, the conclusions entailed by Pollock's approach are identical to the ones entailed by mine, under certain limiting conditions, as described below.

In the face of Reichenbach's reference class problem, Pollock proposes that one directly estimate the value of $freq(T|R_1 \cap R_2)$ by locating the triple consisting of T, R_1, and R_2 among an appropriate set of similar triples. The appropriate reference class, according to Pollock's account, is $\{\langle \tau,\rho_1,\rho_2 \rangle : \tau,\rho_1,\rho_2 \subseteq U \wedge freq(\tau|\rho_1) = v \wedge freq(\tau|\rho_2) = w \wedge freq(\tau|U) = u\}$, where v is $freq(T|R_1)$, w is $freq(T|R_2)$, u is $freq(T|U)$, and U is the set of all objects. Call this reference class R^{Poll}. Pollock shows that $freq(\{\langle \tau,\rho_1,\rho_2 \rangle : freq(\tau|\rho_1 \cap \rho_2) \approx (v \cdot w \cdot (1-u)) / (u \cdot (1-v-w) + v \cdot w)\}|R^{Poll}) \approx 1$, on the assumption that |U| is very large.[11] Based on the preceding frequency statement, it is possible to infer that $PROB(freq(T|R_1 \cap R_2) \approx (v \cdot w \cdot (1-u)) / (u \cdot (1-v-w) + v \cdot w)) \approx 1$, by the following instance of [DI]:

(DI_{Poll}): $\langle T,R_1,R_2 \rangle \in R^{Poll}$ and
$freq(\{\langle \tau,\rho_1,\rho_2 \rangle : freq(\tau|\rho_1 \cap \rho_2) \approx (v \cdot w \cdot (1-u)) / (u \cdot (1-v-w) + v \cdot w)\}|R^{Poll}) \approx 1$
is a reason for concluding that
$PROB(\langle T,R_1,R_2 \rangle \in \{\langle \tau,\rho_1,\rho_2 \rangle : freq(\tau|\rho_1 \cap \rho_2) \approx (v \cdot w \cdot (1-u)) / (u \cdot (1-v-w) + v \cdot w)\}) \approx 1$
(i.e., that $PROB(freq(T|R_1 \cap R_2) \approx (v \cdot w \cdot (1-u)) / (u \cdot (1-v-w) + v \cdot w)) \approx 1$).

One significant worry about Pollock's approach is that it generally yields implausibly strong conclusions about the value of $freq(T|R_1 \cap R_2)$. For example, in the case where $freq(T|U) = 0.5$, $freq(T|R_1) = 0.4$, and $freq(T|R_2) = 0.6$, Pollock's approach yields the conclusion that $PROB(freq(T|R_1 \cap R_2) \approx 0.5) \approx 1$, where the approximation can be made as tight as one likes by assuming that U is sufficiently large. In other words, given the described frequencies and the assumption that U is sufficiently large, it is correct to be virtually certain that $freq(T|R_1 \cap R_2)$ is exactly or almost exactly 0.5. Unlike Pollock's approach, the approach of the present paper avoids the preceding problem, since it is based on making a *deductive* inference to the value of $freq(\{\langle x,y \rangle : x \in T\}|R^*)$ (which is above reproach), rather than an inference to the value of $freq(T|R_1 \cap R_2)$.

Another feature of Pollock's approach is that it depends on the possibility of making a judgment concerning the value of $freq(T|U)$. This limits the applicability of the approach. Beyond this, Pollock's approach frequently yields peculiar conclusions about the value of $freq(T|R_1 \cap R_2)$, depending on one's judgment of the value of $freq(T|U)$. In particular, extreme values of $freq(T|U)$ (i.e., values that are far from 0.5) lead to extreme conclusions regarding the value of $freq(T|R_1 \cap R_2)$. For example, if $freq(T|R_1) = 0.5$, $freq(T|R_2) = 0.5$, and $freq(T|U) = 0.01$ (only 1% of objects are in T), then (DI_{Poll}) tells us to infer that $PROB(freq(T|R_1 \cap R_2) \approx 0.99) \approx 1$. This peculiarity of Pollock's approach is especially awkward inasmuch as $freq(T|U)$ will tend to be extreme-valued, since in typical cases either T or T^C will consist of a very small minority of the set of all objects, U. For example, suppose T is the set of gun owners. Obviously, the frequency of elements of the set of gun owners among the set of all objects is vanishingly small, which leads Pollock's approach to issue peculiar conclusions when T is the set of gun owners.

[11] It is demonstrable that the approximations expressed by the two instances of "≈" approach strict identity as the size of U approaches ∞.

While Pollock's approach delivers peculiar conclusions about the value of freq(T|$R_1 \cap R_2$) in cases where freq(T|U) is extreme-valued, the approach delivers the same results as my approach, regarding the value of PROB($c \in T$), in cases where freq(T|U) = 0.5. Given the divergence of Pollock's approach and my approach in cases where freq(T|U) $\neq$ 0.5, it would be nice if there was a way to adjudicate between the two approaches, beyond appeals to the fact that Pollock's approach yields unintuitive conclusions. A more conclusive argument in favor of one or the other approach would show that the reference class (R* or R^{Poll}) for one of the relevant direct inferences ((DI_{R*}) or (DI_{Poll})) is more specific than the other.[12] Indeed, (DI_{R*}) and (DI_{Poll}) support conflicting conclusions, and the correct way to resolve such conflicts, when they can be resolved, will be to appeal to the claim that the reference class for one of the direct inferences is more specific than the other.

As it turns out, R* *corresponds* to a more specific reference class than R^{Poll}. I say "corresponds", in this instance, since we will need to reformulate R* as a reference class whose elements are quintuples, in order to produce the desired result. The needed reformulation of R*, denoted "R*′", is: $\{\langle x,y,\tau,\rho_1,\rho_2\rangle : x\in\rho_1 \wedge y\in\rho_2 \wedge ((x\in\tau \wedge y\in\tau) \vee (x\notin\tau \wedge y\notin\tau)) \wedge \tau = T \wedge \rho_1 = R_1 \wedge \rho_2 = R_2\}$. It should be observed that the claim that R*′ is a 'reformulation' of R* is not unfairly prejudicial to R^{Poll} or (DI_{Poll}), since absent the claim that R*′ is a reformulation of R*, we could directly formulate a variant of (DI_{R*}) that (i) employs R*′ as its reference class, and (ii) yields a conclusion that is equivalent to the conclusion of (DI_{R*}).[13] In either case, it is crucial to observe that R*′ is more specific than R^{Poll}. That is: $\forall x,y,\tau,\rho_1,\rho_2$: $(x\in\rho_1 \wedge y\in\rho_2 \wedge ((x\in\tau \wedge y\in\tau) \vee (x\notin\tau \wedge y\notin\tau)) \wedge \tau = T \wedge \rho_1 = R_1 \wedge \rho_2 = R_2) \supset (\tau,\rho_1,\rho_2\subseteq U \wedge \text{freq}(\tau|\rho_1) = v \wedge \text{freq}(\tau|\rho_2) = w \wedge \text{freq}(\tau|U) = u)$, but not $\forall x,y,\tau,\rho_1,\rho_2$: $(\tau,\rho_1,\rho_2\subseteq U \wedge \text{freq}(\tau|\rho_1) = v \wedge \text{freq}(\tau|\rho_2) = w \wedge \text{freq}(\tau|U) = u) \supset (x\in\rho_1 \wedge y\in\rho_2 \wedge ((x\in\tau \wedge y\in\tau) \vee (x\notin\tau \wedge y\notin\tau)) \wedge \tau = T \wedge \rho_1 = R_1 \wedge \rho_2 = R_2)$. As with Kyburg and Teng, Pollock is committed to the preference for frequency information for more specific reference classes. So, given that R* (or, strictly speaking, R*′) corresponds to a more specific reference class than R^{Poll}, advocates of Pollock's approach should give up their approach, in favor of mine.[14]

Despite the preceding line of reasoning, which supports my approach over Pollock's, Pollock's idea that the value of freq(T|U) *could* have a bearing on what conclusions we should draw is plausible. In fact, it is quite clear that the value of freq(T|U) can have a bearing on PROB($c\in T$). For example, suppose that freq(T|U) = 2/3, freq(T|R_1) = 0.5, freq(T|R_2) = 0.5, and $U = R_1\cup R_2$. In that case, it is derivable that freq(T|$R_1\cap R_2$) = 0. Note, however, that the present example violates the suppositions under which Reichenbach's reference class problem arises, namely, the assumption that the value of freq(T|$R_1\cap R_2$) is unknown. So the fact that the value of freq(T|U) could have a bearing on the value of freq(T|$R_1\cap R_2$), does not demonstrate a flaw of the proposed approach to Reichenbach's reference class problem.

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[12] Alternatively, we may regard the conflict as being between (DI_{Poll}) and a 'non-classical' direct inference to a conclusion concerning the *expectation* of freq(T|$R_1\cap R_2$) based on freq($\{\langle x,y\rangle : x\in T\}$|R*). A direct inference of latter sort is licensed and takes priority, according to the account of direct inference defended by Pollock (1990, 2011), since R* is more specific than R^{Poll}.

[13] In particular, the result is achieved by a direct inference of the following form: From $\langle c,c,T,R_1,R_2\rangle\in R^{*\prime}$ and freq($\{\langle x,y,\tau,\rho_1,\rho_2\rangle : x\in T\}$|R*′) = r infer that PROB($\langle c,c,T,R_1,R_2\rangle \in \{\langle x,y,\tau,\rho_1,\rho_2\rangle : x\in T\}$) = r (i.e., PROB($c\in T$) = r). Notice that under the described conditions, freq($\{\langle x,y,\tau,\rho_1,\rho_2\rangle : x\in T\}$|R*′) = freq($\{\langle x,y\rangle : x\in T\}$|R*).

[14] In fact, Pollock is committed to a slightly more restrictive account of specificity. In order to meet Pollock's own specificity condition (Pollock 1990, 127, 128, 132), we would need to replace R*′ with $\{\langle x,y,\tau,\rho_1,\rho_2\rangle : x\in\rho_1 \wedge y\in\rho_2 \wedge ((x\in\tau \wedge y\in\tau) \vee (x\notin\tau \wedge y\notin\tau)) \wedge \tau = T \wedge \rho_1 = R_1 \wedge \rho_2 = R_2 \wedge \text{freq}(T|R_1) = v \wedge \text{freq}(T|R_2) = w \wedge \text{freq}(T|U) = u\}$.

5. Conclusion

The proposed solution to Reichenbach's reference class problem turns on the reasonableness of forming a judgment regarding PROB($c \in T$) via (DI_{R*}). In cases where one's relevant information is limited to the statements that appear within the premises of (DI_{R_1}) and (DI_{R_2}), the judgment supported by (DI_{R*}) is reasonable. In other cases, we may possess accurate precise-valued frequency information concerning the value of freq($T|R_1 \cap R_2$). In such cases, inference by (DI_{R*}) will be defeated (in accordance with [SD]). That inference by (DI_{R*}) may be defeated, in some cases, is a natural and appropriate limitation of the proposed approach.*

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