A Proposal for the Classical Limit in Bohm's Theory

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In this paper, I describe a general strategy for deriving Newtonian mechanics from Bohm's theory in the classical macroscopic regime. The strategy is based on two main steps: (i) the use of open systems, leading to the formation of independent subsystems; (ii) the analysis of the quantum potential for the emergent subsystems. The first step leads to and explains (within the Bohmian framework) the well-known phenomenon of decoherence. The second one provides a precise criterion for the emergence of Newtonian trajectories. In particular, I will show that, under reasonable classicality conditions, the quantum potential of the emergent subsystems become negligible and, consequently, they will follow an approximately Newtonian dynamics.

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1 Introduction: the classical limit problem

Quantum mechanics is a fundamental physical theory: it describes the behavior of the microscopic constituents of matter, like molecules, atoms, nuclei. The quantum world has very peculiar features: systems are intrinsically indeterministic, can be in superposition of different states, and non-locally connected with each other. On the other hand, the classical macroscopic world, described by Newtonian mechanics, has a more familiar ontology: classical systems have well-defined positions in space and their dynamics is fully deterministic. Nevertheless, every classical object (tables, chairs, human beings, etc.) is composed of a collection of quantum objects. Therefore, we expect the quantum description of matter approaching the classical one under specific conditions, i.e. under the conditions fulfilled by systems at macroscopic scale.[1] This problem is known as *the classical limit* problem, and the standard answer is given by decoherence theory.

Decoherence theory describes the interaction between a quantum system and external (or internal) degrees of freedom. Usually - and especially in the classical limit - one considers the interaction with the external environment (e.g., light photons, air molecules). If the system is initially in a coherent superposition of different states, the interaction will (locally) destroy the superposition,[2] and the system will be represented by an improper mixture of localized states.[3] The loss of coherence between the different relative states is called *decoherence*, and the theory is named after this characteristic physical effect. However, decoherence theory alone does not provide a satisfactory explanation for the

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[1] These are generally called "classicality conditions".

[2] The relative phase of the superposition is delocalized, after the interaction, from the level of the system (now subsystem of an entangled state) to the level of the entangled system-environment state.

[3] The reduced density matrix of the system, even if diagonal, does not represent a mixed state.

emergence of classical objects: classical systems have well-defined positions in space and move according to Newtonian trajectories, while subsystems described by decoherence - mathematically described by reduced density matrices - do not have well-defined positions and do not move according to trajectories in space, due to the Heisenberg's uncertainty principle. That is: the connection between the quantum systems emerging from the decoherence process and the classical objects of the macroscopic world is far from being trivial. The classical limit problem is therefore not just a technical problem but also (maybe especially) a conceptual one. This aspect of the problem is well-captured in the following passage by Ballentine (2015, p. 388):

> Bohr and Heisenberg stressed the analogy between the limits $c \to \infty$ and $\hbar \to 0$, both of which supposedly lead back to the familiar ground of Newtonian mechanics, in the attempt to convert Einstein to their view of quantum mechanics. Einstein was unmoved by such arguments, and indeed the proposed analogy seriously oversimplifies the problem. Newtonian mechanics and relativistic mechanics are formulated in terms of the same concepts: the continuous trajectories of individual particles through space-time. These trajectories differ quantitatively between the two theories, but the differences vanish in the limit $c \to \infty$. But quantum mechanics is formulated in terms of probabilities, and does not refer directly to trajectories of individual particles. A conceptual difference is much more difficult to bridge than a merely quantitative difference.

The conceptual bridge between quantum and classical mechanics can be filled by adopting a clear interpretation of quantum mechanics. In this paper, in particular, I will analyze the problem of the classical limi in Bohm's theory, which is basically a quantum theory of particles in motion. This way, we will have a common structure for quantum and classical mechanics (individual particles following trajectories in three-dimensional space) and the approach to the classical limit will be much clear from the conceptual point of view.[4]

In Bohm's theory, a general N-particle entangled system is described by the usual wave function together with an actual configuration of N point-particles in three-dimensional space. The wave-function evolves in time according to the Schrödinger equation, and the particles evolve in time according to the the guiding equation: $v = \frac{\nabla S}{m}$, which fixes the velocity of the particles, and the quantum Newton's law: $-(\nabla V + \nabla Q) = ma$, which fixes the acceleration of the particles.[5]

Within Bohm's theory, the quantum to classical transition is conceptually completely equivalent to the transition from relativistic to Newtonian mechanics: Bohm's theory

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[4]However, I am not suggesting here that this is the only strategy/approach one can pursue to set the classical limit problem on a rigorous basis. Relevant work has been done, for example, in the Everett/Many Worlds Interpretation (e.g. Wallace (2012, ch. 3) and GRW theory (e.g. Bassi et al. (2016)).

[5]Q represents the quantum potential associated to the wave fucntion.

describes quantum systems as individual particles following continuous and deterministic trajectories in space, together with the wave function. The problem of the classical limit reduces therefore to the following questions:

1. Why does the wave function disappear in the classical limit?
   The answer to this question is strictly related to the emergence of subsystems from entangled states. In fact, the formation of dynamically independent subsystems - mathematically described by effective wave functions - destroys the typical non-local connections between Bohmian particles, enabling the passage from a "holistic" regime (characteristic of the quantum world) to a"local" regime (characteristic of the classical world). In this regard, I will show that the formation of effective wave functions is usually produced by the interaction with the environment, i.e. by the process of decoherence. For this reason, we expect in the classical limit a continuous emergence of effective wave-functions, and this explains why ordinary classical objects are not quantum mechanically correlated. The main role of decoherence in the Bohmian framework is, in other words, that of producing separate subsystems from the general holistic quantum description.

2. Why do the Bohmian trajectories become (approximately) Newtonian?
   To answer this question, I will follow a two-step strategy, showing that:

   - In the decoherence regime, the effective wave functions are well-localized wave packets (typically Gaussian wave packets);
   - When the systems are macroscopic, the quantum potential (a physical parameter associated to the wave function) of such wave packets becomes negligible, and consequently the dynamics becomes (approximately) Newtonian. Therefore, macroscopic quantum systems (in interaction with the environment) will move according to the classical Newtonian dynamics.

The structure of the paper is as following: in sections 2 and 3, I shall discuss two of the most common standard strategies proposed in the literature to solve the classical limit problem, namely the $\hbar \to 0$ approach and the Ehrenfest theorem, and show why they are not sufficient to the scope. In section 4, I shall describe the process of effective factorization, i.e. the process leading to the formation of independent subsystems in the decoherence regime. In section 5, based on the analysis of the quantum potential for the emerging subsystems, I will show that the Bohmian subsystems will undergo an approximately Newtonian dynamics under reasonable classicality conditions.

2 Negligible quantum action: $\hbar \to 0$

A common strategy for the classical limit is to say that, when the Planck constant "goes to zero", i.e., when the quantum action is negligible with respect to the classical action, the quantum system moves according to Newton's second law of dynamics.[6] This comes

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[6]Sometimes the condition $\hbar \to 0$ is said to be unphysical since the Planck constant $\hbar$ is a constant and does not go to zero. However, this argument misses the physical meaning of the formula, that

from the consideration that, for example, the (quantum) commutators becomes mathematically equivalent to the (classical) Poisson brackets, or that the quantum discrete energy levels becomes continuous. The problem with this strategy is that it works only for special states. In fact, it is possible to consider quantum states whose behavior is not characterized by the quantum action. These states will be basically unaffected by this strategy. One example is that of the Einstein's box (1953). He consider a textbook example, the infinite potential well, and ask the following question: when does the standard quantum description - a stationary wave inside the box - turns into a classical state - a particle moving with a well-defined velocity between the walls?

The problem here is that, as far as the system is described by the stationary wave $\Psi(x) = Bsin(kx)$, it will not become classical even for big mass and small de Broglie wavelength.[7] Einstein was certainly right: until this system is isolated, it is impossible to derive from it a classical state, even when the system is macroscopic. The strategy $\hbar \to 0$ simply does not apply in this case, since the spatial part of the wave function does not depend on $\hbar$ and the quantum interference between the plane waves composing the stationary wave is never turned off. It is worth noting that this problem is not alleviated in Bohm's theory, where the particle remains at rest inside the box (because $v = \frac{\nabla S}{m} = 0$ for each point inside the box).

The example of the Einstein's box shows that the criterion $\hbar \to 0$ is not sufficient for recovering the classical limit, and, moreover, that the interaction with the environment is a necessary conditions for the emergence of classical states.

3 Ehrenfest theorem

Ehrenfest theorem shows that the mean value of the position operator follows a pseudo-Newtonian flow:

$$\frac{d^2}{dt^2} \langle \hat{X} \rangle = \langle \nabla V(x) \rangle \tag{1}$$

where V is the potential term in the Hamiltonian of the Schrödinger equation describing the evolution of the system. This relation looks like a quantum analogue of Newtonian dynamics (in terms of position and momentum operators rather than position and momentum of a material object) when the mean value of the gradient of the potential can be approximated with the gradient of the potential of the mean value of the position operator. That is:

$$\frac{d^2}{dt^2} \langle \hat{X} \rangle = \langle \nabla V(x) \rangle \approx \nabla V(\langle x \rangle) \tag{2}$$

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is basically a shorthand to indicate the physical condition: $\hbar \ll A_{cl} \equiv \frac{\hbar}{A_{cl}} \ll 1$, that in the limit for the classical action going to infinity becomes $\lim_{A_{cl} \to \infty} \frac{\hbar}{A_{cl}} = 0$.

[7]Einstein considered these conditions as the genuine classicality conditions. The importance of the interaction with the environment, and thus decoherence, came only after some decades.

This approximation holds whenever one of the three following conditions is satisfied[8]:

a. The potential term in the Hamiltonian operator is at most quadratic;

b. The wave function of the state is a well-localized wave packet;

c. The scale of variation of the potential V is much bigger than a suitable estimate of the width of the wave packet.

To see this, we expand ∇V in Taylor series up to the third order around $\langle x \rangle = x_0$:

$$\nabla V(x) = \nabla V(x_0) + (x - x_0)\nabla^2 V(x_0) + \frac{1}{2}(x - x_0)^2 \nabla^3 V(x_0) \tag{3}$$

and take the mean value of both sides:

$$\langle \nabla V(x) \rangle = \nabla V(x_0) + \frac{1}{2}\sigma^2 \nabla^3 V(x_0) \tag{4}$$

where $\sigma^2 = \langle (x - x_0)^2 \rangle$ is the (mean value of the) variance of the position operator. From eq. (3), we see that condition (2) is satisfied whenever the third order term of the potential is negligible with respect to the first order term, and this happens when one of the conditions (a) (b) (c) apply. Conceptual remark: it is sometimes said that the problem with Ehrenfest theorem is that it applies only to well-localized wave packets, whereas the wave functions generally spread in time. Nevertheless, we see that this is not a problem for Ehrenfest, insofar either condition (a) or (c) is satisfied. For example, we can apply Ehrenfest theorem to a plane wave, since, for this kinds of states, the third order derivative of the potential is exactly zero, and therefore the (enormous) spread of the wave function does not affect the condition (2).

However, the most serious problem with the Ehrenfest theorem is that it only applies to isolated systems, which is not a realistic condition for the classical limit[9]. As it is well understood from the literature, a fundamental classicality condition is given by the ubiquitous interaction with the environment (open systems), which leads to decoherence effects. In these situations, the standard Ehrenfest theorem is not applicable, and we must find other solutions. Of course, there is an equivalent version of the Ehrenfest theorem for open systems, where the condition (2) holds not for the wave function but for the reduced density matrix of the system. However, the connection between this formalism and the emergence of a classical system is quite problematic: while the wave function mathematically represents the state of a system, and its physical meaning in terms of probability density is commonly accepted as the standard view in quantum mechanics, the density matrix does not have a simple (and conceptually clear) connection with the state of the system, being it best regarded as a mathematical tool for extracting statistical predictions on the measurement outcomes.[10]

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[8] These conditions come from a Taylor expansion of the formula (1), and imposing the condition $\langle \nabla V(x) \rangle \approx \nabla V(\langle x \rangle)$. For a complete mathematical derivation, see for example Shankar (1994)

[9] For a critique to the Ehrenfest theorem as useful tool to characterize the classical regime, see in particular Ballentine et al. (1994), Ballentine (1996) and Ballentine (2015, ch. 15).

[10] For example, Schlosshauer (2009).

4 The emergence of autonomous subsystems

After having analyzed the condition $\hbar \to 0$ and the Ehrenfest theorem and their limits for recovering the classical states from quantum mechanics, I will start now my approach to the classical limit in Bohm's theory. In this section, in particular, I describe the process of effective factorization, i.e. a physical process leading to the formation of emergent autonomous subsystems from general entangled states. This process is the physical basis of decoherence in Bohm's theory and an important step towards the classical limit: it explains why, in the macroscopic classical regime, different systems are not quantum mechanically correlated with each others. In the next section, I will show that the emergent subsystems, when macroscopic, will follow an approximately Newtonian dynamics.

4.1 Real factorization

Consider a 2-particle system: if the wave function of the total system can be written as a product of two wave functions associated to each particle, we will say that the system is *factorizable*, and we can write the total state as a factorized state of the subsystems:

$$\psi_{AB}(x_1, x_2) = \psi_A(x_1)\psi_B(x_2)$$

Factorizability expresses the physical independence of particle 1 and particle 2. Indeed, according to the above formula, it is easy to show that the velocity of particle 1(2) depends only on $\psi_A(\psi_B)$. For example, the guiding equation of particle 1 will be:

$$\frac{dX_1}{dt} = \frac{\hbar}{m_1} Im \frac{\nabla_1(\psi_A(X_1)\psi_B(X_2))}{\psi_A(X_1)\psi_B(X_2)} = \frac{\hbar}{m_1} Im \frac{\nabla_1 \psi_A(X_1)}{\psi_A(X_1)}$$

The physical independence can also be seen in the Hamilton-Jacobi formalism.[11] In particular, the velocity and the quantum potential of particle 1 will be:

$$\frac{dX_1}{dt} = \frac{\nabla_1 S_A(X_1)}{m_1}$$

$$Q_A = -\frac{\hbar^2}{2m_1} \frac{\nabla_1^2 R_A(X_1)}{R_A(X_1)}$$

where R and S are, respectively. the amplitude and phase of the wave function written in polar form: $\psi(x,t) = R(x,t)e^{\frac{i}{\hbar}S(x,t)}$. Therefore, if the wave function is factorized, the trajectory of particle 1(2) will depend only on the wave function $\psi_A(\psi_B)$. In particular, the position of the particle 2(1) will not affect the motion of particle 1(2). In short, particle 1 and particle 2 will become dynamically independent (at least, from the quantum mechanical point of view: they will continue to interact through classical potentials). In general, the possibility to write an N-particle system into a factorized state of N different 1-particle systems directly implies that each particle of the total configuration will behave autonomously. It is worth noting that the condition of factorizability, while useful to

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[11]E.g., see Holland (1993, p. 288).

derive a separable dynamical regime for subsystems, is not sufficient to derive the classical motion, i.e., to yield Newtonian trajectories for the systems. Indeed, each factorized system will follow its own Schrödinger equation, and each particle will move according to the usual Bohmian trajectories, which are generally different from the Newtonian ones. Moreover, there is a further problem: factorizability is a very fragile condition: as soon as two particles interact through a classical potential, the Schrödinger evolution will generally transform an initial factorized state into an entangled state:

$$\psi_A(x_1)\psi_B(x_2) \overset{H(x_1,x_2)}{\to} \psi_{AB}(x_1,x_2)$$

where the entangled state $\psi_{AB}(x_1,x_2)$ will be in general a superposition of product states. Since the macroscopic world is characterized by ubiquitous interactions between systems, the condition of real factorization is totally unrealistic in the classical limit. In fact, I will take the opposite view: what is essential for the emergence of subsystems is not a real factorization between isolated systems, but an effective factorization between open interacting systems.

4.2 Effective factorization

Consider now that particle 1 and particle 2 have interacted with each other and thus formed the following entangled state:

$$\Psi(x_1,x_2) = \psi_A(x_1)\psi_B(x_2) + \psi_C(x_1)\psi_D(x_2)$$

If the summands do not overlap in configuration space,[12] i.e., the relative states $\psi_A\psi_B$ and $\psi_C\psi_D$ have (approximately) disjoint supports, the evolution of the wave-function will be such as to form two non-overlapping channels or branches, and the particles will enter just one of them. For example, suppose they enter into the first channel $\psi_A(x_1)\psi_B(x_2)$. From now on, if the overlap continues to be negligible, we can effectively describe the further evolution of the system just using the relative state $\psi_A(x_1)\psi_B(x_2)$. In this case, the wave function $\psi(x_1,x_2)$ reduces *effectively* to a mixture of states, and we call $\psi_A(x_1)\psi_B(x_2)$ the *effective wave function* of the 2-particle system (and $\psi_A(x_1)$ and $\psi_B(x_2)$ the *effective wave functions* of, respectively, particle 1 and particle 2). Moreover, since the effective wave function of the 2-particle system is an (effectively) factorized state of system A (particle 1) and system B (particle 2), the motions of the particle 1 and 2 are independent with each other.

We call such a state *effectively factorizable*. These states are generally produced during measurement-like interactions, i.e. interactions between systems that produce separation of the total entangled wave function in different wave packets whose overlap is negligible in configuration space. Think, for example, of a typical system-apparatus interaction: the initial state is generally a factorized state between the system, $\psi(x)$, and the apparatus, $\phi(y)$, i.e $\Psi(x,y) = \psi(x)\phi(y)$. During the measurement interaction, the Schrödinger evolution will transform the initial state into an entangled state, generally a superposition

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[12]Since the support of a wave function is typically unbounded, the condition is more precisely expressed as *negligible overlap* in configuration space.

of eigenstates of the observable associated with the physical quantity we are measuring through that interaction:

$$\psi(x)\phi(y) \to \sum_i c_i \psi_i(x)\phi_i(y)$$

If the different pointer states of the apparatus $\phi_i(y)$ have (approximately) disjoint supports in the apparatus configuration space, then the final state $\sum_i c_i \psi_i(x)\phi_i(y)$ reduces to an effectively factorized state. Suppose, for example, that the result of the measurement is the eigenvalue s associated with the eigenstate $\phi_s(y)$ of a certain observable measured by the apparatus: this means that the final superposition effectively reduces to the factorized state $\psi_s(x)\phi_s(y)$, where $\psi_s(x)$ and $\phi_s(y)$ are, respectively, the effective wave functions of the system and the apparatus. Therefore, at the end of the measurement, the system and the apparatus will become independent from each other and the further evolution of the system particle will not be affected by the position of the apparatus particles. In short, system and apparatus will be no longer dependent of each other after the measurement. The main point here is: the production of effectively factorized states - and, therefore, of effective wave functions for the subsystems - is the key to understand why, in the classical regime, the typical non-local quantum behavior disappears and why different systems behave autonomously with each other (apart from interactions through classical potentials, that do not depend on the wave function and, indeed, they remain in classical mechanics.).

4.3 Effective factorization via environmental interaction

The effective factorization of the wave function, that is commonly associated to the measurement interaction with a macroscopic device, is naturally produced by the interaction between a system and its external environment, e.g., with external (or internal) degrees of freedom. In fact, a strong interaction with the environment is generally a function of position, and it will generate a process similar to the interaction with the measurement apparatus described above. In the same manner, these sort of interactions will produce an effectively factorized state of the total system-environment entangled state and, consequently, an effective wave function for the system and for the environment[13].
We can represent the interaction between an entangled 2-particle system and an external system (the "environment") as follows:

$$\Psi(x,y)(\Xi_1(z_1)\dots\Xi_N(z_N)) = \sum_i \psi_i(x)\phi_i(y)(\Xi_1(z_1)\dots\Xi_N(z_N)) \to \sum_i \psi_i(x)\phi_i(y)\xi_i(z_1,\dots,z_N)$$

Question: how much is plausible that an effectively factorized state will be created in this interaction? In the case of the interaction with the measurement apparatus, it was the condition of distinct macroscopic pointer states that guaranteed a final factorized

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[13]From a computational/pragmatic point of view, these correspond to the reduced density matrices used to describe subsystems in decoherence theory.

state. In this case, the situation is much more subtle, because it is not obvious that the different relative states of the environment will create disjoint branches in configuration space. However, the possibility to have effectively factorized states is high when the environment is composed of a huge number of particles, condition that usually applies to the classical limit, where the environment has typically an Avogadro number of particles. Higher is the number of particles interacting with the system, higher the possibility that the total entangled state will generate effective wave functions. After all, this kind of reasoning is similar to that one usually employed in standard decoherence to justify the assumption of orthogonality of states. Two different relative environmental states have not to be orthogonal with each other; however, it is possible to show that this condition will be generally satisfied in the long run, i.e. when the environment is composed of a huge number of particles.

That environmental interactions destroy the Bohmian non-locality and describe the formation of physically autonomous subsystems was already stressed by Bohm (1987, p. 344):

> Quantum non-local connection is fragile and easily broken by almost any disturbance or perturbation. [. . .] we may expect that non-local connection will not normally be encountered under ordinary conditions, in which every system is bathed in electromagnetic radiation and is subject to external perturbations of all kinds as well as random thermal energies.

Effective factorizability of the wave function, and the simultaneous emergence of the effective wave function, is a direct consequence of the interaction between systems. The classical local regime of dynamically separable systems will tend to emerge from measurement-like interactions between systems (typically system-environment interactions). The macroscopic world is certainly characterized by the fact that systems interact with each other[14]: under this condition, effective wave functions for subsystems are continuously produced. Thus, it is natural that classical systems behave autonomously: they just reflect, at the macroscopic scale, the physical independence between systems that is described by effective wave functions at the microscopic scale.

5 Newtonian dynamics

In this section, I describe why macroscopic Bohmian subsystems, mathematically described by effective wave functions, follow an approximately Newtonian dynamics in the classical limit.

5.1 Well-localized effective wave functions

The process of effective factorization through environmental interactions produces a special type of effective wave functions, namely well-localized wave packets. We know this

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[14]After all, this is the main lesson of decoherence theory.

from decoherence theory: when a system interact with the environment, the coherence length of the reduced density matrix shrinks (in opposition to this, the ensemble width becomes larger, since the environment provides new energy to the system). The coherence length describe the minimum length over which it is possible to measure quantum interference between different positions of the system. If the coherence length is very short, this means (indirectly) that the wave function of the system is pretty well-localized in position (and momentum, if this effect remains stable in time).

In particular, Zurek, Habib & Paz (1993) show that the pointer states of the system, i.e. those states that "survive" undisturbed to the interaction with the environment, are minimum uncertainty wave-packets, i.e. Gaussian wave packets. In Bohm's theory, the pointer state of the system is clearly the effective wave function. Based on this result, we assume that the effective wave functions in Bohm's theory produced by the interaction with the environment are mathematically described by Gaussian wave packets. This is an important result: starting from it, it is possible to analyze the dynamics of the emergent subsystems in the classical regime. This will be the scope of the next section.

5.2 The emergence of Newtonian dynamics

Here, I shall describe the dynamics of the emergent subsystems, showing that it becomes Newtonian under reasonable conditions characteristic of the classical regime. Starting point is the important result of the last paragraph: the effective wave functions generated by the interaction with the environment are (likely to be) Gaussian states. Therefore, I assume that a typical effective wave function be described by the following state:

$$\psi(x) = A e^{-\frac{x^2}{4\sigma^2}}$$

where A is the amplitude and σ the standard deviation of the Gaussian. In order to connect the wave function of the system with its dynamics, we may compute the quantum potential $Q = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$ and, from it, the total acceleration of the particle through the formula (the so-called quantum Newton's law):

$$F_{cl} + F_Q = -\nabla V - \nabla Q = m\ddot{x}$$

The quantum potential of the Gaussian state above is given by:[15]

$$Q = \frac{\hbar^2}{4m\beta^2}(1 - \frac{x^2}{\beta^2})$$

where $\beta^2 = \sigma^2[1 + (\frac{\hbar t}{2m\sigma^2})^2]$ and m is the mass of the system.

We note that that the quantum potential of the Gaussian state tends to zero for large mass: $m \to \infty$ and for negligible quantum action: $\hbar \to 0$, which are exactly the conditions characteristic of a macroscopic body for which we expect a Newtonian dynamics. This suggests that an open Bohmian system interacting with the environment, and so

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[15]Ballentine (2015, ch. 14).

described by a Gaussian effective wave function, will undergo an approximately Newtonian dynamics when it is macroscopic, i.e. when it is characterized by a large mass and high energy. The final scheme of the dynamics may be summed up as follows:

$$\text{Bohm's theory} = \begin{cases} \ddot{x} = \frac{1}{m}(F_{cl} + F_Q) & \text{Quantum Newton's law} \\ \\ \dot{x} = \frac{\nabla S}{m} & \text{Guiding equation} \end{cases}$$

becomes, in the classical macroscopic regime (when $m \to \infty$ and $\hbar \to 0$ apply):

$$\text{Newton's theory (plus guiding equation)} = \begin{cases} \ddot{x} \cong \frac{1}{m}(F_{cl}) & \text{Second Newton's law} \\ \\ \dot{x} = \frac{\nabla S}{m} & \text{Guiding equation} \end{cases}$$

That is: a macroscopic Bohmian system in interaction with the environment will move according to Newtonian dynamics.

Final remark: the velocity in Bohm's theory and Newtonian mechanics.
It is worth noting, however, that the velocity plays a different role in Bohm's theory and Newtonian mechanics. While in the former it is fixed by a dynamical law (the guiding equation), in the latter it becomes a free parameter (initial condition). Why the velocity plays such a different role in these two theories remains, for the moment, an interesting open question.

Acknowledgements

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