A Bayesian theory of quantum gravity without gravitons

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Abstract: There currently exists no theory that explicitly demonstrates compatibility of the Standard Model in its operational regime with the general theory of relativity in an appropriate limit. Here we present a novel theory of quantum gravity that meets required compatibility, derived from Bayesian and holographic principles. Gravity is not understood as force mediated by gravitons but fundamentally as emergent spacetime dependent on entanglement. Both spacetime and quantum mechanics are understood as arising out of observer's knowledge of the world, instead of pre-existing before an observer. Spacetime exists as a picture of quantum-mechanical knowledge obtained from signals an observer received, and the general theory of relativity is understood as an equation of state, as in Jacobson (1995). Despite some departures from conventional quantum gravity understanding, the theory maintains holographic roots and upholds AdS/CFT.

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1 Introduction

Theories that recover the general theory of relativity and realistic spacetime in classical limits - most famously, string-theoretic models - exist. Theories that recover the Standard Model in weak gravity regime also exist. However, physicists are yet to demonstrate that some theory recovers both in appropriate limits - a good summary of the current state of quantum gravity research.

Here we present a theory of quantum gravity that does recover both - the Bayesian theory of gravity. The new theory does not present new equations unknown to quantum gravity physicists - thus emphasis is on how different pieces fit together. In particular, we argue that dominant physical interpretations of probability and gravity were what prevented construction of an empirically consistent theory of quantum gravity. In a way, this is like how Lorentz transformation was discovered before Albert Einstein, but he was first to construct a coherent interpretation of a theory such that it can actually be used in full power. Despite some departures from conventional views of quantum gravity, our theory only is a re-interpretation of AdS/CFT[1][2][3] suitably generalized to other contexts not involving boundary-bulk relationship - thus the point of departures is not of mathematical character but of how different pieces are interpreted and fit together. For the question of recovering both the Standard Model and the general theory of relativity in appropriate limits, $\mathcal{A}/4$ theory of spacetime is sufficient - QM-P, which is the other component of the Bayesian theory of gravity, is not needed.

In this article, we interpret the general theory of relativity as an equation of state in thermodynamic contexts - following Jacobson (1995)[4]. But it is one step to derive general relativity as a thermodynamic equation of state, and another leap to go further into general theory of gravity - is gravity of non-equilibrium thermodynamic nature (thus spacetime only exists in restricted circumstances) or beyond thermodynamics (spacetime and gravity may make sense at all times)? The Bayesian theory of gravity states that spacetime does exist at all times, and successfully generalizes Jacobson (1995). Spacetime is understood as a way of picturing a quantum state vector.

The theory relies on two components: 1) objective Bayesian interpretation of probability[5] that includes the principle of maximum entropy and minimum Fisher information, which culminates in QM-P that includes the partition function constraint 2) extracting spacetime out of quantum entanglement by inverse Radon transform, effectively performing tomography, given classical Ryu-Takayanagi relation[6]. ($\mathcal{A}/4$ theory of spacetime)

The "catch phrases" that describe the theory very concisely can be given as follow:

- QM = Bayes (Quantum mechanics = Bayesian statistics with additional constraints and requirements)
- Entanglement = Spacetime (entanglement generates a unique spacetime that is not superimposed - not in superposition.)
- GR = Local thermodynamics, in fashion of Jacobson (1995)[4].
- $\mathcal{A}/4$ theory of spacetime (which is directly utilizing Ryu-Takayanagi relation[6].)
- Gravity purifies an irreducible subsystem.
- Measurement is a continuous never-ending process.

- No collapse (there is no breakdown of Schrödinger equation - it describes fully evolution of a state vector from the beginning of the universe to the end.)

- Strong complementarity (Different observers may have different descriptions and state vectors of the universe.)

In objective Bayesian fashion, quantum state vector is interpreted as representing observer's neutral probabilistic knowledge on possible outcomes of the world. Since entanglement determines quantum gravity, gravity is no longer understood as a physical force but arising out of an observer trying to picture the world consistently, using probabilistic knowledge.

Enforced differentiability of state vector avoids discontinuous state vector collapse, thereby resolving the measurement problem. Avoidance of discontinuous state vector collapse raises the question of what a measurement is, and this is to be tackled before finally presenting the quantum gravity theory.

We first provide two important motivations toward the Bayesian theory of gravity. We then review "controversies" surrounding the measurement problem and nature of quantum gravity, as clarifying nature of measurements and quantum gravity is critical in our theory - however, they do not necessarily have to be read to understand the theory. Next, we comment on how quantum mechanics is derived from more fundamental consistency requirements for probabilistic inference, combined with physical constraints - the theory of **QM-P**. Afterwards, the Bayesian theory of quantum gravity is completed with $\mathcal{A}/4$ theory of spacetime, with demonstration of compatibility of the general theory of relativity with the Standard Model in its operational domain. More precisely, we demonstrate compatibility of general relativity in its operational domain (local equilibrium) with any quantum theory admitting Rindler modular Hamiltonian.

While the Bayesian theory of gravity consists of component theories of QM-P and $\mathcal{A}/4$ theory of spacetime, each component is an independent theory, and does not rely on other components. The component theory that purely concerns with gravity questions is $\mathcal{A}/4$ theory of spacetime. The component theory that concerns with how entanglement changes is QM-P, but other quantum theories would work fine as far as they determine state vector evolution in order to allow calculations in $\mathcal{A}/4$ theory of spacetime. For those preferring to read this article in a non-Bayesian direction, it is recommended to start from the discussion of the classical Ryu-Takayanagi relation[6], though understanding theoretical motivations and the discussion on superposition of gravity may be of additional aids in understanding $\mathcal{A}/4$ theory of spacetime.

We use Planck units (or sometimes referred to as natural units) throughout this article - thus, Boltzmann constant $k_B = 1$, speed of light $c = 1$, reduced Planck constant $\hbar = 1$. Furthermore, we set gravitational constant $G = 1$, and G_{uv} refers to the (component form of) Einstein tensor. Schrödinger picture of quantum mechanics is used unless otherwise noted.

2 Theoretical motivations

The two important theoretical motivations lead toward features of the Bayesian theory of gravity:

- Classical Ryu-Takayanagi (RT) relation[6] without quantum corrections. Technically, the RT equation without quantum corrections already does provide a theory of quantum gravity, but it has never been interpreted this way for "good" reasons. This forms $\mathcal{A}/4$ theory of spacetime part of the Bayesian theory of gravity.
- Black hole complementarity[7]. We argue that a Bayesian interpretation of state vector is more natural with black hole complementarity.

The statement that classical RT relation is enough as a consistent theory of gravity is to be demonstrated when $\mathcal{A}/4$ theory of spacetime is finally presented in this article. Here we focus on why classical RT relation was considered insufficient. As to be discussed again, conventional understanding is that just like different outcomes are superimposed in non-gravitational quantum mechanics (superposition), different gravitational outcomes must be superimposed. The classical RT relation maps area to entanglement entropy, but "area" is ill-defined when different outcomes are superimposed. Semiclassical understanding allows area term to be defined, but it still only counts as an approximation in conventional understanding. From this view, the classical RT relation can never be a consistent theory of quantum gravity. We demonstrate in this article that this is not the case. Just to be clear, the classical RT relation as being fundamental does not refute AdS/CFT[1]. Our theoretical framework is really about refining what theories actually are gravitational theories of our world after generalizing AdS/CFT. That the classical RT relation produces an empirically consistent theory of gravity, when we so far do not have any such theory, is an enough theoretical justification for the theory, but interested readers are referred to the discussion on superposition of gravity in this article, where the "regularization problem" surrounding superposition of spacetime is discussed.

We focus more about black hole complementarity here. What is black hole complementarity? A brief argument, following terminology in the AMPS firewall[8], goes as follows. Let A be a subsystem of the Hawking radiation emitted long time ago, B be a subsystem of the radiation just being emitted, and C be its black hole interior Hawking partner. If one considers A, B and C as "separate" subsystems, then we get the strong subadditivity paradox[8] in case conventional understanding of quantum gravity is correct - this is the modern form of black hole information paradox. Black hole complementarity was understood to fix this issue by arguing that no observer can access all of A, B and C as separate subsystems - though this detail itself is not part of the original postulates of black hole complementarity[7]. The AMPS argument[8] was that an observer can access A, B and C, and thus strong subadditivity paradox is back alive. The questions we can ask are:

- Can AMPS contradiction be fully confirmed in an experiment? And if not, is theoretical inconsistency posed by access to all of A, B and C enough to be problematic?

- Is C really a separate subsystem even before asking about access to A, B and C? For example, the Papadodimas-Raju program[9] argues that it is not - which would dissolve away the strong subadditivity paradox. Black hole complementarity then is understood as C not being a separate subsystem from other subsystems. Just as an observer cannot measure in different bases simultaneously - complementarity, an observer cannot simultaneously access two descriptions involving a black hole simultaneously in black hole complementarity.

If we take state vector or its probabilistic contents as physical (ontic), instead of Bayesian (epistemic), it is unclear how AMPS inconsistency, if it exists and even if an experiment cannot fully confirm contradictions, can be brushed aside. After all, probability is something real, driven by physical stochastic processes in a physical interpretation of probability. If entanglement is of epistemic and Bayesian nature, then as far as an experiment cannot confirm inconsistency, there is no theoretical and empirical contradiction. One can call this "strong complementarity" - the view we follow in this article. A more correct view, though, would be that strong complementarity by nature precludes possibility of experimental inconsistency. Note that the Papadodimas-Raju program is not necessarily at odd with this strong complementarity view.

Even when one takes the Papadodimas-Raju program to be a valid resolution to the strong subadditivity paradox, a Bayesian and epistemic interpretation of probability helps. As to be discussed, one may ask why one description is imposed to some observer, while another is imposed for another observer. Usually, we assume that not only a single observer but also different observers can only access, or be governed by, the same description of the universe. Black hole complementarity suggests that multiple observers may access different descriptions of the universe, thus extending the notion of complementarity. This opens up new consistency questions in case probability is physical. Switching to a Bayesian and epistemic interpretation of probability allows us to simply dissolve consistency questions as non-existent.

Having settled to a Bayesian understanding of probability, we must ask how a unique state vector to an observer can be assigned. Unlike in usual statistics, where any prior and appropriately good posterior results are reasonably fine, physics does not seem to have that luxury - though a different opinion exists, especially from QBism[10]. But even in Bayesian statistics, objective Bayesian analysis exists, arguing that there does exist a unique consistent way of doing statistics, with principles mostly involving the principle of maximum entropy (MaxENT)[11] and minimum Fisher information (minFisher). We utilize both principles, but the latter is used only to derive Schrödinger equation from purely statistical and physical considerations. Thus, as far as one already accept Schrödinger equation, minFisher is not utilized. It is MaxENT that is of critical importance for QM-P.

Bayesian parts of the Bayesian theory of gravity - called QM-P - do not have to be assumed in order for the $\mathcal{A}/4$ theory part to be consistent, as far as appropriate non-gravitational state vector evolution can be provided. Given non-gravitational state vector evolution, the $\mathcal{A}/4$ theory can provide corresponding descriptions of gravity. But since entropy is mapped to spacetime, Bayesian interpretation still is a natural setting for the

$\mathcal{A}/4$ theory.

3 Background: measurement problem

This part of the article is written to provide a background on nature of measurements such that QM-P may be understood more clearly - but it is not necessary to read this part to understand the Bayesian theory of gravity. The measurement problem in quantum mechanics can be understood in three parts:

- Discontinuity of state vector when a measurement is done, restricting Schrödinger equation to piecewise applications (that cannot be applied for entire evolution).
- Even if one accepts discontinuous evolution, quantum probability is very different from classical probability.
- Lack of an explanation as to when a particular measurement occurs, or reality exists.

Let us explore the second part first. Mathematically, the problem is that in many circumstances,

$$P(x_t) \neq \sum_{x_0 \in X_0} P(x_0) P(x_0 \to x_t) \tag{3.1}$$

where x_t is an outcome at time $t > 0$ and X_t is the set of all possible outcomes at time t. $P(x_t)$ refers to probability determined from state vector at time t, $\sum_{x_t \in X_t} P(x_t) = 1$. $P(x_0 \to x_t)$ refers to transition probability, determined from transition amplitude. Equation 3.1 is a consequence of quantum superposition, and it is not consistent with classical probability. In other words, an outcome is not completely separate from others, unlike in classical probability. In this article, we break away from this problem by adopting the Bayesian notion of probability.

In contrast, conventional approaches to quantum mechanics mostly accept discontinuity of state vector, sometimes called collapse of state vector. Reality simply does not exist before one measures some outcome - thus it makes no sense to talk of $P(x_t)$ independently of actual measurement x_t. Amplitude, not probability, matters between measurements, and quantum mechanics only provides correct probabilistic predictions at the moment of the measurement. Schrödinger equation only applies between measurements to provide predictions from one measurement outcome to the next outcome.

In conventional approaches to quantum mechanics, because of Equation 3.1, we cannot understand reality as if it is measurement-independent. There are times reality does not exist - or exists as fully quantum - and when reality suddenly exists. When one observes some definite outcome, instead of being condemned to probabilistic knowledge or belief, is left unexplained.

Is discontinuity of state vector much of a problem? It is not so if one accepts a state vector represents probability of outcomes that is to be updated when new observations or measurements arrive - in other words, Bayesian (epistemic) interpretation of probability. As far as conventional approaches can be reconciled with this Bayesian vision, they are not that different from the theory of quantum gravity to be explored in this article. What is really problematic, then, is the third issue - lack of an explanation as to when reality exists in apparently anti-realist quantum mechanics. As far as we assert ourselves as users of quantum mechanics having freedom to measure or not measure, even this is not a problem. When we choose to measure, we get outcomes and state vector is updated necessarily discontinuously. However, this makes us something of a constrained god - having freedom to choose reality we want under constraints of physics. And one can replace humans with other types of observers as well.

The above point basically was about an intuition that one should be able to define a measurement using physical laws, instead of being an independent notion. The collapse understanding violates such an intuition. There are thus alternative approaches that purport to eliminate collapse completely. One of the most popular understandings is many-worlds interpretation (MWI) - also referred to as Everettian interpretation of quantum mechanics. Because of space constraints, and given that this article is not a quantum foundation paper, we will focus on particular variants of MWI only. We acknowledge that there may even exist variants of MWI accepting the collapse postulate.

We argue that despite the claim that MWI simply is an interpretation of quantum mechanics and therefore gives same predictions of other interpretations, variants of MWI that deny the collapse postulate do make different predictions from other interpretations if possible Hamiltonians are not constrained. The argument is simple: if MWI upholds continuous evolution of state vector and breaks away from the collapse postulate such that a measurement outcome is understood to show one particular branch of entire state vector, then depending on a Hamiltonian, recoherence of different world branches is possible such that it would invalidate predictions made from state vector collapsed to the measurement outcome[12]. Decoherence[13] may explain why one can approximately point out when a world branch can be identified and thus one can form predictions as if state vector has collapsed, but this still leaves open possibility of recoherence.

There are two main roads one can take even in these variants of MWI given above:

- Accept that recoherence of world branches (post-measurement) is possible. This runs contrary to conventional approaches to quantum mechanics that denies such recoherence.
- Physical Hamiltonians would ban possibility of recoherence, making MWI completely identical to predictions in conventional approaches.

We do not take any of these paths in this article. We argue instead that while a state vector is updated for measurement outcomes, this update does not necessarily have to be discontinuous. In fact, we nowadays know that any measurement in reality is not an instantaneous process. An observer continuously receives signals from changes of her own states such that state vector is updated accordingly. When an observer assigns probability of 1 to a particular outcome of a measured subsystem in question, one says that a measurement process is completed, and a state vector dutifully captures this information. In practice though, there never really is an assignment of probability 1 to a particular outcome. A discovery of Higgs boson, for example, is not a definitive discovery in non-statistical sense.

We believe that the path underlined above resolves the measurement problem more nicely. First, evolution of a state vector is continuous and there is no collapse. Second, there is no need to worry about recoherence of world branches that invalidates the use of "collapsed" state vectors. Third, we do not have to worry about defining when and how a measurement occurs - a measurement simply is change of states of an observer which happens continuously. An observer perceives reality as if an observer continuously updates a state vector based on continuous variations of observer states. For matters of actual physics, one can simply say that an observer updates a state vector continuously. Intrinsically, an observer can only probabilistically infer other subsystems - thus state vector only dutifully captures probabilistic confidence one has on states of other subsystems. We underline a unique updating mechanism in this article that eliminates any subjective aspect that may enter into updating a state vector.

The statement that state vector encodes statistical and probabilistic information needs clarification, given that different bases (plural of basis) can be used to measure reality. We resolve this basis selection issue by extending the measurement (observable) postulate to a reduced density matrix of a subsystem being measured. The measurement postulate states that applying an observable O to a state vector $|\Psi\rangle$ ($O|\Psi\rangle$) can only give measurement outcomes as real multiples of eigenvectors of O, given that O is constrained to a self-adjoint operator. Since a density matrix is a self-adjoint operator, it is feasible to attempt applying the postulate. Of course a density matrix typically is not considered an observable, given its dependence on state vector. Thus we note that this is an extension of the usual interpretation of the measurement postulate. Given this postulate, we can give a basis on which an observer would speak of statistical and probabilistic information on a subsystem being measured, given that it is almost impossible to know the subsystem exactly. When state evolves continuously over time, a chosen basis by the measurement postulate in Schrödinger picture transforms continuously over time as well.

Thus we are re-interpreting quantum mechanics as providing a statistical toolkit to infer other subsystems in terms of probabilistic information on other physical subsystems based on variations of observer states. It is thus possible that different observers have different state vectors of the same universe. Because quantum mechanics is thought of as being observer-centric, with state vector being purely epistemic, inconsistency is blocked right from conception.

We emphasize that our interpretation does not add anything to postulates of quantum mechanics (and we eliminate the collapse postulate), other than clarifying and interpreting different concepts used in quantum mechanics.

What follows, after discussing nature of quantum gravity, is refining details on this interpretation. The Bayesian theory of gravity in this article depends on the interpretation. We use quantum reconstruction (or simply derivation from more fundamental understandings) of Schrödinger equation and Born rule to motivate and support the interpretation.

4 Background: superposition of gravity?

This background is not necessary to understand the Bayesian theory of gravity, but it does provide some motivation on why the classical Ryu-Takayanagi relation[6], instead of the one with quantum corrections, is used. In conventional approaches to quantum gravity - especially canonical gravity and string theory - there exists superposition of gravitational outcomes. That is, just as reality does not exist in conventional approaches to quantum mechanics, there exists superposition of spacetime, instead of single spacetime.

The following pictures of how spacetime may be superimposed can be given:

- Superposition of spacetime is always there - collapse suddenly occurs such that one spacetime outcome instantaneously becomes reality.
- While collapse into one spacetime outcome does not occur, superposition of spacetime eventually evolves and converges into particular spacetime such that we may effectively treat it as our spacetime.
- Superposition of spacetime does not exist - gravitons are not valid descriptions of gravity.
- Many-worlds interpretation approach, where different spacetimes exist simultaneously as world branches.

Since we set out to reject the first idea, which involves state vector discontinuity, and the fourth idea because of complications with many-worlds interpretations discussed before, what we focus on is the second and third pictures. The second picture may be reconciled with the preferred interpretation of quantum mechanics in this article. But we actually adopt the third picture in this article, and this is in fact what sets this article as a departure from usual approaches to quantum gravity. The single important reason for adopting this radical change is that this allows us, for the first time, to formulate a consistent theory of quantum gravity that reduces both to the Standard Model and general relativity in appropriate limits. While it may be possible that some alternative approach demonstrates both required reductions as well, we do not currently know whether this is the case.

There does exist a problem to the second picture as well. It suggests that the spacetime we "perceive" is only an approximation to the superposition of spacetimes that may contain spacetimes that are very different from our perceived spacetime. But then, how do we manage to pick this approximate spacetime out of superimposed spacetimes? After all, we do perceive reality in terms of spacetime, even if it may only approximately exists and not all features of reality may be captured by spacetime we perceive. This necessarily asks us the question of how we come to regularize quantum superposition to pick some spacetime to portray reality - and if we have to ask this question anyway, why not simply try an approach that eliminates superposition of spacetime?

There exists a misunderstanding that if spacetime does not exist in superposition, then one must be doing quantum field theory on fixed curved spacetime. We argue that this is not true. In light of holographic spirits, we present a vision of spacetime that directly is created from non-gravitational state vector. Spacetime arises directly from statistical information an observer has of other subsystems - or simply a state vector, and thus superposition of spacetime is eliminated. Spacetime does not exist independently of an observer but is merely an objective picture of statistical information that she has to experience. An observer does not have freedom to choose a rule on how she forms spacetime and statistical knowledge, despite spacetime being observer-dependent. Different new observations arising due to observed changes of observer's own states lead to different spacetimes, thus there is no one fixed background spacetime that is true independently of quantum contents. We explore more details when we discuss our theory fully.

In terms of Ryu-Takayanagi (RT) conjecture[6], this is about using the RT formula relating entanglement entropy to area without quantum corrections. This allows spacetime to vary depending on evolution of state vector, but keeps spacetime not superimposed. It may be useful to note how far the RT formula can relate geometry to quantum information in conventional string theory. String theory assumes that spacetime is superimposed, and eventually we cannot express entanglement entropy in terms of area. Each quantum correction may allow us to relate entanglement entropy with area, but these quantum corrections are perturbative, and cannot converge to non-perturbative analysis. Thus the RT formula that we use is an approximative tool to probe nature in string theory - though there is nothing wrong with this itself, as most tools we use in physics are of approximative nature anyway. In our Bayesian theory of quantum gravity, the RT formula without quantum corrections is considered fundamental instead.

The view advocated in this article thus brings us close to Einstein's picture of gravity as spacetime, not as fundamental force. Also, unlike in other theories of quantum gravity, spacetime always exists, and is not in superposition.

5 QM-P and quantum reconstruction

The two main features of quantum mechanics are: 1) Schrödinger equation, 2) Born rule. There are many ways quantum reconstruction can be done, but we stick with one way of reconstruction following Reginatto (1998)[14], which we believe to be cleanest. We consider how Schrödinger equation may be derived instead of postulated. Schrödinger equation basically is Hamilton-Jacobi equation and continuity equation for probability wave constrained by the principle of minimum Fisher information (MinFisher). Furthermore the approach in Reginatto (1998) also recovers the Born rule.

Afterwards, we provide motivations for MaxENT and the partition function requirement - including them to conventional quantum mechanics results in the new theory of **QM-P**. However, these are not really "additions" to quantum mechanics, if we are to interpret quantum mechanics as a consistent Bayesian inference procedure of nature. After all, these additions are really about determining $H(t)$, the path of Hamiltonians to be used in Schrödinger equation.

5.1 Deriving Schrödinger equation

This derivation is not required for the full Bayesian theory of gravity, consisting of QM-P and $\mathcal{A}/4$ theory of spacetime - the theory can be worked out as long as we assume Schrödinger equation and objective Bayesian interpretation of quantum mechanics. This derivation is provided just as to motivate fully Bayesian understanding of quantum mechanics.

We first start with the action Φ_a that recovers Hamilton-Jacobi equation and continuity equation when minimized with respect to S and P:

$$\Phi_a = \int P \left(\frac{\partial S}{\partial t} + \frac{1}{2} \sum_{i,k=1}^{n} g^{ik} \frac{\partial S}{\partial x^i} \frac{\partial S}{\partial x^k} + V \right) d^n x dt \tag{5.1}$$

where $P(\mathbf{x}, t)$ refers to probability, and "potential" $V(\mathbf{x}, t)$. While coordinates $\mathbf{x}$ is used, it does not represent space, while t does refer to time. It is just like how a quantum operator can be understood in different basis, and $\mathbf{x}$ just is a convenient way to represent P, S, Φ_a and V. Therefore, we will understand derivations in Reginatto (1998)[14] as being general and not necessarily referring to local particles living in spacetime. g^{ik} in this context does not refer to spacetime metric, and just is phase space metric. Hamilton-Jacobi equation is then written as:

$$\frac{\partial S}{\partial t} + \frac{1}{2} \sum_{i,k=1}^{n} g^{ik} \frac{\partial S}{\partial x^i} \frac{\partial S}{\partial x^k} + V \tag{5.2}$$

and continuity equation is written as:

$$\frac{\partial P}{\partial t} + \sum_{i,k=1}^{n} g^{ik} \frac{\partial}{\partial x^i} \left(P \frac{\partial S}{\partial x^k} \right) = 0 \tag{5.3}$$

We now extend Φ_a by the constraint of minimum Fisher information. Fisher information metric I_{ik} is:

$$I_{ik} \equiv \int g^{ik} \frac{1}{P} \frac{\partial P}{\partial x^i} \frac{\partial P}{\partial x^k} d^n x dt \tag{5.4}$$

$$\Phi_b = \sum_i \sum_k I_{ik} \tag{5.5}$$

The final action to be minimized, with respect to P and S, is:

$$\Phi = \Phi_a + \lambda \Phi_b \tag{5.6}$$

Continuity equation remains the same as when Φ_a was minimized instead, but Hamilton-Jacobi equation is replaced with its extension. This is the starting point of what differentiates quantum mechanics from classical statistical mechanics. The resulting equations, with $\lambda = \hbar^2/8$ and $\psi(\mathbf{x}) = \sqrt{P(\mathbf{x})} e^{iS/\hbar}$ gives us Schrödinger equation (again, $\hbar = 1$ by natural units convention):

$$i\hbar \frac{\partial |\Psi\rangle}{\partial t} = H |\Psi\rangle \tag{5.7}$$

A state vector, or wavefunction, is simply $|\Psi\rangle = \int d^n x \; \psi(\mathbf{x}) |\mathbf{x}\rangle$.

ψ has both P and S - thus it is a good representation of a state, capturing both reality and epistemic information. Furthermore, it echoes path integral understanding of quantum mechanics. One may ask why a state has to be ψ instead of other possible variables. In fact, it does not have to be, with all consistent choices of a state variable producing equivalent results. It is just that ψ and Schrödinger equation allow for simplest analysis, having the important property of linearity. When you have linearity, why look for other definition of a state variable? There may be only one consistent choice of a state variable or many others, but this does not matter.

Note that while the original reference[14] casts this Schrödinger equation as being non-relativistic, the Schrödinger equation derived does apply to relativistic cases as well, as far as we allow for general interpretations, instead of referring to particle's position or momentum. It is well-known that Schrödinger equation itself does apply for relativistic physics as well, despite common misconceptions.

We distinguish ψ with $|\Psi\rangle$: the former is a measure of a state containing both probabilistic information and actual state. The latter is a state vector that captures information of all possible states.

In deriving Schrödinger equation, a Bayesian view of probability does not need to enter, except for minimization of Fisher information. While it is possible to reconcile this with a physical view of probability, it is not natural. This motivates a Bayesian view of quantum mechanics, which is to be discussed separately from quantum reconstruction. But rest of quantum reconstruction is founded upon a Bayesian view of probability.

Furthermore, because Schrödinger equation does not specify Hamiltonian to be used, minimization of Fisher information is compatible with maximization of von Neumann entropy - both principles will simultaneously need to be used to pin down complete state vector evolution.

5.2 Why MaxENT?

We follow the Shore-Johnson Bayesian axioms to support the principle of maximum entropy[11]. A review article that we follow can be found in [5]. We start directly from an entropy maximization problem for a system of subsystems but without knowing functional form of entropy. This seems to assume the principle of maximum entropy (MaxENT) from the start. But the real assumptions used are two:

- The idea that a unique objective way of setting posterior probability is there and that this can be found by optimizing some objective function. This objective function $S(p(\vec{x}), q(\vec{x}))$, where $p(\vec{x})$ refers to posterior probability distribution and $q(\vec{x})$ refers to prior probability distribution, is called entropy, with its functional form later revealed to be Shannon entropy.

- The same objective function is to be used with different constraints to determine a posterior probability distribution.

Having accepted these assumptions, we now impose the Shore-Johnson axioms:

- Subset independence. If new information regarding only $p(\vec{x_j})$ and $p(\vec{x_k})$ that should not change $p(\vec{x_j}) + p(\vec{x_k})$ arrives, this should not change rest of $p(\vec{x})$.

- Coordinate invariance. $p(\vec{y})$ obtained from existing posterior solution $p(\vec{x})$ by coordinate transformation should be the posterior distribution solution of the problem transformed with coordinate transformation.

- System independence. If $x_1, x_2, .., x_k$ are independent subsystems, and provided constraints are for a single subsystem that is independent of other subsystems, then $p(\vec{x})/q(\vec{x}) = \prod_i r(x_i)$.

- Uniqueness. Posterior probability distribution must be unique.

Restricted to constraints that are of form $\int \mathcal{D}[x]p(x)a(x) = \bar{a}$, this yields unique entropy form of:

$$S(p(x), q(x)) = -K \int \mathcal{D}[x]p(x) \log (p(x)/q(x)) \tag{5.8}$$

which is Kullback-Leibler divergence with sign flipped after setting $K = 1$. While derived with a particular type of constraints, the form is consistent with other types of constraints that provide unique posterior distribution with entropy form in Equation 5.8. We restrict constraints to such ones.

Consistency means that we arrive at same posterior probability for given observation data, regardless of how data are updated sequentially. This allows us to elevate the principle of maximum relative entropy to simply the principle of maximum entropy (MaxENT).

5.3 Why von Neumann entropy?

So far entropy is assumed to be Shannon entropy of $S = -\sum_i p_i \log p_i$. But in quantum mechanics, measure of entropy used is von Neumann entropy. The question then is whether one can simply invoke arguments based on Shannon entropy when von Neumann entropy is used. We confirm that the answer is yes.

The measurement postulate of quantum mechanics says that any observable must be represented by a self-adjoint operator, with each eigenvalue of an operator understood as a measurement outcome. Any density matrix ρ is a Hermitian operator and can qualify as an observable. When ρ is a reduced density matrix, it is an observable that is applied to a hypothetical state vector of a subsystem being measured by an observer. A state vector is for the entire universe in our framework, so one may wonder what a hypothetical state vector of a subsystem would mean, when a subsystem is in a mixed state. Here, we are thinking as if there only exists the subsystem in the universe, as to allow us to translate reduced density matrix into probability of subsystem states. One may complain how probability can ever be an observable. But then even a “physical” observable does not directly provide how it would physically be measured. From that perspective, there is no reason why probability cannot be represented by an observable. Note that an observable usually is considered state-independent, but a density matrix is state vector-dependent, thus there is slight generalization ongoing.

A density matrix represents degree of statistical knowledge an observer has about a subsystem. And the measurement postulate says that only when state of a subsystem is in an eigenvector can a measurement proceed. Thus, given a density matrix, probability only makes sense for quantum states - in this case, hypothetical reduced ones, from the point of an observer - that are eigenvectors of the density matrix observable.

Having diagonalized a density matrix, von Neumann entropy of $S = -tr\left[\rho \log \rho\right]$ is equivalent to Shannon entropy, confirming equivalence. von Neumann entropy simply allows one to calculate the right measure of entropy without considering basis change. This justifies use of MaxENT with von Neumann entropy instead of Shannon entropy in quantum mechanics.

Thus what remains is why an observable must be a self-adjoint operator, having assumed uniqueness of density matrix as representing probability under the self-adjoint operator constraint. For the purpose here, it is enough to examine why an observable must be a diagonalizable linear operator, with eigenvalues representing measurement outcomes. Also, one can instead argue that any measurement process can be described by a diagonalizable linear operator. This is trivial, as one can simply use all possible orthogonal measurement outcomes to form a diagonal matrix of eigenvalues, which one can transform by basis transformations. Thus what is really special about the measurement postulate (and thus quantum mechanics) is that only one particular basis, chosen by a self-adjoint operator, is privileged for measurements, and it is meaningless to talk of a single observer measuring in other bases (plural of basis) simultaneously.

5.4 Total entropy of the system

If MaxENT is to be used, what should total entropy of the system refer to? von Neumann entropy of the system or sum of von Neumann entropy of each irreducible subsystem? The answer of course is the latter - the former should be zero whenever the state vector of a system is pure, so would not be meaningful. But why would this measure make sense?

In context of information theory, different irreducible subsystems, by definition, are separate information sources. This mandates that von Neumann entropy of an individual subsystem must be summed up to get expected total information of the entire system - or "total entropy". (Entropy is measure of expected information in information theory.) By contrast, von Neumann entropy of the entire system is zero - as long as the pure state vector evolves unitarily. One can trivially translate this argument into the MaxENT context without invoking information theory - though two arguments are actually equivalent.

5.5 Nature of measurements

Treating density matrix as an observable raises the following question: should not probability be 1 when a subsystem is measured? If so, would not this defeat the use of density matrix as an observable for probability? This is not the case.

First, a measurement is simply an observer noticing change of her own state (or equivalently Bayesian updates) that is used to infer state of other subsystems. And it is assumed that an observer continuously notices this change. It is required to avoid discontinuous evolution of a state vector. Note that we distinguish “completed measurements” from general “measurements”.

Second, a completed measurement of a subsystem such that a particular outcome has probability of 1 takes time even in practice. When a measurement is complete, one can say collapse of wavefunction (or state vector) happens. But there actually is no collapse - no discontinuous evolution of state vector - and probability of the measured outcome is 1, as expected from collapse. We explain how this is possible.

An observer only has probabilistic predictive knowledge of future outcomes of a subsystem to be measured. But Bayesian updates add to knowledge an observer has about the subsystem, changing probabilistic inference. When a subsystem being measured can be described by a pure state vector instead of density matrix, a measurement is completed - one particular outcome is surely observed. (An entire system is always described by a pure state vector.) An observer does not have freedom to collapse a subsystem into a definite outcome, and thus what we mean by measuring at some time t' before a measurement is completed can only refer to probabilistic knowledge. This is why the use of density matrix as a probability observable makes sense.

The common definition of the measurement problem is difficulty in explaining how wavefunction collapse occurs. The very point of difficulty lies in discontinuity of state vector evolution required by wavefunction collapse. Thus, if this discontinuity is eliminated, one resolves the measurement problem - and discontinuity was eliminated.

The above understanding means subjecting probabilistic inference of physics to the new physical constraint: observer outcome trajectory. Of course each observer cannot actually know future outcomes - for future predictions, an observer is condemned to Hamiltonian $H(0)$ obtained at present-time $t = 0$ such that we evolve predicted future state vector as $|\psi(\Delta t)\rangle = e^{-iH(0)\Delta t}|\psi(0)\rangle$. However, we do know that $H(\Delta t) \neq H(0)$ for actual state vector in the future. This is just Bayesian updating - nothing more, nothing less. Furthermore, Schrödinger equation still works as the main equation of quantum mechanics for both predicted state vector and actual state vector.

5.6 Irreducible subsystem

MaxENT for quantum mechanics relies on total entropy of the entire system, produced by summing von Neumann entropy of irreducible subsystems. Thus this requires a proper definition of what an irreducible subsystem means.

Technically, each irreducible subsystem would have to be given empirically. We can instead ask how we may find irreducible subsystems from empirical data.

Given some state vector $|\Psi\rangle$ of system U, U is decomposed into irreducible subsystems i when every i exhibits zero entropy $S_{i,EFT} = 0$ in the quantum effective field theory in the current spacetime. Note that $S_i = S_{i,UV} + S_{i,EFT}$, where $S_{i,UV} = \mathcal{A}/4$, with $\mathcal{A}$ understood to be minimal surface area - though discussions, for now, are deferred to when Ryu-Takayanagi[6] and $\mathcal{A}/4$ theory of spacetime are presented.

The above definition requires recognizing that spacetime is assumed not to be in superposition. Also, the above definition is not really fundamental - it is rather a consequence of $\mathcal{A}/4$ theory of spacetimeto be discussed later.

5.7 Strong complementarity and quantum decoherence

How a measurement is defined may be worrisome, because observers may come to use different bases (plural of basis) for measurements of the same subsystem. We argue here that there is nothing wrong with this. In fact, this is what makes quantum-mechanical vision of reality different from classical vision of reality and exactly is about the principle of (strong) complementarity. As long as we dispel the notion that an observer must see exactly one thing for the same subsystem, there is nothing inconsistent about this complementarity vision. Quantum mechanics is just a statistical inference tool - so it is consistent by default.

Inconsistency only arises when we try to use state vectors of more than one observer simultaneously and directly to understand reality. This is a nonsense by definition as far as a Bayesian understanding of quantum mechanics is upheld - state vector represents state of knowledge of a single observer. If information arrives from another information source by interacting with an observer and thus varying observer state, then this information would be updated accordingly by Bayesian and quantum-mechanical principles. The key point here is that an observer can only update by looking at its own state - it can never know directly what other subsystems look like. Thus it makes no sense to update by direct subsystem information, as it cannot happen.

However, it is true that rarely do we have to consider observers measuring in different bases, before we even think of consistency issues, as have been analyzed. Why is this so? It is explained by quantum decoherence[15][16]. While quantum decoherence is often said to be about discovering when "classical reality" emerges, these are specific main points of quantum decoherence literature:

1. Explaining when a density matrix of a subsystem being measured converges to an equilibrium, or more precisely a local equilibrium.

2. Explaining why despite interactions with environment required for measurements, the Born rule is respected, working as if a subsystem being measured evolves as a pure state vector, instead of mixed state evolution. Not all interactions work this way, thus providing forms of interactions that ensure nice properties required for accurate measurements. This overlaps with the first point.

3. Explaining when different observers agree on the same local equilibrium.

The first and second point are well-explained in quantum decoherence literature[17] by a density matrix represented in a particular basis converging toward a diagonal matrix (with entries featuring amplitude magnitude squared), with off-diagonal terms exponentially decaying. The point here is not about whether there exists a basis such that a density matrix in question can be represented as a diagonal matrix - there always is a basis that a density matrix be represented by a diagonal matrix. The point is that evolution of density matrix moves toward a stable equilibrium. If there is such an equilibrium, then we may simply pick the privileged basis of the equilibrium to understand our measurements.

Of course the density matrix of a subsystem does not always remain close to the equilibrium after significant decoherence - thus this equilibrium actually is a local equilibrium.

Eventually, the density matrix moves away from the equilibrium, but for duration of a non-instantaneous measurement process, the local equilibrium can be treated as if it is the final equilibrium, as far as deviations away from an equilibrium after significant decoherence is very slow. When discussing how spacetime arises from entanglement, classical reality would be defined as condition of local equilibrium. For a local equilibrium, Einstein field equations of general relativity is the law of spacetime.

In case different observers have relatively equivalent and sufficient access to subsystems in question, consistency requires that the same local equilibrium understanding, at least with high fidelity, must be shared across different observers. This is why despite strong complementarity, we can ignore its effects most of time, with the most famous example that effects of strong complementarity cannot be ignored being a black hole.

5.8 Nature of an observer

It may be argued that an observer cannot maintain an ideal inference mechanism, as we seem to assume. We instead argue that observer's perception works as if it retains an ideal inference mechanism. That is, an observer does not actually "infer" nature - laws of nature force an observer to behave as if it is ideally observing the universe. With this clarification, the charge of an observer requiring too much computational resource is cleared.

In classical physics, the principle of least action is fundamental - but no one says that because a subsystem acts as if it computes least action, the principle must be wrong. The same point applies here as well.

Also, this epistemic viewpoint is shared by Copenhagen interpretation - the most conventional approach to quantum mechanics - which simply argues that state vector collapses when we measure or equivalently "learn" about a subsystem in question. Such learning was assumed to be ideal as well. We simply required that this learning process be continuous, instead of being discontinuous. An observer is condemned to learn and has no freedom on learning processes.

5.9 Conservation principle: partition function constraint

Partition function Z is defined as $Z(T) = Tr\left[e^{-H/T}\right]$ where H refers to Hamiltonian, where T is temperature and Tr refers to trace. We set temperature $T = T_C$, where T_C refers to "critical temperature", a constant. Furthermore, $Z(T_C)$ is constrained to be constant. We do not specify what value of Z and T_C must be - they have to be determined empirically.

The motivation behind the constraint is simple. First, note that we use time-varying Hamiltonian $H(t)$ - or equivalently and more in line with interpretations in this article, a path of Hamiltonians. Second, we wish to understand each $H(t)$ in the path as degrees of freedom integrated away from "final" theory $H(t_f)$ - meaning that all $H(t \neq t_f)$ are renormalized theories of theory $H(t_f)$. This requires us thinking of what different $H(t)$ would have to conserve.

Essentially, we desire some real-valued functional $Y(x, H)$, with real-valued x such that $Y(x_0) = f(H(t_i))$ is conserved among H that is renormalized theory of $H(t_f)$. But given ignorance of H, we also wish to be able to uniquely identify H from function $Y(x)$ up to change in bases by checking derivatives of $Y(x)$ at $x = x_0$, just as in Taylor series.

Furthermore, $Y(x, H)$ then should be considered more fundamental, as a meta-theory, than theories themselves.

There is no clear answer to what conservation must be done, which would fix the form of functional Y, but we know that the most straightforward example already exists: partition function $Z(T) = Tr[e^{-H/T}]$, with assumption that different H, that are renormalized theories of $H(t_f)$, share same $Z(T_C)$. Conventional uses of partition function, such as GKPW relation[2][3] in AdS/CFT, suggest we set $T_C = 1$, but fixing value of T_C for the purpose of this article is not required. This identification of functional Z to be the above functional Y provides a constraint that is non-trivial that can cleanly mapped to critical fixed points in existing thermodynamics and QFT renormalization. Different values of Z are consistent with currently available evidence, given different potential completions of the Standard Model - thus we do not specify what value of Z must be taken, as our focus is on quantum gravity, not Grand Unification. However, once any single $H(t')$ is known, then value of $Z(T_C)$ would be easily calculated, if T_C is already known and the partition function constraint provides valid restriction to possible Hamiltonians.

If there were no partition function restriction, then determined evolution would not have been plausible, since MaxENT is exploited too quickly. MaxENT, Schrödinger equation, partition function along with continuous observations allow one to determine a unique sensible state vector evolution that an observer assigns to the universe, when provided an initial state vector.

5.10 Physical aspects of quantum mechanics

Here we separate physical (empirical) aspects of quantum mechanics from logical aspects that arise from consistency requirements. Physical aspects of quantum mechanics are:

1. Lagrange multiplier $\lambda = 1/8$ when deriving Schrödinger equation.
2. Required convergence toward non-relativistic classical physics at classical limits. (Hamilton-Jacobi and continuity equation)
3. Continuous evolution of reality.
4. A single observer cannot simultaneously measure in different bases. (Complementarity)
5. Observer outcome trajectory used for Bayesian updating.
6. Constant partition function $Z(T_C) = Tr\left[e^{-H/T_C}\right]$ and value T_C.

Rest of quantum mechanics can be said to arise from Bayesian inference consistency requirements (MaxENT, MinFisher) - thus quantum mechanics just is probabilistic inference with given physical constraints.

5.11 Recovering quantum theories

If this Bayesian view of probability is correct for quantum mechanics, how does one recover usual quantum theories, such as the Standard Model? The point is that conventional quantum theories are more like final theories in a Bayesian learning framework. Under sufficiently many observations of subsystems of equal characteristics, Hamiltonian, reduced to apply only for a subsystem, that will be used to predict future state vector evolution of these subsystems of equal characteristics is almost the same one. This suggests that we really do not need full QM-P most of time. However, if there is some degree of freedom in the universe that is only rarely accessed by an observer subsystem, then QM-P comes to matter, as number of observations is insufficient. Black hole complementarity[7] would count as one important example in this direction.

6 Background: classical Ryu-Takayanagi and tomography

The Ryu-Takayanagi (RT) conjecture[6] says:

$$S_A = \frac{\mathcal{A}_A}{4} \tag{6.1}$$

where S_A represents entanglement (von Neumann) entropy of region A, or in this article subsystem A, and $\mathcal{A}_A$ refers to the minimal surface area of subsystem A in the "bulk", though in this article we generalize into contexts not involving boundary-bulk relations. Equation 6.1 is conjectured to work for time-dependent state vector as well, as far as appropriate changes to the notion of minimal surface is made[18].

Usually entanglement entropy is very difficult to compute in QFT, even after divergence problems are resolved, and the RT formula serves to provide the holographic machinery to compute entanglement entropy. In this article, this is not the direction we focus - we go from entanglement entropy to spacetime. This is because we assume that quantum state vector is available to provide emergent spacetime, provided by QM-P, but one does not have to restrict to QM-P.

Before going into how we translate minimal surface areas into entire spacetime, it may be beneficial to note that Equation 6.1 from conventional string-theoretic point of view is a leading order approximation. In conventional understanding of string theory, area law - that entanglement entropy is proportional to area - eventually breaks down, because quantum superposition of spacetime kicks in heavily. But as an approximation, Equation 6.1 works well. In $\mathcal{A}/4$ theory of spacetime, however, Equation 6.1 is fundamental, and does not just work as an approximation. We can do this because we have already assumed that spacetime never exists in superpositions, and thus we can safely ignore quantum corrections.

6.1 Quantum corrections to RT formula

It is beneficial to see what zero quantum correction to RT formula of Equation 6.1 means. It has been proposed[19] that the one-loop correction to the RT formula is essentially given by the bulk entanglement entropy between two bulk regions separated by a minimal surface. The bulk entanglement entropy can be calculated by computing entanglement entropy of the bulk region A_b connected to the boundary region A in an effective field theory on a background spacetime. (Figure 6.1)

Taking Equation 6.1 as fundamental amounts to requiring this bulk entanglement entropy to be zero. In such a case, region A_b is in pure state, along with other regions

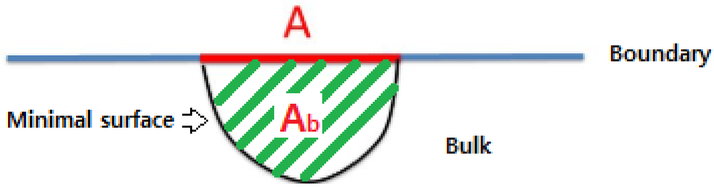

Figure 1. Boundary, bulk and minimal surface in Ryu-Takayanagi and AdS/CFT

enclosed by their minimal surface and boundary. Furthermore, this suggests that spacetime is constructed as to allow for local symmetry without changing underlying quantum physics. Essentially, this echoes the point that each irreducible subsystem is an independent information source - thus total entropy of the universe should add up entropy of each subsystem. This line of thought between entanglement and spacetime has recently been pursued, though with different styles of expositions - see, for example [20].

Gravity thus purifies an irreducible subsystem.

7 Background: tomography - from area to spacetime

A tomographic method that computes entire spacetime using minimal surface area of each irreducible subsystem was suggested in Cao-Carroll (2018)[21]. The point is that when we have surface area terms, we can relate them with metric tensor:

$$\mathcal{A}(\mathcal{C}) = \int_{\mathcal{C}} \sqrt{\det w_{ij}}\; d\sigma \tag{7.1}$$

where $\mathcal{C}$ refer to minimal surface, w_{ij} refers to induced metric tensor from metric tensor g_{ij} of a space manifold, and $\mathcal{A}$ refers to area of minimal surface. The point made in Cao-Carroll[21] is that this is essentially Radon transform, so we can seek for inverse Radon transform method that would recover metric tensor from surface area.

However, (inverse) Radon transform is defined for $\mathcal{C}$ that is a totally-geodesic codimension-1 submanifold of space manifold. To go around limitations, Cao-Carroll (2018)[21] proposes finding a best-fit maximally-symmetric background space manifold for a state vector using MDS, and understand the actual space manifold and state vector as perturbations on the background space manifold and the associated state vector. In Cao-Carroll (2018), perturbations are understood to be small such that first-order approximations are justified. In this article, we drop such restrictions - the inverse Radon transform technique provides a unique spacetime generally as far as the assumed "reference space" (analog to background space in Cao-Carroll) admits inverse Radon transform and state vector does evolve continuously and is differentiable over time - thus no collapse.

Very likely, this reference space will be initial space manifold we associate with initial state vector - or at least the best-fit maximally-symmetric reference space to the initial state vector, but such restriction is not be imposed.

The reference space is restricted to a maximally symmetric space manifold. We assume that the reference space manifold remains fixed throughout the article for well-behaved spacetime evolution. However, even this assumption can be relaxed, with reference

Lorentzian spacetime manifold, consisting of reference space submanifold slices (restricted to maximally symmetric space) tied together, providing reference space at each time t for an observer.

The spacetime update prescription over time works as follows:

- (1) First, determine the reference space manifold to be used. Also determine initial state vector and its consistent initial space, possibly different from reference space.
- (2) Current time is set as $t = 0$. [In case (2) is reached from (5), $t = dt$ at (5) is re-scaled to be $t = 0$.] Space and state vector at $t = -dt$ are called background space and state vector.
- (3) State vector is updated for $t = 0$ accounting for infinitesimal Bayesian updates as well, which induces infinitesimal area perturbations to the background surface area terms. Note again that infinitesimal area perturbations are relative to area term at $t = -dt$.
- (3) Determine the "fake" infinitesimal induced metric tensor perturbation from area perturbations by inverse Radon transform as if background metric tensor is that of the reference manifold. The unique fake infinitesiaml induced metric tensor perturbation is obtained, because the reference space is assumed to be maximally symmetric.[21]
- (4) The fake infinitesimal induced metric tensor perturbation is converted to the actual induced metric tensor perturbation by the re-scaling procedure to be described below.
- (5) The actual metric tensor is computed by summing the background metric tensor with the computed actual metric tensor perturbation to the background. Repeat from (2) at time dt.

Let us work on first-order (infinitesimal) variation and find[21]:

$$d\mathcal{A} = \frac{1}{2}\int_{\mathcal{C}} \sqrt{\det w_{ij}}\; w^{ij} dw_{ij}\; d\sigma \tag{7.2}$$

(dw_{ij} refers to infinitesimal change of induced metric tensor over infinitesimal time.) Notice some difference from Cao-Carroll (2018)[21]. The above equation uses infinitesimal perturbations over infinitesimal time ($d\mathcal{A}$, dw_{ij}), instead of perturbations ($\delta\mathcal{A}$, δw_{ij}) not restricted to infinitesimal perturbation. Since we assumed continuity (and differentiability) of state vector and measurement updates, this allows for infinitesimal analysis, and maps safely to small-perturbation approximation analysis carried out in Cao-Carroll (2018).

Now the inverse Radon transform from area perturbation to induced metric tensor perturbation is carried out with the assumption that w^{ij} is the reference space metric tensor - to avoid confusion we denote it instead as w'^{ij}, which allows for the unique solution. This gives us dw'_{ij}. We then re-scale back entry-wise as:

$$dw_{ij} = dw'_{ij}\frac{w'^{ij}}{w^{ij}}\frac{\sqrt{\det w'_{ij}}}{\sqrt{\det w_{ij}}} \tag{7.3}$$

which is enforced for all sets of coordinate indices, and the above equation does not follow Einstein summation notation, and is in entry-wise notation. Other equations involving tensors always follow Einstein summation notation in this article. Note that one should not transform dw_{ij} to dw_{uv} using previous coordinate system relationship. This is because pre-perturbation $y = y(x)$, where x, y refer to coordinate vector in a different coordinate system, no longer holds.

Also, while we use $d\mathcal{A}$ of minimal surface for purpose of inverse Radon transform, since inverse Radon transform gives us entire "fake" space metric tensor perturbation, not just for minimal surface, we can use same Equation 7.3 for manifold points not in minimal surfaces. Thus, the tomographic procedure fully recovers actual metric tensor.

8 The Bayesian theory of gravity

8.1 Summary: QM-P and $\mathcal{A}/4$ theory of spacetime

Thus, we now have all pieces of the Bayesian theory of gravity. Let current (present) time be $t = 0$, initial condition time be $t = t_0$, and an observing (measuring) irreducible subsystem that "uses" quantum mechanics be j. $Tr[\cdot]$ refers to trace. An observer only really measures her own state, and only knows other subsystems by inferring from change of her states. It consists of:

QM-P:

- Schrödinger equation
- MaxENT: coming from objective Bayesianism
- Partition function constraint: Hamiltonians in the path $H(t)$ are renormalized theories of final theory $H(t_f)$.
- Observation constraint: Observation path (reduced density matrix path) for j: $\rho_j(t)$ for $t_0 \leq t \leq 0$, which must be pure for j, is given. That is, j knows its own pure quantum state exactly from $t_0 \leq t \leq 0$. Subsystem j does not know exact state of other irreducible subsystems, and it only infers states of other subsystems probabilistically.
- State vector and Hamiltonian constraint: state vector and Hamiltonian $|\Psi(t)\rangle$ and $H(t)$ are given for time $t_0 \leq t < 0$ (notice "$<$" instead of $\leq$ for $t < 0$, whereas for observation constraint, it was $t \leq 0$).

$\mathcal{A}/4$ theory of spacetime based on Ryu-Takayanagi[6]:

- Classical Ryu-Takayanagi relation of Equation 6.1: mapping state vector (entropy of subsystem: S_i) with spacetime (surface area: $\mathcal{A}$).
- Metric tensor v_{ij} for initial space manifold at time $t = t_0$ that is consistent with state vector $|\Psi(t_0)\rangle$ and classical Ryu-Takayanagi is considered given. Metric tensor u_{ij} for reference space manifold is also considered given, constrained to be maximally symmetric space manifold. Full metric tensor for entire spacetime is determined

through the tomographic infinitesimal update method aforementioned, which recovers metric tensor perturbation to background from minimal surface area perturbations to background.

To summarize QM-P more concisely, this is an optimization problem for each irreducible subsystem j with MaxENT objective function:

$$\max_{H(0)} \sum_{i \neq j} S_i(t = 0) \tag{8.1}$$

at $t = 0$, where $S_i(t) = -Tr[\rho_i(t) \log \rho_i(t)]$ with ρ_i referring to reduced density matrix of irreducible subsystem i, subject to:

$$i \frac{d}{dt} |\Psi(t)\rangle = H(t) |\Psi(t)\rangle \tag{8.2}$$

which is Schrödinger equation where $|\Psi(t)\rangle$ is state vector at time t, $H(t)$ is Hamiltonian path, and

$$Tr[e^{-H(t)/T_C}] = Z \tag{8.3}$$

which is the partition function constraint where $Z = Z(T_C)$ is time-independent constant, subject to given $|\Psi(t)\rangle$ for $t_0 \leq t < 0$ and given observation $\rho_j(0)$, which may instead be replaced with a pure state vector for j alone. Prediction of future state vector is made by setting $H(t) = H(0)$ for $t > 0$ and evolving state vector using $|\Psi(0)\rangle$ and Schrödinger equation.

Now summarizing the rest of the Bayesian theory of gravity more mathematically.

$$S_i(t) = \frac{\mathcal{A}_i(t)}{4} \tag{8.4}$$

which is Ryu-Takayanagi relation. Initial space is given, which must be consistent with $|\Psi(t_0)\rangle$ and Ryu-Takayanagi. Spacetime is updated infinitesimally over past space manifold using the tomographic procedure - see the discussion before.

8.2 Recovering the Standard Model and general relativity

The Standard Model is trivially compatible with our QM-P framework. Regardless of whether some completion (extension) of the Standard Model is the final theory $H(t_f)$ of the universe in QM-P or not, if it is (or at least approximately very close to) one Hamiltonian of path $H(t)$, we can at least understand it as "local equilibrium" theory in QM-P, where deviations of Hamiltonians from the Standard Model over time are so slow that for most experiments we can consider the Standard Model as valid. This includes scenarios where some completion of the Standard Model is the final theory for currently accessible observations.

Different extensions of the Standard Model are clearly compatible with different values of partition function constant $Z = Z(T_C)$ in QM-P. In case initial evolution of spacetime keeps space to be very close to being maximally symmetric, then empirical cosmological data may sufficiently provide details on how Z and T_C must be set. After all, just figuring out one Hamiltonian $H(t')$ out of entire path $H(t)$ is enough to determine value of Z in case T_C is known, given correctness of the partition function constraint conjecture.

In a way, we are separating the question of Grand Unification and quantum gravity. This article provides a full theory of quantum gravity, but does not answer Grand Unification, which attempts to fill in other non-gravitational holes of the incomplete Standard Model.

Our focus thus is to actually demonstrate that the Standard Model is compatible with general relativity, which currently no theory of quantum gravity demonstrates. (But incompatibility of the Standard Model with general relativity has not been demonstrated for many candidate theories of quantum gravity, especially string theory.) More generally, demonstration is that any quantum theory admitting Rindler modular Hamiltonian is compatible with general relativity. The argument here essentially is re-interpretation of the result in Cao-Carroll (2018)[21], which generalizes Jacobson (2016)[22]. Also, in that general relativity is understood as an equation of state for local equilibrium, this article is in spirit of Jacobson (1995)[4]. (Essentially, Jacobson (2016)[22] re-casted the result in Jacobson (1995)[4] by treating entropy statistically, rather than in thermodynamic fashion - but without use of boundary-bulk holography.) This demonstration does not depend on QM-P, and is compatible with any quantum theory, as aforementioned.

The point is simple - that if a system is in local equilibrium state, with horizons identified by local Rindler horizons, then the equation of state locally must be Einstein field equations[4]. While we mostly follow derivations in Cao-Carroll (2018), some details are different.

First, since a subsystem A is in local equilibrium, entropy of a subsystem is as maximal as it can under constraints at the time. This does not mean that a subsystem A is in maximal entropy states across time. One can write in terms of potential infinitesimal perturbations as:

$$0 = \delta S_A = \delta S_{A,UV} + \delta S_{A,EFT} \tag{8.5}$$

We drop subscripts A whenever the context is obvious from now on. For a very small subsystem A, one can switch coordinate system to set background metric tensor as flat. Initially this assumption seems indefensible, but this is actually required. Equations in classical limit are often determined in terms of the $\hbar \to 0$ limit, which is equivalent to taking size of a very non-classical quantum system to be small.

The linearized (thus, infinitesimal) Einstein field equations go:

$$\delta G_{uv} = 8\pi \delta T_{uv} \tag{8.6}$$

provide the constraint[21] of

$$\delta \mathcal{A} = -8\pi \int_{\mathcal{C}} \int_{x>0} x \delta T_{tt} \; d^n x \tag{8.7}$$

where T_{uv} refers to stress-energy tensor. Though note that T_{uv} in Equation 8.7 is classical, not quantum operator. Thus we now need to distinguish quantum operators from classical counterparts, which is done by adding a hat superscript to quantum operators. Then one sets up Rindler modular Hamiltonian $\hat{H}_{mod}$ which provides the same constraint of Equation 8.7, with identifications:

$$\delta\langle \hat{H}_{mod}\rangle = \delta S_{EFT} \tag{8.8}$$

$$\delta T_{tt} = Tr\left[\delta\rho_{EFT}\hat{T}_{tt}\right] \tag{8.9}$$

$$\frac{\mathcal{A}}{4} = S_{UV} \tag{8.10}$$

We then trace back from Equation 8.7, along with the Lorentzian invariance assumption to obtain the linearized Einstein field equations of Equation 8.6.

Initially Equation 8.10 may seem strange. Did we not define entanglement entropy to be $\mathcal{A}/4$? This is true for actual spacetime, but here we are considering possible spacetime perturbations and how quantum effective field theory states living in background spacetime should respond, and vice versa. In such contexts, Equation 8.10 is the correct equation. In other words, we are calculating gravitational backreactions due to changes in quantum contents. After gravitational backreactions are incorporated into spacetime, $S_{EFT} = 0$ following appropriate change in S_{UV} - since previous calculations were defined for an effective field theory under the old background spacetime.

These calculations describe how spacetime changes as state vector evolves over time, as far as a subsystem in question remains in local equilibrium[4], with general relativity being the law of spacetime at this regime.

Finally, one may ask what happens for cosmological constant Λ. Our equations, for demonstrating recovery of general relativity, so far were for infinitesimal change and perturbations under flat spacetime background. Note again though that despite having assumed flat spacetime background, this choice of background does not affect general validity of resulting equations, because choice of local background can always be set to flat spacetime by coordinate transformation, in case resulting spacetime must be Lorentzian. For infinitesimal perturbations under Minkowski background, the term involving cosmological constant drops out from perturbation form of Einstein field equations[23] - reducing to Equation 8.6. Thus cosmological constant can only be determined from initial space manifold.

9 Conclusion

The full Bayesian theory of gravity consists of QM-P and $\mathcal{A}/4$ theory of spacetime based on classical Ryu-Takayanagi relation[6]. QM-P describes non-gravitational behaviors, which directly are mapped to spacetime by $\mathcal{A}/4$ theory of spacetime holographically. Before QM-P, we review back $\mathcal{A}/4$ theory of spacetime, because it can be considered independently of QM-P.

The use of Ryu-Takayanagi relation as being fundamental without quantum corrections implies that spacetime is not in superposition, though underlying non-gravitational quantum contents are. Also, Ryu-Takayanagi then suggests that gravity purifies each (irreducible) subsystem[19], manifestly demonstrating locality of these subsystems - entanglement is not a spooky action at distance. The assumption that state vector has to evolve continuously and in a time-differentiable way was also made to ensure unique spacetime, as infinitesimal analysis has to be maintained valid. This in turn requires that an observer must receive observation data (Bayesian updates) continuously. This assumption is dutifully captured in QM-P.

QM-P essentially is objective Bayesianism based on the principle of maximum entropy[5] plus the partition function constraint that acts as a conserved quantity to be obeyed by Hamiltonians in a Hamiltonian path $H(t)$, along with some interpretation points. The principle of maximum entropy is straightforward, and the partition function constraint can easily be understood as succeeding theories (Hamiltonians) inverse-renormalizing toward the final theory (Hamiltonian). So the review focuses on interpretation points. Physics behind each irreducible subsystem behaves locally, despite quantum physics seems to be non-local at first sights. This means that the total entropy to be maximized must be the sum of entropy of irreducible subsystems. We then must ask why von Neumann entropy must be used. The answer relies on interpretation of density matrix as providing the right observable for probability - and thus providing the correct description (basis) that an observer must pick. Density matrix has not been used as an observable, and thus this is a non-trivial assertion. This is incompatible with discontinuous state vector evolution, or simply collapse. This in fact is a good thing, and we addressed it by clarifying what a quantum measurement is. A quantum measurement is simply a continuous probabilistic learning process by an observer looking at change of her own state to infer other subsystems. There is no reason why discontinuity must enter. The cost is that we have two state vector evolutions - one describing expected state vector evolution and one that is actual state vector evolution accounting for Bayesian updates or simply measurements in the middle of a measurement process.

QM-P reduces to usual quantum mechanical understanding of a single theory in contexts not requiring invocation of black hole complementarity. By this, we mean that there are sufficient observations of subsystems of same characteristics such that observers surely will use a single theory to describe reality. In case of black holes, there is lack of sufficient observations such that use of different theories will be justified.[24] Since each observer only notices changes of her own states, there cannot be theoretical or empirical inconsistency.

Despite strong complementarity, why can we describe observers as if they are sharing an equal-basis description? This is a consequence of quantum decoherence[17][15] whenever it arises. An important concept here is local equilibrium, where, for some measurement process, deviation away from local equilibrium occurs slowly such that one may describe a subsystem as if it is in local equilibrium. This local equilibrium allows one to provide a basis by which state vector during a measurement process is to be understood approximately, without having to reference density matrix at each time to determine a basis that must be used. Consistency requires that the same local equilibrium state must be shared across observers in most contexts. This local equilibrium is defined as the condition for classical reality to emerge. Einstein field equations thus govern spacetime of local equilibrium states, as in Jacobson (1995)[4], which was verified in $\mathcal{A}/4$ theory of spacetime.

Quantum decoherence also explains why despite measurements of a subsystem requiring interactions with other subsystems, under some interactions one can treat the subsystem as if it evolves purely, instead of mixed state evolution, with the Born rule connecting this "fake" and reduced state vector to probability. (Universe always evolves purely.) However, to determine a basis by which measurements are read, one does anyway need to refer back to reduced density matrix of a subsystem.

We thus demonstrated an empirically consistent theory of quantum gravity.

9.1 Causal inference point of view

While we have so far discussed QM-P from Bayesian principles applied to physics, QM-P really is a statistical framework for Bayesian causal inference - Hamiltonian is exactly about mapping present-time variables with future-time variables! Since our practical observations are done in discrete time, some modifications to QM-P are necessary, along with some generalization, such as observation path trajectory no longer about observer states, but otherwise the QM-P framework remains intact. Viewed from this causal inference point of view, quantum mechanics simply is a specific case of Bayesian causal analysis.

Furthermore, if we view $\mathcal{A}/4$ theory of spacetime from Bayesian perspectives, it is simply about how we may consistently visualize statistical information.

One of the questions that humanity has had over years is why statistics seems to be so powerless in directly deriving laws of nature, rather than just supplying data analysis to inspire laws of nature. So far, no one has really derived laws of nature using a well-known systemic statistical procedure from empirical statistical data. This article, I hope, provides important clues in answering the puzzle. Statistics may also benefit from connections with physics as well.

"Probably," laws of nature are already about statistical analysis at heart.

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