Chapter 32 Scale Modeling

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Word count, excluding figures and references is < 7000

Role of Scale Models in Engineering Practice

Scale models are used in engineering design and analysis today, and have been used in the profession of engineering for well over a century. The methodology of scale modeling is at least potentially applicable to any field of engineering, technology, or science. It is thus a puzzle that many discussions about models in philosophy of science have (mistakenly) assumed that scale modeling is an obsolete methodology that has been replaced by computer models. (e.g., Oreskes 2007) For, not only is experimentation using scale models still employed in many fields of engineering (Sterrett 2017b) , but many of the computer programs used in building and analyzing computer models in engineering rely crucially on data that was generated by extensive scale model experiments set up and performed specifically for the purpose of generating data needed to write those computer programs. So scale modeling is an essential part of much engineering work, even though its involvement in engineering practice is not always obvious.   In addition to the scale models used for research, analysis, and design, there are also configurable scale models that are constructed specifically for educational use in engineering curricula. Such configurable models provide students the opportunity to design, set up and carry out model experiments.[1]

Figure 1. Scale modeling in an educational setting. Configurable (Interactive) Scale Model from Little River Research and Design. "Em4" model. Used with permission. There are descriptions and videos of these models at http://emriver.com/models/em4/

Scale models have become much more sophisticated in recent decades due to significant advancements in measurement technologies (e.g., lasers for measuring distances) and the development of advanced materials. (Sterrett 2017b) These recent advancements have been incorporated into the design of engineering scale model experiments, with the result that some of the scale modeling practiced today was not possible, or even imagined, a hundred years ago.

Scale Models in Philosophy

Most current discussion in philosophy about models has excluded philosophical treatment of accurate accounts of scale modeling used in engineering.[2] For instance, even though Weisburg's widely-read *Simulation and Similarity: Using Models to Understand the World* featured a scale model constructed and used by the Army Corps of Engineers on the cover and in the text, his "weighted feature matching" discussion of similarity is an extension of a psychologically-based conception of similarity (Weisberg 2013), and does not provide a scientific explanation of how and why the methodology of scale modeling worked for that model. As this handbook goes to press, the tide is turning, though, and some recent publications hint at future work underway that may help to rectify the current situation that, other than the few individuals mentioned above, scale modeling is not appropriately recognized in philosophy of science. (Sanchez-Dorado 2019; Oreskes & Bokulich 2017; Pincock forthcoming)

Due to the current lack of engagement with the methodology of scale modeling in the philosophy of science literature, there are not really current debates in the field. There were certainly debates within the profession of engineering about the foundations, merits, and applicability of scale modeling in previous eras, but not within the past half-century. Inasmuch as differences of opinion about scale modeling currently exist in the philosophical community, they are attributable to misconceptions about scale modeling.

Thus, this chapter on scale modeling does not address current debates *per se*, but aims to provide an introduction to the foundations of the methodology and identify misconceptions that currently exist about it in philosophy of science.

What are scale models?

Scale models, as the term is used in engineering, are physical objects or situations, usually specially constructed for the purpose, that are employed experimentally to learn about another imagined or existing physical object or situation. Scale models in engineering are usually constructed by humans, though it's possible to use the methodology of scale modeling to interpret naturally occurring objects or situations as scale models, too. The scale model experiment generally includes the surroundings that influence the behavior of the model, e.g., forces and ambient conditions, and these are designed to be analogous to (i.e., to correspond to) those in the surroundings of what it is intended to model. Construction of the scale model includes determining not only ratios of distances, but ratios of other measureable quantities such as various material properties and forces. Not just any ratio will be of significance in building a model that is informative about the thing it is supposed to model. Which ratios of measurable quantities to use in specifying the model, and how they are used to construct and interpret the model, is determined by employing the theory of dimensions.

After the scale model is constructed, its behavior can be observed, and the observations and measurements made in the model, suitably interpreted, are informative about the object or situation modeled. The formal methodology of scale models not only provides some prescriptions as to how the model is to be constructed, but provides a quantitative translation of the measurements made in the model to the corresponding measureable quantities associated with what it is intended to be a model of. Engineering knowledge is then used to make sense of the results regarding the problem or question being investigated, a process often referred to as "interpretation" of the model experiment.

This methodology is distinctively different from the kind of model-building in which the modeler starts from a mathematical equation describing the model or its behavior. (Sterrett 2002; 2017a) It's a significant philosophical difference, as models in science have generally been associated with scientific equations. (Bailer-Jones 2009) Further, epistemological issues in modeling also differ for scale models. This is because issues important in epistemology associated with scale models, such as evaluation of the external validity of the model, and analyses of how fundamental laws and experimental data on which the model is based are employed in modeling, differ from those that arise in the usual approach on which a model is a mathematical equation. Hence most current philosophical accounts of how models manage to inform us about the world, and what we can conclude from them, are not applicable to scale modeling. They could, however, become enriched by adapting to incorporate the methodology of scale modeling.

Scale Models in Practice -- Unique Challenges, Unique Versatility

The scale model can then be used in other experimental tests. Often, a scale model is useful when we are interested in understanding behavior that results from some unusual event or environmental change: the observations and measurements are informative about how the object or situation modeled will be affected by the corresponding modeled event or changes. Thus, although an experimental test might be designed to model expected normal operation in order to observe the overall behavior of the object or situation modeled (as when used in the pre-construction phase of the design of chemical processing plants), it can likewise involve subjecting the model to the application of an environmental factor such as heat, a temperature difference, a flow process such as an wind, riverflow, or wave motion, or some event (e.g., an impact force, a periodic or nonperiodic motion, the initiation of a landslide, to give a few examples). After the results from the measurements taken in the model have been mapped, i.e., transformed or translated to the object or situation modeled, it is possible to produce tables or graphs of how the object or situation responds to various events or changes, according to the model.

The materials used in a scale model are generally not exactly the same materials that occur in the actual situation that the scale model experiment aims to simulate, for even material properties must be properly scaled. In the kind of model shown in Figure 1, which is used in educational institutions, there might be scaling of material particle size and intergranular friction in order for the model to provide the kind of behavior of interest, such as the progression of material dispersal over long time spans in the river modeled.[3] A common example where the material used in the model can differ from the material in the system modeled is flow in piping systems; water is sometimes used to model a more viscous fluid, such as oil. An example of the kind of difficulties encountered and care taken in getting the crucial material properties right is the case study of modeling ocean cable using plasticized PVC (polyvinyl chloride) piping to get the proper modulus of elasticity in the small scale model -- then, in order to get the appropriate density in the model, the material was impregnated with powdered lead. (Herbich 1998: 331)

Sometimes several different scale models of a given object or situation are built in the course of designing it, as several different scale model experiments are needed to predict the several different kinds of behaviors of an object that are of interest to a modeler, or the different behaviors that are dominant at different scales. The scale model an experimentalist builds to predict the diffusion of heat in a given structure might not work well as an experimental model for predicting other kinds of behavior of the same structure, such as mechanical responses to earthquakes. Also, different phases of the situation modeled, such as the different stages in the life of a volcanic eruption and its aftermath, might require separate scale models, as the behavior of interest to the researcher might differ at different stages as the eruption progresses, and different phenomena will be dominant at different stages.

Scale models are so called because the models usually happen to be built according to a scale that indicates how one should translate measurements of distances from the model to what is modeled. To take a familiar example, a 1/8th scale model of a car would mean that any distance you pick out on the model car corresponds to a distance 8 times as long in the car it is intended to be a model of; thus in the case of geometrical scale, it is easy to comprehend that a 1/8th scale model car that is 1 foot long would indicate that the car it models is 8 feet long. Architectural models of buildings or building complexes are generally scale models in which distances are the *only* thing precisely scaled. However, for the more general notion of scale model used in engineering, other quantities such as velocity and current are generally scaled as well, and more than one quantity is scaled concurrently. Comprehending how scaling works in such complex cases is much more involved.

Scale models are often thought of by the layperson as being constructed as if the model were made by shrinking an object to a smaller size. If only geometrical similarity is to be achieved, rather than, for example, dynamic similarity (in which forces in the model correspond to forces in the situation modeled), that is not inappropriate. However, if similarity of physical behavior (bending, vibrating, buckling, stretching, expanding, cooling, etc.) is desired in a model, then the important interrelationships between all the quantities involved in that physical behavior must change in a coordinated manner. Then, the values of the quantities in the model are related to the quantities in what is modeled in very complex ways. Distance might be translated according to one scale factor, time according to another, and mass according to yet another. Thus, translating a quantity like velocity in a scale model to velocity in the situation it models is not as straightforward as it is for a hobbyist building a 1/8 scale model car where only geometrical similarity is of interest. The quantities in the environment acting on the scale model need to be scaled as well. Thus it is more appropriate to speak of a *physical system*, rather than a physical object such as a ship or plane, when discussing model experiments and the practice of scale modeling.

We then say that, ideally, we aim for the model and what it models to be *physically similar systems*, and we say that we construct a model to be physically similar to what it models. For two things to be "physically similar" or not always needs to be qualified (whether explicitly so stated or not) as similar *with respect to* some behavior considered within the realm of physics. For example the behavior might be the magnitude of a liquid flowrate, electrical charge, or stress in a structural element; or it might be the existence of turbulence, the existence of buckling, or the existence of a phase change.

Most scale models are smaller in size than what they model. There is no reason in principle why a scale model cannot be made on a larger scale than the object or situation it models, though, and in fact some of them are. The advantage of making a scale model is to be able to experiment on a model of something, as a proxy for experimenting on something that cannot itself be experimented upon. Some scale models are tabletop models, as pictured in Figure 1, but there are also some large testing facilities, such as wind tunnels, models of river basins (LSU Center for River Studies) and volcanoes (Sterrett 2017b). These are seldom easily accessible to the public, but there are a few retired models that are. The San Francisco Bay Model discussed in (Weisberg 2013) is one such model. Another place to view scale models is the early facility for testing proposed ship designs that has since been replaced by the current David Taylor Model Basin; it is shown in Figure 2. In such testing facilities, the scale models of ships can be quite large, on the order of 20 feet long. The first experimental facility for testing ships built there was built at the very end of the nineteenth century, in 1896. The current facility on that site contains a shallow water basin, a deep water basin, and a high speed basin. (ASME 1998: 2)

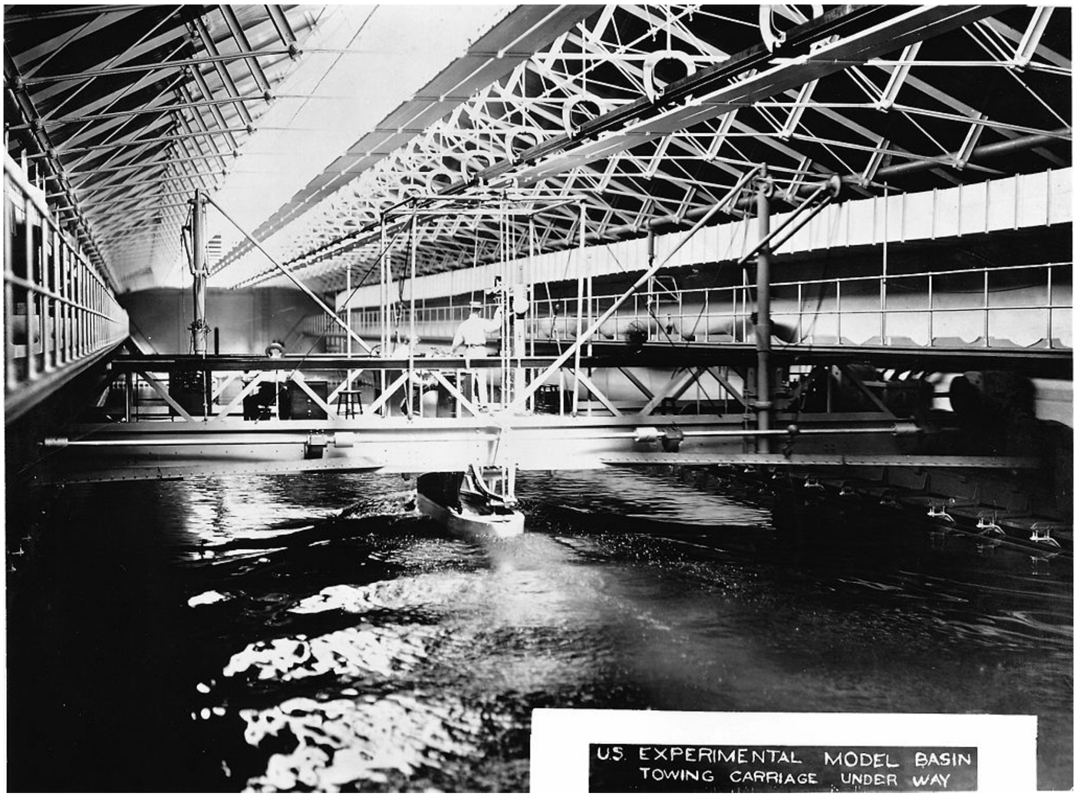

Figure 2. **Experimental Model Basin**, Washington Navy Yard, Washington, DC - interior view, c. 1900. This was the first model basin (towing tank) for the United States Navy. Photo credit: U. S. Navy http://www.dt.navy.mil/div/about/galleries/gallery1/012.html Public domain.

One of the largest scale models, perhaps the largest ever built, is the scale model of the Mississippi River Basin, called the Mississippi Basin Model, or MBM. (Figure 3) Like the David Taylor Model Basin, it holds a special place in the US history of scale models: at 40 acres in size, it is known as "the largest small-scale model" in the world. Many other hydraulic models were built by the same facility (The Waterways Experiment Station.) Historical research into that facility's establishment reveals that there were debates as to the validity of the method of using scale models at that time, around the 1930s (Manders 2011: p. 56); the subsequent investment in and use of the MBM reflects the eventual outcome of that debate. The MBM model has not been preserved, in spite of its significance as a cultural icon, but this is not due to the technology of scale modeling itself becoming obsolete. A new indoor model of part of the basin, costing 4 million dollars and requiring a quarter acre of space, the "Lower Mississippi River Physical Model," has recently been built in a new facility (Lousiana State University's Center for River Studies).

Before the MBM was retired, data were collected from experiments that were specifically designed and carried out to provide data for use in computer programs in the 1970s. The data was incorporated into computer programs used to simulate the flow of water in the Mississippi River Basin. (Foster 1971: vii) Thus the computer model that was used in lieu of the physical MBM scale model after its retirement was not independent of the scale modeling work. When it was in service, the MBM model was used to make predictions, most famously during the 1952 Missouri River Flood. Predictions could be generated from the scale model by controlling water levels in it to correspond with real-time inputs of actual river level measurements. Time in a scale model goes faster; the events of an entire day in the actual river system only took a few minutes in the model. (Foster 1971: 21-27) Likewise, the Center for River Studies housing the current basin model reports that in its model, "one year of the Mississippi River is simulated in one hour." (LSU Center for River Studies website)

Figure 3. Photograph of postcard, personal collection of the author. Text on reverse: "WATERWAYS' EXPERIMENTAL STATION, VICKSBURG, MISSISSIPPI. The most unique Educational attraction in this part of the world is the U. S. Waterways' Experimental Station, located on a reservation four miles south of the city. It employs about one hundred graduate engineers and maintains the largest and best equipped laboratory of its kind in the world. Weighty problems concerning our vast waterways system are under constant study and miniature, scale-built models of our most temperamental streams, have been built for study. Ektachrome by Woody Ogden MADE BY DEXTER, WEST NYACK, NY Pub. by Jackson News Co., Jackson, Miss."

The scale factors that map, or translate, quantities in the model (including the quantities of the modeled environment) to quantities in whatever it is that is modeled, are determined by the ratios used to design the engineering scale model experiment. (Pankhurst 1964) The selected ratios are kept invariant between the scale model and what it models. (That is the aim, at least.) It is in this sense that these ratios are called invariants. The key to understanding how scale model experiments are designed, and why model experiments that work well do so, when they do, is understanding the role of invariants and similarity in the practice of scale modeling.[4] We begin with an extremely simple case in order to make the ideas clear.

Scale models, Invariants, and Similarity: the Basic Ideas

To illustrate the basic ideas behind scale modeling, i.e., the ideas of physical similarity and physically similar systems, we will first make the basic concepts involved clear for the simpler case of geometric similarity. Geometric similarity is generally easy to understand, because we can easily grasp the idea of two figures having the same geometric shape. A major misconception that abounds in philosophy about scale models is that the methodology of scale models is geometric similarity. It is not. The (correct) statement, often found in textbooks on the topic, is that the method of scale models is a *generalization* of geometric similarity. (Sedov 2014: p. 43) This statement seems to have been grossly misunderstood in philosophy, and the misunderstanding is widespread. In the sections that follow, I hope to show the deep analogy between geometric similarity and the kinds of similarity used in scale modeling that are specific instances of physical similarity: kinematic similarity, dynamic similarity, hydrodynamic similarity, and thermal similarity, to name a few. Hence we begin by explicitly setting out the logical structure of reasoning about similarity already familiar from geometry, so as to see how to extend reasoning about similarity to physics.

The Logical Structure of Arguments from Geometric Similarity

One of the simplest examples of geometric similarity is the circle; all circles have the same geometric shape. Any two circles of different sizes are geometrically similar to each other, *in spite of the fact that none of the individual measurements* made on one circle (diameter, area, circumference) will be the same in another circle of a different size. Recall that the ratio of the circumference of a circle to its diameter is invariant no matter how small you shrink a circle in size, nor how large you expand it in size: so long as the figure keeps it shape, i.e., so long as it is a circle, *this* ratio will be the same. Many other ratios of geometrical quantities of a circle are *not* invariant between circles of different sizes: The ratio of circumference to area is *not* the same for all circles, for instance -- *that* ratio *will* vary depending on the size of the circle. Not so for the ratio of circumference to diameter; it's invariant among all circles. We don't even have to know the numerical value of that ratio in order to make the statement that the ratio of the Circumference of Circle#1 to the Diameter of Circle#1 is equal to the ratio of the Circumference of Circle#2 to the Diameter of Circle#2. We can say, whatever that ratio is, i*t doesn't vary* between circles; whatever it is, it is *the same* for every circle. It is *invariant*, from any circle to any other circle.

What is required to establish that two things have the same geometric shape? First, they must be the same *sort of* thing; for example, they must both be closed curves, or both be three-dimensional solids. Secondly, they must be geometrically similar. One way to ensure that two figures are geometrically similar to each other is to *construct* a figure that is similar to a given one. And, that's the general approach taken in scale modeling: to *construct* something that is similar in the relevant ways. However, as we shall see later, the *analogous notion of similarity in scale modeling* has to be generalized quite a bit from the case of *geometrical similarity*.

When there is at least one ratio that is invariant between all geometrical figures of a certain shape, i.e., between all figures that are geometrically similar to a certain figure, and to each other, that invariant ratio can be used to find the value of some distances that are not directly measurable. The method is an extremely simple example of scale modeling: construct a figure that is geometrically similar to one that involves the distance one wishes to know the length of. Then, using the ratio that is invariant between all figures of that shape, construct a proportional equation by equating the ratio expressed in terms of the line segments for one of the figures to the ratio expressed in terms of the line segments for the other figure. If the length of the line segments in the figure you have constructed are known or can be measured, this may allow solving for the distance one wishes to know.

The method is used in a common middle school exercise asking students to determine the height of a tall object such as a tree or flagpole on a bright day, by measuring its shadow and the shadow of their own body. (Figure 4) It will be helpful to identify the structure of the reasoning here, for later use. So long as the area in which the tree and child is sufficiently flat, the right triangle formed by the student, her shadow, and the line connecting them is geometrically similar to the right triangle formed by the tree, its shadow, and the line connecting them. The ratio of [Height of Tree]/[Length of Tree's Shadow] is the same as the ratio of [Height of Student]/[Length of Student's Shadow]. A worksheet prepared for use by middle school teachers illustrates the sun-object-shadow situations in which we find these two similar triangles:

A. The *Terrific Tree*

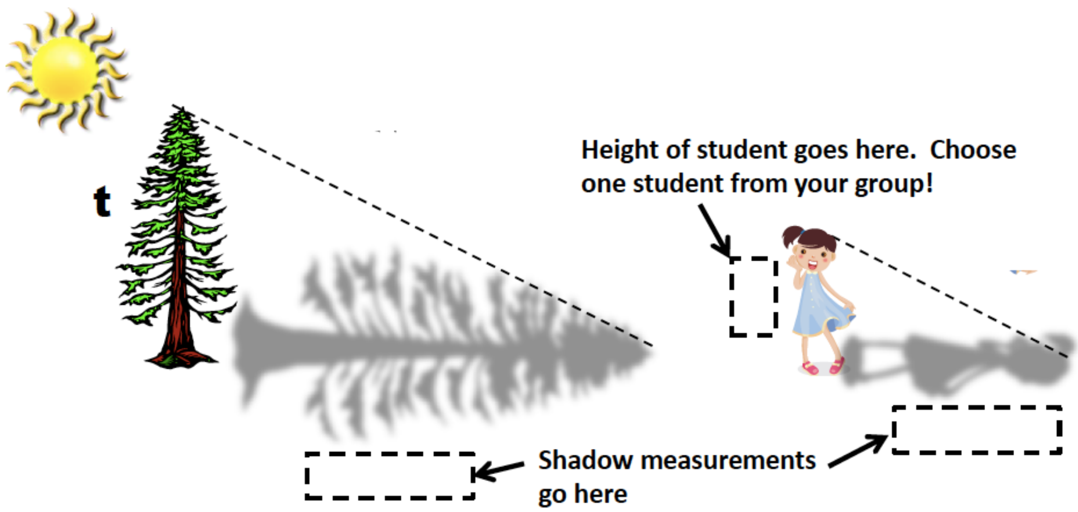

Figure 4. A worksheet designed for use with middle school students showing how to use your own body to determine the height of a tree, from shadow measurements. *Image credit:* "Similar Figures and Indirect Measurement: The Outdoor Lesson", Barry Schneiderman, TeachersPayTeachers.com 2014 Used with permission.

If the height of the student and the lengths of both shadows can be obtained by measurement, the height of the tree can be determined by equating these ratios expressed as follows:

$$\frac{\text{Height of Tree}}{\text{Length of Tree's Shadow}} = \frac{\text{Height of Student}}{\text{Length of Student's Shadow}}$$

Stated in more general terms, the knowledge that this ratio is invariant between the two (sun-object-shadow) situations allows us to equate the ratios. The proportion that results then provides the means to determine the height of the tree, as follows:

t = Height of the Tree =

[Height of Student/Length of Student's Shadow] x Length of Tree's Shadow

One way to look at what we are doing when we indirectly measure the height of the tree this way is that the student-sun-shadow situation has served as a model of the tree-sun-shadow situation, with respect to height.

Note that the criterion of similarity in use here is objective. In spite of the fact that the situations compared have aesthetic aspects and that human cognition is involved in apprehending the two triangular figures associated with the two physical situations, the criterion of geometrical similarity between the two triangular figures indicated in Figure 4 is completely objective. The question of whether two plane triangles are geometrically similar is settled here by the fact that the two triangles are right triangles and the angle at the top of the tree and the angle at the top of the student's head are formed by rays of the sun in the sky hitting them at the same angle. That angle need not even be known in order to conclude that the triangles indicated in Figure 4 above are similar triangles. The reasoning *from* geometric similarity is objective, too, i.e., the *consequence* of the fact that these two triangles have the same shape, i.e., are geometrically similar, is that ratios between corresponding sides are the same. The reasoning from geometric similarity is straightforward reasoning according to the methods of Euclidean geometry. In Euclidean geometry what's similar are two dimensional closed curves (figures), or, if three dimensional, solid figures.

Generalizing Similarity in Geometry: What's Analogous in Physics?

Progressing now from the simple case of geometrical similarity to the more complex case of physical similarity: what could be analogous to geometric shape, for physically similar systems? There isn't really a term for it, but we can explain such a concept in terms of the invariant ratios that remain the same between physically similar systems. (Sterrett 2017a) That is the proper way to think about an analogue of shape in physics: just as we explain geometric shape in terms of the invariant ratios that remain the same between geometrically similar figures, so we conceive of something like shape of a physical system in terms of the value of the invariant ratios that remain the same between physically similar systems. There is a difference, though: geometric shape of closed plane figures is uniquely determined, whereas there are different kinds of similarity in physics. For the more complex kinds of similarity, similarity of physical behavior under gravitational forces, or heating, or cooling, or being set in motion by an earthquake, or undergoing pressurization, and so on, the invariants are certain dimensionless ratios composed of quantities used in physics. Which dimensionless ratios are relevant depends upon what behavior the modeler is interested in modeling. Dimensionless ratios will be explained below; for now we want to state the concept of *physically similar systems* on analogy to *geometrically similar figures*.

To say that two physical configurations or situations S-one and S-two are "physically similar" with respect to a certain kind of behavior (rather than just geometrically similar), is to say that System S-one and System S-two have the same values of the (dimensionless) ratios that determine that kind of behavior. That is, we are considering a case in which a system S-one can have the same ratios of the dimensionless quantities that are relevant to a given behavior as another system S-two has -- for example, the same ratios of certain forces -- even though the values of some or all of its measurable quantities may not be the same system in S-one as the values of the corresponding quantities in system S-two. (Buckingham 1914) So long as the values of the dimensionless ratios are the same, the specific behavior of the two systems on which the similarity of systems was drawn is the same. Specific numerical values of quantities in the model system and in the system it models will differ, of course; these values are related by a scale factor, which is recoverable from the dimensionless quantities.

What is so philosophically significant about scale modeling is that, unlike many other philosophical accounts of models, the methodology of scale models provides a scientific basis for determining that correlation (i.e., the correlation between a certain quantity in the model and a quantity in what it is intended to model (the 'target" system, or any other physically similar system). This is so *even in cases in which the modeler does not know of an equation describing the behavior of the system*. One way to put it is that the method of dimensionless parameters provides (i) a way to construct a model system that is physically similar to the thing of interest, and (ii) a means of interpreting the behavior of the model system in a way that is informative about what it models. Putting the point in terms of the terminology of a "key" as recently employed in philosophy of scientific representation (Frigg & Nguyen 2018), scale modeling *provides its own* "key" by which the results of experimentation on that constructed model are to be interpreted to give quantitative values for the quantities in the thing it models. (Pankhurst 1964) That is truly philosophically significant. It is the holy grail that many other current philosophical accounts of models seek. Often philosophical accounts of modeling leave that aspect to the judgment or knowledge of the modeler, or to experiment. Hence my claim that philosophical accounts of modeling stand to gain much by taking account of how the method of using scale models manages to be as successful as is.

Thus a close study of scale modeling methods allows us to answer the following question: "In what way can the behavior of two systems be said to be the same, if none of the quantities measured in them is the same?" This question often arises in explaining the practice of scale modeling, since the quantities with which physics is concerned will not have the same values in the model as they do in the object or situation modeled. The answer is: the behavior is said to be the same in the model as in the situation modeled, on analogy to the way that two geometrical figures of different size are said to have the same shape. That is, for two figures to have the same geometrical shape, certain ratios of lengths in the figures are the same in both figures. Analogously, for two physical systems to be the same with respect to a certain kind of physical behavior, certain ratios of (measurable) quantities must be the same in both systems. This can be thought of as an analogy between geometric similarity and physical similarity, or as a generalization of geometrical similarity to physics, as shown in Table I. [Table I goes here.]

Table I **GENERALIZATION OF SIMILARITY IN EUCLIDEAN GEOMETRY TO SIMILARITY IN PHYSICS (Mechanics, including heat, fluids, etc.)**	
(Geometrically) Similar Figures	**(Physically) Similar Systems**
Certain *Ratio(s)* of quantities (lengths) are the same in both figures. A proportion holds.	Certain *Dimensionless Ratio(s)* involving quantities used in physics are the same in both systems. So proportion(s) hold.
To establish geometric similarity: *Construct a figure* so that it is geometrically similar to a given figure; or *deduce* that two figures are geometrically similar.	To establish that two systems are physically similar systems: *Construct a system* so that it and the given system are physically similar systems with respect to a certain behavior; or *deduce* that two systems are physically similar systems with respect to a certain behavior.
To reason from geometric similarity of two figures: *From the knowledge of the equality of certain ratios in the given and constructed figures*, knowledge of all the quantities in the constructed figure that occur in those ratios, and of some of the quantities in the given figure, *deduce the value of previously unknown quantities in the given figure.* (Proportional reasoning.)	To reason from the fact that two systems are physically similar systems: *From the knowledge of the equality of certain dimensionless ratios in the given and constructed systems*, knowledge of all the quantities in the constructed system that occur in those ratios, and of some of the quantities in the given system, *deduce the value of previously unknown quantities in the given system.* (Proportional reasoning.)

To offer an example that is easy to grasp visually, one kind of similarity that may hold between two physically similar systems is kinematic similarity. When kinematic similarity holds between two systems, the paths of the particles or bodies in the system trace out figures of the same shape. The paths are said to be homologous, which means that the particles of the two systems have corresponding velocities at corresponding times. (In the simple case where the particles have uniform velocity, the velocities and times will scale linearly between the two systems.) However, not all kinds of similarity in physics lend themselves to such visualization.

Another common way to grasp the physical significance of the nature of the similarity that holds between physically similar systems is to conceptualize the crucial dimensionless ratio or ratios in terms of a ratio of two kinds of forces. Thus the Froude number Fr, which is often expressed in terms of the quantities of velocity, length, and the gravitational constant, is commonly thought of as the ratio of a fluid's inertial force to its gravitational force. The Reynolds number, Re, which is often expressed in terms of the quantities of fluid density, velocity, length, and fluid viscosity, is commonly thought of as the ratio of a fluid's inertial force to its viscous force. The kind of behavior of interest in constructing the model determines which ratios one chooses to keep invariant between the model and the situation or object one wishes to model: the Froude number is used to construct a scale model when wave and surface behavior are important in a situation, as in designing ships for sea travel, while the Reynolds number is used to construct a scale model for a variety of phenomena associated with turbulent flow (examples are flows in piping systems, the response of buildings to high winds, and high speed travel in the atmosphere (aircraft, projectiles)).

The chart below illustrates how the Reynolds number would be used to construct a model of an object or situation such that the Reynolds number is invariant between the model and what it models:

<table>
<tr><th colspan="2"><u>Table 2. Using Dimensionless Ratios In Scale Modeling</u></th></tr>
<tr><th><u>What you want a model of:</u></th><th><u>The model you construct:</u></th></tr>
<tr><td rowspan="2">A system with density ρ1, velocity u1, some characteristic length L1, and dynamic viscosity μ1. Also, some fixed ratios reflecting the physical configuration.

The Reynolds Number characterizing the system, which can be thought of as the ratio of the inertial force to the viscous/frictional force, is expressed as: Re1 = ρ1 u1 L1 / μ1</td><td>The model is a system with density ρ2, velocity u2, some characteristic length L2, and dynamic viscosity μ2. Also, it has the same fixed ratios reflecting the physical configuration as the system you want a model of has.</td></tr>
<tr><td>How you design the model you construct: choose a fluid velocity and a fluid with fluid properties (density and dynamic viscosity) such that the Reynolds number in the model equals the Reynolds number in the system you are modeling. I.e., Choose Re2 = ρ2 u2 L2 / μ2 such that Re2 = Re1.

This will result in a model in which the ratio of the inertial force to the frictional force is the same in the model as it is in the given system.</td></tr>
</table>

Selection of Invariants: Which dimensionless parameters matter?

Since the dimensionless parameters that are kept invariant between the model and what it models are so crucial to the method and its success, the question of where they come from deserves at least a brief answer here. There are two main analytical means of determining the dimensionless parameters relevant to a certain behavior. One method, nondimensionalizing the governing equation to identify the dimensionless parameters that can play the role of invariants for the behavior governed by the equation, relies upon knowing the differential equations governing the phenomenon. The second method, using dimensional analysis and applying the principle of dimensional homogeneity, does not require knowing the actual equation or equations, it only requires knowing what quantities are involved. (Sterrett 2017b)

In the example above, using Reynolds number, the equality of Reynolds number in the model and what it models was sufficient to establish that the model and what it models were physically similar systems. However, in many cases, what is required to establish that two systems are physically similar systems is to show that a certain set of two or more dimensionless parameters has the same value in the model as in the system it models. The theory upon which a set of dimensionless parameters sufficient to establish that two systems are physically similar systems is dimensional analysis, or the theory of dimensions. (Buckingham 1914; Pankhurst 1964; Sterrett 2006; 2009; 2017a; 2017b) The set of parameters is not unique; what is determined is *how many* dimensionless parameters that are required to establish the physical similarity of two given systems with respect to a certain kind of behavior.

In practice, modelers do not usually derive the relevant dimensionless parameters anew for each experiment, or, even, for each kind of experiment. Rather, which dimensionless parameters are appropriate to select as invariant(s) to guide construction of a model is often already established by the community of researchers in which the experimenter is working. Dozens of dimensionless parameters have been identified and given proper names. Though there are by now canonical formulations of each named dimensionless parameter in the community of researchers and practitioners who use that dimensionless parameter, there is not even one unique expression of every dimensionless parameter. Likewise, though there are by now established choices of which set of dimensionless parameters to use to establish similarity of a model to what it models for a certain kind of behavior, there are many different sets of dimensionless parameters that are equivalent in terms of establishing that two systems are physically similar, i.e., that can play the role of invariant(s).

New kinds of experiments are constantly being conceived, too. For these, a combination of analytical approaches and experimentation is used, to determine the appropriateness of the choice of invariant in capturing the kind of behavior one wishes to study, using the kind of model and testing conditions employed. While it is true that experimenter knowledge and practical experience are involved in carrying out these kinds of investigations, the criterion of similarity is still an objective matter; similarity is a matter of a set of relevant invariant dimensionless parameters[5] (the set of parameters might not be unique) being the same between model and situation modeled, a matter of the two systems being physically similar systems. Changes to model materials, testing conditions, and other features of the experiment are evaluated as well, in tandem with the choice of invariants, in order to obtain a sufficiently effective model experiment protocol.

Inherent limitations of the method of physical similarity

In practice, exact similarity of systems in physics is not always achievable. In particular, for dynamic similarity of physically similar systems, exact similarity is in general unachievable unless the model is a full-size scale model.[6] However, most scale models are not full-size scale models, and most scale models are only approximately physically similar to what they model. In this section we briefly indicate the reasons for this.

The reason that it is not in general possible to achieve complete dynamical similarity with a model that is not full-size is that the problem of ensuring that the model and what it models are dynamical (physically) similar systems is *overconstrained*. Solving that problem requires finding a combination of values of all the quantities that appear in all the dimensionless parameters that one needs to keep invariant. It is a simple mathematical matter, an application of linear algebra, to show that in the general case, there are so many constraints that a general solution to the problem of dynamically similar systems is not possible except with a full-size model. Thus, in practice, modelers compromise and construct a model that is only *approximately similar*, rather than *exactly similar*, to what it is a model of. When the scale model is not exactly similar to what it models, scale effects can arise. Part of the modeler's job is to quantify scale effects and design the scale model so that the kind of scale effects that arise in that model are not important to the kind of behavior the model is being used to investigate. As noted earlier, often several different scale models of the same object or situation are made, each one designed to investigate a different kind of physical behavior.

Another reason that exact similarity is not achieved in practice is more deliberate. Most models of large bodies of water, such as large lakes or rivers, are distorted: the vertical dimension of the model corresponding to the depth of the river or lake uses a scale much larger than the horizontal dimensions of the model corresponding to the earth's surface. This is because if the same scale were used, the depth of the water in the model would be impracticably shallow, and the effects of the river bottom or lake bottom would be much exaggerated in the model behavior as a result. Engineering experience gained from experimentation is involved in the process of arriving at a good selection of dimensionless parameters, and engineering expertise is involved in deciding which tradeoffs to make in constructing a model that is only approximately similar. However, the criterion of exact similarity is still well defined, even if seldom obtained. Exact similarity consists in the values of the dimensionless parameters that characterize the model being equal to the values those dimensionless parameters have in the situation it models.

Misconceptions: a brief list

Common misconceptions about scale modeling include (i) confusing the sense in which invariant is used in scale modeling with the sense of invariant used to denote an "invariant of nature" ; (ii) that all dimensions must be expressed in terms of a particular set of base dimensions, such as[M], [L], and [T]; (iii) that the "generalization of geometrical similarity" is another kind of geometrical similarity; (iv) that anything that can be achieved with a scale model can be achieved with a computer simulation; and (v) that scale modeling requires more information than using an equation. (In fact, scale modeling requires less information than numerical simulation, as it does not require that the modeler be in possession of an equation describing the behavior of interest) . (Sterrett 2002; 2017b)

Concluding remarks

Scale modeling is essential in current engineering practice, both in the building of scale models to investigate behavior, and in providing empirical data for use in the design of software for computational simulations. The basis for scale modeling is known as physically similar systems, and can be thought of on analogy to geometrically similar figures, as explained in this article. Instead of ratios of like geometrical quantities, it is dimensionless ratios consisting of ratios of products of physical quantities that play the role of invariants in the theory of scale modeling. In practice, it is often not possible to achieve exact (or full) similitude in a scale model, and empirical investigations are often carried out to make informed judgments about the best compromises to make in the design of a model experiment using an approximately similar scale model.

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Further Reading

Becker, Henry A. (1976) *Dimensionless Parameters: Theory and Methodology*. Wiley. (An extremely concise, insightful, philosophically-minded introduction to the topic with a variety of examples, including some from chemical engineering.)

Buckingham, E. (1914) "On Physically Similar Systems; Illustrations of the Use of Dimensional Equations," *Physical Review*, Vol. 4, p. 345. (This is the classic paper; tightly written by a philosopher-physicist and worth close study; even the more well-known P. Bridgman's *Dimensional Analysis* credits Buckingham as its main resource.)

Pankhurst, R. C. (1964) *Dimensional Analysis and Scale Factors*. Butler and Tanner. (A small, short, clear, concise and comprehensive work covering many kinds of similarity, by the Superintendent of the Aerodynamics Section of the National Physical Laboratory.)

David, F. W. and H. Nolle (1982) *Experimental Modeling in Engineering*. Butterworths. (Excellent textbook that has stood the test of time as a reference for fundamental principles as well as advanced applications. Re-issued in ebook format in 2013.)

Sterrett S.G. (2017) "Experimentation on Analogue Models." In: Magnani L., Bertolotti T. (eds) *Springer Handbook of Model-Based Science*. Springer Handbooks. Springer, Cham. (Discusses the bases for scale modeling as compared to other methods for using concrete physical models in science.)

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[1] Figure 1 shows a configurable model developed for educational use. URLs of time-lapsed videos of the model illustrating its use are provided in the caption.

[2] Kroes, Zwart and Sterrett are some of the few exceptions known to this author.

[3] The spectacular visual effects of using color coded particles can be seen in the videos of progression of sediment transport in the river model, which are available online at the manufacturer's website: http://emriver.com/models/em4/

[4] For a deeper explanation of why keeping these invariants the same actually results in the model behavior reflecting the behavior of what it models, see Sterrett 2009 and Sterrett 2017a.

[5] The set of parameters is not unique, as explained in Buckingham 1914, Sterrett 2006, and Sterrett 2009.

[6] Even when full-size scale models are used, the practice is still referred to as scale modeling, for the principles of scale modeling described above are still involved in setting up the experiment. For example, the same principle is adhered to in determining the fluid properties (density, viscosity) and conditions (temperature, pressure, velocity) to use for the fluid in a flow channel or wind tunnel. The crucial thing is still to keep the appropriate dimensionless parameter(s) (ratio(s)) the same in the model as in what it models.