Symmetry Breaking

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1 Introduction

Symmetry breaking is ubiquitous in almost all areas of physics. It is a feature of everyday phenomenon as well as in more specific contexts within physics when considering elementary particles described by quantum fields, quantum mechanical descriptions of condensed matter systems or general relativistic descriptions of the entire universe. In all of these, symmetry breaking plays an

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essential role. However, one should be careful in understanding ”symmetry breaking” as this *one* thing, e.g. this one mechanism, you can find in all the various physical systems. The reason for this is that the notion of symmetry breaking is very broad, in the sense that many very different scenarios are covered under this name, and also very misleading, as there is often not much that is really being ”broken”.

Symmetry and symmetry breaking are, in a sense, the two faces of the same coin. In terms of the scientific notion of symmetry, i.e. invariance under a group of transformations this can be made very precise. On the one hand, a symmetry of a given order can be seen as the result of a higher-order symmetry being broken to a lower-order symmetry, where the order of a symmetry is the order of the corresponding symmetry group (that is, the number of independent symmetry operations which are the elements of the group). This can be said of any symmetry apart from the “absolute” symmetry, including all possible symmetry transformations. Note that nothing with a definite structure could exist in a situation of absolute symmetry, since invariance under all possible transformation groups means total lack of differentiation. For the presence of some structure, a lower symmetry is needed: in this sense, symmetry breaking is essential for the existence of a structured “thing”.

On the other hand, the breaking of a given symmetry generally does not imply that no symmetry is present; what happens is just that the final configuration is characterised by a lower symmetry than the initial configuration. In other words, the original symmetry group is broken to one of its subgroups. The relations between a group (the unbroken symmetry group) and its subgroups play thus an important role in the description of symmetry breaking.[1]

To find some orientation in this rather confusing state of affair it is useful to specify three aspects of symmetry breaking: (i) What is the *entity* that has the symmetry that is being broken?, (ii) What is the *symmetry* that is being broken? and (iii) What is the *mechanism* by which it is broken? Depending on the answers you give to each of these questions, various subtleties can arise, which have led to an intricate and interesting range of philosophical questions.

(i) System vs Law In Nature, crystals provide a paradigmatic representation of this “symmetry/symmetry breaking” interplay. The many striking sym-

[1]Stewart and Golubitsky (1992) is a clear illustration of how a general theory of symmetry breaking can be developed by addressing such questions as “which subgroups can occur?” and “when does a given subgroup occur?”

metries of their morphology and structure are the remains of the breaking of the symmetry of the initial medium from which they originate, that is, a hot gas of identical atoms. This medium has a very high symmetry, the equations describing it being invariant under all rigid motions as well as under all permutations of the atoms. As the gas cools down, the original symmetry breaks down and the physical system takes up a stable state with less symmetry: this is the final crystal, with its peculiar morphology and internal lattice structure.[2]

Crystals are physical objects. In general, when considering the meaning and functions of symmetry and symmetry breaking it is important and useful to distinguish between the systems, i.e. physical objects and phenomena, and the physical laws governing their behaviour. Historically, symmetry breaking in physics was first considered in relation to properties of objects and phenomena. This is not surprising, since the scientific study of symmetry and symmetry breaking originated with respect to the manifest symmetry properties of familiar spatial figures and physical objects (such as, first of all, crystals). Indeed, the symmetries and dissymmetries of crystals occasioned the first explicit analysis of the role of symmetry breaking in physics,[3] due to Pierre Curie in a series of papers devoted to the study of symmetry and symmetry breaking in physical phenomena towards the end of the nineteenth century.

Curie was motivated to reflect on the relationship between physical properties and symmetry properties of a physical system through his studies of such properties as the pyro- and piezo-electricity of crystals (which were directly related to their structure, and hence their symmetry properties). In particular, he investigated which physical phenomena are allowed to occur in a physical medium (for example, a crystalline medium) endowed with specified symmetry properties. By applying the techniques and concepts of the crystallographic theory of symmetry groups, he arrived at some definite conclusions. In his own words (Curie 1894):

a) When certain causes produce certain effects, the symmetry elements of the causes must be found in their effects.[4]

[2]On this point, and more generally on the role of symmetry breaking in the formation of nature's patterns from the smallest scales to the largest, see for example Stewart and Golubitsky (1992), chapter 3. See also Shubnikov and Kopstik (1974).

[3]The terminological use was the following: "dissymmetry" indicated that some of the possible symmetries compatible with the physical constraints are not present, while "asymmetry" was used to mean the absence of all the possible symmetries compatible with the situation considered. We will follow this usage, here.

[4]This is the statement which has become known as "Curie's principle". On the current debate of Curie's Principle, see Castellani and Ismael (2016); Norton (2016) and Roberts (2016).

(b) A phenomenon may exist in a medium having the same characteristic symmetry or the symmetry of a subgroup of its characteristic symmetry. In other words, certain elements of symmetry can coexist with certain phenomena, but they are not necessary. What is necessary, is that certain elements of symmetry do not exist. *Dissymmetry is what creates the phenomenon.*

Thus, intending the phenomenon as the "effect" and the medium as the "cause", the conclusion is that the symmetry of the medium cannot be higher than the symmetry of the phenomenon.[5] Given that the media in which phenomena occur generally start out in a highly uniform (and therefore symmetric) state, the occurrence of a phenomenon in a medium requires the original symmetry group of the medium to be lowered (broken) to the symmetry group of the phenomenon (or to a subgroup of the phenomenon's symmetry group).[6] In such sense, symmetry breaking is what "creates the phenomenon" as claimed by Curie. For this analysis, Curie is credited as the first one to have recognised the important heuristic, or more generally, methodological role of symmetry breaking in physics.

While Curie considered the concept of symmetry breaking with regard to objects and phenomenon, in modern physics the focus has turned to the symmetries of the laws. This will be the focus in the rest of this article. The physical system under consideration can be described by the Lagrangian or Hamiltonian and so we will often be speaking of the symmetry "of" the Lagrangian or the Hamiltonian. There are still many issues that may affect the possibility and the kind of symmetry breaking that can occur. For one, there are differences in symmetry breaking depending on whether it is a classical, a quantum mechanical or a quantum field theoretical description of the physical system. Another issue, although related, is whether the description has a finite or an infinite number of degrees of freedom. Finally, also of relevance is the dimension of the system under consideration, as there are certain theorems addressing the possibility or impossibility of symmetry breaking given certain dimensions.[7]

[5]For example, the characteristic symmetry of a magnetic field is that of a cylinder rotating about its axis: this means that, for a magnetic field (the effect) to exist, the medium (the cause) must have a symmetry lower or equal to that of a rotating cylinder.

[6]See for example Curie's description of such physical effects as the "Wiedemann effect" and the "Hall effect" .

[7]E.g. Coleman (1973) proves that spontaneous symmetry breaking does not occur in two-dimensional quantum field theories.

(ii) Kind of Symmetry Once we have specified what exhibits the symmetry, the occurrence of symmetry breaking depends also on what the symmetry is that is supposed to be broken. Depending on whether the symmetry is continuous or discrete, a spacetime or internal symmetry or whether the symmetry is global or local will affect the way and kind of symmetry breaking that is possible.[8]

(iii) Breaking Mechanism This leaves us with the third aspect of symmetry breaking, namely the mechanism by which it is broken. There are broadly speaking three kinds of symmetry breaking mechanisms: explicit symmetry breaking (Sect. 2), anomalous symmetry breaking (Sect. 3) and spontaneous symmetry breaking (Sect. 4). We will now discuss each of these in more detail and consider some of the subtleties involved.

2 Explicit Symmetry Breaking

A simple illustration of explicit symmetry breaking is given by starting with some Hamiltonian $\mathcal{H}_0$ which is invariant under a symmetry group G and adding to it an additional term $\mathcal{H}_{ESB}$, such that $\mathcal{H}_0 + \mathcal{H}_{ESB}$ is not invariant under G anymore. In such cases, the symmetry of $\mathcal{H}$ is *explicitly broken* by $\mathcal{H}_{ESB}$ whatever the cause of it may be.

A much discussed example for this kind of symmetry breaking is the Heisenberg ferromagnet given by the Hamiltonian

$$\mathcal{H} = -\frac{1}{2}\sum_{i \neq j} J\, S_i \cdot S_j, \tag{1}$$

where S_i is a three-dimensional spin operator on lattice site i and J is a positive constant which is only non-zero for neighboring sites. The Hamiltonian is invariant under the $SO(3)$ rotation symmetry. Now by turning on an external magnetic field B the Hamiltonian becomes

$$\mathcal{H} = -\frac{1}{2}\sum_{i \neq j} J\, S_i \cdot S_j - B\sum_i S_i, \tag{2}$$

which is not invariant under the $SO(3)$ rotation symmetry anymore. The exter-

[8]E.g. the aforementioned theorem by Coleman in footnote 7 holds for continuous but not discrete symmetries.

nal magnetic field has explicitly broken the symmetry of the original Hamiltonian by introducing a "preferred" direction, namely the direction of the magnetic field. The spin of the electrons will align accordingly.

Note that the breaking of the symmetry in the previous example is simply due to an external magnetic field. As such it is not a conceptually interesting case of symmetry breaking. However, there are also other sources of explicit symmetry breaking. In some circumstances one may have experimental or theoretical reasons to introduce a small term to the Lagrangian, which breaks some symmetry. For instance Lee and Yang (1956) predicted, on the grounds of theoretical development, that parity symmetry could be violated in the weak interactions. This was subsequently experimentally confirmed by Wu et al. (1957). Now, one may argue that this breaking of the symmetry is not really a breaking of the symmetry at all, since it was already there and we just did not know. In some sense the breaking just represents the epistemic change of situation at a certain time: we just assumed the symmetry of the Lagrangian to be bigger than it actually was, and we were shown to be wrong about it.[9]

Historically, it was precisely this kind of change to first trigger the interest in the meaning of the symmetry breaking of laws in physics. Just before the discovery of the violation of parity, in the 1952 seminal book *Symmetry* by Weyl, the issue was still not considered. For Weyl, any form of symmetry breaking was in the phenomena and due to contingency: in his own words, "If nature were all lawfulness, then every phenomenon would share the full symmetry of the universal laws of nature [...]. The mere fact that this is not so proves that *contingency* is an essential feature of the world" (Weyl 1952, p.26).

The discovery of the violation of parity, soon followed by the observation of other violations of the discrete space and time symmetries,[10] brought a change in the above "contingency view". The symmetry violation of a law, such as the parity violation, could now be intended in the sense that what was thought to be a non-observable turned out to be actually an observable, a view particularly defended by Lee himself. From the more general viewpoint of the issue of how to interpret physical symmetries, this is a corollary of an epistemic stance on symmetries, ascribing their significance to the presence of

[9] The issue of whether symmetry breaking is something occurring, for instance temporally, in nature or just representing a change in our epistemic state at a certain time will also come up in the context of spontaneous symmetry breaking and phase transitions (see Sect. 4 and 5).

[10] Namely, the violation of the combination of charge coniugation and parity (CP symmetry) and, therefore, the violation of time inversion (T symmetry) in virtue of the CPT theorem. See ... this volume.

unobservable (or irrelevant features) in the physical description.[11]

Finally, in some circumstances the reason we did not know about the symmetry breaking term is that it was broken only by a small term and one may wonder why does such a small breaking term appear. This initiated the wide-ranging theoretical and philosophical discussion of what counts as a natural parameter.[12]

3 Anomalous Symmetry Breaking

Let us turn to another kind of symmetry breaking, which has so far not received much philosophical discussion, namely anomalies. Anomalies label instances, where the symmetry of the classical theory turn out not to remain the symmetry of the corresponding quantum theory. While "anomaly" may sound very serious, maybe even something that can give rise to scientific revolutions, the name should rather be understood as the consequence of the bafflement physicists found themselves in when they realized that quantum fluctuations can break classical symmetries.[13] A more suitable name may be "quantum mechanical symmetry breaking".

Whether this is something that needs to be cured or not depends most crucially on the kind of symmetry that is being broken in the transition. That the symmetry can break in the transition from classical to quantum becomes obvious if we take a path integral perspective on quantum theory.[14] Let us consider the symmetry transformation of some field ψ:

$$\psi \to \psi' = U\psi. \tag{3}$$

If this is a symmetry of the Lagrangian, then

$$\mathcal{L}(\psi) \underset{U}{\to} \mathcal{L}(\psi'). \tag{4}$$

[11] (Lee 1981) explicitly claims that "the root of all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities? (p. 178). See on this (and, more generally, on the relation between symmetry, equivalence and irrelevance) (Castellani 2003). (Dasgupta 2016) defends an epistemic interpretation of symmetry on a similar basis as Lee.

[12] The original paper by (Hooft 1980) introduced the idea of naturalness and its relation to symmetry breaking. Subsequently, the notion of naturalness was also considered in different contexts. See (Williams 2015, Wells 2015) for more detailed philosophical discussions.

[13] As Zee (2010, p.271) puts it: "[the field theorists] were so shocked as to give this phenomenon the rather misleading name "anomaly", as if it were some kind of sickness of field theory."

[14] This observation is due to Fujikawa (1980).

However, for the quantum theory to be invariant under the symmetry transformation, we need $\int D\psi \; e^{i\int \mathcal{L}(\psi)d^4x}$ to be invariant. But as we know, any coordinate transformation

$$D\psi \underset{U}{\rightarrow} \mathcal{J} D\psi' \tag{5}$$

introduces a Jacobian $\mathcal{J}$. Thus, once the Jacobian is non-unit, the symmetry does not translate to the quantum theory. The actual calculation of $\mathcal{J}$ is quite involved in quantum field theory and requires regularization as the Jacobian diverges.[15]

A nice simple example, which already illustrates the far-reaching consequences of anomalies is given by the Schwinger model.[16] You start with a classical massless charged particle ψ coupled to an electromagnetic field A_μ

$$\mathcal{L}_{\mathcal{S}} = \bar{\psi}(i\gamma_\mu D^\mu)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{6}$$

where $iD_\mu = i\partial_\mu - eA_\mu$. The Lagrangian has a chiral symmetry, i.e. you may rotate the left handed component of the spinor independently from the right-handed component and still keep the Lagrangian invariant. However, the chiral symmetry does not survive quantization as the Jacobian gives rise to a term, which effectively introduces an additional term to $\mathcal{L}_{\mathcal{S}}$. This additional term explicitly breaks the chiral symmetry, leading to a non-interacting theory with massive photons.

Broadly construed, there are now two ways anomalies have been interpreted, depending on whether a global or a gauge symmetry is broken by the quantum fluctuations. Global symmetries that do not survive the quantization lead to possible new effects that are now interpreted as predictions of the theory. This was the case with the first appearance of anomalies in particle physics, namely the problem of understanding the decay rate of the neutral pion ($\pi^0 \rightarrow \gamma\gamma$). The derived decay rate, which was based on the assumption that the chiral symmetry holds, was in disagreement with observations.[17] This deviation was later shown by (Adler 1969) and (Bell and Jackiw 1969) to be due to the breaking of chiral symmetry through one-loop calculations.

Unlike for global symmetries, the anomalies that arise for gauge symme-

[15] Early on, this led theorists to believe that the anomaly may only be due to the choice of regularization. However, one can show that it is actually independent of this choice. See (Jatkar 2016) for calculations using different regularization schemes.

[16] See (Schwinger 1951) for the original paper, (Peskin and Schroeder 1995, Sect.19.1) for a detailed discussion and (Holstein 1993, p.144) for an elementary discussion.

[17] See (Weinberg 1995, Sect. 22.1) for the historical background.

tries, so-called gauge anomalies, can be troublesome. The main reason for this is that gauge symmetries allow one to dispose of negative norm states, which otherwise would render the theory "inconsistent". Thus, unlike the appearance of anomalies for global symmetries, gauge anomalies need to be cured. This imposes strong *theoretical* constraints both on existing theories and on any theory to be developed. For instance, it happens that the particle content of the standard model is appropriately "tuned" to cancel any possible gauge anomaly. If for instance there would be more quarks than leptons, certain gauge anomalies would not cancel. Similarly, strong constraints on the charges of the various particles and their relations are set due to the need to cancel the anomaly.[18]

This gives rise to many interesting philosophical questions, that have not yet received any treatment, with obvious methodological implications for theory development. There is an intricate interplay between inconsistencies of the theory, their quantum origin, and an apparently fine-tuned particle content of the standard model universe. Anomaly cancellation leads to strong restrictions on possible representations of the gauge group for grand unified theories or, similarly, to the need for bosonic string theory to be 26-dimensional and superstring theory to be 10-dimensional. Much of this depends on how problematic the "inconsistency" is and remains an issue that warrants further discussion.[19] As said, this is another illustrative example of the methodological role of symmetry breaking, or more generally symmetry considerations in fundamental physics.

4 Spontaneous Symmetry Breaking

The philosophical discussion of symmetry breaking has mainly been focused on spontaneous symmetry breaking (SSB), which is a rich and subtle topic. SSB occurs when the law governing the behavior of a system has a symmetry which is not shared by its ground state or vacuum. Especially in the context of perturbative quantum field theory, where states are built up from the vacuum, the specification of its symmetry properties is crucial. Depending on whether the symmetry of the law is shared by the vacuum or not, one speaks

[18] See (Schwartz 2014, Sect. 30.4) for a detailed discussion of these constraints.

[19] This becomes apparent when (Guidry 2008, p. 281) speaks of it as a "theoretical prejudice": "The current theoretical prejudice is that gauge theories with incurable anomalies are incorrect because they cannot be perturbatively renormalized."

of different modes or realizations of the symmetry.

Consider a unitary representation of the symmetry group of, say, the Lagrangian. Then, the distinction between the modes relies on a result by Fabri and Picasso (1966), which states that there are, roughly speaking, only two possibilities:[20]

- The symmetry of the Lagrangian leaves also the vacuum invariant, i.e. $U|0\rangle = |0\rangle$ (Wigner-Weyl mode)
- The symmetry of the Lagrangian does not leave the vacuum invariant[21]: $U|0\rangle \neq |0\rangle$, and
  - The symmetry is global (Nambu-Goldstone mode),
  - The symmetry is local (Higgs mode).

Symmetries realized in the Wigner mode can only be broken explicitly or anomalously. We will now turn to the other two modes, which are instances of spontaneous symmetry breaking. A typical and simple illustration of SSB is made in terms of the following Lagrangian, describing a real scalar filed ϕ with a quartic interaction:[22]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4, \tag{7}$$

with $\lambda > 0$. The Lagrangian is invariant under the *discrete* symmetry transformation

$$\phi \to -\phi. \tag{8}$$

For $\mu^2 > 0$ we have a unique minimum with a vacuum expectation value $\langle 0|\phi|0\rangle = 0$, which is invariant under (8) (i.e. the symmetry is realized in the Wigner-Weyl mode). For $\mu^2 < 0$, however, the potential exhibits a degenerate vacuum leading to two minima at $\langle 0|\phi|0\rangle = \pm v$ with $v = \sqrt{\frac{-\mu^2}{\lambda}}$. The Lagrangian remains invariant under the discrete symmetry (8), which however is not shared by any specific vacuum. This is an example of spontaneous symmetry breaking, since one may think of the field "spontaneously choosing" one of the vacua, as it has no reason to prefer one over the other.

[20]See (Nair 2005, Ch.11) and (Aitchison 1982, Sect. 6.1) for a nice discussion on this.

[21]More accurately, $U|0\rangle$ does not exist in the Hilbert space.

[22]This example can be found in many textbooks on quantum field theory; see e.g. (Coleman 1988, Sect. 5.2) or (Guidry 2008, Sect. 8.2), which we are following here.

Now, calling it "spontaneous" may be misleading and some call it rather hidden or secret symmetry.[23] To see why this may be more accurate, let us imagine the field in one of the degenerate vacua, say $\langle 0|\phi|0\rangle = +v$, and consider the construction of the particle spectrum from this vacuum. For this purpose it is useful to redefine the scalar field shifted towards v, i.e.

$$\psi(x) \equiv \phi(x) - v, \tag{9}$$

where we now have $\langle 0|\psi|0\rangle = 0$. Plugging (9) into (7) yields

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \psi)(\partial^\mu \psi) - \lambda v^2 \psi^2 - \lambda v \psi^3 - \frac{1}{4}\lambda v^2 \psi^2. \tag{10}$$

Note that (10) is not invariant under the discrete symmetry of (8), and the field living in the chosen vacuum is not able to "recognize" the more fundamental symmetry of (7). It is in this sense that the symmetry (8) is hidden or a secret symmetry. Note also that the field ψ has a mass $m = 1/2\lambda v^2 = \sqrt{-2\mu^2}$, which therefore differs from the mass of the scalar field ϕ in the unbroken phase.

This simple example exhibits several features which are characteristic for spontaneous symmetry breaking. First, the existence of a degenerate nonzero vacuum expectation value. Second, any of such vacuum states is not invariant under the symmetry of the Lagrangian, but the symmetry transformation relates each vacuum state to each other. Third, on expanding around the chosen vacuum the original symmetry remains hidden.

Nevertheless there are additional features of SSB that do not occur in this simple example. These are special features which occur when you move from discrete to continuous symmetry (Nambu-Goldstone mode) and from global to local symmetry (Higgs mode). Let us consider the Lagrangian in equation (7) with complex scalar fields, i.e.

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - \mu^2 \phi\phi^* - \lambda(\phi\phi^*)^2, \tag{11}$$

where $\phi = \phi_1 + i\phi_2$ and $\mu^2 < 0$. This Lagrangian is now invariant under the global continuous transformation

$$\phi \to \; e^{i\theta}\, \phi. \tag{12}$$

The minima of the potential (See Fig. 1) are now given by $\phi\phi^* = -\mu^2/2\lambda$,

[23]See (Aitchison 1982, p. 69) and (Coleman 1988, Ch. 5).

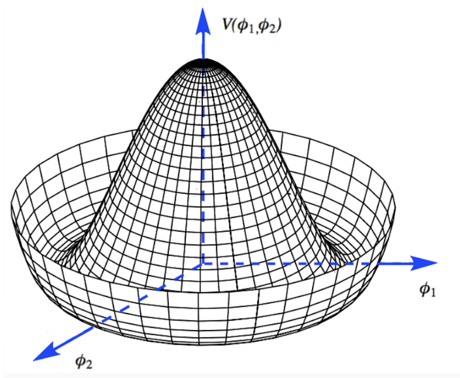

Figure 1: Plot of potential in equation (11). Figure under CC Attribution-Share Alike 3.0 Unported license.

which corresponds to infinitely many possible vacuum states the field can "spontaneously" fall into. This infinite degenerate vacuum is a standard feature of SSB for continuous symmetries. Let us for convenience choose the specific vacuum state $\langle\phi_1\rangle = \sqrt{\frac{-\mu^2}{2\lambda}} = v/\sqrt{2}$ and $\langle\phi_2\rangle = 0$. This solution is now related to all other solutions via (12). If we now expand around this arbitrarily chosen solution

$$\phi(x) = \frac{1}{\sqrt{2}}\Big(v + \psi(x) + i\eta(x)\Big) \tag{13}$$

the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\psi)^2 + \frac{1}{2}(\partial_\mu\eta)^2 - \lambda v^2\psi^2 + \text{cubic and quartic interaction terms.} \tag{14}$$

That is, you have an interacting theory of one massive scalar $\psi(x)$ (corresponding to modes along the radial direction in Figure 1) and one massless scalar η (along the angular direction in Figure 1). The existence of this massless scalar field, called Goldstone boson, is a generic feature of the spontaneous breaking of global continuous symmetries (according to the Goldstone theorem). The theorem states that there are as many Goldstone bosons as there are broken group generators.

Let us now turn from the Nambu-Goldstone realization to the Higgs realization, i.e. let us require the global symmetry transformation from (12) to be a local one.

$$\phi \to e^{ie\theta(x)}\,\phi. \tag{15}$$

The Lagrangian (11) is not invariant under the local transformation. For it to be invariant under local transformations one needs to follow the standard procedure to replace the derivative with the covariant derivative $D_\mu = \partial_\mu + ieA_\mu$, where A_μ is a gauge field, and add the standard kinetic term for the gauge field. If then at the same time $A_\mu \to A_\mu - \partial_\mu \theta(x)$, the Lagrangian will be invariant. Note that the local transformation does not allow a mass term for the gauge field. However, as we are still in the same potential as before, the argument follows analogously with the only difference that no massless particle appears after the expansion around some arbitrarily chosen vacuum. The massless degree of freedom associated with the Goldstone boson now appears as an additional degree of freedom of the gauge field (longitudinal polarization) making it massive. This result is known as the Higgs mechanism proposed by Peter Higgs and others and it provides the mechanism by which mass is generated in the standard model of particle physics.[24]

There is a well-known theorem of lattice gauge theory, which, however, explicitly forbids the possibility of a spontaneously broken local symmetry (Elitzur 1975). This has led to some discussion as to how the spontaneous breaking of a local symmetry does not violate this theorem. For instance, Smeenk (2006) and Friederich (2013) have addressed this issue in more detail relying also on an approach by Fröhlich, Morchio, and Strocchi (1981), where the Higgs mechanism is accounted for in an entirely gauge-invariant approach, i.e. where it is shown that the origin of the Higgs mechanism does not rely on the breaking of a local symmetry.

The philosophical literature on SSB has to a large extent made use of the algebraic formulation of quantum field theory to address the various puzzles that arise for SSB. The reason for this is, as Earman (2003, pg. 344) states that "the algebraic formulation of QFT, though useless for calculations, helps to clarify foundational issues". However, introducing the formalism would go beyond the scope of this entry and Earman (2003) already provides a useful short introduction to the formalism relevant to understand it in the context of SSB.

[24]See (Higgs 1964), (Guralnik, Hagen, and Kibble 1964) and (Englert and Brout 1964). There is extensive historical discussion regarding the development of the Higgs mechanism. See for instance (Guralnik 2009).

5 Spontaneous Symmetry Breaking and Phase Transitions

So far there was no mention of phase transitions. However, they are intricately related to spontaneous symmetry breaking since, in many cases of phase transitions, the system undergoes a change in symmetry as well. Usually, the symmetry of the high energy phase is larger than the symmetry of the low energy phase. More precisely, the system has some *order parameter* (e.g. the magnetization), such that the expectation value of it at the ground state breaks the symmetry (this is the SSB component). The order parameter is also a function of some *control parameter* (e.g. the temperature), which allows it to transition to the broken or unbroken phase (this is the phase transition component). While a detailed analysis would go beyond the scope of this entry, we will illustrate the relation considering examples from above.

Let us first return to the Heisenberg ferromagnet. We saw how we could explicitly break the symmetry of the Hamiltonian with the help of an external magnetic field. The point being, that all electron spins will align along the direction of the magnetic field and thereby break the SO(3)-symmetry. However, this breaking also occurs, now spontaneously, by taking the infinite-volume limit, while letting the magnetisation $B \to 0$. In this limit the ground state expectation value does not vanish, thereby not sharing the SO(3)-symmetry of (1). By increasing the temperature above the Curie temperature the system transitions to the unbroken phase, where the magnetization vanishes, again realising the full symmetry of the system. In the broken phase, there is the interesting feature that the symmetry generators that are broken, if applied to the ground state, provides an infinitely degenerate set of ground states.[25] In the infinite-volume limit, the thus obtained degenerate ground states actually belong to different Hilbert spaces implying unitarily inequivalent representations of the commutation relations. However, this characteristic feature of SSB seems to sensitively depend on the infinite-volume limit. The necessity of this idealizing assumption of an infinite limit has attracted significant treatment within the philosophical literature, showing how one may weaken this assumption (Butterfield 2011, Fraser 2016).

While phase transitions find a natural habitat in the context of condensed matter systems, this is less obvious in the particle physics context. As we

[25] See (Arodz, Dziarmaga, and Zurek 2012, Ch. 1)

mentioned, the control parameter, i.e. temperature, allows one to go from e.g. the unbroken phase to the broken phase in a ferromagnet. But what plays the role of the control parameter in the Higgs mechanism discussed above? Does there need to be one?

This has led to a discussion regarding the ontological status of the Higgs mechanism, which involves also difficulties in understanding gauge symmetries in general (see Nic Teh, this volume). As the Higgs mechanism was presented above, one might consider it as "a mere reshuffling of degrees of freedom" (Lyre 2008, p. 130). The degrees of freedom associated with the massless Goldstone boson provide the needed degrees of freedom to make the gauge boson massive. On the contrary, another view[26] considers the Higgs mechanism in strict analogy to the ferromagnet case, where the two phases are determined by $\mu^2 > 0$ corresponding to the unbroken phase and $\mu^2 < 0$ corresponding to the broken phase. The transition between these two phases then could have occurred physically during the cooling of the early universe. The formal analogy between spontaneous symmetry breaking in these different contexts has received further discussion recently in (Fraser 2012, Fraser and Koberinski 2016).

6 Conclusion

In this Chapter we have given an introduction to the ubiquitous concept of symmetry breaking as it is used in physics and to some of the philosophical discussions it generated. As we saw, symmetry breaking may occur in many different theories with different implications depending on the mechanism that underlies it. One obvious philosophical implication follows directly from its use in the context of laws rather than systems. In the law context a larger interpretational gap needs to be overcome to translate the formal implementation of symmetry breaking to what it corresponds to in the real world. Many of the philosophical issues that arise here are then directly related to the issues that arise for symmetries more generally. That is, the assessment of the impact of symmetry breaking in the context of e.g. global or gauge symmetries is strongly interlinked with the formal, methodological, epistemological and ontological analysis of these respective symmetry concepts.

However, there are certain issues more generally concerned with symmetry

[26]See (Wüthrich 2012) for details.

breaking. As we mentioned, symmetry breaking is a necessary ingredient for the existence of some phenomenon. We, nevertheless, wish to present theories in symmetric form. Spontaneous symmetry breaking is then an ingenious way to account for the lack of symmetry in the real world, while keeping the symmetry of the laws. One contentious way of looking at it, is to ask why we would want to impose a symmetry on the laws of nature that is not observed. Consequently, if symmetry is preferred over asymmetry, then occurrences of asymmetry are in need of explanation. But why is it that we prefer symmetry over asymmetry in the first place?

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