Taming the Delayed Choice Quantum Eraser

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Abstract

In this paper I discuss the delayed choice quantum eraser experiment by giving a straightforward account in standard quantum mechanics. At first glance, the experiment suggests that measurements on one part of an entangled photon pair (the idler) can be employed to control whether the measurement outcome of the other part of the photon pair (the signal) produces interference fringes at a screen after being sent through a double slit. Significantly, the choice whether there is interference or not can be made long after the signal photon encounters the screen. The results of the experiment have been alleged to invoke some sort of 'backwards in time influences'. I argue that in the standard collapse interpretation the issue can be eliminated by taking into account the collapse of the overall entangled state due to the signal photon. Likewise, in the de Broglie-Bohm picture the particle's trajectories can be given a well-defined description at any instant of time during the experiment. Thus, there is no need to resort to any kind of 'backwards in time influence'. As a matter of fact, the delayed choice quantum eraser experiment turns out to resemble a Bell-type measurement, and so there really is no mystery.

Contents

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1 Introduction

Delayed choice scenarios in slit experiments as found in [Wheeler, 1978], and earlier in [von Weizsäcker, 1941] and [Bohr, 1996], have formed a rich area of theoretical and experimental research, as evidenced in the literature ([Eichmann et al., 1993], [Englert and Bergou, 2000], , [Englert et al., 1999], [Mohrhoff, 1999], [Kim et al., 1999], [Walborn et al., 2002], [Kwiat and Englert, 2004], [Aharonov and Zubairy, 2005], [Peres, 2000], [Egg, 2013]). From the results of the original delayed choice experiment Wheeler concluded that 'no phenomenon is a phenomenon until it is an observed phenomenon', and 'the past has no existence except as it is recorded in the present' (ibid.). I shall discuss a modified version of Wheeler's delayed choice experiment, one which was first proposed by [Scully and Drühl, 1982] and later realised in the experiments of [Kim et al., 1999].

2 The Delayed Choice Quantum Eraser

The setup employed by Kim et al. uses double slit interference of photons and raises a conceptual problem, which, according to Wheeler, allegedly implies that there were a change in the behaviour from 'acting like a particle' to 'acting like a wave', or vice versa, well after the particle entered the double slit.

In the old days of quantum mechanics it was believed that the loss of interference in double slit experiments were due to Heisenberg's uncertainty principle, for no measurement device could be so fancy as not to perturb the system observed and destroy coherence. Such a perturbation leads to so-called 'which-path information' that 'collapses the wavefunction', making interference effects disappear. That said, in the delayed choice case the which-path information of the photon is obtained by entanglement without disturbing the wavefunction (cf. Einstein's move in the EPR experiment [Einstein et al., 1935, p. 779]). Significantly, the which-path information can be 'erased' long after the photon encounters the double slit. This was deemed inconceivable in the old picture. The interference pattern, as a result, reappears. Figure 1 illustrates the experimental setup.
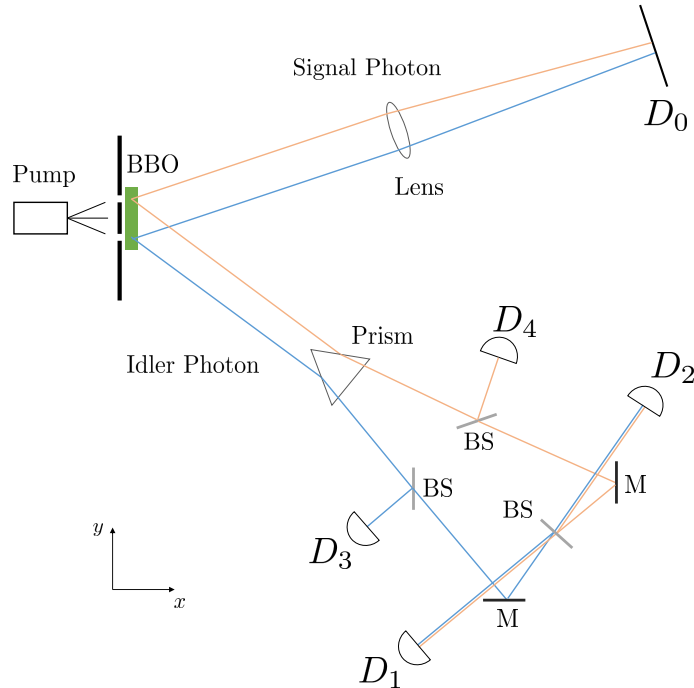
Figure 1: A delayed choice quantum eraser experiment. A laser beam aims photons at a double slit. After a photon passes the slits it impinges on a Barium borate (BBO) crystal placed behind the double slit. The optical crystal destroys the incoming photon and creates an entangled pair of photons via spontaneous parametric down conversion at the spot where it hit. Thus, if one of the photons of the entangled pair can later be identified by which slit it went through, one also knows whether its entangled counterpart went through the one or the other side of the crystal. Whether which-path information about the signal photon arriving at detector D_0 is obtained or erased is decided by manipulating the idler photon well after the signal photon has been registered.

A laser beam (pump) aims photons at a double slit. After a photon passes the slits it impinges on a Barium borate (BBO) crystal placed behind the double slit. The optical crystal destroys the incoming photon and creates an entangled pair of photons via spontaneous parametric down conversion at the spot where it hit. Thus, if one of the photons of the entangled pair can later be identified by which slit it went through, we will also know whether its entangled counterpart went through the one or the other side of the crystal. By contrast, we will have no which-path information if we cannot later identify where either of the photons came from. Even though the entangled photons created at the crystal are now correlated, the experiment can manipulate them differently. We call one photon of the pair the signal photon (sent toward detector D_0) and the other one the idler photon (sent toward the prism). The naming is a matter of convention. The lens in front of detector D_0 is inserted to achieve the far-field limit at the detector and at the same time keep the distance small between slits and detector. The prism helps to increase the displacement between paths. Nothing about these parts gives which-path information and detector D_0 can not be used to distinguish between a photon coming from one slit or the other. At this point we would expect interference fringes to appear at D_0 if we were to ignore that signal photon and idler photon are entangled. The parts of the wavefunction originating at either slit should interfere and produce the well-known pattern of a double slit experiment. On the other hand, quantum mechanics would predict a

typical clump pattern if which-path information were available.

After the prism has bent the idler photon's path, the particle heads off to one of the 50-50 beamsplitters BS. The photon is reflected into the detector D_3 a random 50% of the time when it is travelling on the lower path, or reflected into detector D_4 a random 50% of the time when it is travelling on the upper path. If one of the detectors D_3 or D_4 clicks, a photon is detected with which-path information. That is, we know at which slit both photons of the entangled pair were generated. In that case, the formalism of quantum mechanics predicts no interference at D_0. In all of the other cases the photon passes through the beamsplitter and continues toward one of the mirrors M. Importantly, it does not matter if the choice whether the photon is reflected into the which-path detectors D_3 or D_4 is made by beamsplitters. The original experiment uses beamsplitters and therefore it is randomly decided which kind of measurement is performed. But we could equally replace the beamsplitters by moveable mirrors. In that way the experimenter is free to decide whether which-path information is available by either keeping the mirrors in place or removing them such that the photon can reach the eraser.

After being reflected at one of the mirrors, the photon encounters another beamsplitter BS, which is the quantum eraser. This beamsplitter brings the photon in a superposition of being reflected and transmitted. To that end, for an idler photon coming from the lower mirror the beamsplitter either transmits the photon into detector D_2 or reflects it into detector D_1. Likewise, for an idler photon coming from the upper mirror the beamsplitter either transmits it into detector D_1 or reflects it into detector D_2. If one of the detectors D_1 or D_2 clicks, it is impossible to tell which slit the photon came from. To summarise the above, detectors D_1 and D_2 placed at the output of BS erase the which-path information, whereas a click of detectors D_3 or D_4 provides which-path information about both the idler and the signal photon. Notably, when the photon initially hits D_0, there is no which-path information available, only later when the entangled idler photon is detected at D_3 or D_4.
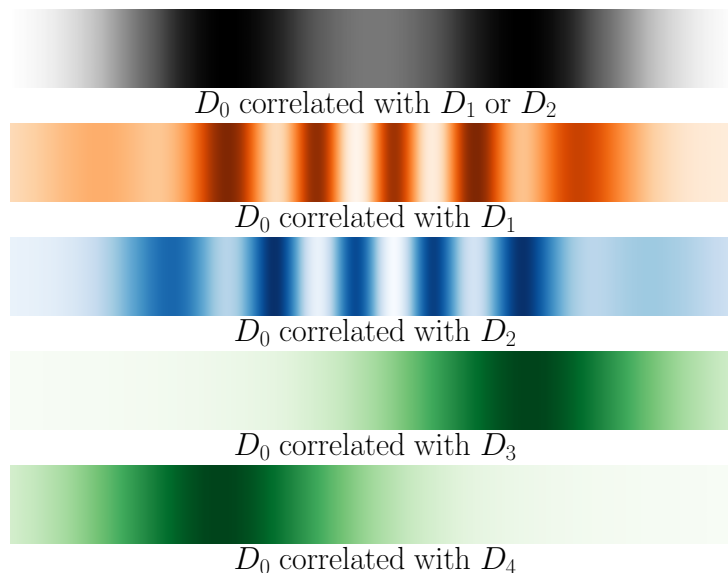

Figure 2: Joint detection events at detector D_0 and detectors D_1-D_4. The figure shows a plot of the bits of Equation 4.7. Records of D_0 and D_1 (D_2) show interference fringes. On the contrary, records of D_0 and D_3 (D_4) show a clump pattern.

This is key. The setup ensures that the which-path information is only erased

or provided, respectively, after D_0 has detected the signal photon. We therefore say the choice is delayed. For each incoming photon from the laser beam there will be a joint detection of the signal photon at D_0 and the idler photon at D_1–D_4. Figure 2 shows the expected results. When which-path information is provided, a clump pattern appears, but when no which-path information is available interference fringes appear. The two interference patterns corresponding to correlation with D_1 and D_2 are out of phase. The reason for that will become clear in the next sections. The results in [Kim et al., 1999] show a single clump as opposed to two clumps in Figure 2. This is simply due to the close distance between the slits Kim et al. chose for their experiments.

Those inclined to instrumentalism might be satisfied at this point, for the predictions of standard quantum mechanics give the desired results to confirm experimental observation. The philosopher, however, might start to worry about what is going on here.

3 Backwards in time influence?

Indeed, it may be tempting to interpret these results as instances of future measurements influencing past events. Seemingly, there is something odd going on in the experiment. The collapse of the wavefunction (either one that shows interference or one that shows a clump pattern) of the signal photon is determined by the way of measurement on the idler photon — an event which occurs after the signal photon has already been detected. Does a measurement cause an entangled particle to collapse retroactively its wavefunction? It seems the detection of the idler photon and thus the choice of which-path information affects the behaviour of the signal photon in the past. Is this a process that reverses causality? Wheeler comments on his original *Gedankenexperiment* as follows:

> '*Does this result mean that present choice influences past dynamics, in contravention of every formulation of causality? Or does it mean, calculate pedantically and do not ask questions? Neither; the lesson presents itself rather like this, that the past has no existence except as it is recorded in the present.*' [Wheeler, 1978]

In comparison, Bohr concludes that understanding of the quantum behaviour of particles is confused by giving pictures which are trying to maintain conceptions of classical physics. He states that a sharp separation of the quantum system and the observing measurement device is impossible [Bohr, 1961]. According to his view there is no point in visualising the process as a path taken by a particle when not in a well-defined state. The only way out of the misery is to 'shut up and calculate' because one will not get answers to such questions. Wheeler refuses this position.

What I think is that one should not expect the formalism of quantum mechanics to provide clear images of what could be 'actually' going on, for at the moment it is a framework with different interpretations. Only if one is to adopt an interpretation, I believe, can a conclusion be meaningful. Many physicists and philosophers did not accept the views of Wheeler or Bohr and have been continuing to debate the delayed choice experiment to seek for possibilities that account for physical intuition.

4 Delayed choice in Collapse interpretation

The first significant point that I found not emphasised in the analyses of [Kim et al., 1999] and others, is that there never appears an interference pattern at D_0 without con-

ditioning on whether we choose which-path information to be available or erased.[1] Technically, by conditioning we mean to constrain the measurement results to the subset of coincidence detections of the signal photon with the idler photon in a chosen detector D_1–D_4. Moreover, it is key to the analysis that the two interference patterns from the joint detection events of D_0 and D_1 or D_2, respectively, obtain a relative phase shift of π and cancel when added together. The analysis of this feature is often left out in the literature (cf. [Kim et al., 1999]).

I shall give an analysis of the experiment proposed by Kim et al. by using standard quantum mechanics. My analysis involves wavefunctions described by the Schrödinger equation, which strictly speaking only applies to massive particles. For a rigorous treatment with photons we would need to avail ourselves of quantum field theory. Nevertheless, we can straightforwardly replace photons with electrons for the sake of a *Gedankenexperiment*. The interference phenomena qualitatively remain the same.

The incoming laser beam can be described as a plain wave

$$\psi = e^{ik_x x} \tag{4.1}$$

impinging on the double slit, where k_x is the wave vector.[2] After the slits the wavefunction can be decomposed into two interfering parts as

$$\psi = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2). \tag{4.2}$$

Wavefunction ψ_1 belongs to the part of the wavefunction emerging from the upper slit and ψ_2 to the part of the wavefunction emerging from the lower slit. We may assume waves of the form

$$\psi_i = \frac{e^{ikr_i}}{r_i}, \tag{4.3}$$

where r_i is the distance from the slit i. These give the well-known two slit interference fringes. The crystal then creates an entangled pair of photons with opposite momenta in the y-direction such that

$$\psi = \frac{1}{\sqrt{2}}(\psi_1 \otimes \psi_1' + \psi_2 \otimes \psi_2'), \tag{4.4}$$

where unprimed wavefunctions correspond to the signal photon and primed to the idler photon. The signal photon sent to detector D_0 is now entangled with the idler photon. This affects the probability amplitudes at D_0, and interference between ψ_1 and ψ_2 vanishes since $\psi_1 \otimes \psi_1'$ and $\psi_2 \otimes \psi_2'$ are orthogonal states (note that ψ_1' and ψ_2' are thought to be non-overlapping and thereby the inner product vanishes). More clearly, the squared norm of the wavefunction yields

$$|\psi|^2 = \frac{1}{2}(|\psi_1|^2|\psi_1'|^2 + |\psi_2|^2|\psi_2'|^2). \tag{4.5}$$

Assuming the signal has not yet reached D_0, if the idler gets reflected into detector D_3 the wavefunction would collapse to $\psi_2 \otimes \psi_2'$, and if reflected into D_4 it would collapse to $\psi_1 \otimes \psi_1'$. In case the idler photon encounters the quantum eraser, the wavefunction undergoes another unitary evolution. The eraser puts the idler photon

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[1]Note that in the experiment of [Kim et al., 1999] the decision is made randomly by the beamsplitters next to the prism, but as I mentioned, they can be replaced with mirrors and allow the experimenter to make this choice.

[2]For the sake of simplicity we can suppress time dependence of the wavefunction since it does not affect the argument. I omit normalisation factors where not stated explicitly.

in a superposition of being transmitted to one detector or reflected to the other. At each reflection at a beamsplitter or mirror the wavefunction picks up a phase of $\frac{\pi}{2}$ (a multiplication of the wavefunction by $e^{i\frac{\pi}{2}} = i$) such that

$$\begin{aligned}
\psi_1' &\mapsto i\psi_{D1} - \psi_{D2} \\
\psi_2' &\mapsto -\psi_{D1} + i\psi_{D2}.
\end{aligned} \tag{4.6}$$

The joint wavefunction then turns into

$$\begin{aligned}
\psi &= \frac{1}{2}(\psi_1 \otimes (i\psi_{D1} - \psi_{D2}) + \psi_2 \otimes (-\psi_{D1} + i\psi_{D2})) \\
&= \frac{1}{2}((i\psi_1 - \psi_2) \otimes \psi_{D1} + (-\psi_1 + i\psi_2) \otimes \psi_{D2})
\end{aligned} \tag{4.7}$$

once the idler photon has passed the quantum eraser. Indices in ψ_{D_1}, ψ_{D_2} refer to which detector the part of the wavefunction is reflected into. In this form state 4.7 makes it clear that when detector D_1 clicks, the wavefunction of the signal photon collapses to $i\psi_1 - \psi_2$, yielding a probability distribution of interference fringes,

$$\begin{aligned}
|\psi_{D_0,D_1}|^2 &= (i\psi_1 - \psi_2)\overline{(i\psi_1 - \psi_2)} \\
&= |\psi_1|^2 + |\psi_2|^2 - 2\operatorname{Im}(\overline{\psi_1}\psi_2).
\end{aligned} \tag{4.8}$$

In the case in which D_2 clicks, the wavefunction collapses to $-\psi_1 + i\psi_2$ and yields a distribution showing anti-fringes:

$$\begin{aligned}
|\psi_{D_0,D_2}|^2 &= (-\psi_1 + i\psi_2)\overline{(-\psi_1 + i\psi_2)} \\
&= |\psi_1|^2 + |\psi_2|^2 - 2\operatorname{Im}(\psi_1\overline{\psi_2}) \\
&= |\psi_1|^2 + |\psi_2|^2 + 2\operatorname{Im}(\overline{\psi_1}\psi_2).
\end{aligned} \tag{4.9}$$

In either case of detection, when travelling on one of the paths, the idler photon is reflected twice, and only once when travelling on the other.

The experiment is designed such that the choice whether the wavefunction collapses to one which produces interference fringes or a clump pattern happens after the signal photon has been detected at D_0. We therefore say the choice is delayed. In the setup of [Kim et al., 1999] the optical length of the idler photon is about 8 ns longer than that of the signal photon.

Crucially, at detector D_0 there never appears an interference pattern, regardless of whether the idler photon reaches the quantum eraser or not. This can readily be seen by adding up the distributions:

$$|\psi_{D_0,D_1}|^2 + |\psi_{D_0,D_2}|^2 = |\psi_1|^2 + |\psi_2|^2. \tag{4.10}$$

The interference terms cancel out when added together which effectively leads to a clump pattern. Each sub-case shows an interference pattern, but the overall statistics adds up to two clumps. Note that there is no way to avoid the phase difference in the interference fringes since any additional device would act symmetrically on both paths. Insert for instance a $\lambda/4$-plate into the paths of the idler photon and it will affect both of the superposed paths reflected into the detectors. Thus, the effect of the plate would cancel out.

Incidentally, the fact that at detector D_0 interference fringes never occur guarantees consistency with no-signalling between D_0 and the other detectors. That is to say, it is not possible to decide what distribution (either an interference pattern or a clump pattern) appears at the detector D_0 by choice of whether the idler

photon will trigger the which-path detectors D_3 and D_4 and thus communicate information. As I noted above, this choice can be realised by replacing the former two beamsplitters by mirrors which can be inserted *ad libitum* by the experimenter (compare no-signalling in *EPR*).

With all this at hand, must we conclude that a measurement in the present retroactively changes the past to make it agree with the measurement outcomes?

4.1 Explanation of the Delayed Choice Quantum Eraser experiment

In the language of collapse interpretations the apparent retroactive action vanishes if a click in D_0 is regarded to collapse the overall wavefunction, not only a click in the detectors D_1–D_4. In the standard explanation, if the detection of the idler photon happens before the detection of the signal photon at D_0, the detectors D_1–D_4 determine what state the wavefunction collapses to. But similarly, in the case when the signal photon is detected at a moment in time preceding the observation of the idler photon, the view that the detected position of the signal photon collapses the wavefunction of the idler photon to trigger one of the detectors D_1–D_4 is just as accurate. However, in this case observation of where the photon lands on the screen — a measurement in the position basis — does not tell which state the overall wavefunction has collapsed to. Technically speaking, a detection of the signal photon in fact produces a mixed state since ψ_1 and ψ_2 are not completely orthogonal. Thus, the state of the signal photon is of the form $\rho = \alpha \left| \psi_1 \right\rangle \left\langle \psi_1 \right| + \beta \left| \psi_2 \right\rangle \left\langle \psi_2 \right|$ (non-selective von Neumann measurement). This is not a threat to the analysis since we can tell the same ontological story for each outcome of the state ρ.

One faces a confusion if one is to stubbornly stick to the notion that a measurement of the idler photon determines the probability distribution at D_0 for the signal photon. In fact, observation of individual subsystems of entangled pairs never determines or changes the probability distribution of the remote particle.[3] The which-path information can be present anywhere in the universe and it is irrelevant whether a future observer decides to acquire it. As soon as the signal photon gets entangled, the photon states lose their coherence.

What this tells us is that no matter how the idler photon gets manipulated, the probability distribution on D_0 is a clump pattern, but when we condition on the outcome of the detectors, which either give which-path information or not, we find correlations as expected. The quantum eraser does not influence the past of the signal photon; rather it reveals the correlations of an entangled photon pair in just another way. Indeed, in Section 6 we will see that two qubits in a Bell-state can be viewed as a simplified version of the delayed choice quantum eraser experiment.

5 Delayed choice in de Broglie–Bohm theory

I will use the term 'de Broglie-Bohm theory' to stand for the interpretation discussed by [Bohm and Hiley, 2006], which has to be distinguished from Bohmian mechanics, as the interpretations differ in some significant ways. Here it is assumed that a particle always travels on only one path. The wavefunction is considered as a quantum potential or pilot wave and used in its polar form

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[3]After all, the conditional probabilities of the measurement outcomes of signal and idler photon are spatio-temporally symmetric. This becomes clear when we look at the reduced states of the system. If we consider the general state of the joint system 4.7, that is, $\rho = \left| \psi \right\rangle \left\langle \psi \right|$, then the state of the signal photon gives an incoherent mixture $\rho_s = \mathrm{tr}_i \left(\rho \right) = \frac{1}{2} \left(\left| \psi_1 \right\rangle \left\langle \psi_1 \right| + \left| \psi_2 \right\rangle \left\langle \psi_2 \right| \right)$ by virtue of the partial trace tr_i over the idler states.

$$\psi(\vec{r}, t) = R(\vec{r}, t)e^{iS(\vec{r}, t)/\hbar}. \tag{5.1}$$

The dynamics of the pilot wave obey the Schrödinger equation

$$i\hbar\partial_t \psi = H\psi \tag{5.2}$$

and the particle's trajectory is determined by

$$\vec{v}(t) = \dot{\vec{x}}(t) = \frac{1}{m}\nabla S(\vec{r}, t)|_{\vec{r}=\vec{x}} \tag{5.3}$$

where m is the mass of the particle. For the sake of simplicity I will set $\hbar = 1$ for the remainder.

Now let us turn to consider how particles behave according to de Broglie-Bohm in this experiment. We construct a set of possible trajectories, each individually corresponding to one initial value of position of the particle within the incident beam. Supposedly, de Broglie-Bohm theory should reveal whether the past is influenced by present observations since it assumes a well-defined path of the particles at all times. Note that the de Broglie-Bohm interpretation does allow us to illustrate such a process and reproduce all the known experimental results in tension with Wheeler's and Bohr's conclusion about these phenomena.

The wavefunction of the incoming laser beam 4.1 is already in polar form and the trajectories in this region are straight lines. First we consider the case without the eraser. To work out what happens we must write the final wavefunction in Equation 4.4 in the form[4]

$$\psi(r, r') = R(r, r')e^{iS(r, r')}. \tag{5.4}$$

The wavefunction is evaluated at the positions of the signal photon r and the idler photon r'. It decomposes as

$$\begin{aligned}\psi(r, r') &= R_1(r)e^{iS_1(r)}R_1'(r')e^{iS_1'(r')} \\ &\quad + R_2(r)e^{iS_2(r)}R_2'(r')e^{iS_2'(r')}.\end{aligned} \tag{5.5}$$

Again, primed variables correspond to the idler photon. For the final amplitude R and the phase S we find

$$R^2 = (R_1 R_1')^2 + (R_2 R_2')^2 + 2R_1 R_1' R_2 R_2' \cos\Delta\phi, \tag{5.6}$$

by the law of cosines, where $\Delta\phi = (S_2 + S_2') - (S_1 + S_1')$. Also,

$$\tan S = \frac{R_1 R_1' \sin(S_1 + S_1') + R_2 R_2' \sin(S_2 + S_2')}{R_1 R_1' \cos(S_1 + S_1') + R_2 R_2' \cos(S_2 + S_2')}. \tag{5.7}$$

We need to evaluate this term for each trajectory. For the photon travelling trough the upper slit the entangled pair is created at this slit, and since the probability of creating an entangled pair at the lower slit is zero when the photon does not pass through it, $R_2' = 0$ (since R_2' has no support in the upper slit). Importantly, $R_2 \neq 0$ at points where R_1 has support. Having said that, vanishing R_2' on this trajectory cancels out overlapping terms, so that $R^2 = (R_1 R_1')^2$ and interference in the quantum potential vanishes. Recall that the quantum potential is evaluated at the positions of all the particles involved. Likewise, if the photon's path goes

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[4]For simplicity I suppress normalisation factors.

through the lower slit, $R_1' = 0$. Thus, $R^2 = (R_2 R_2')^2$ and interference vanishes as before. The guiding phase in the former case yields

$$S = S_1(r) + S_1'(r'). \tag{5.8}$$

That means that the guidance equation for the signal photon becomes independent of S_2 and S_2':

$$p_1 = \nabla_r S = \nabla_r S_1(r), \tag{5.9}$$

with p_1 the particle's momentum.[5] The idler photon then continues to travel to detector D_4 or D_1. Similarly, in the latter case the signal photon is independent of S_1 and S_1'. The idler photon then continues to travel to detector D_3 or D_2. The gradients ∇S_1, ∇S_2 (and consequently the momentum) point in the radial direction away from the slits. All we need to know is that a definite result has actually occurred (such as 'the signal photon has passed the upper slit', or 'the idler photon follows a path towards detector D_4'). Then, all of the other potential states give no contribution to the guidance equation so that the interference term cancels.

I will now turn to the situation where the quantum eraser is present, but we remove the two beamsplitters reflecting the idler photons into the which-path detectors. The question is whether the trajectories change when we consider the quantum potential of the eraser. Recall the wavefunction of the system when the idler photon has passed the eraser:

$$
\begin{aligned}
\psi &= \frac{1}{2}(\psi_1 \otimes (i\psi_{D1} - \psi_{D2}) + \psi_2 \otimes (-\psi_{D1} + i\psi_{D2})) \\
&= \frac{1}{2}((i\psi_1 - \psi_2) \otimes \psi_{D1} + (-\psi_1 + i\psi_2) \otimes \psi_{D2}).
\end{aligned} \tag{5.10}
$$

Or in polar form

$$
\begin{aligned}
\psi = \; & R_1(r)e^{iS_1(r)}(R_{D_1}(r')e^{iS_{D_1}(r')+i\frac{\pi}{2}} \\
& - R_{D_2}(r')e^{iS_{D_2}(r')}) \\
& + R_2(r)e^{iS_2(r)}(-R_{D_1}(r')e^{iS_{D_2}(r')} \\
& + R_{D_2}(r')e^{iS_{D_2}(r')+i\frac{\pi}{2}}).
\end{aligned} \tag{5.11}
$$

Consequently, unlike in the case without the eraser, here the signal photon is guided by a potential with contributions both from R_1 and R_2. Indeed, assume the idler photon to end in the path leading to detector D_1. That means $R_{D_2} = 0$ and the trajectory of the signal photon is determined by

$$
\begin{aligned}
& R_1(r)e^{iS_1(r)}R_{D_1}(r')e^{iS_{D_1}(r')+i\frac{\pi}{2}} \\
& -R_2(r)e^{iS_2(r)}R_{D_1}(r')e^{iS_{D_2}(r')},
\end{aligned} \tag{5.12}
$$

and vice versa by

$$
\begin{aligned}
& - R_1(r)e^{iS_1(r)}R_{D_2}(r')e^{iS_{D_2}(r')} \\
& + R_2(r)e^{iS_2(r)}R_{D_2}(r')e^{iS_{D_2}(r')+i\frac{\pi}{2}}
\end{aligned} \tag{5.13}
$$

if the idler photon travels toward detector D_2. In both cases the paths are those wiggly trajectories which photons take in the usual double slit experiment (up to

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[5] Again, we should talk about massive particles for the guidance equation to make sense. However, the results for photons are equal.

a phase shift). These trajectories produce the same interference patterns that we came across in Figure 2. Bear in mind that if added, they produce a clump pattern.

The eraser drastically changes the wavefunction, but at the same time the signal photon's past trajectory is not influenced by the change. Depending on when the idler photon enters the region between eraser beamsplitter and detectors D_1 or D_2, the signal photon jumps from moving on straight lines to following wavy trajectories typical for interference. This is striking, for the effects on the signal photon are mediated superluminally, in conflict with special relativity. On the other hand, this should not be surprising, for non-locality is one of the features of a hidden variable theory like de Broglie-Bohm's. However, relativistic considerations are beyond the scope of this analysis. In the experiment of [Kim et al., 1999] the moment in time when the idler photon encounters the eraser is always after the signal photon hits the detector. If one adjusted the delay and shorten the optical length of the idler photon such that it passes through the eraser during the signal photon travelling toward D_0, the trajectories would look like those in Figure 3c.

(a)   (b)   (c)
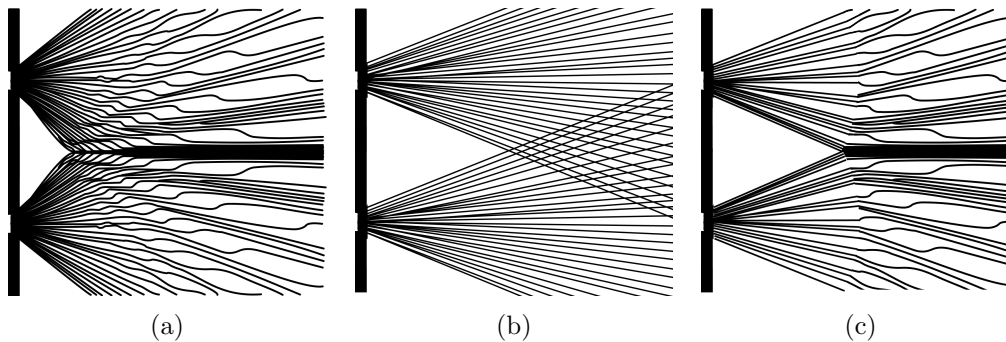
Figure 3: The signal photon follows different trajectories depending on when the idler photon encounters the quantum eraser. **(a)** The well-known wiggly trajectories that lead to an interference pattern in a usual double slit experiment. **(b)** In the case where the idler photon hits the quantum eraser after the signal photon arrives at the screen (which is how the experiment is set up in [Kim et al., 1999]), the signal photon moves on straight lines. **(c)** Before the idler photon has encountered the quantum eraser the signal photon follows straight lines. When the idler photon travels to detector D_1 or D_2, a jump in the guidance relation happens, leading to trajectories as in the interfering case.

Let us recap. There are two ways in which interference fringes can emerge at the detector D_0. When the idler photon arrives at the eraser during the flight of the signal photon, then the signal photon continues to move on wiggly lines giving rise to fringes. There is no change of the past whatsoever. When the idler photon arrives after the signal photon encounters D_0, the trajectories are straight lines (see Figure 3). In this case, selecting out interference patterns by conditioning on D_1 and D_2 does not change trajectories of the past. The reason we can extract interference fringes is that one subset of the trajectories of the signal photon is consistent with the idler photon being detected at D_1 (interference fringes), and another subset is consistent with a detection in D_2 (anti-fringes), and both add up to a clump pattern. This is the case in the experiments of Kim et al. and causes confusion if we do not consider the collapse of the wavefunction due to the signal photon, thus calling for the need of 'backwards in time influence' to restore the interference outcomes. It also trivially follows from my analysis that there is no need to invoke 'entanglement in time'. For I make no use of any non-standard features of standard quantum mechanics or de Broglie-Bohm theory. Pilot wave dynamics restores the

conventional view of the world as particles having a definite trajectory and past. In Wheeler's view the past comes into existence only after the measurement in the present, but my analysis gives an account that consistently attributes a past to the photon's trajectory.

6 Delayed choice and Bell-type experiments

The delayed choice quantum eraser in fact resembles a Bell type experiment. Both Alice (detector D_0) and Bob (detector D_1–D_4) receive one particle of an entangled photon pair. We can directly map the components of the delayed choice experiment to a Bell-type experiment. Figure 4 shows the experiment in the Bell setup.
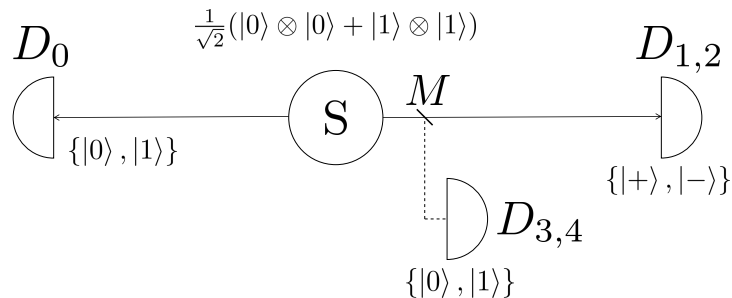

Figure 4: A Bell-type experiment resembles the delayed choice quantum eraser experiment.

The states of the photons are taken to be qubit states. The source S of an entangled pair of photons can be identified with the laser beam, the double slit, and the BBO crystal. M denotes a mirror that can be used to reflect the idler photon into $D_{3,4}$. Here, we can concatenate detectors D_3 and D_4 into one detector, where an outcome $|0\rangle$ would correspond to detection at D_3 and an outcome $|1\rangle$ to detection at D_4. We stipulate that the signal photon is sent towards the lens and the idler photon to the prism. If we are to perform a 'which-path experiment' we measure the idler photon in the computational basis $\{|0\rangle, |1\rangle\}$ at $D_{3,4}$. Detector D_0 measures the signal photon in the computational basis, which corresponds to an interference measurement if the state of the signal photon, for instance, is one of the states of the diagonal basis $\{|+\rangle, |-\rangle\}$. The measurement on the idler photon in the diagonal basis (at $D_{1,2}$) acts as the quantum eraser, i.e. a measurement of the idler photon in the diagonal basis is consistent with the signal photon being in a supersposition of $|0\rangle$ and $|1\rangle$. The results of the detectors D_0 conditioned on the outcome of $D_{1,2}$ show the familiar correlations when compared. The evolution of the system then is

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|\psi\rangle \mapsto CNOT(|\psi\rangle \otimes |0\rangle) = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$$

$$= \frac{1}{2}(|0\rangle \otimes (|+\rangle + |-\rangle) + |1\rangle \otimes (|+\rangle - |-\rangle))$$

$$= \frac{1}{2}((|0\rangle + |1\rangle) \otimes |+\rangle + (|0\rangle - |1\rangle) \otimes |-\rangle). \tag{6.1}$$

The first wavefunction in Equation 6.1 corresponds to the signal photon being send through the double slit. When it has past the BBO crystal the quantum state ends

up in an entangled one. This process is denoted with a CNOT–gate. After rewriting the second slot of the state in the diagonal basis, we recover a wavefunction that is qualitatively identical to Equation 4.7.

If the outcome of measuring the idler at $D_{1,2}$ is, say, $|+\rangle$, would we expect that the measurement has changed the past of the other particle to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$? Certainly not. In the collapse picture, only when the signal photon has not yet encountered detector D_0 would we say it evolved to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ given that the state of the idler photon yielded $|+\rangle$. Otherwise, the wavefunction of the signal photon will first collapse to $|0\rangle$ or $|1\rangle$ and, as a result, leave the state of the idler photon in a mixed state of $|+\rangle$ and $|-\rangle$.

7 Conclusion

We can consistently derive the probabilities for different measurement outcomes in the delayed choice quantum eraser experiment from standard quantum mechanics. The results of the delayed choice quantum eraser experiment are, at first glance, counter-intuitive and striking. When the idler photon is manipulated in a way that provides which-path information about the signal photon, detector D_0 does not show interference, even if conditioned on the idler photon's specific measurement results. On the other hand, if the idler photon is detected such that the measurement irrevocably erases which-path information about the signal photon, then too the interference patterns reappear. Those distributions are complementary in the sense that they add up to a clump pattern. Further, only conditioned on the detector outcomes of the idler photon can the patterns be extracted.

I have shown that both in the collapse interpretation as well as in the de Broglie-Bohm theory the experiment can be understood without invoking 'backwards in time influence'. According to the collapse interpretation the collapse of the wavefunction takes place at the instant of time at which a detector clicks. The collapse is a drastic change of the wavefuntion, but it has no retroactive effect on the past. The seemingly retroactive action disappears if the effects of measurement on the state of the signal photon is considered to also collapse the overall wavefunction.

In the de Broglie-Bohm theory the particle takes one definite trajectory and during its motion does not change its past. However, the idler photon may determine the pilot wavefunction of the signal photon depending on when the idler photon passes the quantum eraser. Most importantly, de Broglie-Bohm theory allows one to consistently construct the trajectories the photons have taken in the past.

Significantly, the delayed choice quantum eraser experiment resembles a Bell-type experiment and thus is not more mysterious than that. If an interpretation of quantum mechanics is adopted, there is no need to invoke a notion such as 'the present action determines the past'. Whether under any other interpretation retro-causality is similarly avoided would need further investigation.

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