Particle Creation and Annihilation: Two Bohmian Approaches

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Abstract

This paper reviews and discusses two extensions of Bohmian Mechanics to the phenomena of particle creation and annihilation typically observed in Quantum Field Theory (QFT): the so-called Bell-type Quantum Field Theory and the Dirac Sea representation. These theories have a secure metaphysical basis as they postulate a particle ontology while satisfying the requirements imposed by the Primitive Ontology approach to quantum physics. Furthermore, their methodological perspective intentionally provides a set of rules to immunize physical theories to the conceptual and technical problems plaguing the standard formulation of Quantum Mechanics and QFT. A metaphysical analysis of both theories will be given, emphasizing the relevant features of each proposal. Finally, it will be acknowledged that, despite the metaphysical virtues and niceties of these frameworks, ultimately they do not provide definitive answers to other cogent foundational issues in QFT. Thus, these theories (as well as the other Bohmian extensions to QFT) should be considered as partial solutions to the problems raised by the quantum theory of fields. This situation can be considered incentive for further research.

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1 Introduction

The Standard Model (SM) of particle physics is the most accurate and successful answer to the fundamental questions concerning the inherent nature of matter, which is explained in terms of elementary fermions (fermionic fields, particles are not localized point-size object but field's excitations) interacting through bosonic fields, and comprehends three of the four fundamental forces in Nature: the electromagnetic, the weak and the strong interactions; only gravitational effects are not taken into account. Furthermore, its predictions have been corroborated with an extreme degree of accuracy. Nonetheless, despite these remarkable triumphs, this theory inherits the conceptual and ontological problems affecting the standard formulation of Quantum Mechanics (QM), such as the measurement problem, the prominent role of observers and measurements in the definition of the theory, etc. In addition, the SM, or more generally Quantum Field Theory (being it the mathematical framework in which the SM is formulated) calls philosophers' attention with new problematical issues absent in non-relativistic quantum theory, e.g. the status and role of cut-offs and renormalization methods, the appearance of infinitely many unitarily inequivalent representations of the canonical commutations relations, a complete agreement between the axioms of QM and those of relativity, etc.

Many remarkable solutions have been advanced to solve these difficulties, but in this paper I will focus on the ontological problems of QFT from the Primitive Ontology perspective[1], showing how it is possible to provide consistent theoretical frameworks without metaphysical ambiguities also in the context of QFT. More precisely, I will review two extensions of Bohmian Mechanics (BM) to the realm of quantum fields, discussing pros and cons of each proposal.

The structure of the paper is the following: in Section 2 I briefly introduce BM and state the motivations to extend this theory to QFT, while in Sections 3 and 4 two Bohmian QFTs with a particle ontology are presented and discussed, the last section contains general remarks on these theories and the conclusions.

2 Review of Bohmian Mechanics

Bohmian mechanics is a quantum theory of particles moving in three-dimensional space following determinate trajectories. This theory is empirically equivalent to the standard quantum

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[1]It should be noted that many prominent philosophers of physics have remarkably different positions concerning the status and the problems of standard QM and QFT. For details see Halvorson and Clifton (2002), Fraser (2006) Kuhlmann (2010), Wallace (2006) and more recently Landsman (2017).

mechanics even though their physical content is remarkably different, since the former is based on a precise metaphysical hypothesis concerning the intrinsic corpuscular nature of matter. Hence, every physical fact is reduced to the motion of the Bohmian particles in physical space.

The equations of motion needed to complete the structure of the theory are the Schrödinger equation for the wave function ψ:

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi$$

and the guiding equation for the particles' motion:

$$\frac{dQ_k}{dt} = \frac{\hbar}{m_k}\text{Im}\frac{\psi^*\nabla_k\psi}{\psi^*\psi}(Q_1,\ldots,Q_N) = v_k^\psi(Q_1,\ldots,Q_N).$$

Since particles moving in physical space have a definite position, BM naturally recovers the notion of trajectory, which is absent in standard QM.

The statistical equivalence is achieved via *equivariance*: if we assume that at any arbitrary initial time t_0 the particle configuration is distributed according to $|\psi_{t_0}|^2$, then it will be so distributed for any later time t, preserving the Born's distribution.

In the literature concerning the foundations of QM, the motivations to consider BM as a serious alternative to the standard quantum theory are well known: not only the notorious measurement problem vanishes, but also its axioms do not contain mathematically ill-defined notions such as *measurement, observer* and *observables*, which are present instead in the standard formulation of QM.[2] This is in virtue of the clear ontology posed at the basis of the theory. However, this successful approach faces two challenges:

1. to explain the phenomena predicted by QFT;

2. to find a relativistic formulation.

These issues find a vivid debate in the pilot-wave community, but unfortunately results concerning the second point are still provisional; therefore, here the issue concerning a relativistic formulation of the BM is left aside.[3]

Physical phenomena typically observed in the context of QFT are the creation and annihilation of particles, yet it is hardly the case that we can properly speak about particles within this theory. As already said above, particles are defined as excitations of quantum fields, objects obtained after the procedure of canonical quantization of a classical field (i.e. imposing the canonical commutations relations to $\phi(x)$ and its conjugate momentum $\pi(x)$): the field's variables become *quantum operators* acting on some Hilbert space, promoting a classical field to an *operator-valued quantum field*:

$$\phi(x,t) \xrightarrow[quantization]{Canonical} \hat{\phi}(x,t).$$

This procedure indicates that the basic notion of QFT depends strictly on the identification between operators and observables and, as a consequence, the concept of quantum field depends on the notions of measurement and observable. Therefore, the problems arising from this dependence are the same ones facing in ordinary QM, and one may conclude that even the basic notions of QFT inherit the same ontological problems of standard QM.[4]

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[2]For an introduction to BM and to these foundational issues see Dürr et al. (2013b) and Bell (1987).

[3]This discussion is carried out in the non-relativistic context and only an *operational* compatibility with special relativity has been achieved so far.

[4]To this regard the reader may refer to Barrett (2014), where the measurement problem is discussed in the context of relativistic quantum mechanics and to Dürr et al. (2004b) for a clear analysis of the operator algebra in QM and BM and the consequences of the identification between operators and observables.

Then, in order to achieve a QFT immune from ill-defined concepts, one may follow the strategy known from non-relativistic QM and pursue a research on the ontology of QFT in the context of the Primitive Ontology (PO) approach, trying to extend BM to the realm of quantum fields.[5]

Furthermore, looking at the foundations of QFT one notes that the notion of physical state becomes secondary: the central objects are the scattering processes since the principal aim of QFT seems to be the calculation of the amplitudes of scattering events. With the extensions of BM to QFT, we cause a paradigm shift: the notion of physical state recovers its centrality. Bohmian QFTs postulate in the first place the primitive ontology of the theory, providing a description of quantum systems in terms of these primitive variables moving according to the equations of motion, giving to the theory the shape of a *mechanical* theory. This is a crucial point: from the scattering-oriented approach to QFT, BQFTs are inverting the current trend through the re-introduction of the familiar notion of evolution of physical states.

3 Bell-type Quantum Field Theory

Dürr et al. (2005) proposed a generalization of a stochastic extension of BM to QFT originally due to J. Bell (Bell (1986)), hence the name *Bell-type QFT* (BTQFT). According to this model a physical system is described by a pair (Q_t, Ψ_t), where the former correspond to a configuration of identical particles[6] and the latter is the state vector which belong to an appropriate Fock space (defined as a $N-$particle Hilbert space): symmetric or anti-symmetric depending on the particles considered, bosons in the former case, fermions in the latter.

The dynamics of the model introduces variations in the particles' number in order to describe the events of particles creation/annihilation. These events are represented by discontinuities in the particles' trajectories, introducing an intrinsic stochasticity in the model, since there is no physical process which causes them.

The picture a) (taken from Dürr et al. (2004a)) represents the emission of a photon at time t_1 (dashed line) from an electron and its absorption at time t_2 by a second electron. These two events correspond to a creation and annihilation event respectively. Between them the photon evolves according to a deterministic trajectory exactly as the electrons do. The picture b) represents a creation of an electron-positron pair at the end of a photon trajectory.

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[5]Considering a physical theory T, its primitive ontology is a *metaphysical* assumption defining the fundamental (or elementary, in the sense used in particle physics) objects postulated by T. These entities are the variables appearing in T's equations with a *direct* physical meaning. In other words, these variables are the formal counterparts appearing in T's mathematical structure referring to *real* objects precisely localized and moving in 3-dimensional physical space. Provided a particular theoretical framework T, the attribute *primitive* of the PO is justified since the PO cannot be further analyzed and/or defined in terms of more elementary notions, so that the primitive variables define a set of entities which are considered fundamental *in the context of T*; furthermore, every physical phenomenon included in T's domain is *ontologically reduced* and explained via the dynamical evolution in space of these fundamental objects according to the particular laws of motion governing the behaviour of the PO. For a careful introduction to the notion of PO the interested reader may refer to Allori et al. (2008), Allori et al. (2014), Bell (1975), Esfeld (2014) and the references therein.

[6]For a detailed account of identical particles in the Bohmian context the reader may refer to Esfeld et al. (2017), and Goldstein et al. (2005). In a nutshell, looking at the particle's trajectory it is possible to recognize to which species a particle belongs to: properties like mass and charge play only a dynamical role, figuring as parameters in the dynamical equations of the theory and specifying the motion of the particles. Contrary to particles' positions, these properties are not intrinsic. In any case the comprehension of the present discussion is not compromised even though the reader is *not* familiar with these metaphysical positions.

(a)　(b)
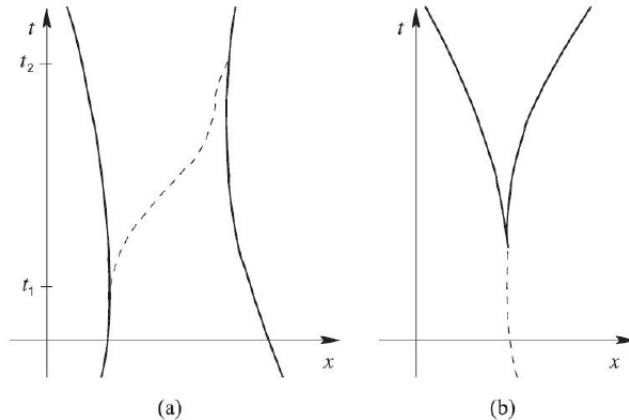
In these examples we can see that the number of the particles is *not* constant and these variations are considered real events in physical space.

Let us see more closely the equations of motion that characterize the model. The state vector evolves according to the Schrödinger equation:

$$i\hbar\frac{d\Psi_t}{dt} = H\Psi_t,$$

where H could be the Schrödinger or the Dirac Hamiltonian. In QFT, the Hamiltonian is a sum of terms: $H_{tot} = H_0 + H_I$, where the first term correspond to free processes and the second term describes the interactions. It is extremely useful to highlight that between the creation and annihilation events the particles follow deterministic trajectories and evolve according to the Bohmian law:

$$\frac{dQ_t}{dt} = v^{\Psi_t}(Q_t),$$

which depends on the free part of H. The interaction Hamiltonian H_I, instead, represents the discontinuities of the particles' trajectories which are represented by *jump rates* $\sigma = \sigma(q', q, t) = \sigma^{\Psi_t}(q', q)$. These jumps correspond to transitions from a given configuration of particles q to another one q' which differs in the particles' number.

Considering the picture a) the event of the photon emission corresponds to a jump of rate $\sigma(q', q, t)$, where the starting configuration is composed by two electrons and the arrival configuration counts also a photon. At time t_2 another jump occurs. These rates give the probability for a configuration q to jump in an interval of time $(t, t + dt)$ into another configuration q'. Between the jumps, the particles return to follow deterministic trajectories. Destinations and times of the jumps are the stochastic elements of the model, and the probabilities governing these jumps are described by Markov processes: they do not depend on the past histories of the particles' trajectories, but only on the present state of the configuration.[7]

Finally, in Dürr et al. (2005) it has been shown that BTQFT preserves equivariance: if the particle configuration $Q(t_0)$ is chosen randomly with distribution $|\Psi(t_0)|^2$, then at any later time $Q(t)$ is distributed with density $|\Psi(t)|^2$. Since both H_0 and H_I are by construction associated with equivariant Markov processes, equivariance is recovered in this extension of BM. Thus, the empirical equivalence has been achieved with any *regularized QFT*. The notions of Equivariance and "process additivity" are the keys features of BTQFT, since they are the guiding principles

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[7]It is important to stress that BTQFT is based on a mathematical framework which is widespread in applied sciences: the piecewise deterministic Markov process proposed by these authors to account for the dynamics of the particles is a standard method used to analyze the evolution of a given class of individuals and its evolution in time, which may well include variations in the number of its components.

in the construction of the dynamics: the processes associated with H_0 and H_I are defined in a manner which allows to yields *typical* histories for the primitive variables compatible with quantum statistics. Thus, it follows that BTQFT is the natural process associated with H in QFT: the sums of equivariant generators for the transition probabilities define a unique equivariant process associated with sums of Hamiltonians.

3.1 Discussion

In the first place, it is important to emphazise that BTQFT meets the requirements of the PO approach:

- A well-defined particle ontology is implemented: according to this Bohmian QFT the primitive variables are particles moving in space following deterministic trajectories. Here fermions and bosons have the same status: both these categories of particles contribute to form the ontology of the theory. The ontological novelty concerns the representation of the phenomena of particles' creation and annihilation.

- The explanation of every physical phenomenon is given in terms of the motion of the primitive variables in physical space. Then, the reductionist program of the PO approach is carried over intact.

- Bell-type QFT reproduces by construction the same statistics of the standard regularized QFT, achieving empirical adequacy and statistical equivalence via equivariance.

- Finally, the theory avoids any reference to physically ill-defined notions within its axioms (e.g. measurement, observer, observables, etc.).

Thus, BTQFT belongs properly to the family of the Quantum Theories Without Observer, where there are both deterministic and stochastic theories. Noteworthily, the stochastic part of the dynamics recalls the processes of the wave function collapses in GRW theories, another well known instance of QTWO, since in both cases these stochastic processes are *spontaneous* in a precise sense: they are not caused or induced by external factors as measurements, observers, forces, etc. More precisely, in the GRW theory the evolution of the wave function is given by stochastic jump processes in Hilbert space which are responsible for the random collapses of the wave function. Between these random processes it evolves deterministically according to the Schrödinger equation. As shown above, the BTQFT replicates this schema. Moreover, as in GRW, the BTQFT formalism provides the rates for these collapses.[8] Thus, there is a structural similarity between the processes associated with the inherent motion of the primitive ontologies of these theories. Therefore, that a Bohmian theory is stochastic does not pose any obstacle to the comprehension and the explanation of the physical phenomena falling within its domain, contrary to the widespread claims according to which it is essential for a Bohmian theory to be deterministic (see to this regard Nikolić (2010), see Oldofredi (2018) for a critical response).

Nevertheless, intuitively one may claim that a stochastic theory may be interpreted as an *incomplete* representation of physical phenomena since some process may have been omitted from the description. Hence, a stochastic theory may supply only *partial* information about the phenomena it should explain. Thus, a stochastic dynamics could be replaced by a continuous (complete) description.

Let us discuss this point. Usually the idea of complete knowledge of the evolution of physical systems is associated to the notion of *complete predictability*. However, it should be noted that determinism and predictability are two *logically distinct* notions: the former is connected

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[8]For details on the similar structures of BM and GWR see Allori et al. (2008) and Allori et al. (2014).

with the *dynamical evolution* of physical states given certain initial and boundary conditions at a certain arbitrary time t_0. A theory is deterministic iff given a physical system S at time t_0 in a certain state $s(t_0)$, and a dynamical law L, the successive states $(s(t_1), \ldots, s(t_n))$ with $(t_0 < t_1 < \cdots < t_n)$ are uniquely determinate by $s(t_0)$ and L. The latter relates with what a specific theory could predict, and looking at the current research concerned with non-linear systems or chaos theory, it is clear how this equivalence is immediately refuted. Often deterministic systems, e.g. physical systems that obey a deterministic dynamics, are extremely sensible to the dynamical perturbation of the initial conditions, so that after extremely short time intervals their behaviors become completely unpredictable. Consider now a physical system S at an arbitrary initial time t_0 in the state $s(t_0)$, and a theory T which governs its behavior via a stochastic law L_S. Now, if one takes into account a set of possible worlds $\{w_1, \ldots, w_n\}$ in which the same initial state $s(t_0)$ of the same system S is given, then it may be possible that the states of S evolve at times $t > t_0$ differently in each world: L_S does not fix a unique evolution for the future states of S. More precisely, given the very same initial state $s(t_0)$ for a system S in every considered world $\{w_1, \ldots, w_n\}$ evaluated at the same initial time t_0, and given a stochastic law L_S, there is no a unique successive fixed state $s(t_1)$ necessarily obtained given both $s(t_0)$ and L_S for every world w_i, implying that it is possible to obtain different evolutions for the very same state.

In this second case at every time t_n the evolution of a given state is independent of its precedent states at times $t_{n-1}, t_{n-2}, \ldots$, but this is also the case for deterministic systems since the complexity of realistic physical situations does not allow us to calculate the dynamical evolution of the system (i.e. every particle trajectory), even if the knowledge of every detail of the its configuration were given. Thus, the information provided by an inherent stochastic theory is as complete as that of a deterministic theory. Therefore, it is not correct to state that a stochastic theory provides incomplete information about the motion of physical systems.

In this regard, it is also interesting to consider an argument contained in Suppes (1993) entitled *The transcendental character of determinism.* Here the author explicitly addresses the logical independence of determinism and predictability. Suppes uses the word *transcendental* in a precise Kantian sense. Recalling Kant's Critique of Pure Reason there are problems - the existence of God, the freedom of the will and the immortality of the soul - cannot be empirically decided, and their solutions must transcend our experience. The same situation arises when someone is trying to figure out whether the intrinsic nature of the world is deterministic or stochastic. The choice, Suppes claims, must transcend experience as scientific practice reveals numerous physical phenomena that may receive both a deterministic and a stochastic treatment: in these cases there is no evidence from experience able to decide if a theory is ultimately deterministic or stochastic.

To argue in favor of this point Suppes introduces a theorem of Ornstein which claims that there are processes which can be equally well analyzed in terms of deterministic systems of classical mechanics or as indeterministic semi-Markov processes. The theorem is particularly strong since its claim is valid independently of the number of observations which are possibly made. This result, then, is important in our discussion, since this is exactly the case of the particles annihilation and creation: both the deterministic and stochastic Bohmian QFTs are *equally* able to explain these physical phenomena, therefore from the experimental evidence one cannot decide whether these phenomena are better described with a deterministic or a stochastic theory.[9] Moreover, since BTQFT is a QTWO fulfilling the requirements of the PO approach there are not metaphysical reasons to reject it in favor of a deterministic view.

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[9]For a critique of this point of view the reader may consider Werndl (2013).

4 The Dirac Sea picture

In this section we will be concerned with a *deterministic* version of a Bohmian QFT based on the ideas of the Dirac Sea (DS) following the usual explanatory schema of the PO approach.

According to this theory, there is a configuration of N *permanent* particles in time. This is the first relevant ontological difference between the DS approach and the BTQFT: in both cases the beables are the particles' positions, but as we have seen in the previous section, according to the latter particles are literally created and annihilated allowing for a variable number of particles in time. On the contrary, in the DS approach the particle number remain fixed and a different explanation for the phenomena of particle creation and annihilation is provided.

In what follows our ontological commitment is only about fermions since, as pointed out by J. Bell, it is the minimal ontological commitment able to explain the measurement outcomes and more generally all the empirical data available. Bosons do not possess the beable status, and this is the second ontological difference between the DS picture and the BTQFT.

4.1 The definition of the Dirac sea model

In order to define the Dirac sea model a set of assumptions is needed:

1. For simplicity, we restrict our attention only to the electron sector of the SM;

2. Only electrodynamics interactions are considered;

3. The interaction with the other particles' sectors of the SM are modeled by a time-dependent external interaction;

4. the Universe is assumed to have a finite volume;

5. The momenta of the electrons are restricted to be lower than some ultraviolet momentum cut-off Λ.

In the first place, it must be specified that even if the set of assumptions contains several simplifications it still provides sufficient structure to describe the phenomena of electron-positron pair creation. Secondly, although we have assumed the total number of particles is M, we consider only a fraction of it since we are interested in the electron sector of the SM. Thus, we consider only a number $N < M$ of particles, where $N =$ electron number. Furthermore, if we would consider only the electrodynamic interaction among electrons, the consequence would be that they would repel each other, implying that the spatial extensions among these fermions will become larger and larger giving rise to an unphysical behavior. Therefore, to avoid this situation, we must consider an external potential which models the interaction between the rest of the particles and the set of the electrons: the motion of the other particles then imposes constraints to the spatial extensions among electrons. In this way their behavior is in agreement with experience. Here, another assumption has to be made:

> this 'external' interaction is reasonably well behaved in the following sense: neither will it dampen the motion of the electrons to such an extent that all electron motion comes to a rest, nor will it drive the electron velocities arbitrarily close to the speed of light. Thus, there is no infinite energy transfer. The idea behind this assumption is that motion should be somewhat conserved among all fermion sectors. Neither does motion arise from nothing nor does it cease to exist, it only varies over the individual particles. (Deckert et al. (2016), p. 5)

The two last conditions have to be introduced in order to obtain a mathematically well-behaved model.

To cast the DS model in Bohmian terms implies the specification of two laws, one for the wave function, one for the particle configuration.

The wave function, which is an anti-symmetric, square-integrable, N-particle spinor-valued function in configuration space in this context, evolves according to

$$i\hbar\partial_t\Psi_t(x_1,\ldots,x_N) = H^N\Psi_t(x_1,\ldots,x_n)$$

where in this case the Hamiltonian H^N has the particular form:

$$H^N = \sum_{k=1}^{N}\Big(H_k^0(x_k) + V_k(t,x_k) + H_k^I(x_k)\Big). \tag{1}$$

The Hamiltonian appearing in the above equation is constituted by the following terms:

- The free Hamiltonian $H_k^0(x) = 1^{\otimes(k-1)}\otimes H^0(x)\otimes 1^{\otimes(N-k)}$, where the $H^0(x) = -ic\alpha\cdot\nabla_x + \beta mc^2$. Here α,β are the 4×4-matrices introduced in the Dirac equation.

- The effective interaction of all the particles on the kth electron is given by the time-dependent potential $V_k(t,x) = 1^{\otimes(k-1)}\otimes V(t,x)\otimes 1^{\otimes(N-k)}$ for some external potential $V(t,x)$.

- The last summand corresponds to the interaction Hamiltonian $H_k^I = \frac{1}{2}\sum_{j\neq k}U(x-x_j)$. The interaction among electrons is modeled by the Coulomb potential $U(x) = \frac{e^2}{4\pi\epsilon_0}|x|^{-1}$: ϵ_0 is the electric constant and e represents the charge of the electron.

The particles follow deterministic trajectories according to a guiding equation which depends on the wave function:

$$v_t(X) = c\Big(\frac{j_t^{(k)}(X)}{\rho_t(X)}\Big)_{k=1,\ldots,N}$$

where

- X is the actual configuration of N electrons which have positions in physical space;

- $\rho(X) = \Psi_t(X)^*\Psi_t(X)$ is the probability density generated by Ψ_t;

- $j_t^{(k)}(X) = \Psi_t(X)^* 1^{\otimes(k-1)}\otimes\alpha\otimes 1^{\otimes(N-k)}\Psi_t(X)$ is the quantum current generated by Ψ_t;

- c represents the speed of light.

The model so defined is able to reproduce the statistics of the standard regularized QFT. The DS picture is empirically adequate since it obtains the statistical equivalence with QFT via equivariance: the form of the guiding equation ensures that if the particles' distribution at an arbitrary initial time Q_0 is randomly distributed in perfect agreement with the Born's rule $|\Psi_0(X)|^2 d^{3N}x$, then this distribution will hold for any future time t.

4.2 Discussion

Having introduced the basic structure of the Dirac sea model (see Colin and Struyve (2007) and Deckert et al. (2016) for a full characterization of the Dirac sea approach), three issues deserve some discussion:

- The definition of the vacuum state;

- The electron-positron pair creation;

- The formalism of the creation and annihilation of the particles.

Looking at the equation of motion for the wave function, the complexity of the system increases exponentially with the increasing of the particle number and depends on the potential U which is responsible for the entanglement of all tensor components of the wave function. To avoid the complexity problem one has to find approximations of the generally very complicated dynamics of the particles. Here the essential idea is to describe this complicated motion of N particles in terms of deviations from a state of equilibrium which will be a suitable vacuum state. Basically, one assumes that the external influence is zero, then it means that $V(t, x_k) = 0$, modeling the motion of the electron only via the Fermi and Coulomb repulsion. But, following this approach, which are the equilibrium states? These states will be the solutions of a simplified equation of motion for the wave function in which the interaction term assumes a constant value. The first step is to consider the term that creates the problem: the Coulomb pair interactions in H_k^I. In order to define these equilibrium states, then, we must consider an approximation in which this term effectively vanishes. Thus, on the level of the wave functions, the electrons do not "take notice" of each other's presence. Effectively they behave as if they were in the vacuum.[10]

Here the vacuum is full of particles moving in a homogeneous manner, without taking notice of each other's motion. The vacuum state in the DS picture is very peculiar, since it is a sum of positive and negative energy states particles which naturally split the total Hilbert space into two subspaces $\mathcal{H}_-$ and $\mathcal{H}_+$, representing positive and negative energy particles respectively, or, following the current jargon, representing particles and antiparticles. It is important to note that according to this view the interpretation of the negative energy states is not problematic, since we take into account a specific ontology for which the only essential property of the particles is the position they have in a given configuration.

According to this view the particles do not have any intrinsic property (exactly as in the case of BTQFT): these are only dynamical parameters appearing in the laws of motion. They play an essential role in determining how the particles move. For instance, the trajectory of the particle itself distinguishes it as either a proton or an electron. Therefore, energy is not an intrinsic property of the particles, this is just a parameter that is useful to disambiguate the category to which a certain particle belong to.[11] In our simple case, the negative energies are a tool to individuate the motion of the *positrons*. Thus, according to this view, the notion of anti-matter in general is only a tool useful to describe the motion of certain species of particles.

Finally, it must be said that all the negative energy states in the vacuum are occupied, and by the Pauli exclusion principle positive energy particles do not fall into lower and lower energy states.

In order to consider the excitations of the vacuum state we have to put the external interaction $V(t, x) \neq 0$, including the external influences of the other particles on the electrons. A single excitation is encoded by a two-particle wave function $\eta_t(x, y)$. Its x tensor component, which we shall refer to as *electron component*, tracks the evolution of the initial excitation χ; its y tensor component, which we shall refer to as *hole component*, tracks the evolution of the corresponding state in the vacuum state.

According to the ontology we have introduced, the negative energies become just an instrument to describe the motion of the positrons, and, therefore, they do not pose in this framework

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[10]For the technical details on the individuation of the vacuum state see Deckert et al. (2016).

[11]For a systematic introduction to these ideas see Esfeld and Deckert (2017).

any interpretational problem. However, we have also claimed that this model does not allow for a variable number of particles, but the phenomenology of the standard model of particle physics suggests that the particle number is not constant. The question now is to understand the explanation of particle creation and annihilation provided by the DS approach.

The first thing to underline is the fact that so far the presentation of this model has been done considering the wave function dynamics defined on a $N-$particle Hilbert space $\mathcal{H}^N$. According to this formalism there are no particle creation and annihilation operators. However, one may recast this model by introducing the Fock space formalism. As already said, $\mathcal{F}$ is a space which allows for the treatment of a variable number of particles; in our case this space keeps track of the wave function excitations with respect to the vacuum state. One can naturally define a Fock space via the introduction of the creation and annihilation operators. Since there is an isomorphism between the (N-particle sector of the) Fock space and the $N-$particle Hilbert space representations, one may recast the N-particle dynamics generated by the Hamiltonian (1) in terms of the creation and annihilation operator formalism, obtaining the canonical second-quantized Hamiltonian one encounters in QED (when neglecting radiation).

One of the main methodological lessons of the PO approach is that we should keep as distinct as possible the mathematical, physical and philosophical aspects of a given physical theory. We should resist the temptation to interpret literally the mathematical structures of physical theories and, as a consequence, one should not infer ontological conclusions directly from the formalism. The main conclusion to draw, thus, is that the terms "creation" and "annihilation" refer to a specific mathematical structure, namely the Fock space representation of a certain dynamics. It is not necessary to claim that since the Fock space formalism implements a mathematical structure which is able to treat a variable number of particles, then there must be real, physical variations in the number of these particles in space. More precisely, we might re-interpret the meaning of the Fock space formalism by saying that it is a *much more effective description of the vacuum state's excitations*. What happens at the level of physical space is only that the particles just arrange spatially in a certain way that can be described by either a Fock space formalism, which provides an efficient description of the variation of the vacuum excitations, or by the N-particles Hilbert space where the dynamics of every single individual object is specified.

5 General Remarks and Conclusion

These two Bohmian QFTs primarily show the concrete possibility to consistently apply the PO methodology to recover the QFT's phenomenology via the definition of ontologically clear alternative formulations. In this paper it has been recalled that stochasticity is not a problem for a well formulated Bohmian theory, as well as it is perfectly coherent to consider a theory with a fixed number of particles to explain and describe the phenomena predicted by QFT.

These theories are particularly interesting since they provide completely different explanations for the events of particle creation and annihilation. On the one hand, BTQFT adheres to the phenomenological evidence and consequently postulates an ontology where objects are randomly destroyed and created. On the other, the DS presents a more familiar metaphysical picture where the number of particles remains fixed: events of creation and destruction of particles are *formal* consequences of the adoption of a certain mathematical structure, i.e. the Fock space formalism, so that they are *not* considered physical events in space and time. Thus, if it is true that the DS does not dramatically depart from our metaphysical intuitions (remaining closer to standard BM with respect to BTQFT), it should be also underlined that this model necessarily yields a more complicated explanation of the observed phenomena, contrary to the case of BTQFT. The choice of which theory is superior to the other is an exercise left to the

reader.

In sum, one has to acknowledge that these theories avoid the conceptual difficulties of QM and QFT by virtue of their unambiguous ontology, and this fact is a encouraging indication that the PO methodology may be a successful approach even in the context of QFT, and may be considered *a guide* in search for a clear ontology in this domain.

Before concluding, I will add a brief discussion of some problematical aspects concerning in general the class of Bohmian QFT.[12] In the first place, it should be noted that the postulation of such primitive ontologies implies remarkable consequences since this class contains non-relativistic theories: equivariance guarantees an operational (statistical) equivalence with the standard approach to quantum field theory, meaning that it ensures only the empirical adequacy of these theories. To be non-relativistic surely allows us to avoid the conclusions of several no-go theorems that show the impossibility of an ontology of localized particles in relativistic QFT (see Malament (1996) and Halvorson and Clifton (2002) and references therein). One should, however, expect and require either a fully relativistic Bohmian QFT or a result showing the impossibility to combine properly the axioms of an ontologically well-defined quantum theory with the axioms of special (and general) relativity (the theorem contained in Gisin (2011) may be a good example). Such a result would be instructive for the foundations of quantum physics, since it could be helpful to understand the possibility or the impossibility to make a certain class of quantum theories genuinely relativistic. Secondly, it may suggest a possible new track for future research, pushing philosophers and physicists to find alternative strategies to combine the first principles of QM and relativity (to this regard the reader may refer to Dürr et al. (2013a)).

Another important issue which is not usually taken into account in discussions on Bohmian QFTs is the treatment of Haag's theorem and the infinitely many unitarily inequivalent representations of the canonical commutation relation, as pointed out correctly by Lam (2015). It is well-known that Bohmian QFTs implement cut-offs in order to have well-defined Hamiltonians; the supporters of the Bohmian approach seem to assume a pragmatic attitude concerning the role and status of cut-offs following exactly what has been argued in Wallace (2006), for which regularization methods do not modify the physical content of a QFT. However, to endorse Wallace's view implies accepting also the second part of his thesis, namely that QFT is not a fundamental theory, but rather it should be considered an *effective* theory emerging from a deeper and yet unknown theory (to this regard see Wallace (2006) and Egg et al. (2017) for a discussion).

It is an opinion of the present author that all this implies the possibility to construct effective quantum field theories with a clear ontology, but also that nowadays there is no possibility to consider any Bohmian QFT as a fundamental theory. Unfortunately, also the algebraic approach to QFT also cannot be considered a valid alternative to the standard formulation of QFT since models describing realistic interactions in space-time, as elegantly argued in Wallace (2011), do not currently exist.

In conclusion, the Bohmian QFTs discussed in this paper reach a remarkable achievement, namely they show that it is possible to formulate QFTs without ontological conundrums. Nonetheless, they should be evaluated as a partial solution to the problems affecting QFT. More generally, it seems fair to claim that currently there are no real alternatives to QFT, but only fragmentary attempts to replace it. This situation, nonetheless, should be interpreted as incentive for further research into the foundations of the quantum theory of fields.

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[12]An introductory overview to the several Bohmian QFTs is contained in Struyve (2010).

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