Probing spacetime with a holographic relation between spacetime and entanglement

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Abstract

This paper introduces and examines the prospects of the recent research in a holographic relation between entanglement and spacetime pioneered by Mark van Raamsdonk and collaborators. Their thesis is that entanglement in a holographic quantum state is crucial for connectivity in its spacetime dual. Utilizing this relation, the paper develops a thought experiment that promises to probe the nature of spacetime by monitoring the behavior of a spacetime when all entanglement is removed between local degrees of freedom in its dual quantum state. The thought experiment suggests a picture of spacetime as consisting of robust nodes that are connected by non-robust bulk spacetime that is sensitive to changes in entanglement in the dual quantum state. However, rather than pursuing the thought experiment in further detail, the credibility of the relation between spacetime and entanglement in this zero entanglement limit is questioned. The energy of a quantum system generally increases when all entanglement is removed between subsystems, and so does the energy of its spacetime dual. If a system is subdivided into an infinite number of subsystems and all entanglement between them is removed, then the energy of the quantum system and the energy of its spacetime dual are at risk of diverging. While this is a *prima facie* worry for the thought experiment, it does not constitute a conclusive refutation.

1 Introduction

A promising recent player on the scene of quantum gravity research is the proposal, by Mark van Raamsdonk (2010; 2011) and collaborators, that spacetime may be regarded as a geometrical representation of entanglement. The proposal has its origin in the so-called AdS/CFT correspondence (Maldacena, 1999); a conjectured duality between theories with gravity and quantum theories without gravity. According to the proposal, the spacetime and spacetime dynamics that account for gravitation can be represented by the entanglement (and dynamics of entanglement) in conformal quantum field theories without gravity. Gravity is a geometrical representation of a system that has an alternative and empirically equivalent representation as entanglement in a quantum field theory.

An enticing prospect of this is that the dual representation in terms of entanglement could perhaps be used as a theoretical probe of the nature of spacetime. This paper seeks to explore the scopes and limits of such an approach. Section 2 introduces the AdS/CFT correspondence and van Raamsdonk's preferred example: the duality between the maximally extended AdS-Schwarzschild black hole and the thermofield double state. Section 3 will then present van Raamsdonk's argument that entanglement in the thermofield double state is closely related to connectivity in the dual spacetime. After generalizing to arbitrary holographic quantum states – states with a spacetime dual – in section 4, section 5 follows the argumentation through to the limit where all entanglement between all local degrees of freedom are removed. It is suggested that in this limit the dual spacetime consists of disconnected, robust spacetime nodes and a non-robust bulk spacetime; thereby indicating a way to use the relation between spacetime and entanglement as a theoretical probe of the nature of spacetime. Rather than developing this thought experiment further, section 6 will undertake a qualitative investigation of the behavior of energy in the limit where all entanglement is removed. It will be argued that the energy density as well as the total energy risks diverging in this limit. The final section discusses how this affects the validity of the thought experiment developed in section 5 and concludes that while the potentially diverging energy raises question about the validity of this thought experiment, it does not, in its current form, constitute a conclusive refutation.

2 AdS/CFT correspondence

Ever since its discovery by Juan Maldecena in 1999, the AdS/CFT correspondence has intrigued researchers of quantum gravity. The correspondence has its origin in string theory and conjectures that certain types of closed string theories in asymptotically Anti de-Sitter (AdS) spacetime are dual to certain non-gravitational conformal field theories (CFT) defined on a fixed spacetime background identical to the asymptotic boundary of the dual AdS spacetime.[1] In particular, the two sides of the duality are conjectured

[1] See Butterfield et al. (2016) for an introduction to AdS/CFT correspondence and some conceptual aspects thereof.

to be empirically equivalent, but different ways to represent the same system.[2] Thus, observables on the CFT side correspond to observables on the AdS side, though the interpretation of the observables on each side of the duality is often different. A notable exception is the energy of a CFT state that corresponds to the energy of its dual spacetime; a result that will be put to use in section 6.

While originally conceived in the context of string theory, the AdS/CFT correspondence reduces in a certain limit[3] to a duality between particular conformal field theories in d dimensions and (semi)classical asymptotically AdS spacetimes[4] in $(d+1)$ dimensions whose gravitational dynamics are described by Einstein's field equations. One example of such a duality between a CFT state and a clasical spacetime is the duality between the maximally extended AdS-Schwarzschild black hole and the thermofield double state (Maldacena, 2003). The maximally extended AdS-Schwarzschild black hole is a particular solution to Einstein's field equations in the presence of a negative cosmological constant consisting of two identical regions, such that from either region the spacetime looks like a AdS-Schwarzschild black hole.

node[scale=0.7,midway, above, inner sep=2mm] $r = 0$;

As seen in figure 1, a light signal from the region denoted I can intersect a signal from region II. We will therefore describe the spacetime as a connected spacetime. However, the light signals can only intersect inside the black hole thereby precluding any causal connection between the two exterior regions I and II. The two regions are causally disconnected. Seen from region I, region II lies behind the black hole horizon ($r = r_h$ in figure 1) and *vice versa*. As such, this interior can be conceived of as a wormhole (a two sided black hole) if it is emphasized that there is no way out of the wormhole once in; it is still a black hole. Adopting the usual terminology, we will subsequently refer to the maximally extended AdS-Schwarzschild black hole as the eternal black hole.[5]

The two identical regions I and II have identical asymptotic boundaries – denoted A and B in figure 1 – with spacetime $\mathbb{R} \otimes S^{d-1}$. These are also causally disconnected. In accord with the general result of the AdS/CFT correspondence, the CFT dual to the eternal black hole must be defined on a spacetime identical to this asymptotic boundary, $A \cup B$.[6] Thus, the full quantum system is comprised of two identical quantum subsystems, Q_A and Q_B.

[2]See de Haro (2015) for a more detailed account of the notion of duality in the context of the AdS/CFT correspondence.

[3]Closed string theory may be approximated by Einstein gravity in the limit where both the string coupling and string length is small such that the super string theory may be approximated by supergravity. See Callan et al. (1987) and Huggett and Vistarini (2015).

[4]More precisely, the spacetime dual is Einstein gravity on $\mathrm{AdS}_{d+1} \times \mathcal{Y}$ where $\mathcal{Y}$ is some compact space that ensures a consistent embedding into string theory. However, the compact space and the embedding into string theory will not play any role in the following.

[5]The name 'eternal black hole' has its origin in the curious property of the maximally extended AdS-Schwarzschild black hole that it can be in an equilibrium with its own Hawking radiation. This is possible because the maximally extended AdS-Schwarzschild black hole is asymptotically global AdS and light rays may travel to the boundary of global AdS and back again in finite time.

[6]Note that for the eternal black hole this is not a contiguous spacetime, but instead a spacetime that consists of two disjoint copies of the same spacetime.

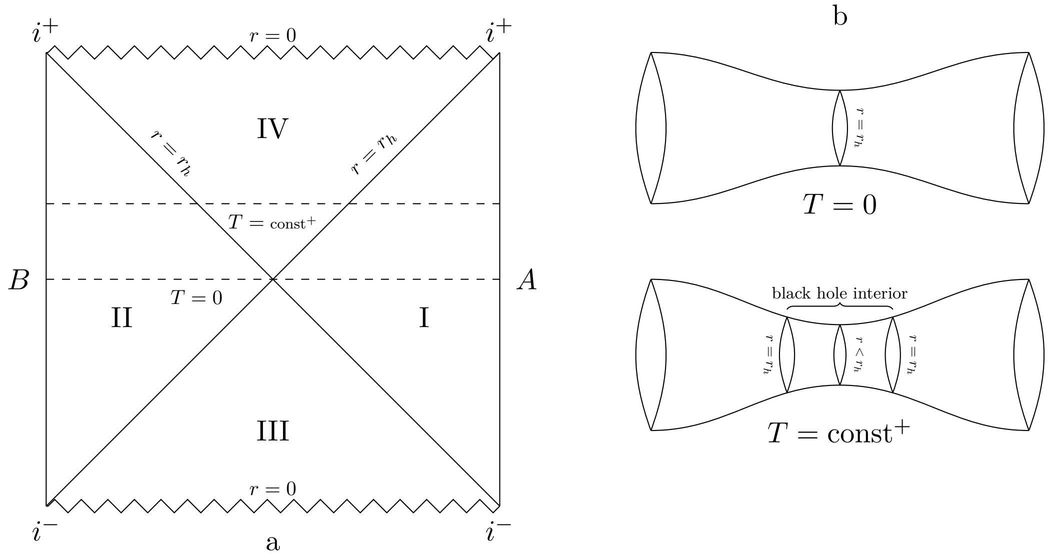

Figure 1: a) Penrose diagram of the eternal black hole with an implicit $d-1$ dimensional sphere over each point that scales as r^2. Regions I and II cover regions that lie outside ($r > r_h$) the black hole covered by region IV. Region III is a white hole. b) Depiction of two spacelike slices of the eternal black hole ($T = 0$ and T equals a positive constant) with one angular coordinate restored.

Since Q_A and Q_B are causally disconnected, the Hilbert space of states of the full quantum system, $\mathcal{H}$, can be decomposed as product of two Hilbert spaces, $\mathcal{H}_A$ and $\mathcal{H}_B$, that are associated with the two subsystems, Q_A and Q_B. $\mathcal{H}_A$ and $\mathcal{H}_B$ may be spanned by an orthogonal basis consisting of (again identical) energy eigenstates $\{|E_i^A\rangle\}$ and $\{|E_i^B\rangle\}$. The thermofield double state, $|\Psi\rangle \in \mathcal{H}$, that is the CFT state dual to the eternal black hole, can then be expressed as

$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_i e^{-\beta E_i/2} \left|E_i^A\right\rangle \otimes \left|E_i^B\right\rangle \tag{1}$$

where β is the inverse temperature of one of the subsystems[7] and $Z = \sum_i e^{-\beta E_i}$.[8]

[7] Since they are identical, the inverse temperature is the same in both subsystems.

[8] A number of authors have recently questioned this duality between the thermofield double state and the eternal black hole (Avery and Chowdhury, 2013; Marolf and Wall, 2013; Mathur, 2014). When considering van Raamsdonk's conclusion drawn from this duality, it is therefore worth keeping in mind that the duality remains disputed. However, even if this duality turns out to be false, it does not disprove the proposed relation between spacetime and entanglement, though it does to some degree compromise the presented qualitative argument and therefore some of the intuitive appeal of the proposal.

3 Entanglement and the Eternal Black Holes

The thermofield double state is a particular state, $|\Psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$, in which we can find the quantum system comprised of the subsystems Q_A and Q_B. Interestingly, the expression of the thermofield double state, eq. (1), in terms of energy eigenstates explicitly unveils the local degrees of freedom of the two subsystems to be entangled through the weighted sum over states $\sum_i e^{-\beta E_i/2} \left|E_i^A\right\rangle \otimes \left|E_i^B\right\rangle$.

For comparison, consider a special state in the Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$:

$$|\Phi\rangle = \left|E_0^A\right\rangle \otimes \left|E_0^B\right\rangle \tag{2}$$

where $|E_0^A\rangle$ is the ground (vacuum) state of $\mathcal{H}_A$ and similarly for $|E_0^B\rangle$. Manifestly, $|\Phi\rangle$ is the product of two pure states and does not contain entanglement between the degrees of freedom in Q_A and Q_B.[9] In this state, therefore, the two systems are completely uncorrelated. Notably, the thermofield double state is identical to this state in the limit where the inverse temperature, β, of the subsystems goes to infinity, i.e. the limit where the temperature goes to zero[10]

$$|\Psi\rangle \overset{\beta \to \inf}{=} |\Phi\rangle \,. \tag{3}$$

Since the local degrees of freedom in Q_A and Q_B are highly entangled in $|\Psi\rangle$ and since they are not entangled at all in $|\Phi\rangle$, it is evident how the inverse temperature, β, controls the entanglement between these degrees of freedom in the two subsystems.

What van Raamsdonk (2010) suggests is that a first indication of the relation between entanglement and spacetime may be obtained from the differences between the spacetime dual of a pure state like Φ and an entangled state like Ψ. The state Φ where there is no entanglement between Q_A and Q_B consists of the tensor product of two identical pure states; more specifically a product of the identical vacuum states of the subsystems. If we suppose that a pure state is dual to some spacetime, then the product of two pure states is dual to the product of two such spacetimes. Thus, the full spacetime consists of the product of two completely uncorrelated spacetimes. It seems, therefore, reasonable to suppose that these two spacetimes are disconnected, i.e. no light signal travelling from one spacetime can intersect a light signal travelling from the other. As already argued, the spacetime dual of the thermofield double state, where Q_A and Q_B are entangled, is a connected spacetime. For the thermofield double state, entanglement between Q_A and Q_B therefore seems like a necessary condition for connectivity between A and B in the dual spacetime.

That bulk spacetime connectivity is related to entanglement in the dual thermofield double state sits well with the Bekenstein-Hawking formula that relates the entropy of a black hole, S_{BH}, with its horizon area, Area_{BH} (Bekenstein, 1973)

$$S_{BH} = \frac{\text{Area}_{BH}}{4G}. \tag{4}$$

[9] This is just one state in a class of product states in $\mathcal{H}_A \otimes \mathcal{H}_B$.

[10] This follows, since the system only occupies the ground state when the temperature goes to zero; as can be seen from the reduced density of state $\rho_A = \frac{1}{Z(\beta)} \sum_i e^{-\beta E_i} \left|E_i^A\right\rangle \left\langle E_i^A\right|$, if one notes that the energy eigenvalue is non-zero for all energy eigenstates other than the vacuum state.

The duality between the eternal black hole and the thermofield double state entails that the black hole entropy – regardless of its origin on the AdS side – is equal to the entanglement entropy of one of the subsystems in the thermofield double state (Emparan, 2006). Employing this relation, it is possible to monitor the horizon area as seen from either side of the eternal black hole, when entanglement between Q_A and Q_B is removed in the dual thermofield double state. As entanglement is removed (i.e. when β is increased), the entanglement entropy decreases and so does the horizon area following eq. (4). Note that when the horizon area decreases, the black hole (and the white hole) gets smaller in the sense that the spacetime singularity comes closer to the horizon.

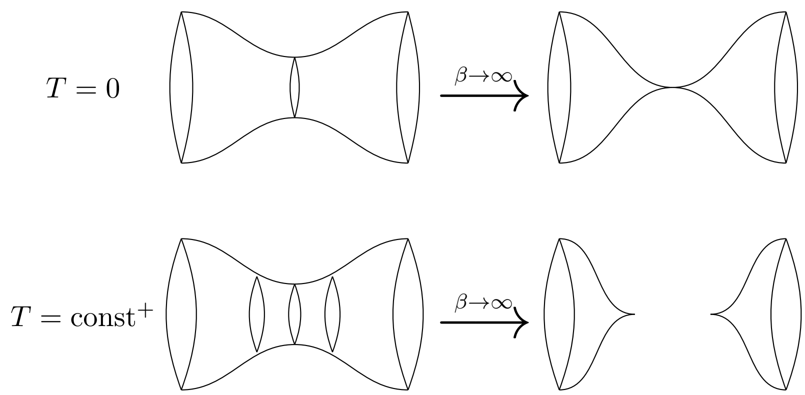

Figure 2: Depiction of the behavior of the spatial slices $T = 0$ and T equals a constant of the eternal black hole when $\beta \to \infty$ in the dual quantum state $|\Psi\rangle$.

In the limit where all entanglement is removed between Q_A and Q_B[11] (i.e. where $\beta \to \infty$), the entanglement entropy goes to zero and so does the horizon area.[12] This limit is depicted in figure 2 for the spatial surfaces $T = 0$ and T equal to a positive constant. As seen, the spacetime regions I and II share no boundary for $T \neq 0$ when $\beta \to \infty$, and for $T = 0$ they share a single point; spacetime pinches. More precisely, the singularity comes closer and closer to the horizon as entanglement is removed in the dual quantum state such that in the zero entanglement limit the spacetime singularity coincides with the horizon. Thus, the entire horizon becomes singular as depicted in figure 3 and in particular the spacetime becomes singular in the single point that connects the two regions I and II.

This does not exactly reproduce the initial assumption that the product of two pure states – such as $|E_0\rangle \otimes |E_0\rangle$ – are dual to two completely disconnected spacetimes.

[11]Obviously, there is no single zero entanglement limit. Rather, any state that is the product of pure states associated with distinct local degrees of freedom is a zero entanglement limit. However, it is a specific class of states in the Hilbert space for the full system.

[12]Some subtleties arise here since there is a critical temperature in AdS spacetime such that black holes must have at least this temperature to exist. While this may compromise aspects of this example with the eternal black hole, it does not affect the generalization to general holographic quantum states in section 4.

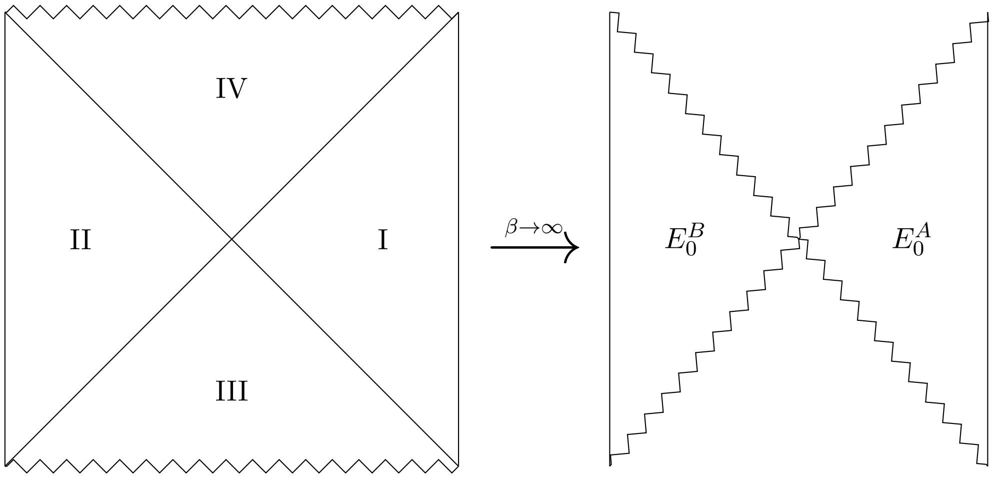

Figure 3: Depiction of the eternal black hole in the limit where all entanglement is removed between Q_A and Q_B in the dual quantum state Ψ. In this limit, the horizon and the spacetime singularity coincides.

A singularity remains, whose geometrical interpretation is uncertain. I will, however, argue that one can give several reasons to disregard this singularity when considering connectivity: 1) It may disappear if one takes into account that black holes in AdS has a minimum non-zero temperature; 2) The mass of a black hole without charge or rotation is proportional to the square-root of the horizon area. The mass of the black hole therefore goes to zero when all entanglement is removed between Q_A and Q_B suggesting that the black hole disappears; 3) Even if the singularity remains, is seems reasonable to suppose that no light signal can cross it. This lends some support to van Raamsdonk who – not considering this complication – concludes: "In this example, *classical connectivity arises by entangling the degrees of freedom in the two components*" (Van Raamsdonk, 2010, 2325). Entanglement between Q_A and Q_B is a necessary condition for spacetime connectivity between regions A and B in the eternal black hole.

4 Beyond the Eternal Black Hole

While conceived in the context of the duality between the eternal black hole and the thermofield doublestate, the relation between entanglement and spacetime connectivity generalises to any quantum state with a classical spacetime dual.[13]

Consider a quantum state $|\Psi\rangle$ with a classical spacetime dual M_Ψ. As required by the AdS/CFT correspondence, $|\Psi\rangle$ is a state in the Hilbert space for a CFT defined on a spacetime identical to the asymptotic boundary of M_Ψ which we denote ∂M_Ψ. To

[13]For a more detailed account of this generalization see Van Raamsdonk (2011); Faulkner et al. (2014); Lashkari et al. (2014).

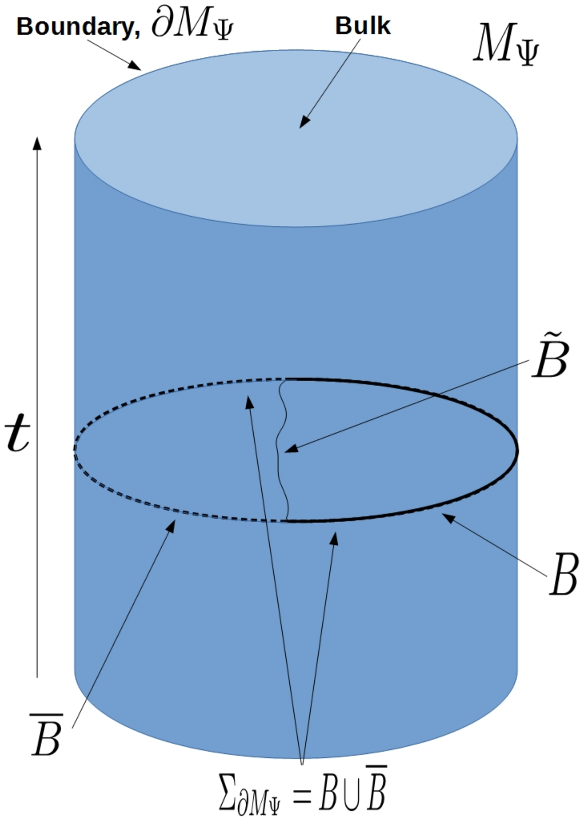

Figure 4: For the purpose of illustration, a spherical space with time, $S^d \times R$, is depicted here.

construct the Hilbert space, one must define on a spatial slice of ∂M_Ψ which will be denoted $\Sigma_{\partial M_\Psi}$. We then have $|\Psi\rangle \in \mathcal{H}_{\Sigma_{\partial M_\Psi}}$. Now, divide $\Sigma_{\partial M_\Psi}$ into two regions B and $\overline{B}$, such that $B \cup \overline{B} = \Sigma_{\partial M_\Psi}$ (see figure 4). Since a CFT is a local quantum field theory, there are specific degrees of freedom associated with specific spatial regions. We can therefore regard the full quantum system as composed of two subsystems, Q_B and $Q_{\overline{B}}$, associated with the two spatially separated regions B and $\overline{B}$. As a consequence, the Hilbert space of the full system can be decomposed as a tensor product of the Hilbert spaces of Q_B and $Q_{\overline{B}}$:[14]

$$\mathcal{H}_{\Sigma_{\partial M_\Psi}} = \mathcal{H}_B \otimes \mathcal{H}_{\overline{B}} \tag{5}$$

$|\Psi\rangle$ can therefore be expressed as a sum over products of states $\left|\psi_i^{\overline{B}}\right\rangle \in \mathcal{H}_{\overline{B}}$ and $\left|\psi_i^B\right\rangle \in$

[14] Some complications are involved in making such a decomposition in a gauge invariant way, but these will not be considered here.

$\mathcal{H}_B$:

$$|\Psi\rangle = \sum_{i,j} p_{i,j} \left|\psi_i^B\right\rangle \otimes \left|\psi_j^{\overline{B}}\right\rangle \tag{6}$$

This will generally not be a product state, i.e. a product of a state in $\mathcal{H}_B$ and one in $\mathcal{H}_{\overline{B}}$. Thus, the local degrees of freedom in Q_B and $Q_{\overline{B}}$ will generally be entangled.

Again, assume that a product state

$$|\Phi\rangle = \left(\sum_i c_i \left|\psi_i^B\right\rangle\right) \otimes \left(\sum_j d_j \left|\psi_j^{\overline{B}}\right\rangle\right) \tag{7}$$

is dual to two disconnected spacetimes. One then obtains the result that entanglement between Q_B and $Q_{\overline{B}}$ in $|\Psi\rangle$ is a necessary condition for the dual spacetime M_Ψ to be a connected spacetime. The duality between the thermofield double state and the eternal black hole is just a particular example of this.

Again, one may monitor this more closely but this time using the Ruy-Takayanagi formula (Ryu and Takayanagi, 2006) that closely resembles the Bekenstein-Hawking formula but applies to general quantum states with a classical spacetime dual. The Ruy-Takayanagi formula reads

$$S_B = \frac{\text{Area}(\tilde{B})}{4G} \tag{8}$$

where S_B is the entanglement entropy of subsystem Q_B, and $\text{Area}(\tilde{B})$ is the area of the smallest bulk surface, $\tilde{B}$, that divides region B from $\overline{B}$, i.e. B from the rest of $\Sigma_{\partial M_\Psi}$ (see figure 4). The black hole horizon in the eternal black hole is exactly such a surface, though this is not immediately obvious. From the Ruy-Takayanagi formula it follows that changing the entanglement between Q_B and $Q_{\overline{B}}$ changes the area of the smallest surface that divides the two corresponding regions in the spacetime dual.

When $|\Psi\rangle \rightarrow |\Phi\rangle$, the state of the full quantum system becomes a product of two pure states such that there is no entanglement between the local degrees of freedom in Q_B and $Q_{\overline{B}}$. Thus, in this limit the entanglement entropy, S_B, goes to zero and so does the area of $\tilde{B}$ according to the Ruy-Takayanagi formula. More explicitly stated, in this limit the bulk metric changes such that the minimal area dividing the two asymptotic regions B and $\overline{B}$ in the spacetime goes to zero; the spacetime dual of the quantum state pinches when $|\Psi\rangle \rightarrow |\Phi\rangle$. For the spatial surface $\Sigma_{\partial M_\Psi}$, figure 5 depicts the limit where all entanglement is removed between Q_B and $Q_{\overline{B}}$.

Again, the limit where all entanglement is removed between the quantum subsystems does not exactly reproduce the expectation that the tensor product of two pure states is dual to a disconnected spacetime. The two regions B and $\overline{B}$ remain connected by a single singular point that has no clear interpretation. Nevertheless, it evident how entanglement between Q_B and $Q_{\overline{B}}$ is related to the connectivity between B and $\overline{B}$ in the spacetime dual. Further support for this is found in an argument from the mutual

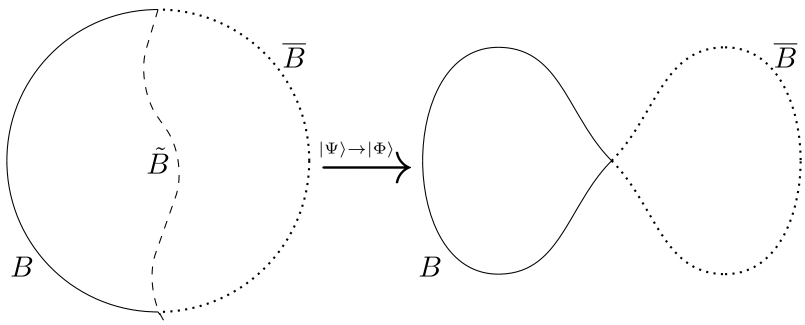

Figure 5: Depiction of the behavior of the spatial slice $\Sigma_{\partial M_\Psi}$ when all entanglement is removed between Q_B and $Q_{\overline{B}}$. Note that the quantum state is defined on a fixed spacetime identical to the asymptotic boundary of the spacetime dual. Thus, the change in $\tilde{B}$ is solely due to changes in the bulk metric despite the appearance to the contrary.

information[15] between a point in B and one in $\overline{B}$ which implies that the proper distance in the dual spacetime between any two such points goes to infinity when the entanglement between Q_B and $Q_{\overline{B}}$ goes to zero (Wolf et al., 2008). As summarised by van Raamsdonk, "the two regions of spacetime pull apart and pinch off from each other" (Van Raamsdonk, 2010, 2327). In other words, the conclusion from section 3 extends even to spacetimes with a contiguous boundary.

5 A Theoretical Probe of Spacetime

Only subdivisions of the full system into *two* subsystems has been studied in the literature so far. However, arguably nothing prevents the subdivision of the full system into more than two subsystems or equivalently the subdivision of subsystems into subsystems of the subsystems. Evidently, if the full system is a local quantum field theory such that it may be divided into two subsystems, one may as well divide it into three subsystems; there is nothing special about subdividing a quantum system in two as opposed to three or any other number. Thus, any subdivision of a local CFT is legitimate under the condition that the subsystems together comprise the full system. It is such further subdivision that will be investigated in the following.

As an example, consider the further subdivision of B into C and $\overline{C}$ and $\overline{B}$ into D and $\overline{D}$. The Hilbert space of the full system then decomposes into a tensor product of the Hilbert spaces of each of the four subsystems

$$\mathcal{H} = \mathcal{H}_C \otimes \mathcal{H}_{\overline{C}} \otimes \mathcal{H}_D \otimes \mathcal{H}_{\overline{D}}. \tag{9}$$

Generally, a state of the full system, $|\Psi\rangle \in \mathcal{H}$, is not a product of pure states of the

[15] Schematically, the mutual information, $I(A,B)$, can be defined as $I(A,B) = S(A)+S(B)-S(A\cup B)$ where $S(M)$ is the entanglement entropy between a region M and the rest of the system (Van Raamsdonk, 2010).

subsystems. Rather, the local degrees of freedom in these subsystems are entangled. When all entanglement is removed between Q_B and $Q_{\overline{B}}$, one finds

$$|\Psi\rangle \to |\Phi\rangle = \left|\psi^B\right\rangle \otimes \left|\psi^{\overline{B}}\right\rangle \tag{10}$$

where $\left|\psi^B\right\rangle \in \mathcal{H}_C \otimes \mathcal{H}_{\overline{C}}$ and $\left|\psi^{\overline{B}}\right\rangle \in \mathcal{H}_D \otimes \mathcal{H}_{\overline{D}}$. In this limit, the subsystems Q_C and $Q_{\overline{C}}$ remain entangled and so do Q_D and $Q_{\overline{D}}$. Following the same procedure, entanglement may again be removed between these subsystems. This takes the full system into a state

$$|\Phi\rangle \to |\Omega\rangle = \left|\psi^C\right\rangle \otimes \left|\psi^{\overline{C}}\right\rangle \otimes \left|\psi^D\right\rangle \otimes \left|\psi^{\overline{D}}\right\rangle \tag{11}$$

where $\left|\psi^C\right\rangle \in \mathcal{H}_C$ etc. Thus, $|\Omega\rangle$ is a product of pure states of the four subsystems.

Again, using the Ruy-Takayanagi formula one can monitor what happens to the dual spacetime in the limit $|\Psi\rangle \to |\Omega\rangle$. When all entanglement is removed between the subsystems Q_C, $Q_{\overline{C}}$, Q_D and $Q_{\overline{D}}$, it follows that the area of the smallest bulk surfaces dividing boundary regions C, $\overline{C}$, D and $\overline{D}$ goes to zero. For a spatial slice of the dual spacetime, this was depicted in figure 5 for the limit $|\Psi\rangle \to |\Phi\rangle$. Having thus removed all entanglement between Q_B and $Q_{\overline{B}}$, only the bulk surface dividing C from $\overline{C}$, $\tilde{C}$, and the surface dividing D from $\overline{D}$, $\tilde{D}$, is non-zero. Taking the further limit $|\Phi\rangle \to |\Omega\rangle$, the area of the surfaces $\tilde{C}$ and $\tilde{D}$ goes to zero. This is depicted in figure 6 for the same spatial surface as in figure 5.

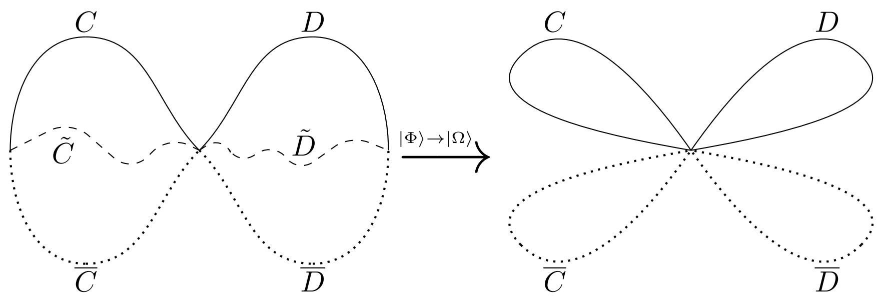

Figure 6: Depiction of the behavior of the spatial slice $\Sigma_{\partial M_\Psi}$ when $|\Phi\rangle \to |\Omega\rangle$. Again, only the bulk metric changes despite the appearance to the contrary.

As seen, the four regions pinch off each other and become disconnected except for a single singular point. Furthermore, they pull apart such that any point on one of the four boundary regions is infinitely far away from a point on one of the other regions (despite the appearance to the contrary for points close to the center of figure 6). It seems, therefore, that one can disconnect any number of spacetime boundary regions of an initially connected spacetime simply by removing all entanglement between the quantum subsystems each living on these boundary regions.

This, I argue, opens interesting perspectives for a thought experiment where the subdivision and entanglement removal is continued until no more subdivisions are possible and the resulting change of the spacetime dual is monitored using the Ruy-Takayanagi formula. Presumably, the subdivision into subsystems can continue until a distinct quantum subsystem is associated with each point of the spacetime on which the full quantum system is defined. Since the CFT must be defined on the asymptotic boundary of the dual spacetime, the maximal number of subsystems is obtained by associating a quantum subsystem to each point on this boundary. We may think of such a construction as analogous to the continuum limit of a quantum field theory on a lattice where we associate with each lattice site a Hilbert space of states, $\mathcal{H}_i$. The Hilbert space of states of the full system, $\mathcal{H}_L$, is then given by the tensor product of the Hilbert space of each lattice site

$$\mathcal{H}_L = \otimes_i \mathcal{H}_i. \tag{12}$$

Taking the continuum limit of this lattice system, one obtains a continuous quantum field theory and $\mathcal{H}_L \rightarrow \mathcal{H}$. Having made this construction, one can remove entanglement between the quantum subsystem until none of the subsystems are entangled, i.e. the full system is a product of pure states of the individual Hilbert spaces $\mathcal{H}_i$.

According to the Ruy-Takayanagi formula and the reasoning above, when entanglement is removed between regions in the CFT then the area of the surfaces dividing these regions in the spacetime dual goes to zero; the regions pinch and pull apart. Thus, if all entanglement is removed between all points of the spacetime on which the CFT is defined, then this entails that all points on the boundary of the spacetime dual pinch off and pull apart. No point on this boundary is connected to any other point. In other words, no signal travelling from one place on the boundary of the spacetime dual can intersect with a signal travelling from another point on the boundary. Thus, on the AdS side one ends up with a number of disconnected spacetime regions equal to the number of boundary points. Supposedly, these regions are still spacetimes in their own right; each dual to the pure state of a quantum subsystem consisting of a single point which must also be the asymptotic boundary of these spacetimes. We may still change these spacetimes by changing the state of the dual quantum subsystem to another pure state, but the spacetimes will remain disconnected. More preciely, if the $\mathrm{i_{th}}$ quantum subsystem, Q_i, is in a state $|\psi_i\rangle \in \mathcal{H}_i$ after all entanglement is removed, one may change this state to $|\phi_i\rangle \in \mathcal{H}_i$ and still have no entanglement between the Q_i and the systems $Q_{j\neq i}$. Such a change of the state of Q_i will entail a change in its spacetime dual. However, it will still have the same boundary - the single point on which Q_i is defined - and the spacetime will still be disconnected from the spacetime duals of the quantum systems $Q_{j\neq i}$.

In summary, the picture emerging is one of spacetime nodes; points on the boundary of the original spacetime that are robust under any change of the dual quantum state of that spacetime. Each of these nodes can be associated with a quantum system, and states of this quantum system are dual to spacetimes that have that particular spacetime node as their asymptotic boundary. Entangling the quantum subsystems associated with spacetime nodes connects the nodes in the dual spacetime, i.e. two signals starting at two spacetime nodes can intersect only if the two associated quantum subsystems are entangled. With this picture, it seems that spacetimes with a connected boundary containing more than one point can be constructed from spacetime nodes by entangling them. Thus, this thought experiment suggests that these spacetime nodes are robust and apparently fundamental entities. The rest of spacetime – bulk spacetime – is not robust but is rather dependent on the properties of the spacetime nodes and their relational structure. Bulk spacetime change and connectivity is controlled by the spacetime nodes and their relations.

From this thought experiment it seems that the relation between entanglement and spacetime provides us with a theoretical handle which may be utilized as a theoretical probe of the deep nature of spacetime.

6 Energy in the Zero Entanglement Limit

Rather than developing the thought experiment in further detail, the remainder of this paper will take on a qualitative investigation of the limit where all entanglement is removed and employ this in a preliminary assessment of the soundness of the thought experiment as a means to uncover the deep nature of spacetime.

In the thought experiment, only details about entanglement are explicit, however, other properties of the quantum system can be derived. A point of interest is how the energy of the quantum system behaves as entanglement is removed between spatially separated subsystems and how this change in energy is manifest is the spacetime dual. The energy-momentum tensor, $T^{\mu\nu}$, for a quantum field theory in Minkowski background is defined in terms of the conserved Noether current under spacetime translations. Thus, it satisfies

$$\partial_\mu T^\mu_\nu = 0 \tag{13}$$

Our primary interest here shall be the time-time component of this tensor – the conserved current under time translations – which is equal to the Hamiltonian density[16] of the quantum field theory. From eq. (13) it follows that the time-time component only depends on the spacial coordinates, $\vec{x}$. For a quantum state, $|\Psi\rangle$, the expectation value of the Hamiltonian density gives the energy density, $\epsilon_\Psi(\vec{x})$, i.e.

$$\langle\Psi|T^{tt}(\vec{x})|\Psi\rangle = \epsilon_\Psi(\vec{x}) \tag{14}$$

The total energy, E, of the quantum state may then be obtained as

$$E = \int d^{d-1}x\epsilon_\Psi(\vec{x}). \tag{15}$$

This total energy is the one obtained in standard quantum mechanics from the expectation value of the Hamiltonian. The ground state of a system is the state with the lowest total energy.

[16]This is the operator, H, related to the Hamiltonian by $H = \int d^dx\mathrm{H}$.

This definition of energy, however, faces complications in spacetimes where there is no timelike Killing vector. Essentially, the problem consists in the choice of vacuum. In spacetimes with a timelike Killing vector, the time direction may be changed by Lorentz transformations but the vacuum state and the particle number operator remain the same under such transformations; every intertial observer will agree on what is the vacuum state, the number of particles and therefore the energy of the system. In spacetimes without a timelike Killing vector, the vacuum state and number operator will change under Lorentz transformations and inertial observers will therefore not agree on what is the vacuum state and the number of particles. This issue will be ignored below. Thus, it will be supposed that the state $|\Psi\rangle$ is defined on a spacetime background with a timelike Killing vector for which it is straightforward to generalize the definition of the energy momentum tensor given above for Minkowski spacetime. Consequently, the boundary of the dual spacetime, ∂M_Ψ, is also supposed to have such a timelike Killing vector since this boundary – according to the AdS/CFT correspondence – is identical to the spacetime on which the quantum state is defined.

Now, what happens to the total energy of the full system when entanglement is removed between two subsystems? Based on the thermofield double state, one could think that the total energy of the system decreases when entanglement is removed. After all, entanglement was removed from the thermofield double state by decreasing the temperature (increasing the inverse temperature); ultimately, in the zero entanglement limit, this resulted in the product of the vacuum states $|E_0^A\rangle$ and $|E_0^B\rangle$.

However, despite being a enticing conclusion, it is generally not the case that the ground state of a full quantum system is a product of pure states of the subsystems; let alone a product of the ground states of the subsystems. As a toy example, consider the coupled linear harmonic oscillator consisting of two harmonic oscillators with the same mass, m, and spring constant, $k_0 \geq 0$, interacting by a harmonic two particle potential with coupling constant, $\kappa \geq 0$ (see figure 7). The Hamiltonian for the system takes the form

$$H = \frac{1}{2}[p_1^2 + p_2^2 + k_0(x_1^2 + x_2^2) + \kappa(x_1 - x_2)^2] \tag{16}$$

where p_1 and p_2 are the momentum of the oscillators and x_1 and x_2 are the positions.

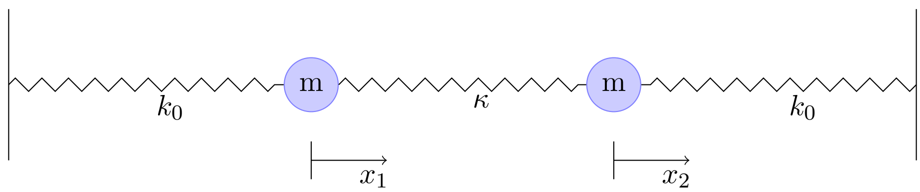

Figure 7: Two coupled harmonic oscillators.

The Hilbert space for this coupled system, $\mathcal{H}_{HO}$, can be decomposed as a tensor product of Hilbert spaces of each oscillator, $\mathcal{H}_{HO} = \mathcal{H}_1 \otimes \mathcal{H}_2$; the oscillators form subsystems that together comprise the whole system. The entanglement entropy quantifying the entanglement between the two subsystems 1 and 2 *for the ground state* of the coupled system is (Srednicki, 1993)

$$S_0 = -\log\left(1-\zeta^2\right) - \frac{\zeta^2}{1-\zeta^2}\log\left(\zeta^2\right) \tag{17}$$

where

$$\zeta = \frac{(k_0+2\kappa)^{\frac{1}{4}} - (k_0)^{\frac{1}{4}}}{(k_0+2\kappa)^{\frac{1}{4}} + (k_0)^{\frac{1}{4}}} \tag{18}$$

such that $0 \leq \zeta \leq 1$.

If and only if this entropy vanishes is the ground state a product state, i.e. a product of pure states of the two oscillators. As seen from the expression of S_0, this obtains only when $\zeta = 0$ which in turn is the case only when $\kappa = 0$. If the full system is such that $\kappa > 0$, then there is entanglement between the subsystems 1 and 2 in the ground state of the coupled system. Thus, for fixed $\kappa > 0$, removing all entanglement between the two subsystems will take the coupled system out of its ground state and therefore increase the energy of the coupled system.

While this is merely a toy example, the conclusion generalizes. For instance, the same result is implied by the Unruh effect (Unruh, 1976). Here the Minkowski vacuum state, $|0\rangle_M$, is expressed in terms of entangled energy eigenstates of the Rindler Hamiltonian, $|E_i\rangle^{Rl}$,

$$|0\rangle_M = \frac{1}{\sqrt{Z}}\sum_i e^{-\pi E_i}\,|E_i\rangle_L^{Rl} \otimes |E_i\rangle_R^{Rl} \tag{19}$$

where L and R denote states of the two subsystems consisting of the right and left Rindler wedge respectively. Generally, $|E_i\rangle^{Rl}$ spans a Hilbert space $\mathcal{H}^{Rl}$ such that $\mathcal{H}_M = \mathcal{H}_L^{Rl} \otimes \mathcal{H}_R^{Rl}$ where $\mathcal{H}_M$ is the Minkowski Hilbert space in which $|0\rangle_M$ is the ground state. Thus, removing entanglement from eq. (19) will result in an excited state of $\mathcal{H}_M$, since $|0\rangle_M$ is the ground state.[17]

Generally, in the ground state of a field theory, the local degrees of freedom will be highly entangled and removing this entanglement will take the system out of the ground state. This becomes very important in the limit where all entanglement is removed from a holographic CFT state. A general continuum field theory – a field theory not defined on a lattice – has an infinite number of local degrees of freedom in any finite volume (Huggett and Weingard, 1994). If disentangling each local degree of freedom from the rest of the system increases the energy, then the total energy risks diverging in the limit where all entanglement between local degrees of freedom is removed. Further, since there is a infinite number of degrees of freedom in any finite volume of the CFT the energy density also risks diverging. Thus, the limit considered in section 5 is one where the CFT tends to a state whose total energy and energy density may approach infinity. Notice that this is not merely an artefact of the well known divergence of the total energy in

[17] Indeed, the Fulling-Rindler vacuum, $|0\rangle^{RL} = |0\rangle_L^{RL} \otimes |0\rangle_R^{RL}$, which is a product of the (pure) ground states of the right and left Rindler wedge respectively, has been shown to have a higher total energy than the Minkowski vacuum state (Parentani, 1993).

continuum quantum field theories. It is the renormalized energy that increases when entanglement is removed.

An objection may be that the above examples imply that *adding* as well as removing entanglement will take a system out of its ground state. Thus, for a system not in the ground state one cannot *prima facie* determine whether one will increase or decrease the energy of the system by removing entanglement. This, however, does nothing to refute the threat of energy divergence in the limit where all entanglement is removed. It may be that the system is initially in a state such that removing entanglement between particular subsystems will decrease the energy of the system. However, as more entanglement is removed, the system will at some point increase its energy again since the ground state is an entangled state between the subsystem and the rest of the system. The only exception is if the system initially is in a state of higher energy than the zero entanglement limit. However, if the energy in the zero entanglement limit is divergent, so is the energy for a state with even higher energy.

As stated in section 2, it is a general result of the AdS/CFT correspondence that the energy of the CFT state corresponds to the energy of the dual spacetime. The energy of a spacetime should here be interpreted as some quasilocal stress energy tensor, since any local operator depending only on the metric and its first order derivatives must vanish in a generally covariant theory. In asymptotically AdS spacetime, one such quasilocal stress energy tensor may be defined in terms of the metric induced on boundary of the spacetime.[18] This stress energy tensor is equal to the expectation value of the energy momentum tensor of the dual quantum state (Balasubramanian and Kraus, 1999). One very crude indication of this correspondence comes from the fact that this stress energy tensor of the spacetime diverges, if the boundary on which the metric is induced is taken to infinity. This corresponds to UV divergences of the energy momentum tensor in quantum field theory, i.e. to the well known divergent vacuum energy in quantum field theory.[19]

The matching between this quasilocal stress energy tensor in the spacetime and the expectation value of the energy momentum tensor in the dual quantum system entails that if the energy density and the total energy diverge in a quantum state even after renormalization, then so does the renormalized time-time component of the stress energy tensor in the dual spacetime and the integral $\int d^d x T^{tt}_{Grav}$, where T^{tt}_{Grav} denotes the stress energy tensor in the spacetime dual to that quantum state. This puts pressure on the

[18] The quasilocal spacetime stress energy tensor is defined as

$$T^{\mu\nu}_{Grav} = \frac{2}{\sqrt{-\gamma}} \frac{\delta S_{grav}}{\delta \gamma_{\mu\nu}} \tag{20}$$

where $\gamma_{\mu\nu}$ is the induced metric on the boundary and S_{grav} is the gravitational action here considered as a functional of $\gamma_{\mu\nu}$. In asymptotically flat spacetimes, this stress energy tensor agrees with the ADM energy (Arnowitt et al., 1959) when $\gamma_{\mu\nu}$ is at spatial infinity (Brown and York, 1993).

[19] Analogously to quantum field theory, the divergences of the spacetime stress energy tensor may be removed by adding local counterterms to the action (Balasubramanian and Kraus, 1999; Skenderis, 2001).

thought experiment developed above. The Ruy-Takayanagi formula, eq. (8), holds only for quantum states with a classical spacetime dual, and spacetimes with large energy cannot be assumed to be classical spacetimes, since high energy induces strong curvature. More precisely, the AdS side may be approximated by Einstein gravity since the Einstein Hilbert action (plus additional fields) appears as the first order approximation in the string length (low energy) of type IIB string theory (Callan et al., 1987; Huggett and Vistarini, 2015). However, at large energies one has to include higher order terms in the string length and the approximation breaks down. It has been argued that these higher order corrections manifest themselves as bulk entanglement correction to the Ruy-Takayanagi formula due to entanglement between the bulk fields over the co-dimension two surface $\tilde{B}$ (Faulkner et al., 2013). Thus, if the reasoning is sound and the energy of the CFT state diverges when all entanglement is removed, then the Ruy-Takayanagi formula cannot be used to justify the behavior of the dual spacetime.

One could try and argue that the conclusion of section 5 can be established without the justification provided by the Ruy-Takayanagi formula. If a state with no entanglement between the local degrees of freedom is dual to a collection of disconnected spacetimes – one for each local degree of freedom – then an initially entangled state where all entanglement is then removed should have that same spacetime dual. However, this assumption is also contestable in the high energy limit as we simply do not know what to expect of the dual spacetime in this regime; if there is such a spacetime dual at all.

An indication to this is implied by a more careful use of the Ruy-Takayanagi formula. From the formula it follows that when the entanglement between two contiguous regions in a CFT tends to zero, then so does the minimal bulk surface connecting the two regions in the spacetime dual. However, as depicted in figure 5, when this limit is taken, the two emerging bulk regions will still be connected by a bulk singularity; though this singularity has no evident geometrical interpretation. In terms of the duality, the bulk singularity corresponds to the entangling surface separating the two subsystems on the CFT side. Every further subdivision into subsystems will introduce another entangling surface between that subsystem and the rest of the system. Removing all entanglement over this surface adds another bulk singularity. When entanglement is removed in this way for the infinite number of local degrees of freedom separated from the rest of the system by entangling surfaces, the spacetime dual fills up with bulk singularities compromising the picture of this spacetime as a classical spacetime.[20]

7 Discussion

The Ruy-Takayanagi formula serves as the foundation of an apparently profound relation between an inherently quantum mechanical phenomenon – entanglement – and spacetime, i.e. the gravitational field. The relation prescribes that gravitational phenomena,

[20] Again, if the Ruy-Takayanagi formula is not valid in this limit, it is most precise simply to insist that one has no resources to assess what spacetime – if any – is dual to a quantum state when all entanglement is removed.

ordinarily accounted for by the dynamics of spacetime, can just as well be described by the entanglement structure of CFT states on fixed spacetimes; these two accounts represent the same physics. In section 5 it was suggested that this relation could serve as a theoretical probe of the deep nature of spacetime beyond our current experimental capabilities. This probe utilizes the rather well understood framework of conformal field theories and a translation manual which has the Ruy-Takayanagi formula at its core to examine spacetime in the regime where all entanglement is removed between local degrees of freedom in the dual quantum system. Emerging from this thought experiment was a structure of spacetime nodes that could be connected by entangling the quantum systems associated with each of them. However, the qualitative argument of section 6 raised the issue that the zero entanglement limit is a high and maybe even divergent energy regime of the quantum system and dual spacetime alike. The validity of the Ruy-Takayanagi formula in the zero entanglement limit is therefore questionable and consequently the foundation of the developed theoretical probe of spacetime is cast into doubt.

In their qualitative form, however, the remarks about energy do not refute this theoretical probe. It may be that the energy turns out to be adequately well behaved in this limit despite the indications to the contrary or it may be that these are special cases where the Ruy-Takayanagi formula is valid beyond its usual domain. Indeed, one could take the remarks about energy in section 6 to be a confirmation that the thought experiment of section 5 does indeed probe the deep nature of spacetime in the sense that it unveils the high energy limit usually regarded as the regime of quantum gravity. Furthermore, even if the Ruy-Takayanagi formula should receive corrections in the zero entanglement limit, the thought experiment may still be of utility in providing a picture of the *leading* order behavior of spacetime in quantum gravity.

These remarks signify the need for a more rigorous and quantitative study of the zero entanglement limit. This, however, is easier said than done. It is in general complicated to write up the full state in terms of states in spatially separated subsystems and further difficulties arise if one wants to monitor what happens when entanglement is removed between the two subsystems. The thermofield double state is a notable exception; here the two subsystems are easily separated and the amount of entanglement between the two subsystems could be controlled by the inverse temperature β. One might therefore suppose that some analogous parameter would control the entanglement between Q_B and $Q_{\overline{B}}$ such that $|\Psi\rangle$ goes to the product state $|\Phi\rangle$ for a particular limit of this parameter. For a quantum state on Minkowski spacetime, such a division into subsystems can be obtained using energy eigenstates of the Rindler Himiltonian as in eq. (19). However, this is not a trivial construction! Furthermore, our thought experiment relies on a continued subdivision until each point of the background spacetime is its own subsystem. One may remove entanglement in eq. (19) by sending the temperature of the Rindler spacetimes to zero (the Minkowski vacuum has Rindler temperature $\frac{1}{2\pi}$). However, to further remove entanglement between local degrees of freedom requires a way to subdivide the Rindler spacetime into spatially separated subsystems and then express the Rindler-Fulling vacuum in terms of states of these subsystems. Eventually obtaining the zero entanglement limit in this way is presumably only viable if some iterative procedure can be developed. Until these difficulties are overcome, the jury will remain out regarding the validity of the theoretical probe of the deep nature of spacetime developed here.

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