Probabilities in deBroglie-Bohm Theory: Towards a Stochastic Alternative

(Version 0.1 beta)

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20. January 2017

"Once we have discarded our rooted predilection for absolute Causality, we shall succeed in overcoming the difficulties."

E. Schrödinger: *What is a Law of Nature? Science Theory and Man*

Abstract:

We critically examine the role and status probabilities, as they enter via the Quantum Equilibrium Hypothesis, play in the standard, deterministic interpretation of deBroglie's and Bohm's Pilot Wave Theory (dBBT), by considering interpretations of probabilities in terms of ignorance, typicality and Humean Best Systems, respectively. We argue that there is an inherent conflict between dBBT and probabilities, thus construed.

The conflict originates in dBBT's deterministic nature, rooted in the Guidance Equation. Inquiring into the latter's role within dBBT, we find it explanatorily redundant (in particular for dBBT's solution of the Measurement Problem, which only requires that the corpuscles possess definite positions), and subject to a number of difficulties. Following a suggestion from Bell, we propose to abandon the Guidance Equation, whilst retaining dBBT's point particle-based Primitive Ontology, with positions as local beables. The resultant theory, which we identify as a stochastic, minimally deBroglie-Bohmian theory, describes a random walk through configuration space. Its probabilities, we propose, are best understood as dispositions of possible corpuscle configurations to manifest themselves. We subsequently evaluate the merits of sdBBT vis-à-vis dBBT, such as the justification of the Symmetrisation Postulate and the violation of the Action-Reaction Principle.

Not only is sdBBT an attractive Bohmian theory that, whilst retaining dBBT's virtues, overcomes many of its shortcomings; it also sparks off a number of exciting follow-up questions, such as a comparison between sdBBT

and other stochastic hidden-variable theories, e.g. Nelson Stochastics, or between sdBBT and the Everett interpretation.

Keywords: Bohmian Mechanics; Probabilities; Typicality; Humeanism; Bell's Formulation of the Everett Interpretation; Primitive Ontology

Content

I. Introduction

Vis-à-vis the hassle in the foundations of quantum mechanics (QM), esp. the measurement problem, the interpretation of the Heisenberg relations and their joint culmination in the EPR "paradox" (1935), the question arises whether QM in its current form is incomplete: Might there exist an element of physical reality that has no counterpart in QM?[1] Einstein, for instance, was "[…] firmly convinced that the essentially statistical character of contemporary quantum theory is solely to be ascribed to the fact that this [theory] operates with an incomplete description of physical systems."[2]

deBroglie-Bohm Theory (dBBT) is an attempt to complete QM: It proffers an account of a deterministic dynamics that describes a sub-quantum particle world, from which QM emerges – as Einstein had hoped for – in a manner "approximately analogous […] to the statistical mechanics within the framework of classical mechanics."[3]

This reference to statistical mechanics (SM) prompts three questions.

1. What does the asserted analogy between dBBT and SM consist in?
2. To what extent is it justified?
3. What precisely does the "statistical character" of QM consist in from the dBBT perspective? What is the role and status of probabilities within dBBT?

Our subsequent pursuit of these questions will put its finger to what we'll argue to be dBBT's biggest shortcoming, namely the joint incompatibility of its deterministic dynamics, its probabilistic Quantum Equilibrium Hypothesis and its aspiration to a thoroughly objective "quantum theory without observer." As a natural and conservative resolution, following a suggestion by Bell, we propose to simply drop the deterministic dynamics, yielding a fundamentally stochastic deBroglie-Bohmian "rump theory". The following paper will attempt to take up the cudgels for this so-far largely neglected theory as superior to dBBT.

We'll proceed as follows: In section II, we briefly review (II.1) and critically examine (II.2) the standard interpretation of dBBT and the status probability has in it, considering interpretations in terms of ignorance as well as typicality and Humean Best Systems (II.3). The

[1] Einstein et al. (1935), cited in Redhead (1987), p.71, who also discusses the EPR argument in detail.
[2] Einstein (1949), p. 666
[3] Ibid.

subsequent analysis of the role the Guidance Equation plays within dBBT (III.1) suggests an alternative stochastic reading of dBBT that dispenses with it: stochastic deBroglie-Bohm Theory (sdBBT). The resulting theory is *irreducibly* stochastic with the probabilities of the Born Rule representing a disposition for a random walk through configuration space (III.2). In IV, we critically examine sdBBT, its status as a minimally deBroglie-Bohmian theory (IV.1), and its status as a fundamental (rather than a merely phenomenalist) theory (IV.2). In IV.3, we address an objection Bell has articulated against sdBBT, namely its "temporal solipsism", i.e. the temporal discontinuity of the world according to sdBBT. We conclude section IV with the completion of our comparison of dBBT's and sdBBT's virtues, respectively, along the lines of metaphysical questions regarding the status of the wavefunction in both, the justification of the Symmetrisation Postulate and claimed conceptual advantages for calculations regarding quantum tunnelling. IV.5.4 discusses the issue of relativity in sdBBT and dBBT, respectively. In the last section (V), we summarise our main findings and sketch promising lines of future enquiry.

II. dBBT and its discontents

II.1. Standard dBBT

For a universe with *N* corpuscles of mass m_i each[4], dBBT consists of three axioms:

(1) The standard, non-relativistic *N*-particle Schrödinger Equation (SE): $i\hbar\frac{\partial}{\partial t}\psi_t(\boldsymbol{q},t) = \hat{H}\psi_t(\boldsymbol{q},t)$ with the wavefunction $\psi_t\colon \mathbb{R}^{3N}\times\mathbb{R} \to \mathbb{C}$ and the *N*-particle Hamiltonian $\hat{H} = -\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\boldsymbol{\nabla}_{\mathrm{i}}^{\,2} + V(\boldsymbol{q},t)$ with $\boldsymbol{\nabla}_{\mathrm{i}} = \frac{\partial}{\partial \mathbf{q_i}}$, $i = 1, \dots, N$.

(2) The Guidance Equation (GE), governing the *i*-th corpuscle's trajectories ($i = 1, \dots, N$): $m_i\frac{d\boldsymbol{Q}_i}{dt} = \hbar\mathfrak{Im}\left\{\frac{\boldsymbol{\nabla}_i\psi}{\psi}\right\}$: Given initial positions of the corpuscles, the GE determines their positions at any other time. Existence and uniqueness of the trajectories are guaranteed under *prima facie* reasonable assumptions (more on this in III.2).

[4] Ascribing the masses to the corpuscles turns out to be subtle – an issue to which we'll return in III.2. For the time being suffice it here to cite the arguably consensus view Esfeld articulates "[…][The corpuscles] do not have any hysical properties over and above their being localised in physical space. Hence, […] these objects cannot be considered as having an intrinsic mass or an intrinsic charge (or an intrinsic spin). They do not have any intrinsic properties", Esfeld (2016), p.4.

(3) The Quantum Equilibrium Hypothesis (QEH): For an ensemble of identical systems with the same wavefunction ψ, the corpuscles' configurations are distributed according to the Born Rule (BR), $\rho = |\psi|^2$.

The continuity equation $\partial_t \rho^\psi + \mathrm{div}\boldsymbol{j}^{\boldsymbol{\psi}} = 0$ obtained from the SE, with the probability density $\rho^\psi = |\psi|^2$ and the probability flux $\boldsymbol{j}^{\boldsymbol{\psi}} = \left(\boldsymbol{j}_k^\psi\right)_{k=1,\dots,N} = \left(\frac{\hbar}{2im_k}\mathfrak{Im}\{\psi^* \boldsymbol{\nabla}_{\mathbf{k}} \psi\}\right)$ implies: If at any instant particles are $|\psi|^2$-distributed, so are they at any other time: If the corpuscles are initially distributed according to the BR, they satisfy it later, too (and vice versa).

For the time being, we'll take the wavefunction to represent a holistic, real property of the system of *N* corpuscles, constituting the basic stuff of reality, located in physical 3-dimensional space. More on this in III.2. Regarding the wavefunction, let's preliminarily accept it as an entity that in some way "pilots" the corpuscles. At this stage, a more detailed discussion of its ontological status – E.g.: A physical field like the electromagnetic one? A law-like abstract entity? A disposition? – needn't delay us yet.

II.2 Critical analysis of dBBT's subjectivist probabilities

Let's first turn to the nature of dBBT's probabilities. Such an inquiry can take two directions:

1. Can the BR somehow be derived/deduced, rather than being just stipulated by *fiat*? E.g. Valentini et al., taking up Bohm's idea from 1953, indeed attempt to explain how a system initially in quantum non-equilibrium relaxes into quantum equilibrium – analogously to Boltzmann's H-Theorem. The results hinge on strong assumptions, however, only little better than just postulating the BR.[5]
2. Following mainstream presentations of dBBT, we therefore include the BR (or QEH) as an independent axiom. The question then remains: What's the status of the probabilities the QEH introduces?

According to Bell, "[…] the only use of probability here is, as in classical [SM], to take account of uncertainty in initial conditions."[6] In the following we assume the *orthodox* interpretation of dBBT's probabilities to be an ignorance interpretation.[7]

[5] Cf. Callender/Weingard (1997), Callender (2006)
[6] Bell (1980), p. 156
[7] E.g. also Esfeld (2016), p. 5

But how convincing is the first part of Bell's answer, viz. the asserted analogy with thermodynamic equilibrium (i.e. the probability distribution of the canonical ensemble, $\rho = e^{-\frac{H}{kT}}/Z$, with Boltzmann's constant k, the classical Hamiltonian/energy-function, the partition function Z and temperature T of the equilibrated system)?[8] At best, we submit, it's heuristic:

- The system's dynamics imposes constraints on the probability distributions in both cases. (More on this in II.3).
- "[I]n both cases it seems natural to try to justify these equilibrium distributions by means of mixing-type, convergence-to-equilibrium arguments […]. [It's] been argued, however, that in both cases the ultimate justification for these probability distributions must be in terms of statistical patterns that ensembles of actual subsystems within a typical individual universe exhibit."[9] The success of such attempts is controversial.[10]

Here the similarities end:

- An immediate crucial difference in terms of physical significance is that, whereas our current universe is far from thermodynamic equilibrium/heat-death (with equilibrium states, of course, observable only locally, e.g. in one's morning *café au lait*) the dBBT universe has *always* been in *global* Quantum Equilibrium, which thus isn't a "quantum heat-*death",* devoid of all structures.[11]
- More importantly, whereas in the thermodynamic case, the classical-mechanical Hamiltonian (operating, with its double role as energy of the macrosystem and generator of the dynamics of its micro-constituents, on both levels) links the micro-dynamics with the macro-system's properties, dBBT's BR-probability distribution contains *only* the wavefunction: Although it, too, occurs on the "macro-", i.e. quantum level, it doesn't genuinely *link two* levels, as dBBT *includes* the SE, as QM's essence, as an axiom and hence as part of the "micro-", i.e. subquantum level description. In other words, since from the dBBT perspective the SE, arguably constituting the essence of the macro-level QM, is an axiom of the micro-level dBBT, both levels are not clearly

[8] Cf. Goldstein (2013), Sect. 9
[9] Ibid.
[10] For the SM case, cf. Sklar (2015), sect. 3.
[11] Cf. Dürr et al. (2003), Ch. 12, 13.
The global nature of quantum equilibrium plays a crucial role for the dynamical systems analysis within dBBT, cf. loc. cit.

separated; QEH thus cannot properly *bridge* them. Rather than as a subquantum theory, which "completes" QM, this suggests to regard dBBT as an alternative theory in its own right, empirically equivalent to QM. Then, however, the analogy, which turns on the idea of one theory emerging from the other, breaks down altogether.

Let's move on to Bell's second assertion, viz. that SM probabilities are epistemic.[12] This is a contentious point [13] – and hence ill-suited to illuminate the interpretation of dBBT's probabilities:

- Popper, for instance, argues that explanations of SM in which epistemic probabilities play a role assert "that irreversibility [as expressed in the 2nd Law of thermodynamics] is a result of our ignorance of the details of the state of the gas." This, he continues, "[…] leads to the absurd result that the molecules escape from our bottle [air-filled and then uncorked in vacuum], *because* we do not know all about them […]"[14]. Popper, in short, claims that epistemic probabilities amount to the absurd belief in telekinesis.
- He also observes the incompatibility between Boltzmannian SM and epistemic probabilities: Firstly, "nescience *always* increases, provided we do not start with complete knowledge. But disorder, or entropy, *decreases at times*; according to Boltzmann, it fluctuates. Secondly nescience does *not* increase if we have complete or perfect knowledge to start with […]. Again this is incompatible with Boltzmann's view; for if ever a system should attain perfect order by a highly improbable fluctuation, it would, in all probability, immediately become disordered again, according to Boltzmann."[15]
- Furthermore, Popper maintains, the probability-subjectivist cannot *explain* the fact and the irreversibility of diffusion of the gas molecules in the example: "[The

[12] According to Uffink (2011), one should distinguish between two forms of non-objective probabilities, where an objective quantity or quality of an object corresponds to an inherent property of the physical object itself, independent of any subject's knowledge of it: *Subjective* probabilities reflect the strength of an individual's belief, i.e. the degree of subjective certainty; by contrast, *epistemic* probability assignments, capture an individual's certainty relative to the information available to them. In other words, epistemic probabilities express an objective or at least inter-subjective evaluation of their knowledge. In the following, we will treat ignorance interpretations of probability as epistemic interpretations.

[13] Cf., e.g. Lavis (2011) for recent defences of objective probabilities in SM.
Contrariwise, e.g. Frigg (2007) or Uffink (2011) make the case for epistemic probabilities in SM.

[14] Popper (1982), pp. 109 (our emphasis)

[15] Op. cit., p. 115 (Popper's emphases)

subjectivist] cannot even say that the gas has in fact expanded. *All he can say is that his state of ignorance has increased* […].”[16]

As Frigg points out, however, a misconception underlies these objections:[17] Espousing an epistemic view on probability doesn’t imply that our beliefs or lack of knowledge *cause* or *bring about* the physical facts. Epistemic probabilities only explain “why or when it is reasonable to expect [gases to disperse, ice cubes to melt, or coffee to mix].”[18]

One may counter that our scientific explanations should go beyond “reasonable expectations”[19]; rather, adhering to a vision of physics close to those of Einstein or Planck[20], we should strive for explanations and interpretations in purely physical terms only, with no reference to subjects and their epistemic states.

And indeed, dBBT expressly aspires after a realist, objectivist “quantum theory *without* observer” (Popper).[21] In particular, dBBT intends to be able to describe fundamental reality even in the absence of any epistemic subjects who could have any “reasonable expectations”, such as in the early phases of the universe. (Note that such deviations from Quantum Equilibrium elicit physical effects, which leave objective, in principle detectable traces in the cosmic microwave background.[22]) Thus, the question still looms: How do epistemic probabilities fit into its otherwise realist, objectivist/subject-free framework? We’ll further pursue that line of thought shortly.

One might bypass the problem if dBBT’s probabilities turned out not really to be probabilities directly expressing chance-related quantities, but something like constraints on all conceivable statistical initial distributions of Bohmian corpuscles. Let’s ponder: How to understand the *continuous* BR-probability distribution $\rho_{th}(\boldsymbol{Q}) \coloneqq |\psi(\boldsymbol{Q})|^2$ vis-à-vis the corpuscles’ *discrete* distributions of the form $\rho_{emp}(\boldsymbol{Q}) := \frac{1}{N}\sum_{i=1}^{N}\delta(\boldsymbol{Q}-\boldsymbol{q_i})$, with $\boldsymbol{q_i}$ the corpuscles’ actual positions? Goldstein declares that the former (which he calls the

[16] Op.cit. p. 116 (Popper’s emphases)
[17] Cf. Frigg (2010), p. 30
[18] Uffink (2011), p. 45
[19] E.g. Bunge (1974) for a thoroughgoingly objectivist philosophical semantics
[20] Cf. Scheibe (2006), Ch. II, III and IX
[21] Allori et al. (2007), sect. 8, expressly classify dBBT, alongside GRW, as a quantum theory without observer. Dürr/Teufel (2009), pp. 177 illustrates the vehemence with which dBBT-adherents dismiss subjectivism in the context of the Measurement Problem.
[22] Cf. Valentini (2010)

"theoretical distribution") must approximate the latter (the "empirical distribution"): "The theoretical distribution is an idealisation providing a good approximation to the empirical distribution, $\varrho_{emp} \approx \varrho_{th}$, in the limit of large ensembles of subsystems."[23]

But what's supposed to be the physical and ontological status of such an idealising constraint? What does the discrepancy between the "theoretical distribution" and the actual "empirical distribution", which ineluctably arises for any Bohmian universe of finite corpuscle number, signify? Two oddities obstruct an interpretation of the QEH as a contingent boundary/initial condition of our universe: Firstly, the constraint itself evolves dynamically – against the intuition that boundary/initial conditions should be fixed. Secondly, the Schrödinger Equation, which governs this dynamics, is a partial differential equation. Hence, it requires the specification of contingent initial data for the wavefunction. In other words: QEH, construed as a contingent constraint, in turn, is subject to another contingent meta-constraint. This seems redundant. (We'll revert to these two peculiarities in IV.5.1) Goldstein's suggestion yields no satisfactory resolution.

Let's therefore bite the bullet: In accordance with dBBT's realist framework, what we're after is a way to accommodate for its probabilities in an *objective* way. What options are on the table?

II.3. Two *culs-de-sac* to objective probabilities for dBBT

In his review of objective interpretations of probability[24], Maudlin discusses two that appear viable for our deterministic dBBT, viz. typicality and Humean Best Systems. How do dBBT's probabilities fare on these?

II.3.1. Typicality

The analogy with SM, fickle as it may be, raises the question: Might Boltzmann's own understanding of probabilities in his approach to SM bail us out?[25] Here, the probability measure figures as a *modal* measure of how typical/common certain sets of phase-space are. A statement involving an equilibrium macrostate with typicality measure ("t-measure", henceforth) close to unity holds *typically*, i.e. for the overwhelming majority of micro-

[23] Goldstein (2011), p. 9
[24] Cf. Maudlin (2011a)
[25] Cf. Goldstein (2001); Lazarovici/Reichert (2015)

configurations – or equivalently, since the dynamics preserves the measure, initial micro-configurations: Most microscopic initial conditions evolve into indistinguishable coarse-grained macrostates. Out of all nomologically possible systems, most behave typically.

Neither randomness nor ignorance becloud the sky of such an interpretation: T-probabilities in SM are compatible with the determinism inherent in its underlying micro-dynamics.

Advocated as an apposite framework for probabilities in SM[26], it's tempting to ponder: Can we transfer the typicality account to dBBT's probabilities? Indeed, standard presentations of dBBT[27] couch the QEH in terms of typicality: The universal wavefunction (of the universe) Ψ induces a t-measure $\mathbb{P}^{\Psi_t} \coloneqq |\Psi_t|^2$. A Law of Large Numbers then establishes that *typical* subsystems of a universe in Quantum Equilibrium, with corresponding "effective" wavefunction ψ are distributed according to $|\psi|^2$, i.e. the BR as probed in laboratory contexts.

More precisely, relaxing the assumption of the wavefunction being factorisable, a subsystem is said to have an effective wavefunction ψ, if the universal wavefunction $\Psi: X \times Y \to \mathbb{C}$, with *X* and *Y* denoting the configuration space of the subsystem and its environment, respectively, can be decomposed as

$$\forall (\mathrm{x}, \mathrm{y}) \in X \times Y: \Psi(\mathrm{x}, \mathrm{y}) = \psi(\mathrm{x})\phi(\mathrm{y}) + \Psi_{\perp}(x, y),$$

where ϕ and $\Psi_{\perp}$ have macroscopically disjoint *y*-support and $Y \subseteq \mathrm{supp}(\phi)$. Subsystems with an effective wavefunction and negligible interaction with its environment can be shown to satisfy the SE for ψ.

For subsystems with the same wavefunction ψ, the $\mathbb{P}^{\Psi}$-measure, conditional on all environmental configurations *Y* that yield to the same effective wavefunction ψ, is determined (independent of *Y*) as:

$$\mathbb{P}^{\Psi}(\{\boldsymbol{Q} = (X, Y): X \in d^n x\} | \{\Psi(.,\mathrm{Y}) = \psi: Y\}) = |\psi|^2 d^n x.$$

[26] Cf. Volchan (2006); Maudlin (2011a); Hemmo/Shenker (2015); Lazarovici/Reichert (2015); Oldofredi et al. (2016)
[27] E.g. Dürr/Teufel (2009), Ch. 8.3. We follow the presentation in Oldofredi et al. (2016), Sect. 3

From this, a Law of Large Numbers follows: For any measurable set $\boldsymbol{A} \subseteq \mathbb{R}^{3n}$ and an ensemble of *N* identically prepared subsystems with the effective wavefunction ψ and the position random variables X_i, it holds that

$$\forall \varepsilon > 0 \colon \mathbb{P}^{\Psi_t}\left(\boldsymbol{Q} \in \mathbb{R}^{3n} \colon \left| \frac{1}{N} \sum_{i=1}^{N} \chi_{X_i \in \boldsymbol{A}}(\boldsymbol{Q}) - \int_{\boldsymbol{A}} d^{3n}Q' \, |\psi(\boldsymbol{Q}')|^2 \right| < \varepsilon \right) \xrightarrow{N \to \infty} 0.$$

In other words: The distribution of corpuscles in sufficiently large ensembles of subsystems, each prepared with the same *effective* wavefunction ψ, typically approximate the statistics of the BR, i.e. $|\psi|^2$, where the measure of typicality is given by the QEH, i.e. $\mathbb{P}^{\Psi} = |\Psi|^2$, with the *universal* wavefunction Ψ.

In light of these results, Oldofredi et al. announce: "Born's rule is thus predicted and explained by [dBBT] as a statistical regularity of typical Bohmian universes."[28] If the QEH is accepted, subsystems necessarily obey the usual quantum mechanical probabilities.

But why buy into the QEH, why accept $\mathbb{P}^{\Psi}$ as its t-measure? In SM, stationarity figures in the usual motivation/justification of the Lebesgue-measure (a justification, however, Frigg has argued to be fundamentally misguided[29]). In dBBT, the notion of equivariance[30] suitably generalises stationarity: Imposing it *uniquely* determines $|\Psi_t|^2$ as the t-measure[31] that depends only locally on Ψ_t and its derivatives – a far more satisfactory picture! Their importance as mathematical theorems notwithstanding, these mathematical *uniqueness* results don't answer the question why we should assume QEH. Dickson succinctly writes: "It's not at all obvious why equivariance is a preferred property of measures over the possible initial distributions. Equivariance is a *dynamical* property of a measure, whereas the question 'Which initial distribution is the correct one?' involves no dynamics, nor is it clear why

[28] Oldofredi et al. (2016), pp.15

[29] Cf. Frigg (2011). One main argument is that although the Lebesgue measure is indeed the only measure invariant under *all* Hamiltonian flows, it's not clear that this property is at all relevant for justifying the choice of the Lebesgue measure, since each system is governed by exactly *one* Hamiltonian. For any *specific* Hamiltonian, there could also be invariant measures other than the Lebesgue measure.

[30] Let Ψ denote the wave function of the universe. It generates the dBBT dynamics in the form of the flow Φ_t^{Ψ}. A Ψ- dependent measure $\mathbb{P}^{\Psi}$ is called equivariant, if for a measurable set A the following equality holds: $\mathbb{P}_t^{\Psi}(A) \coloneqq (\mathbb{P}^{\Psi} \circ (\Phi_t^{\Psi})^{-1})(A) = \mathbb{P}^{\Psi_t}(A)$, cf. Dürr/Teufel (2008), Ch. 11.

[31] Cf. Goldstein/Struyve (2007)

dynamical properties of a measure are relevant".[32] Moreover, we submit, typicality accounts are inept to settle the interpretation of dBBT's probabilities:

- Albeit a measure over *possible* – hence, by standard accounts[33], *non-real* – world-configurations, the t-measure, $\mathbb{P}^{\Psi_\mathrm{t}} = |\Psi_\mathrm{t}|^2$, satisfies a dynamical law – usually characteristic of *real* properties. (We'll revisit this argument in a different context more in detail in III.2.)

  The dynamical nature of the wavefunction has an important implication: Insofar that typicality quantifies how common a trait is amidst the space of nomological possibilities, a typicality statement shouldn't be contingent, that is, vary across different possible worlds. But since the wavefunction, qua SE, requires the specification of initial conditions that characterise a particular world, $\mathbb{P}^{\Psi_\mathrm{t}}$ measure is contingent, as well. Therefore it cannot be a measure of typicality.
- More oddities are in the offing: Even if, as we reported, dBBT's t-measure, $\mathbb{P}^{\Psi_\mathrm{t}} = |\Psi_\mathrm{t}|^2$, is unique and to some extent appears plausible, one must be wary of concluding that sets of measure zero w.r.t. $\mathbb{P}^{\Psi_\mathrm{t}}$ (e.g. the set of initial conditions for which the GE is well-defined, see III.1) are small[34]: Sets of measure zero in general needn't be small at all in any mundane sense: Think, for example, of the *infinitely many* rational numbers in the interval between 0 and 1, whose Lebesgue measure λ is zero: $\lambda(\mathbb{Q} \cap [0;1]) = 0$.

  Moreover, Frigg stresses that "[…] as Sklar […] has pointed out, sets of measure zero needn't even be negligible, if sets are compared with respect to properties other than their measures. For instance, we can judge the 'size' of a set by its cardinality or Baire category rather than by its measure which may lead to different conclusions about a set's 'size'. […] So we face the question of what conveys upon measures a privileged status when it comes to judging typicality."[35]
- Quite concretely, one may ponder: What ensures that typical solutions are *actually* observed? Suppose, we rearrange the first $M \gg 10^{123}$ (the latter being an estimate of the number of bits of our universe) digits of the (infinite) series of digits of a so-

[32] Dickson (1998), p. 123 (Dickson's emphasis)
[33] In non-standard views, such as modal realism, cf. Lewis (1986) and dispositional metaphysics, cf. Mumford (2003), the situation is different. We'll explore the prospect of the latter in III.2.
[34] Cf. Frigg (2011), p.90
[35] Loc.cit.

called normal number $z = (d_1, d_2, \dots)$, i.e. one in which each digit *i* occurs equally often (with equal t-measure $\mu_z(\{i\}) = \frac{1}{10}$), such that of the first *M* digits every other number is seven: $z^* \coloneqq (7, d_1^*, 7, d_2^* \dots)$, where the d_i^*'s are obtained from deleting all 7's from the first *M* d_i's. Of course, the rearrangement z* doesn't change the frequency with which 7 occurs; its t-measure is preserved under finitely many permutations, $\forall i: \mu_{z^*}(\{i\}) = \mu_z(\{i\})$. Yet, by looking at z^*, it *appears* that every other number is 7, i.e. that $\mu_{z^*}(\{7\}) = \frac{1}{2}$.

The immediate lesson is: Without randomisation that suitably *mixes* the results, a high t-measure over the ensemble of possible dBBT systems doesn't explain why our observed statistical patterns *match* typical, i.e. BR-obeying outcomes. The facts of the world (assumed to be, at most, countably inifinite) could perfectly well have the QEH's measure of typicality, whilst still all empirical evidence could violate the BR – if, e.g. a malicious demon decided to suitably re-arrange the facts of the world in a way that all those (finitely many) facts available to us deceive us. Why preclude such a demon? How to ensure that typicality is linked to empirical results?

If the choice of the t-measure can't explain the empirical adequacy of the BR, we could equally well postulate any other t-measure! Hence, without a randomisation, the BR loses its empirical content. But then, what's the point of an allegedly fundamental theory without empirical content? So, we wind up where we started: Whence the randomness in the otherwise deterministic dBBT universe?

- One way out of the preceding problem is to simply *assume* that our dBBT universe or any other system we're interested in is typical, i.e. started from typical initial conditions. But as Frigg poignantly remarks: "Whether the system has a particular initial condition is a factual question, and as such it has to be settled by an appeal to matters of fact and not measures of sets; […] we need an argument for the conclusion that the system indeed started out in a typical initial condition, but that these are of measure (close to) one does not give us such an argument. […] Whether or not this initial condition is also typical is simply irrelevant."[36]

One reply would be to argue that in the absence of further information about the details of a system, it's *rational to expect* the system to be typical.

[36] Op.cit., p.91

We already encountered such an epistemic manoeuvre, and dismissed it as endangering the Bohmian *objectivist* agenda. M

Perhaps more to the point, even any connection between typicality and “rational expectability” is questionable. Conceptually, a t-measure refers to the collection of possible worlds/systems. In essence, such a collection of systems is a canonical/Gibbsian ensemble, as known from Gibbsian SM.[37] Thus, its use is subject to the same criticism as in the Gibbsian case.[38]

In particular, dBBT claims universal applicability, encompassing the universe as a whole. The latter doesn’t have any copies, though. And given that, for all we know, there’s only *one* universe, why should it be “rational to expect” it to be a typical member of all conceivable universes? It’s true that multiverse speculations have become fashionable in contemporary cosmology – however, not without being severely scathed[39]; furthermore, demanding that a consistent interpretation of dBBT’s probabilities be perforce committed to multiverse scenarios seems an unnecessarily compromising deed of desperation.

We conclude that it’s not even clear how typicality should even matter to a substantial interpretation of dBBT – let alone how it would settle the status of its probabilities.

II.3.2. Humean Best Systems

After hitting a blind alley with typicality accounts, what about the other road to objective chances Maudlin considers, Humean Best-System theories (HBSs)?[40]

What underlies them is Lewis’ conception of laws as the systematisation of statements about the “Humean Mosaic” (i.e. the actual/categorical local, only spatiotemporally connected facts making up the world’s history) that strikes the best balance between simplicity, strength (How many phenomena does it cover?) and fit (how exact are the predictions of the systematisation?). These statements can be either deterministic or probabilistic. So, HBSs

[37] Cf. Dürr/Teufel (2008), p. 64
[38] Cf., e.g. Frigg (2008), Ch. 3.3
[39] E.g. Ellis (2014)
[40] Maudlin (2011a), sect. 3; Cf. also Hájek/Hoefer (2006); Frigg/Hoefer (2006)

framework allows for chances even in a fundamentally deterministic world[41] – a conceptual advantage inviting an application to a deterministic dBBT with its probabilistic GE.

What motivates HBSs is to avoid postulating necessary connections and irreducible modalities (about both of which Humeans share their eponym's famous scepticism), as well as a close link between chances and action-guiding credence[42]. Especially attractive with regard to dBBT is that HBSs can accommodate for the QEH as a statement about an initial state of the universe (thanks to the continuity equation for the probability density/flux from the SE), similar to the Past Hypothesis: "[…] [A] stochastic dynamics cannot, by itself, have any implications about what the initial state of the universe is, since that state is not produced by a transition from anything else. But, for the Humean, probabilistic claims are just a means of conveying information about empirical distributions."[43]

Yet, upon closer inspection the optimism that HBS might provide a satisfactory framework for dBBT and its probabilities starts to subside:

- Besides a general worry that HBSs are unable to even provide a metaphysics proper[44], the characterisation of laws as a "best match" between simplicity, fitness and strength is vague: How to flesh out these notions – and their relevant "combination" – in an objective, exact way? One may feel uneasy about this flavour of subjectivism, sticking to this quadruple vagueness – and it's a purely objective, subject-free interpretation of dBBT we'd set out for.
- A more severe problem springs from the non-separable nature of dBBT's wavefunction. To formulate this popular argument, let's inspect Maudlin's more precise contemporary characterisation of Humeanism as the conjunction of three doctrines:

[41] Cf. Hoefer (2011), Frigg (2014)
[42] Cf. Brown (2011)
[43] Maudlin (2011a), p.302
[44] Hájek (2011), sect. 3, for instance, voices this suspicion that HBSs "[…] mistake an idealised epistemology of chance for its metaphysics". And someone desiring a metaphysical framework for dBBT may now interject, it's an objective metaphysics, an attempt to understand and explain its concepts and how a world would look like if it were true, that a fundamental theory such as dBBT calls for: "Humean laws […] can summarise, but not […] explain" Maudlin (2011a), p. 303.

Separability: "The complete physical state of the world is determined by (supervenes on) the intrinsic physical state of each spacetime point (or each point-like object) and the spatio-temporal relations between points."[45]

Physical Statism: "All facts about a world, including modal and nomological facts, are determined by its total physical state."[46]

A third condition specifies the type of facts admissible in Physical Statism: "The intrinsic physical state of the world can be specified without mentioning the laws (or chances, or possibilities) that obtain in the world."[47] The Humean thus abstains from invoking irreducible nomic, modal, dispositional or causal facts in specifying the state of the world.

Entangled states, e.g. the singulet state $\left|\psi_{sing}\right\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$, inherent in generic quantum theories, violate separability[48]; in particular, dBBT does, with (according to Maudlin) the state of the universe being specified by the pair $(\boldsymbol{Q}, \Psi)$. In short, insofar as Humeanism is committed to Separability and an interpretation of dBBT regards the wavefunction as a real, fundamental entity, Maudlin contends that Humeanism and dBBT are incompatible.

Retaining Maudlin's definition of Humeanism, his argument against a Humean interpretation of entanglement-involving quantum theories can be circumvented by demoting the status of the wavefunction to a non-real (more precisely: non-fundamenta) entity[49]. On Humeanism, only the Mosaic and its elements fundamentally exist, i.e. actual, local matters of particular fact in spacetime; all other entities don't: Not having any correlates in fundamental reality, they're useful fictions in our theories to economically summarise or systematise the patterns in the Mosaic, to algorithmically compress the data of the Mosaic. The status of those non-fundamental entities as useful fictions is called "Humean supervenience".

[45] Maudlin (2007), p. 51
[46] Loc.cit.
[47] Op.cit., p. 52
[48] Loc.cit., Ch. 2.1
[49] Cf. Miller (2014); Esfeld (2014), Callender (2015)

By deciding to categorise a certain term as only supervenient, the Humean needn't ban any terms from physical theories as metaphysically illegitimate: Physicists are allowed to continue their business as usual – as long as they don't forget these terms are merely fictional.

Consequently, by declaring the wavefunction supervenient, its non-separability (more precisely, its non-factorisability) becomes innocuous – merely a mathematical peculiarity of a useful fiction, not a feature of reality; the Humean tenet of Separability thus remains unharmed.

In terms of ontological categories, the demotion of the wavefunction to a supervenient fiction amounts treating it as law-like or "nomological", comparable to the status of the Lagrangian in classical field theories. Hence, this ontological re-classification of the wavefunction (in principle, independent from any commitment to Humeanism) is known as the "Nomological View".

Is the Humean Bohmian now off the hook with resorting to the Nomological View? Three objections, we submit, mar her hopes:

- Whilst Humeanism delimits which entities count as fundamental and correspond to something real (viz. the elements of the Mosaic), and which don't (viz. laws, irreducible dispositions, etc.), it procures no criteria for ascertaining *whether* a certain theoretical term should be classified as representing an element of the Mosaic or merely "nomological". In particular, declaring the wavefunction as "nomological" mandates further arguments; just decreeing it in order to preserve Humeanism is question begging.
- Should a term, by fiat declared "law-like", not jibe with characteristics all other *bona fide* laws share, we are all the more entitled to demand good reasons for that fiat. This, we shall see in III.2, is the case for the wavefunction. Downplaying the importance or accuracy of intuitions about laws, as for instance Goldstein and Zanghì parry[50], is a red herring: The issue is not how or whether to revise our intuitions about

[50] Goldstein/Zanghì (2013)

laws, but that the proponent of the Nomological View must proffer arguments for declaring the wavefunction nomological.[51]

- But let's for the time being grant that the wavefunction's status can convincingly be established to be law-like. Within dBBT, this would spawn an odd hierarchy of law-like entities: If the wavefunction is law-like, the SE, as the law that determines the wavefunction, would consequently be "meta-law"-like; in turn, the GE, as the law governing via the wavefunction the dynamics of the corpuscle positions, would ought to be seen as "hypo-law"-like. Both meta-laws and hypo-laws come in two flavours: dynamical (viz. the SE and the GE, resp.) and "static" (viz. the boundary/initial conditions the solution of the SE requires, and the QEH, resp.). Such an explosion of legislature seems unnecessary.

One may try to fend off the preceding objections by construing "nomological" less narrowly: In this vein, Bhogal and Perry[52] suggest that in using the term "law-likeness" one shouldn't be deceived by any received connotations of laws; rather, nomologicality refers to any, merely supervenient theoretical term, regardless of its ontological category as a law, property, etc. In other words: For such a liberalised Humeanism, law-likeness and non-fundamentality are synonymous. (Consequently, even mass or charge in e.g. Esfeld's similar proposal for a liberal "Quantum Humeanism"[53], on which the Mosaic consists only of points in spacetime being either occupied or empty, would count as "law-like".)

However, whether this liberalisation ameliorates the Humean Bohmian's predicament is questionable: The Humean still owes us good arguments that the wavefunction should be regarded as a supervenient entity. The critic of a Humean approach to dBBT may be happy to concede that the wavefunction *could* be nomological, and that consequently such a Humean approach to dBBT is at least conceptually *possible* – but ultimately a critic would demand more than a proof of principle. Neither does a liberal Humeanism *prima facie* curtail the unnecessary proliferation of nomological entities of sub- and superordinate rank.

[51] Callender (2015) illustrates this need for an explicit justification very clearly by pointing to Hamilton-Jacobi Theory to demonstrate that the default argument for the Nomological View, viz. the fact that the wavefunction is defined on configuration space, isn't cogent: Being defined on configuration space, isn't a sufficient condition for "nomologicality".

[52] Cf. Bhogal/Perry (2015)

[53] Cf. Esfeld (2014); Esfeld et al. (2015)

But let's charitably grant that the liberal Humean averts the mentioned problems. Still, she faces yet another counter-argument, which turns on the Bohmian Humean's core claim that dBBT is indeed the best systematisation of the Mosaic. Dewar has recently criticised this claim in detail and its potential defences[54], arguing that the Humean Bohmian shirks their burden of proof of his core claim, with dBBT prevailing also over other approaches to QM, including ordinary QM itself. Here, let's elaborate two lines of thought aiming at very dBBT-specific issues.

- The first one targets the notions of simplicity appealed to when singling out the GE as the allegedly simplest one amongst all empirically equivalent alternative GEs. (We'll return to this underdetermination of the GE in detail in III.1).[55] *Ceteris paribus,* should the Humean Bohmian fail to demonstrate that the GE presented in section II, is the simplest such choice, his own Humean standards would compel him to abandon dBBT in its orthodox form. Humeanism must show the simplicity of the GE, lest it result in a *reductio ad absurdum*. Assaults on dBBT's simplicity can come from three different directions:

  Dürr et al.'s attempts to justify the GE by deriving it from Galilei invariance presuppose that the latter is a necessary *a priori* constraint to impose on any viable dynamics.[56] Whence, though, this distinguished status, especially given that BM's inertial structure is "Aristotelian" (Valentini), rather than Galilean, such as in Newtonian mechanics? An adherent of Brown's Dynamical Approach to symmetry[57], for instance, would furthermore flat-out dismiss that justification as putting the cart before the horse, insisting that the spacetime symmetries be derived from the dynamics/the GE, not the other way around. Quite generally again, one may simply doubt the plausibility of requiring the spacetime symmetries of a less fundamental theory, namely CM, to hold also for the hopefully more fundamental dBBT.

[54] Dewar (2016)
[55] We gloss over here the general vagueness in the notion of simplicity, as is illustrated by Goldstein's "identity-based Bohmian Mechanics", discussed by Esfeld et al. (2015), sect. 3: The Guidance Equation of this identity-based Bohmian Mechanics is considerably more complicated than the one in standard dBBT, "but doesn't necessarily amount to more complicated physics", op.cit., p. 17. Esfeld et al. thus seem to insinuate that in order to assess how simple a theory is, one needs to differentiate between mathematical and some "physical" simplicity, where the latter seems to override the former.
[56] Dürr et al. (2002), Sect. 3
[57] Brown (2007)

A third strategy to undermine the simplicity of the standard GE originates in the non-relativistic limit of the GE, obtained from the Dirac-Bohm Theory (see III.1 and IV.5). It differs from the ordinary GE, despite empirical equivalence. Since the Dirac-Bohm Theory has a considerably larger strength, it seems plausible to consider its non-relativistic limit as "better" in the sense of HBSs. Consequently, dBBT with the standard GE doesn't count as the Best System; hence, the Humean should reject standard BM.

- The underdermination of the GE menaces the Humean Bohmian even more severely: As we'll argue in the subsequent section, the GE is empirically inaccessible, vastly underdetermined and serves no explanatory function – a strong incentive for the Humean to simply abandon the GE, and thereby dBBT altogether, in favour of a statistical theory of random corpuscle jumps – an option we'll explore in detail in section III.

Dewar poignantly adumbrates the same idea: "The problem is that our evidence for quantum mechanics is (famously) *statistical* in nature. It is not that we have direct access to some small number of the Bohmian trajectories, and have successfully stitched those together by overlaying a wavefunction governed by quantum dynamics. What we have instead are individual but imprecise measurements of positions at particular times. […] So what we have really woven together into a quantum tapestry are those probability densities, rather than the trajectories themselves; and *on the Bohmian's own account*, those probability densities represent all that can ever be known for sure about the trajectories."[58]

In short, a Humean, upon re-examining the GE, will have good reasons *not* to consider dBBT the simplest systematisation; a Humean couching of dBBT thus seems self-defeating.

After this brief *tours d'horizon* into Humeanism, we find the latter, too, a dead end for a satisfactory framework for dBBT in general, and its probabilities in particular.

II.3.3. Digression: Objectivism and Heisenberg relations

Germane to dBBT's problems with probabilities and the conflict with its objectivist framework cropping up, is the standard account of the Heisenberg *indeterminacy* relations in dBBT: For

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[58] Dewar (2016), p.13 (Dewar's emphases)

roughly known corpuscle positions their wavefunctions are sharply peaked. "[…] The initial randomness of the particle position translates into the randomness of the particle's asymptotic velocity, […] given by the modulus squared of the Fourier transform of the [...] wave packet. That distribution is all the more spread out as the initial wave packet is sharply localised. This is Heisenberg's uncertainty relation. […] Quantum Equilibrium entails absolute uncertainty about the Bohmian positions."[59]

Two observations regarding this passage are in order:

- An *epistemic* category, uncertainty, invades here the otherwise objective/subject-free account. Other standard reference on Quantum Equilibrium are even more explicit in this regard, identifying the latter as the "origin of absolute uncertainty", conveying the "*most detailed* knowledge *possible* concerning the present configuration of a subsystem (of which the 'observer' or 'knower' is not part […]."[60]

  This starkly contrasts with the objectivism officially professed by many of dBBT's proponents. E.g. Bell red-flags "[…] some words which, however legitimate and necessary in application, have no place in a formulation [of QM] with any pretension to physical precision: *system*, *apparatus*, *environment*, *microscopic*, *macroscopic*, […] *observable*, *information*, *measurement*."[61]
- Secondly, note in Dürr/Teufel's cited account their remark that corpuscles' initial positions are *randomly* distributed.

  Is it actually meaningful to apportion randomness to initial conditions? *Prima facie*, only stochastic processes, e.g. Poisson processes such as radioactive decay, can display randomness; by contrast, initial conditions are brute fact *data*. Moreover, even if one grants the meaningfulness of randomness in initial conditions, we're back in our previous pickle: How can there be randomness in dBBT's *deterministic* world? The

[59] Loc.cit., p. 223

[60] Dürr et al. (2003) , p. 57 (the authors' emphases); cf. also Dürr et al (1995), p. 27.

Dürr et al. (2003), Ch. 11, 12, try to somewhat mitigate the charge of smuggling in epistemic/subjectivist elements: "Whatever we may reasonably mean by knowledge, information, or certainty – and what precisely these do mean is not at all an easy question – it simply must be the case that experimenters, their measuring devices, […] must be a part of or grounded in the environment of these systems. The possession by experimenters of such information must thus be reflected in correlations between the system properties to which this information refers and the features of the environment which express or represent this information. We have shown, however, that given its wave function there can be no correlation between (the configuration of) a system and (that of) its environment, even if the full microscopic environment […] is taken into account" (loc.cit., p. 57).

[61] Bell (1990), p. 34

same authors paradoxically negate that: "It looks as if objective chance is at work, while in truth it is not. There is no chance."[62] How to resolve this contradiction?

Pledging allegiance to its objectivist outlook, might perhaps we re-cast dBBT – or at least, its essential ingredients – as a stochastic/indeterministic theory and thereby resolve the conundrums?

III. Stochastic deBroglie-Bohm Theory (sdBBT)

III.1. Role of the Guidance Equation

Evidently, the GE curtails a stochastic re-conceptualisation of dBBT. The elephant in the room now is: Might it be dispensable? We'll argue in this and the following two sections, it's indeed not only explanatorily redundant, but also spawns a few sources of discomfort.

Let's scrutinise first some of dBBT's often perceived blemishes that involve the GE:

- An immediate inquiry concerns the latter's formal definability.
  In general, a wavefunction possesses zeros. Consequently, for some initial conditions, the GE ($\boldsymbol{Q}_i \propto \mathfrak{Im}\left\{\frac{\nabla_i \psi}{\psi}\right\}$) will steer corpuscles into that set of zeros, where the GE becomes singular. Thus, it isn't well-defined for all of configuration space.
  Although the set of initial conditions that lead to singularities is of measure zero[63], this poses no satisfactory resolution of the problem – a rehearsal the measure-zero problem in SM[64]. In particular, as we saw in II.3.1, sets of measure zero needn't be negligible or even small.
  In consequence, the GE pares down dBBT's domain of applicability.
- Another formal discontent is levelled at GE's non-uniqueness: Are different GEs possible? Indeed, the orthodox GE given above, $\dot{\boldsymbol{Q}} = \frac{\boldsymbol{j}}{\rho}$ with the quantum probability flux $\boldsymbol{j} = \left(\frac{\hbar}{2im_k}\mathfrak{Im}\{\psi^* \boldsymbol{\nabla} \psi\}\right)_k$ and the probability density $\rho = \psi$, is but one of an *infinitude* of viable alternative dynamics[65]: Any other choice $\boldsymbol{j}' = \boldsymbol{j} + \boldsymbol{\jmath}$, with a

[62] Dürr/Teufel (2009) p.64.
[63] Cf. Berndl et al. (1995)
[64] Cf. Frigg (2008), Ch. 3.4, esp. pp. 125
[65] Cf. also Deotto/Ghirardi (2002)

divergence-free vector field $\boldsymbol{j}$, $\mathrm{div}\,\boldsymbol{j} = 0$, for the quantum probability is equally fine and compatible with the QM prediction. Yet, the resulting trajectories will differ. In other words, the choice of no less suitable GEs/dynamics for deterministic trajectories is vastly underdetermined. Which is the right one?

The orthodox GE has been argued to be the simplest such choice compatible with Galilean and time-reversal invariance.[66]

But this is problematic: Firstly, because appeals to simplicity are glaringly hand-waiving: Why should nature care about our preconceptions of simplicity? Secondly, as we saw in II.3.2, the motivation for demanding Galilei invariance is doubtful: It's true, though, that the Schrödinger Equation *turns out* to be Galilei invariant (provided the wavefunction transforms in a particular, non-scalar way), but why expect that from the GE, as well? This seems particularly in need of further arguments, since the motivation of Galilei invariance lies in the Galilean spacetime structure of CM and dBBT seeks to supersede CM: So, why should we assume dBBT's GE to inherit CM's spacetime structure? Thirdly, relativistic generalisations of dBBT in fact speak *against* this putatively simplest choice of the GE: Plugging the quantum probability 4-current of the Dirac equation (see section IV.5.4), i.e. $j^\mu = c\psi^\dagger\gamma^\mu\psi$ (with the Dirac matrices γ^μ, and the Dirac-spinor ψ), into $\dot{\boldsymbol{Q}}^i = \frac{\boldsymbol{c}\boldsymbol{j}^i}{j^0}$ (i=1,2,3), one obtains a relativistic GE for fermions. Its non-relativistic limit[67], however, is *not* our orthodox GE (using spinors which satisfy the Pauli Equation[68]), but contains a spin-dependent additional divergence-free term.

What lesson to draw from this underdetermination of the GE? It seems to betray that the concrete trajectories *per se* play no role in the theory, as long as they are compatible with the Schrödinger dynamics. dBBT's determinism produces no tangible insights into a more fundamental subquantum world – a dead end Einstein seems to have intuited, when dismissing dBBT in a letter to Born form 1952 as "too cheap", likely for the reason that "we would not have discovered statistical mechanics by adding

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[66] Cf. Dürr/Teufel (2009), Ch. 8.1

[67] Cf. Holland/Philippidis (2003). The difference vis-à-vis the original, non-relativistic GE amounts to exactly such an added divergence-free vector-field.

[68] Cf. Dürr/Teufel (2009), Ch. 8.4; Holland (1993), Ch. 9

small corrections to thermodynamics, or by adding hidden variables that were in some way 'guided' by the free energy, or some other thermodynamic quantity."[69]

We can substantiate the suspicion that the GE is explanatorily idle even further:

- As mentioned earlier, dBBT is empirically indistinguishable from QM. How is this achieved? The equivalence in no way depends on the GE.[70] Let's recall the two ingredients for ensuring empirical equivalence with ordinary QM:
    - The BR delivers the right probability for finding a dBBT-corpuscle at a certain position. For position measurements, this coincides with the predictions of standard QM.
    - dBBT's ontology procures the rest: In dBBT, there are *no* dynamical properties other than position; what we usually interpret as such properties, e.g. momentum or spin, are only manifestations of the wavefunction and how it guides the particles positions. When it comes to observable effects, though, the statistics obeys the BR.

  In consequence, the empirical content of dBBT is independent of the deterministic trajectories of the dBBT corpuscles; the GE eschews empirical accessibility.

  Remark: It's instructive to note how dBBT circumvents the two principal no-go theorems for hidden variables theories[71]: Since all the theoretical work is done by the wavefunction, which enters both the GE and the BR, and the latter is "non-local" in that it involves the non-separable/non-factorisable wavefunction, Bell's Theorem is automatically satisfied. As for contextuality, as enforced by the Kochen-Specker-Theorem, it's explicitly implemented in the Bohmian ontology, which construes only position as the only "non-contextual" variables.

- Let's look then beyond empirical equivalence with QM – at the Measurement Problem and how to interpret the macroscopic superpositions the minimally interpreted

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[69] Squires (1996), quoted in Passon (2005), fn. 11

[70] The "familiar statistical description of sub-systems in terms of effective wave-functions […] is really all that matters for most practical purposes", as Esfeld et al. (2015) remark.

[71] Wallace (2008), sect. 2.6.2, cashes out four empirical constraints on any hidden variables theories. The GE only features in the fourth constraint, which requires that hidden variables via their dynamical equation be affected only by "their" branch of the state vector. Crossing out the GE altogether, this constraint is trivially satisfied.

quantum formalism seems to predict. Motivating to a considerable extent the hidden variables agenda, the Measurement Problem more precisely consists in the mutual inconsistency of the three assumptions that the wavefunction is complete, that it evolves in accord with the Schrödinger dynamics and that measurements yield determinate outcomes, respectively.[72]

dBBT has been argued to its most convincing solution – by denying that the wavefunction alone completely specifies the physical state: [73] Only the pair wavefunction *and* corpuscles' position, $(\Psi, \boldsymbol{Q})$, (rather than the wavefunction alone, as e.g. in the Many Worlds Interpretation[74]) represents the *complete* state of the system: Since the positions of the corpuscles are definite at any instance, so is the state of each system – including the various pointer positions of measurement devices. In other words: Although the wavefunction of a measurement device, too, in general is in a superposition, the value-definite/-determinate configuration of the corpuscles of which the device is composed, picks out a definite measurement outcome.

At no point does determinism, i.e. the GE, enter the stage: All the work is done by the presupposed ontology with its position-definiteness and wavefunction-mediated contextuality w.r.t. all other dynamical variables. The GE thus is irrelevant to dBBT's solution of the Measurement Problem.

This is not to downplay a related, but distinct question Maudlin calls the "Problem of Effect": "The result of a measurement […] has predictive power for the future: after the first measurement is completed, we are in a position to know more about the outcome of the second than we could before the first measurement was made."[75] Indeed, the GE accounts for the Problem of Effect[76], allowing for information of the measurement to propagate into the future. As we shall argue in detail in section IV.3, abandoning the GE, and stomaching the absence of a solution to the Problem of Effect is of no empirical or pragmatic consequence: We are licenced to dispense with the requirement that "(t)he result of the first measurement (be) not codified in the subsequent wavefunction […]".[77]

[72]Cf., for instance, Maudlin (1995)
[73]Cf. Dürr/Teufel (2009), Ch. 9 or Esfeld (2014)
[74] Cf., for instance, Wallace (2008), Ch. 2.4
[75] Maudlin (1995), p. 13
[76] Cf. Albert (1992), pp. 147
[77] Maudlin (1995), p. 14

- It has been argued that the GE in some sense explains dBBT's peculiar ontology, in which corpuscles have positions as their only non-contextual dynamical variables: "Bohmian Mechanics should be regarded as a first-order theory, in which it is the velocity […] that is fundamental in that it is this quantity that is specified by the theory, directly and simply […]. […] This is not to say that these second-order concepts [viz. acceleration, force, work and energy] play *no* role in Bohmian mechanics; they are emergent notions."[78]

  However, an ontology cannot be simply extracted from the formalism. It follows from axioms that must be postulated separately[79]; a bare formalism never provides its own interpretation or ontology. Consequently, the formalism cannot explain (in any deductive sense) the ontology: Rather, the two in some cases match better than in others, with some ontological stipulations appearing more plausible than others.

  How well motivated, one may immediately wonder, is dBBT's ontology with its beable status of positions and contextuality of all other variables? We concur here with Bell, who praises its naturalness and cogency[80] without any reference to the GE: "[…] in physics the only observations we must consider are position observations, if only the positions of instrument pointers. It is a great merit of the deBroglie-Bohm picture to force us to consider this fact. If you make axioms, rather than definitions and theorems, about the 'measurement' of anything else, then you commit redundancy and risk inconsistency."[81]
- Does the GE at least help us explain and grasp quantum phenomena? One may well challenge that, arguing that the GE merely garnishes the QEH with a "fictitious determinism"[82] which doesn't enhance our understanding:

[78] Dürr et al. (1995), pp. 7 (Dürr et al.'s emphasis).
Originally, the argument is intended as a criticism of the 2nd-order "quantum potential" formulation of dBBT, espoused e.g. by Holland (1993).
[79] Cf. Bunge (1967); Esfeld, passim
[80] Daumer et al. (1997) elaborate Bell's idea that the *non-contextuality* of variables other than position is rooted in a naïve realism about operators.
[81] Bell (1982), p. 166
[82] Englert (2001) in his review of the German version of Dürr/Teufel (2009) (D. Dürr: "Bohmsche Mechanik als Grundlage der Quantenmechanik", Springer, 2001)

- The idiosyncrasies of dBBT's ontological framework, with its dualist wavefunction-point particle ontology, contextuality, etc., drastically depart from any classical picture, to begin with.[83]
- Also from a less philosophical angle, the "surrealistic" trajectories are misleading: "The Bohm trajectory is […] macroscopically at variance with the actual, that is: observed track"[84]; semi-classical dBBT trajectories in semi-classical situations differ strongly from classical trajectories.[85]

Einstein's objection to dBBT from 1953 takes up this point (already articulated by Pauli much earlier in regards to deBroglie's original pilot-wave theory from 1927) and couples it to a methodological principle.[86] He considers a one-dimensional particle in a perfectly elastic box of length L, centred around zero. Inside the box, the corresponding energy eigenfunctions are superpositions of plane waves travelling to the right and left, respectively:

$$\varphi_n(x,t) = \begin{cases} 2 \nmid n\colon \dfrac{1}{\sqrt{2\mathrm{L}}}\cos\dfrac{n\pi x}{L}\, e^{-i\frac{\hbar n^2\pi^2}{2mL^2}t} \\ 2|n\colon \dfrac{1}{\sqrt{2\mathrm{L}}}\sin\dfrac{n\pi x}{L}\, e^{-i\frac{\hbar n^2\pi^2}{2mL^2}t} \end{cases}.$$

The GE then yields a vanishing velocity at all times – a result that "contradicts the well-founded requirement that in the case of a macro-system the motion should agree approximately with the motion following from classical mechanics."[87]

Similarly, in the case of the ground state of the hydrogen atom, due to its real-valued wavefunction the shell electron, according to dBBT, is at rest[88] – again, at variance with the demand that in the macro-limit one should approximately recover the classical motion.

For the moment, we postpone the discussion of two further types of blemishes to the next subsection. May the aforementioned shortcomings suffice now as a motivation to jettison the redundant GE, leaving us with the dBBT ontology (position determinateness and

[83] Cf. Esfeld et al. (2014)

[84] Englert et al. (1992), quoted in Passon (2005), p. 8, who critically discusses this argument. Note, though, that this discrepancy between the "surrealistic" dBBT trajectories and the "observed tracks" doesn't amount to any *empirical* deviation of dBBT from the well-confirmed quantum mechanical predictions, cf. ibid.

[85] Cf. Matzikin/Nurock (2003)

[86] Cf. Myrvold (2003), esp. sect. 3.1. See also Holland (1993), Ch. 6.5. Both authors criticize these objections.

[87] Einstein (1953), quoted in Myrvold (2003), p. 10

[88] Cf. Holland (1993), Ch. 4.5.

contextuality) and the QEH/BR, whose conjunction alone warrants that all predictions of ordinary QM are reproduced. We dub the resultant theory "stochastic deBroglie-Bohm Theory"(sdBBT) – a name to whose deliberate choice we'll turn in III.3.[89]

In the following, let's take sdBBT seriously – as a fundamentally stochastic micro-theory:

- Corpuscles no longer are assigned continuous, deterministic trajectories, picked out by specifying initial conditions.
- They only have a localisation probability, equal to the BR-probability, quantifying – in a manner to be made precise in the subsequent subsection – the corpuscles' random walk through configuration space: The corpuscles spontaneously jump between possible positions.
- Their random localisations notwithstanding, *at every instance* they have definite positions. All other dynamical properties, like in the standard dBBT ontology, which sdBBT inherits from dBBT, are only derivative/contextualised.

III. 2. Probabilities in sdBBT

Let's examine in more detail now sdBBT's probability space $(\mathbb{R}^{3\mathrm{N}}, \mathcal{A}, \mathbb{P})$, with configuration space $\mathbb{R}^{3\mathrm{N}}$ as the so-called sample space, the σ-algebra[90] of "events" $\mathcal{A}$ generated by $\mathbb{R}^{3\mathrm{N}}$ (i.e. the Borel set) and the Born-probability measure $d\mathbb{P} = |\psi|^2 d^{3N}\boldsymbol{Q}$.[91] We propose the following (non-Popperian[92]) propensity interpretation[93]:

- The sample space is the set of *possible* configurations an *N*-corpuscle universe can occupy.
- As the most "literal" ontology for the probabilistic sdBBT is a "world of propensities" (Popper), we propose the following: Rather than being uniquely determined in their spatiotemporal evolution by initial conditions, the corpuscles possess only a tendency

[89] Bell (1987), Ch.5 considers the same idea as a characterization of the Everett interpretation. We take up the question of whether this identification is justified or not in III.3.
[90] Certain mathematical difficulties prohibit the definability of the probability measure on all partial sets of the sample space. With a σ-algebra of events one constructs a smaller, but simultaneously still sufficiently rich set on which to define the probability measure.
[91] For all formal/mathematical issues, cf., e.g., Georgii (2009), esp. Ch. 1.
[92] Propensity views can vary considerably w.r.t. their referents (propensities *of* what?), their content (propensities *for* what?) and their ontological status (e.g. dispositions?), cf. Torretti (1990), Ch.4.
In particular in the first two respects our propensity view differs from Popper's, which takes propensities to be dispositions of the whole experimental setup to produce long-term frequencies.
[93] We draw partially on Bunge (2011), Ch. 4.

(disposition or propensity) to spontaneously, randomly materialise or jump into existence in a certain configuration at a certain instance in time: The manifestation of the disposition is the localised corpuscle configuration, which we register as frequencies. (We don't assume – and in fact expressly reject, see IV.3 and IV.5.2 – the identity of the corpuscles between different localisations: Corpuscles don't persist in time, lacking diachronic identity.) Thus, a change in a corpuscle's position no longer requires the presence of a triggering mechanism – as a literal reading of sdBBT's formalism seems to call for.

Note that the corpuscles' spontaneous localisation isn't a "collapse of the wavefunction": The disposition continues to evolve unitarily also after its manifestation.

The probability measure *quantifies* the strength of such a propensity[94], with a configuration of zero probability possessing minimal propensity to happen. That doesn't mean, though, that the configuration is impossible. (Recall that generally, for every continuous probability distribution each single, *possible* outcome has probability zero.) Conversely, a configuration with probability 1 isn't necessary/inevitable[95]; it only has the greatest tendency to become actualised. (Dispositionalism suggests a modification of the notion of (nomological) impossibility: An event may be said to be impossible, iff it cannot be assigned a probability value. Adopting this notion of nomological possibility allows us to pre-empt a metaphysical objection against sdBBT's ontology: namely that it postulates an incessant *creatio ex nihilo*, with the corpuscles popping into/out of existence from/back into absolute nothingness, respectively. Such spontaneous materialisations and de-materialisations apparently contradict the received metaphysical principle of substance conservation, i.e. that substances can neither spontaneously emerge nor perish.[96] Dispositionalism avoids the conflict with that principle, though, since firstly the N corpuscles' disposition to localise themselves stretches throughout all of space: The disposition is everywhere (not, of course, its manifestation). Secondly, the number of corpuscles is also conserved: At any instant, there are always N actual corpuscles. Hence, the corpuscles are not popping into /out

[94] The status of the wavefunction may be seen as the gauge field (analogous to the electromagnetic 4-potential in electromagnetism) with the corresponding physical field (analogous to the Faraday tensor).
[95] Cf. Bunge (2011), sect. 6.3.
[96]

of existence out of/into absolute nothingness or empty space. One may, of course, criticise the idea that a disposition elicits spontaneous, random manifestations – but that's a general scepticism of a dispositional metaphysics, not specific to sdBBT.)

Advocating above a single-case propensity interpretation, it now behoves us to address the canonical objections against single-case propensities – an opportunity also to elucidate some of sdBBT's salient features.

- The Reference Class Problem[97] arises for propensity interpretations when, as in Popper's propensity theory from 1957, probabilities are assigned to an experimental setup of physical conditions that generate the observed outcomes: The propensity of an event is thus always *relative* to the generating conditions. These, however, can be incorporated in various, *different* classes. Consequently, one cannot unambiguously assign an event its propensity.
  While the Reference Class Problem notoriously plagues frequentism and long-run propensities, sdBBT – like most variants of QM[98] – escapes it[99]: The system's wavefunction completely determines the dBB-propensities; no further facts of the system as "generating conditions" are necessary. Such propensities are understood as quantities inherent in the corpuscles.
- Hájek criticises this manoeuvre to circumvent the Reference Class Problem as exacting the price of a vacuous notion of propensity:[100] Indeed, propensity approaches are often dismissed as pseudo-explanatory of the type a quack physician dishes up in Molière's *Le Malade Imaginaire*, when "explaining" the sleep-inducing powers of opium through a *virtus dormitiva*.
  This complaint, however, rests on a misunderstanding: Quantification, explanation and interpretation are distinct conceptual operations.
  - Propensity accounts don't pretend to be explanations (i.e. answers to: "How can we *derive* the BR-probabilities from a certain metaphysical or physical

[97] Cf. Hájek (2006)
[98] Cf., e.g. Galavotti (2001)
[99] Cf. Frigg/Hoefer (2010), sect. 3 and 5. The argument carries over verbatim.
[100] Cf. Hájek (2006), pp. 25

theoretical framework" – in the manner of, say, Robb's (failed) attempts to derive the metric structure from causal relations[101]).

- Rather, propensities intend to interpret and ontologically *ground* probabilities: "What could the concept of probability refer to in a world described by sdBBT?" In sdBBT, propensities are inherent, dispositional, real quantities – figuring as truthmakers of probabilistic statements.
- In turn, probabilities formalise and quantify the vaguer, pre-theoretical, qualitative and ontological notion of a propensity: "How to render the concept of a propensity sufficiently mathematically precise?" To this end, Kolmogorow's axioms are imposed as formal desiderata, whose motivation (esp. that of σ-addivity) shall not concern us here. Once imposed, probabilistic statements, thus rendered quantitative, can subsequently be subjected to empirical tests.[102]

- Arguably "devastating for views that take propensities to involve weakened or intermittent causation"[103] is Humphreys' Paradox. It boils down to the incompatibility between inverse conditional probabilities, understood as propensities, and the time-asymmetry of causality.
  Consider a partition $\{B_i : i \in I\}$ of the sample space of physical states of some system, $\Omega = \bigcup_{i\in I} B_i$, with $\forall i \neq k: B_i \cap B_k = \emptyset$, and let $\mathrm{A} \in \Omega$ be an event. Then, Bayes' Theorem purports: $\forall i \in I: \mathbb{P}(B_i|A) = \frac{\mathbb{P}(A|B_i)\mathbb{P}(B_i)}{\sum_{j\in I}\mathbb{P}(A|B_j)\mathbb{P}(B_j)}$. What does this signify in terms of propensities?
  It seems natural to interpret the conditional probability $\mathbb{P}(A|B_i)$ as the propensity of the system to undergo the transition $B_i \rightsquigarrow$ A. This suggests to understand B_i as the cause of the effect A, and hence conditional probabilities as representing a probabilistic form of causation.
  The conditional probability for the inverse transition, $\mathbb{P}(B_i|A)$, definable via Bayes' Theorem, would thus quantify the tendency for the effect A to *bring about* the cause B_i.

[101] Cf. Sklar (1992), pp. 83
[102] Cf. Georgii (2011), Ch.10 &11; Gillies (2000), Ch. 6,7; Beisbart (2011), esp. 4.2; Suárez (2014)
[103] Eagle (2004), p. 402

In consequence, not only would smoking cause lung cancer, but also conversely would lung cancer cause smoking. This seems absurd: Only causes produce their effects, not vice versa; causal chains of events cannot be inversed. Probabilities therefore, the argument concludes, cannot be understood in terms of propensities.

A closer inspection of the paradox is apposite. Three groups of posits enter it:

(i) We adopt standard probability calculus, with probability measures obeying Kolmogorow's axioms.

(ii) The second group of posits comprises assumptions about the nature of propensities (denoted by $\mathfrak{P}$, time-indexed w.r.t. $t_1 < t_2 < t_3$) in a causal process $A(t_1) \rightsquigarrow B(t_2) \rightsquigarrow C(t_3)$ (i.e. chain of events, A, B and C):

   a. The propensity of the process $A(t_1) \rightsquigarrow B(t_2)$ is nontrivial:
   $$1 > \mathfrak{P}_{t_1}(B(t_2)|A(t_1)) > 0$$

   b. The propensity of the causal chain isn't minimal :
   $$\mathfrak{P}_{t_1}\big(C(t_3)|A(t_1) \wedge B(t_2)\big) > 0.$$

   c. In the absence of B, the process $A(t_1) \rightsquigarrow C(t_3)$ has minimal propensity:
   $$\mathfrak{P}_{t_1}\big(C(t_3)|A(t_1) \wedge \neg B(t_2)\big) = 0.$$

   d. Future events are causally neutral/irrelevant for past events:
   $$\mathfrak{P}_{t_1}\big(B(t_2)|A(t_1) \wedge C(t_3)\big) = \mathfrak{P}_{t_1}\big(B(t_2)|A(t_1) \wedge \neg C(t_3)\big) = \mathfrak{P}_{t_1}\big(B(t_2)|A(t_1)\big)$$

(iii) The last posit bridges *(i)* and *(ii)*, with an identity thesis, according to which all propensities $\mathfrak{P}_t$ can be uniquely associated with probabilities, with the strength σ of the propensity being identified with the probability:
$$\mathfrak{P}_t \to \mathbb{P}_t, \forall x\colon\ \mathfrak{P}_t(x) \mapsto \sigma[\mathfrak{P}_t(x)] \equiv \mathbb{P}_t(x)$$

Humphreys' Paradox now consists in the inconsistency that arises from the conjunction of *(i)*, *(ii)* and *(iii)*. Which premise therefore to relinquish in order to overcome the contradiction?

In contradistinction to Humphreys himself, we wish to remain as conservative as possible, and retain the standard probability calculus.

Accepting the causal intuitions of propensities captured in *(ii)*, Suárez has pled for rejecting the identity thesis[104] – in particular, its first part: Not all probabilities are amenable to a propensity interpretation; some conditional probabilities in particular don't refer to factually possible transitions in the world. This isn't unfamiliar from other theories whose formalism also treats physically impossible situations, forbidden by extra-mathematical selection rules. E.g. in standard QM, *N*-particle wavefunctions transform either symmetrically or anti-symmetrically under permutations; *formally*, nothing prevents mixed-symmetric transformation behaviour. Nature, though, doesn't seem to realise this option.

In their defence of propensity interpretations in GRW collapse-theories, Hoefer and Frigg have indeed abolished the identity thesis by asserting "that GRW propensities are all forward-looking in time"[105] – a "response that any advocate of objective quantum probabilities will wish to make."[106]

In addition to abandoning the identity thesis (iii), sdBBT compels us to abandon *(iic)*: It captures the intuition that in a causal chain $A(t_1) \rightsquigarrow B(t_2) \rightsquigarrow C(t_3)$ the intermediary event *B* is indispensable; i.e. in *B*'s absence, the chain has minimal propensity to occur. In sdBBT, however, the corpuscles' configurations spontaneously and randomly jump from one instant to another, independently of previous configurations; sequences of configurations are no longer causally connected. In other words, there exists no triplet of events that satisfies (iic); in sdBBT (iic) is violated. Humphreys' Paradox thus lapses.

Spurred by those doubts regarding the conceptual tenability of propensity interpretations *simpliciter*, as well as perhaps by discomfort about the lack of *direct*, empirical falsifiability of propensities[107], one might generally baulk at our invocation of a metaphysics of *dispositions*. What swayed us? We'll consider first reasons to regard the wavefunction real ("wavefunction

[104] He furthermore makes a case against conversely identifying all propensities with probabilities.
[105] Frigg/Hoefer (2007), p.385
[106] Loc.cit.
[107] As well as the "inferential link" between frequencies as estimates for probabilities and the "decision-theoretical link" concerning the role probabilities play in decisions and action-relevant choices, cf. e.g., Brown (2011), esp. sect. 3.
Such issues are intimately tied up with the debate between Humeanism and Governing Law Approaches, which lies outside our current scope. We refer to e.g. Maudlin (2007), esp. Ch. 2 for further details.

realism"); we'll then argue that viewing the wavefunction more specifically as a real, *dispositional* quantity overcomes the key problems of wavefunction realism.

Above, we declared the corpuscles' propensity a *real/*physical property, represented by the wavefunction. This wavefunction realism naturally accounts for the wavefunction's contingency, structural complexity and time-evolution[108]:

- The wavefunction has dynamical degrees of freedom of its own, governed by the SE and dependent on initial/boundary conditions. Varying across different worlds that differ in those conditions, the wavefunction consequently represents a contingent quantity – as opposed to absolute objects, which don't vary across possible worlds, e.g. the Minkowski spacetime metric in Special Relativity. Contingency is seen as characteristic of real entities – as opposed to merely conventional ones.
  As Brown and Wallace suggestively point out, "(h)istorically, it was exactly when the gravitational and electric fields began to be attributed independent dynamics and degrees of freedom that they were reified: the Coulomb or Newtonian 'fields' may be convenient mathematical fictions, but the Maxwell field or the dynamical spacetime metric are almost universally accepted as part of the ontology of modern physics."[109]
- Its contingency also implies that we can to some extent control the wavefunction, insofar as we can prepare a physical system via initial and boundary conditions. This controllability speaks strongly against a nomological interpretation of the wavefunction[110], which conceive of the latter as a law. As Goldstein/Zanghì admit: "And laws are not supposed to be things that we can control – we're not God."[111]

[108] Cf. Brown/Wallace (2004), pp. 12
[109] Loc.cit.
[110] E.g. Esfeld et al. (2014)
[111] Goldstein/Zanghì (2013), p.99. Their counterargument is "[…] that there is only one wave function we should be worrying about, the fundamental one, the wave function Ψ of the universe" (loc.cit.), and that the latter is not controllable. Taking up their invocation of God, one needs to distinguish between two forms of Divine intervention: On the one hand, God's thaumaturgical ability to suspend the extant laws of nature, and on the other hand his *weakly* and *strongly* demiurgical ability to create worlds: The former refers to the ability to create distinct nomologically possible worlds, differing only w.r.t. contingent elements, such as initial/boundary conditions, while the non-contingent ("nomological") elements remain the same; strong demiurgy refers to God's ability to create worlds in which different laws of nature hold.
The kind of uncontrollability on which the intuitive notion of a law turns is weak demiurgy. In consequence, assuming that He shares our ontological intuitions, God, too, arguably would reject the nomological view.

- Like other real quantities, such as the metric in General Relativity, subject to a set of 16 coupled hyperbolic-elliptic nonlinear PDEs, sdBBT's wavefunction-dependent propensity is structurally very rich.

  In both sdBBT and dBBT, this structural complexity mirrors its ontological importance in that the wavefunction is the only property (other than positions) that can be assigned to corpuscles.
- sdBBT's propensity evolves in time according to the SE. It seems unnatural to regard something that thus changes over time as not real; in fact, mutability has even been proposed as the essential difference between purely conceptual/"Platonic" *abstracta* and physical *concreta.*[112]

Despite indeed suggesting that the wavefunction represents a *real* entity (rather than, say, a law-like *abstractum*, as in the nomological interpretation) – Brown and Wallace's "reification" argument remains still silent on the ontological category under which to subsume the "reified" entity: Is it a *substance* or *property*?

For an answer within a traditional ontological framework, let's analyse the extent to which the wavefunction carries itself properties or not; the former would suggest its status as a substance.

Given the dependence of the wavefunction – as a solution to the SE – on parameters such as mass or charge, Holland has proposed that the wavefunction itself should be regarded as massive, charged etc.[113] Discussing neutron interferometric thought experiments, Brown et al. elaborate this proposal to attribute the state-independent, non-dynamical properties mass, charge, magnetic moment, etc. to *both* the wavefunction and the Bohmian corpuscles.[114]

One of the core features of the traditional notion of substance[115] is "ultimate subjecthood" (i.e. being predicable without itself being predicated of something else). It suggests that the wavefunction should be regarded as a substance, yielding, *prima facie, two* substances, Ψ and the corpuscles. One may find this substance-dualism already unpalatable enough.

[112] Cf. Bunge (1981)
[113] Cf. Holland (1993), pp. 78
[114] Cf. Brown et al. (1995); Brown et al. (1996), sect. 3 and 4
[115] Cf., e.g., Kuhlmann (2010), Ch. II.7

Ontologically yet more disconcerting is that in dBBT the corpuscles, albeit ultimate subjects of position ascription, ignore another traditional category feature of substances, viz. independence: Via QEH and the GE, respectively, the corpuscles' initial configurations and behaviour are determined by the wavefunction. (A converse dependence of the wavefunction on the corpuscles doesn't obtain – an issue we'll discuss in III.3.). In consequence, dBBT's ontology is a substance-dualism consisting of one *bona fide* substance, viz. the wavefunction, and one entity, viz. the corpuscles, whose ontological category is ambivalent; the fact that the wavefunction and the corpuscles arguably share the state-independent properties mass, charge, etc. further exacerbates this ontological uneasiness.

We submit, sdBBT's commitment to a metaphysics of dispositions ("dispositionalism") offers a neater ontological picture: While concurring with the aforesaid proposal to ascribe the non-dynamical properties also to the wavefunction, sdBBT needn't classify the latter as a substance: The wavefunction represents corpuscles' disposition of spontaneously popping into existence, both localising themselves at certain sites with certain masses, charges, magnetic momenta, etc. Thus, sdBBT's corpuscles are the *only* substances. They are ascribed properties, which fall into two inter-related types – the disposition of spontaneous materialisation, represented by the wavefunction, and the corresponding manifestation of that disposition of the corpuscles to randomly localise themselves at certain positions, with certain state-independent properties. That the wavefunction is defined on configuration space, and that the corpuscles and the wavefunction can both be attributed the state-independent properties, now simply reflects the ontological dependence between dispositions and their manifestations.

To some, our subscription to such a "revisionary metaphysics" (Strawson) that posits also dispositions, rather than exclusively categorical properties, may seem appalling, e.g. on the grounds of dispositions defying direct empirical access – an objection, however, that applies *generically* to theoretical terms and runs into the pitfalls already the Logical Positivists had to face with their observational/theoretical dichotomy.[116]

Dispositionalism has merits both specifically in the germane context of GRW theories[117] (arguments that carry over *verbatim* to sdBBT*)*, as well as generally in its own right. Amongst

[116] Cf., for instance, Suppe (1974), esp. pp. 85

[117] Cf. Dorato/Esfeld (2010); for a slightly different view, cf. Frigg/Hoefer (2006), esp. sect. 5

them, the literature, to which we refer for details[118], extolls the following: "[…] a clear sense in which quantum systems in entangled states possess properties even in the absence of definite values; […] a clear transition from quantum to classical properties; […] a clear transition from quantum to classical structures; and […] [the grounding of] the arrow of time."[119]

The crucial advantage of dispositionalism in our context at hand, we submit, is that the dispositional character of the wavefunction can straightforwardly account for the 3*N*-dimensionality of the configuration space, on which the wavefunction is defined.[120]

By means of contrast, consider two alternate proposals:

- "Configuration Space Realism" regards the wavefunction as a real object in configuration space. It faces "the Problem of Perception": If the fundamental quantum world lives in 3N- dimensional configuration space, how is this compatible with the world ostensibly unfolding itself in 3*D* physical space? How to adjudicate between the rivalling claims of reality being 3*N*-dimensional vs. 3-dimensional?
- "Multifield Realism" tries to avoid the Problem of Perception by regarding *both* configuration space and 3-space as *equally* real; the wavefunction, acting like an invisible hand on the particles and fields by guiding their dynamics, encodes a multitude of fields in ordinary 3*D*-space. Multifield Realism faces "the Problem of Communication", though: How does the Multifield Wavefunction impart on the objects in 3-space the relevant physical information? What's the mechanism by which it affects them?

How does sdBBT ward off these problems and position itself?

The above description of the Problem of Perception implicitly presumed that wavefunction realism entails that the wavefunction is "fundamental" in the sense of a substantial, physical field, i.e. a spatially extended body, which hence indeed one would expect to be defined on 3-space. Recall, however, our previous remark that wavefunction realism only asserts the *reality*, not necessarily the ontological *fundamentality*/substantiality of the wavefunction.

[118] Cf., for instance, Choi/Fara (2012) or Esfeld (2008)
[119] Dorato/Esfeld (2010), p. 41
[120] Cf. Suárez (2015).

SdBBT thus remains wavefunction realist, whilst conceiving of the wavefunction as a (dispositional) *property*. Consequently, the intuition that the wavefunction, no longer understood as a substantial/physical field, must be defined on physical space loses its plausibility; the wavefunction represents a property of the *N*-corpuscle universe. Since the ontologically subordinate wavefunction and the ontologically primary corpuscles don't compete for fundamentality, the spaces they each inhabit – configuration space vs. 3-space, respectively – can peacefully coexist.

Put differently, our actual perceptions are based on the actual positions the corpuscles occupy in 3-space; these positions in turn are the manifestations of the disposition, represented by the wavefunction. Consequently, the compartmentalisation of reality into dispositions (where the wavefunction belongs) and their manifestations (where all actual configurations belong, including those we detect and observe) resolves the Problem of Perception.

The Problem of Communication is overcome in tandem: The wavefunction and the actual configurations fall into different ontological compartments. Consequently, the wavefunction can't act on the configurations in any causal form.

Dispositionalism avoids the conflict with that principle, though, since firstly the *N* corpuscles' disposition to localise themselves stretches throughout all of space: The disposition is everywhere (not, of course, its manifestation). Secondly, the number of corpuscles is also conserved: At any instants, there are always *N* actual corpuscles. Hence, the corpuscles are not popping into /out of existence out of/into absolute nothingness or empty space.

Let's close this subsection with a remark on the global nature of the disposition at hand. The wavefunction Ψ_t , representing the propensity of the *N*-corpuscle universe to pop into existence with a certain configuration $\boldsymbol{Q} := (\boldsymbol{Q}_1, \dots, \boldsymbol{Q}_N) \in \mathbb{R}^{3N}$, is a *holistic* (non-separable, but *not* non-local) property of the whole system. It doesn't supervene on the properties of each individual corpuscles.[121] Rather, the propensity $\mathbb{P}^{\Psi_t}\big(\boldsymbol{Q}_j \in \mathcal{Q}\big)$ of the *i*-th corpuscle to localise itself in the region $\mathcal{Q} \subset \mathbb{R}^3$ in physical 3D space, derives from the global, usually entangled wavefunction via:

[121] Cf. also Esfeld et al. (2014)

$$\mathbb{P}^{\Psi_t}(\boldsymbol{Q}_i \in \mathcal{Q}) = \int_{\mathcal{Q}} \prod_{\substack{j=1 \\ j\neq i}}^{N} d^3\boldsymbol{Q}_j |\Psi_t(\boldsymbol{Q})|^2.$$

Deferring for the moment a detailed discussion of the non-separability of the wavefunction to section IV.5.4, let's see how our proposed interpretation of sdBBT's wavefunction as a holistic disposition overcomes the alleged inability to account for interactions that Esfeld and Gisin have recently diagnosed for primitive ontologies, with discrete beables whose temporal sequences are no longer continuous[122] ("flash ontologies"):

According to Maudlin, one form of the problem, as it arises in GRW-flash, is that "(f)or to accept this theory is to accept that microscopic reality is nothing at all like what we took it to be – not even the parts that we naively accepted as 'revealed' to us by microscopes. The ball-and-stick models of DNA – which do an admirable job of accounting for cellular behaviour – would be wildly misleading: a strand of DNA would show up in space-time as a sparsely scattered set of flashes, which would hardly suggest, over reasonable time periods, a double helix. Accepting the flash ontology entails rejecting the space-time picture of cellular structure that has guided the great advances in medicine and biochemistry. The theory does, of course, account for the practical success of that picture."[123] In other words, Maudlin appeals to the realist principle that the accounts the various disciplines envision of their scrutinised fragments of reality be mutually consistent, rather than contradict each other; the GRW-flash theory, Maudlin's objection asserts, flouts this principle: According to GRW-flash theory, a DNA strand materialises approximately only once per day, whereas the DNA model biologists successfully utilise in their research assumes that the DNA strand persists.

Despite its likewise "flashy" ontology, sdBBT obeys Maudlin's principle: As a result of the entire *N*-corpuscle universe performing a random jump through configuration space, the macroscopic objects of our stochastic Bohmian world are indeed composed of *actual* corpuscles. To be sure, sdBBT radically revises our picture of the world, namely implying that we most probably only exist for one instant in the whole history of the universe (more on this "temporal solipsism" in section IV.3); surprisingly, though, as we shall see, this doesn't

[122] Esfeld/Gisin (2014), sect. 5, whom we follow here.
[123] Maudlin (2011b), p. 257

contradict the picture of the world we get from other sciences, as long as this is understood as the best guess of how the world looks, *given our current evidence*.

Arguably a more serious version of the problem concerns measurements: What is a measurement device supposed to interact with, if the measured quantum object, say a trapped electron, only materialises once in a blue moon? Esfeld and Gisin write: "Let us take for granted that the flash ontology can account for macroscopic objects such as measuring apparatuses in terms of 'galaxies of flashes'. But on the flash ontology, there is nothing between the source of the experiment and the measurement apparatus. In other words, there is nothing with which the apparatus could interact: there is no particle that enters it, no mass density and no field that gets in touch with it, either."[124] They conclude that the flash ontology is too sparse to account for interactions. How to get out of this pickle? The previous argument is of no avail in this regard: Although at each instant of time our ambient macrocosmos is made up of actual corpuscles, positing an interaction between these corpuscles would amount to an instantaneous action-at-a-distance, which not only falls short of a plausible mechanism, but also rubs against the relativity of simultaneity at the heart of Special Relativity.[125]

Our dispositionalist interpretation of sdBBT resolves the problem: The dispositional nature of the wavefunction warrants that both the measurement system and the quantum system indeed always exist, albeit not necessarily *actualiter*. To establish the connection between measurement system and observed system, now the wavefunction's holistic nature comes into play: Both systems are fundamentally inseparable, with only the wavefunction of the compound system being the complete quantum state, representing the holistic disposition of the pair (configuration of the measurement apparatus, configuration of the measured system) popping into existence, with its statistical correlations as brute facts of the manifestation of the holistic disposition.

IV: Critical Analysis of sdBBT

After expounding in the previous section some conceptual and metaphysical idiosyncrasies of sdBBT and addressing some enquiries regarding its consistency, let's now turn to two sorts of further qualms:

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[124] Esfeld/Gisin (2014), p. 12
[125] E.g. Maudlin (2011)

- A first one concerns the taxonomisation of sdBBT as a deBroglie-Bohmian theory: Does the family resemblance between dBBT and sdBBT sanctify or do the differences contest it?
- Such a comparison evokes a second question: Is sdBBT merely a phenomenological theory, and hence not on a par with theories, such as dBBT, that lay claim to being fundamental? And furthermore, if we grant that sdBBT is a fundamental theory, what are its decisive advantages over dBBT that might justify taking sdBBT indeed *seriously*?

IV.1: sdBBT as a minimally deBroglie-Bohmian theory

A reader who has charitably followed our discussion of sdBBT so far might yet hesitate: To what extent may sdBBT still be classified as a *deBroglie-Bohmian* theory? Is sdBBT sailing under false colours?

Note first that sdBBT differs from ordinary QM in its ontological premises, all of which sdBBT shares with dBBT (except for determinism): In QM, no pair of conjugated dynamical variables is ontologically (or otherwise) preferred over another; in particular, position and momentum are on a par[126]. (In the case of Everett's Many World Interpretation of QM, this symmetry, i.e. the absence of any intrinsically distinguished dynamical variables, has occasionally led to the charge of the so-called Problem of Preferred Basis.) By contrast, sdBBT, albeit empirically equivalent to standard QM, breaks this symmetry of all pairs of conjugated variables by distinguishing position as the sole "beable" (Bell), i.e. the dynamical variable that is *always* value-definite/-determinate; furthermore, position is the only non-contextual variable, i.e. in both dBBT and sdBBT corpuscles have no dynamical properties other than position. In consequence, Bell's identification [127] of sdBBT as a version of Everett's Many Worlds Interpretation of QM is mistaken.

This distinction of position as a beable can be seen to be motivated physically by the fact that decoherence – the entanglement of systems with their ambient environment – acts as an effective superselection rule, with position as a preferred variable, relatively stable/robust

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[126] Pauli and Heisenberg reiterated this difference in their criticism of dBBT as introducing an "artificial asymmetry", cf. Myrvold (2003), sect. 3, who also critically evaluates it.

[127] *Contra* Bell (1987), Ch. 5, for whom "keeping the instantaneous configurations, but discarding the trajectory is the essential […] of the theory of Everett", loc.cit., p. 133.
By contrast, Daumer et al., cf. the chart on p. 393 implicitly distinguish between the Everett interpretation and sdBBT (despite labeling the latter "Bell's reformulation of Everett's theory", cf. op. cit., fn 13).

under interference, and in particular, entanglement effects, and commuting with all other observables. Conversely, such a distinction by stipulation has explanatory surplus value: It supplies a clear primitive ontology whose merits, such as an advantage in the debate on scientific realism, have been extolled elsewhere.[128]

In sdBBT, we just dismantled from the orthodox dBBT the "superfluous 'ideological super-structure'"[129] that undermined the consistency of an objectivist interpretation of all its three axioms and was the culprit, subject of a number of further complaints. Sharing the ontological framework of dBBT, which we deem the hallmark of the deBroglie-Bohmian theory paradigm, sdBBT is thus fully-fledged "deBroglie-Bohmian", of which it may be seen as a minimal version.[130] (N.B.: Other theories might as well count as minimally deBroglie-Bohmian theories, Nelson Stochastics being a potential candidate, see sect. VI.) More formally, we stipulate that a theory be called deBroglie-Bohmian, iff it's a so-called "primitive-ontology theory"[131] (as opposed to a so-called "wavefunction ontology" theory, such as the Everett interpretation) with the following features[132]:

(1) Its objects are particles or corpuscles – as opposed to e.g. primitive ontology theories with strings, flashes or matter fields as fundamental stuff. (N.B.: Wavefunction ontology theories postulate the wavefunction as the only fundamental constituent of reality, with no need for stuff over and above it.[133])

[128] Cf. Allori et al. (2008); Allori (2013, 2016)

[129] Heisenberg (1953), quoted in Myrvold (2003), p. 12

[130] Fine (1996) comes to the same conclusion and encourages a realist understanding of sdBBT (*avant la letter*): "At the heart of Bohmian mechanics ist the wavefunction and determinate particle positions, and perhaps we need be realists about nothing else", p. 249.

[131] Cf. Allori et al. (2008); Allori (2015)

[132] Cf. also Dürr et al.'s criteria for a theory to count as deBroglie-Bohmian: "A Bohmian theory should be based upon a clear ontology, the primitive ontology, corresponding roughly to Bell's local beables. […] For the nonrelativistic theory […] the primitive ontology is given by particles described by their positions, but we see no compelling reason to insist upon this ontology for a relativistic extension of Bohmian Mechanics. […] There should be a quantum state, a wave function, that evolves according to the unitary quantum evolution and whose role is to somehow generate the motion for the variables describing the primitive ontology. The predictions should agree (at least approximately) with those of orthodox quantum theory – at least to the extent that the latter are unambiguous." Dürr et al. (1995), sect. 5

[133] It deserves to be pointed out that despite some important similarities, viz. the assumptions that the world is made up of discrete events in spacetime and that these are stripped of their spatiotemporal continuity, the ontology of the GRW *flash* theory differs (*contra* e.g. Esfeld et al. (2015), p.5) from sdBBT's in that the latter is a particle-based primitive ontology, whereas the former falls under wavefunction ontology approaches; by contradistinction, with its interpretation of the wavefunction as a mass density field of matter in physical space, the GRW *matter* theory is a field-based primitive ontology, cf., for instance, Esfeld et al. (2015). In short, sdBBT's stuff consists of particles, GRWf's in events of the wavefunction.

(2) The positions of these corpuscles have beable status – as opposed to e.g. fermion number in Bell's merely Bohm-*like* lattice Quantum Field Theory.[134]
(3) To secure compatibility with the Kochen-Specker-Theorem, all dynamical variables other than position are contextual.
(4) It's fully empirically equivalent with QM – as opposed to objective collapse theories, which tamper with the Schrödinger dynamics.

Within this family of theories, sdBBT seems to be the ontologically most parsimonious member.

IV.2: sdBBT and realism

One might object: Wasn't one of the key points of the hidden-variables agenda to re-inaugurate realism by dispensing with QM's probabilities? E.g., according to von Neumann, the refusal to accept indeterminism inherent in QM underlies the search for hidden variable theories.[135] So, doesn't sdBBT, as an irreducibly stochastic theory, defeat its purpose?

The question rests on a misunderstanding that conflates determinism and realism. Dürr and Teufel rectify this view: "It is often said that the aim of (dBBT) is to restore determinism in the quantum world. That is false. […] What is 'out there' could just as well be governed by stochastic laws […]. A realistic quantum theory is a quantum theory which spells out what it is about. (dBBT) is a realistic quantum theory. It *happens* to be deterministic […]. The merit of (dBBT) is *not* determinism, but the refutation of all claims that quantum mechanics cannot be reconciled with a realistic description of reality."[136] In short: In giving up dBBT's "fictitious determinism", we needn't succumb to anti-realism.

Note, however, as Fine points out, that dBBT compels us to disentangle the two claims underlying *classical* realism[137]: firstly, the metaphysical one, that there exists an observer-independent world; and secondly, the epistemological one, that our measurements and

[134] Cf. Dürr et al. (2004)
[135] Von Neumann (1932), Ch. III.2
[136] Dürr/Teufel (2009), pp. 10 (our emphasis).
In the same spirit, Goldstein et al. declare that the goal of dBBT "[…] is to replace the measurement postulate of standard quantum mechanics with postulates that refer to electrons and nuclei instead of observers, axioms from which the measurement rules can be derived", Goldstein et al. (2009), p. 11.
[137] Cf. Fine (1996), Sect. 6

observations disclose features of the world as they had existed before and independent of the measurement act.

While dBBT and sdBBT doubtlessly both embrace the metaphysical component of classical realism ("objectivism"), a measurement act in dBBT – itself a physical process – can occasionally disturb the state of the measured system, so that its pre-measurement state and the state revealed through the measurement differ: Measurements can be invasive, and not merely passive records of the state of affairs. Already Bohm, in his treatment of the 1-dimensional particle in a box (see Sect. III.3) acknowledged this turning away from the epistemological component of classical realism: "According to the Bohmian prescription for velocity, the unmeasured particle is actually standing still! Measurement disturbs the situation, freeing the wave function, which guides the particle into motion."[138]

A generic feature of theories in which the wavefunction plays a prominent role for the state of a system, the interaction with a measurement device also in sdBBT can considerably affect the wavefunction of the measured system: Measurements thus can no longer be seen as passively reading out pre-existing values. This is vividly demonstrated by the Quantum Zeno Effect, where the physical interaction with a measurement device inhibits the decay of an unstable particle. But already thoroughly classical systems, once they reach a certain complexity, can flout the epistemological claim of classical realism, as the butterfly effect popularly illustrates: Any tiny perturbation of any observation *qua* physical interaction with the observational instruments may be amplified, so that what the presence measurement reveals about the system's state can considerably differ from the state in which the system would have been in isolation, i.e. without the measurement. (It's in fact such an – in essence classically optical – perturbation of the system through the measurement apparatus that Heisenberg invoked in his microscope thought experiment as an argument or explanation for his uncertainty relations.)

In light of the fact, however, that not even classical physics always epistemologically conforms to classical realism, it seems that epistemologically the latter demands too much. It therefore seems well motivated to rest content with the merely metaphysical realism, which both sdBBT and dBBT exhibit.

[138] Loc.cit., pp. 245

IV.3: "Temporal Solipsism" (Bell)

IV.3.1 sdBBT's many worlds

More disquieting is sdBBT's departure from another traditional metaphysical intuition, namely the temporal continuity of configurations: Since the corpuscles perform random jumps through configuration space, the connection between their past and future states through continuous trajectories is severed. By contradistinction, in dBBT, micro- (and, *a fortiori*, macro-) configurations are continuously linked in time.

More precisely, Bell apprehends that sdBBT posits many worlds, which "[…] exist, not at the same time, but one after another."[139] Indeed, an sdBBT universe typically keeps jumping between even macroscopically distinct configurations. Any minute, thus, a universe could pop into existence in which dinosaurs aren't extinct yet. Strictly speaking, each such "world" endures only for infinitesimal instants of time – a peculiarity Bell lampoons as "temporal solipsism".[140] Yet, striking our common sense intuitions as absurd, as sdBBT doubtlessly does in this regard, bears little argumentative weight – especially not if the theory is expressly conceded empirical adequacy[141], i.e. being experimentally indistinguishable from QM; as Lewis is reported to have said: "I do not know how to refute an incredulous stare."

Indeed, Bell hints at a more systematic reason[142], worrying that "[…] if (sdBBT) were taken seriously, it would hardly be possible to take anything else seriously."[143] This can be explicated in two ways:

- One source of Bell's worry may be that sdBBT astronomically exacerbates our uncertainty of any past or future we're capable to reconstruct or anticipate. Since the state of the world is the outcome of an irreducibly stochastic jump, it's extremely probable that already in the next second the universe will visit a region of configuration space corresponding to a world in which not even the Milky Way has formed. Conversely, it seems overwhelmingly *im*probable that the world in which our fiancées have just accepted our marriage proposal still exists tomorrow! How can we trust that

[139] Allori (2015), p.17
[140] Cf. Bell (1987), pp 135
[141] E.g. Allori (2015), p.17; Allori (2016), esp. sect. 5.1, 6.1; For general criteria for the empirical adequacy of primitive ontological approaches to QM, cf. Allori et al., (2007), sect.7.3
[142] Cf. loc.cit., sect. 8.1.
[143] Bell (1987), p.136

our memories and future anticipations retro-/predict the *real, actual* future? SdBBT simply thwarts this trust, transforming it even into utter unlikeliness.

- Bell now links this maddening uncertainty, rooted in the objective randomness of the world, to the reliability of our current knowledge of the world: In particular, how can we trust that our memories anticipate the real future? In other words, he assumes a tight connection between the uncertainty about our world's existence and the relevance of information: How can our current knowledge be *relevant* for decisions whose consequences lie in a future that might – with overwhelming probability – *not* happen? If we assume that memories and knowledge supervene on current microphysical states, then the loss of temporal continuity entails the possibility of worlds spontaneously popped into existence and containing archaeological evidence, neuronal configurations, etc. that tell of a past which has never occurred: We might suddenly wind up in a world in which we *actually* remember yesterday's unpleasant encounter with a stegosaurus in our bathroom! Temporal solipsism thus, Bell suggests, evinces a form of solipsism even more radical in that regard than the traditional one: Not only is everything outside our own minds unreliable, but also everything outside the *present* content of our minds, i.e. momentary perceptions.
- As a third reason to reject sdBBT, Bell accuses its temporal solipsism of being irrefutable. We construe this as a methodological ramification of the conjunction of the previously claimed unreliability of our memories and the doubts regarding their relevancy for future-directed actions: If contemporaneous configurations are no longer reliably linked to past configurations, with memories, archaeological and historical records being "entirely unreliable"[144], the same must apply to measurements – or even perceptions: They, too, forfeit their reliability and relevancy. SdBBT thus becomes untestable, and hence arguably unscientific.

How lethal is Bell's criticism? Our strategy will be to block it by denying that sdBBT's stochastic jumping between worlds in any way impinges on the relevancy of the information available to us.

[144] Allori (2015), p. 20

To elaborate that thought, we first have to clarify the Achilles' heel of Bell's above criticism, namely its lax notion of a "world".

A sideway glance at Everett's Many Worlds Interpretation will prove rewarding: How do Everettians define worlds? [145] According to Wallace, a "world" isn't a fundamental, but an emergent concept, defined only pragmatically as the components of the wave function of the universe in some decomposition into dynamically approximately independent wavepackets, narrowly localised in momentum and position space, and hence approximately stable and robust. These components ("branches") and their trajectories are picked out by decoherence[146], which also enables their re-identification over time. Such worlds approximate well "for all practical purposes" (FAPP) classical, Newtonian trajectories. As Wallace succinctly puts it: "And if there are multiple wave-packets, the system is dynamically isomorphic to a collection of independent classical systems."[147]

In the following, "worlds" will always refer to corpuscle configurations in such emergent, pragmatically defined "branches".

Neither decoherence nor the functionalism employed in the above pragmatic concept of world are committed to any specific interpretation of QM. Thus, with impunity we may also avail ourselves of them to clarify sdBBT's many worlds character, which Bell had correctly diagnosed: Whereas in Everett all such worlds exist simultaneously, in sdBBT, each is spontaneously realised only one at a time, randomly "selected" by the configuration into which the "supercorpuscle" jumps. The Everettian picture thus resembles an ill-attuned TV that displays several channels *simultaneously*[148]; the corresponding sdBBT picture is that of a TV randomly switching between different channels.

Note that on this understanding of a world, sdBBT's temporal solipsism involves "jumps between *worlds*", not merely jumps between (macro-)configurations *simpliciter*: Being associated with decoherence-induced wavepackets, which one can track through time, worlds display some form of diachronic stability (we'll elaborate on this in a moment); consequently, jumps between worlds amount to the random actualisation of *parts* of multiple "parallel"

[145] Cf. Vaidman (2014) and Wallace (2001, 2011), whom we follow.
[146] Bacciagaluppi (2012)
[147] Wallace (2011), p. 11
[148] We take this simile from Allori (2013), sect. 4.4

world histories. The focus on jumps between configurations *simpliciter* prescinds from any trans-temporal, historical aspects.

By definition, "*our* memories" refer to macro-configurations that belong to a world indexically pointed out by the speaker. Everettians would paraphrase this as the indexical selection of the branch of the wavefunction that the observer inhabits. In the following, we'll also borrow from the Everett literature the notion of personal identity, distinguishing between multiple (actualised or non-actualised) copies of the present authors in various worlds, and "us", i.e. particular (and actualised) copies, associated with a particular past.[149]

Memories (barring psychologically flawed ones), archaeological evidence, etc. are parts of the history of that world. Since any such world behaves FAPP-classically, and in particular is causally FAPP-closed, the histories we reconstruct from our available, present knowledge about the world will be causally closed: All the data within one world, i.e. our perceptions, memories and future anticipations, are consistent and even coherent, with no a-causal or otherwise absurd irregularities. In short, any snapshot of the history of a world (including perceptions, memories, etc.) will appear completely normal – FAPP-indistinguishable from a suitable snapshot of a classical world. Consequently, assuming that QM is empirically adequate, for all practical purposes no empirical evidence (including measurement records) consistent with QM can ever contradict sdBBT, which, after all, simply consists in positing a stochastic occurrence of such snapshots of world histories, allowed by the universal wavefunction. In short, sdBBT is capable of accounting for all empirical phenomena QM predicts.

Whether a world is actualised at other points in time or not, is a separate question; if a nanosecond later a world is actualised with completely *different* historical records, then simply a different world has jumped into existence, notwithstanding our linguistic habit to then indexically refer to that world, too, as "ours". In short, the reply to Bell's question: "How can a temporal solipsist take seriously what he or she remembers and perceives?" is simply that – with the usual epistemological and methodological caution – on *that* basis we reconstruct/anticipate *our* world.

[149] Saunders/Wallace (2008), esp. Section 2.1

What, then, about the relevance of our current memories and perceptions for future-directed actions? Doesn't sdBBT's temporal solipsism undermine any such relevancy of current information for future-directed actions? Why contract a life insurance in *this* world, when the latter's persistence already until next Saturday is doubtful? Rephrasing our reconstruction of Bell's criticism, such an argument assumes that the following two questions are related:

1. Does our world actually persist "without interruptions"? Or does the world we (believe to) inhabit fade in/out of actuality or existence only for finite periods or moments, with times of non-actuality/non-existence in-between? We'll call this the "Problem of Persistence".
2. Is data within one world – namely the one we happen to inhabit at some arbitrary instant – relevant to actions? Should we pursue our duties, dreams and hopes that pertain to a still-to-come future of *our* world? We'll call this the "Problem of Relevancy".

We deny that the Problem of Persistence has any logical bearing on the Problem of Relevancy: The temporal discontinuity of our actual world doesn't impinge on the relevancy of taking an action (or adopting a certain belief) now, on the basis of current knowledge.

- Even if the event I remember or as a result of which I'm taking action (or adopt a belief) wasn't/isn't actualised, it still belongs to *my* world. (Recall the above stance on personal identity we imported from the Everett literature.) If at any later point *my* world again pops into existence, I will face the consequences of forbearing a necessary action – or conversely, harvest the fruits of having done it. In that respect, I should act/believe: If I exist again, then I benefit from doing *A*. If I don't do *A*, and I don't exist again, I won't have the chance to regret having done *A*, either.
- More formally, let's tentatively take an action *A* relative to a certain goal *G* to be relevant, represented by the holding of the relation $r(.,.)$ if and only if *A* is an appropriate means to reach *G*, i.e. *ceteris paribus* brings about the intended goal: $G \rightarrow A$: $r(G, A) \Leftrightarrow G \rightarrow A$.

  This definition of *r* remains silent on the actuality $\alpha(\mathcal{W})$ of the world $\mathcal{W}$ to which *G* belongs, i.e. that $\alpha(\mathcal{W})$ holds, such that $G \in \mathcal{W}$.

  Such reference is straightforwardly implemented by stipulating that an action *A* relative to goal *G* be considered relevant, i.e. that $\tilde{r}(A, G)$ holds, iff *G* isn't an appropriate means for A, i.e. $\neg(G \rightarrow A)$, only if the G-containing world $\mathcal{W}$ isn't actual, either $\neg\alpha(\mathcal{W})$. That is: $\tilde{r}(G, A) \Leftrightarrow \neg(G \rightarrow A) \rightarrow \neg\alpha(\mathcal{W})$. By contraposition, thus:

$$\tilde{r}(G, A) \Leftrightarrow \alpha(\mathcal{W}) \rightarrow (G \rightarrow A).$$

This bi-conditional remains true, even if the world $\mathcal{W}$ isn't actual, $\neg\alpha(\mathcal{W})$.

In conclusion, the relevancy of an action relative to a goal is independent from the actuality of a world in which the goal exists.

By decoupling the reliability of our memories or future anticipations from the ontological Problem of Persistence, Bell's worry of sdBBT undermining the relevancy of our actions is blocked.

Consequently, also his claim of sdBBT's irrefutability becomes moot – as far as the reliability of measurements is the issue: They are just as reliable and relevant as any other historical records.

As far as the issue is irrefutability, it's either trivially true – or beside the point: Given its empirical equivalence with ordinary QM, any violation of a quantum-mechanical prediction would refute sdBBT, too. Insofar, however, that Bell criticises that a defence of temporal solipsism can't resort to empirical arguments, the objection is beside the point: Firstly, should empirical indistinguishability be the issue, it suffices to recall that under-determination by the empirical data is a generic scientific phenomenon[150], for which one cannot reproach any specific theory. Secondly, should the issue be the need for trans-empirical arguments, then to assess empirically indistinguishable theories, one needs trans-empirical criteria, such as internal consistency, relationship to other theories or explanatory value.[151]

Temporal solipsism certainly goes against our metaphysical grain, prompting the question: Why take sdBBT seriously? Why assume that sdBBT offers an account closer to reality than others? E.g. Collins requires that "in order to have sufficient reason to believe that a theory is (approximately) true, the theory must at least offer some truth-indicating theoretical advantage – that is, make some explanatory progress – over its phenomenalist counterpart

[150] Cf., for instance, Stanford (2013)
[151] Cf. Esfeld (2014)

[i.e. sdBBT]."[152] We'll postpone a critical discussion of whether Collins' classification of sdBBT as "phenomenalist" is justified or not, to section IV.4.

Relevant for this section is that his epistemological critique of dBBT concludes that vis-à-vis sdBBT, "we do not have sufficient reason to believe Bohm's theory"[153]. This concords with the results of our analysis so far, which this section reaffirmed by rebutting Bell's methodological criticism of sdBBT, and vindicating sdBBT's empirical adequacy and conceptual consistency: The picture of the world, i.e. the idea of us being trapped in an infinitesimal "time capsule" (Julian Barbour), may be uncanny; but, we argued, it can satisfactorily account for all available empirical data and their role as reliable evidence.

IV.3.2 Bohm Brains vs. Boltzmann Brains

It's instructive to compare sdBBT's multiverse nature, rooted in its temporal solipsism, to Boltzmann Brains scenarios in SM, the possibility of worlds randomly jumping into existence, filled with memories and records of a non-existent past.

Boltzmann Brain scenarios arise in the context of Boltzmann's H-theoretical attempts to explain the entropy increase, captured in the 2nd Law of Thermodynamics. Not hinging on a preferred direction of time, his argument also holds for time-reversal. (Loschmidt's *Umkehreinwand*). Consequently, low entropy states most likely are random fluctuations amidst thermal equilibrium, islands in an ambient high-entropy ocean.

This consequence also should apply to our brains, which then are likewise to be seen as low entropy fluctuations: "We pop into existence as thermal fluctuations with our brains full of memories of a nonexistent past. […] In the Boltzmann brain scenario, you most likely came into being by a fluctuation in this instant. So, your recollection of reading the sentence before this one is just as fabricated as every other memory."[154]

How to respond to this bizarre speculation?[155]

[152] Collins (1996), p. 266
[153] Loc.cit.
[154] Norton (2015), p. 3
[155] We shall gloss over the controversial question whether such an interpretation of the H-theorem in terms of a theory of *random* fluctuations is justified or not, two important issues in this context being the identification of the uniform measure with a probability measure and the status of ergodicity.
Lazarovici/Reichert (2014) argue for an interpretation in terms of typicality, instead.

Norton has pointed out that in addition to Boltzmann's argument that a non-equilibrium state arose with some, albeit small probability from a fluctuation away from equilibrium requires an extra step, namely the existence of a *particular* low entropy state, i.e. a brain (including its memories) just like ours. He contemplates two main *conceivable* arguments for this extra step:

The first one appeals to Poincaré recurrence, which states that eventually the evolution of a thermal system will return to an arbitrarily small neighbourhood of its starting point. However, "(t)he recurrence argument cannot give us isolated brains that exist momentarily, since we would have to assume their possibility in a [sic] the first place."[156] Hence, Norton discards this argument as circular. Furthermore, he remarks that the Poincaré recurrence theorem presupposes antecedent conditions, such as finite phase space volume and finitely many degrees of freedom, likely not to be met for our universe.

The second argument appeals to the time-reversal invariance of the underlying micro-dynamics: Since an isolated brain of a sentient observer can degrade thermally into a final equilibrium heat death, by time-reversal, a state of thermal equilibrium can also evolve – no matter how improbably – into an isolated brain of a sentient being. Norton also repudiates this argument, pointing out that the time-reversal invariance of CM's micro-dynamics no longer needs to carry through from a more fundamental quantum perspective, with the weak interaction (mediating radioactivity and hence arguably also biologically non-negligible) violating time-reversal invariance and "[…] the standard account of quantum mechanics (involving a time-irreversible collapse of a quantum wave on measurement. If that collapse is a real process, then the microphysics is very far from time reversal invariant."[157]

How do Norton's objections against Boltzmann Brain scenarios fare in the sdBBT case? We submit, they don't carry over.

Poincaré recurrence isn't satisfied for generic quantum theories (the quantum counterparts assume discrete energy spectra, a far too restrictive assumption for the general case[158]), hence, it can't be invoked. Neither does the time-reversibility argument work in the sdBBT case: Although Norton's second objection to it, in terms of a time-irreversible collapse of the

[156] Norton (2015), p. 5
[157] op.cit., p. 8
[158] Cf. Wallace (2013)

wavefunction, is blocked in sdBBT, as a non-collapse theory, the time-irreversibility of weak interactions persists.

The pivotal reason, however, for the futility of the argument invoking Poincaré recurrence was the "initiation problem" (Norton): "While the time reversed constitution of brains is possible, without a Poincaré recurrence theorem, what assurance is that this particular fluctuation will be initiated, even if only in probability?"[159] In contrast to the Boltzmann Brains case, sdBBT overcomes the Initiation Problem by its fundamental stochasticity: A "Bohm Brain", indiscernible from ours, complete with spurious memories, *could* indeed just happen to randomly materialise.

Besides the above problems related to the generation of a Boltzmann Brain, which sdBBT thus seems to be able to escape, Norton elaborates a second, arguably more detrimental argument against Boltzmann Brains: "Our memories are of a relatively orderly past, full of regular occurrences conforming to discoverable laws. Nothing forces a Boltzmann brain to have such regular memories. […] (I)t is vastly improbable that a Boltzmann brain might have memories of a regular past just like ours."[160] In other words, to the mind of a Boltzmann Brain, the world, complete with memories and archeological evidence, ought to resemble rather Borges' *Ciudad de los Inmortales* than, say, the rock city of Petra. Consequently, the uniformity of the world we experience – i.e. a world that seems to admit of perfectly coherent consistent stories – empirically contradicts what Boltzmann Brain scenarios would have us expect, namely "batty brains" (Norton), typically hallucinating chaotic worlds. In short, Boltzmann Brains are empirically inadequate: The theory would predict batty memories and perceptions.

Of course, one could just stipulate that a "mind-bogglingly rare" (Myrvold) fluctuation generated our brains, such that our world simply happens to appear regular to us. But on what grounds should we believe in such a miraculous coincidence – other than in order to restore empirical adequacy of our initial hypothesis in an ad-hoc way? It's exactly this (and *only this*) move that would render the whole Boltzmann Brain scenario self-refuting, since it relies on

[159] Norton (2015), p.7
[160] op. cit., pp. 8

science, whilst simultaneously reducing all empirical evidence, on which our trust in science rests, to illusions, thereby undermining the very reliability of science.[161]

sdBBT's Bohmian Brains avoid this self-refuting dodge, as we saw: The worlds that randomly pop into existence are by construction regular, with for all practical purposes uniform histories; unlike in the case of Boltzmannian Brains, sdBBT predicts that batty brains are very unlikely – in line with sdBBT's empirical adequacy.

IV.4: sdBBT as a phenomenalist theory?

Let's briefly, however, pause to gainsay Collins' filing of sdBBT as a phenomenological theory, i.e. "an account that merely takes the phenomenological laws of quantum mechanics as its unexplained given."[162] This characterisation is objectionable:

- Firstly, as we reiterated above, sdBBT differs in important ways from ordinary QM, conceptually going far beyond any account of QM *simpliciter*.
- Secondly: What is actually meant by "phenomenological laws", such as Hubble's Law, describing the expansion of the universe with a (highly idealised!) linear correlation between the distance and observed recession velocities of far-away galaxies? On common notions, phenomenological models link observable properties, without providing any mechanism in terms of explanations from fundamental first principles.[163] (In the case of Hubble's Law, of course, after Hubble's formulation of his phenomenological law, Lemaître succeeded in deriving it as an approximation from General Relativity, promoting thereby its status from phenomenological law to (approximate) theorem.)

  SdBBT sits squarely with such a notion of "phenomenological": SdBBT doesn't restrict itself to more or less directly observable properties: This is evident for the wavefunction, but also applies to the corpuscle positions – and the contextual nature of all other dynamical variables.

[161] E.g. Myrvold (2014) for such a criticism of Boltzmann Brains (echoing Bell's that we saw in the preceding subsection).
[162] Op.cit., p. 265
[163] E.g. Bunge (1997)

- Moreover, with its clearly delineated, objective ontology of the basic constituents of reality, sdBBT qualifies as a candidate for a *fundamental* account of reality no less apt than dBBT or other primitive ontology approaches.[164]

  In other words: sdBBT, as a primitive ontology in general, and as a theory with copious advantages over dBBT, offers much to justify the claim that it actually provides first principles, and thereby skirts also the second part of the above characterisation of a phenomenological theory.

IV.5: sdBBT vs. dBBT

The case for sdBBT's advantages over dBBT can indeed be further strengthened in at least three regards:

1. Firstly, with now all the conceptual paraphernalia in our hands, we can see how sdBBT overcomes three often perceived ontological shortcomings of dBBT, one related to the status of empty wavefunctions, and the other two related to the odd double role of dBBT's wavefunction.
2. Secondly, we need to address an intriguing objection that dBBT's GE after all *does* possess explanatory surplus value – viz. by allowing a justification of the Symmetry Postulate for composite *N*-corpuscle systems. An objection pulling in the same direction is that dBBT offers some conceptual advantages for the treatment of *i.a.* tunnelling times.

   sdBBT, we'll argue, will prove no less successful w.r.t. these features.
3. Thirdly, we'll argue that sdBBT's prospects of a satisfactory, relativistic treatment look promising, with dBBT's essential impediments being removed in sdBBT.

IV.5.1: Metaphysical quarrels with wavefunctions in dBBT

Having deepened our understanding of sdBBT's conceptual and metaphysical structure, we're now in a position to resume our previous discussion of what many have perceived as dBBT's ontological shortcomings. We'll see how sdBBT overcomes them.

- In dBBT, by specifying initial corpuscle positions one selects one of the possible *trajectories* as the actual one. The positions thus distinguish those branches of the wavefunction that encode the actual state of the system. Yet, all other branches of the

[164] Cf. Allori (2013ab, 2015ab, 2016)

wavefunction, into which no trajectories lead, are equally real: Myriads of empty wavefunction branches, which in general no longer affect the system, thus populate space – an unsavoury feature.[165] It's unclear what their status is supposed to be; furthermore, if they are irrelevant for the dynamics of the corpuscles, then one would rather dispense with them.

SdBBT removes this blemish: Since the modulo square of the wavefunction represents the corpuscles' localisation probability, conceived of as a real property, the notion of an "empty wavefunction" loses its meaning – or at least its sting: Corpuscles simply have a propensity to localise themselves *anywhere*. Whether this propensity actually manifests itself or not, has no bearing on its reality.

Of course, the price for this solution is the postulate of a disposition, which by its metaphysical nature is empirically rather elusive. However, we argued in III.2, a number of advantages commend dispositionalism, compensating for this elusiveness.

- Another ontological oddity has struck many of dBBT's detractors: While via the GE the wavefunction guides the trajectories of the corpuscles, the latter don't conversely influence the wavefunction. The Action-Reaction-Principle thus seems violated.[166] This is seen as undesirable: Rather, a substance should only be a potential *agens*, i.e. act, if it can equally be a potential *patiens,* i.e. be in turn acted upon. Einstein, for instance, valued this Action-Reaction-Principle as a key (and, vis-à-vis Special Relativity (SR), novel) virtue of General Relativity[167], in which, as Wheeler famously put it, spacetime tells matter how to move, whereas matter tells spacetime how to curve.

  With the abolition of deterministic trajectories in sdBBT, the wavefunction no longer acts on the corpuscles. Of course, they still have the dispositional *property* to localise themselves, but the relation between dispositions and manifestations is not one of action, usually understood as changes in substances interdepend: Objects simply *possess* a property; the meaning of a property acting on the substance that bears it to

[165] Brown/Wallace (2005) and Brown (2009) ratch up this unease with empty wavefunctions to the charge that dBBT is "Everett in denial": In essence they argue that dBBT merely posits corpuscles on top of the branches of the multiverse, as "mere epiphenomenal 'pointers', being relegated to picking out one of the many branches, defined by decoherence, while the real story – dynamically and ontologically – is being told by the unfolding evolution of those branches", op.cit., pp.8. This indexical role is then criticized as an ad-hoc ingredient. Callender (n.d.) critically examines this "redundancy argument".

[166] Cf. Brown/Anand (1995)

[167] Cf. Brown/Lehmkuhl (2013)

us appears obscure. By dint of the dispositional nature of the wavefunction, the validity of the Action Reaction Principle remains unscathed.

- At the heart of dBBT's trouble with the Action-Reaction Principle lies the double role of the wavefunction in both guiding the corpuscles' trajectories and in representing probabilities for their localisation – two roles that are logically distinct. This double role makes dBBT look contrived.[168]
  Eliminating the GE trivially disposes of the *double* nature, *tout court*.

IV.5.2: Justification of the Symmetrisation Postulate

Linking such metaphysical aspects and what might be seen as explanatory, even predictive power, Brown et al. have claimed that, whereas standard QM must posit it as a contingent axiom[169], via the topological approach, dBBT can naturally justify the Symmetrisation Postulate for identical particles.[170] It states that wavefunctions, $\Psi(\boldsymbol{Q}, \dots, \boldsymbol{Q_N})$, of bosonic (fermionic) *N*-particle systems behave (anti-)symmetrically under permutations $\hat{\pi}_{ij}$ of particles, labelled *i* and *j*: $\hat{\pi}_{ij}\Psi(\boldsymbol{Q_1}, \dots, \boldsymbol{Q_N}) = \pm\Psi(\boldsymbol{Q_1}, \dots, \boldsymbol{Q_N})$. Such an astonishing claim would contradict the redundancy of the GE, from which sdBBT draws its appeal. With our grasp of sdBBT's conceptual structure sharpened, we're now in a position to address the claim. We submit, sdBBT is equally able to motivate the Symmetrisation Postulate.

The technical details of the topological approach, which turns on the non-trivial topology (multiple connectedness) of the reduced configuration space (more on that below) shall not detain us here; instead, we'll focus on the crucial step, viz. the removal of the "coincidence points" $\Delta \coloneqq \{\boldsymbol{Q} \coloneqq (\boldsymbol{Q_1}, \dots, \boldsymbol{Q_N}) \in \mathbb{R}^{3N} \colon \exists \mathrm{i} \neq \mathrm{j} \colon \mathbf{Q_i} = \mathbf{Q_j}\}$ from configuration space $\mathbb{R}^{3N}$. Brown et al. claim dBBT naturally justifies this removal. Let's recapitulate their chain of reasoning:

(1) The corpuscles being identical (more precisely: indistinguishable), their index labels possess no intrinsic meaning. Hence, since the configuration **Q** of corpuscle positions completely specifies the actual state of a system, for a given wavefunction Ψ, a

[168] E.g. Timpson (2011), pp. 14
[169] In relativistic QFT, however, the Symmetrisation Postulate, i.e. the connection between spin and (anti-) symmetry can be proven as a theorem, cf. Streater/Wightman (1964), Ch. 4.4
[170] Cf. Brown et al. (1998).

permutation $\hat{\pi} \in \Sigma_{\mathrm{N}}$ of indices shouldn't alter the state of the system: $(\hat{\pi}\boldsymbol{Q}, \Psi)$ and $(\boldsymbol{Q}, \Psi)$ are physically equivalent. The redundancy of configuration space can thus be purged by transition to the reduced configuration space $\mathfrak{D} := \mathbb{R}^{3N}/\Sigma_{\mathrm{N}}$, where configurations that differ from one another only by a permutation are identified, preserving the physical information.

(2) The GE being 1st-order, corpuscles coincide either always or never: $\mathfrak{D}\backslash\Delta$ and Δ are invariant under the action of the dBBT dynamics.

   a. This implies that, consistently with the dBBT dynamics, we may remove the coincidence points Δ from reduced configuration space $\mathfrak{D}$.

   b. This removal "[…] seems physically well motivated, since they correspond to motions for which [the involved corpuscles] coincide for all times – which would appear as the motion of one particle of M-fold mass and charge."[171]

(3) The removal doesn't affect dBBT's observable/statistical predictions, for the set of coincidence points is of measure zero, $\int_{\Delta} \mathrm{d}^{3\mathrm{N}}\mathbf{Q}\, |\Psi|^2 = 0$.

Let's analyse these arguments. What's the role of the GE in them?

- (1) only requires the position definiteness and dBBT ontology; hence, it carries over to sdBBT.
- (3) is a mathematical fact, independent of the GE: In sdBBT, too, the removal of Δ doesn't affect the observations.
- (2), of course, lapses in sdBBT.

Does that ruin the motivation for removing Δ?

- The first part of (2) is only needed to ensure consistency with the GE. With no GE in sdBBT, we trivially needn't worry about consistency with it.
- Regarding the second part, note first that (2b) doesn't hinge on the GE specifically, only on it being 1st order.

  More to the point: Why is it actually implausible to regard two particles with the same spatiotemporal trajectories as two? The tacit metaphysical premise in the background is a spatiotemporal *principium individuationis* (PI)[172], according to which

[171] Loc.cit., p.7

[172] Schopenhauer uses this term in connection with Kant, e.g. Kant (1781), A263.

spatiotemporal distinctness constitutes individuality/identity and thus grounds numerical distinctness. In other words, (2b) assumes that spatiotemporal distinctness is a necessary condition for individuating two otherwise indistinguishable particles. And since the number of particles is fixed in a deBroglie-Bohmian theory, the removal of Δ follows.

However, in the stochastic world of sdBBT, corpuscles no longer have continuous spatiotemporal paths: They localise/delocalise spontaneously at random points. One thus faces two options: Either one continues to adhere to the spatiotemporal PI and allows for multiply occupied spacetime *points*, thereby indeed forgoing the justification for removing Δ. Equally well, however, given that nowhere do Brown et al. explicate the above (PI), let alone argue for it, one could adapt an alternate *principium individuationis* (PI*), arguably more natural to a discontinuous world of corpuscle jumps: Two corpuscles are identical, if they coincide at one point in spacetime. *PI** then *directly* motivates the removal of coincidence points, thereby replacing (2b).

In short, what does the essential work in the topological motivation for the Symmetrisation Postulate for dBBT are a dBBT-independent, measure-theoretical statement, and a Principle of Individuality. Modifying the latter in a plausible manner, we conclude that sdBBT is no less apt to motivate it.

IV.5.3: Quantum tunnelling

Our claim of the GE's redundancy is likewise disputed by asserted conceptual advantages in experimental contexts whose treatment doesn't fit comfortably within standard QM, e.g. regarding dwell and tunnelling times, escape times and escape positions, scattering theory and quantum chaos.[173]

Each such application deserves an investigation in its own right; here, we'll exemplarily examine the arguably clearest case, viz. quantum tunnelling[174] – a phenomenon important, e.g., for describing α-decay: Quantum particles can penetrate a potential barrier classically insurmountable, with the particles' energy being below the potential barrier. The question then naturally arises: How much time does it take a particle to cross the barrier in such a

[173] Cf. Goldstein (2013), Sect. 15, also for further literature on these issues.
[174] Cf. Holland (1993), Ch. 11 and Passon (2010), Ch. 7.5, whom we closely follow.

scenario? Ordinary QM boycotts the very question: Firstly, time is not an observable, but a parameter; secondly, standard QM doesn't assign particles definite trajectories, depriving statements about how long it takes a particle to traverse a certain distance of any immediate meaning. In order to define such time scales, one must therefore employ surrogate methods that only use well-defined operators[175], by tracking wave packets via the evolution of their maxima. The results obtained from different methods, however, turn out to be not always mutually compatible.

In dBBT, it is claimed, a straightforward, unambiguous picture emerges: The GE pilots corpuscles that start from some initial positions deeply inside the barrier and beyond; other corpuscles with different initial positions get reflected. At each moment, the corpuscles have both, a definite position Q and velocity $\dot{Q}$, for simplicity assumed to be time-independent. Let each α-th particle with initial position $Q_0^{(\alpha)}$ reach a given point Q at time $t^{(\alpha)}$. (In general, solutions of the GE, $Q = Q(t; Q_0^{(\alpha)})$, won't be invertible in closed form.) For the corpuscle with initial position $Q_0^{(\alpha)}$ it will take the time $\Delta t(Q_A, Q_B; Q_0^{(\alpha)})$ to travel from sites Q_A to Q_B, on opposite ends of the potentials barrier each. Since, according to QEH, the initial positions are unknown, only their statistical distribution, $|\psi|^2$, the observable, average "dwell-time" for a tunnelling process, $\langle \Delta t_{AB} \rangle$, can be readily determined as:

$$\langle \Delta t_{AB} \rangle = \sum_{\alpha} \int \mathrm{d}Q_0^{(\alpha)} \left| \psi(Q_0^{(\alpha)}) \right|^2 \Delta t(Q_A, Q_B; Q_0^{(\alpha)}).$$

The deterministic trajectories play a crucial *heuristic* role here, suggesting an intuitive way to define measurable quantities. But might this advantage be merely illusory? Could the (empirically elusive) trajectories make certain choices of how to define, say, dwell-time, only *appear* more natural than others? The alternative constructions of dwell-time variables, e.g. via the evolution of wave packets, are still equally legitimate and adequate.

But let's grant for the sake of the argument that the conceptual advantage is real. Then, sdBBT is no worse off than dBBT, suggesting an equally appealing instruction of how to define dwell-time, viz. as the *average* time one needs to wait until a corpuscle localises itself first at Q_A and then at Q_B.

[175] Cf. Landauer/Martin (1994) and Chiao (1998)

In conclusion, for tunnelling times, the conceptual advantages accredited to dBBT are of a heuristic nature; should one indeed deem such heuristic guidance an advantage, sdBBT's more thrifty primitive ontology is no less apt to provide it.

IV.5.4: sdBBT, dBBT and relativistic QM

Pride of place in our comparative analysis of sdBBT and dBBT shall be a glance at the most frequent criticism of dBBT – namely the (still largely unresolved) issue of its relativistic generalisation. The locus of those problems with a relativistic dBBT, we'll presently argue, again lies in the GE. Dispensing with the latter thus makes sdBBT an attractive alternative to dBBT: We submit that sdBBT, indeed, is free from non-locality, i.e. a spooky action-at-a-distance, but exhibits non-separability.

Let's briefly recall the situation in ordinary QM with the collapse postulate and its relativisation. There, the following quandary looms[176]: Either the collapse is instantaneous (as standard formulations seem to suggest) – highly problematic for SR, as superluminal signal transfer arguably gives rise to paradoxes, involving causal loops. Furthermore, the instantaneous propagation of an effect privileges a reference frame – in conflict with SR's Principle of Relativity, which postulates the equivalence of all reference frames [177]; alternatively, contemporary QM needs to be modified *ad-hoc* so as to suitable account for a collapse mechanism. Either choice seems hard to swallow.

By denying the existence of the collapse, non-collapse theories, such as dBBT, avoid this dilemma. The price dBBT has to pay, though, is its manifest action-at-a-distance: Each corpuscle's velocity field depends on the configuration of all other, even space-like separated corpuscles, which in light of SR one wouldn't expect to exert any influence upon each other.

Not only should the intimated conflict between an action-at-a-distance be fleshed out, but also, one may object that dBBT, as introduced so far, is a non-relativistic theory – and hence unsurprisingly conflicting with SR. Therefore, let's examine some expressly relativistic proposals.

[176] Cf. Maudlin (2011b)

[177] E.g. for Popper, an action-at-a-distance, as a possible interpretation of an instantaneous collapse "[…] would mean that we have to give up Einstein's interpretation of special relativity and return to Lorentz's interpretation and with it to Newton's absolute space and time", Popper (1982), p. 29.

Associated with the Klein-Gordon Equation for the complex scalar function ϕ, describing a spin 0-particle of mass *m* and charge *q* in the external electromagnetic potential A^μ,

$$\left[m^2 + \left(\partial_\mu + iqA_\mu\right)\left(\partial^\mu + iqA^\mu\right)\right]\phi(\boldsymbol{Q}, t) = 0,$$

is the conserved 4-current $j^\mu = \frac{1}{2m}\left(\phi^* \overleftrightarrow{\partial}^\mu - 2qA^\mu\right)\phi$, satisfying $\partial_\mu j^\mu = 0$.

Based on j^μ, one now may be tempted to stipulate a GE for the Klein-Gordon scalar, analogous to the non-relativistic case, via $\frac{dQ^i}{dt} = \frac{j^i}{j^0}$. In the absence of an external field, for a plane wave of positive energy, this appears reasonable, with the 4-current future-pointing, $j^0 > 0$, and time-like, $j_\mu j^\mu > 0$. In the general case, though, such a proposal isn't viable for two main reasons[178]: Firstly, j^0 is of indefinite-sign, not even for free solutions of positive energy states, and hence defies a particle or probability density. Secondly, j^μ generically isn't time-like. Holland concisely comments: "A theory of material objects in which an initially time-like, future-pointing trajectory may pass through the light cone to become space-like, and even move backwards in time, is clearly unacceptable."[179]

In conclusion, the whole idea of a dBBT version of the Klein-Gordon Theory can't get off the ground, for the Klein-Gordon Theory doesn't admit of a satisfactory single-particle interpretation to begin with. (Besides of some formal difficulties with suitable position operators and localizability in relativistic QM, in general, Holland reminds the reader that, unless supplemented by further *ad-hoc* constraints, Lorentz-covariant wave equations, such as the Klein-Gordon Equation, are well-known to exhibit superluminal transmission and backwards-causation – both consequences highly problematic in their own right, if not downright paradoxical.)

The current absence of a consistent *particle* interpretation of the Klein-Gordon Equation seems to preclude a satisfactory deBroglie-Bohmian theory, with its commitment to a particle-based primitive ontology.[180]

[178] Holland (1993), Ch. 12.1

[179] Op.cit., p. 500

[180] More satisfactory *field*-based bosonic theories exist, cf. e.g. Dürr et al. (2004); Nikolic (2005). But they go beyond our current scope – in two regards: Firstly, in that we are focussing on deBroglie-Bohmian approaches, narrowly construed as primitive ontologies based on *particles/corpuscles* with *position* as beable; secondly, we restrict our analysis to the domain of relativistic/non-relativistic QM. An extension to quantum field theory deserves an investigation in its own right.

How does the situation look for fermions? Here, we'll argue, sdBBT will unbosom its advantages over dBBT w.r.t. a relativistic extension. Exemplarily, we'll now analyse the arguably most successful proposal for a dBBT of Dirac-particles, viz. so-called Hypersurface Bohm-Dirac Models.[181]

Bohm himself proposed to derive the relativistic counterpart of the GE from the *N*-fermion Dirac Equation (for ease of notation, $\hbar = c = 1$) for each particle $k \in \mathbb{N}^{\leq N}$:

$$\left[i\gamma_k^0\partial_t + \sum_{\substack{j=1\\ i=1,2,3}}^{N}\left(i\gamma_k^i\partial_i + e\gamma_k^i A_i(\boldsymbol{Q}_k,t) - e\gamma_k^0\Phi(\boldsymbol{Q}_k,t)\right) - m\right]\psi(\boldsymbol{Q},t) = 0$$

Here, the γ_k^μ denotes the μ-th Dirac matrix, acting on the *k*-th particle (in the following, we'll use the following conventions: $\boldsymbol{\gamma}_k \coloneqq \left(\gamma_k^i\right)_{i=1,2,3}$ and $\gamma^0 = \otimes_{k=1}^N \gamma_k^0$), *m* and *e* denote the mass and the charge of the Dirac particles, respectively, $\psi\colon \mathbb{R}^{3N}\times\mathbb{R} \to (\mathbb{C}^4)^{\otimes N}$ the *N*-particle spinor and $A^\mu = (\Phi, \mathbf{A})$ the electromagnetic potentials of the external field.

The probability 4-current for each particle *k*,

$$j_k^\mu = (\rho_k, \boldsymbol{j}_k) = \left(\bar\psi\gamma^0\psi, \bar\psi \otimes_{j=1}^{k-1}\gamma_j^0\otimes\boldsymbol{\gamma}_k\otimes \otimes_{j=k-1}^{N}\gamma_j^0\psi\right)\Big|_{(\boldsymbol{Q},t)},$$

with the adjoint spinor $\bar\psi = \psi^\dagger\gamma^0$, is conserved, $\partial_\mu j^\mu = 0$. In complete analogy to the non-relativistic GE, we obtain the relativistic GE (rGE) for the k-th corpuscle from the temporal and spatial components of the 4-current:

$$\frac{d\boldsymbol{Q}_k}{dt} = \frac{\boldsymbol{j}_k}{\rho_k}\bigg|_{(\boldsymbol{Q},t)} = \frac{\bar\psi \otimes_{j=1}^{k-1}\gamma_j^0\otimes\boldsymbol{\gamma}_k\otimes \otimes_{j=k-1}^{N}\gamma_j^0\psi}{\bar\psi\gamma^0\psi}\bigg|_{(\boldsymbol{Q},t)}$$

In the single-particle case, $N = 1$, this rGE reduces to ($\mu = 0, \dots, 3$):

$$\frac{dQ^\mu}{dt} = \frac{\bar\psi\gamma^\mu\psi}{\bar\psi\gamma^0\psi}\bigg|_{(\boldsymbol{Q},t)}$$

[181] Cf. Dürr et al. (2013)

It's Lorentz invariant: Due to the physical insignificance of re-scaling the time parameterisation, one obtains, for some parametrisation s of the particle's worldline, a manifestly covariant and geometric reformulation:

$$\frac{dQ^\mu}{ds} \parallel \bar{\psi}\gamma^\mu\psi.$$

The complicacies surge in the many-particles case, $N > 1$. Then, the rGE, by using a common time for all particles, stipulates a hyperplane of simultaneity of all *N* corpuscles on whose positions the l.h.s. of the rGE depends, thereby defining a distinguished reference frame *K*. In particular, the relativistic QEH, with the probability density, i.e. the 0-component, of the above Dirac 4-current, no longer holds in all frames. Although *K* turns out not to be detectable, i.e. experimentally, all reference frames remain indistinguishable[182], the postulate of an absolute reference frame violates SR's Principle of Relativity, which declares the equivalence of all reference frames. In short: While the statistical predictions coincide with those of standard Dirac theory, on the level of the individual particles SR is violated.

One might try to dodge the absolute simultaneity by moving to a multi-time wavefunction $\psi(\boldsymbol{Q}_1, t_1; \dots \boldsymbol{Q}_N, t_N)$ on $(\mathbb{R}^3 \times \mathbb{R})^{\otimes N}$, which assigns each particle a time of its own and satisfies the multi-time Dirac Equation:

$$\left[i\gamma_k^\mu\left(\partial_{\mu,k}^2 - ieA_\mu\right) - m\right]\psi = 0.$$

And indeed, the rGE, constructed via the above scheme from the 4-current, turns out to be Lorentz invariant for factorisable states, i.e. multi-time wavefunctions of the form $\psi(\boldsymbol{Q}_1, t_1; \dots \boldsymbol{Q}_N, t_N) = \prod_{k=1}^{N} \varphi_k(t_k, \boldsymbol{Q}_k)$.

What about entangled/non-factorisable states? The resulting 4-current of the *k*-th particle, $j_k^\mu(\boldsymbol{Q}_1, t_1; \dots \boldsymbol{Q}_N, t_N) = \bar{\psi} \otimes_{j=1}^{k-1} \gamma_j^0 \otimes \gamma_k^\mu \otimes \otimes_{j=k-1}^{N} \gamma_j^0 \psi\big|_{(\boldsymbol{Q}_1, t_1; \dots \boldsymbol{Q}_N, t_N)}$ then likewise is no longer separable. For a viable GE, we thus need to connect the velocity of the *k*-th particle with the *N*-1 tuples $(t_j, \boldsymbol{Q}_j)$, with $j \neq k$. One could now consider a hyperplane Σ_η on which $t_1 = t_2 = \cdots = t_N$, the multi-time Dirac Equation then reduces to the familiar one. Let η^μ be

[182] Cf. Berndl et al. (1996); Dürr et al. (1999)

an observer at rest in this reference frame *K* thus distinguished, $\eta^{\mu} \perp \Sigma_{\eta}$. Then, the 4-current becomes:

$$j_k^{\mu}(\boldsymbol{Q}_1, t_1; \dots \boldsymbol{Q}_N, t_N) = \bar{\psi} \otimes_{j=1}^{k-1} (\gamma_j^{\nu} \eta_{\nu}) \otimes \gamma_k^{\mu} \otimes \otimes_{j=k-1}^{N} (\gamma_j^{\nu} \eta_{\nu}) \psi \big|_{(\boldsymbol{Q}_1, t_1; \dots \boldsymbol{Q}_N, t_N)}.$$

The resulting rGE for the *k*-th particle position $\boldsymbol{Q}_k$ is:

$$\frac{dQ_k^{\mu}}{dt} = \frac{j_k^{\mu}}{j_k^{\nu} \eta_{\nu}}.$$

The probability density in the denominator is now obviously a scalar, but furthermore easily verified to be independent of the particle index. Consequently, a re-parametrisation yields the manifestly covariant rGE:

$$\frac{dQ_k^{\mu}}{ds} = j_k^{\mu} \big|_{\mathbf{Q}(\Sigma)}.$$

But how to construct the preferred reference frame? Which vector field η^{μ} to take? Lest the thus attained Lorentz covariance be hazarded, η^{μ} must be built via a Lorentz invariant law. A popular proposal is to choose the rest frame of the total energy-momentum of the universe, $P^{\mu} = \int_S \; d\sigma_{\nu}(x) \langle \Psi | t^{\mu\nu} | \Psi \rangle$, with the wavefunction of the universe Ψ, the total energy-momentum tensor $t^{\mu\nu}$ in the Heisenberg picture and S an arbitrary space-like hypersurface. (Due to the conservation of energy-momentum and Stoke's Theorem, P^{μ} doesn't depend on *S*.) The preferred vector field then is $\eta^{\mu} \coloneqq \frac{P^{\mu}}{\|P^{\mu}\|}$.

Have we thus finally achieved a satisfactorily relativistic dBBT? Although Lorentz covariance has been restored, the model still lacks what Bell calls "serious Lorentz invariance": It needs to postulate *extra* structure in spacetime – an addition SR *per se* doesn't warrant. Furthermore, it deserves to be pointed out that it's not clear how to extend the model to incorporate interaction potentials.

In conclusion, the quest for a rGE heaves on us the burden of extra structure in spacetime; SR *simpliciter* and dBBT seem incompatible.[183]

[183] Cf. also Maudlin (1996), Sect. 2

Where now does sdBBT stand? Two points are pertinent:

Firstly, as compared to standard QM, sdBBT, as a non-collapse theory, avoids the full-front collision between *instantaneous* collapses of the wavefunction and Relativity, as well as the awkwardness of an ad-hoc modification of some principles of QM so as to incorporate a collapse. But isn't the spontaneous manifestation of a corpuscle configuration tantamount to form of instantaneous collapse? This is mistaken: The dispositional wavefunction continues to evolve unitarily according to the Schrödinger dynamic, even after a manifestation. The (absence of a) manifestation of a disposition has no bearing on the disposition and its evolution as such.

As compared to dBBT, with the abolition of a GE, the need for preferred reference frames or extra structure lapses – and thereby dBBT's obstacle for "serious Lorentz covariance". But what about an action-at-a-distance, stemming from QM's allegedly inherent non-locality, which, as Bell argued, originates in the fact "(t)hat the [wavefunction] […] propagates not in ordinary three-space, but in a multidimensional configuration space[…]"?[184] Indeed, actions-at-a-distance would threaten the compatibility with SR in a manner resembling what we witnessed in the Dirac-Bohm case. However, Bell's diagnosis of QM's innate non-locality, which the violation of his famous inequalities are supposed to attest to, is too hasty.

Bell famously elucidates locality (or "local causality") for two measurement outcomes *A* and *B* of spacelike separated measurements, with measurement settings ***a*** and ***b*** as follows: "[…] once all the possible common causes of the two events are taken into account (which, guided by classical relativistic intuitions, we take to reside in their joint past), we expect the probability distributions for the measurement outcomes to be independent and no longer display any correlations."[185] Formally, with λ denoting all relevant causal factors, i.e. the common causes in the overlap of the past cones of the measurement events, the probability distribution $\mathbb{p}_{\boldsymbol{a}/\boldsymbol{b}}$, expressing the correlations between the measurement outcomes, dependent only locally on the measurement setups **a**/**b**, factorises:

$$\mathbb{p}_{\boldsymbol{a},\boldsymbol{b}}(A \wedge B|\lambda) = \mathbb{p}_{\boldsymbol{a}}(A|\lambda)\mathbb{p}_{\boldsymbol{b}}(B|\lambda).$$

[184] Bell (1987), p. 115
[185] Brown (2005), p.184

From this Bell derives his inequalities, which QM generically violates for entangled states.

However, Brown remarks that "[…] failure of local causality in Bell's sense does not entail the presence of non-local causes. In arriving at the requirement of factorisability it is necessary to assume something like Reichenbach's principle of the common cause; namely that if correlations are not due to a direct causal link between two events, then they must be due to common causes, such causes having been identified when conditionalization of the probability distribution results in statistical independence." [186] The violation of quantum correlations satisfying Bell's local causality thus could simply imply that quantum correlations aren't always apt for causal explanations; violation of local causality on its own needn't imply non-locality in the guise of an instantaneous action-at-a-distance.

sdBBT takes that route: It blocks the very formulation of Bell's local causality by rejecting the Principle of Common Cause (whose scope of validity remains contented on independent grounds, also in classical physics[187]). Recall from our discussion of Humphreys' Paradox that we embraced from the outset sdBBT's stochastic, a-causal nature.

This a-causality is rooted in sdBBT's temporal solipsism, i.e. the absence of any diachronic identity of corpuscles. It defies *both* sdBBT's locality and non-locality. Not persisting beyond an infinitesimal instant of time, corpuscles can neither themselves traverse arbitrary distances, nor can they act on each other: An influence denotes a correlation between a change in the state of one thing and a change in the state of another. But, lacking diachronic identity, no pair of successive positions occupied by corpuscles can be attributed to the same corpuscle; no corpuscle at a different, later position can be re-identified with the corpuscle at an earlier stage. Conceptually, no corpuscle can change: The possibility of mutual influence or action thus is blocked. sdBBT is neither an action-at-a-distance nor a local theory!

How then to understand entangled states? We submit that sdBBT simply postulates brute fact statistical correlations that reflect the wavefunction's non-separability.

More in detail, following Einstein's view on QM[188], entangled corpuscles can be understood "literally" – namely as failure of separability of their propensities: Separated subsystems of a

[186] Op.cit. p. 185
[187] Cf. Arntzenius (2010)
[188] Cf. Howard (1985, 1989)

joint system typically no longer possess separate wavefunctions. The wavefunction thus represents a holistic, dispositional property. When performing a measurement, we register correlations between manifestations of these non-separable dispositions. The correlations, by themselves relativistically innocuous, then are brute facts of a fundamental, non-separable reality. Only when trying to further explain them in terms of a common cause does the pickle regarding compatibility with SR start – but there is neither need nor justification (beyond the merely heuristic fecundity) to impose the *a priori* demand all correlations require explanation.[189] Fine offers a helpful analogy: "[Such a demand] is like the ideal that was passed on in the dynamical tradition from Aristotle to Newton, that motion *as such* requires explanation."[190]

It deserves to be mentioned how sdBBT escapes Einstein's transcendental criticism of non-separability.[191] Distinguishing meticulously between non-separability and non-locality, he argued, that *were* QM complete, we'd have to abandon separability, i.e. the independent existence of separated systems; and "(i)f this axiom were to be completely abolished, the idea of the existence of (quasi-) enclosed systems, and thereby the postulation of laws which can be checked empirically in the accepted sense, would become impossible."[192] sdBBT blocks this worry in a twofold way: Firstly, QM isn't regarded as complete: As a minimal extension of QM, sdBBT supplements it by the primitive ontology in terms of corpuscles and their positions as beables. Secondly, the actual configurations of one subsystem exist indeed independently from those of a remote subsystem: The actual, local configurations completely define the actual state of each subsystem; nonetheless, there exist correlations between them that betoken the non-separable disposition of the joint system.

Finally, one might worry about a metaphysical incompatibility between SR and sdBBT's stochasticity. Arguably, SR is best understood in terms of a Block Universe View[193], with no temporal part of an object's 4-dimensional worldline being ontologically distinguished as "the present" and the present only an indexically designated point on the worldline. By contrast,

[189] Cf. Fine (1989) for an elaboration of this for QM
[190] Op.cit., p. 192
[191] Cf. Howard (1985,1989); also Brown (2005), Appendix B3
[192] Einstein (1948), quoted in Brown (2005), p. 187
[193] Cf. e.g. Petkov (2009), esp. Ch. 5 and 6; Cf., however, Dickson (1989), Ch. 8 for a criticism

the strong intuition prevails, articulated e.g. by Popper[194], that in an indeterministic world the future is "open", i.e. un-determined, while the past is "fixed", unalterable; consequently, the present is the ontologically distinguished moment, where pure potentiality of the open future turns into the congealed facticity of the past. By introducing such an ontologically distinguished hypersurface that defines the present, this ("Growing Block Universe") view has been argued to collide with SR. Consequently, as a stochastic and hence indeterministic theory, sdBBT entails a Growing Block Universe Theory of Time – in conflict with SR.

The conflict can be resolved by challenging the claim that stochastic theories are committed to the kind of indeterminism that implies an open future in the above sense. Dispositionalism offers a straightforward alternative conceptualisation of indeterminism: The 4-dimensional picture of *actual* reality in such an indeterministic world isn't that of continuous trajectories of corpuscles in Minkowski spacetime; rather, the corpuscles are distributed in spacetime like dust.[195] In other words: Whereas the disposition evolves deterministically, its manifestation takes the form of dust sprinkled over spacetime, their distribution typically approximating the relativistic analogues to the Born Rule, relative to an observer. Such a view is effortlessly compatible with a Block Universe View, and might aptly be called a Dust Universe View. (A perhaps helpful paraphrase is that in the Dust Universe View, the 4-dimensional spacetime isn't pervaded by continuous particle trajectories; rather, the "dust of events" is randomly distributed.)

In summary, we have seen that -within the framework (and confines) of relativistic QM- sdBBT suggests considerable advantages over dBBT; a comparative evaluation of quantum field theoretic extensions of both therefore seems promising.[196]

V. Summary and outlook

We started our investigation with examining the conceptual tension between ambitions for an inveterately objectivist "quantum theory without observers", the probabilistic QEH, guaranteeing dBBT's empirical adequacy, and its determinism, as introduced via the GE.

[194] Cf. Popper (1988), Ch. III, sect. 18. Popper adduces this "argument from the asymmetry of the past and the future" to support his indeterminism.
[195] Cf. also Petkov (2009), Ch. 10, who calls this view "4-atomism".
[196] It seems plausible, however, that the deBroglie-Bohmian framework, narrowly understood as a *particle*-based primitive ontology with *position* as beables, needs to give way to, say, a *field*-based primitive ontology with e.g. *fermion number*, as Bell's model for a Hamiltonian QFT suggests, cf. Bell (1984).

Further analysis identified the latter as the source of even more problems, related to dBBT's formal definability, uniqueness, the violation of the Action Reaction Principle and the contrived-looking double role of the wavefunction. Fortunately, the GE turned out to be redundant: All explanatory work w.r.t. solving the Measurement Problem, empirical equivalence with QM or the natural justification of the Symmetrisation Postulate, is done by dBBT's particle primitive ontology with position as beables and contextuality of all other dynamical variables. This suggested to excise the GE, whilst keeping dBBT's aforementioned ontological framework, yielding a realist, albeit fundamentally stochastic/indeterministic theory – sdBBT – the probabilities of which we proposed to construe as an *N*-corpuscle universe's disposition to spontaneously/randomly actualise a certain configuration. We tried to make the case that sdBBT counts as a minimally deBroglie-Bohmian theory, deserving to be taken seriously as a potentially fundamental account of microscopic reality. It turned out to be a many worlds theory – however with the arguably slightly eccentric feature of "temporal solipsism": FAPP-classical worlds typically exist *actualiter* only for an instant, before absconding again into potentiality. Future research regarding sdBBT, especially along the lines of its relativistic field-theoretical extension, is promising.

Its relation, too, to other theories sparks off a number of exciting questions:

- Above we presented sdBBT as a minimally deBroglie-Bohmian theory and argued for its superiority vis-à-vis dBBT. Another theory seems to qualify equally well as minimally deBroglie-Bohmian – namely Nelson Stochastics, which aims to derive the wavefunction and its Schrödinger dynamics from a classical Wiener-process in configuration space, subject to certain natural constraints.[197] It would be highly interesting to compare sdBBT to Nelson Stochastics: Might sdBBT prove advantageous also vis-à-vis other minimally deBroglie-Bohmian Theories?
- More generally, how does sdBBT fare vis-à-vis other primitive ontology approaches to QM?[198] Of special interest is a comparison with "Schrödinger's Many Worlds Interpretation" (Allori et al.[199]), an arguably minimally "continuous-matter-field

[197] Cf. Bacciagaluppi (2005) for a review, as well as a comparison with dBBT.
[198] Cf. Esfeld (2014) for "guidelines for an assessment of the proposals". He argues for the superiority of dBBT. Given our result of sdBBT's superiority over dBBT, sdBBT's superiority seems to follow straightforwardly, but a direct comparison might yield instructive details.
[199] Cf. Allori et al. (2014)

primitive ontology" (Allori et al.). It can be viewed as an Everettian many worlds counterpart to sdBBT, in which temporal solipsism is overcome.

- Perhaps even more alluring will be the comparison with proposals from the rivalling "wavefunction ontology" (Allori) camp, a strictly monist approach which postulates nothing (e.g. corpuscles, fields, etc.) on top of the wavefunction. Of special interest will, of course, be the comparison with Everett's Many Worlds Interpretation.
  Besides its many worlds character being perceived as metaphysically too sumptuous, criticism of the Many Worlds Interpretation has tended to focus on the role and status of probabilities and the Problem of the Preferred Basis, both of which sdBBT eschews *ab initio*.

Acknowledgement

We owe thanks to Harvey Brown (Oxford, UK), Michael Esfeld (Lausanne, CH), Dustin Lazarovici (Lausanne, CH), Niels Linnemann (Geneva, CH), Vera Matarese (HKU, HK), Oliver Pooley (Oxford, UK), James Read (Oxford, UK), Thomas Møller-Nielsen (Oxford, UK), Darrell Rowbottom (Lingnan, HK) Simon Saunders (Oxford, UK) and Jiji Zhang (Lingnan, HK) for helpful discussions and feedback on earlier versions of the manuscript.

Patrick Duerr gratefully acknowledges the generous financial support of the British Society for the Philosophy of Science.

Alexander Ehmann gratefully acknowledges the generous financial support of the Research Grants Council through the Hong Kong PhD Fellowship Scheme.

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