Some historical aspects concerning the rise of the first exact measurements of the anomalous magnetic moment of the muon

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Abstract

In this paper, we wish to outline the main historical moments which have led to the first exact measurements of the anomalous magnetic moment of the muon.

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1. Historical introduction

In this section, we recall the main events and facts of that historical path which goes from the introduction of the spin to the notion of anomalous magnetic moment, with particular attention to the leptonic case. The necessarily limited historical framework so outlined in this section, covers a temporal period which roughly goes up from early 1920s to 1960s.

1.1 On Landé separation factors

Following (Muirhead 1965, Chapter 2) and (Tomonaga 1997), when a fundamental interaction is taken into account then the experimental determination of the basic particle data, like masses, lifetimes, spins and magnetic moments, is necessarily required. The most accurately known properties of the particles are those which can be associated with their magnetic moments. Magnetic properties of elementary particles have been and yet are of paramount importance both to theoretical and experimental high energy physics. One of the main intrinsic properties of the elementary particles is the spin, which can be inferred from the conservation laws for angular momentum. Following (Landau 1982, Chapter VIII), in both classical and quantum mechanics, the laws of conservation of angular momentum are a consequence of the isotropy of space respect to a closed system, so that it depends on the transformation properties under rotation of the coordinate system. Therefore, all quantum systems, like atomic nuclei or composite systems of elementary particles, besides the orbital angular momentum, show to have as well an intrinsic angular momentum, called *spin,* which is unconnected with its motion in space and to which it is also associated a magnetic moment whose strengths are not quantized and may assume any value. The spin disappears in the classical limit $\hbar \to 0$ so that it has no classical counterpart. The spin must be meant as fully distinct from the angular momentum due to the motion of the particle in space, that is to say, the orbital angular momentum. The particle concerned may be either elementary or composite but behaving in some respect as an elementary particle (e.g. an atomic nucleus). The spin of a particle (measured, like the orbital angular momentum, in units of $\hbar$) will be denoted by $\vec{s}$. Following (Rich and Wesley 1972), (Bertolotti 2005, Chapter 8), (Miller et al. 2007) and (Roberts and Marciano 2010, Chapter 1), the physical idea that an electron has an intrinsic angular momentum was first put forward independently of each other by A.H. Compton in 1921 to explain ferromagnetism[1] and by G. Uhlenbeck and S. Goudsmit

[1]Furthermore, Compton acknowledges A.L. Parson for having first proposed the electron as a spinning ring of charge. Compton modified this idea considering a much smaller

in 1925 to explain spectroscopic observations in relation to the anomalous Zeeman effect, while spin was introduced into quantum mechanics by W. Pauli in 1927 as an additional term to the *Pauli equation* which is obtained by the non-relativistic representation of the Dirac equation to small velocities (see (Jegerlehner 2008, Part I, Chapter 3, Section 3.2)) to account for the quantum mechanical treatment of the spin-orbit coupling of the anomalous Zeeman effect (see also (Haken & Wolf 2005, Chapter 14, Section 3)). An equation similar to the Pauli's one, was also introduced by C.G. Darwin in 1927 (see (Roberts and Marciano 2010, Chapter 3, Section 3.2.1)).

Following (Jegerlehner 2008, Part I, Chapter 1), (Melnikov and Vainshtein 2006, Chapter 1) and (Shankar 1994, Chapter 14), leptons have interesting static (classical) electromagnetic and weak properties like the magnetic and electric dipole moments. Classically, dipole moments may arise either from electrical charges or currents. In this regards, an important example which may turns out to be useful to our purposes is the circulating current, due to an orbiting particle with electric charge Q and mass m, which exhibits the following orbital magnetic dipole moment

$$\vec{\mu}_L = \frac{Q}{2c}\vec{r} \wedge \vec{v} = \frac{Q}{2mc}\vec{L} = \Gamma\vec{L} \tag{1}$$

where $\Gamma = Q/2mc$ is the *classical gyromagnetic ratio*[2] and $\vec{L} = m\vec{r}\wedge\vec{v} = \vec{r}\wedge\vec{p}$ is the orbital angular momentum whose corresponding quantum observable is the operator $-i\hbar\vec{r} \wedge \nabla = \hbar\vec{l}$, so that we have the following orbital magnetic dipole moment operator (see (Jegerlehner 2008, Part I, Chapter 3) and (Shankar 1994, Chapter 14))

$$\vec{\mu}_l = g_l \frac{Q\hbar}{2mc}\vec{l} \tag{2}$$

where g_l is a constant introduced by the usual quantization transcription rules. For $Q = e$, the quantity $\mu_0 = e\hbar/2mc$ is normally used as a unit for the magnetic moments and is called the *Bohr magneton.* The electric charge Q is usually measured in units of e, so that $Q = -1$ for leptons and $Q = +1$ for antileptons; therefore, we also may rewrite (2) in the following form

$$\vec{\mu}_l = g_l \frac{Qe\hbar}{2mc}\vec{l} = g_l Q\mu_0\vec{l}. \tag{3}$$

distribution of charge mainly concentrated near the center of the electron. The Compton's paper is almost unknown (see (Compton 1921)) albeit it is quoted by the 1926 Uhlenbeck and Goudsmit paper. Following (Roberts and Marciano 2010, Chapter 3, Section 3.2.1), also R. Kronig proposed, in 1925, the spin as an internal angular momentum responsible for the electron forth's quantum number (see (Bertolotti 2005, Chapter 8).

[2]Usually, the gyromagnetic ratio is denoted by lower case γ, but here we prefer to use the upper case Γ to distinguish it by the well-known *Lorentz factor* $\gamma = 1/\sqrt{1-\beta^2}$ with $\beta = v^2/c^2$.

Both electric and magnetic properties have their origin in the electrical charges and their currents, apart from the existence or not of magnetic charges. Following (Jegerlehner 2008, Part I, Chapter 1) and (Muirhead 1965, Chapter 9, Section 9.2(d)), whatever the origin of magnetic and electric moments are, they contribute to the electromagnetic interaction Hamiltonian (interaction energy) of the particle with magnetic and electric fields which, in the non-relativistic limit, is given by

$$\mathcal{H}_{em} = -(\vec{\mu}_m \cdot \vec{B} + \vec{d}_e \cdot \vec{E}) \tag{4}$$

where $\vec{\mu}_m$ and $\vec{d}_e$ are respectively the magnetic and electric dipole moments (see (Jegerlehner 2008, Part I, Chapter 1)).

If one replaces the orbital angular momentum $\vec{L}$ with the spin $\vec{s}$, then we might search for an analogous (classical) magnetic dipole moment, say $\vec{\mu}_s$, associated with it and, therefore, given by $(Q/2mc)\vec{s}$. Nevertheless, following (Born 1969, Chapter 6, Section 38) and (Muirhead 1965, Chapter 2, Section 2.5)), to fully account for the anomalous Zeeman effect, we should consider this last expression multiplied by a certain scalar factor, say g_s (often simply denoted by g), so that

$$\vec{\mu}_s = g_s \frac{Q}{2mc}\vec{s} \tag{5}$$

which is said to be the *spin magnetic moment.* Now, introducing, as a corresponding quantum observable, the *spin operator* defined by $\vec{S} = \hbar\vec{s} = \hbar\vec{\sigma}/2$, where $\vec{\sigma}$ is the *Pauli spin operator,* it is possible to consider both the *spin magnetic moment operator* and the *electric dipole moment operator* (see (Jegerlehner 2008, Part I, Chapter 1)), respectively defined as follows

$$\vec{\mu}_s \doteq g_s Q \mu_0 \frac{\vec{\sigma}}{2}, \qquad \vec{d}_e \doteq \eta Q \mu_0 \frac{\vec{\sigma}}{2}, \tag{6}$$

where η is a constant, the electric counterpart of g_s. Following (Caldirola et al. 1982, Chapter XI, Section 3), the attribution of a $s = 1/2$ spin value to the electron, led to the formulation of the so-called *vectorial model* of the atom. In such a model, amongst other things, the electron orbital angular moment $\vec{L}$ composes with the spin $\vec{s}$ through well-defined spin-orbit coupling rules (like the Russell-Saunders ones) to give the (classical) *total angular moment* defined to be $\vec{j} \doteq \vec{L} + \vec{s}$, while the (classical) *total magnetic moment* is defined to be $\vec{\mu}_{total} \doteq \vec{\mu}_L + \vec{\mu}_s$, so that, taking into account (3) and (6), the corresponding quantum observable counterpart, in this vectorial model, is

$$\vec{\mu}_{total} \doteq \vec{\mu}_l + \vec{\mu}_s = g_l Q \mu_0 \vec{l} + g_s Q \mu_0 \frac{\vec{\sigma}}{2} = Q\mu_0(g_l \vec{l} + g_s \vec{S}) \tag{7}$$

which is said to be the *total magnetic moment* of the given elementary particle with charge Q and mass m; since $g_s \neq 1$, it follows that it is not, in general, parallel to the total angular moment operator $\vec{J} \doteq \vec{l} + \vec{S}$, so that it undergoes to precession phenomena when magnetic fields act.

The existence of the various above constants g_l, g_s and η is mainly due to the fact that, in the vectorial model of anomalous Zeeman effect, the direction of total angular moment $\vec{j}$ does not coincide with the direction of total magnetic moment, so that these scalar factors just take into account the related non-zero angles which are called *Landé separation factors* because first introduced by A. Landé (1888-1976) in the early 1920s (see (Born 1969, Chapter 6, Section 38)). To be precise, only the parallel component of $\vec{\mu}_{tot}$ to $\vec{j}$, say $\vec{\mu}^{\|}_{tot}$, is efficacious, so that we should have

$$\vec{\mu}^{\|}_{tot} = g_j \frac{Q\hbar}{2mc}\vec{j} \tag{8}$$

where the scalar factor g_j (or simply g) takes into account the difference between the vectorial model of anomalous Zeeman effect and the theory of the normal one. To may computes this factor, we start from the relation

$$\mu^{\|}_{tot} = \mu_l \cos(\widehat{\vec{l}, \vec{j}}) + \mu_s \cos(\widehat{\vec{s}, \vec{j}}) \tag{9}$$

with

$$\mu_l = g_l \frac{Q\hbar}{2mc} l, \qquad \mu_s = g_s \frac{Q\hbar}{2mc} s \tag{10}$$

where g_l and g_s are known to be respectively the *orbital* and *spin factors,* which, in turn, represent the ratios respectively between the orbital and spin magnetic and mechanic moments. Replacing (10) into (9), we have

$$g_j = g_l \frac{l}{j} \cos(\widehat{\vec{l}, \vec{j}}) + g_s \frac{s}{j} \cos(\widehat{\vec{s}, \vec{j}}) \tag{11}$$

from which (see (Born 1969, Chapter 6, Section 38)) it is possible to reach to the following relation

$$g_j = g_l \frac{(j^2 + l^2 - s^2)}{2j^2} + g_s \frac{(j^2 + s^2 - l^2)}{2j^2} \tag{12}$$

Experimental evidences dating back to 1920s and mainly related to the anomalous Zeeman effect, seemed suggesting that $g_l = 1$ and $g_s = 2$ for the electron, that is, the atomic vectorial model explains the fine structure features of alkali metals and the anomalous Zeeman effect if one supposes to be $g_s \neq 1$, that is to say, a spin intrinsic gyromagnetic ratio anomalous respect to the orbital one ($g_l = 1$), so speaking of a *spin anomaly.* Following (Bohm 1993, Chapter IX, Section 3), the deviations from the $g_s = 2$ value for the electron comes from the radiative corrections of quantum electrodynamics and is of the same order as, and of analogous origin to, the Lamb shift. The value $g_s = 2$ was first established as far back as 1915 by a celebrated experiment of A. Einstein and W.J. de Haas which led to the formulation of the so-called *Einstein-de Hass effect* and that was also incorporated in the spin hypothesis put forward in the 1920s (see (Špolskij 1986, Volume II, Chapter VII, Section 70)). Following (Jegerlehner 2008, Part I, Chapter 1), the anomalous magnetic moment is an observable which may be studied through experimental analysis of the motion of leptons. The story started in 1925 when Uhlenbeck and Goudsmit put forward the hypothesis that an electron had an intrinsic angular momentum of $\hbar/2$ and that associated with this there were a magnetic dipole moment equal to $e\hbar/2mc$, i.e. the Bohr magneton μ_0. According to E. Back and A. Landé, the question which naturally arose was whether the magnetic moment of the electron $(\mu_m)_e$ is precisely equal to μ_0, or else $g_s = 1$ in $(10)_2$, to which them tried to answer through a detailed study of numerous experimental investigations on the Zeeman effect made in 1925, reaching to the conclusion that the Uhlenbeck and Goudsmit hypothesis was consistent although they did not really determine the value of g_s. In 1927, Pauli formulated the quantum mechanical treatment of the electron spin in which g_s remained a free parameter, whilst Dirac presented his revolutionary relativistic theory of electron in 1928, which, instead, unexpectedly predicted $g_s = 2$ and $g_l = 1$ for a free electron. The first experimental evidences for the Dirac's theoretical foresights for electrons came from L.E. Kinster and W.V. Houston in 1934, albeit with large experimental errors at that time. Following (Kusch 1956), it took many more years of experimental attempts to descry that the electron magnetic moment could exceed 2 by about 0.12, the first clear indication of the existence of a certain anomalous contribution to the magnetic moment given by

$$(13) \qquad a_i \doteq \frac{(g_s)_i - 2}{2}, \qquad i = e, \mu, \tau.$$

With the new results on renormalization of QED by J. Schwinger, S.I. Tomonaga and R.P. Feynman of 1940s, the notion of *anomalous magnetic moment* (AMM) falls into the wider class of QED radiative corrections.

1.2 On Field Theory aspects of AMM

Following (Jegerlehner 2008, Part I, Chapter 3), for the measurement of the anomalous magnetic moment of a lepton, it is necessary to consider the motion of a relativistic point-particle i (or Dirac particle[3]) of charge $Q_i e$ and mass m_i in an external electromagnetic field $A_\mu^{ext}(x)$. The equations of motion of a charged Dirac particle in an external field are given by the *Dirac equation*

$$\big(i\hbar\gamma^\mu\partial_\mu + Q_i\frac{e}{c}\gamma^\mu(A_\mu + A_\mu^{ext}(x)) - m_i c\big)\psi_i(x) = 0, \tag{14}$$

and by the second order wave equation

$$\big(\Box g^{\mu\nu} - (1-\xi^{-1}\big)\partial^\mu\partial^\nu)A_\nu(x) = -Q_i e\bar{\psi}_i(x)\gamma^\mu\psi_i(x). \tag{15}$$

The first step is now to find a solution to the relativistic one-particle problem given by the Dirac equation (14) in the presence of an external field, neglecting the radiation field in first approximation. In such a case, the equation (14) reduces to

$$i\hbar\frac{\partial\psi_i}{\partial t} = \big(-c\vec{\alpha}(i\hbar\vec{\nabla} - Q_i\frac{e}{c}\vec{A}) - Q_i e\Phi + \beta m_i c^2\big)\psi_i \tag{16}$$

where

$$\beta = \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \vec{\alpha} = \gamma^0\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \tag{17}$$

are the Dirac matrices, $A^{\mu\ ext} = (\Phi, \vec{A})$ is the electromagnetic four-potential with scalar and vector potential respectively given by Φ and $\vec{A}$ (of the external electromagnetic field) and $i = e, \mu, \tau$. For the interpretation of the solution to the last Dirac equation (16), the non-relativistic limit plays an important role because many relativistic QFT problems may be most easily understood and solved in terms of the non-relativistic problem as a starting point. To this end, it is helpful and more transparent to work in natural units, the general rules of transcription being the following: $p^\mu \to p^\mu, d\mu(p) \to \hbar^{-3}d\mu(p), m \to mc, e \to e/(\hbar c), \exp(ipx) \to \exp(ipx/\hbar)$ and, for spinors, ${}^t(u,v) \to {}^t(u/\sqrt{c}, v/\sqrt{c})$; furthermore, we shall consider a generic lepton e^-, μ^-, τ^- with charge Q_i, dropping the index i. Moreover, to get, from the Dirac spinor ψ, the two-component Pauli spinors φ and χ in the non-relativistic limit, one has to perform an appropriate unitary transformation,

[3] That is to say, a particle without internal structure.

the so-called *Foldy-Wouthuysen transformation*[4], upon the Dirac equation (16) rewritten as follows

$$i\hbar\frac{\partial\psi}{\partial t} = \vec{H}\psi, \qquad \vec{H} = c\vec{\alpha}\big(\vec{p} - \frac{Q}{c}\vec{A}\big) + \beta mc^2 + Q\Phi, \tag{18}$$

with $\vec{\alpha}$ and β given by (17) (see (Bjorken and Drell 1964, Chapter 1, Section 4, Formula (1.26)).

Then, following (Bjorken and Drell 1964, Chapter 1, Section 4) and (Jegerlehner 2008, Part I, Chapter 3), in order to obtain the non-relativistic representation for small velocities, we should split off the phase of the Dirac field ψ, which is due to the rest energy of the lepton

$$\psi = \tilde{\psi}\exp\big(-i\frac{mc^2}{\hbar}t\big), \qquad \tilde{\psi} = \begin{pmatrix} \tilde{\varphi} \\ \tilde{\chi} \end{pmatrix} \tag{19}$$

so that the Dirac equation takes the form

$$i\hbar\frac{\partial\tilde{\psi}}{\partial t} = (\vec{H} - mc^2)\tilde{\psi} \tag{20}$$

and describes the following coupled system of equations

$$\big(i\hbar\frac{\partial}{\partial t} - Q\Phi\big)\tilde{\varphi} = c\vec{\sigma}\big(\vec{p} - \frac{Q}{c}\vec{A}\big)\tilde{\chi}, \tag{21}$$

$$\big(i\hbar\frac{\partial}{\partial t} - Q\Phi + 2mc^2\big)\tilde{\chi} = c\vec{\sigma}\big(\vec{p} - \frac{Q}{c}\vec{A}\big)\tilde{\varphi} \tag{22}$$

which, respectively, provide the Pauli description in the non-relativistic limit and the one of the negative-energy states. As $c \to \infty$, it is possible to prove that

$$\tilde{\chi} \cong \frac{1}{2mc}\vec{\sigma}\big(\vec{p} - \frac{Q}{c}\vec{A}\big)\tilde{\varphi} + O(1/c^2), \tag{23}$$

[4]It is a unitary transformation introduced around the late 1940s by L.L. Foldy and S.A. Wouthuysen to study the non-relativistic limits of Dirac equation as well as to overcome certain conceptual and theoretical problems arising from the relativistic interpretations of position and momentum operators. Following (Foldy and Wouthuysen 1950), in the non-relativistic limit, where the momentum of the particle is small compared to m, it is well known that a Dirac particle (that is, one with spin 1/2) can be described by a two-component wave function in the Pauli theory. The usual method of demonstrating that the Dirac theory goes into the Pauli theory in this limit makes use of the fact that two of the four Dirac-function components become small when the momentum is small. One then writes out the equations satisfied by the four components and solves, approximately, two of the equations for the small components. By substituting these solutions in the remaining two equations, one obtains a pair of equations for the large components which are essentially the Pauli spin equations. See (Bjorken and Drell 1964, Chapter 4).

by which we have

$$(24)\qquad \big(i\hbar\frac{\partial}{\partial t}-Q\Phi\big)\tilde{\varphi}\cong\frac{1}{2m}\big(\vec{\sigma}(\vec{p}-\frac{Q}{c}\vec{A})\big)^2\tilde{\varphi}$$

and since $\vec{p}$ does not commute with $\vec{A}$, we may use the relation

$$(25)\qquad (\vec{\sigma}\vec{a})(\vec{\sigma}\vec{b})=\vec{a}\vec{b}+i\vec{\sigma}(\vec{a}\wedge\vec{b})$$

to obtain

$$(26)\qquad \big(\vec{\sigma}(\vec{p}-\frac{Q}{c}\vec{A})\big)^2=\big(\vec{p}-\frac{Q}{c}\vec{A}\big)^2-\frac{Q\hbar}{c}\vec{\sigma}\cdot\vec{B}$$

where $\vec{B}=rot\ \vec{A}$, so finally reaching to the following 1927 *Pauli equation*

$$(27)\qquad i\hbar\frac{\partial\tilde{\varphi}}{\partial t}=\tilde{H}\tilde{\varphi}=\Big(\frac{1}{2m}\big(\vec{p}-\frac{Q}{c}\vec{A}\big)^2+Q\Phi-\frac{Q\hbar}{2mc}\vec{\sigma}\cdot\vec{B}\Big)$$

which, up to the spin term, is nothing but the non-relativistic Schrödinger equation. Following too (Muirhead 1965, Chapter 3, Section 3.3(f)), the last term of (27) has the form of an additional potential energy. Now, by (4), since the potential energy of a magnet of moment $\vec{\mu}_m$, in a field of strength B, is $-\vec{\mu}_m\cdot\vec{B}$, equation (27) shows that a Dirac particle with electric charge Q should possess a magnetic moment equal to $(Q\hbar/2mc)\vec{\sigma}=2Q\mu_0\vec{\sigma}/2$ that, compared with $(6)_1$, would imply $g_s=2$. This is what Dirac theory historically provided for an electron. Later, Pauli showed as the Dirac equation could be little modified to account for leptons of arbitrary magnetic moment by adding a suitable term.

Indeed, in[5] (Pauli 1941, Section 5)), the author concludes his report with some simple applications of the theories discussed in (Pauli 1941, Part II, Sections 1, 2(d) and 3(a)), concerning the interaction of particles of spin 0, 1, and 1/2 with the electromagnetic field. In the last two cases we denote the value $e\hbar/2mc$ of the magnetic moment as the normal one, where m is the rest mass of the particle. The assumption of a more general value $g_s(e\hbar/2mc)$ for the magnetic moment demands the introduction of additional terms, proportional to g_s-1, into the Lagrangian or Hamiltonian. Pauli concludes his report with some simple applications of the theories discussed in (Pauli 1941, Part II, Sections 1, 2(d) and 3(a)) concerning the interaction of particles of spin 0, 1, and 1/2 with the electromagnetic field. In the last two cases, Pauli denotes the value $e\hbar/2mc$ of the magnetic moment as the normal one, where m is the rest mass of the particle. The assumption

[5]See also (Pauli 1973, Chapter 6, Section 29).

of a more general value $g(e\hbar/2mc)$ for the magnetic moment demands the introduction of additional terms, proportional to $g-1$, in the Lagrangian or Hamiltonian. To be precise, following (Dirac 1958, Chapter 11, Section 70), (Corinaldesi and Strocchi 1963, Chapter VII, Section 4), (Muirhead 1965, Chapter 3, Section 3.3(f)) and (Levich et al. 1973, Chapter 8, Section 63 and Chapter 13, Section 118), Pauli modified the basic Dirac equation, written in scalar form as follows

$$i\hbar\gamma_\mu \frac{\partial}{\partial x_\mu}\psi + mc^2\psi - i\hbar\frac{Q}{c}\gamma_\mu A_\mu \psi = 0, \tag{28}$$

to get the following Lorentz invariant *Dirac-Pauli equation*

$$\begin{aligned} & i\hbar\gamma_\mu \frac{\partial}{\partial x_\mu}\psi + mc^2\psi - i\hbar\frac{Q}{c}\gamma_\mu A_\mu \psi - i\hbar a_\mu \gamma_\mu \gamma_\nu (A_{\mu,\nu} - A_{\nu,\mu}) \\ & = i\hbar\gamma_\mu \frac{\partial}{\partial x_\mu}\psi + mc^2\psi - i\hbar\frac{Q}{c}\gamma_\mu A_\mu \psi - i\hbar a_\mu \sigma_{\mu\nu} q_\nu A_\mu = 0 \end{aligned} \tag{29}$$

replacing the gauge invariant interaction term $-i\hbar\sigma_{\mu\nu}q_\nu A_\mu$ with the following phenomenological term (see also (Sakurai 1967, Chapter 3, Section 3-5) $-i\hbar a_\mu \sigma_{\mu\nu} q_\nu A_\mu$ called an *anomalous moment interaction* (or *Pauli moment*), where a_μ represents the anomalous part of the magnetic moment of the particle, q is the momentum transfer and $\hat{\sigma} = -(i/2)[\vec{\gamma}, \vec{\gamma}]$ is the spin 1/2 momentum tensor. In the non-relativistic limit, this last expression reduces to the following equation (compare with (27))

$$i\hbar\frac{\partial\psi}{\partial t} = \Big(\frac{1}{2m}\big(\vec{p} - \frac{Q}{c}\vec{A}\big)^2 + Q\psi - \big(a_\mu + \frac{Q\hbar}{2mc}\big)\vec{\sigma}\cdot\vec{B}\Big) \tag{30}$$

so justifying the use of the term 'anomalous' to denote a deviation from the classical results. Thus, the transition from the non-relativistic approximation of the Dirac equation goes over into the Pauli equation; furthermore, from this reduction there results not only the existence of the spin of particles but also the existence of the intrinsic magnetic moment of particle and its anomalous part. Namely, we should have $g_s = 2(1 + a_\mu)$, where its higher order part $a_\mu = (g_s - 2)/2 \geq 0$ just measures the deviation's degree respect to the value $g_s = 2$ (Dirac moment) as predicted by the 1928 Dirac theory for electron[6] as well as by H.A. Kramers in 1934 (see (Farley and Semertzidis 2004, Section 1)) developing Lorentz covariant equations for spin motion in a moving system. Later, this Pauli *ansatz* was formally improved

[6]Following (Roberts and Marciano 2010) and (Miller et al. 2007, Section 1), the non-relativistic reduction of the Dirac equation for an electron in a weak magnetic field $\vec{B}$, is as follows $i\hbar(\partial\psi/\partial t) = [(p^2/2m) - (e/2m)(\vec{L} + 2\vec{S})\cdot\vec{B}]\psi$, by which it follows that $g_s = 2$.

and generalized by L.L. Foldy and S.A. Wouthuysen in the forties to obtain a generalized Pauli equation which will be the theoretical underpinning of further experiments. Indeed, at the first order in $1/c$, the lepton behaves as a particle which has, other than a charge, also a magnetic moment given by $\mu_m = (Q\hbar/2mc)\vec{\sigma} = (Q/mc)\vec{S}$, as said above. Following (Corinaldesi and Strocchi 1963, Chapter VII, Section 5), (Bjorken and Drell 1964, Chapter 4, Section 3) and (Jegerlehner 2008, Part I, Chapter 3), from an expansion in $1/c$ of the Dirac Hamiltonian given by $(18)_2$, we have the following effective third order Hamiltonian obtained applying a third canonical Foldy-Wouthuysen transformation to $(18)_2$

$$\begin{aligned} \vec{H}'''_{FW} &= \beta\Big(mc^2 + \frac{\big(\vec{p}-(Q/c)\vec{A}\big)^2}{2m} - \frac{\vec{p}^4}{8m^3c^2}\Big) + Q\Phi - \beta\frac{Q\hbar}{2mc}\vec{\sigma}\cdot\vec{B} + \\ &\quad -\frac{Q\hbar^2}{8m^2c^2}div\vec{E} - \frac{Q\hbar}{4m^2c^2}\vec{\sigma}\cdot\big[(\vec{E}\wedge\vec{p} + \frac{i}{2}rot\vec{E})\big] + O(1/c^3) \end{aligned} \tag{31}$$

where each term of it, has a direct physical meaning: see (Bjorken and Drell 1964, Chapter 4, Section 3) for more details. In particular, the last term takes into account the spin-orbit coupling interaction energy and will play a fundamental role in setting up the experimental apparatus of many $g-2$ later experiments. The last Hamiltonian, to the third order, gives rise to the following *generalized Pauli equation* $i\hbar(\partial\tilde{\varphi}/\partial t) = \vec{H}'''_{FW}\tilde{\varphi}$, which is a generalized version, including high relativistic terms via the application of a Foldy-Wouthuysen transformation, of the first form proposed by Pauli in 1941 (see (Pauli 1941)) and that leads to the second approximation Schrödinger-Pauli equation as a non-relativistic limit of the Dirac equation (see (Corinaldesi and Strocchi 1963, Chapter VIII, Section 1)).

Our particular interest is the motion of a lepton in an external field under consideration of the full relativistic quantum behavior which is ruled by the QED equations of motions (14) and (15) that, in turn, under the action of an external field, reduce to (16). For slowly varying field, the motion is essentially determined by the generalized Pauli equation which besides also serves as a basis for understanding the role of the magnetic moment of a lepton at the classical level. The anomalous magnetic moment roughly estimates the deviations from the exact value $g_s = 2$, because of certain relativistic quantum fluctuations in the electromagnetic field (initially called *Zitterbewegung*) around the leptons and mainly due, besides weak and strong interaction effects, to QED higher order effects as a consequence of the interaction of the lepton with the external (electromagnetic) field and which are usually eliminated through the so-called *radiative corrections.* At present, we are interested to QED contributions only. Following (Muirhead 1965, Chapter

11, Section 11.4), (Jegerlehner 2008, Part I, Chapter 3) and (Melnikov and Vainshtein 2006, Chapter 2), the QED Lagrangian of interaction of leptons and photons is (see also (Muirhead 1965, Chapter 8, Section 8.3(a)))

$$\mathcal{L}_{int}^{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma_\mu\partial_\mu - m)\psi - QJ^\mu A_\mu \tag{32}$$

where ψ is the lepton field, $A^\mu = (\Phi, \vec{A})$ is the vector potential of the electromagnetic field, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the field-strength tensor of the electromagnetic field, $J^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$ is the electric current and Q is the lepton charge. Let us consider an incoming lepton $l(p_1^\mu, r_1)$, with 4-momentum p_1^μ, rest mass m, charge Q and r_1 as third component of spin, which scatters off the external electromagnetic potential A_μ towards a lepton $l(p_2^\mu, r_2)$ of 4-momentum p_2^μ and third component of spin r_2. To the first order in the external field and in the classical limit of $q^2 = p_2^2 - p_1^2 \to 0$, the interaction is described by the following scattering amplitude

$$\mathcal{M}(x;p) = \langle l(p_2^\mu, r_2)|J^\mu(x)|l(p_1^\mu, r_1)\rangle \tag{33}$$

where $\vec{q} = \vec{p}_2 - \vec{p}_1$ is the momentum transfer. In practice, it will be more convenient to work, through Fourier transforms, with invariant momentum transfers rather than spatial functions. So, in momentum space, due to space-time translation invariance for which $J^\mu(x) = \exp(iPx)J^\mu(0)\exp(-iPx)$, and to the fact that the lepton states are eigenstates of 4-momentum, that is to say $\exp(-iPx)|l(p_i, r_i)\rangle = \exp(-ip_i x)|l(p_i; r_i)\rangle, i = 1, 2$, we find the following Fourier transform of the scattering matrix

$$\begin{aligned}\tilde{\mathcal{M}}(q;p) &= \int \exp(iqx)\langle l(p_2, r_2)|J^\mu(x)|l(p_1, r_1)\rangle d^4x = \\ &= \int \exp[i(p_2 - p_1 - q)x]\langle l(p_2, r_2)|J^\mu(0)|l(p_1, r_1)\rangle d^4x = \\ &= (2\pi)^4\delta^{(4)}(q - p_2 + p_1)\langle l(p_2, r_2)|J^\mu(0)|l(p_1, r_1)\rangle \end{aligned}\tag{34}$$

which is proportional to the Dirac δ-function of 4-momentum conservation. Therefore, the T-matrix element is given by

$$\langle l(p_2, r_2)|J^\mu(0)|l(p_1, r_1)\rangle. \tag{35}$$

Via the current conservation law $\partial_\mu J^\mu(\vec{x}) = 0$ and the parity conservation in QED, the most general parametrization of the T-matrix element has the following QED relativistically covariant decomposition

$$\langle l(p_2)|J^\mu(0)|l(p_1)\rangle = \bar{u}(p_2)\Gamma^\mu(p_2, p_1)u(p_1) \tag{36}$$

where Γ^μ, called lepton-photon *vertex function,* is any expression (or group of expression) which has the transformation properties of a 4-vector and is also a 4×4 matrix in the spin space of the lepton. Following (Muirhead 1965, Chapter 11, Section 11.4(c)) and (Roberts and Marciano 2010, Chapter 2, Section 2.2; Chapter 3, Section 3.2.2), we shall have the following Lorentz structure for the scattering amplitude

$$(37)\qquad \bar{u}(p_2)\Gamma^\mu(p_2,p_1)u(p_1) = -iQ\bar{u}(p_2)\Big(F_D(q^2)\gamma^\mu + F_P(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}\Big)u(p_1)$$

where $u(p)$ denotes the Dirac spinors, while $\sigma^{\mu\nu} = (i/2)(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) = (i/2)[\gamma^\mu,\gamma^\nu]$ are the components of the Dirac spin operator $\hat{\sigma} = -(i/2)\vec{\gamma}\wedge\vec{\gamma}$ or else the spin 1/2 angular momentum tensor. $F_D(q^2)$ (or $F_E(q^2)$) is the *Dirac* (or *electric charge*) *form factor,* while $F_P(q^2)$ (or $F_M(q^2)$) is the *Pauli* (or *magnetic*) *form factor,* which roughly are connected respectively with the distribution of charge over the lepton and with the anomalous magnetic moment to the interaction lepton-electromagnetic field. We now need to know the relationships between these form factors and the anomalous part of the lepton magnetic moment.

In the non-relativistic quantum mechanics, a lepton interacting with an electromagnetic field is described by the Hamiltonian

$$(38)\qquad H = \frac{(\vec{p}-Q\vec{A})^2}{2m} - \vec{\mu}_s\cdot\vec{B} + Q\Phi, \qquad \vec{B} = rot\vec{A}$$

which is nothing that $\tilde{H}$ of (27). To find the relations between the lepton magnetic moment μ_s and the Dirac and Pauli form factors, we consider the scattering of the lepton off the external vector potential A_μ in the non-relativistic approximation, using the Hamiltonian (38) and comparing the results with (33). Following (Melnikov and Vainshtein 2006, Chapter 2), the non-relativistic scattering amplitude in the first order Born approximation is given by

$$(39)\qquad \Omega = -\frac{m}{2\pi}\int\bar{\psi}(\vec{p}_2)V\psi(\vec{p}_1)d^3\vec{r}$$

where $\psi(\vec{p}_1) = \tilde{\varphi}\exp(i\vec{p}_1\cdot\vec{r})$ and $\psi(\vec{p}_2) = \tilde{\chi}\exp(i\vec{p}_2\cdot\vec{r})$ are the wave functions of the lepton described by the two components of Pauli spinors (see (19)) $\tilde{\varphi}$ and $\tilde{\chi}$, and

$$(40)\qquad V = -\frac{Q}{2m}(\vec{p}\cdot\vec{A} + \vec{A}\cdot\vec{p}) - \mu_s\vec{\sigma}\cdot\vec{B} + Q\Phi.$$

By a Fourier transform, we have

$$(41)\qquad \Omega = -\frac{m}{2\pi}\tilde{\chi}\Big(-\frac{Q}{2m}\vec{A}_q\cdot(\vec{p}_2+\vec{p}_1) + Q\Phi_q - i\mu_s\vec{\sigma}\cdot(\vec{q}\wedge\vec{A}_q)\Big)\tilde{\varphi}$$

where Φ_q and $\vec{A}_q$ stands for the Fourier transforms of the electric potential Φ and of the vector potential $\vec{A}$. Therefore, we will derive (41) starting from the relativistic expression for the scattering amplitude (33) and taking then the non-relativistic limit. If the Dirac spinors are normalized to $2m$, the relation between the two oscillating amplitudes in the non-relativistic limit, is given by

$$-i \lim_{|\vec{p}| \ll m} \mathcal{M}(x;p) = 4\pi\Omega. \tag{42}$$

To derive the non-relativistic limit of the scattering amplitude $\mathcal{M}$, we use the explicit representation of the Dirac matrices, given by

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad i = 1,2,3, \tag{43}$$

and the Dirac spinors $u(p)$. Using these expressions in $\mathcal{M}$ and working at first order in $|\vec{p}_i|/m$ $i = 1,2$, we obtain

$$\begin{aligned} \mathcal{M} &= -2iem\tilde{\chi}\Big[F_D(0)\Big(\Phi_q - \frac{\vec{A}_q\cdot(\vec{p}_1+\vec{p}_2)}{2m}\Big) + \\ &\quad -i\frac{F_D(0)+F_P(0)}{2m}\vec{\sigma}\cdot(\vec{q}\wedge\vec{A}_q)\Big]\tilde{\varphi}. \end{aligned} \tag{44}$$

Using (41), (42) and (44), we find

$$F_D(0) = 1, \qquad \mu_s = \frac{Q}{2m}\big(F_D(0) + F_P(0)\big) \tag{45}$$

which compared with (5) and (6), give

$$g_s = 2(1 + F_P(0)) \tag{46}$$

so that, if the Pauli form factor $F_P(q^2)$ does not vanish for $q = 0$, then g_s is different from 2, the value predicted by Dirac theory of electron. It is conventional to call this difference the muon anomalous magnetic moment and write it as

$$a_\mu = F_P(0) = \frac{g_s - 2}{2} \tag{47}$$

so that, in the static (classical) limit we have too

$$F_D(0) = 1, \qquad F_P(0) = a_\mu \tag{48}$$

where the first relation is the so-called *charge renormalization condition* (in units of Q), while the second relation is the finite prediction for a_μ in terms of the pauli form factor. In QED, a_μ may be computed in the perturbative expansion in the fine structure constant[7] $\alpha = Q^2/4\pi$ as follows

$$a_\mu^{QED} = \sum_{i=1}^{\infty} a_\mu^{(i)} = \sum_{i=1}^{\infty} c_i \left(\frac{\alpha}{\pi}\right)^i. \tag{49}$$

The first term in the series is $O(\alpha)$ since, when radiative corrections are neglected, the Pauli form factor vanishes. This is easily seen from the QED Lagrangian $\mathcal{L}_{int}^{QED}$ given by (32), which implies that, through leading order in α, the interaction between the external electromagnetic field and the lepton, is given by $-iQ\bar{u}(p_2)\gamma^\mu u(p_1)A_\mu$. A consequence of the current conservation, is the fact that the Dirac form factor satisfies the condition $F_D(0) = 1$ to all orders in the perturbation expansion. The renormalization constants influence the Pauli form factor only indirectly, through the mass, the charge and the fermion wave function renormalization, because there is no corresponding tree-level operator in QED Lagrangian. Therefore, the anomalous magnetic moment is the unique prediction of QED; moreover, the $O(\alpha)$ contribution to a_μ has to be finite without any renormalization. The QED radiative corrections provide the largest contribution to the lepton anomalous magnetic moment. The one-loop result was computed by J. Schwinger in 1948 (see (Schwinger 1948)), who found the following lowest-order radiative (or one-loop) correction to the electron anomaly (see (Rich and Wesley 1972) and (Roberts and Marciano 2010, Chapter 3, Section 3.2.2.1))

$$a_e^{(2)} = F_P(0) = \alpha/2\pi \cong 0.00116. \tag{50}$$

In 1949, F.J. Dyson showed that Schwinger's theory could be extended to allow calculation of higher-order corrections to the properties of quantum systems. Since Dyson showed too that the one-loop QED contribution to the anomalous magnetic moment did not depend on the mass of the fermion, the Schwinger's result turned out to be valid for all leptons, so that we have $a_i^{(2)} = F_P(0) = \alpha/2\pi,\ i = e, \mu, \tau$. Currently, QED calculations have been extended to the four-loop order and even some estimates of the five-loop contribution exist. It is interesting however to remark that Schwinger's calculation was performed before the renormalizability of QED were understood

[7]Following (Muirhead 1965, Chapter 1, Section 1.3(b)), the interaction of the elementary particles with each other can be separated into three main classes, each with its own coupling strength. To be precise, the common parameter appearing in the electromagnetic processes is the *fine structure constant* $\alpha = e^2/4\pi\hbar c$; the strength of strong interactions is characterized by the dimensionless coupling term $g^2/4\pi\hbar c$, while the weak interactions are ruled by the Fermi coupling constant G_F.

in details; historically, this provided a first interesting example of a fundamental physics result derived from a theory that was considered to be quite ambiguous at that time. Therefore, the anomalous magnetic moment of a lepton is a dimensionless quantity which may be computed order by order as a perturbative expansion in the fine structure constant α in QED and beyond this, in the Standard Model (SM) of elementary particles or extensions of it. As an effective interaction term, the anomalous magnetic moment is mainly induced by the interaction of the lepton with photons or other particles, so that it has a pure QED origin. It corresponds to a dimension 5 operator (see (51)) and since any renormalizable theory is constrained to exhibit terms of dimension 4 or less only, it follows that such a term must be absent for any fermion in any renormalizable theory at tree (or zero-loop) level. It is the absence of such a *Pauli term* that leads to the prediction $g_s = 2 + O(\alpha)$. Therefore, at that time, it was necessary looking for other theoretical tools and techniques to experimentally approach the determination of the anomalous magnetic moment of leptons. Following (Jegerlehner 2008, Part I, Chapter 3), in higher orders the form factors for the muon in general acquires an imaginary part. Indeed, if one considers the following effective dipole moment Lagrangian with complex coupling

$$\mathcal{L}_{eff}^{DM} = -\frac{1}{2}\left[\bar{\psi}\sigma^{\mu\nu}\left(D_\mu\frac{1+\gamma_5}{2} + \bar{D}_\mu\frac{1-\gamma_5}{2}\right)\psi\right]F_{\mu\nu} \tag{51}$$

with ψ the muon field, we have

$$\Re D_\mu = a_\mu\frac{Q}{2m_\mu}, \qquad \Im D_\mu = d_\mu = \frac{\eta}{2}\frac{Q}{2m_\mu}, \tag{52}$$

so that the imaginary part of $F_P(0)$ corresponds to an electric dipole moment (EDM) which is non-vanishing only if we have T violation. The equation (51) provides as well the connection between the magnetic and electric dipole moments through the dipole operator D. As we will see later, the incoming new ideas on symmetry in QFT will turn out to be of extreme usefulness to approach and to analyze the problem of determination of the anomalous magnetic moment of the leptons, the equation (51) being just one of these important results.

1.3 Experimental determinations of the lepton AMM: a brief historical sketch

1.3.1 On the early 1940s experiences

Following (Kusch 1956), (Rich and Wesley 1972), (Farley and Picasso 1979), (Hughes 2003) and (Jegerlehner 2008, Part I, Chapter 1), in the same period in which appeared the famous 1948 Schwinger seminal research note, thanks to the new molecular-beams magnetic resonance spectroscopy methods mainly worked out by the research group leaded by I.I. Rabi in the late of 1930s, P. Kusch and H.M. Foley detected, in 1947, a small anomalous g_L-value for the electron within a 4% accuracy (see also (Weisskopf 1949)), analyzing the $^2P_{3/2}$ and $^2P_{1/2}$ state transition of Gallium: to be precise they found the values $g_s = 2.00229 \pm 0.00008$ and $g_l = 0.99886 \pm 0.00004$; later, J.E. Nafe, E.B. Nelson and Rabi himself were able, in May 1947, to detect a discrepancy between theoretical and predicted values of about 0.26% by the measurements of the hyperfine structure level splitting of hydrogen and deuterium in the ground state on the accepted Dirac g-factor of 2, which was quickly confirmed in the same year by D.E. Nagle, R.S. Julian and J.R. Zacharias (see also (Schweber 1961, Chapter 15, Section d)). In this regards, in September 1947, G. Breit (1947a,b) suggested that such discordances between theoretical expectations and experimental evidences could be overcome if one had supposed $g \neq 2$. Independently by Breit, also J.M. Luttinger (1948) (as well as T.A. Welton and Z. Koba - see (Rich and Wesley 1972) and references therein - between 1948 and 1949) stated that some experiments of then, seemed to require a modification in the g-factor of the electron. In this regards, Schwinger suggested that the coupling between the electron and the radiation field could be the responsible of this, calculating the effect on the basis of a general subtraction formalism for the infinities of quantum electrodynamics. Luttinger, instead, shown that the possible change in the electron magnetic moment could be derived very simply without any reference to an elaborate subtraction formalism. Soon after, P. Kusch, E.B. Nelson and H.M. Foley presented, in 1948, another precision measurement of the magnetic moment of the electron, just before Schwinger's theoretical result whose 1948 paper besides quotes them, which together the discovery of the fine structure of hydrogen spectrum (*Lamb shift*) by W.E. Lamb Jr. and R.C. Retherford in 1947, as well as the corresponding calculations by H.A. Bethe, N.M. Kroll, V. Weisskopf, J.B. French and W.E. Lamb Jr. in the same period, were the main triumphs of testing the new level of QED theoretical understanding with precision experiments. All that was therefore a stimulus for the development of modern QED. These successes had a strong impact in establishing the QFT as a general formal framework for the theory of elementary particles and for our understanding of fundamental interactions. The late 1940s were characterized by a close intertwinement between theory and experiment which greatly stimulated the rise of the new QED. On the theoretical side, a prominent role was gradually undertaken by the new non-Abelian gauge theory proposed by C.N. Yang and R.L. Mills in 1954 as well as by the various relativistic local QFT symmetries amongst which the discrete ones of charge conjugation (C), parity (P) and time-reversal (T) reflection which are related amongst them by the well-known CPT theorem, according to which the product of the these three discrete transformations, taken in any order, is a symmetry of any relativistic QFT (see (Streater and Wightman 1964)). Actually, in contrast to the single transformations C, P and T, which are symmetries of the electromagnetic and strong interactions only (d'après T.D. Lee and C.N. Yang celebrated work), CPT is a universal symmetry and it is this symmetry which warrants that particles and antiparticles have identical masses as well as equal lifetimes; but also the dipole moments are very interesting quantities for the study of the discrete symmetries mentioned above.

1.3.2 Some previous theoretical issues

The celebrated 1956 paper of T.D. Lee and C.N. Yang (see (Lee and Yang 1956)) on parity violation, has been an invaluable source of theoretical insights. The paper discusses the question of the possible failure of parity conservation in weak interactions taking into account what experimental evidences existed then as well as possible proposal of experiments for testing this hypothesis. Amongst these last, they discuss, since the beginning, on some experiments concerning polarized proton beams which would have led to an electric dipole moment if the parity violation were occurred. The related important consequences were too discussed, like the proton and neutron EDM, taking into consideration the previous early 1950s experiences made by E.M. Purcell, N.F. Ramsey and J.H. Smith for the proton who made an experimental measurement of the electric dipole moment of the neutron by a neutron-beam magnetic resonance method, finding a value less than 10^{-20} e-cm ca. in agreement with parity conservation for strong and electromagnetic interactions. Nevertheless, Lee and Yang argued that yet lacked valid experimental confirmations of parity conservation for weak interactions suggesting, to this end, to consider the measure of the angular distribution of the electrons coming from β decays of oriented nuclei like those of Co^{60}, thing that will be immediately done, with success, by C.S. Wu and co-workers, furnishing a first experimental evidence for a lack of parity conservation in β decays. Subsequently, Lee and Yang also argue on the question of parity conservation in meson and hyperon decays, as well as in those strange particle decays having the following features: 1) the strange particle involved has a non-vanishing spin and (2) it decays into two particles at least one of which has a non-vanishing spin or rather it decays into three or more particles. Thus, what conjectured by Lee and Yang could be also applied to the decay processes a) $\pi \to \mu + \nu$ and b) $\mu \to e + 2\nu$. So, in the sequential decay $\pi \to \mu \to e$, starting from a π meson at rest, one might study the distribution of the angle θ between the μ-meson momentum and the electron momentum, the latter being in the center-of-mass system of the μ meson. The decay b) is then a pure leptonic one, so no hadronic phenomenon is involved, this making easier the related calculations (see (Okun 1986, Chapter 3)). Lee and Yang then argue that, if parity is conserved in neither a) nor b), then the distribution will not in general be identical for θ and $\pi - \theta$ directions. To understand this, one may consider first the orientation of the muon spin. If a) violates parity conservation, then the muon would be in general polarized along its direction of motion. In the subsequent decay b), the angular distribution problem with respect to θ is therefore closely similar to the angular distribution problem of β rays from oriented nuclei, as discussed before, so that, in this way, it will be also possible to detect possible parity violations in this type of decays. These last remarks on $\pi\mu e$ sequence will be immediately put in practice in the celebrated 1956 experiences pursued by R.L. Garwin, L.M. Lederman with M. Weinrich and by J.L. Friedman with V.L. Telegdi, which will further confirm Lee and Yang hypothesis of parity violation in weak interactions. Following (Sakurai 1964, Chapter 7, Section 2) and (Schwartz 1972, Chapter 4, Section 11), polarized muons slow down and stop before they decay, but depending on the material (graphite, aluminium, etc.) the muon spin direction is still preserved, so we have a source of polarized muons. Negative muons are emitted with their angular momenta pointing along their directions of motion, whereas positive muons are emitted with their angular momenta pointing opposite to their directions of motion. Furthermore, if these positive muons were stopped in matter and allowed to decay, then the direction of this angular momentum (or spin) at the moment of decay could be determined by the distribution in directions of the emitted decay electron which follow the former. If parity is not conserved in muon decay either, then there will be a forward-backward asymmetry in the positron distribution with respect to the original μ^+ direction. The just above mentioned experiences showed more positrons emitted backward with respect to the μ^+ direction, showing that parity is not conserved in both π and μ decays.

As it has said above, Lee and Yang already argued on electric dipole moments in relation to parity conservation law for fundamental interactions, in some respects enlarging the discussion to the general framework of discrete symmetry transformations. To understand about the properties of the dipole moments under the action of such transformations, in particular the behavior under parity and time-reversal, we have to look at the interaction Hamiltonian (4) and, above all, at the equations (6) which both depend on the axial vector $\vec{\sigma}$, so that also $\vec{\mu}_m$ and $\vec{d}_e$ will be also axial vectors. On the other hand, the electric field $\vec{E}$ and the magnetic one $\vec{B}$ transform respectively as a (polar) vector and as an axial vector. Then, an axial vector changes sign under T but not under P, while a (polar) vector changes sign under P but not under T. Furthermore, since electromagnetic and strong interactions are the two dominant contributions to the dipole moments, and since both preserve P and T, it follows that the corresponding contributions to (4) must conserve these symmetries as well. Indeed, following (Muirhead 1965, Chapter 9, Section 9.2(d)), we have

$$\begin{gathered} P\vec{\sigma}P^{-1} = \sigma, \qquad T\vec{\sigma}T^{-1} = -\vec{\sigma}, \qquad P\vec{H}P^{-1} = \vec{H}, \\ T\vec{H}T^{-1} = -\vec{H}, \qquad P\vec{E}P^{-1} = -\vec{E}, \qquad T\vec{E}T^{-1} = \vec{E}, \end{gathered} \tag{53}$$

whence it follows that

$$\begin{gathered} P(\vec{\sigma}\cdot\vec{H})P^{-1} = \vec{\sigma}\cdot\vec{H}, \qquad\qquad T(\vec{\sigma}\cdot\vec{H})T^{-1} = \vec{\sigma}\cdot\vec{H}, \\ P(\vec{\sigma}\cdot\vec{E})P^{-1} = -\vec{\sigma}\cdot\vec{E}, \qquad\qquad T(\vec{\sigma}\cdot\vec{E})T^{-1} = -\vec{\sigma}\cdot\vec{E}. \end{gathered} \tag{54}$$

Therefore, as L.D. Landau and Ya.B. Zel'dovich pointed out (see (Landau 1957) and (Zel'dovich 1961)), due to these symmetry rules on P and T, the magnetic term $-\vec{\mu}_m \cdot \vec{B}$ is allowed, while an electric dipole term $-\vec{d}_e \cdot \vec{E}$ is forbidden so that we should have $\eta = 0$ in $(6)_2$. Now, T invariance (that, by CPT theorem, is equivalent to CP invariance) is also violated by weak interactions, which however are very small for light leptons. Nevertheless, for non-negligible second order weak interactions (as for heavier leptons - see (Chanowitz et al. 1978) and (Tsai 1981)), an approximate T invariance will require the suppression of electric dipole moments, i.e. $d_e \to 0$. Thus, electric dipole interaction cannot occur unless both P and T invariance breaks down in electrodynamics. Following (Roberts and Marciano 2010, Chapter 1, Section 1.3), P.A.M. Dirac discovered, in 1928, an electric dipole moment term in the relativistic equations involved in his electron theory. Like the magnetic dipole moment, the electric dipole moment had to be aligned with spin, so that we have an expression of the type $\vec{d} = \eta(Q\hbar/2mc)\vec{s}$ (see $(6)_2$) where, as already said, η is a dimensionless constant which is the analogous to g_s. Whilst the magnetic dipole moment is a natural property of charged particles with spin, electric dipole moment are forbidden both by parity and time reversal symmetries as said above. Nevertheless, from a historical viewpoint, the search for an EDM dates back to suggestions due to E.M. Purcell and N.F. Ramsey since 1950 who however pointed out that the usual parity arguments for the non-existence of electric dipole moments for nuclei and elementary particles, albeit appealing from the standpoint of symmetry, weren't necessarily valid. They questioned about these arguments based on parity and tried, in 1957, to experimentally measure the EDM of the neutron through a neutron-beam magnetic resonance method, finding a value for d of about $(-0.1 \pm 2.4) \cdot 10^{-20}$ e-cm. This result was published only after the discovery of parity violation although their arguments were provided in advance of the celebrated 1956 T.D. Lee and C.N. Yang paper on parity violation for weak interactions. Once parity violation received experimental evidence, other than L.D. Landau, soon after also N.F. Ramsey, in 1958, pointed out that an EDM would violate both P and T symmetries.

1.3.3 Further experimental determinations of the lepton AMM

A) Some introductory theoretical topics

i) On resonance spectroscopy methods. Amongst special devices and techniques of experimental physics, a fundamental role is played by magnetic resonance spectroscopic techniques through which Zeeman level transitions are induced by magnetic dipole radiations by means of the application of an external static magnetic field $\vec{B}$. The spontaneous transitions with $\Delta l = \pm 1$ (electric dipole) are more probable than those with $\Delta l = 0$ and $\Delta m = \pm 1$ (magnetic dipole). Nevertheless, the presence of a resonant electromagnetic field increases the latter. With the action of this perturbing field the probability of induced transitions is proportional to the square of the intensity of the electromagnetic field, so that magnetic dipole transitions may be easily induced through suitable radio-frequency (RF) values provided by a RF oscillator with an imposed constant magnetic field which has the main role to select the desired RF frequencies to be put in resonance with the precession ones. As an extension of the original method of the famous Stern-Gerlach experiment, the above mentioned technique was first proposed by I.I. Rabi, together his research group at Chicago around the late 1930s, who made important experiments on atomic beams that, amongst other things, led to the precise determination of the atomic hyperfine structure; in particular, the Lamb shift between hydrogen $2S_{1/2}$ and $2P_{1/2}$ gave an accurate measurement of the electron anomalous magnetic moment. Independently by Rabi's research group works, also L.W. Alvarez and F. Bloch set up, in 1940, a similar technique. The nuclear magnetic moments have been measured through *nuclear magnetic resonance* (NMR) techniques that, thanks to relaxation mechanisms which release thermal energy in such a manner to warrant a weak thermal contact between nuclear spins and liquid or solid systems to which they belong, allow to determine fundamental physical properties of the latter. The *electron paramagnetic resonance* (EPR) or *electron* *spin resonance* (ESR) refers to induced transitions between Zeeman levels of almost free electrons in liquids and solids. It has been first observed by E.K. Zavoiskij in 1945 and usually runs into the microwaves frequencies and it has been applied to determine anomalous magnetic moment values. Both in NMR and EPR, in which an external inhomogeneous magnetic field $\vec{B}_0$ is acting, the transitions between Zeeman levels are induced by an additional homogeneous alternating weak magnetic field $\vec{B}_1$ (for instance, acting upon a x-y plane), oscillating transversally to $\vec{B}_0$ (for instance, directed along the z axis) with an angular frequency ω_1 which may be, or not, in phase with Larmor precession frequency; for instance, if $\vec{B}_1$ acts along the x axis, then an induced e.m.f. will be detectable along the y axis. Thanks to the 1949 N.F. Ramsey works, it is also possible to apply a second alternating static magnetic field $\vec{B}_2$, even perpendicularly to $\vec{B}_0$ (*double resonance* techniques), and so on (*multiple resonance* techniques); the possible reciprocal geometrical dispositions of the various involved magnetic fields $\vec{B}_0, \vec{B}_1, \vec{B}_2$ and so on, give rise to different resonance experimental methods also in dependence on the adopted relaxation methods and related detected times: amongst them, the *Bloch decay* and the *spin echoes.* In single resonance techniques, the perturbing alternating field $\vec{B}_1$ must be in resonance with the separation between two adjacent Zeeman levels (i.e. with $\Delta m = \pm 1$). The resulting statistical coherence will imply a macroscopic value (roughly $N\mu_{ct}$) quite high to may be detected by a coil, with the symmetry axis belonging in the equatorial plane and, for instance, oriented along the y axis, also thanks to electronic devices which will amplify the initial value.

Following (Dekker 1958, Chapter 20), (Kittel 1966, Chapter 16), (Kastler 1976, Part III, Chapter V), (Cohen-Tannoudij et al. 1977, Volume I, Complement F_{IV}), (Bauer et al. 1978, Chapters 12 and 13), (Pedulli et al. 1996, Chapters 7, 8 and 9), (Humphreis 1999, Chapter 14), (Bertolotti 2005, Chapter 9) and (Haken and Wolf 2005, Chapter 12), for particles having a non-zero spin, the application of the field $\vec{B}_0$ only, implies a torque acting upon the cyclotron (or orbital) magnetic moment $\vec{\mu}_L$ so giving rise to two non-zero components, namely a longitudinal component $\vec{\mu}_{cl}$ (directed along $\vec{B}_0$) and a transversal one $\vec{\mu}_{ct}$ (belonging to the plane having $\vec{B}_0$ as normal vector). This torque will imply too a Larmor precession, with angular frequency given by $\omega_0 = g(eB_0/2mc)$ (for elementary particles with rest mass m), that causes a rotation of $\vec{\mu}_{ct}$ in the equatorial plane around the z axis. Nevertheless, in general there is no statistical coherence amongst these transversal components, also due to the thermal excitation. But, as showed by F. Bloch, W.W. Hansen and M. Packard as well as by E.M. Purcell, H.C. Torrey, N. Bloembergen and R.V. Pound in the years 1945-46, the application of a perturbing (alternating) magnetic field $\vec{B}_1$, transversally arranged respect to $\vec{B}_0$ and usually induced by the passage, along a transmissive spire, of a direct current (DC) into a variable RF oscillator, gives rise to a coherent and ordered precession of the transversal components of magnetic moment when the frequency of the perturbing field, say ω_1, is equal to ω_0 (*magnetic resonance* condition or *resonance equation*); this, in turn, will imply either spin-orbit decouplings as well as resonating Zeeman magnetic level transitions, in agreement with the well-known Bohr's correspondence principle according to which the concept of quantum level transition should correspond, in the classical electrodynamics, to the periodic variation either of an atomic electric or magnetic moment (in our case, the rotation of $\vec{\mu}_{ct}$ in the equatorial plane). The weak perturbing magnetic field $\vec{B}_1$ is usually applied, above all in NMR techniques, in such a manner that its values verify $B_1 \ll B_0$ which nevertheless imply long storage times; often, as in the original (Chicago) I.I. Rabi research group experiences, a second opposed (to $\vec{B}_0$) inhomogeneous magnetic field is also applied next to the RF oscillator group, to refocalize the particle beam until the receiver device. In such a manner, a very weak rotating magnetic field is able to reverse the spin direction of the beam particles, whilst $\vec{\mu}_L$ precesses (*Rabi's precession*), in the rotating frame, about a well-precise 'effective' magnetic field $\vec{B}_{eff}$, given by the superposition of the various applied magnetic fields, according to particular equations of motion called *Bloch's equations.* In dependence on the RF oscillator chosen as an energy source, we have either *continuous wave* (CW) or *pulsed wave* (PW) resonance techniques: the intensity of the resulting signal is measured in function of the magnetic field or frequency values for the former and in function of the time for the latter. As we shall see later, the resonance spectroscopy methods have played a fundamental role in determining magnetic ed electric properties of atomic and nuclear systems (see, for instance, (Bloch 1946)): for instance, through a suitable formulation of a resonance condition, it will be possible to experimentally determine the anomalous magnetic moment of elementary constituents as electrons, neutrons, protons and muons.

ii) On spin precession motion. Following (Schwartz 1972, Chapter 4, Section 10), (Rich and Wesley 1972, Section 3.1.1), (Cohen-Tannoudij et al. 1977, Volume I, Complement F_{IV}), (Ohanian 1988, Chapter 11, Section 11.1), (Kinoshita 1990, Chapter 11, Sections 1-4), (Picasso 1996, Section 2), (Farley and Semertzidis 2004, Section 3) and (Barone 2004, Chapter 6, Section 6.10), a general *precession problem* is identified by a kinematical equation of the form $d\vec{\Phi}/dt = \vec{\Omega}(t) \wedge \vec{\Phi}$, where $\vec{\Phi}$ is the vectorial quantity that precesses around the given vector $\vec{\Omega}$; for instance, $\vec{\Phi}$ may be a magnetic moment, an angular momentum or the spin, which precesses around the direction given by the force lines of the perturbing field $\vec{\Omega}$ (as, for example, a magnetic field), with angular velocity $\Omega(t)$. The related experienced torque $\vec{\tau}$, is given by $\vec{\Omega}(t) \wedge \vec{\Phi}$. In case of an elementary spinning particle having charge Q and mass m, in a (uniform) magnetic field $\vec{B}$, we may put $\vec{\Phi} = \vec{\mu}_s$, where $\vec{\mu}_s$ is the spin magnetic moment given by $g_s Q \mu_0 \vec{\sigma}/2$ the $(6)_1$. In this case, $\vec{\Omega} = k\vec{\mu}_s = (gQ/2mc)\vec{\mu}_s$, so that we have, in the particle rest frame, the following Larmor precession equation $d\vec{\mu}_s/dt = k\vec{\mu}_s \wedge \vec{B}$ (see (Cohen-Tannoudij et al. 1977, Volume I, Complement F_{IV}), (Bloch 1946, Equation (11)) and (Bargman et al. 1959, Equation (3))) related to the precession of $\vec{\mu}_s(t)$ around $\vec{B}$; $\vec{\sigma}$ is said to be the *polarization vector.* The relativistic generalization of the last precession equation will lead to the so-called *Bargman-Michel-Telegdi equation* (see (Bargman et al. 1959)). Following (Gottfried 1966, Chapter VI, Section 49), for beams of elementary particles, said $\vec{\sigma}$ the Pauli operator whose components are the Pauli matrices, the *beam polarization* is defined to be $\langle \vec{\sigma} \rangle$ and shall often be written as $\vec{P}$; it is zero for an incoherent and equal mixture of $|1/2\rangle$ and $|-1/2\rangle$, whereas $|\vec{P}| = 1$ for pure spin states.

B) The first experimental determinations of the electron AMM

Following (Kusch 1956), (Rich and Wesley 1972), (Crane 1976), (Farley and Picasso 1979), (Combley et al. 1981), (Kinoshita 1990, Chapters 8 and 11) and (Jegerlehner 2008, Part I, Chapter 1) and as it has already said above, P. Kusch and H.M. Foley, in November 1947, measured a_e for the electron with a precision of about 5%, obtaining the value $a_e = 0.00119(5) = 0.00119 \pm 0.00005$ at one standard deviation. The establishment of the reality of the anomalous magnetic moment of the electron and the precision determination of its magnitude, was part of an intensive programme of post-war research with atomic and molecular beams which seen actively involved P. Kusch at Columbia, together to I.I. Rabi research group. All that was crowned by success with the assignment of Nobel Prize for Physics in 1955, shared with W.E. Lamb, whose related Nobel lecture is reprinted in (Kusch 1956). Other attempts to estimate the anomalous magnetic moment either of the electron and of the proton were carried out by J.H. Gardner and E.M Purcell in 1949 and 1951, by R. Karplus and N.M. Kroll in 1950, by S.H. Koenig, A.G. Prodell with P. Kusch in 1952, by R. Beringer with M.A. Heald and by J.B. Wittke and R.H. Dicke in 1954, by P.A. Franken and S. Liebes Jr. in 1956 as well as by W.A. Hardy and E.M. Purcell in 1958, in any case reaching to an accuracy of about 1% for the various anomalous moment values. The Gardner and Purcell experiments (see (Gardner and Purcell 1949) and

(Gardner 1951)) introduced, for the first time, a new experimental method to determine a_e, based on a comparison of the cyclotron frequency of free electrons with the nuclear magnetic resonance (NMR) frequency of protons, so opening the way to the application of resonance techniques to measure the lepton anomalous moments on the wake of the pioneering Rabi's molecular beam resonance method for measuring nuclear magnetic moments (see (Rabi et al. 1938, 1939)) recalled above. To be precise, an experimental determination of the ratio of the precession frequency of the proton, $\omega_p = \mu_p H_0$, to the cyclotron frequency, $\omega_e = eH_0/mc$, of a free electron in the same magnetic field, was carried out. The result, ω_p/ω_e, is the magnitude of the proton magnetic moment, μ_p, in Bohr magnetons μ_0. Finally, by the comparison between μ_p/μ_0 and μ_e/μ_p, it was possible to determine μ_e/μ_0. Possible sources of systematic error were carefully investigated and in view of the results of this investigation and the high internal consistency of the data, it was felt that the true ratio, uncorrected for diamagnetism, lie within the range $\omega_e/\omega_p = 657.475 \pm 0.008$. If the diamagnetic correction to the field at the proton for the hydrogen molecule was applied, the proton moment in Bohr magnetons became $\mu_p = (1.52101 \pm 0.00002) \times 10^{-3}(e\hbar/2mc)$. In (Koenig et al. 1952), the ratio of the electron spin g_e value and the proton g_p value was measured with high precision. It was found that $g_e/g_p = 658.2288 \pm 0.0006$, where g_p is the g value of the proton measured in a spherical sample of mineral oil. This result, when combined with the previous measurement by Gardner and Purcell of the ratio of the electron orbital g_e value and the proton g_p value, yielded for the experimental value of the magnetic moment of the electron $\mu_s = (1.001146 \pm 0.000012)\mu_0$. The result was in excellent agreement with the theoretical value calculated by Karplus and Kroll, namely $\mu_s = (1.0011454)\mu_0$. However, all these methods were related to electrons bound in atoms, this implying, amongst other things, a lower accuracy level due to the corrections necessary to account for atomic binding effects. Thus, anomalous moment experimental determinations on free electrons were more suitable.

Following (Rich and Wesley 1972), (Kinoshita 1990, Chapter 8), in the years 1953-54, H.R. Crane, W.H. Louisell and R.W. Pidd at Michigan, for the first time, determined a_e for free electrons from measurements of $g - 2$ (not g itself) by means of the precession of the electron spin in a uniform magnetic field, obtaining the result $g = 2.00 \pm 0.01$, that is to say, g must be within 10% of 2.00. They introduced, on the basis of the previous basic work made by N.F. Mott in 1930s, a new pioneering technique which will be later called the $(g-2)$ *precession method,* so opening the way to the precession methods for determining lepton g factors. Following (Louisell et al. 1954), (Hughes and Schultz 1967, Chapter 3), (Rich and Wesley 1972), (Combley and Picasso 1974) and (Crane 1976), we briefly recall the main stages which led to the experimental methods for measuring the magnetic moment of the free electron according to this $(g-2)$ precession method. A first attempt was based, after a N.H. Bohr argument[8], on a statistical fashion of the well-known 1924 Stern-Gerlach experiment on the atomic magnetic moments, applied to free electrons and consisting in sending a large number of electrons through a magnetic field and by attempting to use the detailed line shape to reveal the effects of the magnetic moment. Nevertheless, such a method appeared particularly unpromising in connection to a precise solution to the electron moment problem. A second attempt, instead, was based on the previous 1929 N.F. Mott double-scattering method for studying the polarization of particles beams. The Louisell, Pidd and Crane principle of the method employed a Mott double-scattering method roughly consisting in producing polarized electrons by shooting high-energy electrons upon a gold foil; hence, the part of the electron bunch which is scattered at right angles, is then partially polarized and trapped in a constant magnetic field where spin precession takes place for some time. The bunch is afterwards released from the trap and allowed to strike a second gold foil, which allows to analyze the relative polarization. To be precise, this method depend on the fact that a beam of electrons is partially polarized along a direction normal to the plane defined by the incident beam and the emerging scattering direction. Furthermore, a second scattering process exhibited an azimuthal asymmetry in scattering intensity, if measured in the same plane, mainly due to polarization perpendicular to the plane of the incident and scattered beams. Mott defined the amplitude of this asymmetry as δ and provided some its estimates. To explain this effect, both on the basis of the above Bohr' argument and in taking into account the Stern-Gerlach results, Mott put forward the hypothesis that electron spins had to be thought of as precessing around the direction of a magnetic field rather than as aligned parallel or anti-parallel to this, like in the Stern-Gerlach experiment[9]. There-

[8]Arguing upon the unobservability of the magnetic moment of a single electron on the basis of the well-known Heisenberg indetermination principle. Therefore, we must consider a statistical approach in such a manner that the average behavior of the spins of a large ensemble of particles can be treated, to a large extent, as a classical collection of spinning bar magnets.

[9]Following (Miller et al. 2007) and (Roberts and Marciano 2010, Chapter 1), the study of atomic and subatomic magnetic moments began in 1921 first with a paper by O. Stern then with the famous 1924 O. Stern and W. Gerlach experiment in which a beam of silver atoms was done pass through a gradient magnetic field to separate the different magnetic quantum states. From this separation, the magnetic moment of the silver atom was determined to be one Bohr magneton μ_0 within 10%. This experiment was carried out to test the Bohr-Sommerfeld quantum theory. In 1927, T.E. Phipps and J.B. Taylor

fore, the asymmetry observed along the second scattering should be due to this precession because, if the spin were aligned parallel and anti-parallel to the direction of a magnetic field parallel to the beam incident on the scatterer of the experimental apparatus, then it would be enough to apply a weak magnetic field to remove such an asymmetry effect. In this sense, the spin had to be meant as a physical observable rather than a mathematical device (d'après Pauli). Furthermore, since this 1954 Louisell-Pidd-Crane method essentially requires a simultaneous measurement of the electron position and of a single spin component, it follows that the uncertainty principle is not violated. Crane says that Mott's way out of his dilemma was, perhaps, the first break toward thinking of electrons as precessing magnets. Nevertheless, this far seeing Mott's hint didn't took by nobody at that time until the 1953-54 pioneering works of Louisell, Pidd and Crane. They extended this Mott double-scattering method inserting, between the first and second scatterers, a constant magnetic field, parallel to the path to the path between the scatterers, in the form of a magnetic mirror trap which permitted the electrons to undergo several hundred $(g-2)$ precessions between scatterings. This causes the electron to precess and rotates the polarization plane of maximum asymmetry after the second scattering no longer coincides with the plane of the first scattering. By measuring the angle of rotation and knowing the magnetic field, the electron energy and the distance, the gyromagnetic ratio for the electron may be found. A fact which had a dominating influence was that the orbital, or cyclotron, angular frequency of the electron in the magnetic field differs from the angular frequency of precession of the spin direction although in higher-order correction terms, these respectively being given by $\omega_o = eB/(2mc)$ and $\omega_s = g(eB/(2mc))$ with $g = 2(1 + \alpha/2\pi + ...)$ (d'après Schwinger). This fact turns out to be useful to determine g whose value may be therefore determined from a direct comparison of the rotation of the plane of polarization and the cyclotron rotation. Moreover, all observed asymmetries in the beam, whether they are associated with the spin or not, rotate around together, so that it was needed for discriminating amongst them. Certain sources of asymmetry have nothing to do with the polarization effect notwithstanding they follow the polarization asymmetry itself as it rotates around. However, Louisell, Pidd and Crane were able to determine and isolate the non-spin asymmetry, mainly due to scattering

repeated the experiment with a hydrogen beam and they also observed two bands from whose splitting they concluded that, like silver, the magnetic moment of the hydrogen atom was too one μ_0. Subsequently, in 1933, R.O. Frisch and O. Stern determined the anomalous magnetic moment of the proton, while in 1940, L.W. Alvarez and F. Bloch determined the anomalous magnetic moment of the neutron, and both turned out to be quite different from the value 2, because of their internal structure.

nonlinearities, from spin asymmetry that was experimentally detected with very small measurement errors. Due to the action of the Lorentz force, if ϕ_c (or ϕ_o) is the cyclotron (or orbital) rotation angle between scatterers, ϕ_d is the sum of deflection angles at entry and exit to the solenoid field, and ϕ_s is the angle through which the spin asymmetry was rotated relative to the direction of the beam before entry into the solenoid field, then an estimate to g is given by $2(\phi_s - \phi_d)/\phi_c$, whose experimentally detected values were reported in Table I of (Louisell et al. 1954), computed at different values of B. Nevertheless, Louisell, Pidd and Crane concluded that the precision of which their method is capable (they obtained an accuracy of 1%) was not enough to reveal the correction to the g factor at about one part in a thousand, so that their result wasn't sufficiently precise to be useful in comparison with the theoretical prediction. Meanwhile, or in parallel, the results so found have been ascertained to be coherent with Dirac theory of electron by H. Mendlowitz with K.M. Case, who also calculated the possible effects of a uniform magnetic field on a Mott double-scattering experiment showing that they can be used to measure a_e as in the Louisell-Pidd-Crane experience. Coherence with Dirac theory also came from a previous 1951 work of H.A. Tolhoek and S.R. De Groot which concerned another parallel research area on hyperfine structures oriented towards precision measurements on g of the free electron; the latter proposed, in 1951, a scheme in which a magnetic field and a RF field were interposed between the first and second Mott scatterers, and in which destruction of the asymmetry indicated resonance. A notable research group based at the University of Columbia and directed by I.I. Rabi since 1940s, followed another line of attack to measure the gyromagnetic ratio for the free electron, based upon the magnetic resonance method, proposing new experiments in two somewhat different forms respect to the previous research line based on Mott scattering method. In both these forms, polarized electrons are trapped in stable orbits into a magnetic field. A radio-frequency (RF) perturbing field is then applied and the frequency which destroys the polarization is determined. From the frequency which destroys the polarization and the strength of the magnetic field, the value of the gyromagnetic ratio is obtained. Since 1956, H.G. Dehmelt group at Washington demonstrated that spin-exchange collisions between oriented sodium atoms and free, thermal energy electrons could be used to measure a_e via a direct RF resonance technique, so contributing to the first determinations of the free electron anomalous magnetic moment.

The two above mentioned methods mainly differ in the way in which the electrons are polarized, giving priority to trapping, and in the way in which the presence or absence of polarization is determined after the application of the magnetic or RF perturbing field and the subsequent escaping from the trapping phase carried out by the latter. The essence of the method consists essentially in finding the frequency of the feeble beat between the rotation of the spin direction (in the trap or well) and the orbital, or cyclotron, rotation when the particles are trapped in a well-determined magnetic well. Afterwards, a careful determination of electron energies as well as a precise control of fields and potentials are also demanded. Forerunners of resonance methods, other than the above mentioned one, may be also retraced in some previous experiences made by R.H. Dicke and F. Bloch in the early 1940s. In any case, following (Louisell et al. 1954), in both methods in which resonance is involved, the strong coupling to the cyclotron motion due to the fact that the required perturbing frequency is almost identical to the cyclotron one with consequent transfer of energy from the perturbing field to the cyclotron motion, might introduce serious difficulties in order to achieve the right accuracy with the increasing of the cyclotron revolutions. Furthermore, it is very difficult to control the particle while it is into the trap inside which it oscillates (along the Z direction, parallel to the perturbing field). Nevertheless, Louisell, Pidd and Crane state that the magnetic resonance methods, together their experimental extension to the Mott double-scattering method, seem to be the only ones[10] able to give really quantitative results of sufficient accuracy to reveal the correction to the electron moment. Some problems occur when we consider electrons and positrons which both require to be previously polarized: for the former, the above mentioned Mott scattering method is used, while for the latter, a suitable radioactive source is used for their initial polarization whereas the final one is found through a clever scheme first proposed by V.L. Telegdi (see (Grodzins 1959, Section 5.1, p. 219)). As regards muons, instead, this last problem does not subsist since them born already polarized and reveal their final polarization through the direction of the related decay products. Following (Crane 1976) and (Hughes and Schultz 1967, Chapter 3, Section 3.5.3.1), in 1958, P.S. Farago proposed a method[11] for comparing the orbital and the spin precession of electrons moving in a magnetic field, which will turn out to be useful to directly measure radiative corrections to the free-electron magnetic moment. Indeed, the Farago's principle of the method consisted in considering initially polarized electrons, emitted by a β active source and moving perpendicular to a strong uniform magnetic field $\vec{B}$, hence using a Mott scattering for analysis. A uniform weak vector field $\vec{E}$ is also applied perpendicularly to $\vec{B}$ in such a manner that the beam walks enough to miss the back of the source

[10]Besides some other experimental attempts to get polarized beams of electrons, by F.E. Myers and R.T. Cox as well as by E. Fues and H. Hellman, at the end of 1930s.

[11]Besides also quoted by (Bargmann et al. 1959, Case (E))).

of the first turn. The beam continues walking towards right for a distance almost equal to the orbital diameter. After the order of about some hundreds of revolutions, it then encounters a Mott scattering foil at which the final direction of polarization perpendicular is determined from the intensity asymmetry in the direction perpendicular to the orbit plane. If the final polarization direction is measured as a function of the transit time between source and target (consisting of about 250 orbital revolutions or turns), then a sine curve is obtained whose frequency is equal to the difference between the spin precession frequency and the orbital frequency of the circulating electrons. To the extent that $E/B \ll 1$ (electron trochoidal motion), this difference frequency is proportional to $(\mu_e/\mu_0 - 1) = g/2 - 1 = a_e$, so that the Farago's method measures directly the radiative correction to the free electron magnetic moment μ_e, hence a_e (see (Farago 1958)). The Farago's method was later improved and experimentally realized by his research group at the University of Edinburgh (see (Farago et al. 1963)); it constituted, at that time, the first method that allowed a continuous measurement rather than by pulses. Nevertheless, the Farago's method couldn't compete in accuracy with experiments in which the particles are trapped and allowed to make a far larger number of revolutions. In any case, its principle of the method, in some respects, has preempted certain basic methods underpinning some later storage techniques (amongst which the one based on polynomial magnetic fields). Other determinations of a_e were later realized, in the early 1960s, by D.T. Wilkinson, D.F. Nelson, A.A. Schupp, R.W. Pidd and H.R. Crane (*Michigan group*) even improving their principle of the method of 1954 and mainly based upon the remark that, if polarized electrons were caused to move with their velocities perpendicular to a uniform magnetic field, then, at a fixed azimuth on the cyclotron orbits, one would observe the polarization precessing at a rate equal to the difference between the spin precession rate (ω_s) and the orbital cyclotron rate (ω_c), just this difference precession rate (anomalous or spin-cyclotron-beat frequency $\omega_a = \omega_s - \omega_c$) being directly proportional to a_e. This method will be generically called the (*Michigan*) *principle of* $(g-2)$ *spin motion,* or simply *spin precession method* (or also *free-precession method*), and will lead to the next basic equation (59).

Following (Rich and Wesley 1972) and (Crane 1976), meanwhile the spin precession methods were further pursued as a result of the pioneering works made by the above Michigan group, other techniques were employed to approach $g-2$, above all for electrons. As it has already said above, H.A. Tolhoek and S.R. De Groot proposed, since 1951, a scheme in which a magnetic field, coupled with a RF field, would be interposed between the first and the second Mott scatterers, even if themselves were aware that such an apparatus wasn't able to provide enough cycles of the spin precession to give a well defined frequency, mainly because of the absence of a trap. In 1953, F. Bloch proposed a novel resonance-type experiment to measure a_e using electrons occupying the lowest Landau level in a magnetic field. In the years 1956-58, H.G. Dehmelt performed an experiment in which free thermal electrons in argon buffer gas, at the mean temperature of $400^o K$, become polarized in detectable numbers by undergoing exchange collisions with oriented sodium atoms during which the atom orientation is transferred to the electrons. Such collisions establish interrelated equilibrium values among the atom and the electron polarizations which depend on the balance between the polarizing agency acting upon the atoms (optical pumping) and the disorienting relaxation effects acting both on atoms and electrons. When the electrons were furthermore artificially disoriented by gyromagnetic spin resonance, an additional reduction of the atom polarization ensued, which was detected by an optical monitoring technique (with an optical pumping cell rather than a quadrupole trap), so allowing to the determination of the free-electron spin g factor and opening the way to experimentally use the so-called *Penning trap* consisting of a uniform axial magnetic field $\vec{B} = B_{0z}\hat{z}$ and a superimposed electric quadrupole field generated by a pair of hyperbolic electrodes surrounding the storage region. The magnetic field confines the electrons radially, while the electric field confines them axially. The essential novel feature of the this Dehmelt's techniques consisted, following an idea of V.L. Telegdi and co-workers (see (Ford et al. 1972)), in the fact that a RF induced pulse (or beat) frequency, rather than a spin precession frequency, was the main responsible to rotate the polarization. The principle of the method is quite similar to the known *spin echoes* of E.L. Hahn (1950) in which an intense RF power in the form of pulses is applied to an ensemble of spins in a large static magnetic field. The frequency of the pulsed RF power is applied through a RF current circulating in a wire stretched along the center axis of the trapping chamber, producing lines of force that are circles concentric with the orbits. If the RF is held on for the right length of time, then the polarization is turned from the plane perpendicular to the applied magnetic field towards the direction parallel to it. Afterwards, it comes back again if the RF pulse is held on twice as long, just like spin echoes.

Following (Gräff 1971), (Rich and Wesley 1972) and (Holzscheiter 1995), the precision measurements of lepton g-factor anomalies can be classified as being either *precession experiments* and *resonance experiments* in dependence on the technique employed, in both of which the main involved problem being that concerning the trapping of polarized charged particles. The main dynamical features of the problem are as follows: the momentum $\vec{p}$ of the particle, which is exactly perpendicular to $\vec{B}$, revolves with the cyclotron (or orbital) angular frequency $\omega_c = QB/mc$, the spin precesses about $\vec{B}$ with Larmor angular frequency $\omega_s = (1 + a_l)\omega_c$ with $a_l = (g-2)/2$, while the difference between these angular frequencies is the one at which the spin rotates about the momentum, that is to say $\omega_{a_l} = \omega_s - \omega_c = a_l QB/mc = \theta/T$ where θ is the angle between spin and momentum and T the time. Consequently, to get the lepton anomaly a_l, it is thus necessary to measure the quantities ω_{a_l} and B, assuming Q/mc to be known. Thus, we have $a_l = \omega_{a_l}/\omega_c$ (see also (Kinoshita 1990, Chapter 11, Section 4.1, Equation (4.8)). If the particle velocity has a small angle relative to the orbital plane x-y of motion particle, then the particle will follow a spiral path, along the axial direction given by the z-axis, with *pitch angle* ψ, spiralling in the main (not necessarily constant) magnetic field B_z; the $(g-2)$ frequency is consequently altered. In any real storage system, the pitch angle is corrected by suitable vertical focusing forces which prevent the particles to be lost. Furthermore, the pitch angle changes periodically between positive and negative values, so that the correction to the $(g-2)$ frequency become more complex. All the $(g-2)$ experiments for electrons and muons are in principle subject to a pitch correction and, as we will see later, this problem will be successfully overcome, for the first time, with the introduction of the so-called *polynomial magnetic fields.* An arbitrary experiment which attempts to measure the anomalous magnetic moment of a free lepton necessarily encounters the following problems: *a)* trapping of the particle; *b)* measurement of the trapping field either by nuclear magnetic resonance (NMR) or by measuring ω_s or ω_c; *c)* polarization of the spin of the particle; *d)* determination of the anomaly frequency either *i)* by detection of the spin polarization vector relative to the momentum vector of the particle as a function of the time in a magnetic field, calling this type of experiment a *geometrical experiment*[12], or, alternatively, *ii)* by induction and detection of the relevant RF transition ω_s and ω_c or, if possible, ω_s or ω_c and the difference angular frequency ω_a directly, calling this type of experiment a *RF spectroscopic experiment*[13]. To trap particles, it has been used: 1) the *magnetic bottle method* consisting in imposing a homogeneous magnetic field with a superimposed relatively weak inhomogeneous magnetic field as first used by the above mentioned Michigan group; 2) a RF quadrupole trap starting from the first studies on electric quadrupole mass separator made by F. v. Bush, W. Paul, H.P. Reinhard with U. v. Zahn and by E. Fisher, in the 1950s, for separating isotopes. To detect the ions, a resonance detection technique is used, taking advantage of the fact that for given parameters of the trap each charge-to-mass ratio exhibits a certain

[12]Roughly corresponding to the above precession experiment type.

[13]Roughly corresponding to the above resonance experiment type.

unique ”eigenfrequency”. In addition to the radio-frequency quadruple field, a RF dipole field at the frequency ω_{res} is applied as well to the end caps. If through proper choice of the parameters a and q, respectively representing the amplitudes of the RF component and the direct current (DC) component of the quadruple field, the ions are brought to resonance with this dipole field, then the amplitude of the ion motion is increased, absorbing energy from the drive field, and can be detected. The important fact is that different ions will have different frequencies for a given set of a and q, or, that at a fixed frequency, one can bring all different ion species to resonance subsequently by slowly varying the DC potential at a constant RF amplitude. This made the quadruple trap an ideal tool for precision mass spectrometry or residual gas analysis, areas in which RF traps have gained high respect over the last decades. At first glance, the RF drive field seems to be a disturbance to the system, and in effect it is. Due to the continuously applied drive force stored particles are heated permanently, leading to 2nd order doppler broadening of spectral lines. This effect can be counteracted by cooling mechanisms, either collisions with residual gas molecules, or far more powerful and selective than this, by laser cooling. Nevertheless, due to this ”micromotion”, the Paul's research group trap has always been a second choice respect to the so-called *Penning trap* if one desired an ultrahigh precision work. Based on this last new device, dating back to the late 1930s F.M. Penning works, D.H. Dehmelt group at Seattle (Washington), P.S. Farago group at Edinburgh and G. Gräff group at Bonn/Mainz have performed various electron $g-2$ experiments.

As concerns, instead, the polarization problem, in experiments of geometrical type, polarized muons are produced by the forward decay of pions, polarized electrons by Mott double-scattering and polarized positrons by beta decays, while, as regards experiments of RF spectroscopic type, electrons are polarized by means of spin exchange with a polarized atomic beam as well as electrons of low energy are created in pulses in a high magnetic field. Finally, as regards the determination of the lepton anomaly, in the geometrical experiments the angle θ between the spin vector and momentum of the particle is measured at a fixed orbital point as a function of time. The polarization of electrons is detected by Mott double-scattering, the polarization of positrons by exploiting the spin dependence by ortho- and para-positronium formation, whilst the muon polarization is measured using the fact that in the rest frame, the decay electrons are preferentially emitted along the spin direction. As the momentum of a particle in a magnetic bottle is no longer perpendicular to the magnetic field, the Bargmann-Michel-Telegdi (BMT) formula for ω_a (see (Bargmann et al. 1959, Equation (9))) has to be used. Instead, in the RF spectroscopic measurements, the transition at frequency ω_a has to be induced and observed. Nevertheless, this level transition corresponds to a combination of a magnetic and electric dipole transition with $\Delta n = \pm 1$ and $\Delta m_s = \pm 1$ at the same time[14]; such a transition if forbidden to first order, but it can be enforced by an inhomogeneous magnetic RF field which, in turn, necessarily must be accompanied by a homogeneous magnetic RF field. This last field, nevertheless, may produce line shifts and line asymmetries. Furthermore, the transition at frequency ω_a involves a jump from one cyclotron orbit to another with a spin flip at the same time; likewise for the induction of the Larmor frequency. The main limitations of RF spectroscopic experiments lie just in this transition prohibition and in the presence of unwanted homogeneous magnetic RF fields; another limitation is also provided by the limited energy of the trapped particles. In conclusion, the principle of the method of almost all $g-2$ experiments roughly consists in measuring the interaction between the magnetic moment of the particle and a homogeneous magnetic field superimposed by an inhomogeneous magnetic or electric trapping field. The latter, nevertheless reduces the accuracy of the experiments which may be improved decreasing the relative inhomogeneity even if, for technical reasons, this is not possible in the $g-2$ experiments of the muons through further substantial increase of the homogeneous magnetic field. Therefore, to sum up (following (Rich and Wesley 1972)), the precession experiments include measurements of the electron, positron and muon anomalies, the distinguishing feature of these experiments (as those made at Michigan for electrons and at CERN for muons) being a direct observation of the spin precession motion of polarized leptons in region of static magnetic field. The resonance technique instead has mainly been used to measure lepton anomaly (prior to electrons), its characteristic feature being the presence of an oscillating electromagnetic field used to induce transitions between the energy eigenstates of a lepton interacting with a static magnetic field by applying a microwave field at the spin precession frequency ω_c and subsequently a RF field at the spin-cyclotron difference frequency ω_a.

c) Towards the first experimental determinations of the muon AMM

In the same period in which the above mentioned electron AMM determinations were achieved, many further experimental evidences were also accumulated in confirming that the muon behaved as a heavy electron of spin 1/2, so that the former were taken as models to set up possible experiences for

[14]For instance, a quantum state transition from $|n, m_s = -1/2\rangle$ to $|n-1, m_s = +1/2\rangle$ is forbidden being a second order (two-photon) transition because it involves a simultaneous change of the spin quantum number (m_s) and of the orbital (or cyclotron) quantum number (n). But, with a proper choice of the electromagnetic configuration by means of the application of a suitable perturbing field, this transition can be driven.

the latter. But, before to outline these, what were the theoretical motivations underlying the researches towards muon? In 1956, V.B. Berestetskii, O.N. Krokhin and A.X. Klebnikov, in providing, through processes involving photons and leptons, a sensitive test of the limit for the (R.P. Feynman) UV cut-off (or QED-breaking) Λ_l, which represents a measure for the distance at which QED breaks down, pointed out that the measurement of the muon anomalous magnetic moment could accomplish this in a more sensitive manner than that of the electron. Indeed, if one supposes that the muon is not completely point-like in its behavior, but has a form factor[15] $F_\mu(q^2) = \Lambda_\mu^2/(q^2+\Lambda_\mu^2)$, then it can be show that an expression for the sensitivity of a_μ is given by

$$\frac{\delta a_\mu}{a_\mu} = -\frac{4m_\mu^2}{3\Lambda_\mu^2} \tag{55}$$

which may be generalized for leptons as follows

$$\frac{\delta a_l}{a_l} \sim \frac{m_l^2}{\Lambda_l^2}, \qquad l = e, \mu, \tau. \tag{56}$$

Berestetskii, Krokhin and Klebnikov emphasized that the high muon mass could imply a significant correction to a_μ even when Λ_μ is large. Therefore, due to its high mass, the muon allows to explore very small distances (of the order of 10^{-15} cm) because of the simple fact that $q^2 \sim m$ and the higher it is the momentum q^2, the higher it is the energy involved and, therefore, the shorter it is the involved distance scale due to uncertainty principle. Furthermore, mainly because of the vastly different behavior of the three charged leptons mainly due to the very different masses m_l implying completely different lifetimes $\tau_e \simeq \infty$ and $\tau_l = 1/\Gamma_l \propto 1/(G_F^2 m_l^5)$ $l = \mu, \tau$, as well as vastly different decay patterns, it was clear that the anomalous magnetic moment of the muon would be a much better probe for possible deviations from QED. In 1957, J. Schwinger thought that the muon could have an extra interaction which distinguished it from the electron and gave it its higher mass. This could be a coupling with a new massive field or some specially mediated coupling to the nucleon. Whatever the source be, the new field would have had its own quantum fluctuations, and therefore gives rise to an extra contribution to the anomalous moment of the muon. Thus, the principle of $(g-2)$ spin motion was also recognized as a very sensitive test of

[15]The dependence on q^2 of the form factors, experimentally enables us to get information about charge radial distributions and magnetic moments of charged leptons (see (Povh et al. 1995, Part I, Chapter 6, Section 6.1)). For instance, for a generic Dirac particle, we have $F(q^2) = 1$.

the existence of such fields and potentially a crucial signpost to the so-called $\mu - e$ puzzle (see later). But, at that time, there wasn't any possibility to descry some useful principle of the method for pursuing this[16], so that nobody had an idea how to measure a_μ. Albeit the $(g-2)$ spin motion principle will turn out to be, a priori, very similar to those later developed to measure a_μ, nevertheless it was immediately realized that handling the muons in a similar way was impossible, and this raised the difficult task of how to may polarize such short lived particles like muons, in comparison with the long lifetimes of electrons which allowed to measure a_e directly by atomic spectroscopy in magnetic fields. As we shall see later, this was pursued, for the first time, by the pioneering works of the first CERN research groups on $g-2$ since the late 1950s, above all thanks to new magnetic storage techniques set up just to this end. Nevertheless, behind this last pioneering research work, there was a great and considerable previous work of which a brief outline we are however historically obliged to remember.

The principle of the method of the Michigan group experiments has been applied to determine the muon g-factor in some experiments performed, since the middle 1950s, by a notable research group of the Columbia University headed by L.M. Lederman in the wake of the previous work of his maestro I.I. Rabi (see (Lederman 1992)). The first works on the muon g-factor. In 1958, T. Coffin, R.L. Garwin, S. Penman, L.M. Lederman and A.M. Sachs (see (Coffin et al. 1958)) made a RF spectroscopic experiment with stopped muons in which the magnetic moment of the positive μ meson was measured in several target materials by means of a solid-state nuclear magnetic resonance technique with perturbing RF pulses. Muons were brought to rest with their spins parallel to a magnetic field. A radio-frequency (RF) pulse was applied to produce a spin reorientation which was detected by counting the decay electrons emerging after the pulse in a fixed direction. The experimental results were expressed in terms of a g-factor which for a spin 1/2 particle is the ratio of the actual moment to $e\hbar/2m\mu c$. The most accurate result obtained in a $CHBr_3$ target, was $g = 2(1.0026 \pm 0.0009)$ compared to the theoretical prediction of $g = 2(1.0012)$, while less accurate measurements yielded $g = 2.005 \pm 0.005$ in a copper target and $g = 2.00 \pm 0.01$ in a lead target.

After the well-known above mentioned 1956 proposal of parity violation in weak transitions by T.D. Lee and C.N. Yang, it was immediately realized that muons produced in weak decays of the pion $\pi^+ \to \mu^+ + \nu_\mu$ (see Section 1) could be longitudinally polarized, while the decay positron of the

[16] For instance, the parity violation of weak interactions was not yet known at that time.

muon $\mu^+ \to e^+ + 2\nu_\mu$[17] could indicate the muon spin direction. This was confirmed by R.L. Garwin, L.M. Lederman and M. Weinrich (see (Garwin et al. 1957)), as well as by J.I. Friedman and V.L. Telegdi (see (Friedman and Telegdi 1957)), in the same year of[18] 1957. The first researchers, who achieved an accuracy of 5%, started from certain suggestions, made in the remarkable works of T.D. Lee, R. Oehme and C.N. Yang, according to which their hypotheses on violation of C, P and T symmetries had to be sought in the study of the successive reactions 1) $\pi^+ \to \mu^+ + \nu_\mu$ and 2) $\mu^+ \to e^+ + \nu_\mu + \bar{\nu}_\mu$. To be precise, they pointed out that the parity violation would have implied a polarization of the spin of the muon emitted from stopped pions in the first decay reaction along the direction of the motion; furthermore, the angular distribution of electrons in the second decay reaction could serve as an analyzer for the muon polarization. Moreover, in a private communication, Lee and Yang also suggested to Garwin, Lederman and Weinrich that the longitudinal polarization of the muons could offer a natural way of determining their magnetic moment, partial confirmations of the validity of this idea having already been provided by the preliminary results of the celebrated C.S. Wu and co-workers experiments on Co^{60} nuclei. By stopping, in a carbon target puts inside a magnetic shield, the polarized μ^+ beam formed by forward decay in flight of π^+ mesons inside the cyclotron, Garwin and co-workers established the following facts: *i)* a large asymmetry was found for electrons in 2), establishing that the μ^+ beam was strongly polarized; *ii)* the angular distribution of the electrons was given by $1 + a\cos\theta$ where θ was measured from the velocity vector of the incident muons, founding $\theta = 100^o$ $a = -1/3$ with an estimated error of 10%; *iii)* in both reactions, parity was violated; *iv)* by a theorem of Lee, Oheme and Yang (see (Lee et al. 1957)), the observed asymmetry proves that invariance under charge conjugation is not conserved; *v)* the g value for free μ^+ particles was found to be $+2.00 \pm 0.10$; and *vi)* the measured g value and the angular distribution in 2), led to the very strong probability that the μ^+ spin was 1/2. The magnetizing current, induced by applying a uniform small vertical field in the magnetic shielded enclosure about the target, produced as a main effect the precession of muon spins, so

[17] Only after 1960, it was ascertained that $\nu_\mu \neq \bar{\nu}_\mu$, whereupon we might more correctly write $\mu^+ \to e^+ + \nu_\mu + \bar{\nu}_\mu$ (see Section 1).

[18] For technical reasons, the paper of Friedman and Telegdi was delayed to the Physical Review Letters issue next to the one in which was published the paper of Garwin, Lederman and Weinrich, notwithstanding both papers were received almost contemporaneously, the former on January 17, 1957 and the latter on January 15, 1957. Nevertheless, following (Cahn and Goldhaber 2009, Chapter 6), the Friedman and Telegdi emulsion experiment at Chicago was started before others but has employed more time to be completed because of the laborious scanning procedure.

that a road based on muon spin precession principle to seriously think about the experimental investigation of a_μ, was finally descried. Amongst other things, the work of Garwin, Lederman and Weinrich opened the way to the so-called *muon spin resonance* (μSR), a widespread tool in solid state physics and chemical physics. In 1957, their result was improved to an accuracy of about 4% by J.M. Cassels, T.W. O'Keele, M. Rigby, A.M. Wetherall and J.R. Wormald.

Likewise, following the celebrated suggestion of Lee and Yang on non-conservation of parity in weak interactions, Friedman and Telegdi (1957) investigated the correlation between the initial direction of motion of the muon and the direction of emission of the positron in the main decay chain $\pi^+ \to \mu^+ \to e^+$ produced in nuclear emulsions just to detect a possible parity non-conservation in the latter decay interactions. Following Lee and Yang arguments, violation of parity conservation may be inferred essentially by the measurement of the probability distribution of some pseudoscalar quantity, like the projection of a polar vector along an axial vector. For instance, Lee and Yang themselves suggested several experiments in which a spin direction is available as a suitable axial vector; in particular, they pointed out that the initial direction of motion of the muon in the decay process $\pi^+ \to \mu^+ + \nu_\mu$ can serve for this purpose, as the muon will be produced with its spin axis along its initial line of motion if the Hamiltonian responsible for this process does not have the customary invariance properties. If parity is further not conserved in the decay process $\mu^+ \to e^+ + 2\nu_\mu$, then a forward-backward asymmetry in the distribution of angles, say $W(\theta)$, between this initial direction of motion and the moment of the decay electron, is predicted. To this end, positive pions from the University of Chicago synchrocyclotron were brought to rest in emulsion carefully shielded from magnetic fields, as well as over 1300 complete decay events were measured. A correlation $W(\theta) = 1 + a\cos\theta$ was found, with $a = -0.174 \pm 0.038$, clearly indicating a backward-forward asymmetry, that is to say a violation of parity conservation in both decay processes. Following an argument of T.D. Lee, R. Oehme[19] and C.N. Yang, this asymmetry would have implied a non-invariance of either decay reactions

[19]Reinhard Oehme (1928-2010) was an influential theoretical physicist who gave notable contributions mainly in mathematical and theoretical physics. Amongst these, Oehme was the first to realize that every time the CPT symmetry must be obeyed, then if P was violated, C and/or T had to be violated as well. He proved that if the various experiments suggested by Lee and Yang showed a P violation, then C had to be violated too. In this regards, Oehme sent a letter to Yang and Lee explaining this insight, and they immediately suggested that all three together would have written a paper (Lee et al. 1957)). See above all (Yang 2005) where this historical event, often misunderstood, has definitively been clarified.

with respect to both space inversion P and charge conjugation C, taken separately. Furthermore, Friedman and Telegdi given a detailed discussion of a depolarization process specific to μ^+ mesons, i.e. the possible formation of muonium ($\mu^+ e^-$). The results of this and similar experiments were also compared with those obtained with muons originating from p^+ decays in flight and the implications of such a comparison were discussed too. Therefore, the Friedman and Telegdi work, for the first time, pointed out, also thanks to a private communication with R. Oehme, that P and C were violated simultaneously, or rather, to be precise, P was normally violated while CP was to very good approximation conserved, in the decay processes analyzed by them.

Following (Farley and Picasso 1979) and (Jegerlehner 2008, Part I, Chapter 1), it should be mentioned that until the end of 1950s, the nature of the muon was quite a mystery. In that period, the possible deviations from the Dirac moment $g = 2$ were ascribed to the interaction of leptonic particle with its own electromagnetic field. Any other field coupled to the particle would produce a similar effect and, in this regards, the calculations have been made for scalar, pseudoscalar, vector and axial-vector fields, using an assumed small coupling constant f to a certain boson of mass M. For example, for the case of a vector field, the above mentioned work of Berestetskii, Krokhin and Klebnikov as well as the 1958 work of W.S. Cowland, provided the estimate $\delta a_\mu^{Vec} = (1/3\pi)(f^2/M^2)m_\mu^2$ so that a precise measurement of a_μ could therefore reveal the presence of a new field, but, before this, it had to be discovered all the known fields, comprising the weak and strong interactions, and hereupon taken into account. Following (Picasso 1996) and references quoted therein, the theoretical value for a_μ can be expressed as follows $a_\mu^{(th)} = a_\mu^{QED(th)} + a_\mu^{QCD(th)} + a_\mu^{Weak(th)}$. In the 1950s, the only contribution which could be measured with a certain precision was the QED one, while both the strong and weak interaction contributions will be determined only later[20]. In any case, the QED contribution turns out to be the dominant one for a_e while as of today, good estimates have been achieved for weak interaction contributions to a_μ but not for the hadronic ones. While today it is well-known that there exist three lepton-quark families with identical basic properties except for differences in their masses, decay times and patterns, at that time it was very hard to believe that the muon is just a heavier version of the electron, so giving rise to the so-called *$\mu - e$ puzzle,* paraphrasing the previous well-known *$\theta - \tau$ puzzle* which was brilliantly solved by the cele-

[20]The first ones who pointed out on the importance of hadronic vacuum-polarization contributions to a_μ were C. Bouchiat and L. Michel in 1961 as well as L. Durand in 1962 (see (Roberts and Marciano 2010, Chapter 3, Section 3.2.2.2)).

brated work of T.D. Lee and C.N. Yang on the parity violation for weak interactions. For instance, it was expected that the muon exhibited some unknown kind of interaction, not shared by electron and that would have due to explain the much higher mass. All this motivated and stimulated the experimental research to explore a_μ. As it has already been said above, the big interest in the muon anomalous magnetic moment was motivated by the above mentioned Berestetskii, Krokhin and Klebnikov argument in relation to the main fact according to which the anomalous magnetic moment of leptons mediates spin-flip transitions whose amplitudes are proportional to the masses of particles, so that they are particularly appreciable for heavier ones via a generalization of (55) given by

$$\frac{\delta a_l}{a_l} \propto \frac{m_l^2}{M_l^2} \qquad (M_l \gg m_l) \tag{57}$$

where M_l is a parameter which may be either an energy scale or an ultraviolet cut-off where QED ceases to be valid (QED-breaking) or as well the mass of a hypothetical heavy state or of a new heavier particle. The relation (57) also allows us to ascertain whether an elementary particle has an internal structure: indeed, if the lepton l is made by hypothetical components of mass M_l, then the anomaly a_l would be modified by a quantity δa_l given by the relation $\delta a_l = O(m_l^2/M_l^2)$ so that the measurements of a_l might provide a lower limit for M_l which, at the current state of research, has a magnitude of about 1 TeV, which imply strong limitations to the possible hypotheses on the internal structure of a lepton (see (Picasso 1985)). On the other hand, the relation (57) also implies that the heavier the new state or scale, the harder it is to see. Therefore, from (57), it follows that the sensitivity to high-energy physics grows quadratically with the mass of the lepton, which means that the interesting effects are magnified in a_μ compared to a_e by a factor of about $(m_\mu/m_e)^2 \sim 4 \cdot 10^4$, and this is just what has made and still makes a_μ the elected monitoring fundamental parameter for the new physics also because of the fact that the measurements of a_τ go out of the present experimental possibilities due to the very short lifetime of τ.

As also reported in (Garwin et al. 1957), if $g = 2$ then the direction of muon polarization would remain fixed relatively to the direction of motion throughout the trajectory, while if $g \neq 2$ then a phase angle δ opens up between these two directions. Following (Muirhead 1965, Chapter 2, Section 2.5(a,e)), (Farley and Picasso 1979) and (Picasso 1996), to estimate δ, let us assume that we have longitudinally polarized charged leptons slowly moving in a magnetic field and we know their direction of polarization. If they are allowed to pass into a system with a magnetic field of strength B, they experience a torque given by $\vec{\tau} = \vec{\mu}_s \wedge \vec{B}$ which, in turn, implies the execution of helical orbits about the direction of $\vec{B}$ which lead to a *Larmor precession* about the direction of $\vec{B}$ with the following angular velocity (in natural units) calculated in the particle rest frame

$$\omega_s = g\frac{Q}{2mc}B = \Gamma B \tag{58}$$

where $\Gamma = g(Q/2mc)$ is the *gyromagnetic ratio.* If the charged particle is also in motion, then it will execute spiral orbits about $\vec{B}$ which possess the characteristic *cyclotron frequency* ν_c given by $\omega_c = 2\pi\nu_c = (Q/mc)B$. In one defines the laboratory rotation frequency of the spin relative to the momentum vector as $\omega_{a_i} \doteq \omega_s - \omega_c$, then the phase angle δ, after a time t, is given by

$$\delta = \omega_{a_i}t = (\omega_s - \omega_c)t = \frac{g-2}{2}\frac{Q}{mc}Bt = a_i\frac{Q}{mc}Bt \tag{59}$$

where $g = 2(1+a_i)$ $i = e, \mu, \tau$. Hence, if $g = 2$, then $\omega_s = \omega_c$ and the charged leptons will always remain longitudinally polarized. But if $g > 2$ as predicted, then the spin starts to precess and turns faster than the momentum vector. Therefore, it is immediately realized that a measurement of the phase angle δ after a time t, may estimate the magnitude of the deviation of the g-value from 2. Equation (59) will be the basic formal tool for the so-called $(g-2)$ experiments and that will be carried out later: if the charged lepton is kept turning in a known magnetic field $\vec{B}$ and the angle between the spin and the direction of motion is measured as a function of time t, then a_i may be estimated. The value of Q/mc is obtained from the precession frequency of the charged leptons at rest, via equation (58). Furthermore, the fundamental equation (59) has been derived only in the limit of low velocities but it has been proved to be exactly true as well at any speed as, for example, made in (Bargmann et al. 1959) using a covariant classical formulation of spin-motion. It has also been proved that the $(g-2)$ precession is not slowed down by time dilation even for high-velocity muons.

Following (Farley and Picasso 1979) and (Brown and Hoddeson 1983, Part III, Chapter 8), after the celebrated experience made by Garwin, Lederman and Weinrich in 1957, the possibility of a $(g-2)$ experiment for muon was finally envisaged. In 1959, as recalled by (Jegerlehner 2008, Part I, Chapter 1), the Columbia research group made by L.M. Lederman, R.L. Garwin, D.P. Hutchinson, S. Penman and G. Shapiro, performed a measurement of a_μ with a precision of about 5%, even using a precession technique applied to a polarized muon beam whose directions are determined by means of their asymmetric decay modes. In the same years, many other research groups at Berkeley, Chicago, Liverpool and Dubna started as well to study the problem. If the muon had a structure giving a form factor less than one for photon interactions, then the value of a_μ should be less than predicted. Nevertheless, compared with the measurement on the electron, the muon $(g-2)$ experiment was much more difficult because of the low intensity, diffusive nature and high momentum of available muon sources. All this, together the possibility to get a reasonable number of precession cycles, entailed, amongst other things, the need to have large volumes of magnetic field. One solution, adopted by A.A. Schupp, R.W. Pidd and H.R. Crane in 1961, was to scale up the original Michigan $(g-2)$ method for electrons whose spin directions was established with the aid of a double scattering experiment in which the first and second scatterings were performed respectively before and after the passage of the electrons through a solenoid. However, out of the many attempts to approach such a problem (see also (Garwin 2003)), the first valuable results were achieved by the first CERN $(g-2)$ team composed in alphabetic order by G. Charpak, F.J.M. Farley, T. Muller, J.C. Sens and A. Zichichi (credit by CERN-BUL-PHO-2009-017), formalized the 1st of January 1959 but already operative since 1958. As recall (Combley and Picasso 1974), (Farley and Picasso 1979), (Combley et al. 1981) and (Jegerlehner 2008, Part I, Chapter 1), the breakthrough experiment which made the direct attack on the magnetic moment anomaly of muons was performed at CERN synchrocyclotron (SC) by the first $(g-2)$ team mentioned above. As a result of this measurement, the experimental accuracy in the value of the muon anomalous magnetic moment was reduced to 0.4% from the level of 15% at which it had previously stood. Following (Brown and Hoddeson 1983, Part III, Chapter 8), the CERN experiments performed from 1961 to 1965, have been based on the main idea according to which, roughly speaking, the muons produced by a beam of pions decaying in flight are longitudinally polarized; furthermore, in the subsequent decays, the electrons reveal the direction of the muon spins because they are preferentially emitted along the spin direction at the momentum of decay. Hence, a $(g-2)$ experiment may be performed trapping the longitudinally polarized muons in a uniform magnetic field and then measuring the precession frequency of the spins. It has only to be added that, due to the very short muon lifetime, it was necessary to use high-energy muons in order to lengthen their decay times using the relativistic time dilation effect. The results reduced the error in the measure of $(g-2)$ from the previous 15% to 0.4%.

Following (Jegerlehner 2008, Part I, Chapter 1), surprisingly nothing of special was observed even within 0.4% level of accuracy of the experiment; it was the first real evidence that the muon was just a heavy electron, so reaching to another celebrated experimental evidence of the validity of QED. In particular, this meant that the muon was point-like and no extra short distance effects could be seen. This latter point was however a matter of accuracy and therefore the challenge to go further was quite evident; in this regards, see the reviews (Farley and Semertzidis 2004) and (Garwin 2003). As recalled in (Cabibbo 1994, Part I), G. Bernardini, then research director responsible for the SC at CERN, remembers as, around the end of 1950s, there were many ideas for the high precision measurements of the anomalous magnetic moment of the muon, two of them having been that of the *screw magnet* and that of the *flat magnet.* Gilberto Bernardini consulted the greatest magnet specialist, Dr. Bent Hedin, who said that would have been necessary some years to fully carried out one of this project, the flat magnet one, so that it was initially chosen the screw magnet project. In the meanwhile, A. Zichichi had the ingenious idea to trying a new very simple technique consisting in shaping a flat pole with very thin iron sheets, glued together by means of the simplest possible method, the scotch tape. In this way, instead of six years, a few months of hard work allowed Zichichi to built up particular high accuracy magnetic fields, based on the theoretical notion of *Garwin-Panofsky-Zichichi polynomial magnetic fields,* which constitute just those experimental tools that needed for attaining high measurements of a_μ. The so-called *six-meters long flat magnet* providing an injection field, followed by two transitions, hence a storage, then another transition and finally an ejection field, became the core of the first high precision measurement of the muon $(g-2)$. Likewise, R.L. Garwin, in (Cabibbo 1994, Part I), remembers that, in achieving this, it was determinant the special responsibility of Zichichi profused by him in producing the bizarre magnetic field in their storage magnetic system, accomplished with imagination, energy and efficiency. Again, in (Garwin 1986, 1991, 2001) and (Garwin 2003), the author recalls that the 80-ton magnet six-meters long was shimmed in a wondrous fashion under the responsibility of Nino Zichichi who did a wonderful job in doing this, while the polarization was measured as the muons emerged from the static magnetic field thanks a system perfected by G. Charpak; F.J.M. Farley was instead in charge to develop the computer program which would take the individual counts from the polarization analyzer done by Charpak, while T. Muller played the electronic work with the help of C. York. Following (Jones 2005), the six-meters magnet came to CERN as the heart of the first $g-2$ experiment, the aim of which was to measure accurately the anomalous magnetic moment, or g-factor, of the muon. This experiment was one of CERN outstanding contributions to fundamental physics and for many years was unique to the laboratory.

To this point, it is need to retake the equations of motion of a charged particle in a magnetic field $\vec{B}$ from a relativistic viewpoint. Following (Combley et al. 1981), (Picasso 1996) and (Jegerlehner 2008, Part II, Chapter 6), the cyclotron (or orbital) frequency is given by

$$\vec{\omega}_c = \frac{Q}{\gamma mc}\vec{B} \tag{60}$$

where $\gamma = 1/\sqrt{1-\beta^2}$ and $\vec{\beta} = \vec{v}/c$. When a relativistic particle is subject to a circular motion, then it is also need to take into account the so-called *Thomas precession,* which may be computed as follows. The particle rest frame of muon rotates around the laboratory frame with angular velocity $\vec{\omega}_T$ given by

$$\vec{\omega}_T = \Big(1 - \frac{1}{\gamma}\Big)\frac{Q\vec{B}}{mc} \tag{61}$$

and it is different from the direction of the angular velocity with which the muon's spin rotates in the rest frame, so that the angular velocity of spin rotation in the laboratory frame is given by

$$\vec{\omega}_s \doteq \vec{\omega}_L - \vec{\omega}_T = \Big(a_\mu + \frac{1}{\gamma}\Big)\frac{Q\vec{B}}{mc} \tag{62}$$

which shows that the angular frequency of anomalous magnetic moment is, in relativistic regime, equal to the angular frequency at very low energies, that is to say

$$\vec{\omega}_{a_\mu} = \vec{\omega}_s - \vec{\omega}_c = a_\mu \frac{Q\vec{B}}{mc}. \tag{63}$$

To argue upon the electric dipole moment of the muon, we should consider the relativistic equations of the muon in the laboratory system in presence of an electric field $\vec{E}$ and of a magnetic field $\vec{B}$. In this case, under the conditions of purely transversal fields $\vec{\beta}\cdot\vec{E} = \vec{\beta}\cdot\vec{B} = 0$, following (Bargmann et al. 1959), the cyclotron angular velocity is given by

$$\vec{\omega}_c = \frac{Q}{mc}\Big(\frac{\vec{B}}{\gamma} - \frac{\gamma}{\gamma^2-1}\vec{\beta}\wedge\vec{E}\Big) \tag{64}$$

while the spin angular velocity is given by

$$\vec{\omega}_s = \frac{Q}{mc}\Big(\frac{\vec{B}}{\gamma} - \frac{1}{1+\gamma}\vec{\beta}\wedge\vec{E} + (\vec{B} - \vec{\beta}\wedge\vec{E})\Big) \tag{65}$$

so that the angular frequency of the muon anomalous magnetic moment, related to the spin precession, is given by

$$\vec{\omega}_{a_\mu} = \vec{\omega}_s - \vec{\omega}_c = \frac{Q}{mc}\left(a_\mu\vec{B} + \Big(\frac{1}{\gamma^2-1} - a_\mu\Big)\vec{\beta}\wedge\vec{E}\right) \tag{66}$$

which is the key formula for measuring a_μ; $\omega_a = |\vec{\omega}_a| = \omega_s - \omega_c$ is the anomalous frequency difference or spin-flip transition. If a large enough electric dipole moment given by $(6)_2$ there exists, then either the applied field $\vec{E}$ (which is zero at the equilibrium beam position) and the motional electric field induced in the muon rest frame, say $\vec{E}^* = \gamma\vec{\beta} \wedge \vec{B}$, will add an extra precession of the spin with a component along $\vec{E}$ and one around an axis perpendicular to $\vec{B}$, that is to say

$$\vec{\omega} = \vec{\omega}_{a_\mu} + \vec{\omega}_{EDM} = \vec{\omega}_{a_\mu} + \frac{\eta Q}{2mc}\left(\vec{E} + \vec{\beta} \wedge \vec{B}\right) \tag{67}$$

or else

$$\Delta\omega_{a_\mu} \cong d_e(\vec{E} + \vec{\beta} \wedge \vec{B}) \tag{68}$$

which, for $\beta \sim 1$ and $d_e\vec{E} \sim 0$, yields

$$\omega_{a_\mu} \cong B\sqrt{\left(\frac{Q}{mc}a_\mu\right)^2 + (d_e)^2}. \tag{69}$$

The result is that the plane of precession is no longer horizontal but tilted at an angle

$$\theta \equiv \arctan\frac{\omega_{EDM}}{\omega_{a_\mu}} = \arctan\frac{\eta\beta}{2a_\mu} \cong \frac{\eta}{2a_\mu} \tag{70}$$

and the precession frequency is increased by a factor

$$\omega'_{a_\mu} = \omega_{a_\mu}\sqrt{1+\delta^2}. \tag{71}$$

The angle θ produces a phase difference in the $(g-2)$ oscillation. It is therefore important to determine whether there is a vertical component to the precession in order to separate out the effect of an electric dipole moment from the determination of ω_{a_μ}. The angle of tilt θ given, in the small angle approximation, by (70), may be detected by looking for the time variation of the vertical component of the muon polarization with the same frequency as the $(g-2)$ precession of the horizontal polarization. Therefore, in order to eliminate the electric dipole moment as a source of any discrepancy which might appear in $(g-2)$ direct measurements of higher precision is preliminarily required. In any case, the main determination in the electric dipole moment of the muon is not merely this last clarification of the $(g-2)$ measurements. Indeed, it is also of fundamental importance in itself since the existence of such a static property for any particle would imply the lack of invariance for the electromagnetic interaction under both P and T, as recalled above. Some of the theories unifying the weak and electromagnetic interactions predict a small electric dipole moment for some particles including the muon and a precise measurement of this property would tighten the constrains within which such theories might operate, so that precise measurements of the electric dipole moment of the muon as of other particles were and still are highly desirable.

........(to be inserted the dependence on r by B following Picasso 1985 and Jegerlehner on muon storage experiments) ....

2. The bases for the first exact measurements of anomalous magnetic moment of the muon

The first works of A. Zichichi concerned cosmic ray experimental physics and were carried out until the end of 1950s. From this period onwards, A. Zichichi was involved, as briefly said above, in some crucial experiments concerning the muon $(g-2)$ measurements and carried out at CERN of Geneva. The first work on muon anomalous magnetic moment in which he was involved is (Charpak et al. 1960) where a precise measurement of the electric dipole moment of the muon was obtained within the QED context only. The work starts from the above mentioned Michigan spin precession method used to measure a_e which exploits the possibility to have beams of polarized leptons underwent to asymmetric decay. With this method, i.e. the spin precession methods (see previous Section), one can measure $(g-2)$ by storing the particles for some time in a magnetic field and then measuring the relative precession angle between the spin and the angular momentum which serves as a reference vector. As in the electron experiments, the primary requirement was in being able in injecting the muons into a magnetic field so that they could circulate on essentially periodic orbits, hence to trap them in this field for a large number of orbit periods as possible. Nevertheless, at that time, the available muon beams exhibited, in comparison with the electron case, very low fluxes, high momenta and large extensions in position and momentum space (hence, low density in phase space) which implied many other new difficulties besides the above mentioned primary requirement. On the other hand, the muons did not require the analysis of the spin polarization by scattering since the asymmetric electron decay reveals the spin deviation; indeed, as said above, the electrons were emitted along the spin direction at the moment of decay. Starting from the principle of the method of the experimental apparatus used in (Garwin et al. 1957), the essence of this idea had already been established in (Berley et al. 1958) where the existence of longitudinally polarized beams of μ mesons and the availability of muon decay electron asymmetry as a polarization analyzer suggested this method by means of which one may search for a muon electric dipole moment. A discussion of the results achieved in (Berley et al. 1958) was then made in (Garwin and Lederman 1959) from which turns out that several practical methods for overcoming these difficulties were either experimentally and theoretically undertaken before this work of Charpak, Lederman, Sens and Zichichi, but without succeed in the enterprize. Instead, this research group was able, for the first time, to trap 85 MeV/c momentum muons for 28 turns, i.e. orbit periods, with no pulse magnets. Their results clearly suggested too that minor modifications in their method were enough to enable one in achieving storage for several hundreds of turns. Well, all this was made possible, as also recalled in the previous section, just thanks to the ingenious technical and experimental ability of A. Zichichi in building up suitable polynomial magnetic fields of high precision and thanks to which it was possible to obtain thousand muon turns (see also (Farley 2005)); in turn, all this was carried out on the basis of the theoretical framework mainly worked out on previous remarkable studies made by R. Garwin and W.K.H. Panofsky, upon which we shall in-depth return later. The extreme importance and innovativeness of this experimental technique was successfully carried out later, at a technical level, in producing the so-called *six-meters long flat magnet* which, in turn, was mainly built up by A. Zichichi starting from a suitable modification of a previous magnet provided by the University of Liverpool (see (Zichichi 2010) and (CERN 1960)). Seen the fundamental importance of this event, it is necessary to outline the early works and ideas which came before the dawning of this experimental apparatus, and mainly worked out, for the first time, in the paper (Charpak et al. 1960) on whose content we now will briefly argue.

The principle of the method consists in injecting, say along the Y axis, a muon beam into a median (X, Y) plane of a flat magnet gap. A moderator (or absorber) M, centered on the origin of the (X, Y) plane, will contain such a beam through a suitable reduction of the momentum beam p and of the mean vertical (i.e. along Z direction) field value B_{z0}. So, the muons lost energy and consequently follow small and more sharply orbits which will be contained within the magnetic field region, and to prevent a reabsorption by moderator after one turn, a small transverse linear gradient of the magnetic field is inserted, causing an orbit drift along the X axis in the direction opposed to *sign* a. The magnetic field configuration is therefore planned to produce such a drift of the muon orbits along the X axis away from the moderator M, focusing the muon beam in the median (X, Y) plane. The magnetic field therein used has the following polynomial form

$$B_z = B_{z0}(1 + aY + bY^2) \tag{72}$$

along the median plane, where $a, b \in \mathbb{R}$ have to be small (Garwin-Panofsky). If r is the distance from the origin and $ar \ll 1$ and $br^2 \ll 1$, then the muons emerging from M will move on nearly circular orbits of radius r. A linear gradient alone leads to a step-size drift of these orbits along the X-direction by an amount equal to

$$s = \pi r^2 \langle grad_Y \frac{B_z}{B_{z0}} \rangle = \pi r^2 a \quad \text{per turn} \tag{73}$$

where $\langle \ , \ \rangle$ denotes average over one orbit loop. This drift will enable some muons to get over M after their first turn, whereupon they go on along a trochoidal orbit. Moreover, following previous basic and notable studies made by R.L. Garwin and W.K.H. Panofsky[21], the linear gradient also produces a weak vertical focusing with wavelength given by

$$\frac{\lambda_\nu}{2\pi} \cong \frac{0.76}{a}. \tag{74}$$

Taking into account equation (73), because we want to be $r/s \gg 1$ in order to store as large as possible a number of turns in a magnet of given finite size, it follows that this focusing is very weak either because of sensitive variations of the field index n and since $(r/s \gg 1) \Rightarrow (\lambda_\nu/2\pi r \gg 1)$ which implies low frequencies and consequently a weak focusing, hence a poor storage. Nevertheless, as was pointed out by R.L. Garwin (see his 1959 CERN Internal Report), one can improve the vertical focusing while maintaining a given large value of r/s by the addition of a quadratic term of the type by^2 and indeed, for a polynomial magnetic field of the type (72) with a and b small, one has

$$\frac{\lambda_\nu}{2\pi} \cong \frac{1}{\sqrt{b + 1.74a^2}} \sim \frac{1}{\sqrt{b}} \tag{75}$$

while the drift step-size is still given by (73), so that we can handle a and b in such a manner to have high values of the former and low values of the latter. For example, by taking $b = 50a^2$, one can, while maintaining the same r/s of above (for such orbits), improve the focusing to 1 oscillation per 7 turns. Therefore, the intensity of stored muons is increased by a factor $38/7 \sim 5$ by

[21] See R.L. Garwin, *Numerical calculations of the stability bands and solutions of a Hill differential equation,* CERN Internal Report (October 1959) and W.K.H. Panofsky, *Orbits in the linear magnet,* CERN Internal Report (October 1959).

the addition of the quadratic term to the magnetic field. Thus, to sum up, the term ay produces the X axis drift of an orbit of radius r in step-sizes of magnitude $a\pi r^2$ per turn[22]. The next by^2 term adds vertical focusing in such a manner that the wavelength of the vertical oscillations are about $2\pi/\sqrt{b}$; it has as well the useful function to fix more firmly the magnetic median plane around the center of the magnet gap because just the median plane begins to touch the poles, then all the particles will go lost. In any case, it is not allowed to choose b arbitrarily large for vertical defocusings minimizing λ_ν because this would lead to a spread in the drift step-size and hence in storage times. Indeed, orbits emerging at an angle ϕ with respect to the Y axis would have a step-size given by

$$s(\phi) = \pi r^2(a - 2br\phi) \tag{76}$$

so that the magnitude of b may be chosen in order to maximize the number of particle stored for a given number of turns.

Once having established these fundamental theoretical points, mainly due, as recalled above, to previous works of R.L. Garwin and W.K.H. Panofsky, the next step was to practically realize such polynomial magnetic fields, far from being an easy task. This primary work was masterfully and cleverly accomplished by A. Zichichi starting from a previous magnet provided by the University of Liverpool for whose technical details we refer to the Section 2 - Injection and Trapping, of the original work (Charpak et al. 1960). He was very able to set up a complex but efficient experimental framework that provided suitable polynomial magnetic fields for the magnetic storage of muon beams. The experimental results are of historical importance and were represented in the Figures 2. and 3.a)-b) of (Charpak et al. 1960) whose characteristics were adequately theoretically explained in the above mentioned Section 2 of (Charpak et al. 1960). These results were the first valuable experimental evidence of the fact that particles turning several times inside a small magnetic arrangement was pursuable, so endorsing that presentiment according to which longer magnetic systems of this type could give further and more precise measurements. All this was in fact done in the subsequent experiments made by A. Zichichi and co-workers and that will be described later. The final section of the work (Charpak et al. 1960) deals then with attempts to measure the electric dipole moment of the muon starting from the experimental results achieved by the previous works (Berley et al. 1959) and (Garwin and Lederman 1959) and whose principle of the method was mainly based on the determination of the phase angle given by (59) through the so-

[22] According to a principle of the method almost similar to the one proposed by P.S. Farago in (Farago 1958) for the free electron case.

called *up-down asymmetry parameter*[23] α, taking into account the original theoretical treatment given by (Bargmann et al. 1959) and briefly recalled in the previous Section 3. To this end, Charpak, Lederman, Sens and Zichichi used their innovative experimental arrangement to storage polarized muon beams, just to determine this EDM of the muon. The related value so found was consistent with time reversal invariance and could be considered equal to zero within the experimental errors which have been considerably reduced respect to those of the above mentioned previous works on muon EDM determination. To be precise, their formal treatment is that of (Bargmann et al. 1959) in which are considered the covariant classical equations of motion of a particle of arbitrary spin moving in a homogeneous electromagnetic field. As it has already been said, the theoretical considerations made in (Bargmann et al. 1959) include too the relativistic case because of a remark due to F. Bloch. We consider longitudinally polarized muons possessing an EDM given by $(6)_2$, which move in a magnetic field $\vec{B}$ in a plane perpendicular to the latter. In their instantaneous rest frame, they experience an electric field given by $\vec{E}^* = \gamma\vec{\beta} \wedge \vec{B}$ which causes a precession of the EDM. In the laboratory frame, the spin precesses around $\vec{v} \wedge \vec{B}$ (hence, out of the orbit plane in which relies $\vec{v} \wedge \vec{B}$) by an angle $\Theta_s = \omega_s t$ when the orbit has gone through an angle $\Theta_o = \omega_c t$ (or Θ_c) on its orbital plane (see Equation (59)). The polarization (perpendicular to the orbit) thus produced, is detected by stopping the muons after a known Θ_o and measuring the up-down asymmetry of the electrons emerging from the muon decay with respect to the orbit plane (placed in the median plane of the storage magnet set up in (Charpak et al. 1960) and detected by the scintillator No. 4 of their apparatus). This determination, successfully achieved by Charpak, Lederman, Sens and Zichichi, was different from the previous ones only in the magnitude of Θ_o, in which it was assumed to be $\Theta_o \in]0, 2\pi[$, whereas they used the new storage device based on polynomial magnetic fields to get $\Theta_o = 2n\pi$ with $n \geq 28$, just thanks to the multiple turns that their arrangement was able to provide. The principle of the method consisted in analyzing two range of flight times of particles, a *group A* of *early* particles having made few turns in the storage magnet and which are used for calibration, and a *group B* of *late* particles which have made many revolutions. In turn, the measurements were divided into three groups in dependence on the mean turn index $\langle n \rangle$ of late particles, this being fixed for the early ones and equal to $\langle n \rangle \approx 1$. The Group I concerns late muons with $\langle n \rangle \approx 11.5$; the Group II concerns late muons with $\langle n \rangle \approx 16.5$, while Group III concerns muons with $\langle n \rangle \approx 19.5$. For each of these groups, the difference in up-down asymmetry, say $\Delta^{(i)} = a^{(i)}_{early} - a^{(i)}_{late}$ $i = I, II, III$,

[23]It is given by $\alpha = (N_{up} - N_{down})/(N_{up} + N_{down})$ respect to the median plane.

between the early and late ones, is evaluated. The values so found are reported in the Table I of (Charpak et al. 1960) and from these it is then possible to estimate the angle $\Theta_s^{(i)}$, through which the spin has rotated out of the median plane, as $\Delta^{(i)}/a_{max}^{(i)}$ where $a_{max}^{(i)}$ is the maximal obtainable value of asymmetry in the given ith group. Then $\Theta_o^{(i)} \approx \omega_c \langle t^{(i)} \rangle$ where $t^{(i)}$ is the beam flight time detected by the final median plane scintillator. Furthermore, to improve distribution calculations and to reduce systematic errors, the EDM telescope was also symmetrically displaced at different heights with respect to the magnet median plane. Finally, combining the three values of $\Theta_s^{(i)}/\Theta_o^{(i)} \;\; i = I, II, III$ (listed in the above mentioned Table I), it was possible to estimate η of $(6)_2$, whence to deduce the upper limit for the EDM of the muon.

Following (Lee 2004, Chapter 2), the accelerator physics principles involved in the work (Charpak et al. 1960) mainly concern with transverse particle motion in the sense as first outlined in the 1941 seminal paper (Kerst and Serber 1941) for the betatron case. In Frenet-Serret coordinates (x, s, z) (s is oriented as the tangent, x as the normal and z as the binormal respect to the orbit plane) and in zero electric potential, we have a two-dimensional magnetic field given by $\vec{B} = B_x(x, z)\hat{x} + B_z(x, z)\hat{z}$ where $\hat{z} = \hat{x} \wedge \hat{z}$. In straight geometries, we have a magnetic flux density given by

$$B_z + iB_x = B_0 \sum_{n\in\mathbb{N}_0} (b_n + ia_n)(x + iz)^n \tag{77}$$

where a_n, b_n are called $2(n + 1)$th *multipole coefficients* and are given by (Lee 2004, Chapter 2, Section I.3, Equations (2.26)). The expression (77) is said to be the *Beth representation* (see (Beth 1966, 1967)). For example, in discussing the focusing of atomic beams, the sextupole terms are show to be able to make high spin focusings (see (Lee 2004, Chapter 2, Exercise 2.2.18)). In such a case, some historical predecessors of these techniques to obtain polarized ions may be found in (Haeberli 1967) where, among other things, are discussed too some previous experiences with separate magnets operating at the quadrupolar or sextupole order, due to H. Friedburg, W. Paul and H.G. Bennewitz in the early 1950s. In certain sense, looking at the (77), the Garwin-Panofsky-Zichichi polynomial magnetic fields might be considered as special cases forerunner of such Beth representations. The work (Charpak et al. 1960), which has been submitted to the redaction of the Nuovo Cimento Journal on April 4, 1960, has been therefore the milestone for the next phase of the exact measurements of the anomalous magnetic moment of the muon because, thanks to the introduction of the polynomial magnetic fields, it will be possible to carry out, after a series of ever best measurements, the first historical exact measurement of the muon AMM, the one made in (Charpak et al. 1965). Indeed, the polynomial magnetic field technique developed in (Charpak et al. 1960) to measure the EDM of the muon, will be usefully employed later to measure the AMM of the muon, as made in (Charpak et al. 1965). In conclusion, it has been thanks to the *Garwin-Panofsky-Zichichi magnetic polynomial fields* that the experimental results have been achieved. The theoretical work of R.L. Garwin, together the one made by W.K. Panofsky (presented in the CERN Memorandum SC/9976/Rapp/81 of 28th October 1959), has been crucial to achieve this: it has not officially published (like the Panofsky's one) but worked out in certain remarkable CERN memorandums written in 1959, to be precise the CERN Memorandums Ref. RLG/1 (October 1, 1959), Ref. RLG/2 (October 7, 1959), Ref. SC/9976/Rapp/81 and, above all, Ref. RLG/3 (October 21, 1959), where the theoretical bases for the magnetic polynomial fields has been casted. These last documents have been kindly provided to me by Professor Garwin himself, to whom I'm grateful for this. Nevertheless, nobody has, so far, highlighted this last notable historical fact, not even the protagonists themselves of these pioneering physical experiments which have provided the first exact experimental confirmations of QED.

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