Fuzzy Clustering Image Segmentation Based on Particle Swarm Optimization

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Abstract
Image segmentation refers to the technology to segment the image into different regions with different characteristics and to extract useful objectives, and it is a key step from image processing to image analysis. Based on the comprehensive study of image segmentation technology, this paper analyzes the advantages and disadvantages of the existing fuzzy clustering algorithms; integrates the particle swarm optimization (PSO) with the characteristics of global optimization and rapid convergence and fuzzy clustering (FC) algorithm with fuzzy clustering effects starting from the perspective of particle swarm and fuzzy membership restrictions and gets a PSO-FC image segmentation algorithm so as to effectively avoid being trapped into the local optimum and improve the stability and reliability of clustering algorithm. The experimental results show that this new PSO-FC algorithm has excellent image segmentation effects.

Keywords: image segmentation, fuzzy clustering, particle swarm optimization

1. Introduction
Image segmentation is not only one of the study hotspots of image processing and computer vision, but it is also the significant foundation of image recognition. The purpose of image segmentation is to segment the objective region people need from the background [1]. As a common data analysis tool, clustering analysis is the process to divide data sets into enormous clusters constituted by similar data. Clustering analysis has been widely used in image segmentation, data mining, pattern classification, medical diagnosis and machine learning [2]. Among numerous image segmentation algorithms, the image segmentation algorithm based on clustering analysis is an extremely important and widely used algorithm in image segmentation. The traditional clustering algorithm is a kind of deterministic clustering algorithm. One of the many samples belongs to a certain class to a certain extent and it can also belong to another class or other classes at the same time due to the uncertainty and fuzziness of its class, therefore, it is difficult to divide it into the only class exactly. The most generally-used algorithm is fuzzy clustering algorithm and it has excellent convergence as an unsupervised clustering algorithm [3].

Fuzzy set is suitable to process the relevant problems to uncertainty and fuzziness and it has been extensively applied. Fuzzy clustering comes into being by integrating fuzzy clustering and the concept of fuzziness. Fuzzy clustering makes it possible for the clustering samples to belong to multiple classes and it uses membership to refer to the size of the degree of membership. As a widely used method, FC algorithm has been successfully applied in image analysis, medical diagnosis, target identification and image segmentation. FC algorithm performs fuzzy clustering on the consistent pixels in the image through membership matrix and segments the image through iterative optimization of the objective function. However, FC algorithm also has many shortcomings [4]. For instance, it is greatly affected by noise. It is very sensitive to the initial value and it depends greatly on the selection of the initial clustering center. When the initial clustering center severely deviates from the global optimal clustering center, it may cause the algorithm trapped in local optimum, especially in the case of numerous clustering samples. Therefore, this kind of problems can be solved by improving the membership function in FC algorithm and introducing PSO with strong global optimization ability [5].

Compared with FC algorithm, PSO can seek the global optimal solution at a short time; it allows selecting initial value randomly and it can obtain the optimal solution; therefore, it can greatly reduce the pre-phase work [6]. By using such characteristics as ergodicity and
randomness of PSO and integrating the optimization characteristic of particle swarm, this paper proposes a PSO-FC algorithm, namely the fuzzy clustering algorithm based on particle swarm optimization with global optimization ability by integrating PSO algorithm and FC algorithm. PSO-FC can not only search the optimal solution within the global range, but it can also exert the accuracy of the local optimization ability of FC algorithm and obtain the global optimal solution. This paper firstly introduces the application analysis of image segmentation. Then it discusses the fundamental theories of fuzzy clustering and particle swarm algorithm. Next, it proposes the image segmentation algorithm based on particle swarm and fuzzy clustering. Finally, it is the experimental analysis.

2. Image Segmentation

So far, it is very difficult to find an effective common image segmentation method to make various images reach the optimal segmentation quality. Many image segmentation algorithms are meant for certain type of image or certain specific segmentation. From the type of image, image segmentation includes gray image segmentation, color image segmentation and texture image segmentation. According to the definition of image segmentation, the image segmentation algorithms are divided into two types: one is based on region and it uses the regional similarity, namely assuming that the neighborhood pixels in the same region have similar characteristics such as gray, color or texture while the other is based on boundary and it uses the discontinuity between regions [7].

In the image study and application, people name the part they are interested in objective or foreground and the rest background. Image segmentation is to express the image into the sets meeting uniformity, connectivity and consistency, namely to mark and position the objective and background in the image and then segment the objective from the background or other pseudo-objectives according to the prior knowledge of objective and background. The regional uniformity refers to certain similarity criterion based on gray, texture, color and other characteristics that all pixels in this region meet. Connectivity means that this region has the path to link any two points. And consistency is the topology accuracy of the segmentation shape in medical image segmentation. In other words, it needs to consider not only the local property, but also the connectivity of the global geometrical shape. People have come up with different descriptions on image segmentation and they have given image segmentation the following definition with the concept of set: use set \( R \) to refer to the entire image region; divide \( R \) into \( C \) non-empty sub-regions (sub-sets), namely \( R_1, R_2, \ldots, R_c \) and meet the following five conditions:

\[
\begin{align*}
(1) & \quad \bigcup_{i=1}^{c} R_i = R . \\
(2) & \quad \text{If } i = 1, 2, \ldots, C, R_i \text{ are connected regions.} \\
(3) & \quad \text{To all } i \text{ and } j, i \neq j . \ R_i \cap R_j = \phi . \\
(4) & \quad \text{If } i = 1, 2, \ldots, C , \text{ then } P(R_i) = \text{True} . \\
(5) & \quad \text{If } i \neq j , \text{ then } P(R_i \cup R_j) = \text{False} .
\end{align*}
\]

Among these conditions, \( P(R_i) \) is the logic predicate to the elements in set \( R_i \). Condition(1) requires that the union set of all sub-region in the segmentation result shall include all the pixels of the original image; Condition(2) requires that the pixels of the same sub-region in the segmentation result are connected; Condition(3) requires that any pixel of the original image can’t belong to two different sub-regions; Condition(4) requires that the pixels which belong to the same region have certain same characteristics and Condition(5) requires that the different sub-regions in the segmentation result have different characteristics [8].

In one word, image segmentation is to segment the regions with certain similar properties into the same region and segment those with different properties into different regions. An effective segmentation shall preserve the important characteristics of the image in a possibly short time and get the consistent outline or complete area of the objective in the image [9].
3. Fundamental Theories of Fuzzy Clustering and Particle Swarm Optimization

3.1. Fundamental Principle of Fuzzy Clustering (FC)

FC algorithm is a method to automatically classify the samples. It determines the membership of every sample to the class center by minimizing the objective function so as to determine the class of the sample. FC divides the set \( Z \) in a fuzzy manner and determines the degree every data point belongs to every class with the membership value between 0 and 1. In line with the introduction of fuzzy division, membership matrix allows the circumstances with the value of 0 and 1; however, one membership unification restriction shall be added, that is, the sum of the memberships that one data point belongs to different classes shall be 1.

\[
\sum_{j=1}^{n} u_{ij} = 1, \forall j = 1, \ldots, n
\]  

(1)

Then, the objective function of FC is as follows:

\[
J_m(U, P) = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{ij}^m d_{ij}^2
\]  

(2)

In this formula, \( u_{ij} \) can be any real number between 0 and 1 and \( m \in [1, \infty) \) is a weighted index called fuzziness index, fuzzy control parameter and weighted parameter [10].

Construct new objective function through lagrangian multiplier and according to Formula (1) and (2) and get the necessary conditions to make Formula (2) get the minimum value [11].

\[
J(U, P, \lambda_1, \ldots, \lambda_n) = J(U, P) + \sum_{j=1}^{n} \lambda_j \left( \sum_{i=1}^{n} u_{ij} - 1 \right) = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{ij}^m d_{ij}^2 + \sum_{j=1}^{n} \lambda_j \left( \sum_{i=1}^{n} u_{ij} - 1 \right)
\]  

(3)

In this formula, \( \lambda_j, j = 1, \ldots, n \), is the lagrangian multiplier of \( n \) constraint formulas in Formula (1). It can be seen from lagrangian multiplier that the solutions of Formula (1), (2) and (3) are equivalent. Seek the partial derivatives of all input parameters and make the result 0 and get the necessary conditions to make Formula (2) get the minimum by integrating constraint condition Formula (1).

\[
c_i = \frac{\sum_{j=1}^{n} u_{ij}^m x_j}{\sum_{j=1}^{n} u_{ij}^m}
\]  

(4)

and

\[
u_{ij}^m = \frac{1}{\sum_{j=1}^{n} \left( \frac{d_{ij}}{d_{ij}} \right)^{2(m-1)}}
\]  

(5)

3.2. Basic FC Flow

The core of FC algorithm is the estimations of the clustering center matrix \( P = [c_i]_{n \times d} \) and fuzzy membership matrix \( U = [u_{ij}]_{n \times m} \). The estimation of FC parameters can be determined by Formula (4) and (5) and they are mutually integrated; therefore, perform estimation solution through alternative iterative algorithms [12].

Step 1: Initialize the parameters of the clustering center matrix. As for the given number of clustering classes \( c (2 \leq c \leq n) \), \( n \) is the sample size. Assuming that the iterative stopping
threshold is $\varepsilon$, which ranges from 0.001 to 0.01. Set the maximum iterations as and initialize the clustering center matrix $P^{(0)}$ with $t = 0$.

Step 2: Update the fuzzy division matrix. Calculate the distances $d_{ij}^{(t)}$ from every sample $x_j$ to the clustering center $c_i$ through the clustering center matrix $P^{(t)}$, update the fuzzy division matrix $U^{(t)}$ according to Formula (5) and get

$$u_{ij}^{(t)} = \frac{1}{\sum_{l=1}^{c} \left( \frac{d_{ij}^{(t)}}{d_{lk}^{(t)}} \right)^{2(\kappa-1)}}$$

(6)

Step 3: Update the clustering center matrix, update the clustering center matrix $P^{(t+1)}$ according to Formula (4) and get:

$$c_i^{(t+1)} = \frac{\sum_{j=1}^{n} \left[ u_{ij}^{(t)} \right]^{\kappa} x_j}{\sum_{j=1}^{n} \left[ u_{ij}^{(t)} \right]^{\kappa}}$$

(7)

Step 4: Judge the threshold. For the given threshold $\varepsilon$, if $\left\| P^{(t+1)} - P^{(t)} \right\| \leq \varepsilon$, or the number of iterations is larger than or equal to the maximum iterative counts $(t \geq t^*)$, stop the iteration; otherwise, make $t = t + 1$ and turn to Step 2.

When a certain sample $x_j$ overlaps with a certain clustering center $c_i$, $d_{ij} = \left\| x_j - c_i \right\| = 0$.

In order to avoid $d_{ij} = 0$; specify that when $d_{ij} = 0$, the membership that this sample belongs to this class is 1 and the membership it belongs to other classes is 0. When the iteration process stops, get the fuzzy clustering center and the estimation value of the parameters of fuzzy division matrix. The judgment evidence FC algorithm determines the clustering class of sample $x_j$ is

$$k = \arg \max_{i=1,\ldots,c} u_{ij}$$

(8)

As for the above iterative algorithms, pre-initialize the fuzzy division matrix and then implement the iteration process [13],[14].

3.3. Fundamental Principle of PSO

The fundamental principle of PSO comes by stimulating group feeding. They perform simple behaviors on account of the following three rules: (1) movement towards the nearest companion; (2) movement towards the destination; (3) movement towards the group center [15].

Assume that $X_i = (x_{i1}, x_{i2}, \ldots, x_{in})$, $V_i = (v_{i1}, v_{i2}, \ldots, v_{in})$, $P_i = (p_{i1}, p_{i2}, \ldots, p_{in})$ are the current position, flying speed and global optimal position of particle respectively. In order to make the particle to find food at the shortest time, it needs to find a solution in the question domain to minimize the value of the objective function, namely that the objective function $F(x)$ is the biggest and then the current optimal position of the $i$th particle can be determined by Formula (9).

$$p_i(t+1) = \begin{cases} p_i(t) & \text{if } F(x_i(t+1)) \geq F(p_i(t)) \\ x_i(t+1) & \text{if } F(x_i(t+1)) < F(p_i(t)) \end{cases}$$

(9)
Assume that the number of individuals in the group is \( S \) and the optimal position all individuals can find is \( P^* \), as indicated by Formula (10):

\[
P^*_g(t) \in \{ p_g(t), p_1(t), \ldots, p_N(t) \} \mid f(P^*_g(t)) = \min \{ f(p_g(t)), f(p_1(t)), \ldots, f(p_N(t)) \}
\]

(10)

According to the definition of PSO, the speed and position update of every particle can be described by Formula (11) and (12):

\[
v_j(t+1) = v_j(t) + c_1 r_{ji}(t) (p_j(t) - x_j(t)) + c_2 r_{gi}(t) (P^*_g(t) - x_j(t))
\]

(11)

\[
x_j(t+1) = x_j(t) + v_j(t+1)
\]

(12)

In these formulas: the subscript "i" means particle \( i \), "j" refers to the \( j \)th dimension of the particle; \( t \) is the number of iterations where the group is located; \( c_1 \) and \( c_2 \) are acceleration constants, which are within \([0,2]\) and \( c_1 = c_2 = 1 \) and \( r_1 \) and \( r_2 \) are two random numbers within \([0,1]\).

It can be seen by analyzing Formula (11) and (12) that \( c_1 \) and \( c_2 \) are the parameters to be used to adjust the position of the particle and the global optimal position respectively. In the meanwhile, in order to avoid the particle from deviating from the search space, it needs to constrain \( v_j \) and \( x_j \) in the group evolution, namely \( v_j \in [-v_{max}, v_{max}] \) and \( x_j \in [-x_{max}, x_{max}] \).

The particle speed update in Formula (11) is divided into three parts: the first part is its current speed; the second part is its own experience and the third part is the social recognition. If the particle updates its speed only through its own experience, as indicated in Formula (13).

\[
v_j(t+1) = v_j(t) + c_1 r_{ji}(t) (p_j(t) - x_j(t))
\]

(13)

In this way, the performance of PSO will be very bad because without exchange of group information among different particles, it will lead to the independent movement of \( N \) particles in a group and it is easy to be trapped into local optimum. If only social recognition is added, see Formula (14):

\[
v_j(t+1) = v_j(t) + c_2 r_{gi}(t) (P^*_g(t) - x_j(t))
\]

(14)

Then the movement of the particle will become a social behavior with self-recognition. Although it can expand the search range of the particle and improve the convergence speed, the algorithm will be easy to be trapped into local optimum [16],[17].

The following is the fundamental principle of particle movement in PSO, as indicated in Figure 1.
The general process of PSO is indicated in Figure 2 [18].

![Figure 2. Basic flowchart of PSO](image)

### 4. Image Segmentation Algorithm Based on Particle Swarm and Fuzzy Clustering

#### 4.1. Theoretical Idea of PSO-FC Algorithm

When the standard FC algorithm segments the gray image, it divides the classes of pixels according to the gray characteristics of image and continuously updates the membership matrix and clustering center until the algorithm convergence through gradient descent so as to realize the segmentation of gray image. However, in daily life, the pixels points of a common image are enormous. For example, a common 256×256 gray image has 65,536 pixels points and the pixels points to be classified are 65,536 according to the standard FC algorithm. For the standard FC algorithm to divide according to pixel points, the number of iterations and the time it takes are undoubtedly very huge. Therefore, the standard FC algorithm is not suitable for real-time application [19].

#### 4.2. Flow of PSO-FC

Step 1: Read in the original image to be segmented. Assume that the number of clustering is \( c \), the population size is \( N \), the learning factors are \( c_1 \) and \( c_2 \), the inertia weight factor \( \omega = \omega_{\text{max}} - \frac{\omega_{\text{max}} - \omega_{\text{min}}}{\text{iter}_{\text{max}}} \times t \); the maximum inertia weight factor is \( \omega_{\text{max}} \) and the minimum weighted factor is \( \omega_{\text{min}} \); the maximum iterations \( \text{iter}_{\text{max}} \) and the maximum speed is \( v_{\text{max}} \) [20].

Step 2: Initialize the particle warm. Randomly generate \( z_{i1}, z_{i2}, \ldots, z_{ik} \). The number of dimensions is \( D \) dimension and the vector \( z_i = (z_{i1}, z_{i2}, \ldots, z_{ik}) \) with every component value is within 0–1. Take the valuation range of every component carrier and particle swarm of \( z \) as \( x_{i,j} = a + (b - a) z_{i,j}, j = 1, 2, \ldots, D, i = 1, 2, \ldots, N \), and calculate the objective function[21],[22].

Step 3: Evaluate the fitness of every particle and update the individual extremum \( p_i \) and the global extremum \( p_g \) according to Formula (9) and (10).

Step 4: Randomly generate a \( D \)-dimensional vector \( s_0 = (s_{0,1}, s_{0,2}, \ldots, s_{0,D}) \) with every component value within 0–1.

Step 5: Update its own speed according to Formula (11) in PSO and restrict it within \( [v_{\text{min}}, v_{\text{max}}] \).
Step 6: According to Formula (5) and (6), update the membership matrix \( \mu_{ij} \) and clustering center \( Z_{i}^{(t)} \). In the meanwhile, update the speed and position of every particle and generate the next-generation particle swarm [23].

Step 7: Judge whether it has reached the iteration termination condition. If the iteration terminates, jump out from circulation and output the global optimal solution \( p_{g} \), namely the clustering center and output the segmented image, otherwise, turn to Step 3.

5. Experimental Result & Analysis

In order to verify the effectiveness of the algorithm proposed in this paper, compare it with fuzzy clustering algorithm and multi-threshold Otsu image segmentation method. The algorithm of this paper doesn’t need to determine the number of classes in advance and it can self-adaptively divide into several classes in the clustering process. Comparative experiments need to determine the number of classes in advance and the number of classes are determined according to the number of classes of the algorithm in this paper.

Simulate this experimental result and data in the environment of Windows 7 with Intel (R) Core2CPU 1.33G and a memory of 2.5G through Matlab R2012a. Set the algorithm parameters as follows: the size of the particle swarm is 100; the maximum iterations is 200; the learning factors are: \( c_1 = 1.9 \) and \( c_2 = 1.8 \), the number of clusters \( c = 3 \), the maximum inertia weight factor is \( \omega_{\text{max}} = 0.9 \), the minimum inertia weight factor is \( \omega_{\text{min}} = 0.1 \), and the fuzzy index is \( m = 3 \).

Figure 3 is the segmentation effects by different algorithms.

![Figure 3](image-url)

(a) Original image                         (b) Fuzzy clustering algorithm

(c) Multi-threshold Otsu method           (d) Algorithm of this paper

Figure 3. Segmentation results by different algorithms

Figure 3 (a) is the original image to be processed while Figure 3 (b), (c) and (d) are the results to segment (a) by using fuzzy clustering algorithm, multi-threshold Otsu algorithm and
PSO-FC segmentation algorithm. It can be seen through comparison that with determined number of classes, these three segmentation methods can segment the tree in the middle of the image; however, as for the trees and plants in the background, the segmentation effect by fuzzy clustering algorithm is the worst since the original tree can’t be identified. Next comes multi-threshold Otsu method, which can segment some trees. The algorithm of this paper has the best segmentation effect and it can segment every objective completely. To sum up, the improved algorithm proposed in this paper has better image segmentation effects.

6. Conclusion
Since image has many uncertainties and inaccuracy, people find that fuzzy theory has excellent description ability on such uncertainties. Image segmentation is to classify the image pixels. Apply fuzzy clustering into image segmentation to get better effects than the traditional image processing methods. However FC algorithm has bad anti-noise ability or robustness and it is easy to be converged to local minimum. Therefore, this paper integrates PSO and FC and applies it into image segmentation to get excellent segmentation effects through comparative study of the experimental results.

Acknowledgments
This work was supported by the Science and technology research projects in Henan province department of education (grant No.12A520038).

References


