

Instituut voor Cultuurtechniek en Waterhuishouding  
Wageningen

INFILTRATION FROM SOME DEFINED SOURCE  
INTO SOILS WITH DEEP GROUNDWATER TABLE

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CENTRALE LANDBOUWCATALOGUS

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## 1. INTRODUCTION

When designing infiltration works for the sake of filtering drinking water or waste water the area to be reserved has to be known. This area depends on the infiltration rate and the quantity to be filtered. If no clogging of pores occur the dimensions of the works can be calculated in some analytical way (POLUBARINOVA, 1962). Even in the case of inhibited infiltration the analytical solutions of the problem can be used if the bottom width of the source is relatively large with respect to the waterdepth. In this case the inhibition can be translated into an apparent lower waterdepth. The condition has to be met however that the soil under the source is saturated. If not so numerical solutions have to be applied, as only analytical solutions for the infiltration into a complete unsaturated soil can be used under condition the source having only one dimension (RAATS, 1970). Numerical solutions have to be applied if one is interested in the pressure head distribution, rejecting the classic boundary condition that the velocity, perpendicular to the 'free surface' is zero. This implies that there is a flow both into the saturated and into the unsaturated zone. Knowledge of the flow distribution over the saturated and unsaturated zone, completed with the distribution of the air content in the unsaturated zone, which is deducted from the pressure head distribution, can be of great importance when predicting the purification rate by oxidation (BAKKER en WIND, 1974).

Using such numerical solutions, the relation between the hydraulic conductivity and the pressure head has to be described. The solution is simplified introducing an exponential relationship. In the case study this relationship is obtained from field measurements (BOELS, 1974).

The described computer program has been developed for calculating the steady state infiltration rate from a large source, lying in a region with a very deep groundwater table.

## 2. THE FORMULATION OF THE PROBLEM

### 2.1. The basic equation

From some source waterflows under steady state conditions into an anisotropic soil where the groundwater table is very deep. In the flow region a saturated as well as an unsaturated flow may occur.

In each point in the flow region the continuity law has to be fulfilled:

$$\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Darcy's law is assumed to be valid as well in the saturated as in the unsaturated zone:

$$V = -K \cdot \nabla \phi \quad (2)$$

The hydraulic head  $\phi$  exists of a component due to the pressure head in the soil,  $h$ , and another due to gravity,  $y$ , both expressed in m watercolumn:

$$\phi = h + y \quad (3)$$

In the unsaturated region the hydraulic conductivity is a function of the pressure head. According to RIJTEMA (1965) an exponential relationship between  $K$  and  $h$  is assumed:

$$K(h) = K(o)e^{\alpha(h-e)} \quad (4)$$

The soil is anisotropic in the  $x$ - and  $y$ -direction, so

$$K(o,x) = \beta K(o,y) \quad (5)$$

The value of  $\alpha$  (eq. (4)) is supposed to be the same in the vertical and in the horizontal direction.

In the transition zone the flow becomes unsaturated. RAATS (1974,b), assumes that the hydraulic head,  $\phi$ , is continuous in the transition zone, so

$$\phi^+ = \phi^- \quad (6)$$

and based on field measurements of BOELS (1974)

$$\nabla \phi^+ = \nabla \phi^- \quad (7)$$

From eq. (2), (4) and (7) it can be concluded that the hydraulic conductivity in the saturated zone equals  $K(o)$  in the unsaturated zone. Substituting eq. (3) in eq. (2), and the new equation in eq. (1) gives:

$$K(y) \cdot \frac{\partial^2 h}{\partial y^2} + K(x) \frac{\partial^2 h}{\partial x^2} + \frac{dK(y)}{dy} \left( \frac{\partial h}{\partial y} + 1 \right) + \frac{\partial h}{\partial x} \cdot \frac{dK(x)}{dx} = 0 \quad (8)$$

$$\text{In the saturated zone } \frac{dK(x)}{dx} = \frac{dK(y)}{dy} = 0$$

Inserting eq. (5) into eq. (8) and dividing by  $K(y)$ , the differential equation in the saturated zone reads

$$\frac{\partial^2 h}{\partial y^2} + \beta \frac{\partial^2 h}{\partial x^2} = 0 \quad (9)$$

Inserting eq. (5) and (4) into eq. (8) the following differential equation in the unsaturated zone is obtained:

$$\frac{\partial^2 h}{\partial y^2} + \beta \frac{\partial^2 h}{\partial x^2} + \alpha \frac{\partial h}{\partial y} \left( \frac{\partial h}{\partial y} + 1 \right) + \beta \frac{\partial h}{\partial x} \cdot \alpha \frac{\partial h}{\partial x} = 0 \quad (10)$$

## 2.2. The boundary conditions

In  $x = 0$  a symmetric axis or plane exists, so that the derivative of  $h$  with respect to the  $x$ -coordinate equals 0.

If at an arbitrary distance from the center of the source the flow region is supposed to be bounded, say at a distance  $L$ , then in  $x = L$ ,  $\frac{\partial h}{\partial x} = 0$ . If a constant rainfall,  $N \text{ m day}^{-1}$ , is assumed, the vertical velocity perpendicular to the surface and the velocity in  $x = L$ , equals  $N$ . Along the wet surface of the source the hydraulic head is constant:  $\phi_s$ .

Below  $y = 0$  an aquifer is assumed in which  $\frac{\partial h}{\partial x} = -1$  (only a flow in horizontal direction). In (L.o) the pumping plant withdraws per time period the same amount of water as flows from the source into the soil. As a consequence the watertable in (L.o) is held constant. Summarized the boundary conditions areas follows (see fig. 1).

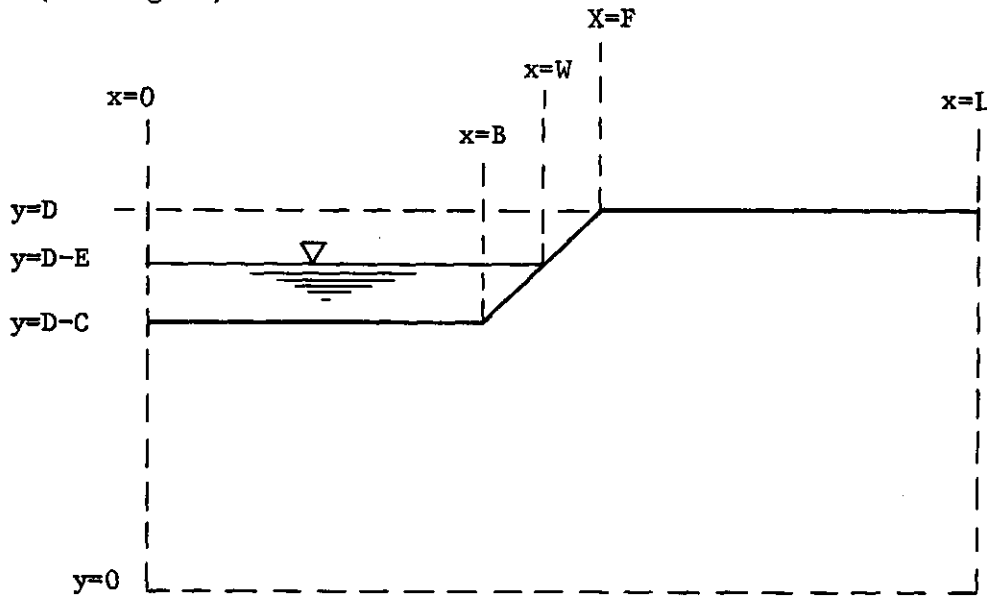


Fig. 1. Crosssection, belonging to the definition of the boundary conditions

$$\frac{\partial h}{\partial x} = \begin{cases} 0 & , x=0 \text{ or } x=L \\ -1 & y=0 \end{cases} \quad (11)$$

$$V_y = N \begin{cases} y = (D-E) + (x-w) \operatorname{tg}\alpha, & W < x < F \\ y = D & F < x < L \\ x = L & 0 < y < D \end{cases} \quad (12)$$

$$h = 0 \quad y = D \quad x = L \quad (13)$$

$$\phi = \text{constant} \begin{cases} y = (D-C) + (x-B) \operatorname{tg}\alpha, & B < x < W \\ y = (D-C) & 0 < x < B \end{cases} \quad (14)$$

It is possible that the hydraulic conductivity just below the source decreases due to the deposition of impurities. The thickness of the layer in which this occurs will not exceed 0,1 m however (BOELS, 1974, REYMERINK, 1975). To deal with this phenomenon a pressure head just below the source may be introduced. The conductivity in this layer is calculated after the infiltration rate has been determined.

### 2.3. The numerical solution of the difference equation

As it is impossible to solve the difference equations (9) and (10) in an analytical way a numerical solution has to be found.

Using the TAYLOR's series expansion for the solution of a function at a certain point, neglecting the derivatives of the higher order, the first derivative in a point,  $a$ , using the forward differentiation, can be written as

$$f'(a) \approx \frac{f(a+\Delta a) - f(a)}{\Delta a} \quad (15)$$

and the second derivative as

$$f''(a) \approx \frac{f(a+\Delta a) + f(a-\Delta a) - 2f(a)}{\Delta a^2} \quad (16)$$

In these equations  $\Delta a$  is a small increment of  $a$ .

With the aid of the eq. (15) and (16) it is possible to write the pressure head at a certain point as a function of the pressure head in the surrounding points. The following equations are substituted in eqs. (9) and (10):

$$\frac{\partial^2 h}{\partial y^2} \approx \frac{1}{\Delta y^2} \{h(x, y+\Delta y) + h(x, y-\Delta y) - 2 h(x, y)\} \quad (17)$$

$$\frac{\partial^2 h}{\partial x^2} \approx \frac{1}{\Delta x^2} \{h(x+\Delta x, y) + h(x-\Delta x, y) - 2 h(x, y)\} \quad (18)$$

$$\begin{aligned} \frac{\partial h}{\partial x} &\approx \frac{1}{\Delta x} \{h(x+\Delta x, y) - h(x, y)\} \approx \frac{1}{\Delta x} \{h(x, y) - h(x-\Delta x, y)\} \\ &\approx \frac{1}{2\Delta x} \{h(x+\Delta x, y) - h(x-\Delta x, y)\} \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{\partial h}{\partial y} &\approx \frac{1}{\Delta y} \{h(x, y+\Delta y) - h(x, y)\} \approx \frac{1}{\Delta y} \{h(x, y) - h(x, y-\Delta y)\} \\ &\approx \frac{1}{2\Delta y} \{h(x, y+\Delta y) - h(x, y-\Delta y)\} \end{aligned} \quad (20)$$

To approximate the first derivatives in the best way, a centered difference quotient has been used (FORSYTHE and WASOW, 1960). After substituting eqs. (17), (18), (19) and (20) into eq. (10),  $h(x, y)$  is found to be a function of the pressure head in the surrounding gridpoints. In the saturated zone, when  $\alpha=0$ , is:

$$\begin{aligned} h(x, y) &= \frac{\Delta x^2}{\Delta x^2 + \beta \Delta y^2} \left\{ \frac{1}{2} (h(x, y+\Delta y) + h(x, y-\Delta y)) \right\} + \\ &\frac{\Delta y^2}{\Delta x^2 + \beta \Delta y^2} \left\{ \frac{\beta}{2} (h(x+\Delta x, y) + h(x-\Delta x, y)) \right\} \end{aligned} \quad (21)$$

The approximation of the first and second derivative with eq. 17 through 20, is not accurate enough in the unsaturated zone.



If between two planes the horizontal flow is relatively small, the mean vertical velocity between these planes can be calculated by integrating

$$\bar{v} = -K\left(\frac{\partial h}{\partial y} + 1\right) \quad (22)$$

for  $\bar{v} = \text{constant}$  when  $y < z < y + \Delta y$ ,

and  $h(z) = h(y)$ ,  $z = y$  and  $h(z) = h(y + \Delta y)$ ,  $z = y + \Delta z$

Using the exponential relationship between  $K$  and  $h$ , according to eq. (4) gives

$$\alpha \int_y^{y+\Delta y} dz = - \int_{k(y)}^{K(y+\Delta y)} \frac{1}{\bar{v} + K(z)} dK \quad (23)$$

so

$$\bar{v} = \frac{K(y+\Delta y) - K(y) \exp(-\alpha \Delta y)}{\exp(-\alpha \Delta y) - 1} \quad (24)$$

Combining eq. (22) and (24) and solving the first derivative in  $(y + \Delta y)$ :

$$\left(\frac{\partial h}{\partial y}\right)_{y+\Delta y} = \frac{1}{\exp(\alpha \Delta y) - 1} \left[ 1 - \exp\{\alpha(h(y) - h(y + \Delta y))\} \right] \quad (25)$$

The discrepancy between the first derivative, approximated by the TAYLOR's expansion series and the one obtained by applying eq. 25 is made clear in fig. 2.

From fig. 2 it can be concluded that the approximation according to the TAYLOR's expansion series is fairly good if  $\{h(y) - h(y + \Delta y)\}$  is very small.

Abundant errors are made in the transition zone using eq. (20) as an approximation of the first derivative. For this reason a KIRCHHOFF type equation will be used in the unsaturated zone.

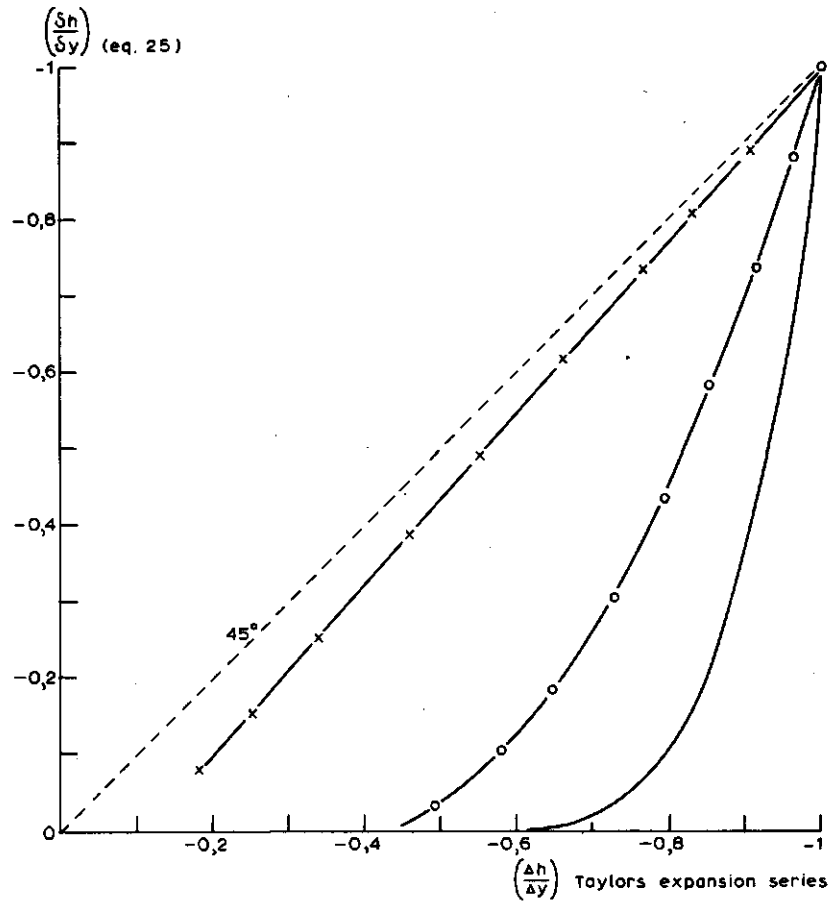


Fig. 2. Discrepancy between the first derivative using TAYLOR's expansion series

and eq. 25 for  $\alpha = 30 \text{ m}^{-1}$  and

—  $h(y) - h(y + \Delta y) = 0.30 \text{ m}$   
 ○ —  $h(y) - h(y + \Delta y) = 0.10 \text{ m}$   
 × —  $h(y) - h(y + \Delta y) = 0.01 \text{ m}$

The flow area can be considered as a resistance network, where the KIRCHHOFF law is valid in the gridpoints

$$\bar{v}_{y,i} \cdot \Delta X - \bar{v}_{x,i} \cdot \Delta Y = \bar{v}_{y,0} \cdot \Delta X - \bar{v}_{x,0} \cdot \Delta Y \quad (26)$$

Here is

$$\bar{v}_{y,i} = \frac{K(y + \Delta y) - K(y) \cdot \exp(-\alpha \Delta y)}{\exp(-\alpha \Delta y) - 1} \quad (27)$$

$$\frac{v}{v_{y0}} = \frac{K(y) - K(y-\Delta z) \exp(-\alpha\Delta z)}{\exp(-\alpha\Delta z) - 1} \quad (28)$$

$$\frac{v}{v_{xi}} = \frac{K(x-\Delta x) - K(x)}{\alpha\Delta x} \quad (29)$$

$$\frac{v}{v_{x0}} = \frac{K(x) - K(x+\Delta s)}{\alpha\Delta s} \quad (30)$$

Eq. 29 and 30 are got by integrating

$$v = -K \frac{\partial h}{\partial x} \quad (31)$$

for  $v = \text{constant}$ ,  $K = K(x)$ ,  $x = X$  and  $K = K(n+\Delta n)$ ,  $n = X + \Delta x$

Substituting eq. (5), 27, 28, 29 and 30 into eq. (26) and deviding the result by  $K(0.4) \exp(-\alpha e)$  gives:

$$\begin{aligned} \exp \alpha h(x,y) &= \left[ \frac{\beta}{\alpha} \frac{\Delta Y}{\Delta s} + \frac{\beta}{\alpha} \frac{\Delta Y}{\Delta x} + \frac{\Delta X \cdot \exp(-\alpha\Delta z)}{1-\exp(-\alpha\Delta z)} + \frac{\Delta X}{1-\exp(-\alpha\Delta y)} \right]^{-1} \\ &\left[ \frac{\Delta Y}{\Delta x} \cdot \frac{\beta}{\alpha} \cdot \exp \alpha h(x-\Delta x, y) + \frac{\Delta Y}{\Delta s} \cdot \frac{\beta}{\alpha} \cdot \exp \{ \alpha h(x+\Delta s, y) \} + \right. \\ &\left. \frac{\Delta X \exp(-\alpha\Delta z)}{1-\exp(-\alpha\Delta z)} \cdot \exp \{ \alpha h(x, y-\Delta z) \} + \frac{\Delta X}{1-\exp(-\alpha\Delta y)} \cdot \exp \alpha h(x, y+\Delta y) \right] \quad (32) \end{aligned}$$

Where:  $\Delta z < \Delta Y$        $\Delta x < \Delta X$   
 $\Delta y < \Delta Y$        $\Delta s < \Delta X$

These restrictions have been made to account for the situation in which one or more nodes except that one in  $(x,y)$  ly in the saturated zone.

The following situations can be distinguished

a)  $(x-\Delta X, y)$  lies in the saturated zone.

If the flow direction between two nodes in the same horizontal plane is assumed to be only horizontal, the flow velocity in the saturated zone equals the velocity in the unsaturated zone.

$$V_{\text{sat}} = -K_{o,x} \frac{\partial h}{\partial x} = V_{\text{unsat}} = \frac{K(x-\Delta x, y) - K(x, y)}{\alpha \Delta x} \quad (33)$$

as  $K = K_{o,x} \cdot h > e$  and  $\frac{\partial h}{\partial n} = e - \frac{h(x-\Delta X, y)}{\Delta X - \Delta x}$

in the saturated zone, from eq. (33) can be deduced that:

$$\Delta x = \frac{1 - \exp \alpha(h(x, y) - e)}{1 + \alpha(h(x-\Delta X, y) - e) - \exp \alpha(h(x, y) - e)} \Delta X \quad (34)$$

In the case  $(x-\Delta X, y)$  lies in the saturated zone, the pressure head in  $(x-\Delta x, y)$  equals  $e$  and the distance  $\Delta x$  is calculated with eq. (34).

The same expression can be used if  $(x+\Delta X, y)$  lies in the saturated zone.

b)  $(x, y-\Delta Y)$  lies in the saturated zone.

Neglecting the change of the flow velocity between  $(x, y)$  and  $(x, y-\Delta Y)$  we have:

$$V_{\text{sat}} = -K \left( \frac{\partial h}{\partial y} + 1 \right) = V_{\text{unsat}} \quad (35)$$

In the saturated zone  $\frac{\partial h}{\partial y} = \frac{e - h(x, y-\Delta Y)}{\Delta Y - \Delta z}$

Substituting eq. (24) the following equation is obtained:

$$\frac{e - h(x, y-\Delta Y)}{\Delta Y - \Delta z} + 1 = \frac{\exp -\alpha \Delta z - \exp \alpha(h(x, y) - e)}{\exp -\alpha \Delta z - 1} \quad (36)$$

from which  $\Delta z$  can be solved, using an iteration procedure.

As in the saturated zone:

$$\frac{h(x, y-\Delta Y) - e}{\Delta Y - \Delta z} = \frac{\bar{v}}{K_o} + 1 \quad (37)$$

it can be shown that if  $\bar{v}$  is very small and  $K_o$  is large  $\frac{\bar{v}}{K_o} + 1 \approx 1$ . In most practical problems the ratio  $\frac{\bar{v}}{K_o}$  in the described situation is small. So for practical purposes, the magnitude of  $\Delta z$  equals  $\Delta Y - (h(x, y-\Delta Y) - e)$ . This is built in into the program.

c)  $(x, y+\Delta Y)$  lies in the saturated zone.

If there is no change in the vertical flow between  $(x, y+\Delta Y)$  and  $(x, y)$ , then in  $(x, y)$  only a saturated situation can exist. However, in the steady state situation there will be some change. For practical reasons the pressure head between these two points is assumed to change linearly with height.

### 3. THE COMPUTER PROGRAM

#### 3.1. General remarks

The program starts by defining two grids: a condensed one, containing part of the bottom, the talud of the source and the adjoining flow region and a coarse one, containing the total flow region. Then an initial pressure head distribution in each gridpoint is calculated, in such way, that in the region  $\{x, y | 0 < x < (BB+HBRON/HTAL); BODD < y < AGRW\}$  the flow is a saturated one and the pressure head is linearly distributed at  $BODD < y < HGRW$ , while  $h=POTBR$  in  $y=BODD$  and  $h=0$  in  $y=HGRW$ .

In the region  $\{x, y | (BB+HBRO/HTAL) < x < L, 0 < y < HGRW\}$  the flow is unsaturated and the vertical velocity equals the mean rainfall intensity (NEERSL). The pressure head distribution in this region is deducted from eq. (24):

$$h(x, y) = E + \frac{1}{2} \ln \left\{ \frac{NEERSL}{K_0} \left( \frac{1}{\exp(\alpha y)} - 1 \right) + \frac{1}{\exp(\alpha y)} \right\} \quad (42)$$

Now the iteration procedure starts. First in the coarse grid and then in the fine grid, after adjusting the pressure head in the boundary nodes of this grid to the head in the coarse grid.

The infiltration from the source is calculated and if this differs less than  $0.001 \text{ m}^3/\text{day}$  from the last calculated infiltration, the iteration procedure stops and the pressure head distribution in the coarse and the condensed grid is printed. The infiltration also stops if the number of iterations exceeds a maximum number, defined by NRITER.

### 3.2. T h e i n p u t

The input of the program includes:

- Maximum number of iterations, NRITER
- half the bottom width of the canal, BB
- the depth of the source bottom, BODD
- the water depth in the source, HBRON
- the pressure head just below the bottom, POTBR
- the tangent of the angle between the x-axis and the talud, HTAL
- the width of the approximated flow region, next to the source, SIDZO
- the depth of the original groundwater table, HGRW
- the average rainfall, NEERSL (negative value)
- the hydraulic conductivity in horizontal direction, KOX
- the same in vertical direction, KOY
- the air entry value, ENTRV
- the constant in the RIJTEMA formula, ALFA
- the thickness of the underlying aquifer times its conductivity, KADE
- the width of the meshes in the coarse grid in the horizontal direction, GRIDX
- the same but in the vertical direction, GRIDY
- a relaxation parameter, OMEG with a value between 1 and 2.

The unit length of the variables are given in m and the unit time in days.

GRIDX should be chosen in such a way that  $BB/GRIDX$  is just an integer and  $BODD/HTAL$  is smaller than GRIDX.

The relaxation parameter OMEG must have a value  $1 < OMEG < 2$ .

This parameter can be used to over-estimate the new calculated pressure head in some iteration to accelerate the convergence of the procedure.

### 3.3. T h e f o r m u l a t i o n o f t h e g r i d s

Part of the bottom, the talud and the adjoining region lies in the condensed grid. The width of the meshes of this grid are in the horizontal direction  $STEPX = 0.2 * GRIDX$  and in the vertical direction  $STEPY = 0.1 * GRIDY$ .

If BB is greater than GRIDX, JEBOD is the integer of the quotient  $\text{GRIDX}/\text{STEPX}$ . If BB is smaller than GRIDX, JEBOD is the integer of the quotient  $\text{BB}/\text{STEPX}$ . IEBO is defined as integer of the quotient  $\text{BODD}/\text{STEPX}$ .

The maximum number of grid points in the horizontal direction is:  $\text{JEMAX} = \text{JEBOD} + \text{GRIDX}/\text{STEPX}$ . The maximum number of gridpoints in the vertical direction is  $\text{IEMAX} = \text{IEBO} + \text{GRIDY}/\text{STEPY}$ .

The gridpoints in the condensed grid:  $(\text{IEBO}, \text{JEBOD})$ ,  $(\text{IEBO}, 1)$ ,  $(\text{IEMAX}, 1)$ ,  $(\text{IEBO}, \text{JEMAX})$ ,  $(\text{IEMAX}, \text{JEMAX})$  coincide with the gridpoints in the coarse grid, denoted by  $(\text{IBOD}, \text{JBOD})$ ,  $(\text{IBOD}, \text{JIN})$ ,  $(\text{IBOD} + 1, \text{JIN})$ ,  $(\text{IBOD}, \text{JIT})$ ,  $(\text{IBOD} + 1, \text{JIT})$  respectively.

The maximum number of grid points in the horizontal direction (JMAX), is the integer of the quotient  $(\text{BB} + \text{SIDZO})/\text{GRIDX}$  and the maximum number of grid points in vertical direction is:

$$\text{IMAX} = \text{HGRW}/\text{GRIDY}.$$

The first grid point in each row in the condensed grid is denoted by  $(\text{I}, \text{JERAND}(\text{I}))$  and the first point in the coarse grid by  $(\text{I}, \text{JRAND}(\text{I}))$ .

These points are calculated as well as the horizontal and vertical distance between these points and the talud of the source. The grid point in the condensed grid just above and besides the watertable in the source is denoted by  $(\text{IEW}, \text{JEW})$ .

### 3.4. The iteration procedure

#### 3.4.1. The iteration in the coarse grid

The iteration is identified by  $\text{N}=\text{land}$  starts in  $(1, \text{JRAND}(1))$ . If the pressure head in the node  $(\text{I}, \text{J})$  is less than the air entry value,  $\text{ENTRV}$ , eq. (32) is applied in a way that if some adjoining node lies in the saturated zone the distance between  $(\text{I}, \text{J})$  and the transition zone is calculated as is pointed out in 2.3. Instead of using the pressure head in that node, the pressure head in the transition zone,  $\text{ENTRV}$ , is introduced. Eq. (21) is applied if the pressure head in  $(\text{I}, \text{J})$  equals or is greater than  $\text{ENTRV}$ . A GAUSS-SEIDEL type of iteration procedure is applied.

To account for the boundary condition the coarse grid is provided with an additional row and column. The row is defined along the surface to account for the condition that  $V_y = \text{NEERSL}$ .

The pressure head in this row,  $UHS(j)$  is defined in such a way that the mean velocity between the additional node and the node  $(1,j)$  in the coarse grid equals  $\text{NEERSL}$ . The pressure head  $UHS(j)$  is calculated with eq.(24). In the column  $j=\text{JAMX}$ , it is assumed that the vertical velocity in each grid point equals  $\text{NEERSL}$ . The pressure head distribution in this column remains constant during the iteration procedure.

For reasons of symmetry in  $j=1$  an additional column is defined for  $i>\text{IBOD}$ , where the pressure head  $IHX(i)=U(1,J)$ .

The row  $i=\text{IMAX}$  represents the top of an unconfined aquifer, where only a horizontal flow occurs. To fulfill the condition that in a steady state situation inflow = outflow, the pressure head in  $(\text{IMAX},\text{JMAX}-1)$  is adjusted with

$$U(\text{IMAX},\text{JMAX}-1) = U(\text{IMAX},\text{JMAX}) + (\text{GRIDX}/\text{KADE}) * \text{QINF} \quad (43)$$

Here  $U(\text{IMAX},\text{JMAX})$  represents the pressure head in  $(\text{IMAX},\text{JMAX})$ , while  $\text{QINF}$  is the calculated inflow.

The quantity that flows from  $(\text{IMAX},\text{JMAX}-1)$  into the aquifer is calculated after which  $\text{QINF}$  is diminished with the same amount consequently the pressure head in  $(\text{IMAX},\text{JMAX}-2)$  is calculated with eq. (43).

#### 3.4.2. The iteration in the condensed grid

The iteration in the condensed grid is identified by  $N=0$  and starts by equalizing the pressure head in the nodes of the fine grid to the pressure head in the corresponding nodes of the coarse grid.

The pressure head distribution at the bounds:  $j=1$ ,  $I=\text{IEMAX}$  and  $J=\text{JEMAX}$  are calculated by linear interpolation between the nodes in which the pressure head is known. Along the talud, above the watertable,  $\frac{\partial h}{\partial x} = 0$  and the vertical velocity equals  $\text{NEERSL}$ .



Along the surface an additional row, similar to the one in the coarse grid, is introduced. In the same way as is done in the coarse grid, the pressure head in this additional row,  $EHUS(j)$ , and along the talud is calculated.

After one iteration the pressure head  $THX(I)$  in the coarse grid is adjusted for the region  $I=1, IBOD$ . If there an  $EU(i,j)$  exists that lies on the row  $I$  of the coarse grid at a distance of  $GRIDX$  from  $(I, JRAND(I))$ , then  $THX(I) = EU(i,j)$ . If such an  $EU(i,j)$  does not exist  $THX(I) = EU(i, JERAND(i))$ . N.B.  $i$  and  $I$  ly on the same row.

### 3.4.3. Some remarks on the iteration in the transition zone

In 2.3 is pointed out in which way errors can be avoided when calculating  $h(x,y)$  from  $h(x-\Delta x,y)$ ,  $h(x+\Delta x,y)$ ,  $h(x,y-\Delta y)$  and  $h(x,y+\Delta y)$  if  $(x,y)$  lies in the unsaturated zone and one or more adjoining nodes ly in the saturated zone.

Applying eq. (21) in the situation where  $(x,y)$  lies in the saturated zone and one or more adjoining nodes ly in the unsaturated zone, will result in an overestimating of  $h(x,y)$ , especially if  $(x,y+\Delta y)$  lies in the unsaturated zone.

In this special case the velocity between  $(x,y+\Delta y)$  and  $(x,y)$  can be calculated, using eqs. (24) and (37).

The apparent pressure head in  $(x,y+\Delta y)$  can now be calculated from:

$$h(x,y+\Delta y) \approx h(x,y) - \frac{\Delta y}{K_0} (V+K_0) \quad (44)$$

where  $V$  is the mean flow velocity.

A correct estimation of  $h(x,y)$  can be made, using the calculated apparent pressure head instead of the real pressure head. Eq. (44) has been built into the program.

### 3.5. Some results

The infiltration from a canal lying in a sandy soil has been calculated. The original groundwater table was 40 m below surface. The bottom width is 40 m, the bottom depth 2,5 m below surface and the water depth in the canal 2,0 m.

The saturated conductivity in horizontal direction is 15 m per day and in vertical direction 10 m per day. The unsaturated conductivity is described with  $K=K_0 \exp 30(h+0,08)$  (BOELS, 1974). The pressure head  $h$ , is expressed in m watercolumn. The physical data are derived from infiltration experiments on a sandy soil (Veluwe).

To account for clogging of pores the pressure head just below the bottom of the canal is assumed to be: 2,0; 0,1 -0,08 -0,10 m and - 0,20 m watercolumn. As the thickness of the clogged layer is about 0,1 m, the conductivity of this layer can be calculated from the infiltration rate and the gradient of the pressure head.

Table 1. Calculated infiltration rate and conductivity in the clogged layer, due to the degree of clogging

Pressure head just below bottom, m	Infiltration rate this model m/day		Infiltration rate acc. Polubarinova		Conductivity clogged layer		Clogging degree
	abs.	rel.	abs.	rel.	abs.	rel.	
2,0	11,2	100%	12,4	100%	10	100%	0
0,1	9,6	86	10,0	81	0,48	4,8	0,952
-0,08	9,4	84	-	-	0,46	4,6	0,954
-0,10	5,1	46	-	-	0,456	4,56	0,953
-0,20	0,03	0,27	-	-	0,435	4,35	0,965

The clogging degree is defined as:

$$C = \left(1 - \frac{K_c}{K_o}\right) * 100\%$$

where:

$K_c$  = conductivity in the clogged layer

$K_o$  = original conductivity

From table 1 it can be concluded that a decreasing conductivity in the clogged layer has relative small influence on the infiltration rate as far as the decreased conductivity is more than 0,46% of the original conductivity. The soil below the clogged layer remains saturated. A small incremental decrease in conductivity in the clogged layer results into an unsaturated situation below this layer and a sharp decrease in infiltration rate.

When designing infiltration works it is of great importance to know which clogging degree has to be considered. Probably the bottom of the source can be cleared from time to time, so that the infiltration rate remains at a high level.

In fig. 3a and b the pressure head distribution is drawn in the situation where the pressure head just below the bottom is 0.10 m water column. From this figure it can be concluded, that the zone besides the source where the pressure head is influenced by the infiltration is relatively small. The pressure head -0.387 m is the one that occurs due to rainfall only.

The area next to the canal where the water supply to the ecological system is influenced by the infiltration, is determined as follows: It is assumed the pressure head in the rooting zone equals the wilting point ( $\approx 15$  bar), while the effective rooting depth is 1 m. Furthermore it is assumed that the system is influenced if the flow rate to the rooting zone is more than  $0.005 \text{ m day}^{-1}$ .

From RIJTEMA (1965) it can be derived that a vertical flow of  $0.005 \text{ m day}^{-1}$  will occur if 0.4 m below the rooting zone a pressure head -0.387 m is found. The flow is less than  $0.005 \text{ m day}^{-1}$  if this pressure head is found at a greater depth below the rooting zone.

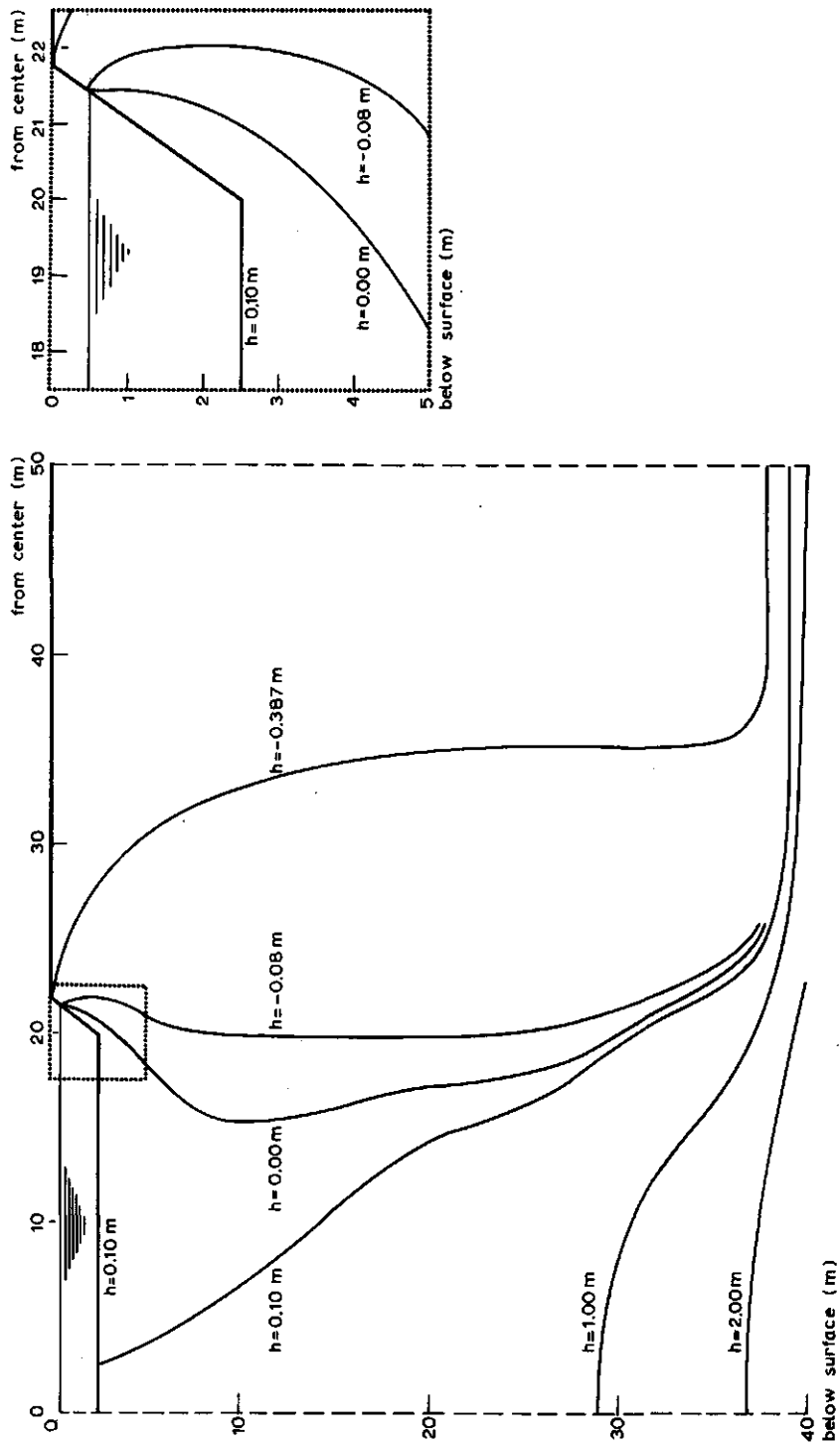


Fig. 3a. Pressure head distribution in a cross section perpendicular to the canal

Fig. 3b. the same, calculated in the condensed grid

From the pressure head distribution it can be seen, that where the equipotential line  $h = -0.387$  m lies more than 1.4 m below surface, the ecological system will not be influenced by the infiltration. So the width of the zone where this system is influenced is about 5 m.

#### 4. SUMMARY

When designing infiltration works the area to be reserved has to be known. This area depends upon the hydrological features of the soil and the occurrence of depositing impurities in the pores, causing an increasing resistance to the flow and resulting in a decreasing infiltration rate. Due to clogging of pores just below the bottom of the source, an unsaturated flow may occur.

As no analytical solutions were available to predict the infiltration rate when the soil in the flow region is partly saturated, a numerical solution has been developed for steady state conditions.

In the saturated region a La Place type difference equation has been applied, whereas in the unsaturated zone a Kirchoff type equation is used.

To deal with an increasing resistance due to clogging of the pores just below the bottom of the source, a pressure head, less than the water depth, just below the source may be introduced.

Calculations were carried out to predict the infiltration rate from a canal, lying in a sandy soil with a groundwater table 40 m below surface.

If no clogging occurs the infiltration rate is  $1.12 K(o)$ , where  $K(o)$  is the saturated vertical conductivity. If the conductivity just below the bottom of the source is  $0.046 K(o)$ , the infiltration rate is  $0.94 K(o)$ . However if the conductivity in this layer is  $0.043 K(o)$ , the infiltration rate is  $0.003 K(o)$ .

The explanation of the phenomenon is that as long as the soil just below the bottom of the canal remains saturated, the infiltration rate is hardly influenced by the pressure head. If the soil becomes

unsaturated, as is the case when the conductivity just below the bottom of the source is less than  $0.046 K(o)$ , the infiltration rate decreases very quickly.

From the pressure head distribution the possible flow rate to the rooting zone can be approximated. If it is assumed that the ecological system will not be distorted if the flow rate to the rooting zone is less than  $0.005 \text{ m day}^{-1}$ , the width of the zone next to the canal where the ecological system is influenced will not exceed 5 m.

Summarized it can be concluded that if the soil remains saturated just below the bottom of the source, the infiltration rate is about the saturated conductivity, while the ecological system is influenced in a zone which has a width of less than 5 m.

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