NOTA 952

Instituut voor Cultuurtechniek en Waterhuishouding
Wageningen

CALCULATION OF CAPILLARY CONDUCTIVITY AND

CAPILLARY RISE FROM GRAIN SIZE DISTRIBUTION

I. REAL AND THEORETICAL VALUES OF THE EXPONENT IN A FORMULA OF

BROOKS AND COREY FOR THE CALCULATION OF HYDRAULIC CONDUCTIVITIES

Ing. G.W. Bloemen

I N H O U D

1. SIMPLE METHODS FOR THE DETERMINATION OF $k - \Psi$ RELATIONS

A simplified procedure for the calculation of hydraulic conductivity, in unsaturated soils was proposed by BROOKS and COREY (1964). By making use of some simplifying assumptions in a generalised Kozeny equation they obtained

$$k = k_s \qquad\qquad \Psi < \Psi_a \tag{1}$$

$$k = k_s (\frac{\Psi_a}{\Psi})^n \qquad \Psi > \Psi_a \tag{2}$$

Here k_s is the experimentally determined permeability of the fully saturated medium, Ψ_a is the air entry point and n is given by

$$n = 2 + 3 \lambda \tag{3}$$

Here λ is a parameter which is related to the pore size distribution of the medium. It is determined from the moisture tension curve as

$$\lambda = \log \frac{S - S_r}{1 - S_r} / \log \frac{\Psi_a}{\Psi} \tag{4}$$

S in eq. (4) is saturation, that is

$$S = \theta/\theta_s \tag{5}$$

where θ and θ_s are respectively any volumetric water content and the maximum volumetric water content. Residual saturation S_r in eq. (4) is the saturation at which the permeability of the soil

becomes essentially zero. S_r is assumed to be equivalent to that part of the pore volume which does not contribute to the flow of water in the soil. S_r is determined as the value which gives the best fit of data points on a straight line when $\log \frac{S - S_r}{1 - S_r}$ is plotted against $\log \Psi$ and saturation values near unity are excluded. The slope of this straight line yields λ according eq. (4). The intersection point of the straight line with the $\log \Psi$ scale is considered the approximate value of $\log \Psi_a$.

CAMPBELL (1974) obtained eq. (2) from a formula to calculate $k - \Psi$ relations from pore size distributions. The exponent n in eq. (2) is defined as

$$n = 2 + 3/b \tag{6}$$

and the parameter b is calculated from that part of the moisture tension curve that produces a straight line when plotted on log scales as

$$b = \log \frac{\Psi_a}{\Psi} / \log \frac{\theta}{\theta_s} \tag{7}$$

From eq. (4) and eq. (7) it follows that $\lambda = \frac{1}{b}$ when S_r is zero. The CAMPBELL method is a simplification of the BROOKS and COREY method because it does not recognize the necessity of the calculation of S_r. Which part of the moisture tension curve is representative for the pore size distribution is decided on arbitrarily. The advantage of the calculation of S_r, however, is that there is a statistical expedient to decide on the best value of the exponent n.

The use of a formula like eq. (2) is attractive because of its simplicity. The formula allows the difference between soils to be expressed in different values of only three parameters. When these are known of a soil the hydraulic conductivity at any moisture tension can be calculated very fast with no other equipment than an electronic slide rule calculator.

This paper investigates into the possibility of deriving the values of the exponent n in eq. (2) from moisture tension curves.

2

2. CALCULATED k - Ψ RELATIONS COMPARED WITH MEASUREMENTS

The methods of BROOKS and COREY, and of CAMPBELL were applied to moisture tension curves of a number of soil samples. These data were collected in literature and in the archives of the laboratory of the Institute for Soil and Water Management in Wageningen (Netherlands). The data had to include determined k - Ψ relations to verify the usefulness of both models. Data were only used if the measurements were performed with plain water.

In table 1 the collected sets of data are specified. Also values of the highest suction that k was measured at and the matching satu-ration values are given. The values are given of S_r and n which were calculated with a computor program for non-linear parameter optimization (STOL, 1975). In chapter 5 is to be explained why the percentage of particles <2 μm is also entered in table 1 in those cases that it was known.

S_r appears to have substantial variation and also to be zero in a number of cases. This possibility was recognized by LALIBERTE, BROOKS and COREY (1968, page 60). Also in table 1 are values of n according to CAMPBELL's method, that is when S_r is assumed to be zero, even if it is not.

When there is a difference between the result of CAMPBELL's method and that of BROOKS and COREY, n must always be the highest according to the last method. This is arithmetical because with a constant value of S, increasing values of S_r systematically de-crease the value of $\frac{S - S_r}{1 - S_r}$ and that more so when S is small, that is when suction is high. The effects of increasing S_r on the slope of the straight line which gives the pore size distribution index and on the intersection point when the straight line is extropola-ted to $\frac{S - S_r}{1 - S_r} = 1$, are shown in figure 1. This diagram represents nr. 6 of table 1 and shows clearly why S_r is 0.06.

In the diagrams in the figures 2 and 3 is visualized how accord-ing eq. (1) and eq. (2) calculated k - Ψ relations agree with sepa-rate measurements of k as far as those were available for the cases

Table 1. Specification of collected testdata and values of exponent n according the methods of BROOKS and COREY (N'_B) and CAMPBELL (N'_C) or calculated from measured R − Ψ relations (N). Ψ is the highest suction measured at and θ/θ is the matching saturation. S_r is residual saturation.

| Specification	Ψ	θ/θ$_s$	S$_r$	N'$_B$	N'$_C$	N	%<2μm
BRUCE (1972)*							
1. Cecil sandy loam Ap	2 500	0.18	0.03	2.92	2.82	2.01	5
2. " " " B$_2$	2 100	0.65	0.10	2.19	2.16	1.75	26
3. " " " B$_3$	9 000	0.60	0.0	2.21	2.21	1.36	39
ELRICK a.o. (1964)							
4. Guelph loam	540	0.52	0.0	2.61	2.61	2.21	−
WATSON (1966)							
5. Botany sand, 300-150 μm	50	0.22	0.10	30.18	23.80	30.0	0
RUBIN a.o. (1964)							
6. Rehovoth sand	340	0.06	0.06	8,5	5.49	5.91	1.5
KUNZE a.o. (1968)							
7. Ida silt loam	11 000	0.25	0.15	3.01	2.69	2.32	23
8. Webster silty clay loam	6 500	0.35	0.15	2.63	2.43	2.49	33
9. Colo clay	10 000	0.37	0.25	2.73	2.38	2.43	−
CHOW a.o. (1972)							
10. Silt loam	25 000	0.38	0.0	2.68	2.68	2.15	7.4
HENSELER a.o. (1968)*							
11. Stark toniger Schluff	600	0.0	0.0	2.88	2.88	2.38	23
12. Lehmiger sand	300		0.08	3.69	3.10	3.39	8
ARCHIVES OF THE I.S.W.M. Laboratory*							
13. Loess	530		0.0	2.59	2.57	2.43	17
14. Sandy clay	280		0.0	2.69	2.67	1.99	18
15. Medium fine sand	95		0.04	4.14	3.38	4.49	2
16. Fine sand	120		0.05	3.67	3.20	3.90	0.6
17. Cover sand	120		0.12	3.48	2.89	2.58	
18. "	150		0.09	3.90	3.26	3.42	
19. "	130		0.0	2.99	2.99	2.98	
20. "	100		0.06	3.75	3.42	2.92	
21. "	90		0.04	4.63	3.58	2.46	
22. "	135		0.07	3.44	3.25	3.13	

* = undisturbed samples
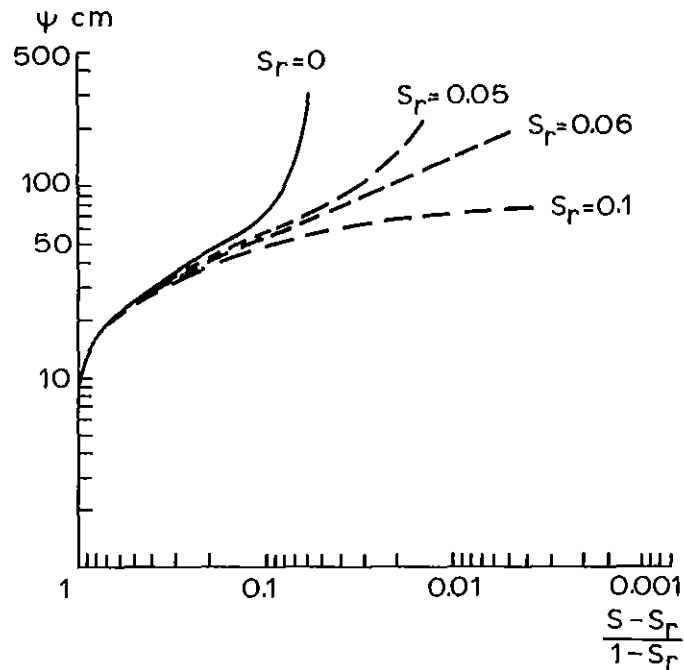
Fig. 1. Log Ψ plotted against log $\dfrac{S - S_r}{1 - S_r}$ with increasing values of S_r. The soil is Rehovoth sand (RUBIN a.o. 1964)

in table 1. Relations obtained with the BROOKS and COREY method are shown as dashed lines, solid lines represent the CAMPBELL method. For the silt loam (CHOW and DE VRIES, 1972) in fig. 3 a value of k_s was estimated.

It is striking that generally speaking the slope of the calcu-lated hydraulic conductivity function agrees well enough with that of the measured function, while when the calculated function does not fit the measurements it is largely because it has shifted to a too high or too low level of Ψ. This is obviously caused by over- or underestimation of Ψ_a and will be discussed on a later occasion. Now a comparitive evaluation of both methods of calculation of the exponent n will be given.
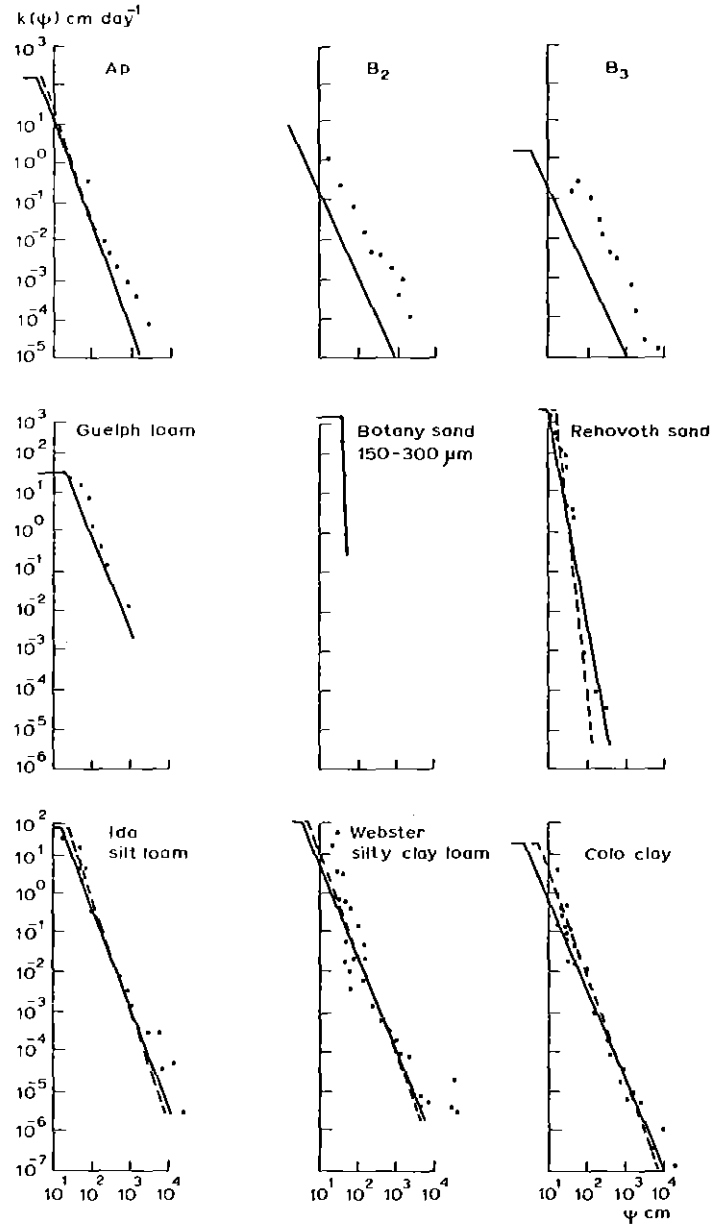
5

Fig. 2. Calculated k - Ψ relations and measurements of k(Ψ) for the numbers 1 to 9 inclusive of table 1

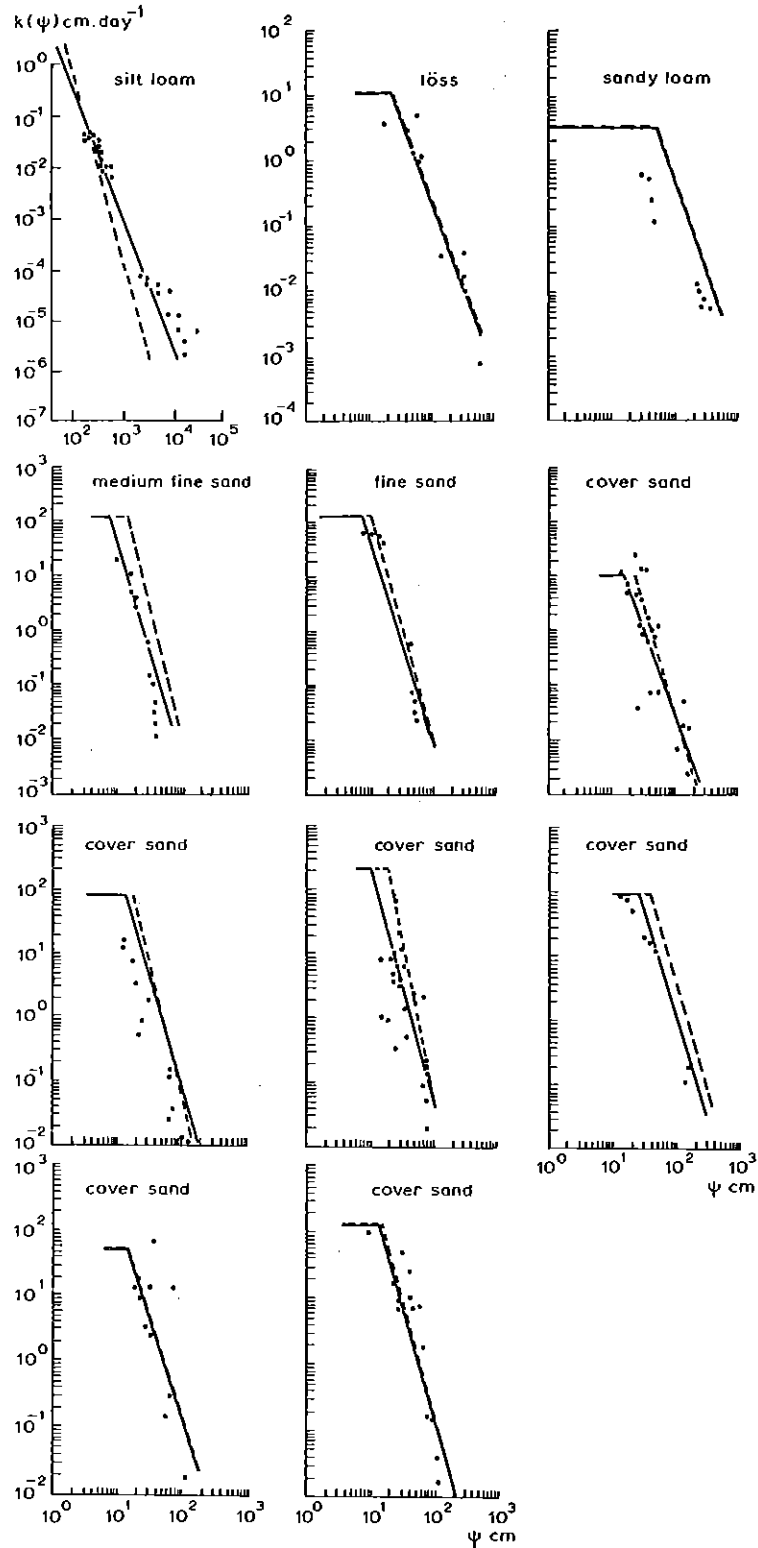Fig. 3. Calculated k - Ψ relations and measurements of k(Ψ) for the numbers 10 to 22 inclusive of table 1.

3. A TEST ON THE RESULTS OF TWO METHODS OF CALCULATION OF THE EXPONENT n

Values of n which are derived from moisture tension curves and hereafter to be defined as n' with indexes B for BROOKS and COREY (1964) and C for CAMPBELL (1974), were compared with values of n which were calculated from measurements of k and Ψ. If data points were given in the literature concerned, values of k and Ψ were read in the diagrams; if not data points were read from the presented curves.

For the calculation of n from measured values of k and Ψ, log k/k_s was plotted against log Ψ. Calculating the linear regression of log k/k_s on log Ψ yields a regression coefficient representing n. As k/k_s can not exceed unity the intersection point of the regression line and the log Ψ scale is an estimation of Ψ_a for

$$\log k/k_s = n(\log \Psi_a - \log \Psi) \tag{8}$$

The value of n may also be determined when k_s is unknown by estimation of k_s. In that case the resulting estimation of Ψ_a does not have any significance. In table 1 the values of n as obtained according to eq. (8) are given. The regressions of n'_C and n'_B on n satisfy the equations

$$n'_C = 0.743 \, n + 0.919 \tag{9}$$

and

$$n'_B = 0.989 \, n + 0.634 \tag{10}$$

Correlation coefficients are very high, 0.996 and 0.993 respectively. The tests of Student have been used to conclude if the departures of these equations from n'_C = n have significancy. From the departure of the regression coefficients from unity it holds for eq. (9) that the test value t = 1,946, and for eq. (10) it holds that t = 0.423. For the departure of the constant from zero it holds for eq. (9) that t = 9.30 and for eq. (10) that t = 3.396. The crit-

8

ical value of t = 2.086 when the test is twosided and the confi-
dence interval is 95%. It follows that the differences between n'_C
and n have significancy. CAMPBELL's method is not reliable, though
in a number of cases it gives the same results as BROOKS and COREY's
method. The latter method gives relative values of N'_B which are
perfectly proportionate but systematically too high. This would
mean that real values of n can be lower than the minimum value acc-
ording to eq. (3). More data are available to support this assump-
tion.

4. A RELATIONSHIP BETWEEN THE VALUE OF EXPONENT n AND CLAY PERCEN-
    TAGE

In literature k - Ψ relations are given by authors who unfortun-
ately did not supply moisture retention curves so the data could
not be included in the comparative evaluation in chapter 4. However
in most cases data on particle size distribution were supplied. In
table 2 a set of data from this literature is given. The exponent n
is again derived from measured values of k and Ψ, as shown in chap-
ter 4. It is entered in table 2 with the clay percentage. This fea-
ture serves to distinguish between the soils in tables 1 and 2. In
fig. 4 the exponent n is plotted against the clay percentage. Dis-
turbed and undisturbed samples are separated because LALIBERTE and
COREY (1969) have shown that sample treatment may affect pore size
distribution on which the value of n is depending. They found a
mean ratio between exponent n of disturbed and undisturbed samples
of about 1 : 0.65. Fig. 4 shows a distinct relationship, between
exponent n and clay percentage of the disturbed samples despite a
considerable scatter which will be due to incompatibility of data
because of different sample treatment etc. of the various authors.
Nevertheless a mean curve can easily be fitted in diagram 4a. When
this curve is transferred to the diagram 4b it shows that undisturb-
ed samples of soils with equal clay percentage have as a mean lower
values of n. The ratio is about 1 : 0.7 if samples with less than
4% clay are neglected.

Table 2. Specification of collected testdata and clay percentages
plus values of the exponent n in BROOKS and COREY's formu-
la as calculated from measured k - Ψ relations

Specification		%<2μm	n
RICHARDS a.o. (1952)			
Superstition sand		5.6	4.76
Coachello loamy fine sand		4.4	3.90
Pachappa fine sandy loam		4.7	4.50
Millville silt loam		14.8	2.45
Chino sandy clay loam		36.1	1.50
Presties clay		45.6	1.95
GARDNER a.o. (1958) *			
Chino-clay		55	1.98
Pachappa sandy loam		8	2.91
VETTERLEIN a.o. (1964)			
Staubsand		2.46	12.40
Staub		1.95	14.40
Anlehmiger sand		4.76	3.80
Schwach lehmiger sand		7.99	3.13
Lehmiger sand		11.59	2.83
Sandiger lehm		18.95	2.51
Lehmiger Ton		36.38	2.41
Siltiger Ton		47.30	2.10
Ton		57.30	1.96
GIESEL a.o. (1972) *			
Kiesiger sand		0	4.80
Mittelsand		1.6	3.20
Feinsand		3.5	2.85
Lehmiger sand		7.1	2.30
Sandiger lehm		20.0	2.00
Sandig-toniger lehm		36.5	1.80
Schluffiger ton (hohe Lagerungsdichte)		47.8	1.45
Schluffiger ton (mittleredichte)		46.0	1.35
Toniger Schluff (hohedichte)		20.0	2.60
Toniger Schluff (mittleredichte)		19.5	2.55
BECHER (1971) *			
Sand	Bv	3	3.05
Parabraunerde	Al	7	2.54
Parabraunerde	Agl	9	2.20
Parabraunerde	Bt	18	1.65
Parabraunerde	Bv	14	1.94
Marsch		54	1.00

* = undisturbed samples
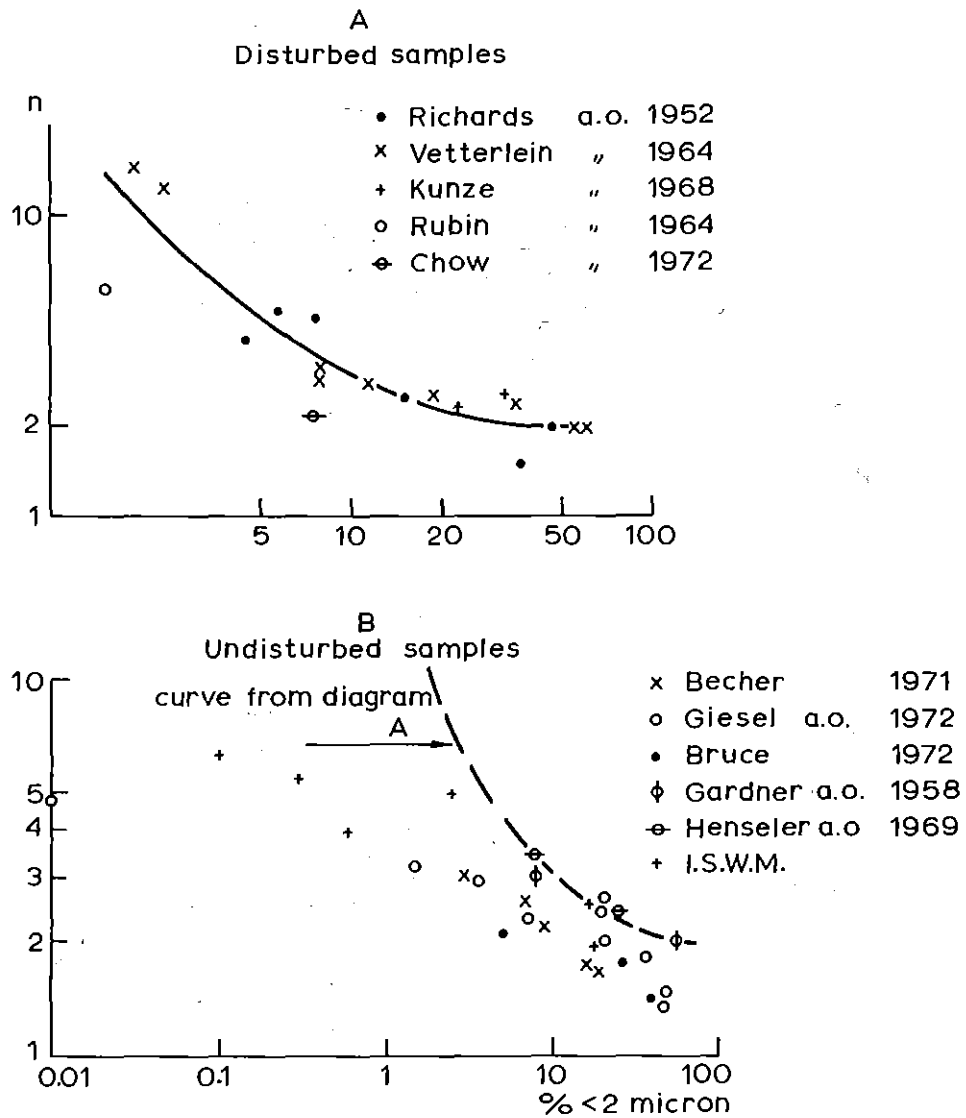
A
Disturbed samples

B
Undisturbed samples

Fig. 4 Relationship between clay percentage and value of exponent n
in BROOKS and COREY's formula

In this range there is a lack of observations in both diagrams.
The lowest value of n for undisturbed samples would as a mean be
about 1.4. This is fairly well in agreement with the constant of
0.63 in eq. (10) which should be an estimation of the overvaluation

of n with eq. (3). A relationship between the value of n and clay percentage of the soil is obvious.

BROOKS and COREY (1964, pages 2, 4) have put some restrictions on the validity of their formula. It is not always valid for saturation below field capacity or for porous media having only extremely small pores e.g. the spaces between clay platelets or when with decreasing saturation the porous matrix changes in geometry.

The significance of a diagram like fig. 4b is perhaps that it is the most practical and direct way to assess the value of n in eq. (2) empirically for a large variety of soils.

5. SUMMARY AND CONCLUSION

Making use of data from literature it was ascertained that a method recommended by CAMPBELL (1974) fails in determining the value of the exponent n in a formula of BROOKS and COREY (1964) for the calculation of hydraulic conductivity as a function of moisture tension. The method of calculation recommended by BROOKS and COREY themselves gives relative values of n which are perfectly proportionate but systematically too high. This paper gives an empirical approach for the assessment of the value of n for a large variety of soils. It is suggested that this approach should comprise as many observations as possible. A more complete soil characteristic than clay percentage should be used to reduce scatter as much as possible, so the accuracy of the estimation of n would be optimal. In a following paper this will be elaborated and discussed.

REFERENCES

BECHER, H.H., (1971). "Ein Verfahren zur Messung der ungesättigten
        Wasserleitfähigkeit". Zeitschr. Pflanzenern. Bodenk. 128.
        Band. Heft 1.

BROOKS, R.H. and H.T. COREY, (1964). "Hydraulic properties of porous
        media". Hydrology paper no. 3. Colorado State University,
        Fort Collins, Colorado.

BRUCE, R.R., (1972). "Hydraulic conductivity evaluation of the soil
        profile from soil water retention relations". Soil Sci.
        Soc. Am. Proc. 36 nr. 4.

BURDINE, N.T., (1953). "Relative permeability calculations from pore
        size distribution data". Petroleum Transactions, A.I.M.E.,
        vol. 198.

CAMPBELL, G.S., (1974). "A simple method for determining unsatura-
        ted conductivity from moisture retention data". Soil Sci.
        vol. 117, nr. 6.

CHOW, T.L. and J. DE VRIES, (1972). "Dynamic measurements of hydro-
        logic properties of a layered soil during drainage and
        evaporation, followed by wetting". Proc. 2nd Symp. Fund.
        Transp. Phen. in Por. Media. I.A.H.R./I.S.S.S. Guelph,
        Ontario.

ELRICK, D.W. and D.H. BOWMAN, (1964). "Note on an improved appara-
        tus for soil moisture flow measurements". Proc. Soil Sci.
        Soc. Am. vol. 28 nr. 8.

GARDNER, W.R. and M. FIREMAN, (1958). "Laboratory studies of evapo-
        ration from soil columns in the presence of a watertable".
        Soil Sci. 85 pp. 224-249.

GIESEL, W., M. RENGER und O. STREBEL, (1972). "Berechnung des
        kapillaren Aufstiegs aus dem Grundwasser in der Wurzelraum
        unter Stationären Bedingungen". Zeitschr. Pflanzen ern. und
        Bodenk. 132. Band. Heft 1.

HENSELER, K.L. und M. RENGER, (1969). "Die Bestimmung der Wasser-
        durchlässigkeit im wasserungesättigten Boden mit den
        Doppelmembran Druckapparatur". Zeitschr. Planzenern. und

Bodenk. 122. Band. Heft 1.

KUNZE, R.J., G. UEHARA and K. GRAHAM, (1968). "Factors important in
the calculation of hydraulic conductivity". Soil Sci. Soc.
Am. Proc. 32 no. 6.

LALIBERTE, G.E., R.H. BROOKS and A.T. COREY, (1968). Permeability
calculated from desaturation data. Journ. of the Irr. and
Drain Division of the A.S.C.E. Vol. 94 nr. IR.1.

LALIBERTE, G.E. and A.T. COREY, (1969). "Hydraulic properties of
disturbed and undisturbed soils". Permeability and Capilla-
ry of Soils, A.S.T.M.S.T.P. 417, Am. Soc. Testing Mats.
p. 56.

RICHARDS, L.A. and D.C. MOORE, (1952). "Influence of capillary con-
ductivity and depth of wetting on moisture retention in
soil". Trans. Am. Geoph. Un. Vol. 33, nr. 4.

RUBIN, J., R. STEINHARDT and P. RENNIGER, 1964. "Soil water rela-
tions during rain infiltration. II. Moisture content pro-
files during rains of low intensities". Soil Sci. Soc. Am.
Proc. Vol. 28 no. 1.

STOL, Ph.Th., (1975). A contribution to theory and practice of non-
linear parameter optimization. Agricult. Res. Repo 835
Pudoc. Wageningen.

VETTERLEIN, E., R. KOITZSCH, (1964). "Ergebnisse von Untersuchungen
über die Wasserbewegung in ungesättigten Böden mittels
Durchfluszverfahren". Trans. 8th Intern. Congress of Soil
Science, Bucharest, Romania.

WATSON, K.K., (1966). "An instantaneous profile method for determ-
ining the hydraulic conductivity". Water Res. Res. Vol. 2.
no. 4.

14