## WORKABLE TIME AND THE WEATHER

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The estimation of the probability distributions of the workable time for farm operations raises several questions. Some of these questions are discussed, leaning on the literature on the subject and the estimation of distributions of workable time for combine harvesting of wheat in the Netherlands.

## INTRODUCTION

The short and long term decisions on a farm strongly depend on the amount of workable time that will be available for the individual operations. This time is determined directly and indirectly (i.e. via the crop and soil) by the weather and just as difficult to predict. However, it is possible to estimate the probability distributions of workable time on the basis of observations made in the past. Given these distributions, the farmer is able to estimate the risk related to his decision and thus to make an optimum choice. The estimation of these probability distributions raises several questions, which are answered differently by the various authors. What is workable time ? How can or should it be measured ? How many observat.ions are needed to make accurate estimates of the distributions ? What is the relationship between this distribution and the weather, and hence the period (of the year) and the geographical location (for the same operation) ? These and related questions are discussed below.

## WORKABLE TIME

There appear to be almost as many definitions of workable time in the literature as there are authors. The investigations by Roth, Anton and Beyse (1), Lermer (2), Hesselbach (3), Reboul (4), Al Hamchari, Desbrosses and Mamoun (5), De Wiljes and Zast (6), and Bischoff and Knecht (7) can be defined as follows.

Observations on one more weather variables and on the workable time for a given (type of) operation in a given period are made during a small number of (calender) years on a relatively large number of sites (farms). They thus have the observations:

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\(\left\{\left(\underline{x}_{i j}, x_{i j}\right), i=1,2, \ldots \ldots, n_{j} ; j=1,2, \ldots . ., m\right\}\), where
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$\underline{y}_{i j}=$ number of workable time units (hours, days) at site no $i$ in year no $j$
$\underline{x}_{i j}=$ weather $=$ a vector of weather variables, such as the number of dry hours, the rainfall, the mean radiation intensity, etc. in the period under examination at site no. i in year no. $j$.

What is meant by "the weather" $\underline{x}_{i j}$ varies from author to author. The content of $\underline{\underline{y}}_{i j}$ varies as well. Roth, Anton and Beyse (1), and Hesselbach (3) observe the time during which the job is interrupted by rain, dew or frost, while the rest of the given period is defined as workable time. Reboul (4) and Al Hamchari, Desbrosses and Mamoun (5) take the time during which according to work records of farmers consulted the operation has been executed. Lermer (2), De Wiljes and Zaat (6), and Bischoff and Knecht (7) take the time which is said to be workable in the judgement of the farmer (whether the operation is executed or not).

The best fitting curve $\hat{\underline{x}}_{i j}=t\left(\underline{x}_{i j}\right)$ is drawn through the observations $\left\{\left(\hat{\mathbf{y}}_{i j}, x_{i j}\right), i=1,2, \ldots \ldots, n_{j} ; j=1, \ldots \ldots, m\right\}$ (according to some curve-fitting procedure).

Finally, the probability distribution of the transformation $\hat{\underline{y}}=t$ ( $\underline{x}$ ) is estimated on the basis of:
(a) the assumptions: for all $i=i, 2, \ldots \ldots, n_{j}$, and $j=1,2, \ldots \ldots, m$, $\underline{x}_{i j}$ has the same distribution as $\underline{x}$, and $\operatorname{Prob}[\underline{x} \leqslant \bar{x}]=$ Prob [
(b) the observations $\left\{\underline{x}_{i j}, i=1,2, \ldots \ldots, m, m+1, \ldots \ldots, m\right\}$.

They, thus, change over from the observations $\left\{\underline{y}_{i j}, i=1,2, \ldots \ldots, n_{j}\right.$; $j=1,2, \ldots \ldots, m\}$ to the observations $\left\{\hat{\mathrm{y}}_{i j}, i=1,2, \ldots \ldots, n_{j}\right.$; $j=1,2, \ldots \ldots, m, m+1, \ldots \ldots, M\}$. By doing so, many (M) observations on $\hat{\mathbf{y}}$ are created. Now two problems arise.
The first concerns the interpretation of $\hat{\hat{y}}$, or the model wherein $\hat{\underline{\hat{y}}}$ as an estimator is imbedded. Although the authors are not very explicit on this interpretation it would be defined by the following four points.
(1) The number of workable hours $\underline{W}$ is a transformation of the weather $\underline{x}: \underline{W}=T(\underline{x})$. The weather is not known a priari and is therefore seen as a random variable. The events $[\underline{x}=x]$ and $[\underline{W}=T(x)]$ are equivalent.
(2) The observations (on $\underline{x}$ ) $\underline{x}_{i j}$, $i=1,2, \ldots \ldots, n_{j}$ and $j=1,2, \ldots \ldots, m$ are mutually independent and identically distributed (so, the $n_{j}$ locations of observation are assumed to lie in a homogeneous area.)
(3) The observation $\underline{\underline{y}}_{i j}$ differs from $T\left(\underline{x}_{i j}\right)$ with an error $\varepsilon_{i j}$ : $\underline{y}_{i j}=T\left(\underline{x}_{i j}\right)+\underline{\varepsilon}_{i j}$, where $\underline{\varepsilon}_{i j}$ is (assumed to be) normally distributed with an expectation $\mathrm{B}_{\mathrm{E}_{\mathrm{ij}}}=0$ and variance Var $\underline{E}_{\mathrm{ij}}=\sigma^{2}$ for all $i$ and $j$.
(4) Given $\underline{x}=x, \underline{\hat{y}}=t(x)$ is an estimate of $W=T(x)$, i.e. the coëfficients of $t(\underline{x})$ are estimates of the coëfficiënts of $T(\underline{x})$.

Indeed, the number of workable time units is not only a function of the weather (and the crop and soil), but also of a set of workability criteria (technical, economical, etc.) of the farmer. The workable time forms part of the management decision process, and depends on the decision criteria and constraints. In other words: every farmer has his own definition of workable time, and hence his own probability distribution of workable time. This view clearly disagrees with (a) the interpretation of $\varepsilon_{i, j}$ as an error of observation or judgement, and (b) the assumption that the probability distribution of $y_{i j}$ does not depend on $i$. Another difficulty is the accuracy with which the probability distribution of $\underline{\underline{W}}=\mathrm{T}(\underline{x})$ is estimated. This accuracy is a function of both the number of observations on $\underline{x}$ and the (in)accuracy of the coëfficiënts of $t(\underline{x})$ (whict, in turn, is a function of the number of paired observations on $\underline{x}$ and $\underline{y}$, and variance of $\underline{y}$ ). In most cases, however, the latter source of uncertainty is not taken into account. Very important, of course, is the choice of the general form and the factors of $T(\underline{x})$, which is fairly arbitrary in this approach.

In the most recent literature, we see a different approach. A further analysis is made of the workable time function, i.e. $T(x)$. This approach (see Smith (8), Kish and Privette (9), Baier (10), Hassan and Broughton (11), Elliot, Lembke and Hunt (12), Ayres (13), and Portiek (14)) can be summarized as follows:
(1) The relevant state $\underline{s}_{j}(t)$ - at time $t=1, \ldots \ldots, K$, in year no. $j=$ $1, \ldots \ldots, m$ - of a given soil-crop-weather system is estimated by $\hat{\underline{s}}_{j}(t)=$ $f\left(\underline{x}_{j}(t)\right)$, where $\underline{x}_{j}(t)=$ weather at time $t$ in year no $j$.
(2) The researcher chooses some workability criteria. These criteria divide the possible values of $\mathbf{s}_{j}(t)$ into a set of workable states and a set of unworkable states.
(3) The time interval ( $t-p \Delta t, t+(1-p) \Delta t), 0<p<1, \Delta t>0$, is said to be workable if (and only if) $\hat{\mathbf{s}}_{j}(t)$ belongs to the set of workable states.
The values of p and $\Delta t$ are chosen by the researcher; the most common values of $p$ are 0 , $\frac{1}{2}$ and 1 ; the most common value of $\Delta t$ is 1 (day or hour)
(4) The number of workable hours (days) in a given period in year no $j, \hat{\mathrm{y}}_{j}$, is found by counting the number of workable intervals in that period.
(5) The probability distribution of $\underline{y}$, the number of workable hours (days) in a year, is estimated on the basis of the observations $\left\{\hat{\underline{y}}_{j}, j=1, \ldots, m\right\}$.

The advantages of this approach lie in the fact that the workability criteria are stated explicitly. Objective observations can be made on the soil-cropweather system and the influence of diverse workability criteria on the probability distribution of workable time can be examined easily.
Of course, the problems concerning the interpretation and accuracy of estimation are shifted to the formulation of $\hat{\mathbf{s}}_{j}(t)=f\left(\underline{x}_{j}(t)\right)$.
To find ${\underset{\underline{s}}{j}}(t)$, some researchers take a small sample, $\left\{\left(\underline{s}_{j}(t), \underline{x}_{j}(t)\right), t=\right.$ $1,2, \ldots \ldots, K ; j=1,2, \ldots \ldots, m\}$, and then apply a curve-fitting procedure.
Others make a further analysis of $s_{j}(t)$ where, at the most elementary level of analysis, the coëfficiënts are estimated by a curve-fitting procedure, established by direct observation or deduced from the laws of nature. In most cases, the empirical basis of the models is very small. Apparently (and for obvious reasons) the researcher's attention was devoted primarily to the building and subsequent use of the model. For the future however, the primary task seems to be the gathering of empirical data.

## Concepts and data

An hour is said to be workable for combine-harvesting if:

- the amount of rain in that hour $\leqslant 0.1 \mathrm{~mm}$.
- the moisture attached to the plants due to rain in that hour $\leqslant 0.5 \mathrm{~kg} / \mathrm{ha}$.
- the moisture attached to the plants due to condensation $\leqslant 0.5 \mathrm{~kg} / \mathrm{ha}$.
- the kernel moisture content $\leqslant q=17,19,21,23,25,27 \%$. The moisture state of the crop (wheat, combine ripe) is calculated using a model described by Van Elderen and Van Hoven (15), with the input variables: rain, cloudiness, vapour pressure, temperature, radiation, and wind velocity (at hour $t$ ). The weather data are taken from De Bilt. 1)

For every hour in the period between July 16 th and September $30 t h$, in the period 1957 - 1968, the rain data and the calculated moisture states are compared with the workability criteria. The numbers of workable hours in periods of $1,2,3,4$ and 5 half-months, in the 24 hours day and parts of the day, are then established. The half-months are:
July II: 16. - 31. July
Aug $I$ : 1. - 15. August
Aug II: 16. - 31. August
Sept I : 1. - 15. September
Sept II: 16. - 30. September
The numbers of workable hours in different years at the same place and in the same period of the year are assumed to be mutualiy independent and identically distributed. The first part of this assumption (mutual idependence) has been tested on the observations and not rejected at the $5 \%-1 e v e l$ of significance. (Series test on observations, De Jonge (16)). The second part (identical distribution) could not be tested, but seems to be acceptable, since these numbers of workable hours are generated by the same criteria, the same crop and (practically) the same climate system.
Figs. $1-5$ show the cumulative frequency distributions of the workable hours in July II, Aug I, Aug II, Sept I, and Sept II, respectively for combine harvesting at maximum kernel moisture contents of $17,19,21,23,25$ and $27 \%$. The small numbers are year numbers: $1=1957$, etc.

1) Meteorological station in the centre of The Netherlands,







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\text { Fig. } 2 \text { Cumulative frequency distributions of the }
$$

$$
\begin{aligned}
& \text { numbers of workable hours for combine harvesting } \\
& \text { at maximum kernel moisture contents of } 17,19, \\
& 21,23,25 \text { and } 27 \% \text { in Aug I (all hours). De Bilt, } \\
& \text { years } 1=1957, \ldots, 12=1968
\end{aligned}
$$

$$
\begin{array}{llllll}
\begin{array}{l}
\text { Cumulative } \\
\text { frequency }
\end{array} & 17 & 19 & 21 & 2325 & 27
\end{array}
$$


Fig. 4 Cumulative frequency distributions of the

Fig. 3 Cumulative frequency distributions of the numbers of workable hours for combine harvesting at maximum kernel moisture contents
 hours). De Bilt, years $1=1957$,


Fig. 5 Cumulative frequency distributions of the numbers of workable hours for combine harvesting at maximum kernel moisture contents of $17,19,21,23,25$ and $27 \%$ in Sept II (all hours). De Bilt, years $1=1957, \ldots .$. ,
$12=1968$.


Table 1. Means and standard deviations (s.d.) of the numbers of workable hours for combine-harvesting at maximum kernel moisture contents of $17,19,21,23,25$ and $27 \%$, in the half-months July II, Aug I, Aug II, Sept I and Sept II. De Bilt, 1957-1968.

|  |  | Maximum kernel moisture content (\% w.b.) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 17 | 19 | 21 | 23 | 25 | 27 |
| July II | mean | 49.3 | 93.3 | 128.7 | 156.1 | 172.7 | 182.9 |
|  | s.d. | 56.6 | 69.1 | 65.7 | 60.5 | 52.7 | 48.6 |
| Aug I | mean | 30.4 | 74.7 | 109.6 | 134.3 | 151.5 | 165.5 |
|  | s.a. | 33.0 | 37.3 | 37.8 | 37.3 | 36.3 | 35.0 |
| Aug II | mean | 26.6 | 73.6 | 106.9 | 131.5 | 146.6 | 159.8 |
|  | s.d. | 29.3 | 57.2 | 67.0 | 71.0 | 68.9 | 65.2 |
| Sept I | mean | 38.8 | . 61.8 | 83.5 | 101.8 | 118.2 | 132.8 |
|  | s.d. | 73.1 | 85.1 | 86.9 | 83.9 | 77.7 | 71.5 |
| Sept II | mean | 13.9 | 33.5 | 58.5 | 89.3 | 108.3 | 122.4 |
|  | s.a. | 36.5 | 62.8 | 66.1 | 62.7 | 63.9 | 65.1 |

Table 2. Mean numbers of workable hours for combine-harvesting at maximum kernel moisture contents of $17,19,21,23,25$ and $27 \%$ in a half -month in four 6-hour parts of the day, in percentages of the mean number of workable hours in all hours of the day. De Bilt, 1957-1968.

\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{Half-month} \& \multirow[t]{2}{*}{hours of the day} \& \multicolumn{6}{|r|}{Maximum kernel moisture content (\% w.b.)} <br>
\hline \& \& 17 \& 19 \& 21 \& 23 \& 25 \& 27 <br>
\hline \multirow{5}{*}{July II} \& 0-6 \& 12 \& 12 \& 11 \& 12 \& 13 \& 12 <br>
\hline \& 6-12 \& 23 \& 21 \& 23 \& 25 \& 24 \& 25 <br>
\hline \& 12-18 \& 32 \& 35 \& 36 \& 35 \& 35 \& 35 <br>
\hline \& 18-24 \& 33 \& 32 \& 30 \& 28 \& 28 \& 28 <br>
\hline \& 0-24 \& 100 \& 100 \& 100 \& 100 \& 100 \& 100 <br>
\hline \multirow{5}{*}{Aug I} \& 0-6 \& 13 \& 12 \& 10 \& 10 \& 10 \& 10 <br>
\hline \& 6-12 \& 18 \& 19 \& 22 \& 25 \& 26 \& 26 <br>
\hline \& 12-18 \& 35 \& 37 \& 37 \& 37 \& 36 \& 37 <br>
\hline \& 18-24 \& 34 \& 32 \& 31 \& 28 \& 28 \& 27 <br>
\hline \& 0-24 \& 100 \& 100 \& 100 \& 100 \& 100 \& 100 <br>
\hline \multirow{5}{*}{Aug II} \& 0-6 \& 16 \& 13 \& 12 \& 12 \& 12 \& 12 <br>
\hline \& 6-12 \& 16 \& 16 \& 21 \& 22 \& 22 \& 22 <br>
\hline \& 12-18 \& 34 \& 39 \& 38 \& 38 \& 39 \& 40 <br>
\hline \& 18-24 \& 34 \& $\bigcirc$ \& 29 \& 28 \& 27 \& 26 <br>
\hline \& 0-24 \& 100 \& 100 \& 100 \& 100 \& 100 \& 100 <br>
\hline \multirow{5}{*}{Sept I} \& 0-6 \& 15 \& 15 \& 15 \& 13 \& 12 \& 12 <br>
\hline \& 6-12 \& 25 \& 23 \& 23 \& 22 \& 23 \& 23 <br>
\hline \& 12-18 \& 33 \& 36 \& 37 \& 40 \& 40 \& 41 <br>
\hline \& 18-24 \& 27 \& 26 \& 25 \& 25 \& 25 \& 24 <br>
\hline \& 0-24 \& 100 \& 100 \& 100 \& 100 \& 100 \& 100 <br>
\hline \multirow[b]{5}{*}{Sept II

4} \& 0-6 \& 25 \& 18 \& 16 \& 15 \& 15 \& 14 <br>
\hline \& 6-12 \& 16 \& 17 \& 19 \& 18 \& 19 \& 20 <br>
\hline \& 12-18 \& 27 \& 36 \& 38 \& 40 \& 41 \& 42 <br>
\hline \& 18-24 \& 32 \& 29 \& 27 \& 27 \& 25 \& 24 <br>
\hline \& $0-24$ \& 100 \& 100 \& 100 \& 100 \& 100 \& 100 <br>
\hline
\end{tabular}

These frequency distributions vary with the maximum kernel moisture content, both with respect to their location and shape. (Figs. 1-5).

Table 1 gives the arithmetic means and the standard deviations. The mean increases with increasing maximum kernel moisture content, while the standard deviation varies only slightly.
In the course of the season (from July II to Sept II), the number of workable hours for combine-harvesting in a half-month appears to decrease.

Table 2 gives the mean percentage division of the number of workable hours in a half-month into four 6 -hour parts of the day, i.e. $100 \cdot \hat{\underline{y}}_{j} / \mathbf{z}$,

$$
\begin{aligned}
& \text { where } \underline{z}={ }_{i}{\underset{\sum}{\stackrel{4}{2}}}_{1} \hat{\mathrm{y}}_{i} \text {, and } \hat{\mathrm{y}}_{i}=1 / 12 \sum_{j=1}^{12} y_{-i j} \text {, } i=1, \ldots \ldots \text {, 4, where: }
\end{aligned}
$$

$$
\begin{aligned}
& \text { 15(16) } 24 \\
& \underline{y}_{4 j}=\sum_{k=1} \sum_{m=19} \underline{x}_{j k m} \\
& \mathrm{X}_{\mathrm{jkm}}=\text { numbers of workable hours in a half-month in year no } j \text {, in } \\
& \text { hour no } m \text { of day no } k \text {. }
\end{aligned}
$$

Only little over $50 \%$ of the number of workable hours are daytime hours (0600-1800 hours). This percentage of daytime hours increases with maximum kernel moisture content.

## Estimating probabilities: accuracy and number of observations

The observed cumulative relative frequencies (Figs. 1-5) are considered as estimates of the cumulative probabilities of the number of workable hours. What is the accuracy of these estimates ? rake as a measure of accuracy the $95 \%$ confidence intervals of the estimated cumulative probabilities. (See Fraser (17)).
Let $y_{(1)}, y_{(2)}, \ldots \ldots, y_{(12)}$ be the realisations - ranked in order of magnitude - of the number of workable hours $\underline{y}$. The $95 \%$ confidence interval for $p_{i}=\operatorname{Prob}\left[\underline{y} \leqslant y_{(i)}\right], i=1, \ldots ., 12$, is constructed as follows.

Consider the test with nullhypothesis $\left(H_{0}\right): p_{i}=p_{0}$, alternative hypothesis $\left(H_{1}\right): p_{i} \neq p_{0}$, and teststatistic $\underline{i}=$ the number of realisations that are smaller or equal to $\underline{y}_{(i)}$. Under $H_{0}$, this teststatistic $\underset{i}{ }$ has a binomial distribution with parameters $p_{i}=p_{0}$ and $n=12$.
$H_{0}$ is rejected if Prob $\left[\underline{i} \leqslant i ; H_{0}\right] \leqslant 0.025$ or Prob $\left[i \geqslant i ; H_{0}\right] \leqslant 0.025$. The upperbound, $\underline{b}_{2}$, of the $95 \%$ confidence interval for $p_{i}$ is the smallest $p_{0}$ for which Prob $\left[i \leqslant i ; H_{0}\right] \leqslant 0.025$.
The lower bound, $\underline{b}_{1}$, is the largest $p_{0}$ for which Prob $\left[\underline{i} \geqslant i ; H_{0}\right] \leqslant 0.025$ Thus, we find: $\left(\underline{b}_{1} \leqslant p_{i} \leqslant b_{-2}\right)$.

| 0 | $\leqslant p_{1} \leqslant 0.34$ |
| :--- | :--- |
| 0 | $\leqslant p_{2} \leqslant 0.44$ |

$0.02 \leqslant p_{3} \leqslant 0.58$
$0.09 \leqslant p_{4} \leqslant 0.66$
$0.15 \leqslant p_{5} \leqslant 0.73$
$0.21 \leqslant \mathrm{p}_{6} \leqslant 0.79$
$0.27 \leqslant p_{7} \leqslant 0.85$
$0.34 \leqslant p_{8} \leqslant 0.91$
$0.42 \leqslant \mathrm{p}_{9} \leqslant 0.95$
$0.52 \leqslant \mathrm{p}_{10} \leqslant 0.98$
$0.67 \leqslant p_{11} \leqslant 1$
$0.78 \leqslant p_{12} \leqslant 1$

Tnis accuracy leaves much to be desired.
Moreover, these interval show that a test based on the 12 observarions is not very powerful. (Power defined as the probability of rejecting $H_{0}$ in favour of $H_{1}$, when $H_{0}$ is false). This means that only large differences between the null- and the alternative hypothesis can be shown.
In order to estimate with greater accuracy and test with more power, more observations are needed. The minimum number of observations required for greater accuracy and power is given by:
$n=\left[\frac{T_{1-\alpha} \cdot \sqrt{p_{0}\left(1-p_{0}\right)}+T_{1-\beta} \cdot \sqrt{p_{1}\left(1-p_{1}\right)}}{p_{0}-p_{1}}\right]^{2}$,

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where: n = number of observations required
    1-\alpha = probability of not rejecting H}\mp@subsup{H}{0}{}\mathrm{ , when }\mp@subsup{H}{0}{}\mathrm{ is true.
    1-\beta = probability of rejecting H}\mp@subsup{H}{0}{}\mathrm{ , when }\mp@subsup{H}{0}{}\mathrm{ is false.
    T T-\alpha, T T-\beta}\mp@code{=(1-\alpha) and (1-\beta) percentage points of the standara
        normal distribution.
    p},\mp@subsup{p}{1}{}=\mathrm{ values of p = Prob [y 
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The values of $n$ for $p_{0}=0.5, \alpha=\beta=0.05$ and 0.10 , and several values of
$p_{1}$ are as follows:

|  | $\alpha=\beta=$ | 0.05 |
| :--- | ---: | ---: |
| $p_{1} \alpha$ |  | 0.10 |
| 0.45 | 1536 | 655 |
| 0.40 | 376 | 161 |
| 0.35 | 163 | 69 |
| 0.30 | 88 | 37 |
| 0.25 | 53 | 22 |
| 0.20 | 34 | 14 |

## Differences between the half-months

There are two reasons for studying more closely the differences between the observations of workable hours in different half-months. In the case of non-systematic differences, the observations may be considered to have the same distribution and (1) we have more than one observation per year, and (2) the user needs to apply only one distribution.

Table 1 and Figs. 1-5 suggest the hypothesis that the probability distribution of the number of workable hours changes systematically in the course of the period July II - Sept II. To test this hypothesis, Terpstra's test is applied (16).

The test:
Given: $k$ random samples: $\left\{\underline{y}_{i j}, j=1, \ldots \ldots, n_{i}\right\}, i=1, \ldots, k$.

Nullhypothesis, $H_{0}$ : the samples are from the same population
Alternative,$H_{1}$ : the samples are not from the same population and show a decreasing (increasing) trend in the order 1, 2, ..., k.

Teststatistic: form pairs of the samples: $(1,2),(1,3), \ldots \ldots,(1, k)$, $(2,3), \ldots \ldots,(2, k), \ldots \ldots,(k-1, k)$. Assign to the observations of pair $(i, m), i=1, \ldots \ldots, k-1 ; m>i$, ranks from 1 to $\left(n_{i}+n_{m}\right)$. Where $n_{i}=$ number of observations in sample i).
The teststatistic is then:

$$
\underline{W}=\sum_{i<m} \frac{n_{i}\left(n_{i}+n_{m}+1\right)-2 S_{i, m}}{n_{i} n_{m}},
$$

where $S_{i, m}=$ the sum of the ranks assigned to sample $i$ in the pair ( $i, m$ ). The case $n_{1}=n_{2}=\ldots . n_{k}=n$ yields:

$$
\begin{aligned}
& \underline{W}=\frac{n k(k-1)(2 n+1)-4, \sum_{i<j} \underline{S}_{i}, j}{2 n^{2}} \\
& \text { with expectation } E \underline{W}=0 \text { and variance }
\end{aligned}
$$

$$
\sigma_{W}^{2}=\frac{\sum_{i=1}^{k}(k+1-\hat{2} i)^{2}+\frac{k(k-1)}{2}}{3 n^{2}}
$$

The random variable $\underline{T}=\frac{W}{\sigma_{W}} \quad$ is $N(0,1)$ distributed.
Applications of Terpstra's test:
(a) Observations: the number of workable hours for combine-harvesting at a maximum kernel moisture content of $23 \%$ in the $k=5$ half-months, July II

- Sept II, in the $n=12$ years, 1957 - 1968, at De Bilt.

Results: $\underset{\sim}{W}=-3.1527, \sigma_{W}^{2}=1.1342$, and $\underset{\sim}{T}=-2.9603$.
Since Prob $[I \leqslant-2.9603]^{W}=0.00154, H_{0}$ is rejected.
(b) Observations: as (a), except Sept II, so $k=4$.

Results: $\underset{W}{W}=-0.1736, \sigma_{W}^{2}=0.5694$, and $\underline{T}=-0.2308$.
Since Prob $[T \not T-0.2308]=0.4090, H_{0}$ is not rejected.

Now, it is interesting to examine two related (and relevant) weather variables.
(c) Observations: the mean daily rainfall (mm day ${ }^{-1}$ ) in Juiy II, Aug I, Aug

II, Sept I and Sept. II in the 12 years 1957 - 1968 at de Bilt (Table 3).
Results: $\underset{W}{W}=-0.9062, \sigma_{W}^{2}=1.1342$, and $\underset{\underline{T}}{ }=-0.8509$.
Since $\operatorname{Prob}[T \leqslant-0.8509]=0.1977, H_{0}$ is not rejected.
(e) Observations: as (d), except Sept II. Results: $\underset{\underline{W}}{ }=-3.9722, \sigma_{W}^{2}=0.5694$, and $\underset{\underline{T}}{ }=-5.2807$. Since Prob $[\mathrm{T} \leqslant-5.2807]^{\mathrm{K}} \leqslant 10^{-6}, \mathrm{H}_{0}$ is rejected.
(f) Observations: as (d), except Sept I and Sept II.

Results: $\underset{W}{W}=-1.6320, \sigma_{W}^{2}=0.2292$, and $T=-3.4092$. Since Prob $[\underline{T} \leqslant-3.4092]=0.000337, H_{0}$ is rejected.

We may conclude that the number of workable hours for combine-harvesting per half-month tends to decrease systematically, in the course of the cereal harvesting period, i.e. September is likely to have fewer rather than more workable hours than August.
In this harvesting period, the "wetting conditions" (rain) are nearly the same, but the "drying conditions" (radiation) get worse.

Relatoinship between the number of workable hours and the weather

There is, of course, a relationship between the number of workable hours and one or several of the factors "rain", radiation", and "wind velocity". But to what extent and is there a (practically acceptable and usable) simple formula for the estimation of the workable time from weather data ? Given are 12 observations of:
$y=$ number of workable hours for combine-harvesting in a given period
$\underline{x}_{1}=$ rainfall ( mm water) in the same period
$\underline{x}_{2}=$ aceumulated hourly measurements of radiation, cal $\mathrm{cm}^{-2}$.
$\underline{x}_{3}=$ accumulated hourly measurements of the wind velocity, cm sec ${ }^{-1}$.

Table 5 gives Spearman's rank correlation coëfficients (16) for the relationship between the number of workable hours ( $\underline{y}$ ) and rain ( $\underline{x}_{1}$ ), radiation ( $\underline{x}_{2}$ ) and wind velocity ( $\underline{x}_{3}$ ) respectively.
Table 5 shows that the number of workable hours is to a significant degree governed by the factors rain and radiation, and not by the factor wind velocity. Some results of a curve fitting analysis are given in Table 6. The analysis is carried out on the observations (12 years, De Bilt) of:

Table 3. Mean daily rainfall at De Bilt in July II, ....., Sept II. (mm day ${ }^{-1}$ )

| Year | July II | Aug I | Aug II | Sept I | Sept II |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1957 | 4.33 | 5.12 | 5.27 | 6.25 | 7.20 |
| 58 | 4.20 | 2.88 | 2.46 | 1.57 | 4.78 |
| 59 | 2.28 | 1.47 | 0.20 | 0.01 | 0.20 |
| 60 | 1.23 | 4.23 | 5.26 | 1.33 | 0.79 |
| 61 | 1.43 | 2.03 | 3.14 | 3.01 | 1.45 |
| 62 | 2.79 | 2.69 | 1.94 | 2.13 | 1.47 |
| 63 | 0.45 | 4.52 | 7.29 | 2.88 | 2.55 |
| 64 | 1.84 | 1.87 | 3.32 | 3.17 | 1.26 |
| 65 | 4.85 | 1.96 | 5.91 | 4.01 | 0.85 |
| 66 | 10.50 | 3.27 | 1.56 | 3.23 | 0.01 |
| 67 | 1.02 | 5.37 | 1.84 | 2.52 | 2.33 |
| 68 | 1.43 | 4.82 | 2.65 | 2.32 | 6.25 |
| -2.0 | 3.35 | 3.41 | 2.70 | 2.43 |  |

Table 4. Mean of hourly measurements of radiation at De Bilt in July II, ......, Sept II. (cal $\mathrm{cm}^{-2}$ )

| Year | July II | Aug I | Aug II | Sept I | Sept II |
| :---: | :--- | :---: | :---: | :---: | :---: |
| 1957 | 15.32 | 12.69 | 11.41 | 9.41 | 6.05 |
| 58 | 15.79 | 12.66 | 13.93 | 11.69 | 7.78 |
| 59 | 20.64 | 13.16 | 17.14 | 15.58 | 9.86 |
| 60 | 13.98 | 13.16 | 10.17 | 10.63 | 8.53 |
| 61 | 15.15 | 13.58 | 11.63 | 9.54 | 8.74 |
| 62 | 16.71 | 13.98 | 15.27 | 12.91 | 8.90 |
| 63 | 19.98 | 12.06 | 9.82 | 11.31 | 6.49 |
| 64 | 17.58 | 14.02 | 14.32 | 11.39 | 10.45 |
| 65 | 14.06 | 17.72 | 13.13 | 9.98 | 9.05 |
| 66 | 14.53 | 14.29 | 14.05 | 10.53 | 8.02 |
| 67 | 19.16 | 13.66 | 13.68 | 9.19 | 7.92 |
| 68 | 16.51 | 12.07 | 13.72 | 10.53 | 6.61 |
| -16.62 | 13.59 | 13.19 | 11.06 | 8.20 |  |

$\underline{y}=$ number of workable hours for combine-harvesting at a maximum kernel moisture content of $23 \%$ in a given period.
$\underline{x}_{1}$ and $\underline{x}_{2}=$ rain and radiation, as defined above, in the same period. The values $\hat{a}, \hat{b}$ and $\hat{c}$ are the least square estimates of the model coëfficiënts $\mathrm{a}, \mathrm{b}$ and c , respectively. Transformations are $\ln \mathrm{y}=\ln \mathrm{a}+\mathrm{b} \ln \mathrm{x}$, for $y=a x^{b}, \ln y=\ln a+b x$ for $y=a e^{b x}$, and $\ln y+\ln a+b \ln x_{1}+c \ln x_{2}$ for $y=a x_{1}^{b} x_{0}^{c}$. When $A$ is the least square estimate of $\ln a$, then $\hat{a}=e^{A}$. Very instructive additional information is given in Figs. 6 and 7 , showing the relations between $\underline{y}$, the number of workable hours for combine-harvesting at a maximum kernel moisture content of $23 \%$, and the rainfall ( $\underline{x}_{1}$ ) and radiation $\left(\underline{x}_{2}\right)$, in the period July 16th - September 30th. Spearman's rank correlation coëfficients are -0.65 (Fig. 6) and +0.65 (Fig. 7), indicating that the best fitting curve greatly depends on the presence of the two extreme points. Without these extremes, as is the case in most periods of one, two or three half-months, the relationship is very poor. (The rankcorrelation coëfficients are not sensitive to level differences in the observations).

Apparently, the number of workable hours in a period is not a simple function of some simple representatives of the weather in that period. The addition of other factors ioes not give much better results. Presumebly, the most important "factor" in addition to rain $\left(\underline{x}_{1}\right)$ and radiation $\left(\underline{x}_{2}\right)$ is the distribution of these weather factors over time.

Table 5. Spearman's coëfficiënts ofrank vorrelation between numbers of workable hours for combine-harvesting at maximum kernel moisture contents of $17,19,21,23,25$ and $27 \%$ and (1) the rainfall, mm, (2) the radiation, cal $\mathrm{cm}^{-2}$, and (3) the wind velocity, $\mathrm{cm} \mathrm{sec}{ }^{-1}$, in half-month periods.
Observations from De Bilt in the years 1957 - 1968.

|  |  | Meximum kernel moisture content (\% w.b.) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 17 | 19 | 21 | 23 | 25 | 27 |
| July II | rain <br> radiation | $\begin{aligned} & -.50^{\star} \\ & .88^{\star \star} \end{aligned}$ | $\begin{array}{r} -.63^{\star *} \\ .84^{* *} \end{array}$ | $\begin{array}{r} -.76^{\star \star} \\ .79^{\star t} \end{array}$ | $\begin{gathered} -.76^{* \pi} \\ .79^{* *} \end{gathered}$ | $\begin{array}{r} -.84^{\star \star} \\ .78^{\star \star} \end{array}$ | $\begin{array}{r} -.85^{* *} \\ .76^{* *} \end{array}$ |
|  | wind | -. 44 | -. 41 | -. 48 | -. 48 | -. 50 | -. 41 |
| Aug I | rain | . 06 | -. 06 | -. 23 | -. 30 | . 24 | . 30 |
|  | radiation | -. 10 | -. 14 | . 10 | . 29 | . 30 | . 19 |
|  | wind | -. 28 | . 09 | . 10 | . 05 | . 10 | . 33 |
| Aug II | rain radiation | $\begin{aligned} & -.51^{\star} \\ & .79^{\star t} \end{aligned}$ | $\begin{gathered} -.66^{* *} \\ .83^{* *} \end{gathered}$ | $\begin{gathered} -.76^{\star t} \\ .91^{\star t} \end{gathered}$ | $\begin{aligned} & -.76^{\star *} \\ & .89 \end{aligned}$ | $\begin{array}{r} -.77^{\star 夫} \\ .90^{* t} \end{array}$ | $\begin{array}{r} -.74^{\star t} \\ .88^{* *} \end{array}$ |
|  | wind | -. 11 | -. 16 | -. 18 | -. 03 | -. 03 | . 03 |
| Sept I | rain ${ }^{\text {radiation }}$ | $\begin{aligned} & -.50^{\star} \\ & .76^{* *} \end{aligned}$ | $\begin{array}{r} -.63^{* *} \\ .66^{* *} \end{array}$ | $\begin{aligned} & -.55^{*} \\ & .76^{* *} \end{aligned}$ | $\begin{array}{r} -.63^{* *} \\ .85^{\star *} \end{array}$ | $\begin{aligned} & -.62^{\star *} \\ & .88^{\star *} \end{aligned}$ | $\begin{aligned} & -.53^{\star} \\ & .87^{\star k} \end{aligned}$ |
|  | wind | . 49 | . 41 | . 43 | . 48 | . 45 | . 46 |
| Sept II | rain | -. 33 | $-.69^{\text {x* }}$ | -.81** | -.76 ** | -.81** | -. 81 ** |
|  | radiation | . 53 | . 73 ** | . 71 ** | . 48 | . 48 | . 48 |
|  | wind | . 00 | $-.60^{\star *}$ | -. 27 | -. 24 | -. 23 | -. 23 |

* significant at $10 \%$ level
** significant at $5 \%$ level

Number of
workable hours


Fig. 6 Relation between the number of workable hours
for combine-harvesting at a maximum kernel
moisture content of $23 \%(y)$ and the rainfall
$\left(x_{1}\right), \mathrm{mm}$. in the period July 16 th -
September 30th.
Observations De Bilt 1957-1968


Fig. 7 Relation between the number of workable hours
for combine-harvesting at a maximum kernel
moisture content of $23 \%$ ( $y$ ) and the radiation $\left(x_{2}\right)$, cal $\mathrm{cm}^{-2}$, in the period July 16 th -
September 30th.
Observation De Bilt 1957-1968.

Table 6. Results of a curve fitting analysis. (De Bilt, 1957-1968)
$y=$ number of workable hours for combine-harvesting in the given period.
$\underline{x}_{1}=\operatorname{rain}, \underline{x}_{2}=$ radiation (see text).
$\hat{a}, \hbar$ and $\hat{c}$ are the least square estimates of the model coëfficiënts a, b, c.
$r^{2}=$ coëfficient of determination.


The amount of workable time is a function of both the possibility and the utility of cultivating the crop and the soil within a farm. Thus, every farmer has his own definition of workable time, so that the researcher has to consider several definitions of workable time.
The location and shape of the probability distribution of workable time depend on the exact definition of workable time.

Under the climatic conditions of the Netherlands, the variance of the number of workable hours is very large. A sufficiently accurate estimation of the probability distribution of workable time needs, therefore, many observations: over many years and in many places.
The probability distribution of workable time depends on the weather, and hence on the period of the year (and the geographical location).
Not only the amounts of rain, radiation, wind velocity, etc. determine the number of workable hours in a given period, but also the distribution of these amounts over time.

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